EMI : SOLUTIONS

Ex I : SINGLE CORRECT MCQs

Q1. ✓ (1)

instantaneous angular speed of the e, $\omega = \alpha t$,

Therefore ~~induce~~ magnetic field at the center of its CM $B = \frac{\mu_0 \{e/(2\pi/\omega)\}}{2R}$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{e\omega}{R}\right)$$

Therefore, magnetic flux thru' the loop

$$\phi_B = B \times \pi r^2 = \frac{\mu_0 e \omega r^2}{4R}$$

∴ induced emf

$$\mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \frac{\mu_0 e r^2}{4R}\left(\frac{d\omega}{dt}\right)$$

$$\boxed{\mathcal{E} = \frac{\mu_0 e r^2 \alpha}{4R}} \Rightarrow (B)$$ ✓

Q2. ✓ (2)

$R = (R_0 + t) \Rightarrow$ Area $A = \pi R^2 = \pi(R_0+t)^2$

$\Rightarrow$ magnetic flux $\phi_B = BA = B\pi(R_0+t)^2$

∴ induced e.m.f, $\mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \frac{d}{dt}\left[B\pi(R_0+t)^2\right]$

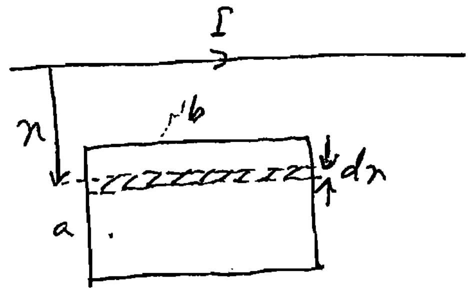
$$= B\pi \times 2(R_0+t)$$

$$\boxed{\mathcal{E} = 2\pi B(R_0+t)}$$

Now, since the ϕ_B (into the plane of the diagram) is increasing, by application of Lenz's law, the induced current should be counter-clockwise ✓

Q3. ③ $\mathcal{E} = -\frac{d\Phi_B}{dt}$, Now from the given graph, for the interval of time $t=0$ to $t=t_1$, B increases linearly $\Rightarrow \mathcal{E}$ is 'negative' and constant. For time $t=t_1$ to t_2, B is constant $\Rightarrow$ B = 0 and $t=t_2$ to $t=t_3$, B decreases linearly and therefore $\mathcal{E}$ = positive and constant. After $t=t_3$ again $\mathcal{E}$ is zero. (C) ✓

Q4. ②



The magnetic flux through a differential strip of thickness 'dx'

$$d\Phi_B = B \times dS = B \times b\,dx = \frac{\mu_0 I b\,dx}{2\pi x}.$$

Therefore the total flux,

$$\Phi_B = \int_{x=d}^{x=(d+a)} \frac{\mu_0 I b}{2\pi x}\,dx = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

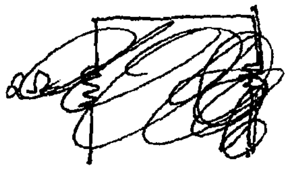
$$\Rightarrow \Phi_B = \frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d}\right) I_0 e^{-t/\tau}$$

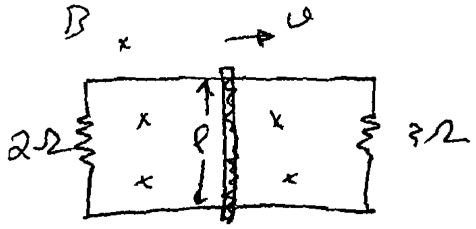
$\therefore$ induced voltage $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d}\right) I_0 \frac{d}{dt}\left(e^{-t/\tau}\right)$

$$\Rightarrow \mathcal{E} = \frac{\mu_0 b}{2\pi} \frac{I_0}{\tau} \ln\left(\frac{d+a}{d}\right) e^{-t/\tau}$$

$$\Rightarrow \boxed{\mathcal{E} = \frac{\mu_0 b I}{2\pi \tau} \ln\left(\frac{d+a}{d}\right)}$$ (B) ✓

Q5. (5)

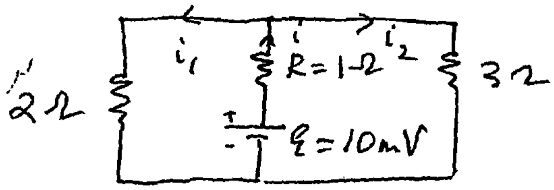




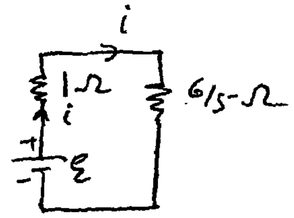
$v = 1\,m/s$
$l = 10\,cm$
$B = 0.1\,T$

The moving connector will act like a voltage source of emf $\mathcal{E} = Blv = 0.1 \times 0.1 \times 1 = 0.01$ Volts and a resistor $R = 1\Omega$ in series with it.

The equivalent circuit is therefore,



$\simeq$

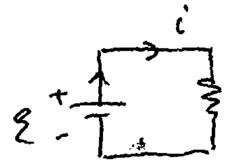


$\cancel{\mathcal{E} - (1 + \frac{6}{5})i = 0}$

$\cancel{i = \frac{\mathcal{E}}{11/5} = \frac{10^{-2}}{11/6} = \frac{63}{1100}}$ ~~Amp~~

$\cancel{i = \frac{3}{530}}$ ~~Amp~~

$\simeq$



$\frac{11}{5} = 2.2\Omega = R_{eq}.$

$\therefore \quad i = \frac{\mathcal{E}}{R_{eq}} = \frac{0.01\,V}{2.2\,\Omega} \Rightarrow \boxed{i = \frac{1}{220}\ \text{Amp}}$

(R)

Q6.

(6) $y = ax^2$

$\frac{d^2y}{dt^2} = w \Rightarrow \frac{dy}{dt} = wt \Rightarrow y = \frac{1}{2}wt^2$

$\Rightarrow \frac{1}{2}wt^2 = ax^2$

$\Rightarrow x = \pm\sqrt{\frac{w}{2a}}\, t$

Therefore at any given instant of time $t=t$, the upward velocity of the rod, $v = wt$ and the length of the rod (between the contact points with the parabolic frame) $l = 2\sqrt{\frac{w}{2a}} \cdot t$

$\therefore$ the induced voltage $\mathcal{E} = Blv = B \times 2\sqrt{\frac{w}{2a}}\, t \times wt$

$\Rightarrow \mathcal{E} = B\sqrt{w^3}$

$\Rightarrow \mathcal{E} = Bw\sqrt{\frac{2w}{a}}\, t^2$

$y = \frac{1}{2}wt^2$

$\therefore \mathcal{E} = 2By\sqrt{\frac{2w}{a}}$

(A) ✓

Q7. Cancelled

Area $= 10^{-2}\, m^2$

$B = 0.1$ Tesla $= 0.1\,T$

$R = 0.1\,\Omega$

Final area $= 0.5 \times 10^{-2}\, m^2 \Rightarrow \Delta A = 0.5 \times 10^{-2}\, m^2$

time $\Delta t = 0.1$ sec

$\therefore \mathcal{E} = \frac{\Delta\phi_B}{\Delta t} = \frac{B \times \Delta A}{\Delta t} = 0.1 \times \frac{0.5 \times 10^{-2}}{0.1}$

$\mathcal{E} = 0.5 \times 10^{-2}$ Volts

$\therefore$ average current $i = \frac{\mathcal{E}}{R} = 5 \times 10^{-2}$ Amp.

(B)

Q8. ~~(8)~~ (7)

Area : A

Mag. Field : B

Resistance : R

$\theta = 0$ to $\theta = 180°$

$\therefore \Delta\Phi_B = |\Phi_2 - \Phi_1| = |(BA\cos 180° - BA\cos 0)|$

$\Delta\Phi_B = 2BA$

$\therefore$ Total Charge $\Delta Q = \int i\,dt$

$= \int \frac{\mathcal{E}}{R} dt$

$= \frac{1}{R}\int \frac{d\Phi_B}{dt}.dt$

$\Delta q = \frac{\Delta\Phi_B}{R} = \frac{2BA}{R}$

(C) ✓

Q9. (8) $\mathcal{E} = 0$

The magnetic field lines due to the ~~current carrying~~ current carrying wire along the negative Z-axis do not intersect the square frame (they only 'graze' its surface tangentially). Hence, $\Phi_B = 0$

$\Rightarrow \mathcal{E} = \frac{d\Phi_B}{dt} = 0$ (C) ✓

Q10. (9.) ~~$a_1 < g$ ⟹ The falli~~

For fig-I the falling magnet will induce a current in the loop which in turn will create a magnetic field which will "retard" the motion of the magnet, hence $a_1 < g$. ✓

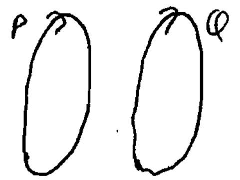
For fig-II the falling loop will experience an induced current due to the magnetic flux through it increasing. This induced current will also interact with magnetic field to produce an 'upward' force on it. Hence $a_2 < g$. ✓

(C) ✓

Q~~1~~. (REMOVE)

Cancelled → Repeated in Section II (Q12 has same concept)

Soln:-



If ~~loop~~ P approches Q, the flux through Q will increase, the induced EMF in Q should oppose this and therefore "reduce" current in it. Invert the statement and same applies to P.

∴ (A) ✓ ~~Q~~ cancelled

Q~~12~~. Application of Faraday's Law, the current 'I' does not
(10) create any flux in the circular coil. $\phi_B = 0 \Rightarrow \frac{d\phi_B}{dt} = 0$

$\Rightarrow$ no induced current.

(D) ✓

Q~~13~~. Application of Lenz's law, as the magnet falls, induced
(11) current will appear in the metal pipe that 'opposes' the fall, therefore magnetic force acting on magnet due to the induced current will "oppose" it's motion and ~~thereby~~ increase in magnitude proportionately to velocity after a ~~few~~ time. ~~Hence~~ thereby achieving terminal velocity

∴ (C) ✓

~~Q15~~.

Q~~14~~. The flux through the loop is "coming-out" of the
(12) plane, therefore as it increases, induced current should appear clock-wise.

∴ (A) ✓

Q~~15~~.

13 ∴ i_2 is constant and from c to d (clockwise)

∴ anti-clockwise ~~current is~~ flux is increasing at a constant rate or clockwise flux is decreasing at a constant rate.

∴ i_1 should be clockwise i.e. positive and decreasing uniformly.

or i_1 . .. anti-clockwise (negative) and increasing uniformly.

∴ (D) ✓

Q~~16~~ Cancelled REMOVE

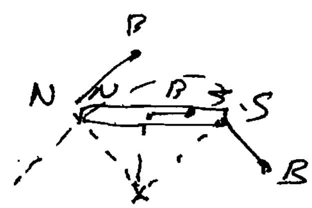
↳ Repeated concept as Q13.

↳ Soln.. : the induced current in the copper tube will create a force on the bar magnet that opposes it's motion and increases uniformly with it's speed. Therefore terminal velocity will be achieved after some time.

∴ (B) ✓ Cancelled

Q~~17~~. 14 Torque $= \vec{\tau} = (\vec{M} \times \vec{B})$

Now ~~here~~ here $\vec{B}$ and $\vec{M}$ are co-linear so $\tau = 0$



Alternately from the direction of $\vec{B}$ at the N and S poles, the direction of Force and Torque (if any) can be shown to be.

$F \neq 0$ and $\tau = 0$

∴ (D) ✓

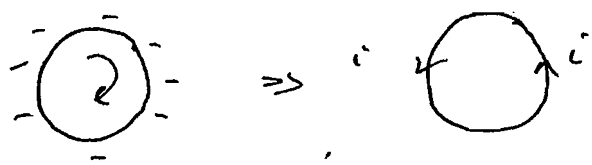
Q~~18~~. (REMOVE)
Cancelled

↳ Same concept in Q 12

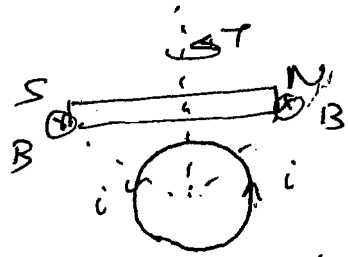
Solution : $\phi_B = 0 \Rightarrow \frac{d\phi_B}{dt} = 0$

$\Rightarrow$ (D) ✓ Cancelled

Q~~19~~.
15



$\therefore$



from the direction of magnetic field, the 'N' pole experiences a force "into" the plane and the 'S' pole a force "coming-out".

$\therefore$ (B) ✓

Q~~20~~.
16

$\mathcal{E} = Blv_0 = \text{Constant}$

$\therefore$ charge in the capacitor $q = C\mathcal{E} = \text{Constant}$

$\therefore \; i = \frac{dq}{dt} = 0$

$\therefore$ (C) ✓

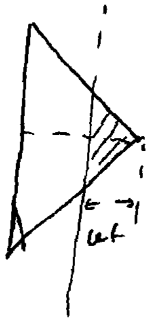
Q~~21~~.
17

Apply lenz's law. Force on 'Q' due to induced current in P should "oppose" change in flux.

$\therefore$ (A) ✓

~~Q22.~~

(18)



Area of shaded portion $= \frac{1}{2} \times (\omega t) \times (2\omega t) \propto t^2$

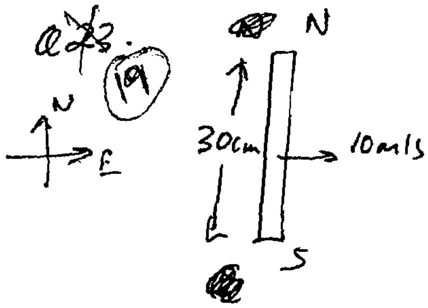
$\therefore$ flux $\phi_B = B \times (\omega t)^2 \Rightarrow \phi_B \propto t^2$

$\therefore \mathcal{E} = \left|\frac{d\phi_B}{dt}\right| \propto t$

$\therefore i = \frac{\mathcal{E}}{R} \propto t$

$\therefore$ (D) ✓

~~Q23.~~ (19)



$\otimes B = 4T$

$\mathcal{E} = Bl v = \cancel{4 \times 0.3 \times 10} = 4 \times 0.3 \times 10 = 12$ Volts.

and $(\vec{v} \times \vec{B})$ is upwards $\therefore V_N - V_S = +12\,V$

$\therefore$ (A) ✓

~~Q24.~~ (20)

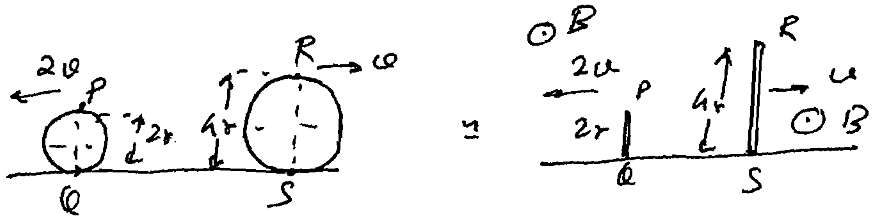
$i = \frac{\mathcal{E}}{R_{eq}} \Rightarrow i = \frac{Blv}{3\Omega} \Rightarrow \frac{2 \times 0.1 \times v}{(3+1)} = 1 \times 10^{-3} \Rightarrow v = 2 \times 10^{-2}$ m/s

$\Rightarrow$ ~~$v = 1 \times 10^{-2}$ m/s~~ $\Rightarrow \boxed{v = 2 \text{ cm/s}}$

~~= 1~~

$\therefore$ (C) ✓

Q~~20~~: (21)



Now, from motional emf concept,

$$V_P - V_Q = +(B\times 2r\times 2v) = +(4Brv)$$

and $$V_R - V_S = -(B\times 4r\times v) = -(4Brv)$$

$$\therefore \quad V_P - V_R = 8Brv$$

$\therefore$ (C) ✓

Q~~26~~. (22) $\mathcal{E}_y = M\left(\frac{di}{dt}\right)_x$ and $\mathcal{E}_x = M\left(\frac{di}{dt}\right)_y$

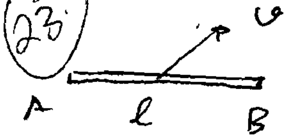
Now given $\mathcal{E}_y = E$ when $\left(\frac{di}{dt}\right)_x = \dot{I}$

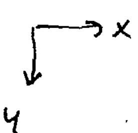
$$\therefore \quad M = \left(\frac{E}{\dot{I}}\right)$$

$\therefore$ Flux linkage x $\quad (\phi_B)_x = M I_y \Rightarrow (\phi_B)_x = \left(\frac{E}{\dot{I}}\right) I_0$

$\therefore$ (B) ✓

Q~~27~~. (23) $\vec{v} = v_0(\hat{i} - \hat{j})$, $\vec{B} = B_0(\hat{i} + \hat{j})$





Here the component of $\vec{v}$ $\perp$r to the rod is v_0 and the component of $\vec{B}$ mutually $\perp$r to both these two quantities (~~or~~ plane of motion) is zero.

