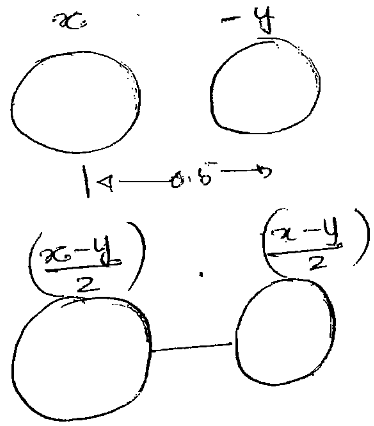
Electrostatics.

Ex-I

(1) 1. (B)



$$\frac{kxy}{(0.5)^2} = 0.108.$$

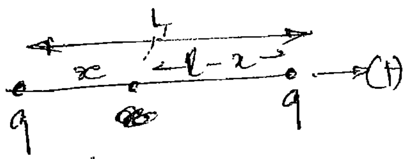
$$\frac{k(x-y)^2}{4(0.5)^2} = 0.036.$$

Eliminating k.

$$3x^2 + 3y^2 = 10xy.$$

Eliminate.

(2) 2. (d)

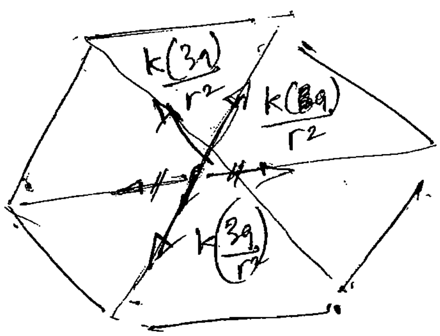


$$E = \frac{kq}{x^2} - \frac{kq}{(l-x)^2}$$

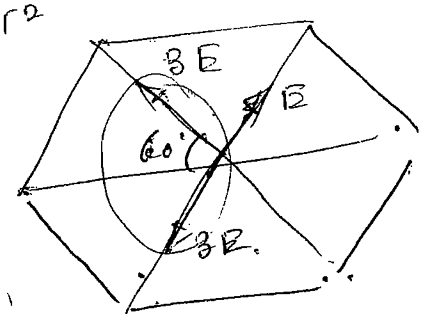
$$= \frac{kql(l-2x)}{[x(l-x)]^2}.$$

check the sign scheme of $l(l-2x)$ & note that E is undefined for ~~x=l~~ $x=0, l$.

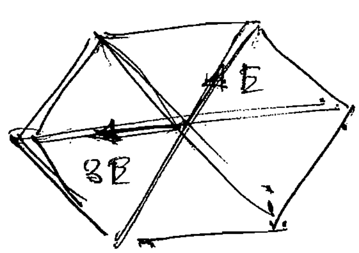
(3) 3. (D)



$$\frac{kq}{r^2} = E$$

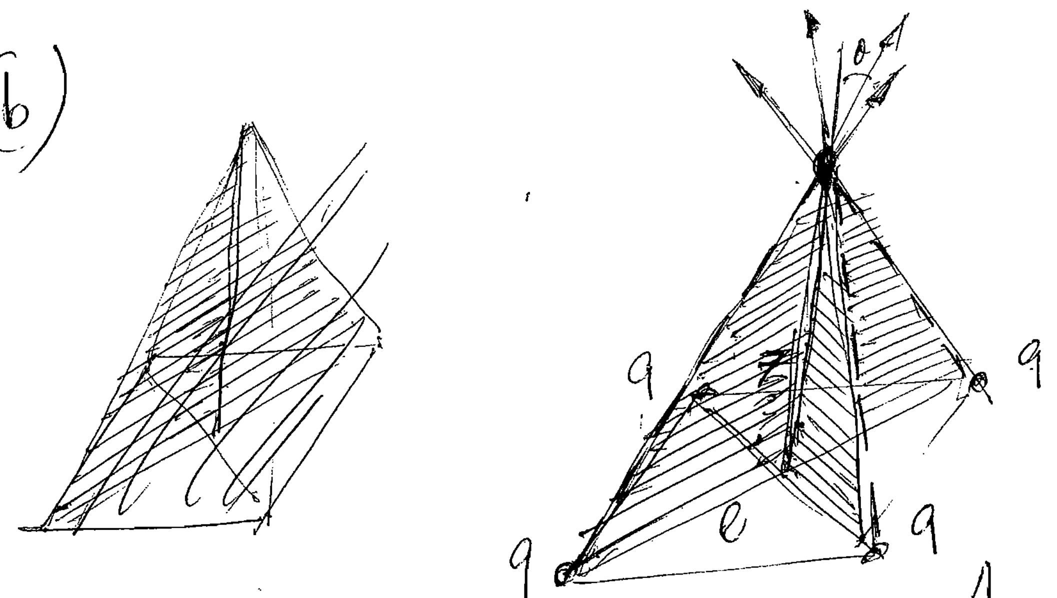


=



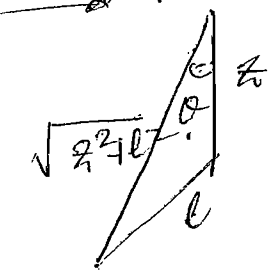
Resultant isn't along any of the given lines.

4 (b)



$$E = 4E_0 \cos\theta$$

$$= 4\frac{kq}{(z^2+l^2)}\frac{z}{\sqrt{z^2+l^2}}$$



$$= \frac{4kqz}{(z^2+l^2)^{3/2}}$$

$$\frac{dE}{dz} = \frac{4kq}{(z^2+l^2)^3}\left[(z^2+l^2)^{3/2}\cdot 1 - \frac{3}{\cancel{2}}(z^2+l^2)^{1/2}\cdot \cancel{2}z\cdot z\right]$$

$$= \frac{4kq}{(z^2+l^2)^3}\left[\sqrt{z^2+l^2}\,(z^2+l^2-3z^2)\right]$$

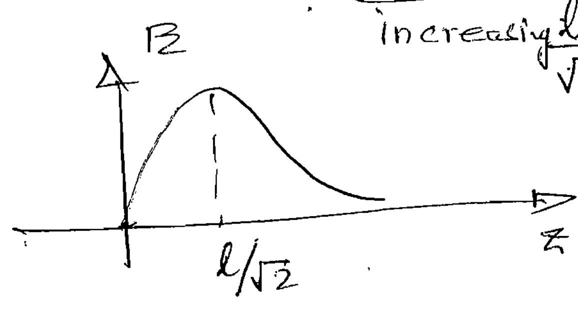
$$= \frac{4kq}{(z^2+l^2)^{5/2}}(l^2-2z^2)$$

sign scheme of $\frac{dE}{dz}$.

+ve -ve

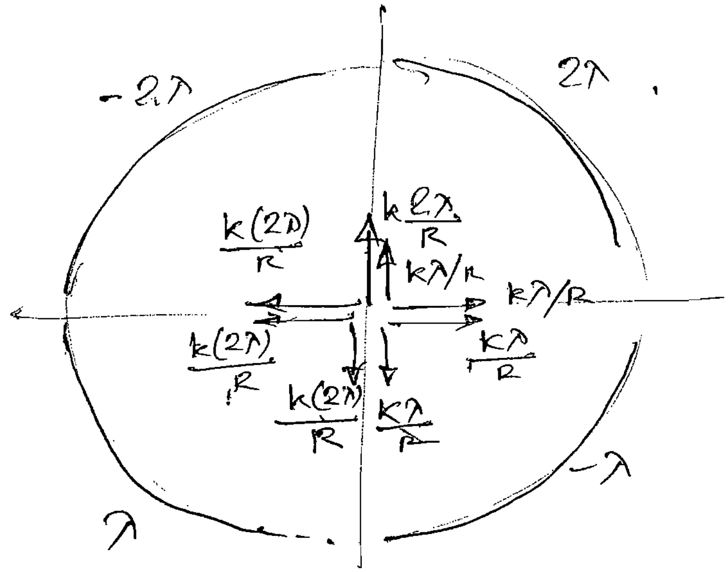
increasing $\frac{l}{\sqrt{2}}$. decreasing

E



5. (a)

(5)