→ Can be REMOVED : EASY.

Q28: Cancelled

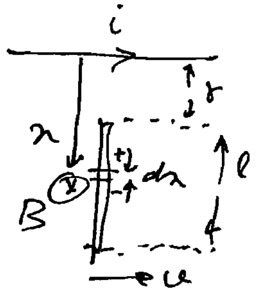
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\therefore \varepsilon = Blv = \frac{\mu_0 I l}{2\pi r} v$$

$\therefore$ (B) Cancelled

Q29: Cancelled REMOVE → REPEATED from SOLVED EXAMPLE 7

Soln: $B = \frac{\mu_0 i}{2\pi x}$



$$\therefore d\varepsilon = Bv\,dx = \frac{\mu_0 i v}{2\pi}\frac{dx}{x}$$

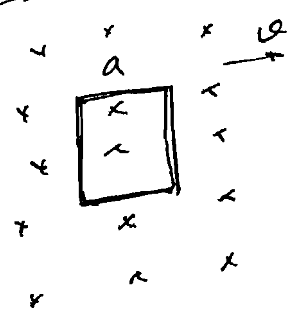
$$\therefore \varepsilon = \int_{x=r}^{x=r+l} \frac{\mu_0 i v}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \varepsilon = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{r+l}{r}\right)$$

$\therefore$ (D) Cancelled

→ Can be REMOVED : EASY

Q30. Cancelled



$W = 0$

as $\phi_B = B \times l^2 = \text{constant}$.

$\therefore \varepsilon = \frac{d\phi_B}{dt} = 0$

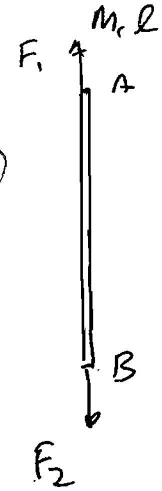
$\therefore$ magnetic forces $= 0$

$\therefore$ No external force needed $\therefore W = 0$

(D) Cancelled

Q3.

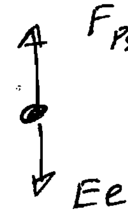
24.



$F_1 \neq F_2 \quad (F_2 > F_1)$

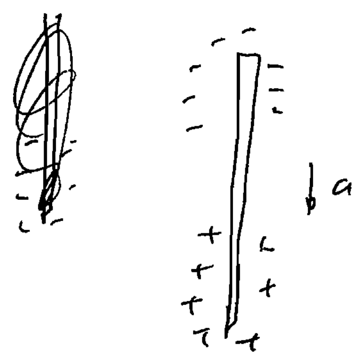
$\downarrow a \qquad \therefore \quad a = \left(\dfrac{F_2 - F_1}{M}\right)$

$\therefore$ for an FBD of a free electron in steady state in the non-inertial frame of the rod.



$F_{Pseudo} = ma = m\left(\dfrac{F_2 - F_1}{M}\right)$

where E : induced electric field due to 'polarization' of charge.



$\therefore \quad E = \dfrac{ma}{e} = \dfrac{m}{e}\left(\dfrac{F_2 - F_1}{M}\right)$

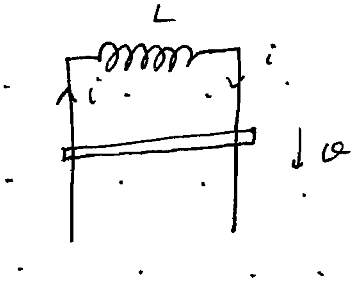
$\therefore$ potential difference btw the end-points of the rod,

$$|\Delta V| = |El| = \left|\frac{m}{e}\left(\frac{F_2 - F_1}{M}\right)l\right|$$

$\therefore$ (A) ✓

PTO

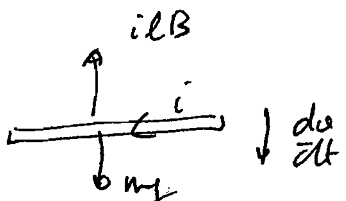
Q ~~32~~. 25.



$\mathcal{E} = Blv$

∴ from Kirchoff's loop law,

$$Blv - L\frac{di}{dt} = 0 \quad \text{— (I)}$$



Now, from Newton's 2nd law

$$mg - ilB = m\frac{dv}{dt} \quad \text{— (II)}$$

diffⁿ (I) : $Bl\frac{dv}{dt} - L\left(\frac{d^2i}{dt^2}\right) = 0$

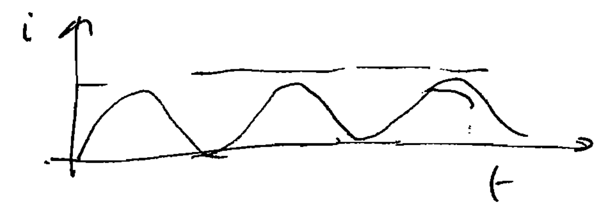
$Bl\left[\cancel{m}g - \frac{Bl}{m}i\right] = L\frac{d^2i}{dt^2}$

Substitute $\frac{dv}{dt} = \frac{mg-(Bl)i}{m}$ from (II)

$$\boxed{\frac{d^2i}{dt^2} = -\left(\frac{B^2l^2}{mL}\right)i + \frac{Blg}{L}}$$

Therefore $i = i_0\sin\{\omega t + \phi\} + \underbrace{\text{const.}}_{= i_0}$

∴ i vs t



∴ ∴ (A) ✓

Q ~~32~~. 26. $Q = i^2R = \left(\frac{\mathcal{E}}{R}\right)^2R = \frac{\mathcal{E}^2}{R} = \frac{(Blv)^2}{R}$

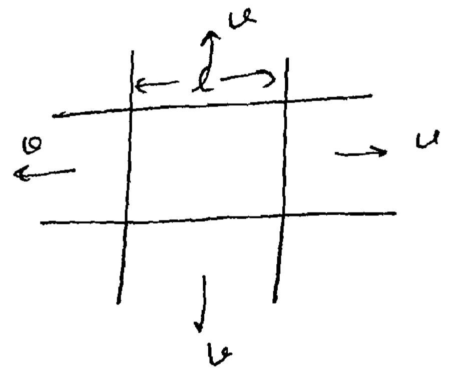
$a = 0 \Rightarrow F - ilB = 0$

$\Rightarrow F = ilB = \frac{B^2l^2v}{R} = \frac{Q}{v}$

∴ (B) ✓

(or use UDM method.)

Q3~~4~~ 27.



$$l = (a + 2vt)$$

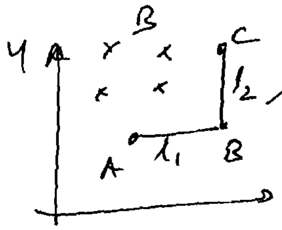
$$\therefore \phi_B = Bl^2 = B(a+2vt)^2$$

$$\Rightarrow \mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = 2B(a+2v)\times 2v = 4B(a+2v)v = 4Blv$$

$$R_{eff} = r \times 4l$$

$$\therefore i = \frac{\mathcal{E}}{R_{eff}} = \frac{Bv}{r} \quad \therefore (A) ✓$$

Q~~35~~ 28.



$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

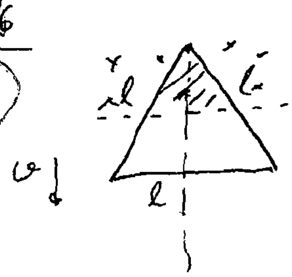
$$\mathcal{E}_1 = (V_C - V_B) = Bl_2 v_x$$

and $\mathcal{E}_2 = (V_A - V_B) = Bl_1 v_y$

$$\therefore (V_A - V_C) = B(l_1 v_y - l_2 v_x)$$

$$\therefore (C) ✓$$

Q~~36~~ 29.



$\phi_B \propto$ Area of shaded part

and Area $\propto \left(\frac{\sqrt{3}}{2}l - vt\right)^2$

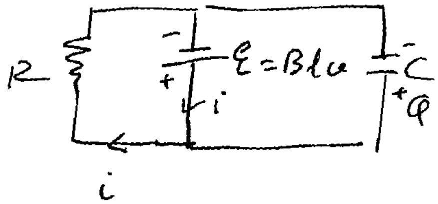
$$\therefore \mathcal{E} = \frac{d\phi_B}{dt} \propto \left(\frac{\sqrt{3}}{2}l - vt\right)$$

$$\therefore i = \frac{\mathcal{E}}{R} \propto \left(\frac{\sqrt{3}}{2}l - vt\right)$$

~~Q3~~

30

$\mathcal{E} = V_E - V_H = Bl\omega$



$Q = C\mathcal{E} = BlC\omega$: constant $\Rightarrow$ current ~~thos~~ through the capacitor $C = 0$

and $i = \frac{\mathcal{E}}{R} = \frac{Bl\omega}{R}$

$\therefore$ (D) ✓

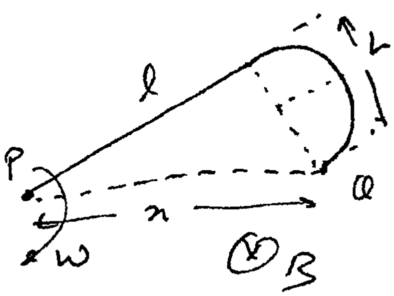
~~Q~~

31

$\mathcal{E} = V_Q - V_P = \frac{B\omega x^2}{2}$

$\Rightarrow \mathcal{E} = \frac{B\omega(l^2+L^2)}{2}$

$\therefore$ (C) ✓



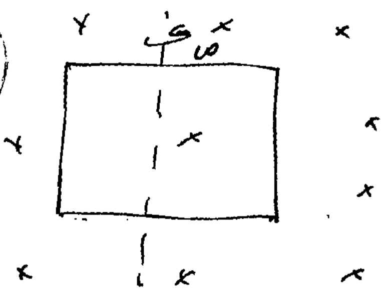
Use superposition as explained in ~~Lecture~~ ~~Sol~~ Illustration ______ .

~~Q~~ Cancelled

REMOVE $\longrightarrow$ Same as Q29 in ~~Sol~~ Exercise II .

~~Q~~

32



$\mathcal{E}_{avg} = \left|\frac{\Delta\phi_B}{\Delta t}\right|$: for 90° rotation

$|\Delta\phi_B| = (B \times A)$

$\Delta t = \frac{\pi}{2\omega}$

$\therefore \mathcal{E}_{avg} = \frac{2\omega BA}{\pi}$

~~Q1~~

33. Amplitude $\mathcal{E}_0 = NAB\omega$

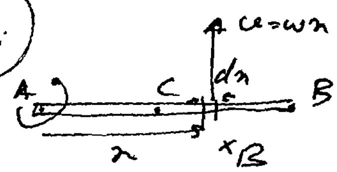
$$= 100 \times \pi \times (0.1)^2 \times (10 \times 10^{-3})(100 \times 2\pi) = 2 \times \pi^2 \simeq 20\,V$$

$$\therefore \quad i_0 = \frac{\mathcal{E}_0}{R} = \frac{20}{10} = 2\,\text{Amp.} \quad \therefore (B)$$

$10^{2-2-2+2}$

~~Q2.~~

34.



$$d\mathcal{E} = Bv\,dx = B\omega x\,dx$$

$$\therefore \quad V_c - V_B = \int_{x=l/2}^{x=l} B\omega x\,dx = \frac{B\omega}{2}\left(l^2 - \frac{l^2}{4}\right)$$

$$\mathcal{E} = \frac{3}{8} B\omega l^2$$

$\therefore$ (D) ✓

~~Q3.~~

35. $a = \frac{eE}{m}$

$$\oint E\,dl = \frac{d\phi_e}{dt} = \pi r^2 \left(\frac{dB}{dt}\right) = \cancel{\text{const}}$$

$\therefore \quad E \times 2\pi r \propto r^2$

$\Rightarrow \quad E \propto \frac{1}{r}$

$\therefore \quad a \propto \frac{1}{r}$

and sense of emf = anti-clock wise

$\therefore \quad a$: towards right.

(B) ✓

Q44.

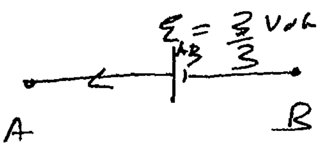
36. $\frac{dB}{dt} = \sqrt{3}\ (T/s)$

$\mathcal{E} = \left(\frac{dB}{dt}\right) \times \left(\frac{1}{2} \times l \times \frac{\sqrt{3}}{2} l\right)$

$= \sqrt{3} \times \frac{\sqrt{3}}{4} l^2$

$\mathcal{E} = \frac{3}{4} \times 2^2 = 3$ Volt, $\therefore\ \mathcal{E}_{AB} = \mathcal{E}_{AC} = \mathcal{E}_{CB} = \frac{\mathcal{E}}{3} = 1$ Volt

$\therefore$



, $i = \frac{\mathcal{E}}{R_{Total}} = \left(\frac{3}{1+2+2}\right) = 0.6$ Amp

$\therefore\ V_A - V_B = \mathcal{E}_{AB} - iR_{AB} =$

$\Rightarrow\ V_A - V_B = 1 - (0.6 \times 1)$

$= 0.4$ Volt

$\therefore$ (A) ✓

Q45.

37. $i_{min} = \frac{10\,V}{10\,\Omega}$ (with S closed at $t = 0$, $L \to$ open switch)

$i_{min} = 1$ Amp

$i_{max} = \frac{10\,V}{5\,\Omega}$ (with S closed @ $t \to \infty$, $L \to$ closed switch)

$= 2$ Amp.