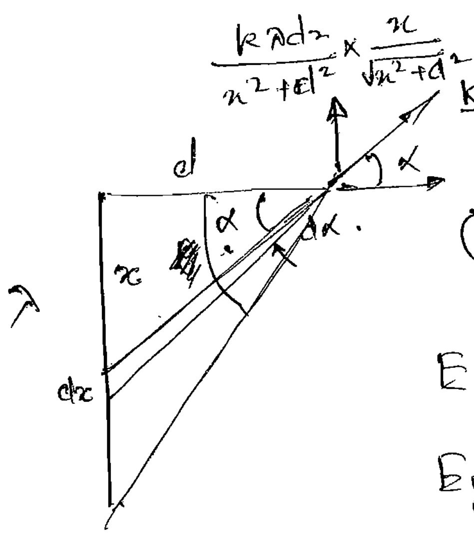
$$\frac{k(2\lambda)}{R}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R}$$

$$= \frac{\lambda}{2\pi\epsilon_0 R}.$$

6. (a)

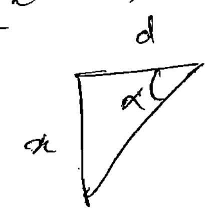
(6)



$$\frac{k\lambda dx}{(d^2+x^2)}\cos\alpha = \frac{k\lambda dx}{(d^2+x^2)}\frac{d}{\sqrt{d^2+x^2}}.$$

$$E_x = k\lambda d \int \frac{1}{(d^2+x^2)^{3/2}}dx.$$

$$E_y = k\lambda \int \frac{x}{(x^2+d^2)^{3/2}}dx.$$



$$\int \frac{1}{(d^2+x^2)^{3/2}}dx$$

$$= \int \frac{1}{d^3\sec^3\alpha}\, d\sec^2\alpha\, d\alpha.$$

$$\tan\alpha = \frac{x}{d}$$

$$x = d\tan\alpha.$$

$$dx = d\sec^2\alpha\, d\alpha.$$

$$= \frac{1}{d^2}\int_0^{60'} \cos\alpha\, d\alpha.$$

$$= \frac{\sqrt{3}}{2d^2}$$

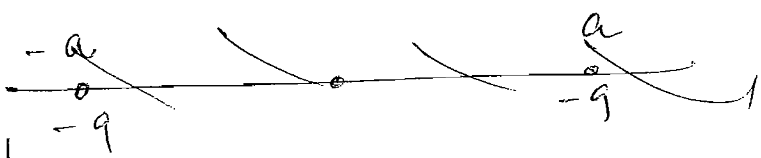
$$E_x = \cancel{k\lambda d^2\sqrt{3}} \quad \frac{k\lambda\sqrt{3}}{2d}.$$

$$\int \frac{x}{(x^2+d^2)^{3/2}}dx. = \frac{1}{2}\int \frac{d(x^2+d^2)}{(x^2+d^2)^{3/2}} = \frac{1}{2}\int z^{-3/2}\,dz$$

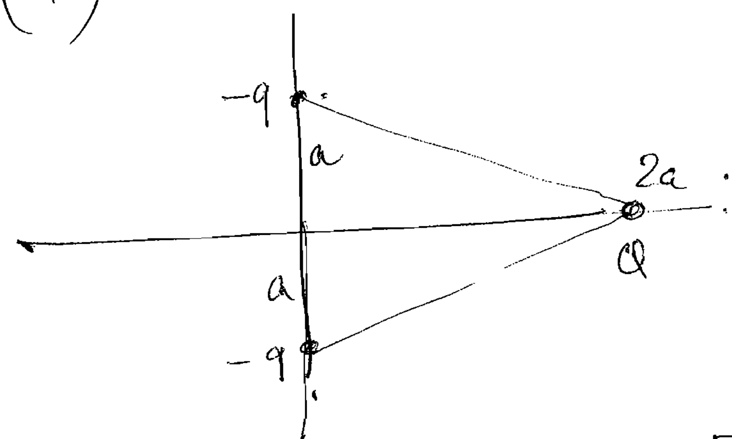
$$= -\left[\frac{1}{\sqrt{z}}\right] = -\left[\frac{1}{\sqrt{x^2+d^2}}\right]^{d\sqrt{3}}_{0}$$

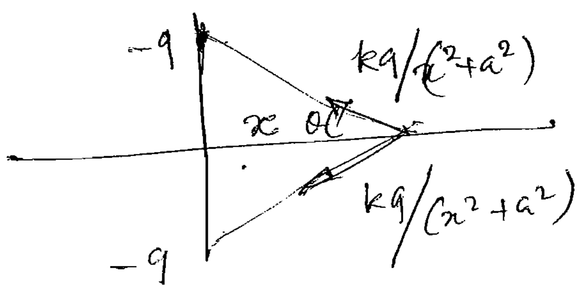
$$= -\left[\frac{1}{2d} - \frac{1}{d}\right] = \frac{1}{2d}.$$

$$E_y = k\lambda \qquad E_y \quad k\lambda/2d \quad = 1$$



7. 7.(d)



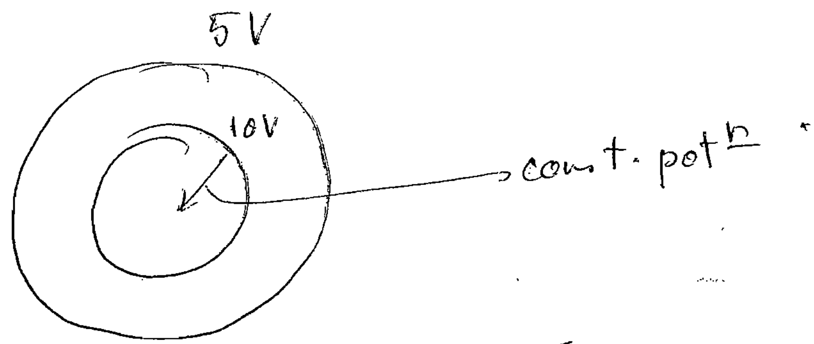


$$F = \left[2 \frac{kq}{(x^2+a^2)} \times \frac{x}{\sqrt{x^2+a^2}} \right] Q$$

$$F = \frac{2kqQx}{(x^2+a^2)^{3/2}}$$

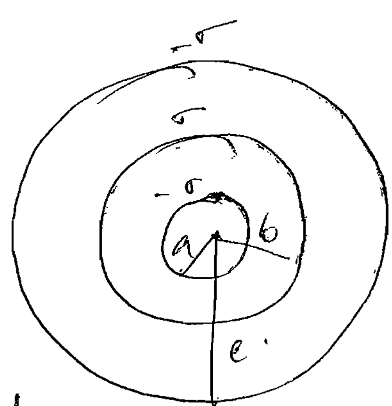
$F \not\propto x$ except for small oscillation.

8. 8.(a)



9. 9(c)

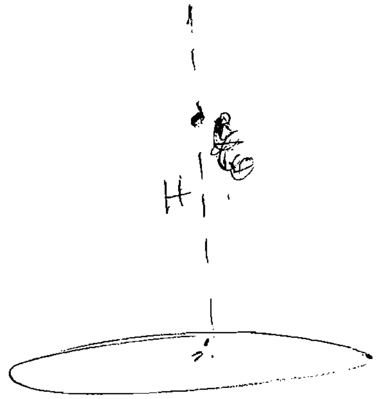
$$\frac{\sigma}{\epsilon_0}(-c) + \frac{\sigma}{\epsilon_0} b + \frac{\sigma}{\epsilon_0}(-a)$$

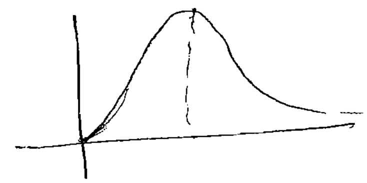


$\sigma \left[b - a - c \right]$

10. (b)

(10)



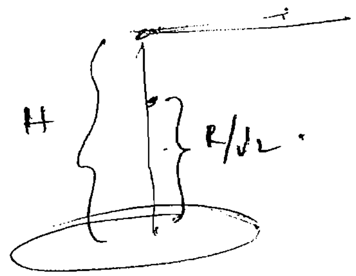


For ring E is max. at $\frac{R}{\sqrt{2}}$

$$\frac{KQHq}{(R^2+H^2)^{3/2}} = mg$$

mg

If $H > \frac{R}{\sqrt{2}}$

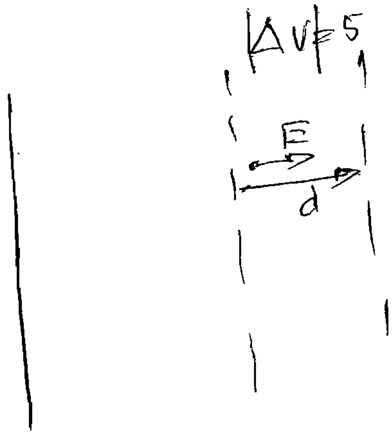


↑ Taken up, $E\downarrow$, mg is same, accelerat down

↓ " down, $E\uparrow$ mg is same, acceler up.