$\therefore\ i_{max} - i_{min} = 1$ Amp

(C) ✓

Q~~46~~.
38

$$\tau_{charge} = \frac{L}{2R}$$

$$\tau_{Disch} = \frac{L}{3R}$$

$$\therefore \frac{\tau_{charg}}{\tau_{Dn}} = \frac{3}{2}$$

∴ (B) ✓

Q~~47~~.
39

$\mathcal{E} = \mathcal{E}_0 e^{-t/\tau}$ $\quad$ $\mathcal{E}_0$: EMF of battery / source

and $\quad$ and $\mathcal{E} = V_L = \left|-L\frac{di}{dt}\right|$

$$i = i_0(1 - e^{-t/\tau})$$

$$i = \frac{\mathcal{E}_0}{R}(1 - e^{-t/\tau})$$

$$\therefore i = \frac{\mathcal{E}_0}{R} - \frac{1}{R}\mathcal{E} \Rightarrow \mathcal{E} = \mathcal{E}_0 - iR$$

∴ (A) ✓

Q~~48~~. → REMOVE Question

Cancelled

$i = (2 + 4t)$ Amps

$L = 2H$

$$\therefore U_L = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times (2+4t)^2$$

$$\therefore \frac{dU_L}{dt} = 2(2+4t) \times 4$$

$$\left(\frac{dU}{dt}\right)_{t=0} = 16 \text{ J/s}$$

PTO

Q49.
40.

$\left|\frac{di}{dt}\right|$ is greater for (1)

$\therefore$ self-induced voltage $V_L = \left|L\frac{di}{dt}\right|$ is greater for (1)

$\therefore$ (A) ✓

Q50.
41.

$\frac{di}{dt} = 4$ Amp/s

@ $t=t$, $i = 2$ Amp, $q = ?$

$$\Rightarrow \quad \mathcal{E} - iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$\Rightarrow \quad 4 - (2\times1) - (1\times4) - \frac{q}{C} = 0$$

$$\Rightarrow \quad q = |-2C| \quad , \quad C = 3\mu F$$

$$q = 6\mu C$$

(C) ✓

Q51. ~~cancelled~~ Remove $\rightarrow$ Repeated ~~concept~~ question (Q25 of Section II)

Q52.
42.

$L = 5H$

$\mathcal{E} = 6V$

$R = 10\Omega$

$\tau = \frac{L}{R} = 0.5$ sec.

$V_L = \mathcal{E}e^{-t/\tau}$

$$\therefore (V_L)_{t=\ln\sqrt{2}} = (6V)e^{-\frac{1}{0.5}\ln\sqrt{2}} = (6V)e^{-\ln(2)} = (6V)\,e^{\ln(1/2)}$$

$$= 3 \text{ Volts}$$

$\therefore$ (A) ✓

~~Q 53~~

43

A, N, L

$\phi_B = Li$

$\Rightarrow N \times B \times A = Li$

$\Rightarrow$ ~~B = L~~

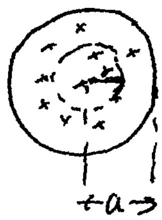
$\Rightarrow i = \dfrac{NBA}{L}$

$\therefore$ (A) ✓

~~Q 54.~~

44

for $r < a$

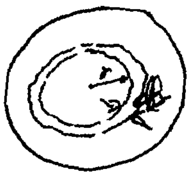


$$B = \frac{\mu_0 i (r^2/a^2)}{2\pi r}$$

$$\Rightarrow B = \frac{\mu_0 i r}{2\pi a^2}$$

Energy density $u_B = \dfrac{1}{2}\dfrac{B^2}{\mu_0} = \dfrac{(\mu_0 i r / 2\pi a^2)^2}{2\mu_0}$

$$= \frac{1}{2\mu_0} \times \frac{\mu_0^2 i^2 r^2}{4\pi^2 a^4}$$



$$u_B = \left(\frac{\mu_0 r^2}{8\pi^2 a^4}\right) i^2$$

Now the energy stored in a differential cylindrical shell of length l, radius r and thickness dr,

$$dU = u_B \, dV = u_B (2\pi r \, dr \, l)$$

$$= \frac{\mu_0 r^2 i^2}{8\pi^2 a^4} 2\pi r l \, dr$$

$$dU = \frac{\mu_0}{4\pi} \frac{i^2 l}{a^4} r^3 dr$$

$$\therefore U = \int_0^a \frac{\mu_0}{4\pi} \frac{i^2 l}{a^4} r^3 dr = \frac{\mu_0}{4\pi} \frac{i^2 l}{a^4} \cdot \frac{a^4}{4}$$

$$U = \mu_0 i^2 l$$

Therefore energy per unit length:

$$\boxed{\frac{\partial U}{\partial l} = \frac{\mu_0 i^2}{16\pi}}$$

$\therefore$ (B) ✓

Q55.

45. @ $t=0$ (with K closed), $i = \frac{2V}{(10+20)\Omega} = \frac{2}{30}$ Amp

(L → open switch,

@ $t \to \infty$, $i = \frac{2V}{20\Omega} = \frac{1}{10}$ Amp (L → short ckt)

$\therefore$ (A) ✓

~~Q56.~~ Cancelled (REMOVE)

↳ Repeated ~~Ques~~ Concept in Subjectives.

~~Q57.~~ Cancelled

(DISCUSS)

→ REMOVE

Q~~48~~. 46.

V_{max} (for L) = V_{max} (for C)

$\therefore$ $\frac{Q}{C} = 16$ volts Q : max charge on capacit

U_{max} (for L) = U_{max} (for C)

$\therefore$ $\frac{Q^2}{2C} = 160\ \mu J$

$\therefore$ $\left(\frac{Q}{C}\right)^2 \times \frac{C}{2} = \frac{Q^2}{2C}$

$\Rightarrow (16)^2 \times \frac{C}{2} = 160 \times 10^{-6}$

$\Rightarrow C = \frac{160 \times 2}{16^2} \times 10^{-6}$ $\frac{5}{4}$

$C = \frac{20}{16}\ \mu C = 1.25\ \mu C$ $\therefore$ (D)

Q~~49~~. Cancelled → REMOVE

→ ~~Soln~~ Soln

$V_L = L\frac{di}{dt} = \varepsilon$

$\Rightarrow \int_0^5 di = \frac{\varepsilon}{L}\int_0^T dt$ $\Rightarrow T = \frac{5 \times 4}{2} = 10$ sec

$\therefore$ (D)

Q~~50~~ 47.

$V = V_0 - \left(\frac{V_0}{T}\right)t$

$\therefore$ $L\frac{di}{dt} = V \Rightarrow$ $di = \frac{V}{L}\,dt \Rightarrow \int_0^i di = \int_0^t \frac{1}{L}\left\{V_0 - \left(\frac{V_0}{T}\right)t\right\}dt$

$\Rightarrow i = \frac{V_0}{L}\left\{t - \frac{t^2}{2T}\right\}$

∴ ~~total~~ current @ t=T

~~$i_T = \frac{V_0}{L}\left\{T - \frac{T}{2}\right\} = \frac{V_0 T}{2L}$ and @ t=0, $i_0 = \frac{V_0 T}{L}$~~

~~∴ $i_{avg} = \frac{i_0 - i_T}{2} = \frac{V_0}{2L}$~~

and since ~~current is a linear function of time, $i_{avg} = \frac{i_0 + i_T}{2} = \frac{V_0 T}{L}\left(\frac{1+1/2}{2}\right)$~~

$$i = \frac{V_0}{L}\left(t - \frac{t^2}{2L}\right)$$

∴ in time $t=0$ to $t=T$, charge flowing through the circuit $\Delta Q = \int i\,dt \Rightarrow \Delta Q = \int_{t=0}^{t=T} \frac{V_0}{L}\left(t - \frac{t^2}{2L}\right) dt$

$$\Rightarrow \Delta Q = \frac{V_0}{L}\left\{\frac{T^2}{2} - \frac{T^3}{6T}\right\}$$

$$\Rightarrow \Delta Q = \frac{V_0 T^2}{2L}\left(\frac{2}{3}\right) = \frac{V_0 T^2}{3L}$$

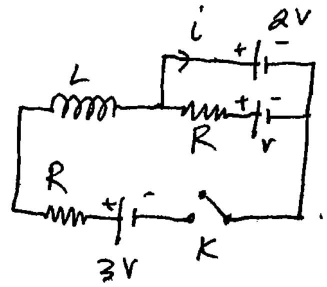
∴ avg current $i_{av} = \frac{\Delta Q}{\Delta t} = \frac{V_0 T^2/3}{T}$

$$\Rightarrow \boxed{i_{avg} = \frac{V_0 T}{3L}} \quad \text{Ans}$$

∴ (B) ✓

QG1

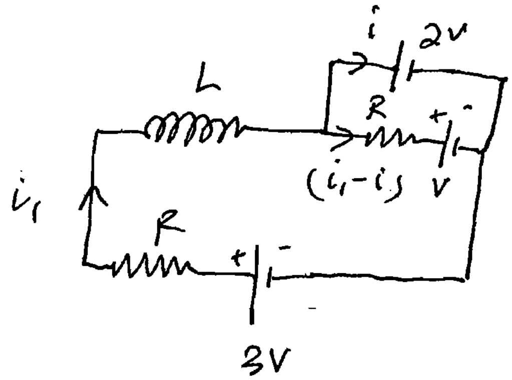
48.



@ $t=0$ (just before the switch

$i = \frac{-V}{R}$ (only the ~~s...~~ smaller loop operates)

when k is closed @ $t=t$



Let currents and voltages as show.

$$3V - i_1 R - L\frac{di_1}{dt} - R(i_1 - i) - V = 0$$

$$\Rightarrow 2V - L\frac{di_1}{dt} - 2Ri_1 + Ri = 0 \quad \text{— (I)}$$

$$-2V + V + R(i_1 - i) = 0$$

$$\Rightarrow R(i_1 - i) = V$$

$$\Rightarrow Ri = Ri_1 - V \quad \text{— (II)}$$

$\therefore$ from (I) and (II)

$$2V - L\frac{di_1}{dt} - 2Ri_1 + Ri_1 - V = 0$$

$$\Rightarrow V - L\frac{di_1}{dt} - Ri_1 = 0$$

$$\Rightarrow i_1 = \frac{V}{R}(1 - e^{-tR/L})$$

$\therefore$ ~~$i = i_1 \frac{V}{R} = \frac{V}{R}(2 - e^{-tR/L})$~~

Now $i = i_1 - \frac{V}{R}$ (from Eq (II))

$$\therefore \boxed{i = -\frac{V}{R}e^{-Rt/L}}$$

'–' sign showing reverse sense.

$\therefore$ (C) ✓

Q62.

49. Heat produced = Potential Energy in the inductor initially (just before switch is toggled from 4 to 7.)

$$= \frac{1}{2} L i_0^2 = \frac{1}{2} L \left(\frac{E}{R_1}\right)^2$$

$\therefore$ (A) ✓

Q63.

50. $\frac{1}{2} L i^2 =$ [illegible] 32 J. for $i = 4$ Amp $\Rightarrow L = \frac{2 \times 32}{16} = 4H.$

$i^2 R =$ [illegible] 320 watts for $i = 4$ Amp.

~~for i = 4 Amps~~

$\therefore \quad R = \frac{320}{16} = 20\,\Omega$

$\therefore \quad \tau = L/R = 4/20 = 0.2$ sec $\quad \therefore$ (A) ✓

Q64

57. $V_B - V_A = -(I \times 1\Omega) - 15V - L\left(\frac{dI}{dt}\right)$

$\Rightarrow V_B - V_A = -(5V) - (15V) - (5 \times 10^{-3} \times (-10^3))$

$\Rightarrow V_B - V_A = -20 + 5$

$\Rightarrow (V_B - V_A) = -15V$

$\therefore$ (C) ✓

Q65

52. $i = Ie^{-t/\tau}$

$\Rightarrow \Delta Q = \int i\,dt = \int_0^t Ie^{-t/\tau} dt = I\tau(1 - e^{-t/\tau})$

$\Delta Q = \frac{IL}{R}(1 - e^{-t/\tau})$

Now at $t = t_0$ $U = \frac{1}{4} U_0 \Rightarrow i = \frac{1}{2} I \Rightarrow e^{-t/\tau} = \frac{1}{2}$

Q.

53)

$C = 2\mu F$

$V_0 = 12V$

$Q = 24\mu C$

$U_c = \frac{1}{2}CV_0^2 = \frac{1}{2}\frac{Q^2}{C} = 144\mu J$

$L = 0.6mH$

when $V_c = 6V$, $U_c = \frac{1}{2}CV^2 = 36\mu J$

$\therefore \quad U_L = \frac{1}{2}Li^2 = \left(\frac{1}{2}CV_0^2 - \frac{1}{2}CV^2\right) = (144-36)\,\mu J$

$\Rightarrow \quad \frac{1}{2}Li^2 = 108\,\mu J$

$\Rightarrow \quad \frac{1}{2} \times 0.6\times10^{-3}\, i^2 = 108\times10^{-6}$

$i^2 = \frac{108\times10^{-6}}{0.3\times10^{-3}} = \frac{\cancel{108}^{36}}{\cancel{3}}\times10^{-2}$

$\Rightarrow \quad i = 6\times10^{-1} = 0.6\ Amp.$

$\therefore$ (D) ✓

Q.

54)

$q = q_0 \sin\{wt + \alpha\}$: LC oscillation ckt.

$\therefore \quad i = -\frac{dq}{dt} = -q_0 w \cos\{wt + \alpha\}$

$\therefore \quad |i_{max}| = q_0 w = \frac{q_0}{\sqrt{LC}}$

$\therefore \quad \frac{di}{dt} = -q_0 w^2 \sin(wt + \alpha)$

$\therefore \quad \left|\frac{di}{dt}\right|_{max} = q_0 w^2 = \frac{q_0}{LC}$

$\therefore$ (A) ✓

Q48.
55

$\frac{1}{2}Li^2 = U \Rightarrow L = \frac{2U}{i^2}$

$Ri^2 = P \Rightarrow R = P/i^2$

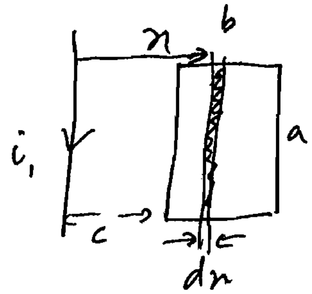
$\Rightarrow \quad \therefore \tau = L/R = \frac{2U}{P}$

$\therefore$ (C) ✓

Q49.
56.

$\phi_B = M i_1$

where ϕ_B = flux through the Rectangular loop



$d\phi_B = B\,dS = \frac{\mu_0 i_1}{2\pi x} \cdot a\,dx$

$\therefore \phi_B = \frac{\mu_0 i_1 a}{2\pi}\int_{x=c}^{x=b+c}\frac{dx}{x} \Rightarrow \phi_B = \mu_0$

$\phi_B = \left[\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)\right] i_1$

$\therefore \boxed{M = \frac{\mu_0 a}{2\pi}\ln(1+b/c)}$

$\therefore$ (D) ✓

Q70 Cancelled → REMOVE → Repeated Concept.

$\phi_B = 0$ (the current in the long conductor does not produce any flux through the circular ring)

$\therefore$ $\underline{M = 0}$

(D)

EMI : EXERCISE-II

Multiple option MCQs

Q1. ✓ $\frac{[\phi_B]}{[R]} = \frac{[B][A]}{[E/I]}$

$\left[\frac{d\phi_B}{dt}\right] = [\mathcal{E}]$

$\Rightarrow \frac{1}{[R]}\left[\frac{d\phi_B}{dt}\right] = [i]$

$\therefore \frac{[d\phi_B]}{[R]} = [i][dt] = [\text{charge}]$

(B) ✓

Q2. ✓ Since a clockwise current in I indicates clockwise in both loops.