Stable.

(11). (b)



$$|\Delta V| = Ed = \frac{\sigma}{2\epsilon_0} d.$$

$$5 = \frac{10^{-7}}{2 \cdot 8.85 \times 10^{-12}} d.$$

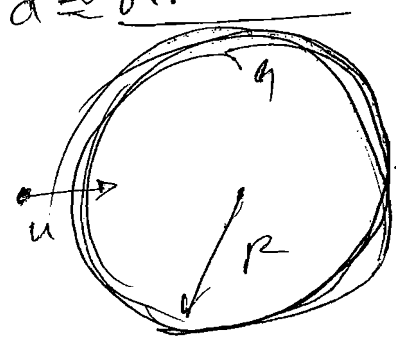
$$2 \cdot 8.85 \times 10^{-4}.$$

$$d = 8.85 \times 10^{-3}\ m$$

$$= 0.885 \times 10^{-3}$$

$$d \approx 0.88\,mm.$$

(12) (b)

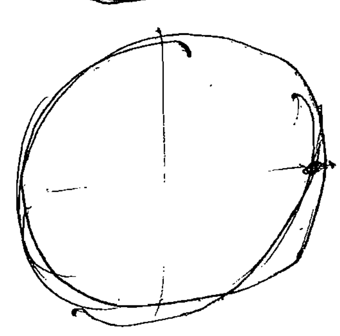


$$\frac{1}{2}mu^2 + \frac{Kq^2}{R} = \left(\frac{3}{2R}Kq\right)q$$

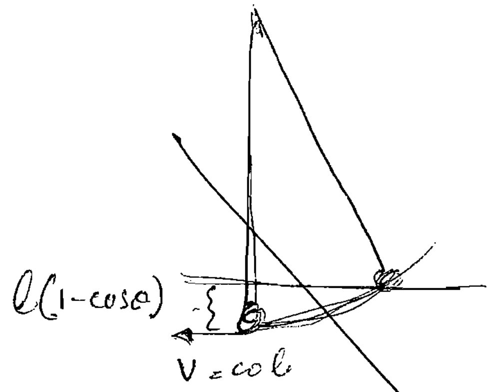
$$\frac{mu^2}{2} = \frac{Kq^2}{2R}.$$

$$u = q\sqrt{\frac{k}{mR}}$$

$$u = \frac{q}{\sqrt{4\pi\epsilon_0 mR}}.$$



13.

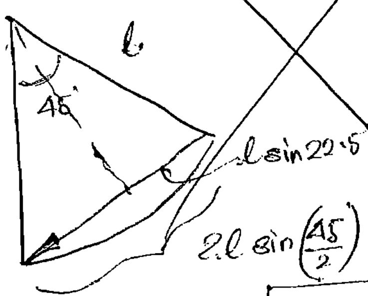


$\Delta E = W_E$.

$\frac{1}{2} \cancel{m}\omega^2 l^2 - 0 - \cancel{m}gl(1-\cos\theta)$

$= \cancel{q}\frac{\cancel{mg}}{\cancel{q}} 2l\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$.

$\omega^2 l^2 = 2gl\left(1-\frac{1}{\sqrt{2}}\right)$

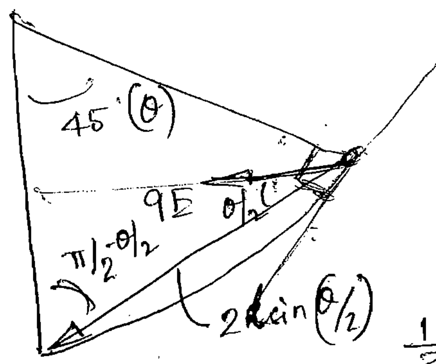


$2l\sin\left(\frac{45}{2}\right)$

$2l\sqrt{\frac{1-\cos 45}{2}}$

$2l\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

17. (B)



$W_E = qE\, 2l\sin(\theta/2)\cos(\theta/2)$

$= qEl\sin\theta$

$= q\cdot\frac{mg}{q}\, l\sin\theta$

$\Delta E = W_E$

$\frac{1}{2}m\omega^2 l^2 - mgl(1-\cos\theta)$ ~~$qEl\sin\theta$~~.

$= mgl\sin\theta$

$\omega^2 l^2 = 2gl(1-\cos\theta) + 2gl\sin\theta$

$\omega^2 = \frac{2g}{l}(1+\sin\theta-\cos\theta)$

$\theta = 45$

$\omega = \sqrt{\frac{2g}{l}}$.

14.(a)

14

$$\frac{1}{2}mv^2 + \frac{kQq}{4a} = \frac{kQq}{a}$$

$$\frac{1}{2}mv^2 = \frac{3kQq}{4a}$$

$$\frac{1}{2}mv^2 = 3Vq$$

$$v = \sqrt{\frac{6Vq}{m}}$$

$$\frac{kQ}{4a} = V$$

$\cancel{kQq}$

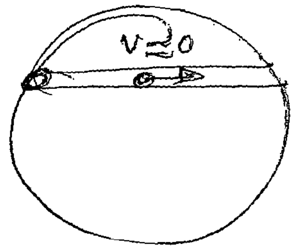
15.(a)

15

$$\frac{kQq}{R} + \frac{1}{2}mu^2 = \frac{11}{8}\frac{kQq}{R} \Rightarrow \frac{1}{\cancel{2}}mu^2 = \frac{3kQq}{4\cancel{8}R}$$

$$u^2 = \frac{3kQq}{4mR} = \frac{\cancel{3}\rho\cdot\frac{4}{\cancel{3}}\pi R^3\cdot q}{4\cdot\cancel{4\pi}\epsilon_0 mR}$$

$$u = \sqrt{\frac{\rho q R^2}{4\epsilon_0 m}}$$

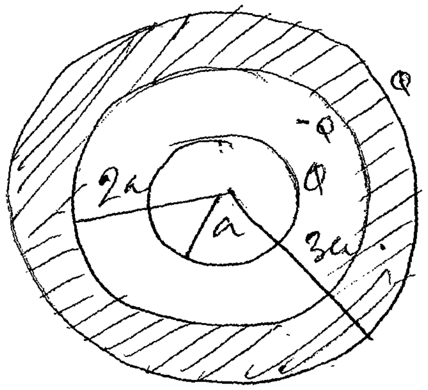


$\because q=1$

$$u = \sqrt{\frac{\rho R^2}{4\epsilon_0 m}}$$

16

16.(a)

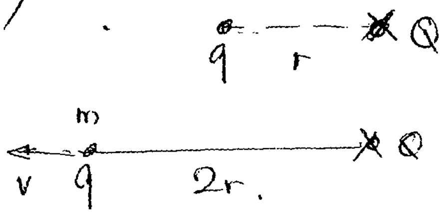


$$U = \frac{kQ^2}{2a} - \frac{kQ^2}{4a} + \frac{kQ^2}{3a}$$

$$= \frac{5kQ^2}{12a}$$

17.(b)

17



$$\frac{1}{2}mv^2 + \frac{kQq}{2r} = \frac{kQq}{r}$$

$$\frac{mv^2}{\cancel{2}} = \frac{kQq}{\cancel{2}r}$$

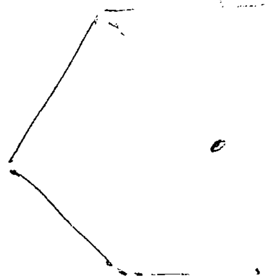
$$v = \sqrt{\frac{kQq}{mr}}$$

Impulse = mv

$$= \sqrt{mkQq} = \sqrt{Qqm}$$

18. (b)

(18)