$\phi_B = B\times(L^2+l^2)$ for case I

But in case II, clockwise current in loop abgh indicates anti-clockwise current in fedc

$\therefore \phi_B = B\times(L^2-l^2)$ for case II

(C) ✓

Q3. ✓ Flux 'into' the plane is ~~decreasing~~ decreasing.

Therefore current should be clock-wise in both loops for case I.

(C) ✓

Q4. ✓ Again since flux into the plane is decreasing the current in larger loop abgh is clockwise and smaller loop cdef is anti-clockwise

Q5: ✓

for Case I

$\mathcal{E}_{ind} = \frac{dB}{dt} \times (L^2+l^2)$

$i_{ind} = \frac{1}{R}\frac{dB}{dt}(L^2+l^2)$

for Case II

$\mathcal{E}_{ind} = \frac{dB}{dt}(L^2-l^2)$

$\therefore\ i_{ind} = \frac{1}{R}\frac{dB}{dt}(L^2-l^2)$

$\therefore\ I_2 < I_1$

(B) ✓

Q6. ✓ There will be an induced EMF of $\cancel{\frac{dB}{dt}\times \pi a^2)}$ $\left(\frac{dB}{dt}\times \pi a^2\right)$ in each loop where 'a' is the radius and given the field B is diminishing (into the plane), the direction of induced voltages will be clockwise for both loops.

(A), (C), (D) ✓

(Since both are closed loops, assume they operate independently as circuits neglecting the effects of mutual inductance)

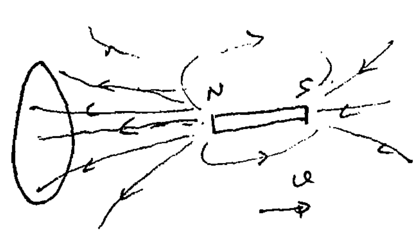
Q7. ✓ The triangle containing CD (larger one) will dominate over the smaller one in deciding the 'sense' or direction of induced current.

Flux 'into' the plane is increasing so the larger △ must have anti-clockwise current (Lenz's Law)

∴ (A) ✓

Q8 ✓ By application of Lenz's Law, induced voltage, induced current and the resultant magnetic force ~~betw~~ experienced by 'B' when the current in 'A' is changing will all ~~tend~~ tend to 'oppose' the change. Hence when 'i' is increased, the

9. ✓



As the bar magnet is moved away, as seen from the position of the magnet, flux "going into" the circular coil is decreasing and ~~the~~ therefore by application of Lenz's Law there will be an induced "clockwise" current.

(B) ✓

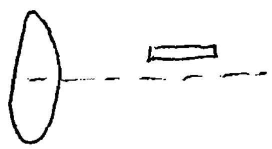
10. ✓ The anticlockwise current observed indicates two possibilities about the magnetic flux through the loop.

(i) If the flux is "into" the loop, it must be increasing. (~~can~~ or north pole faces the ring and moves towards it)

(ii) If the flux is ~~"out wards of"~~ "coming out" of the loop, it must be decreasing. (or south pole facing the ring and moving away from it)

(B), (C) ✓

11. ✓ If the magnet is placed "off-axis" as shown, the magnetic field lines ~~on~~ acting on it are both of non-uniform ($\vec{B}$ varies with position) and non-colinear ($\vec{B}$ is slightly "bent" off axis).

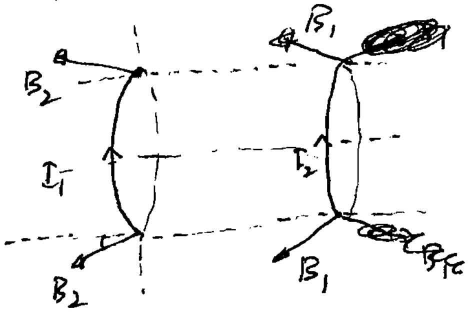


Therefore it experiences both ~~as~~ a net force and a ~~torque~~ torque.

(C) ✓

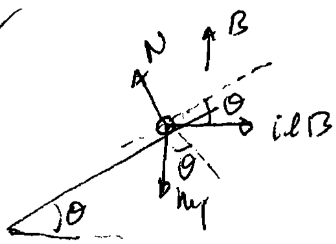
12. ✓ If one of the loops has no current and the other (with a current in it) moves towards it the induced current in the first will be of "opposite" sense by application of Lenz's Law.

'radial' and an 'axial' component. Now, the axial component will ~~not~~ produce any net force on the first loop, however the radial component will produce a net force which will attract the loop if it has current in same direction and oppose it if vice versa.



(B), (D) ✓

13. ✓



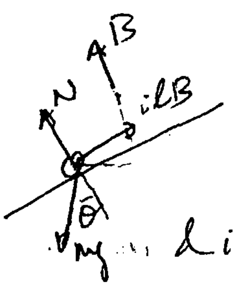
i : into the plane

Equilibrium conditions : $ilB\cos\theta - mg\sin\theta = 0$

$\Rightarrow \boxed{ilB = mg\tan\theta}$

(A), (B) ✓

14 ✓



i: into the plane.

$ilB - mg\sin\theta = 0$

$\Rightarrow \boxed{ilB = mg\sin\theta}$

(B) ✓

15. ✓

$\mathcal{E} = Blv = 4 \times 1 \times 20 = 80$ Volts.

$\therefore \quad q = C\mathcal{E} = 10\mu F \times 80 V = 800 \mu C$

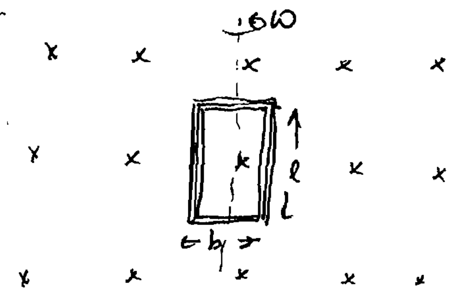
From lenz's law a right hand rule

$(V_P - V_Q) = +Blv = +80$ volts

$\therefore$ Plate A has $+800 \mu C$
and B has $-800 \mu C$ charge.

(A) ✓

16. ✓



Consider a rectangular coil of N turns of dimensions $(l \times b)$ rotating with angular speed 'ω' in a uniform field B.

By application of Faraday's law

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\left(N(l \times b)B(\hat{n}\cdot\hat{k})\right)$$

where $\hat{n}$: area vector's direction

$$\mathcal{E} = NlbB\omega \cos(\omega t)$$

$\therefore$ $\mathcal{E}$ is independent of only 'R'

(D) ✓

17. ✓ The only type of motion that generates a change in flux and therefore an induced emf for a closed conducting loop inside a uniform magnetic field is "rotation".

Therefore (A), (B), (C) and (D)

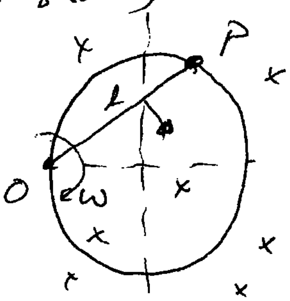
18. ~~For any section~~

~~For the voltage~~ inc ~~of~~ any

18. ✓ For the induced emf across any section of the wire join the end points with a straight line and apply the formula (as shown)

$$\mathcal{E} = \frac{B\omega l^2}{2} = V_P - V_O$$

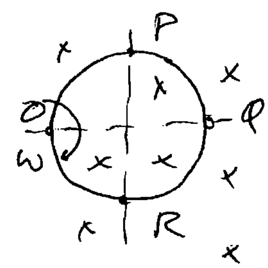
with polarity as per right hand rule.



$$\therefore (V_P - V_O) = +\frac{B\omega (R\sqrt{2})^2}{2}$$

$$(V_R - V_O) = +\frac{B\omega (R\sqrt{2})^2}{2}$$

$$(V_Q - V_O) = \frac{B\omega (2R)^2}{2}$$



$\therefore (V_Q - V_P) = \frac{B\omega R^2}{2}(4-2) = +B\omega R^2$ etc.

$\therefore$ (B), (D) ✓

19. ✓ Same as above

(C) ✓

20. ✓ For the entire ring the net emf across the closed loop will be <u>zero</u> (Flux is ~~uniform~~ constant $\phi_B = B\times \pi R^2$)

$\therefore$ (D) ✓

Q21. ✓ $\tau = L/R$ and $i_0 = E/R$ where i_0 = steady state current.

Now, ~~i_0 is greater for ckt (b) than for~~

from the graph, i_0 is same for circuits (b) and (c). Therefore $\frac{V}{R_1} = \frac{V}{R_2} \Rightarrow R_1 = R_2$

~~whereas~~ whereas time constant τ is ~~greater~~ smaller for circuit (b) as compared to (c)

$$\therefore \quad \frac{L_1}{R_1} < \frac{L_2}{R_2} \Rightarrow L_1 < L_2$$

$\therefore$ (B), (D) ✓

Q22. ✓ From the graph of current Vs time it is evident that the steady state current $i_0 = E/R$ remains the same after making the changes whereas the time constant $\tau = (L/R)$ increases.

$\therefore$ (A), (C) ✓

Q23. ✓ The current passing through is constant, therefore for a resistor the voltage $V = iR$ would be constant, for an inductor $V = -L\frac{di}{dt}$ would be zero, and for a capacitor $V = \frac{q}{C}$, $q = i\Delta t$ would increase with time linearly.

$\therefore$ (D) ✓

Q24. ✓ The presence of the iron rod will increase the intensity of the magnetic field inside the solenoid, hence the flux and hence the self-inductance ($\Phi_B = Li$), however it has no effect on the "resistance" of the coil.

Q25: ✓ $[RC] = T$

$[L/R] = T$

$[\sqrt{LC}] = T$

$\therefore$ (A), (B), (C) ✓

Q26. ✓ The back emf @ $t=0$ = $\mathcal{E}$. $\therefore$ the inductor "blocks" current @ $t=0$

$\therefore$ (D) ✓

Q27. ✓ for RC charging ckt $q = C\mathcal{E}(1-e^{-t/RC})$
and $i = \mathcal{E}/R \, e^{-t/RC}$

whereas for LR charging ckt $i = \mathcal{E}/R(1-e^{-Rt/L})$

$\therefore$ (B), (D) ✓

~~Q28~~ ~~(REPLACE QUESTION)~~ ~~For LC oscillations $q = q_0\cos(\omega t)$, $i = i_0\sin(\omega t)$~~
~~$\therefore U_C = \frac{1}{2}\frac{q_0^2}{C}\cos^2(\omega t)$ and~~
~~$U_L = \frac{1}{2}L i_0^2\sin^2(\omega t)$~~
~~$\therefore$ (B), (D)~~ ✓

~~Q29~~. ✓
Q28. ✓ $\tau = L/R$, $U_L = \frac{1}{2}Li^2$

$P_R = i^2R$

$\therefore \tau = (2\times U_L)/P_R \Rightarrow$ (A) ✓

Q29. ✓
~~Q30~~. ✓ The inductor initially "blocks" current through 'B_1' but eventually in steady state acts as a short circuit.
$\therefore$ the current through B_1 will be zero initially while the current through $B_2 = \mathcal{E}/R$ (constant)
as the time passes and the circuit goes to steady state, the current through B_1 becomes $i = \mathcal{E}/R$

$\therefore$ (A), ✓

Q30. ~~Q31.~~

$$[R] = \frac{[V]}{[I]} = \frac{[J/Q]}{[Q][t]} = Kg\, m^2 sec^{-2} \times (Coulomb)^{-2} \times (sec)^{-1} = \frac{Kg\, m^2 sec^{-2}}{Amp\, sec} \times \frac{1}{Amp}$$

$$= Kg\, m^2 A^{-2} sec^{-3}$$

$$[L] = [V] \times \left[\frac{di}{dt}\right]^{-1} = \frac{Kg\, m^2 sec^{-2}}{(Amp\text{-}sec)} \times \frac{sec}{Amp} = Kg\, m^2 A^{-2} s^{-2}$$

$$[C] = \frac{[V]}{[Q]} = \frac{Kg\, m^2 sec^{-2}}{(Amp\, sec)} \times \frac{1}{(Amp\, sec)} = Kg\, m^2 A^{-2} sec^{-4}$$

$$[\phi_B] = [B][Area] = \frac{[F]}{[il]} \times [L^2] = \frac{Kg\, m\, sec^{-2}}{Amp\, m} \times m^2 = Kg\, m^2 A^{-1} s^{-2}$$

$\therefore$ (A)

Q31 ~~Q32.~~ When the switch 'S' is opened, the magnetic flux through the inductor tends to change at a very rapid rate (since 'i' starts to decay fast) creating a strong back emf.

$\therefore$ (C)

Q32 ~~Q33.~~ The induced "back" emf in the inductor initially tends to block current through the lamp but as the circuit achieves steady state the back emf minimizes and the current through the lamp stabilizes.

$\therefore$ (B)

Q33 $E_{ind} = -\left(\frac{d\phi_B}{dt}\right)$, now $\left(\frac{\Delta\phi_B}{\Delta t}\right)_A = 0.4$ Wb/sec due to current in 'B' changing.

$(E_{...}) = -M\left(\frac{di}{...}\right) \Rightarrow 0.4 = M \times \left(\frac{\Delta i}{...}\right)$ ...

~~Moved to~~

~~Q21~~. (MOVE TO AC Ckts) : Theory Question.

(B), (D)

EMI Solutions

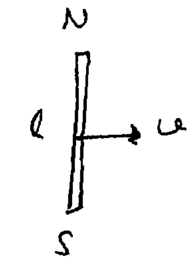
~~Exercise - III~~ EXERCISE IV - SUBJECTIVES

Q1. ✓ $B_H = 3\times10^{-4}$ T

dip = $\tan^{-1}(4/3)$

$l = 0.25$ m

$v = 10$ cm/s



The induced voltage $\mathcal{E} = Blv$ where B: Component of the earth's magnetic field 'perpendicular' to the plane of the rod's motion or B_v, $B_v = B_H \tan\theta = (3\times10^{-4} \times \frac{4}{3})$

$$\Rightarrow \mathcal{E} = B_v l v = \underset{(T)}{4\times10^{-4}} \times \underset{(m)}{0.25} \times \underset{(m/s)}{0.1} = 10^{-5}\ \text{Wb/m}^2\text{s}$$

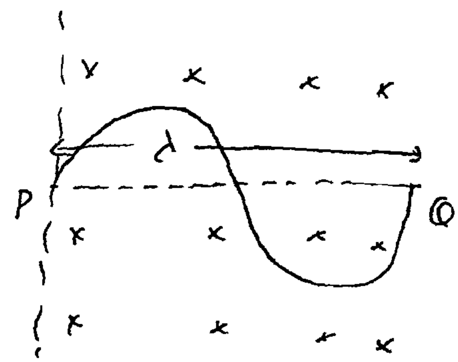
$$\Rightarrow \mathcal{E} = 10\times10^{-6}\ \text{T/s}$$

$$\boxed{\mathcal{E} = 10\,\mu V}$$

Q2. ✓ $\vec{v} = v_x\hat{i} + v_y\hat{j}$

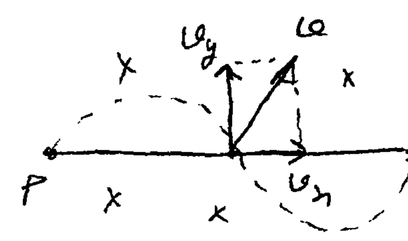
$\vec{B} = -B_0\hat{k}$

induced motional emf $\mathcal{E}$?

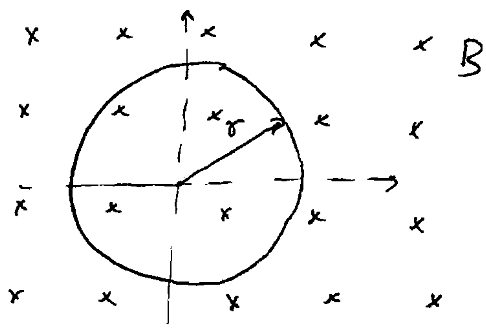


If we were to join the points P and Q with a straight wire (assume no electrical contact at the mid-point of PQ), the resulting closed loop would experience 'zero' induced voltage (flux through it will not change due to the translational motion)

Hence by application of superposition the induced voltage across the sin-curve shaped conductor is equal (and opposite) ... imaginary straight wire



Q3. ✓



$B = 0.02\,T$, $r = (r_0 - 0.1t)$

~~$r = r_0 - 0.1t$~~ where 0.1 (mm/s)
t (sec)
r (mm)

$\therefore$ induced emf $\mathcal{E} = \left|-\frac{d\Phi_B}{dt}\right| = B\frac{d}{dt}(\pi r^2)$

$$= 2\pi r \frac{dr}{dt} \times B$$

$$\boxed{\mathcal{E} = 2\pi r \times (0.1) \times B}$$

~~$\therefore$ when $r = 4$ mm~~

~~$\mathcal{E} = 2\pi \times 4 \times 0.1 \times 0.02 \times 10^{-6}$ Volts~~

~~$\Rightarrow \boxed{\mathcal{E} = 16\pi \times 10^{-7}}$ Volts~~

when $r = 4$ cm

$\mathcal{E} = 2\pi \times (4\times10^{-2}) \times (0.1\times10^{-3}) \times 0.02$

$\Rightarrow \mathcal{E} = 16\pi \times 10^{-7}$

$\Rightarrow \mathcal{E} \approx 50 \times 10^{-7}$ Volts

$$\boxed{\mathcal{E} = 5\ \mu\text{Volts}}$$ ✓

Q4. $\left[\frac{L}{RCV}\right] = \left[\frac{[ML^2T^{-2}A^{-2}]}{[T][ML^2T^{-3}A^{-1}]}\right]$ $\qquad [RC] = [\tau] = T$

$\Rightarrow \left[\frac{L}{RCV}\right] = M^0L^0T^0A^{-1}$

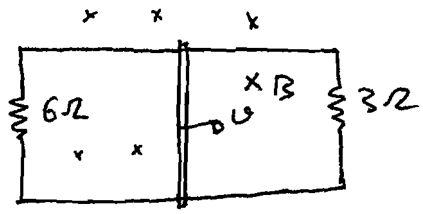
Q5

$l = 1.0\,m$

$B = 2.0\,T$

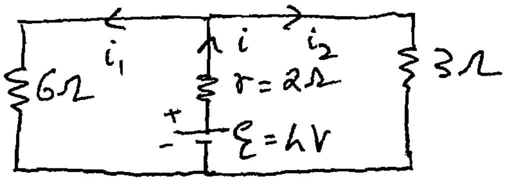
$r = 2\Omega$, $v = 2\,m/s$

$F = ?$

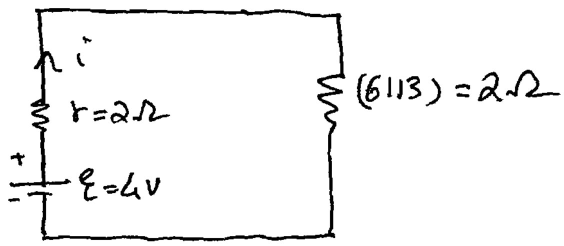


The moving rod will develop an induced ~~volt~~ voltage of $\mathcal{E} = Blv = 2 \times 1 \times 2 = 4$ volts.

It can therefore be replaced with a d.c. source of $\mathcal{E} = 4V$ ~~and~~ in series with a 2Ω resistor in the equivalent circuit diagram. (with polarity as per right-hand rule)



which can be further simplified to



Therefore $i = \frac{\mathcal{E}}{R_{eq}} = \frac{4V}{4\Omega} = 1$ Amp.

Now, the magnetic force $F_B = ilB = 1 \times 1 \times 2 = 2$ Newtons acting on the moving rod due to the magnetic field acting on the current will have to be balanced with an external force $F = 2N$ to keep the rod in uniform motion

$= 1 \times 2$

Q6. The magnetic flux at the center of the two coils due to an instantaneous current 'i' in the outer coil.

$$B = \frac{\mu_0 i}{2b}$$

Since $a \ll b$, the magnetic flux through the smaller one, $\phi_B \simeq B \times \pi a^2 = \frac{\mu_0 i \pi a^2}{2b}$

$\therefore$ induced voltage $\mathcal{E} = \frac{d\phi_B}{dt} = \frac{\mu_0 \pi a^2}{2b}\left(\frac{di}{dt}\right)$

$\therefore$ induced current $i_{ind} = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi a^2}{2bR}\left(\frac{di}{dt}\right)$

$\therefore$ charge $\Delta Q = \int i_{ind}\, dt = \int \frac{\mu_0 \pi a^2}{2bR}\left(\frac{di}{dt}\right) dt$

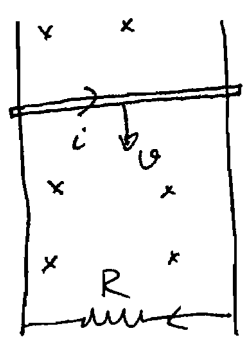
$$= \frac{\mu_0 \pi a^2}{2bR} \int di$$

$$= \frac{\mu_0 \pi a^2 \Delta i}{2bR}$$

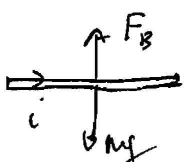
$$\boxed{\Delta Q = \frac{\mu_0 \pi a^2 i}{2bR}}$$

(as current changes from '0' to 'i')

Q7. At some instant $t=t$ when the rod has achieved its terminal velocity 'v', the ~~acc~~ net force on it should be zero $\Rightarrow$ $F_B = mg \Rightarrow iLB = mg$



$t=t$, $a=0$



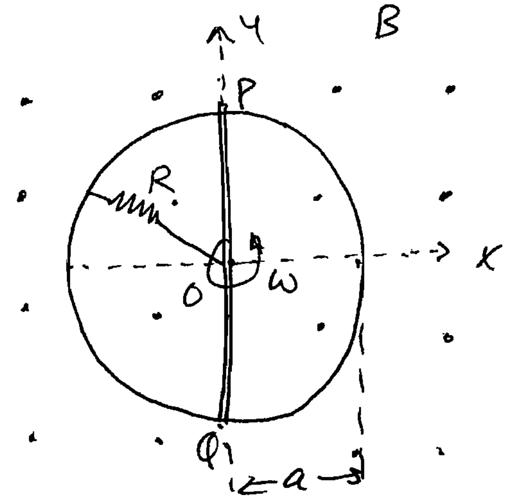
Now, $i = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$ (motional emf) $\Rightarrow \frac{B^2 l^2 v}{R} = mg \Rightarrow \boxed{v = \frac{mgR}{B^2 l^2}}$

Q8. Q8. ✓

$\vec{B} = 50\hat{k}$ (T)

$\omega = 20$ (rad/s)

$R = 10\,\Omega$, $a = 0.1\,m$

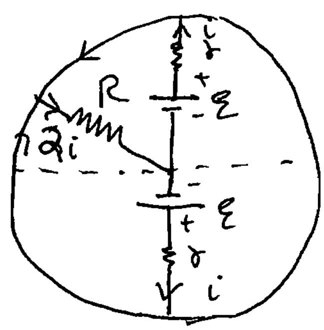


The rotation of the metallic rod attached along the diameter PQ will create motion generated emf such that,

$$(V_P - V_O) = +\frac{B\omega a^2}{2}$$

and $$(V_Q - V_O) = +\frac{B\omega a^2}{2}$$

Therefore in an equivalent circuit, two voltage sources $\varepsilon = \frac{B\omega a^2}{2} = \frac{50\times 20\times (0.1)^2}{2} = 5$ Volts and two resistors $r = 10\,\Omega$ can be placed as shown below



$$\varepsilon - ir - 2iR = 0$$

$$\Rightarrow i = \frac{\varepsilon}{r+2R} = \frac{5}{10+20} = \frac{1}{6}\text{ Amp}$$

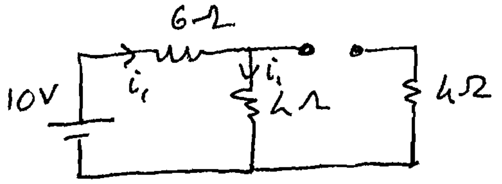
∴ the current through 'R',

$2i = \frac{1}{3}$ Amp $\simeq 0.33$ Amp

Q9.

$i_1 = i(t=0)$

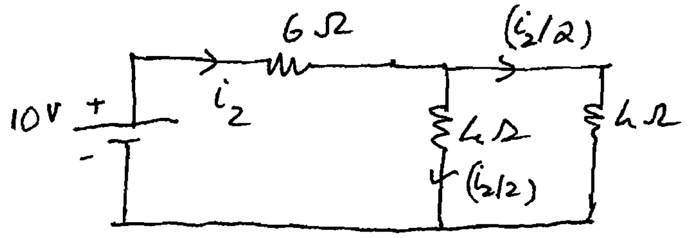
The equivalent circuit at time $t=0$



(inductor acts like an 'open' circuit)

$\Rightarrow \quad i_1 = \frac{10V}{10\Omega} = 1$ Amp.

@ $t \to \infty$ (steady state), the inductor will act like a 'short circuit'. Hence the equivalent ckt.



$i_2 = \frac{10V}{8\Omega} = 1.25$ Amp.

$\therefore \quad \boxed{i_1 : i_2 = 4:5}$ Ans

Q10

When the swith is closed in position (1) for a long time, the inductor stored potential energy $U = \frac{1}{2}Li^2$

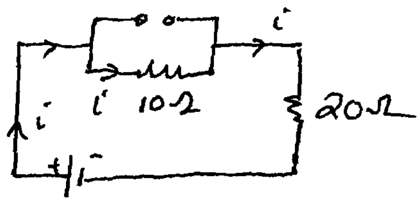
$\Rightarrow U = \frac{1}{2}L\left(\frac{\varepsilon}{R_1}\right)^2$

When the switch is moved to position (2) the d.c. source ε disengages and the inductor dissipates it's stored potential energy through R_2, therefore heat produced

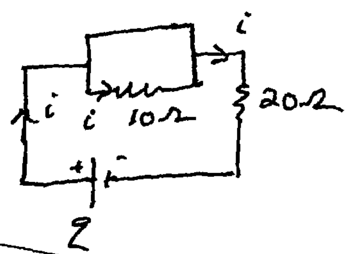
$$\boxed{H = U = \frac{1}{2}\frac{L\varepsilon^2}{R_1^2}}$$

Q11.

At $t=0$, "L" acts as an open-ckt. Therefore from the ~~Equivalent~~ Equivalent ckt diagram $i = \frac{2V}{30\Omega} = 0.067$ Amp.



At steady state, $t \to \infty$, "L" acts as a short-ckt. Therefore from Equivalent ckt diagram $i = \frac{2V}{20\Omega} = 0.1$ Amp.



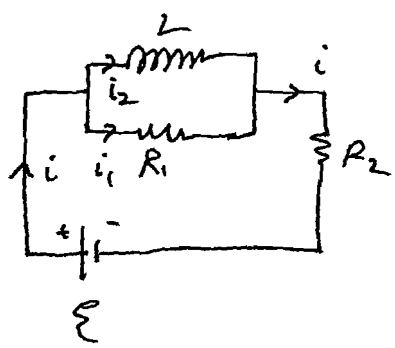
(OPTIONAL)

(To add part about time constant 'τ')

$$\mathcal{E} - L\frac{di_2}{dt} - iR_2 = 0 \quad \text{—— (I)}$$

$$\mathcal{E} - i_1R_1 - iR_2 = 0 \quad \text{—— (II)}$$

$$i = (i_1 + i_2) \quad \text{—— (III)}$$



$$\Rightarrow \quad L\frac{di_2}{dt} = i_1R_1$$

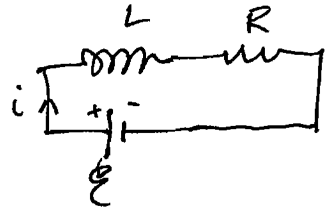
$$\Rightarrow \quad i = \left\{\frac{L}{R_1}\frac{di_2}{dt} + i_2\right\} \rightarrow \text{Substituting in Eq (I)}$$

$$\mathcal{E} - L\frac{di_2}{dt} - \left\{\frac{L}{R_1}\frac{di_2}{dt} + i_2\right\}R_2 = 0$$

$$\Rightarrow \quad \mathcal{E} - L\left(1 + \frac{R_2}{R_1}\right)\frac{di_2}{dt} - i_2R_2 = 0$$

$$\Rightarrow \quad \mathcal{E} - L'\left(\frac{di_2}{dt}\right) - R_2 i_2 = 0 \quad \text{—— (IV)}$$

Compare Eq (10) with a standard L-R (charging) circuit's transient equation where



$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$

$\tau = L/R$

and $\mathcal{E} - L\frac{di}{dt} - Ri = 0$

Since $\tau = L/R$

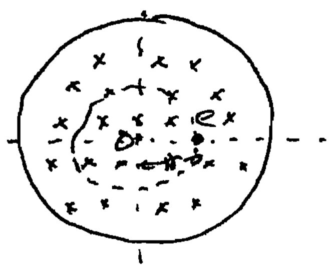
for our given circuit $\tau = L'/R_2 = L(1+\frac{R_2}{R_1})/R_2$

$10^{17} \times \left(\frac{30}{200}\right)$

$$\boxed{\tau = L\left(\frac{R_1+R_2}{R_1R_2}\right)}$$

$\tau \simeq 1.5$ secs

Q12



$B = kt$

By application of Faraday's Law over closed loop

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{r} = \left|-\frac{d\phi_B}{dt}\right| = \pi r^2 \frac{dB}{dt} =$$

$$\Rightarrow \quad E \times 2\pi r = \pi r^2 \times k$$

$$\Rightarrow \quad \boxed{E = \frac{kr}{2}}$$

E : Non-conservative, non-electrostatic electric field generated due to the time varying magnetic field. B

$\therefore$ just after it is released, the acceleration of the e^-, $a = \frac{eE}{m}$

$$\Rightarrow \boxed{a = \frac{ekr}{2m}}$$

Ans

Q13.

~~$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau}$~~

$$i(t) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \quad , \quad \mathcal{E}/R = 1.5 \text{ Amp}$$

$$\tau = L/R = 0.5 \text{ sec}$$

$$i(t \to \infty) = \mathcal{E}/R = 1.5 A$$

$$i(t=1) = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = 1.5(1 - e^{-1/0.5})$$

$$\boxed{i_1 = 1.5\left(1 - \frac{1}{e^2}\right)}$$

$$\therefore \quad \frac{i_\infty}{i_1} = \frac{1}{\left(1 - \frac{1}{e^2}\right)} = \left(\frac{e^2}{e^2 - 1}\right) \simeq 9/8 \text{ (Approx)}$$

Q14.