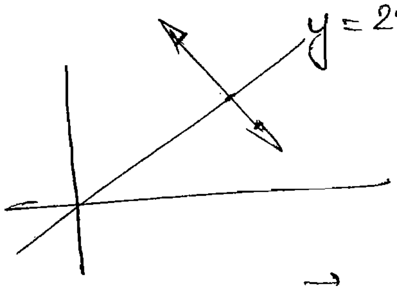
$E = \frac{kq}{r^2}$

$V = (n-1)\frac{kq}{r}$

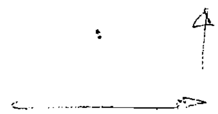
$\frac{V}{E} = r(n-1)$

19. (d)

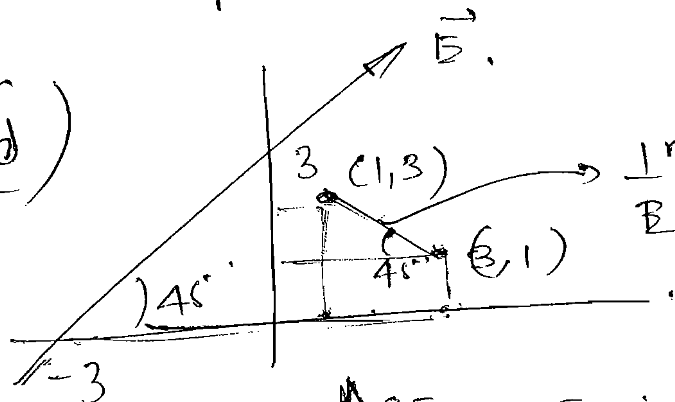
(19)



E ⊥ equipotential line.
only (d)



(20) (d)

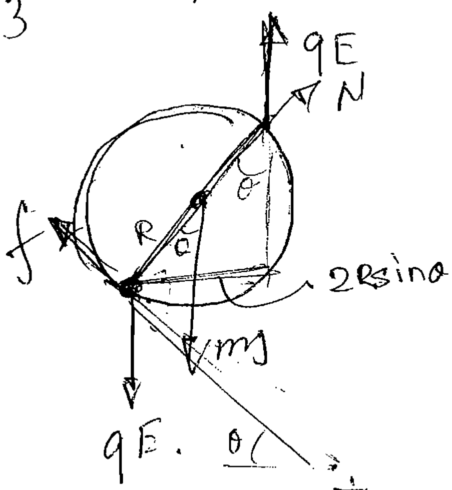


$\perp^r$ to $\vec{E}$

Equipotential.

21. (b)

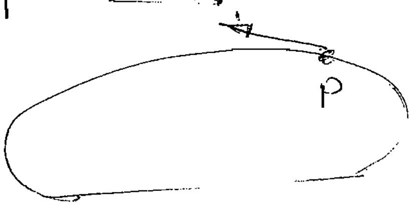
(21)



Taking torque abt bottommost pt. $qE\,2R\cancel{\sin\theta} = mg\,R\cancel{\sin\theta}$

$E = \frac{mg}{2q}$.

(22) 22. (b)



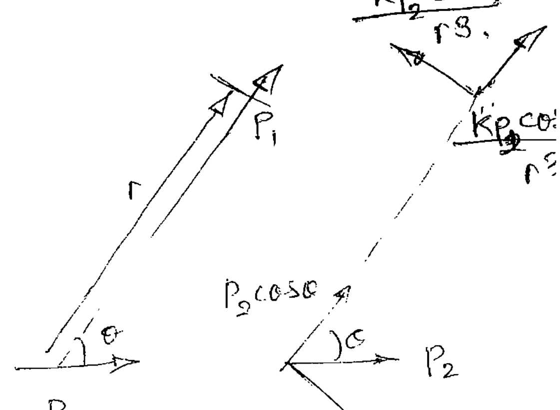
23. (b)

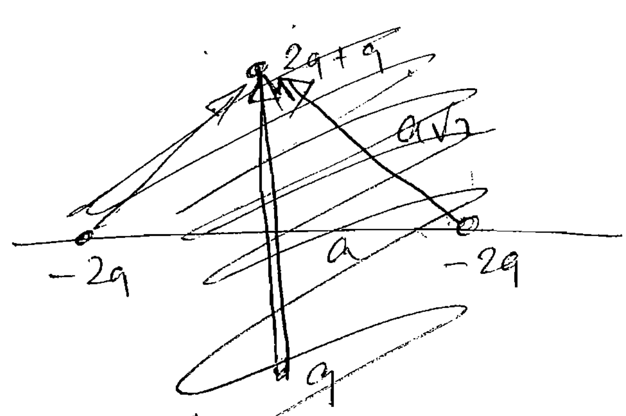
(23)

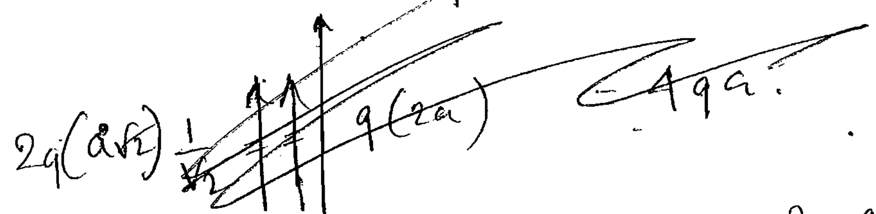
$U = -\vec{p}_1 \cdot \vec{E}_2$

$\cancel{= \vec{p}_1 [E_r \hat{e}_\theta}$

$= -p_1 \hat{e}_1 \cdot (E_r \hat{e}_r + E_\theta \hat{e}_\theta)$

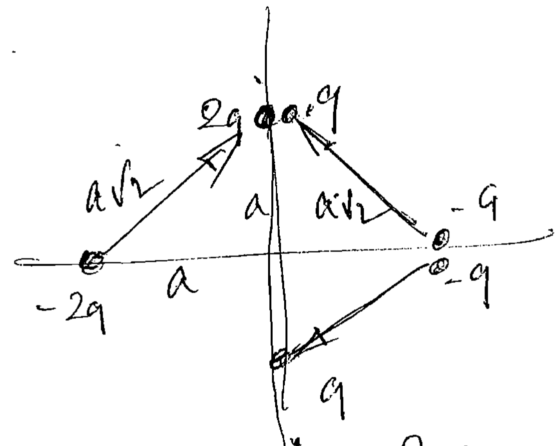


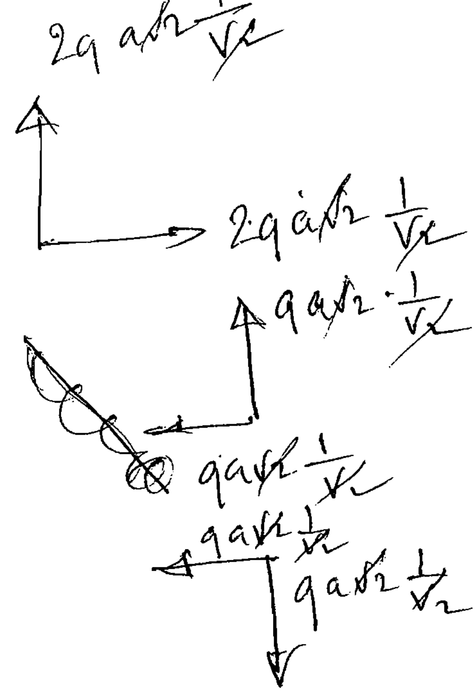


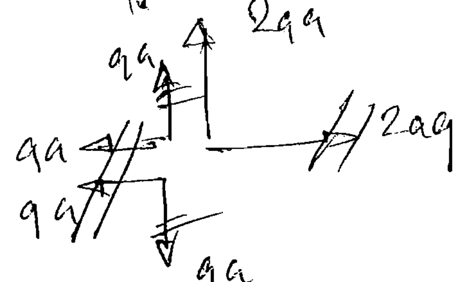


24. (a)

24







25. A(~~a~~) b

25 A

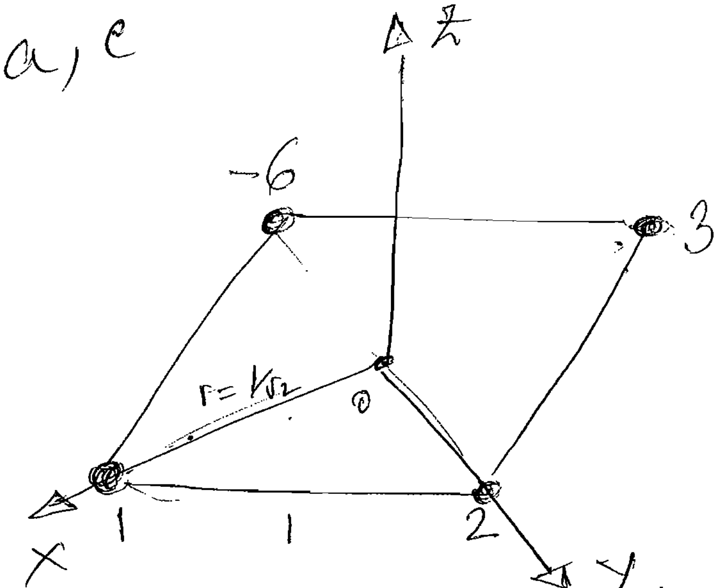
B (B) (d) Pot^n only due to $-q+Q$.
rest cancelled

$K\frac{(Q-q)}{}$

~~26. (a)~~ Apply superposition. Exercise #2
Electrostatics

27. a, e

(1)



$$V_0 = \frac{k(1)}{r} + \frac{k(2)}{r} + \frac{k(3)}{r} + \frac{k(-6)}{r}$$

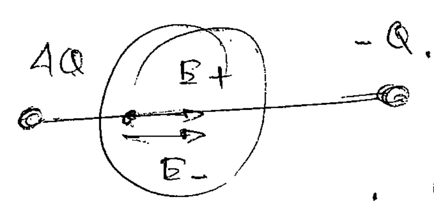
$$= 0$$

Dist. of any pt. on z axis from each charge is same

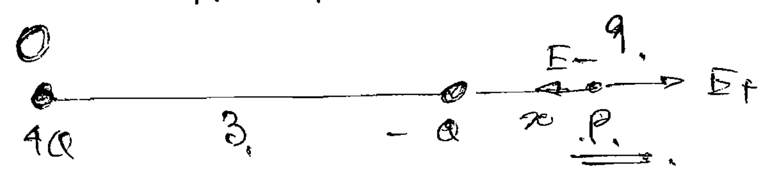
& $V_z = 0$

28.

(2)



Add up. ∴ (a)

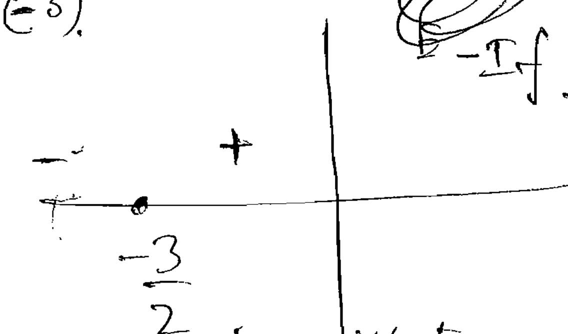


$$E = \frac{k4Q}{(3+x)^2} - \frac{kQ}{x^2}$$

$$= \frac{kQ(2x+3)(x-3)}{x^2(3+x)^2}$$

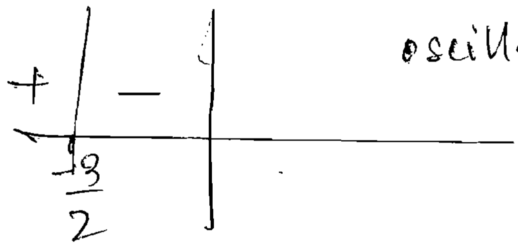
If $q > 0$

$F = qE$



oscillate.

If $q<0$



oscillates .

(3) 29 (a, c) check signs of E as above .

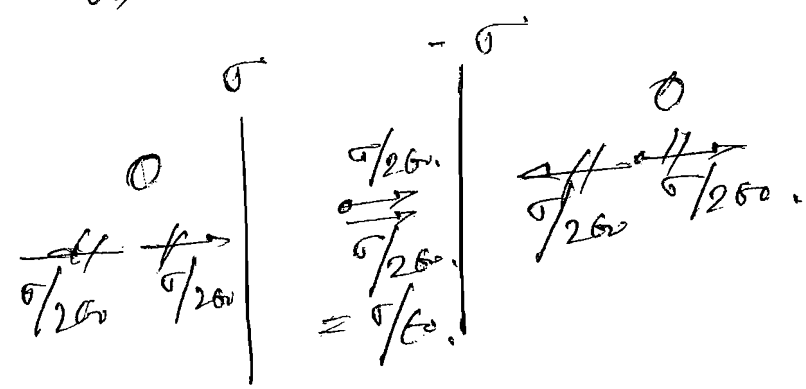
(4) 30 (b),(c) check configs.

U is min

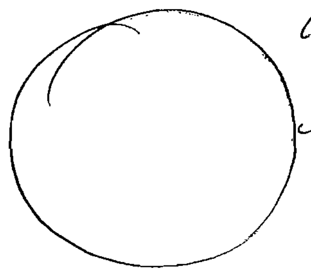
$\Rightarrow \frac{dU}{dx} = 0$

$E = -\frac{dU}{dx} = 0 \Rightarrow E = 0$.

(5) 31 (a, c)



32. (d)



$q \rightarrow$ Equipotential .

$E_{inside} = 0$, irrespective of E_{ext}

Distribution of charge uniform / non uniform based on $E_{int} = 0$

33 (b)(a)

6

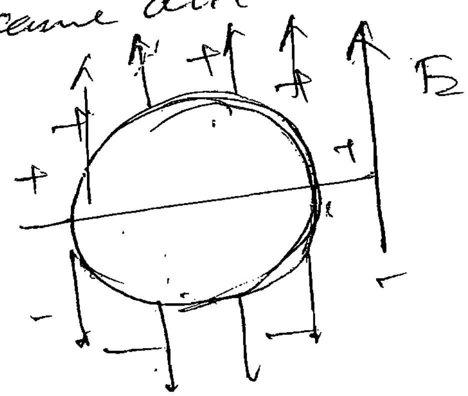
$$E = \frac{kQx}{(x^2+R^2)^{3/2}}$$

$$V = \frac{kq}{\sqrt{x^2+R^2}}$$ decreasing.

34 (a)

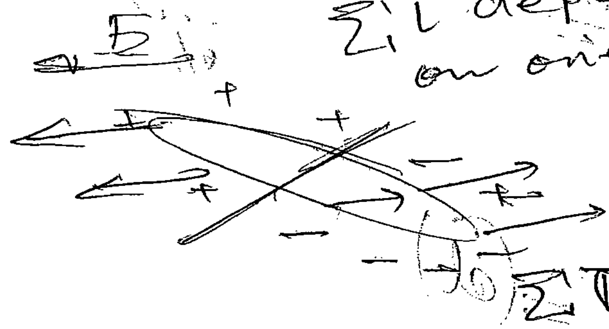
Pot is scalar. Each 1/2 is at same dist

$V = 0$



$\sum_i F = 0$

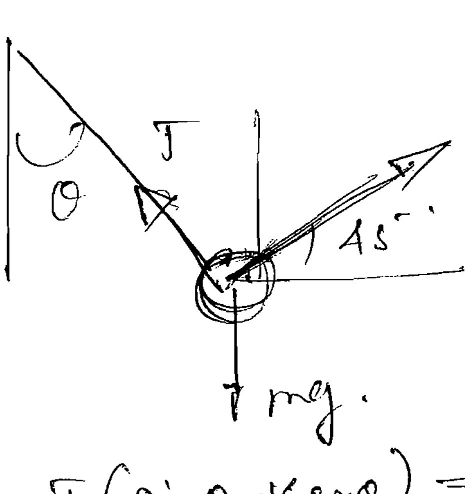
$\sum_i \tau$ depends on orientation.