$B_0 = 0.08 T$

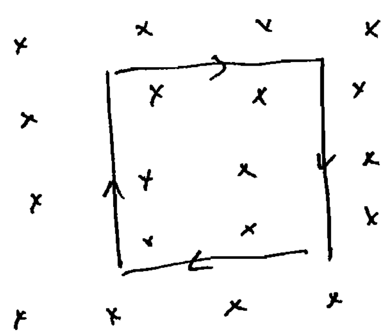
$A = 0.01 m^2 = (0.1 \times 0.1) m^2$

$-\frac{dB}{dt} = 3.0 \times 10^{-4} \ T/s \quad \therefore \quad B = B_0 - kt$

where $k = 3.0 \times 10^{-4}$

$\therefore$ Magnetic Flux $\Phi_B = BA = (B_0 - kt) A$

$\therefore$ induced voltage $\mathcal{E} = -\frac{d\Phi_B}{dt}$



$$\mathcal{E} = -\frac{d}{dt}(B_0 - kt)A$$

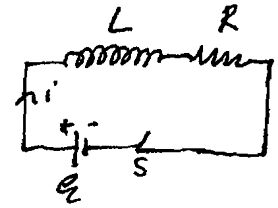
$$\Rightarrow \mathcal{E} = kA$$

$$\Rightarrow \mathcal{E} = 3.0 \times 10^{-4} (T/s) \times 0.01 (m^2)$$

$$\Rightarrow \boxed{\mathcal{E} = 3 \times 10^{-6} (W/s) \text{ or } V} = 3 \mu V$$

$\Rightarrow$ Orientation of the '$\mathcal{E}$' will be Clock-wise since the Φ_B associated with a field "into" the plane of the diagram is decreasing with time.

Q15.



$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$

$$\Rightarrow \int_0^Q dq = \int_0^\tau \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\,dt$$

$$\Rightarrow Q = \frac{\mathcal{E}}{R}\left[\tau - \tau(1 - \tfrac{1}{e})\right]$$

$$\Rightarrow \boxed{Q = \frac{\mathcal{E}}{R} \times \frac{\tau}{e} = \frac{\mathcal{E}L}{eR^2}}$$

Q16.

$U_i = \frac{1}{2}Li_0^2$ ~~when $U = \frac{1}{4}\cdot\frac{1}{2}Li_0^2$~~ ~~Energy dissipated through Resistor~~ ~~$H = U_i - U$~~ ~~$= \frac{3}{4}\cdot\frac{1}{2}Li_0^2$~~

Alt method $i = i_0 e^{-t/\tau}$

$\therefore$ @ some time $t = t$, $U = \frac{1}{2}Li_0^2 e^{-2t/\tau}$

$$\text{if } U = \frac{1}{4}U_i \Rightarrow e^{-2t/\tau} = \frac{1}{4}$$

$$\Rightarrow \frac{2t}{\tau} = \ln 4$$

$$\Rightarrow \boxed{t = \tau \ln 2}$$

~~Heat dissipation through resistor~~

~~$H = \int i^2 R\,dt = \int_0^\tau i_0^2 e^{-2t/\tau} R\,dt$~~

~~$= \ldots R$~~

therefore, charge flown through the resistor,

$$Q = \int_0^{t=\tau\ln 2} i\,dt = \int_0^{\tau\ln 2} \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\,dt$$

$$\Rightarrow Q = \frac{\mathcal{E}}{R}(t - \tau e^{-t/\tau})\Big|^{t=\tau\ln 2} = \frac{\mathcal{E}}{R} \times \tau(1 - e^{-2})$$

Q16.

Initially, the energy stored in the inductor,

$$U_i = \frac{1}{2} L i_0^2 ,$$

~~if at time~~

At time $t = t$, $i = i_0 e^{-t/\tau} \Rightarrow U = \frac{1}{2} L i_0^2 e^{-2t/\tau}$

$$\Rightarrow \text{ if } U = \frac{1}{4} U_i \Rightarrow e^{-2t/\tau} = (1/4)$$

$$\Rightarrow 2t/\tau = \ln 4 \Rightarrow \boxed{t = \tau \ln 2}$$

$$\Rightarrow \cancel{t = 2\ln 2}$$

∴ the charge flown through the resistor from $t=0$ to $t = \tau \ln 2$ $\quad Q = \int i\,dt = \int_{t=0}^{t=\tau\ln 2} i_0 e^{-t/\tau} dt$

$$= i_0 \left[\frac{e^{-t/\tau}}{-1/\tau} \right]_0^{\tau \ln 2}$$

$$= i_0 \tau \left[1 - e^{-\ln 2} \right]$$

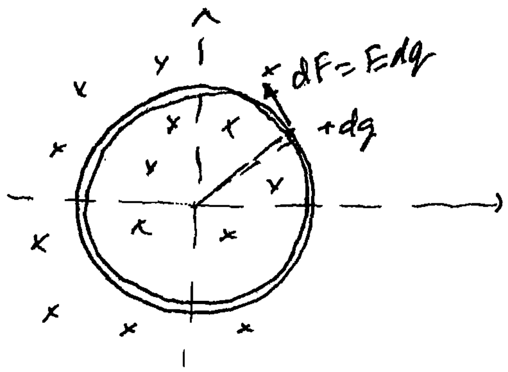
$$\boxed{Q = \frac{i_0 \tau}{2}} \Rightarrow \boxed{Q = \frac{L i_0}{2R}}$$

If $i_0 = \mathcal{E}/R$, & $\tau = L/R$ $\quad \boxed{Q = \frac{\mathcal{E} L}{2R^2}}$

Q17. The magnitude of the Electric Field (of circular geometry) (non-conservative, non-electrostatic) generated at any point on the circumference of the ring due to the time varying field B can be derived from $\oint \vec{E} \cdot d\vec{l} = \left| \frac{d\phi_B}{dt} \right| \Rightarrow E \times 2\pi R = \pi R^2 \times \frac{dB}{dt}$, $B = (0.2t)$

$$\Rightarrow E = \frac{R}{2} \times \frac{dB}{dt} = 0.2 \text{ (V/m)}$$

Now, considering a differential length element of length 'dx' and charge $dq = \lambda dx$ on the



$$d\tau = R\,dF = R\,dq\,E$$

$$\Rightarrow d\tau = R\lambda\,dx \times \frac{R}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow d\tau = \frac{R^2}{2}\lambda\left(\frac{dB}{dt}\right)dx$$

$$\Rightarrow \text{net torque } \tau = \int d\tau = \frac{R^2\lambda}{2}\left(\frac{dB}{dt}\right)\int_{x=0}^{x=2\pi R} dx$$

$$\Rightarrow \tau = \frac{R^2}{2}(\lambda \times 2\pi R)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \tau = \frac{R^2}{2} \times Q \times \left(\frac{dB}{dt}\right)$$

Therefore, it produces a uniform angular accel' equal to $\alpha = \frac{\tau}{I} = \frac{R^2/2\, Q\,(dB/dt)}{mR^2} = \frac{Q}{2m}\left(\frac{dB}{dt}\right)$

$\therefore$ angular speed after $t = \Delta t = 10\,\text{sec}$

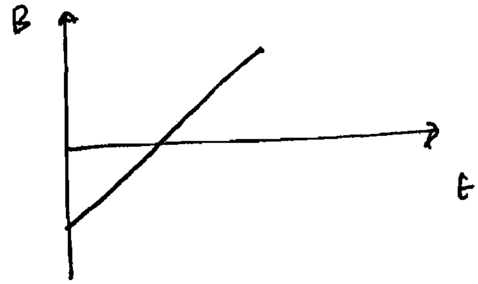
$$\Rightarrow \omega = \alpha\Delta t = \frac{Q}{2m}\left(\frac{dB}{dt}\right)\Delta t$$

$$\omega = \frac{2}{2 \times 50 \times 10^{-3}} \times (0.2) \times 10$$

$$\boxed{\omega = 40\ \text{rad/sec}}$$

Q18.

$B = (kt - c)$; $(0 \le t \le c/k)$



$$\mathcal{E} = A\frac{dB}{dt} = (\pi a^2 \times k)$$

$$i = \frac{\mathcal{E}}{R} = \frac{\pi a^2}{R}k \Rightarrow \text{charge } Q = \int i\,dt = \frac{\pi a^2}{R}k\int_{t=0}^{t=c/k} dt$$

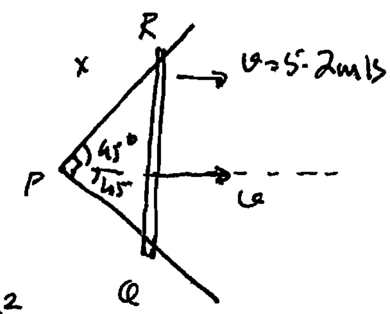
$$\boxed{Q = \frac{c\pi a^2}{R}}$$

Q19.

$B = 0.35\,T$

at $t = 0$, $s = (5.2 \times 3)$

$s = 15.6\,m$

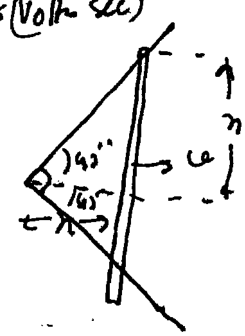


$\therefore$ Area of the ΔPQR

$$\text{Area} = 2 \times \left[\tfrac{1}{2} \times 15.6 \times 15.6\right] m^2$$

$$\therefore \phi_B = B \times \text{Area} = 0.35 \times (15.6)^2 = 85.18 \text{ (Volt sec)}$$

$\cancel{\mathcal{E} = \left(\frac{d\phi_B}{dt}\right)}$



At a general time 't'

$x = vt$

$$\therefore \text{Area} = 2 \times \left(\tfrac{1}{2} \times x^2\right) = x^2$$

$$\therefore \phi_B = B \times x^2$$

$$\therefore \text{induced emf. } \mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = B \times 2x\frac{dx}{dt}$$

$$= B \times 2x \times v$$

$$= 0.35 \times 2 \times 15.6 \times 5.2$$

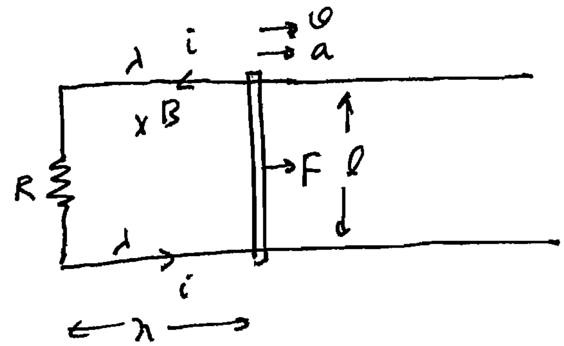
$$\mathcal{E} = 56.78 \text{ Volts}$$

$\mathcal{E} = 2Bxv \quad , \quad x = vt$

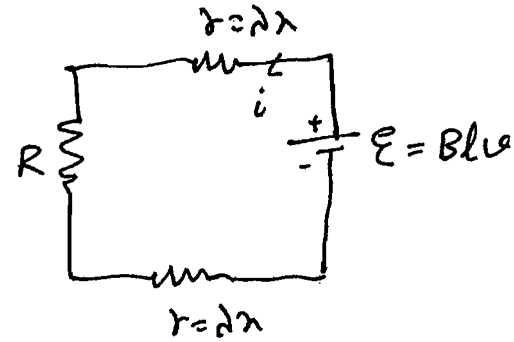
$\Rightarrow \mathcal{E} = (2Bv^2)t$

$\therefore$ '$\mathcal{E}$' varies "linearly" w.r.t 'time'

Q.20.



As the rod moves with velocity 'v', and acceleration 'a', the equivalent circuit diagram for any instant $t=t$ when $x=x$ is as shown



now $\mathcal{E} - i(2r+R) = 0$

$$\Rightarrow i = \frac{\mathcal{E}}{(2r+R)}$$

$r = \lambda x$, Since 'i' is constant

$$\Rightarrow \left(\frac{\mathcal{E}}{2\lambda x+R}\right) = i \quad \Rightarrow \quad \frac{Blv}{2\lambda x+R} = i = \text{Constant} \quad \Rightarrow \quad \boxed{v = \frac{(2\lambda x+R)\,i}{Bl}}$$

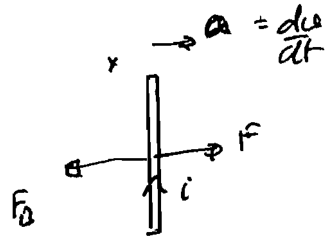
$$\Rightarrow Bl\left(\frac{dx}{dt}\right) = (2\lambda x+R)\,i$$

To determine the relation between the velocity $v = \frac{dx}{dt}$ and displacement 'x', first integrate the above equation to get 'x' as a function of time.

$\cancel{\left(Bl\frac{dx}{dt}\right) = (2\lambda x+R)\,i \times dx}$

$$\int_{x=x_0}^{x=x} \frac{dx}{(2\lambda x+R)} = \int_{t=0}^{t=t} \frac{i}{Bl}\,dt$$

$$\Rightarrow \ln(2\lambda x+R) = \frac{i}{Bl}\,t$$



from the FBD above $F - F_B = m\left(\frac{dv}{dt}\right)$

$$\Rightarrow F = F_B + m\left(\frac{dv}{dt}\right)$$

$$= ilB + m\frac{d}{dt}\left\{\frac{(2\lambda x + R)i}{Bl}\right\}$$

$$= ilB + m \times \frac{2\lambda}{Bl}\left(\frac{dx}{dt}\right)$$

$$= ilB + \frac{2m\lambda v}{Bl}$$

$$= ilB + \frac{2m\lambda i}{Bl}\left\{\frac{(2\lambda x + R)i}{Bl}\right\}$$

$$\Rightarrow \boxed{F = \frac{2m\lambda i^2(2\lambda x + R)}{B^2 l^2} + ilB}$$

Q21. (i) $\vec{B} = (B_0 + 3t)\hat{i}$, $r = 0.1\text{m}$

For the given loop, the total flux

$$\phi_B = \left(B \times \frac{\pi r^2}{4}\right)$$

(This field will produce flux only through the quadrant (quarter circle) lying in the Y-Z plane as it is "tangential" to the surfaces of the other 2 quadrants)

$$\therefore \ \mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \frac{\pi r^2}{4} \times \frac{dB}{dt} = \left\{\pi\frac{(0.1)^2}{4} \times 3 \times 10^{-3}\right\} V = 0.024 \times 10^{-3}$$

$$\boxed{\mathcal{E} = 2.4 \times 10^{-5}\ V}$$

(ii) Since the ϕ_B for $\vec{B}$ along the x-axis increases with time, ~~loo~~ by application of Lenz's Law the induced current in the loop will be "Clockwise" i.e. along cbac.

Q.22.

(i) $B = 10\,\mu T$

$l = 3.0\,m$

$F = 10 \times 10^3\,N$

$\therefore\ F = ilB \Rightarrow 10^4 = 10 \times 10^{-6} \times 3 \times i$

$\therefore\ i = \left(\frac{10^9}{3}\right)$ Amp

$\boxed{i \simeq 3.3 \times 10^8 \text{ Amp}}$

(ii) Power dissipated $= i^2R$, $R = 1\Omega$

$= (3.3 \times 10^8)^2 \times 1 \approx 10^{17}$ watts.

(iii) Not realistic

Q.23.