$\sum \tau \neq 0$

35 (a,b)

7



$$T\sin\theta + \frac{qE}{\sqrt{2}} = mg$$

$$\frac{qE}{\sqrt{2}} = T\cos\theta$$

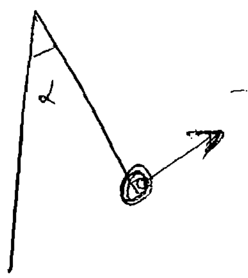
$\therefore T(\sin\theta + \cos\theta) = mg$

$2mg(\sin\theta + \cos\theta) = mg$

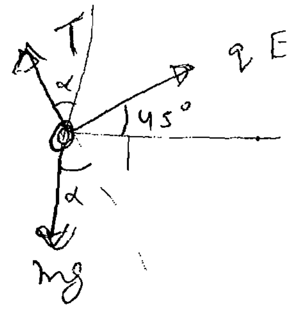
$\theta = 30°$

$60°$

(35) (1)



$\vec{E} = (\hat{i} + \hat{j}) \times 10^5$



Applying lami's theorem,

$$\frac{T}{\sin(135^\circ)} = \frac{mg}{\sin(45^\circ + \alpha)}$$

$$\Rightarrow \frac{2\cancel{mg}}{(1+\sqrt{3}) \cdot \frac{1}{\cancel{\sqrt{2}}}} = \frac{\cancel{mg}}{\frac{1}{\cancel{\sqrt{2}}} \cos\alpha + \frac{1}{\cancel{\sqrt{2}}} \sin\alpha}$$

$$\Rightarrow \therefore \cos\alpha + \sin\alpha = \frac{(\sqrt{3}+1)}{2}$$

$$=$$

$$\therefore \alpha = 30^\circ \text{ or } 60^\circ.$$

~~36~~

(8)

$\vec{P} = (2\hat{i} + 3\hat{j})\ \mu\text{Cm}.$

$\vec{E} = (3\hat{i} + 2\hat{k}) \times 10^5\ \text{NC}^{-1}.$

$$\vec{\tau} = \vec{P} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \end{vmatrix} = \begin{pmatrix} \hat{i}(6) - \hat{j}(4) \\ + \hat{k}(-9) \end{pmatrix} \times 10^{-1}$$

(b) Potential Energy

$$= -\vec{p}\cdot\vec{E}$$

$$= -(2\hat{i}+3\hat{j})\cdot(3\hat{i}+2\hat{k}) \times 10^{-6} \times 10^{5}$$

$$= -(6)\times 10^{-1}$$

$$= -0.6\ J$$

(c) Pot Energy $= |\vec{P}||\vec{E}|\cos\theta$

Max value $= |\vec{P}|\,|\vec{E}|$

$$= \sqrt{2^2+3^2}\ \sqrt{3^2+2^2} \times 10^{-6} \times 10^{5}$$

$$= 13\cdot\times 10^{-1} = 1.3\ J.$$

9 (37)

Potential decreases in the direction of electric field. $\therefore V_A < V_B$. (d)

More dense the electric field lines, more the electric field at that point.

$E_A > E_B$ (b)

10 (38)

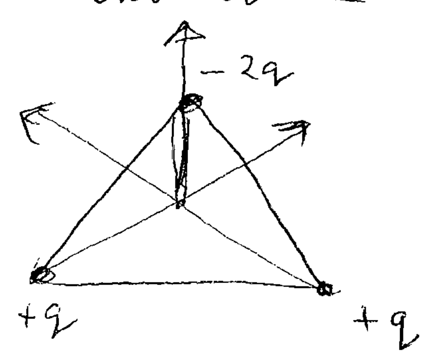
Pot at centre

$$= k\left[\frac{-2q}{r} + \frac{q}{r} + \frac{q}{r}\right] = 0$$

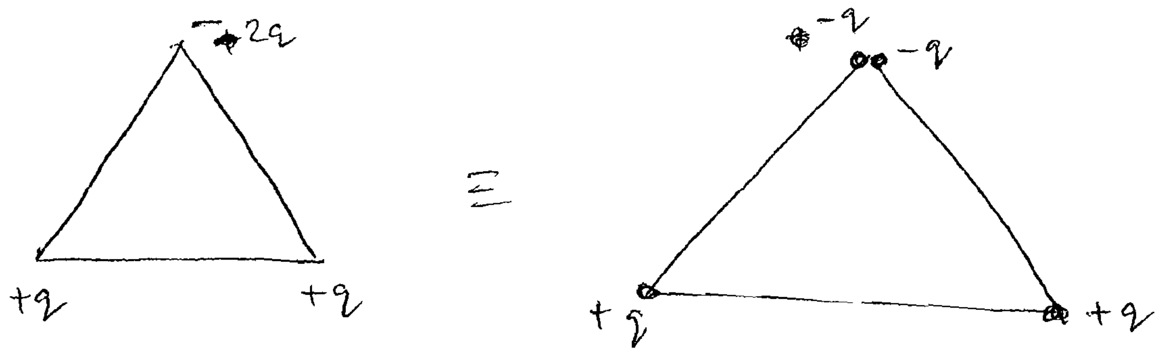
(where r is the distance of any one of the vertices from the centroid).

For Electric field, we have to add individual electric fields vectors.

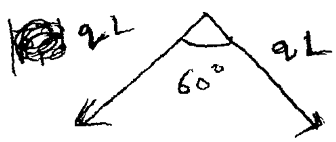
We see that all electric fields will add up. So, sum cannot be zero.



Dipole moment.



$\equiv$



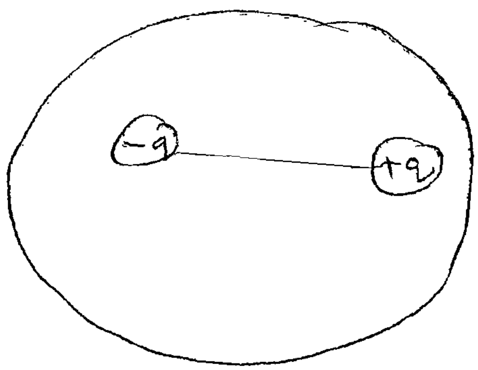
Magnitude of resultant

$$= \sqrt{(qL)^2 + (qL)^2 + 2(qL)(qL)\cos 60°}$$

$$= qL\sqrt{1+1+2\tfrac{1}{2}} = \sqrt{3}\,qL.$$

options a, d.

12

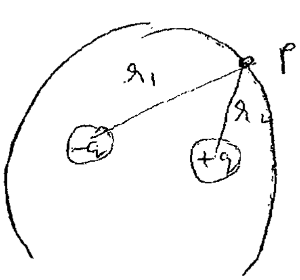


net charge enclosed = 0

∴ Net flux = 0.

Electric field at any point will not be zero.

Say, Electric potential will be zero at point P on surface.

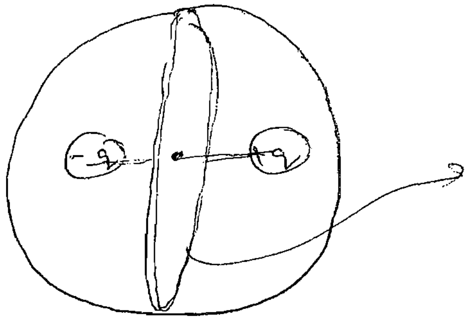


Then,

$$k\left[\frac{-q}{r_1} + \frac{q}{r_2}\right] = 0.$$

$$\therefore\ r_1 = r_2.$$

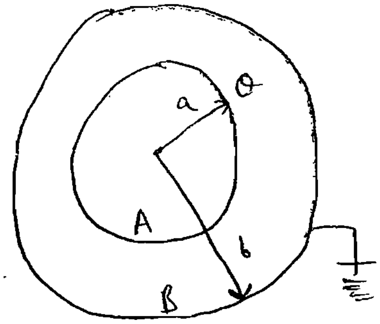
whose centre lies midway b/w $-q$ and q



circle on which potential $= 0$.

40

12



Shell B is earthed.

$\therefore$ Potential at B will be zero.

$$\therefore \frac{kQ}{b} + \frac{kq}{b} = 0$$

$$\therefore \boxed{q = -\frac{Q}{\cancel{b}}}$$

where q is the charge on the outer shell.

Electric field at $a \le r \le b$

$$= |\vec{E}| = k\frac{Q}{r^2} \quad \text{(by gauss law)}$$

Option (a)

Potential at r, $a \le r \le b$ will be

Potential due inner shell + Potential due to outer

$$= \frac{kQ}{r} + \frac{k(-Q)}{b} = kQ\left(\frac{1}{r} - \frac{1}{b}\right).$$

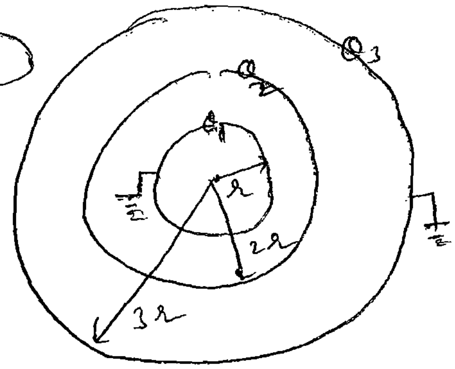
Option (d)

Potential difference b/w A and B

is $Pot_A - \cancel{Pot_B}^{\to 0}$

(41)

(13)



~~$V_A = 0$~~

$V_1 = 0$ and $V_3 = 0$.

$V_1 = 0$

$\therefore \quad K\left[\frac{Q_1}{r} + \frac{Q_2}{2r} + \frac{Q_3}{3r}\right] = 0 \quad$ —(1)

Also $\quad V_3 = 0$

$\therefore \quad k\left[\frac{Q_1}{3r} + \frac{Q_2}{3r} + \frac{Q_3}{3r}\right] = 0 \quad$ —(2)

From (1) and (2), we have,

$Q_1 + \frac{Q_2}{2} + \frac{Q_3}{3} = 0$

and $\quad Q_1 + Q_2 + Q_3 = 0$. $\quad$ (option (a) is right.

$\therefore \quad \frac{Q_2}{2} + \frac{Q_3}{3} = Q_2 + Q_3$.

$\Rightarrow \quad -\frac{2Q_3}{3} = +\frac{Q_2}{2}$.

$\therefore \quad \boxed{\frac{Q_3}{Q_2} = -\frac{3}{4}} \rightarrow$ (option (d) is wrong).

Also $\quad Q_1 + Q_2 + \left(-\frac{3}{4}Q_2\right) = 0$

$\therefore \quad Q_1 + \frac{Q_2}{4} = 0$

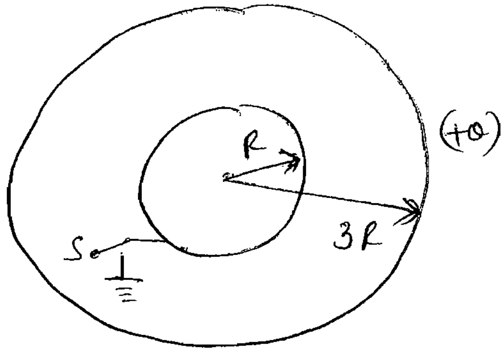
$\therefore \quad \boxed{\frac{Q_1}{} = -\frac{Q_2}{4}} \rightarrow$ option (b) is right

and $\quad \frac{Q_3}{Q_1} = \frac{-3/4\, Q_2}{-Q_2/4} = 3$.

$\therefore$ option (c) is also right

~~42~~

14



When switch is open,

$$V_{inner} = V_{\text{due inner charge}} + V_{\text{due outer}}$$

$$= 0 + \frac{Q}{3R}$$

$$V_{outer} = V_{\text{due inner}} + V_{\text{due outer}}$$

$$= 0 + \frac{Q}{3R} \quad \text{(equal)} \qquad \text{(a)}$$

Switch closed,

$V_{inner} = 0$ (connected to earth) (b)

Let q Charge attained by inner (switch closed).

then $V_{in} = 0$

$$\therefore \frac{Kq}{R} + \frac{KQ}{3R} = 0$$

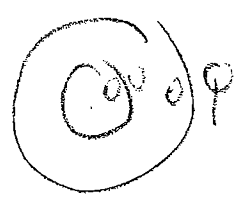
$$\therefore q = -\frac{Q}{3} \qquad \text{(c)}$$

Capacitance initial :→

$C_{in} = 4\pi\epsilon_0 3R$

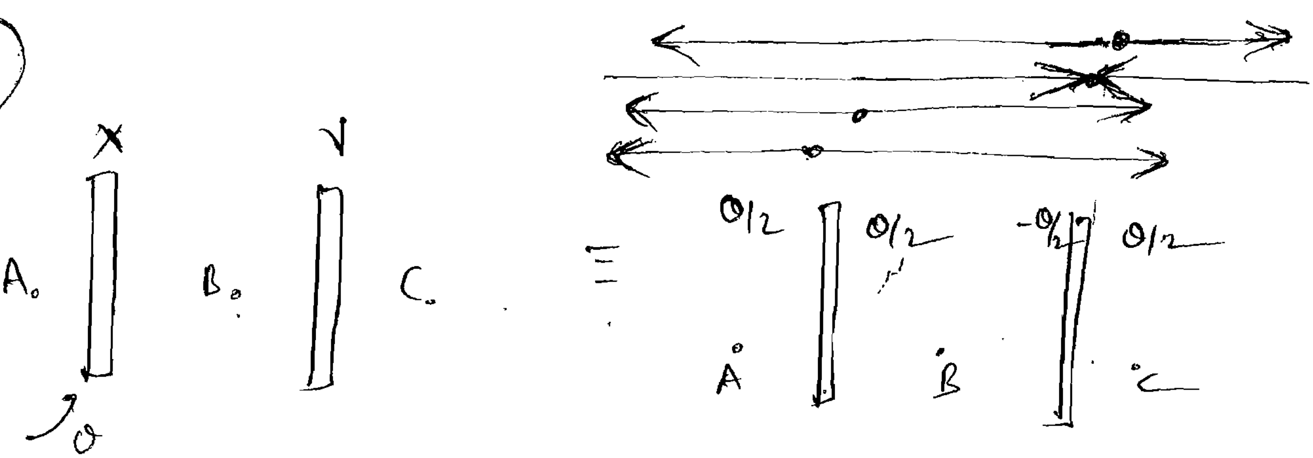
$$C_f = \frac{4\pi\epsilon_0 R \cdot 3R}{3R - R} + 4\pi\epsilon_0 3R$$

$= 6\pi\epsilon_0 R + 12\pi\epsilon_0 R$



$= 18\pi\epsilon_0 R$

15 (43)



Then, $E_A = \dfrac{Q/2}{2\varepsilon_0 A} + \dfrac{Q/2}{2\varepsilon_0 A} = \dfrac{Q}{2\varepsilon_0 A}$

$$E_B = \frac{Q/2}{2\varepsilon_0 A} + \frac{Q/2}{2\varepsilon_0 A} = \frac{Q}{2\varepsilon_0 A}$$

$$E_C = \frac{Q/2}{2\varepsilon_0 A} + \frac{Q/2}{2\varepsilon_0 A} = \frac{Q}{2\varepsilon_0 A}.$$

(Fields due to individual surfaces are represented by arrows).

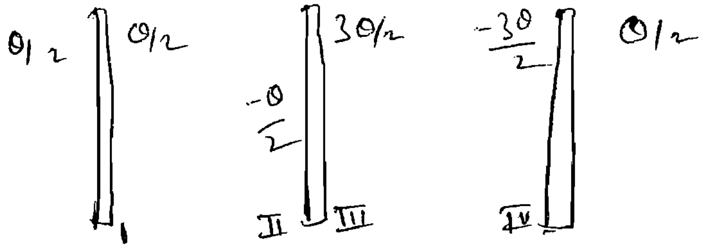
16 (44)

a, c, d

Charge on outer surfaces →

$$\frac{Q + Q - Q}{2} = \frac{Q}{2}.$$

∴ Charge distribution :→



(a) ✓

(b) ✓

(c) ✓

$$V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

$$= \left(\frac{Q/2}{\epsilon_0 A}\right) d + \left(\frac{3Q/2}{\epsilon_0 A}\right) d$$

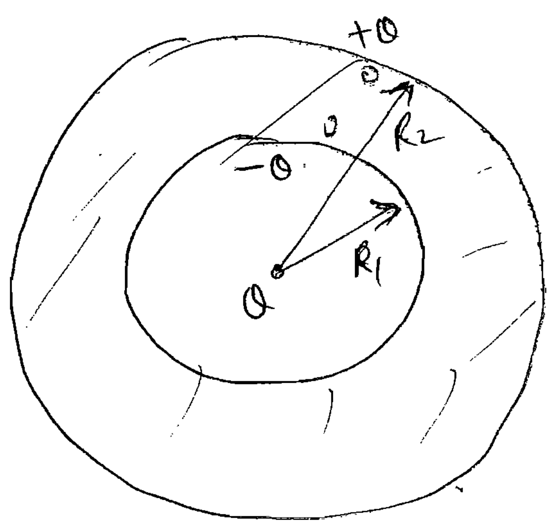
$$= \frac{2Qd}{\epsilon_0 A} \neq V_C - V_B$$

Hence wrong option is (d).