(i) induced voltage $\mathcal{E} = \frac{d\Phi_B}{dt} = \left|\frac{1}{2} \times (2m)^2 \times \left(\frac{dB}{dt}\right)\right|$, $B = (0.042 - 0.87t)\,T$

$\therefore\ \mathcal{E} = (2 \times 0.87) = 1.74$ Volts.

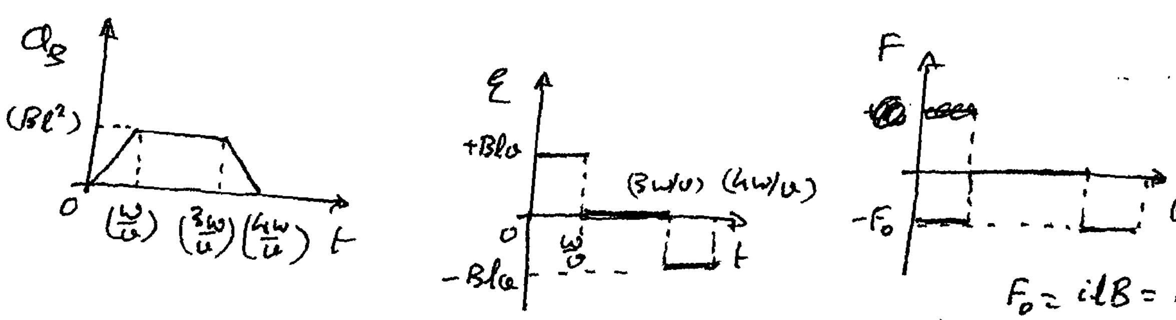
Now, by application of lenz's law, ~~it is~~ it can be shown that the induced voltage will ~~create~~ lead to create an anti-clockwise induced current (same orientation as the battery)

Therefore, net voltage $= 1.74 + 20 = 21.74$ Volts.

(ii) Through the battery (inside it) charge will flow from -ve terminal to +ve.

~~Out~~ In the circuit it will be anti-clockwise.

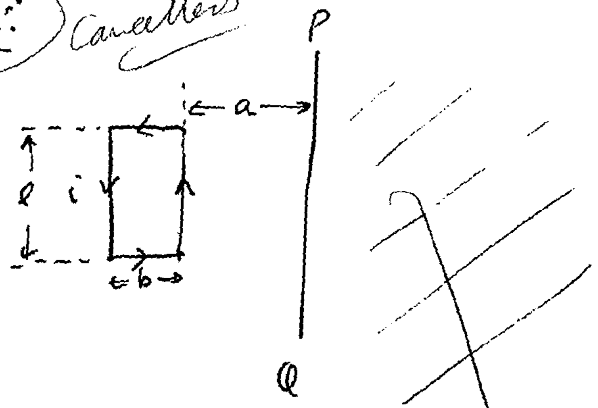
Q24.



$F_0 = ilB = \frac{B^2l^2v}{R}$

-ve : force toward

Q25. Cancelled



Flux through loop in shaded region.

Use superposition. Place infinite straight conductor along PQ with current i'. Find the flux due to this through the loop.

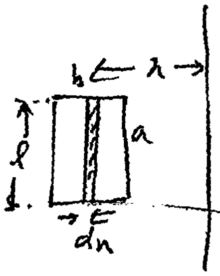
$$\phi = M_{12} i' = M_{21} i = \phi$$

Flux through the loop

↳ Flux through the shaded region.

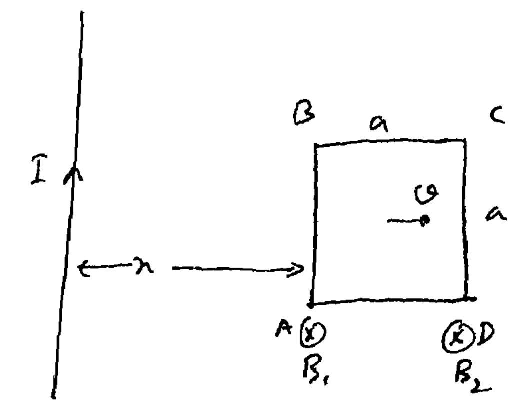
(if $i = i'$)

$\therefore \quad \phi = \frac{\mu_0 i}{2\pi}$



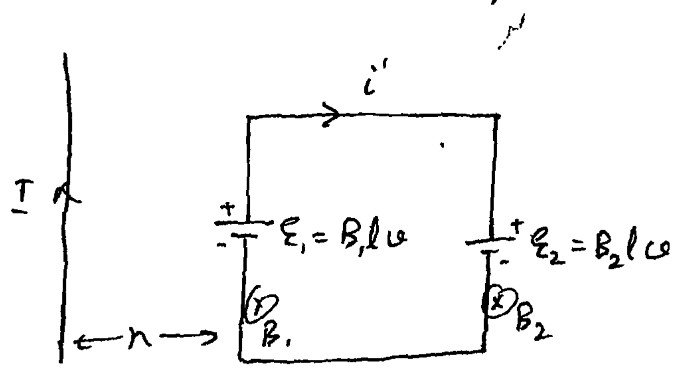
25.

Q.26



Resistance R
side length a

As the loop moves away there is an induced current flowing (clockwise) through it due to the induced voltage from EMI. The instantaneous equivalent circuit diagram is as shown



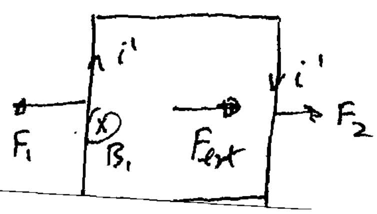
induced current $i' = \left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R}\right) = \frac{(B_1 - B_2) l v}{R}$

where $B_1 = \frac{\mu_0 I}{2\pi x}$

$B_2 = \frac{\mu_0 I}{2\pi(x+a)}$

$$\therefore \quad i' = \frac{\mu_0 I l v}{2\pi R}\left[\frac{1}{x} - \frac{1}{(x+a)}\right], \quad \frac{dx}{dt} = v$$

Therefore magnetic forces ~~Fext~~ $F_1 = i' l B_1$ and $F_2 = i' l B_2$ have to be balanced by the external force F_{ext} to keep the loop moving with a uniform velocity 'v'.



$\Rightarrow \quad F_{ext} = (F_1 - F_2)$

$\Rightarrow \quad F_{ext} = i' l (B_1 - B_2)$

$$\Rightarrow F_{ext} = \frac{\mu_0^2 I^2}{4\pi^2} \frac{l^2 v^2}{R} \left\{ \frac{1}{x} - \frac{1}{(x+a)} \right\}^2$$

$$\therefore \text{Workdone } W = \int F_{ext}\, dx = \int_{x=a}^{x=2a} \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \left\{ \frac{1}{x} - \frac{1}{(x+a)} \right\}^2 dx$$

$$\Rightarrow W = \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \int_{x=a}^{x=2a} \left\{ \frac{1}{x^2} + \frac{1}{(x+a)^2} - \frac{2}{x(x+a)} \right\} dx$$

$$\Rightarrow W = \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \int_{x=a}^{x=2a} \left\{ \frac{1}{x^2} + \frac{1}{(x+a)^2} - 2\left(\frac{1}{ax} - \frac{1}{a(x+a)} \right) \right\} dx$$

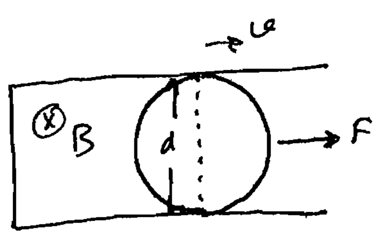
$$\Rightarrow W = \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \left[-\frac{1}{x} - \frac{1}{(x+a)} - \frac{2}{a} \ln\left(\frac{x}{a+x}\right) \right]_{x=a}^{x=2a}$$

$$\Rightarrow W = \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \left[\frac{1}{2a} + \frac{1}{6a} - \frac{2}{a} \ln(4/3) \right]$$

$$\Rightarrow W = \frac{\mu_0^2 I^2 l^2 v^2}{4\pi^2 R} \left[\frac{2}{3a} - \frac{2}{a} \ln(4/3) \right]$$

$$\Rightarrow \boxed{W = \frac{\mu_0^2 I^2 l^2 v^2}{2\pi^2 R a} \left\{ \frac{1}{3} - \ln(4/3) \right\}}$$

Q 27.



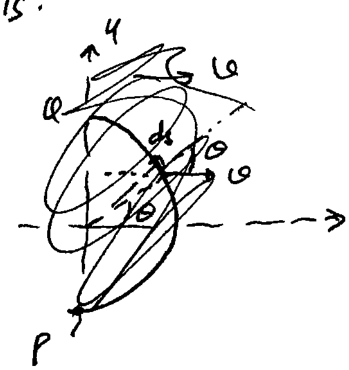
$F = ilB$, $l = d$, $i = \frac{Blv}{R} = \frac{Bdv}{R}$, $R = \frac{\lambda}{2} \times \left(\pi \frac{d}{2}\right)$

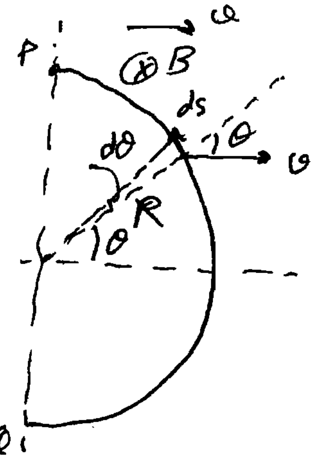
$$\Rightarrow F = \frac{B^2 d^2 v}{R} = \frac{4 B^2 d^2 v}{\pi \lambda d}$$

27 Alternate method

To calculate the induced voltage across the two-semi circular metallic sections between the two contact points on the parallel rails.

The voltage across the diff section of the semi-circle





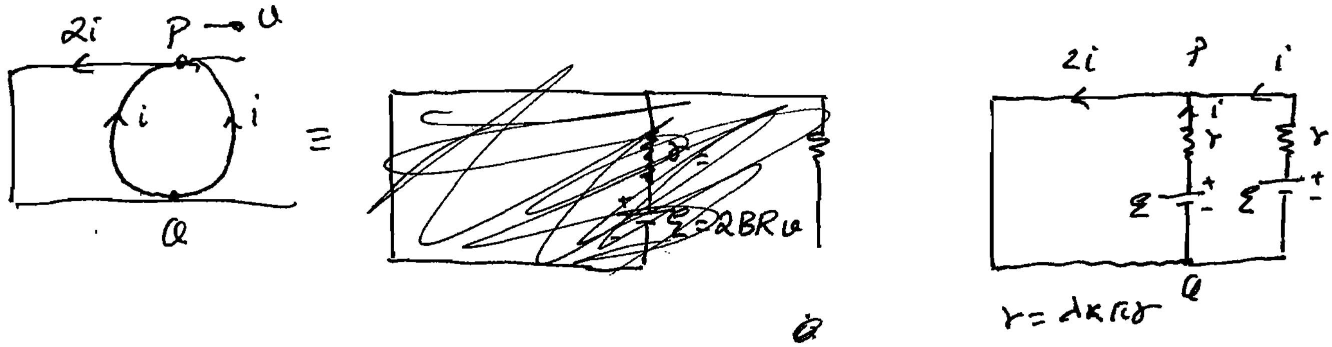
$ds = Rd\theta$,

$d\mathcal{E} = v\cos\theta\, B\, Rd\theta$

$\therefore$ net induced voltage $(V_P - V_Q) = \mathcal{E} = \int_{\theta=\theta-\pi/2}^{\theta=\pi/2} vBR\cos\theta\, d\theta$

$$\Rightarrow \boxed{\mathcal{E} = 2vBR}$$

Now, since there are two such semi-circles (part of the ring) in the actual circuit connecting points P and Q, the equivalent ~~ckt diag~~ circuit diagram is as follows.



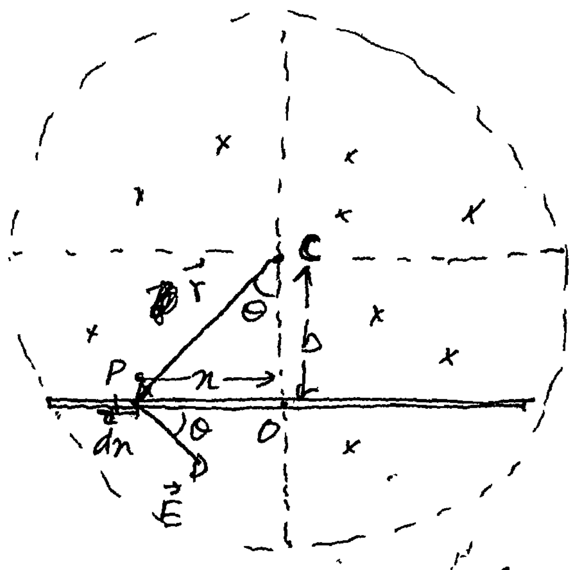
take $r = (d/2)$ $\therefore$ $\mathcal{E} = vBd$, $r = \lambda\frac{\pi d}{2}$

Therefore the current through each of the semi-circular branches $i = \mathcal{E}/r = \frac{2vB}{\pi\lambda}$

Now each of the branches will experience a magnetic force $F_B = idB = \frac{2vB^2d}{\pi\lambda}$

Therefore the external Force needed to balance these $F_{ext} = 2F_B \Rightarrow \boxed{F_{ext} = \frac{4vB^2d}{\pi\lambda}}$ ✓

Q28.



At any point 'P' on the rod, the distance from 'P' to the mid-point of the rod ~~being~~ 'O' being $PO = x$ and the distance from the cylinder's axis C being $PC = r$ the Electric field $E = \frac{r}{2}\left(\frac{dB}{dt}\right)$ {from app. of Faraday's law and direction perpendicular to $\vec{r}$ as shown. $\oint \vec{E}\cdot d\vec{l} = \frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$ }

Therefore for a diff' section 'dx', induced emf.

$d\mathcal{E} = |\vec{E}\cdot d\vec{x}| = E\,dx\cos\theta = \frac{r}{2}\left(\frac{dB}{dt}\right) \times dx \times \frac{b}{r} \Rightarrow d\mathcal{E} = \frac{b}{2}\left(\frac{dB}{dt}\right)dx$

Therefore emf induced $\mathcal{E} = \int_{x=(-l/2)}^{x=(+l/2)} \frac{b}{2}\left(\frac{dB}{dt}\right)dx = \frac{b}{2}\left(\frac{dB}{dt}\right)l$

Since $b = \sqrt{R^2 - \frac{l^2}{4}}$

$$\Rightarrow \boxed{\mathcal{E} = \frac{l}{2}\left(\frac{dB}{dt}\right)\sqrt{R^2 - \frac{l^2}{4}}}$$

PTO

Q29. REMOVED

The net EMF $\mathcal{E}$ across closed conducting loop ABCDA will be uniformly distributed over the circumference therefore

$$\mathcal{E}_{ABC} = \mathcal{E} \times \left(\frac{\text{length of sector ABC}}{\text{circumference}}\right)$$

and $$\mathcal{E}_{CDA} = \mathcal{E} \times \left(\frac{\text{length of sector CDA}}{\text{circumference}}\right)$$

$$\therefore \quad \mathcal{E}_{ABC} + \mathcal{E}_{CDA} = \mathcal{E}$$

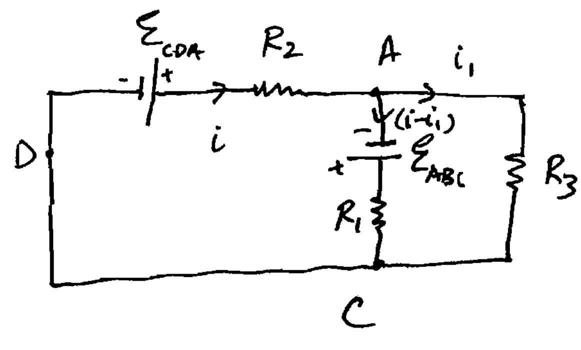
and $\mathcal{E}_{ABC} : \mathcal{E}_{CDA} = (\text{length of sector ABC}) : (\text{sector CDA})$

$$\Rightarrow \quad \mathcal{E}_{ABC} : \mathcal{E}_{CDA} = R_1 : R_2 \qquad \left(\text{Resistance} = \frac{\rho l}{A}\right)$$

$$\therefore \quad \mathcal{E}_{ABC} = \left(\frac{R_1}{R_1+R_2}\right)\mathcal{E}$$

$$\mathcal{E}_{CDA} = \left(\frac{R_2}{R_1+R_2}\right)\mathcal{E}$$

$\therefore$ Equivalent ckt diagram



$\Rightarrow \quad i_1 = ?$

$$\mathcal{E}_{CDA} - iR_2 - i_1R_3 = 0$$

$$\mathcal{E}_{ABC} - (i-i_1)R_1 + i_1R_3 = 0$$

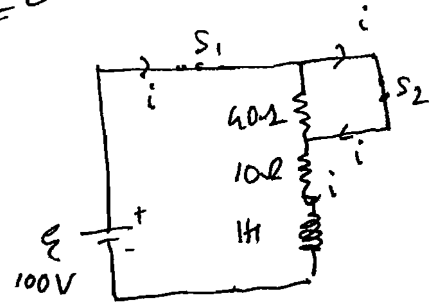
$$\Rightarrow \quad \frac{\mathcal{E}_{CDA} - i_1R_3}{R_2} = \frac{\mathcal{E}_{ABC} + i_1R_1 + i_1R_3}{R_1}$$

$$\Rightarrow \quad i_1 = \frac{(R_1\mathcal{E}_{CDA} - R_2\mathcal{E}_{ABC})}{(R_1R_2 + R_2R_3 + R_1R_3)} = 0$$

Question Cancelled

~~Q30~~ for $t=0$ to $t=0.1\ln(2)$

Q29.



$R_{eq} = 10\,\Omega$

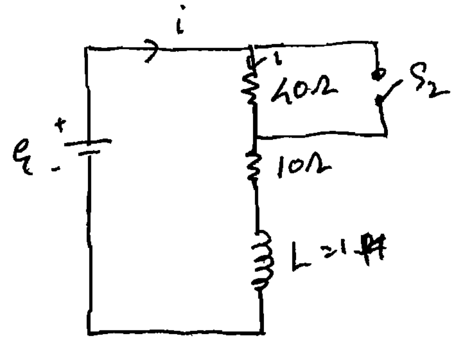
$\therefore \tau = L/R = 1H/10\Omega = 0.1\,sec$

$\therefore \quad i = \frac{\varepsilon}{R}(1-e^{-t/\tau}) = 10(1-e^{-t/0.1 sec})$

$\therefore$ @ $t = 0.1\ln(2)$ sec

$i = 10(1-e^{-\ln 2}) \Rightarrow i = 5\,Amp$

Now @ $t=0.1\ln(2)$ the switch S_2 is opened. Therefore at some time $t > 0.1\ln 2$



$\varepsilon - iR - L\frac{di}{dt} = 0$ where $R = 50\,\Omega$, $L = 1H$

$$\Rightarrow \int_{i=5\,Amp}^{i=?} \frac{di}{(\varepsilon - iR)} = \int_{t=0.1\ln(2)}^{t=0.2\ln(2)} \frac{dt}{L}$$

$$\Rightarrow -\frac{1}{R}\ln(\varepsilon - iR)\Big|_{5}^{i} = \frac{t}{L}\Big|_{0.1\ln(2)}^{0.2\ln(2)}$$

$$\Rightarrow \ln\left(\frac{\varepsilon - iR}{\quad}\right) = -R \times 0.1\ln(2)$$

$\Rightarrow$ $\cancel{\left(\frac{100-50i}{100-250}\right) = \left(\frac{150}{1}\right)}$

$\Rightarrow \left(\frac{\mathcal{E} - iR}{\mathcal{E} - 5R}\right) = e^{-\{R/L \times 0.1 \ln(2)\}}$

$\Rightarrow \left(\frac{100 - 50i}{100 - 250}\right) = e^{-\{\frac{50}{1} \times 0.1 \ln(2)\}}$

$\Rightarrow \left(\frac{100 - 50i}{-150}\right) = e^{-5\ln(2)}$

$\Rightarrow \frac{100 - 50i}{-150} = \left(\frac{1}{2^5}\right)$ $\qquad 2 + \frac{3}{32}$

$\Rightarrow i = \frac{\left[100 + \left(\frac{150}{32}\right)\right]}{50} \Rightarrow \boxed{i = 2.09 \text{ Amp}}$ or $i = \left(2 + \frac{3}{32}\right)$

$\boxed{i = \left(\frac{67}{32}\right)}$ Amp ✓

Q31. $\longrightarrow$ ~~REMOVE OR REPLACE (Repeated Concept)~~

Q32 $\longrightarrow$ FIGURE MISSING $\longrightarrow$ REMOVE or REPLACE

Q30. For the loop $V - L\frac{di}{dt} = 0$

$\Rightarrow \frac{di}{dt} = \frac{(2t+3t^2)}{0.5} \Rightarrow i = 2\{t^2 + t^3\}$

Now i increases monotonically, therefore

$\Rightarrow i_{max} = i(t=2) = 2(2^2 + 2^3) = 24$ Amps. ✓

and $U_L = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.5 \times 24^2 = 144$ Joules ✓

Q31.

For the half of the circular loop XY plane, ~~...~~ area vector $\vec{A_1} = \left(\frac{\pi a^2}{2}\right)\hat{k}$ and for the half bent at an angle of 60° to the horizontal (X-Y Plane), $\vec{A_2} = \frac{\pi a^2}{2}\left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{k}\right)$

~~Therefore sing~~ Therefore since the magnetic field is uniform ~~...~~ in space and given by $\vec{B} = (B_0 t)\hat{k}$, the total flux, $\phi_B = \vec{B}\cdot(\vec{A_1} + \vec{A_2}) = \left(B_0 t \frac{\pi a^2}{2}\right) - \left(B_0 t \frac{\pi a^2}{4}\right)$

$$\Rightarrow \phi_B = \frac{1}{4} B_0 t \pi a^2$$

$\therefore$ Emf induced $\boxed{\mathcal{E} = \left|-\frac{d\phi_B}{dt}\right| = \frac{1}{4} B_0 \pi a^2}$

Now, total resistance of the loop $R = r_0 \times 2\pi a$

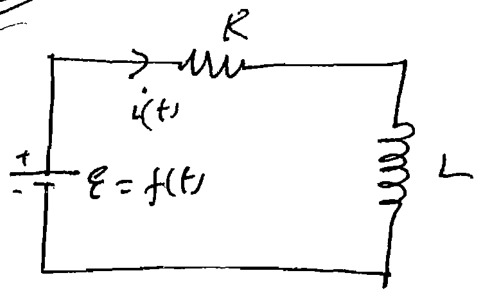
$\therefore$ charge flowing through $\Delta Q = i\Delta t$

$$= \frac{\mathcal{E}}{R}\Delta t$$

$$\Rightarrow \boxed{\Delta Q = \frac{B_0 a t}{8 r_0}}$$

~~directi~~ direction of induced current : RQPSR

~~Q32.~~ Q33



$i = 3+5t$
$R = 4\,\Omega$
$L = 6\,H$
$\mathcal{E} = ?$

By application of Kirchoff's loop law,

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

$$\Rightarrow \mathcal{E} - \{(3+5t)\times 4\} - \{6\times \frac{d}{dt}(3+5t)\} = 0$$

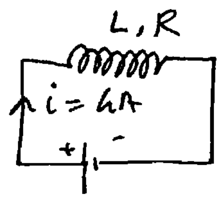
$$\Rightarrow \mathcal{E} - (12+20t) - 30 = 0$$

$$\Rightarrow \boxed{\mathcal{E} = (42+20t)\ \text{Volts.}}$$

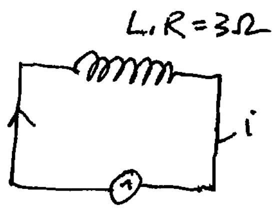
Ans

Q~~34~~. → SHIFT TO AC CIRCUITS → Q4 AC Ckts Ex3

(a)



$\Rightarrow$ Resistance of the coil $R = \frac{12\,V}{4\,Am} = 3\,\Omega$



$V = V_0 \sin(\omega t)$
$V_0 = 12V$
$\omega = 50$ rad/s

$i = i_0 \sin(\omega t + \phi)$

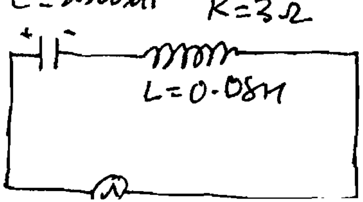
$$i_0 = \frac{V_0}{\sqrt{(\omega L)^2 + R^2}} \Rightarrow 2.4 = \frac{12}{\sqrt{(50L)^2 + 9}}$$

$$\Rightarrow (50L)^2 + 9 = 25$$

$$\Rightarrow 50L = \sqrt{16}$$

$$\Rightarrow L = 4/50 = 0.08\,H$$

(b)



$$\bar{P} = \frac{1}{2} V_0 i_0 \cos\phi$$

$$i_0 = \frac{V_0}{Z}, \quad Z = \sqrt{(X_c - X_L)^2 + R^2}$$

$$= \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$= \sqrt{(4-8)^2 + 3^2}$$

$$Z = \sqrt{16+9} = 5$$

$$\frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = \frac{10^6}{125 \times 10^3} = \frac{1000}{125} = 8$$

$$\therefore \; i_0 = \frac{V_0}{Z} = \frac{12}{5} = 2.4 \text{ Amp.}$$

$$\cos\phi = \frac{R}{Z} = \frac{3}{5}$$

$$\therefore \; \bar{P} = \frac{1}{2} \times 12 \times 2.4 \times \frac{3}{5}$$

$$\boxed{\bar{P} = 8.64 \text{ Watts}}$$

$$12 \times \frac{12}{2} \times \frac{3}{5}$$

$$\frac{144 \times 3}{10}$$

$$\frac{432}{10} \quad \frac{1}{2} \times \frac{12^2}{5} \times \frac{3}{5}$$

$$\frac{432 \times 2}{100}$$

Note:- Question might have assumed $V_{rms} = 12V \;\; \therefore \; i_{rms} = \frac{V_{rms}}{Z} = 2.4$ Amp

$$\therefore \; \text{Avg. Power} = \bar{P} = V_{rms} i_{rms} \cos\phi$$

$$= 12 \times 2.4 \times \frac{3}{5}$$

$$\Rightarrow \boxed{\bar{P} = 17.28 \text{ Watts}}$$

Q3. → (SHIFT TO AC circuits) → Q5 AC Ckts Ex3

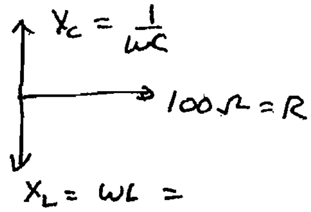
$R = 100\,\Omega$

$V_{rms} = 200$ Volts $\Rightarrow V_0 = 200\sqrt{2}$ Volts.

$\omega = 300$ rad/s

$C = ?$

$L = ?$



when $C = 0$, $\phi = -60°$

$$\Rightarrow \tan 60° = \frac{|X_L|}{R} \Rightarrow \frac{\omega L}{R} = \sqrt{3} \Rightarrow L = \frac{\sqrt{3}\times 100}{300} = 0.57\,H$$

$$\Rightarrow \boxed{L = 0.57\,H}$$

when $L = 0$, $\phi = +60°$

$$\Rightarrow \tan 60° = \frac{X_C}{R} \Rightarrow \frac{(1/\omega C)}{R} = \sqrt{3}$$

$\frac{\sqrt{3}}{9}$ $\frac{1.73}{9}$

$$\Rightarrow C = \frac{1}{\sqrt{3}R\omega} = \frac{1}{\sqrt{3}\times 3\times 10^4}$$

$$\Rightarrow C = 0.19\times 10^{-4}$$

$$\Rightarrow \boxed{C \simeq 19\,\mu F}$$

Since $X_C = X_L \Rightarrow$ Resonant Ckt (for the LCR circuit)

$Z = R = 100\,\Omega$

$\phi = 0 \Rightarrow \cos\phi = 1$, $i_{rms} = \frac{200}{100} = 2$ Amp

$$\therefore \bar{P} = V_{rms} i_{rms} = \frac{V_{rms}^2}{Z} = \frac{40000}{100}$$

$$\Rightarrow \boxed{\bar{P} = 400\text{ Watt}}$$

PTO

Q ~~36~~. → move to AC → $\underline{\text{Q6 AC Ckts Ex3}}$

$R = 120\,\Omega$

$2\pi f_0 = 4\times10^5$ rad/sec $\Rightarrow \frac{1}{\sqrt{LC}} = 4\times10^5$

$V_R = iR = 60$

$V_L = i\times\omega L = 40$

$\Rightarrow \omega L = \frac{2}{3}\times R = 80$

$\Rightarrow L = \frac{80}{4\times10^5} = 2\times10^{-4} \Rightarrow \boxed{L = 2\times10^{-4}\,H}$

$\therefore C = \frac{1}{(4\times10^5)^2}\times\frac{1}{(2\times10^{-4})} =$ ~~$\frac{10^{-6}}{32} = 0.125\,\mu F$~~ ~~$\Rightarrow C = 0.125\,\mu F$~~

$\Rightarrow C = \frac{10^{-10}\times10^4}{32}$

$\Rightarrow \boxed{C = \frac{1}{32}\,\mu F}$

for $\phi = -45^\circ$

$\left(\omega L - \frac{1}{\omega C}\right) = R$

$\Rightarrow \omega^2 LC - \omega RC - 1 = 0$

$\Rightarrow \omega = \frac{RC \pm \sqrt{(RC)^2 + 4\times LC}}{2LC} =$ ~~120×10⁻⁶~~

$\Rightarrow \omega = \frac{\left(\frac{30}{8}\times10^{-6}\right) \pm \sqrt{\frac{900}{64}\times10^{-12} + \frac{1}{4}\times10^{-10}}}{2\times\frac{1}{16}\times10^{-10}}$

$\Rightarrow \omega = \frac{\frac{3}{8}\times10^{-5} \pm \sqrt{\left(\frac{9}{64}+\frac{1}{4}\right)\times10^{-10}}}{\frac{1}{8}\times10^{-10}}$

$\Rightarrow \omega = \left(\frac{3}{8} \pm \sqrt{\frac{25}{64}}\right)\frac{\times10^{-5}}{\frac{1}{8}\times10^{-10}} = (3\pm5)\times10^{+5}$

$RC = 120\times\frac{1}{\cancel{8}\,\cancel{32}}\times10^{-6}$ ($\cancel{30}$ 3.75)

$LC = \frac{1}{\cancel{32}}\times10^{-6}\times2\times10^{-4}$ (16)

$= \frac{1}{16}\times10^{-10}$