~~45~~

~~107~~

29



(Figure shows the charge distribution

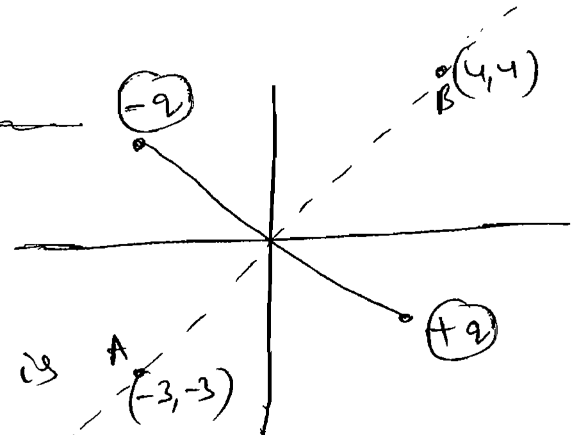
Now, $V_{\text{inner surface}} = V_{\text{due point charge } Q}$

$+ V_{\text{due inner surface charge}}$

$$= \frac{KQ}{R_1} - \frac{KQ}{R_1} + \frac{KQ}{R_2}$$

$$= \frac{KQ}{R_2} \quad \text{(none of these)}$$

(30) $q = 2\mu C$

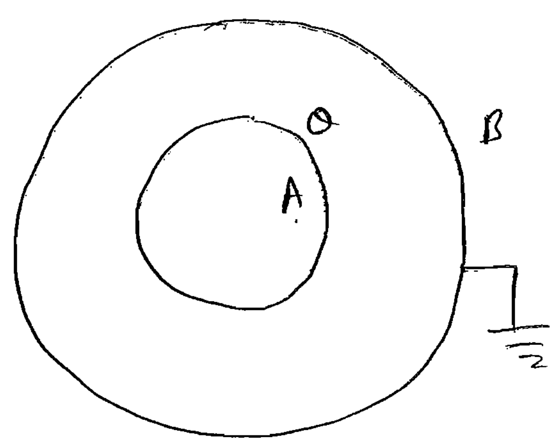


The dashed line is the equipotential surface. each point having the same potential on the dashed line.

So, No work done for displacement from A to B.

(31) B earthed

$\therefore V_B = 0$



But $V_B = K\left[\frac{Q}{r_B} + \frac{q}{r_B}\right] = 0$

(where q is the charge which comes on B).

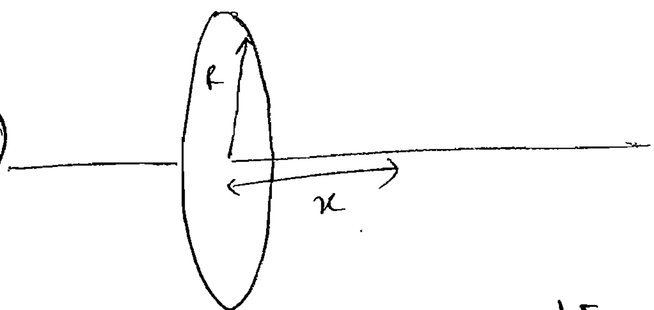
$\therefore Q + q = 0 \quad \therefore \boxed{q = -Q}$

field inside A = 0 (Gauss law)

field ~~inside~~ b/w A and B $\neq 0$ (Gauss law charge Inclosed $\neq 0$).

(c) ✓

(32)



we have

$$E_x = \frac{K Q x}{(a^2+x^2)^{3/2}}.$$

For E_x to be max,

$$\frac{dE_x}{dx} = 0.$$

$$\therefore x = \frac{a}{\sqrt{2}}.$$

and $(E_x)_{max} = (E_{a/\sqrt{2}}) = K \cdot \dfrac{2q}{3\sqrt{3}R^2}$.

(Read theory).

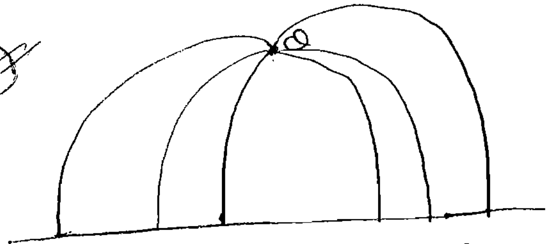
(33)

Potential energy at A is same as that at B and C.

$\therefore$ Work done in taking the charge from P to any of A or B or C is same.

$W_A = W_B = W_C$.

(34)



Lines of force will be

(i) perpendicular to infinite metal plate

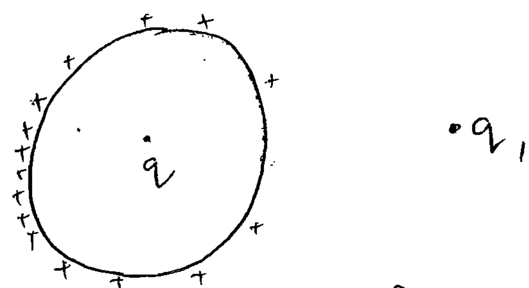
Also option (D) is wrong because there cannot be any 'kink' in lines of force as they would mean 2 directions at a particular point. (6)

~~Hence~~.

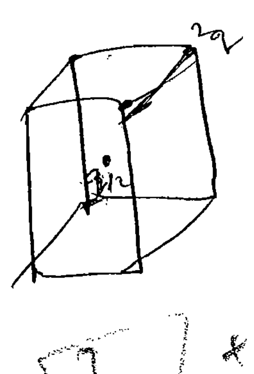
In option (B), fields are going into the charge Q. So, incorrect.

Option (A) is right.

~~53~~
35 The charge on the shell will distribute till that time and in that way such that the final electric field at any point inside the conductor becomes zero due to charges present on the shell as well as outside the shell. Thus, q is shielded from any electric field and force on q is zero



~~54~~
36



flux through cube

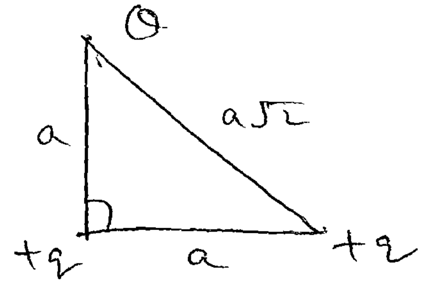
$$= \frac{q_{enclosed}}{\varepsilon_0}$$

$$= \frac{q/12}{\varepsilon_0}$$

$\frac{q}{2}\left(\frac{1}{16\varepsilon_0}\right)$ $\left[\frac{1}{4\varepsilon_0}\right] + \frac{2q}{8\varepsilon_0}$

Option (a)

37 ~~55~~



Net E. energy

$$= k\left[\frac{q^2}{a} + \frac{Qq}{a} + \frac{Qq}{a\sqrt{2}}\right]$$

$$= 0.$$

$$\therefore \frac{q}{1} + \frac{Q}{1} + \frac{Q}{\sqrt{2}} = 0$$

$$\therefore q + Q\left(\frac{1}{1} + \frac{1}{\sqrt{2}}\right) = 0$$

$$\therefore \frac{-q}{1+\frac{1}{\sqrt{2}}} = Q$$

$$\Rightarrow \boxed{\frac{-\sqrt{2}\,q}{\sqrt{2}+2} = Q} \quad \therefore (6)$$

38 ~~58~~

2.24 mg

$= 2.24 \times 10^{-3}$ g

Atomic mass of Fe = 56 amu.

6×10^{23} atoms $\rightarrow$ 56 g

$\therefore$ 1g $\rightarrow \frac{6 \times 10^{23}}{56}$ atoms.

2.24×10^{-3} g $\rightarrow \frac{6 \times 10^{23}}{56} \times 2.24 \times 10^{-3}$

$= 6 \times 10^{23} \times 10^{-5} \times 4$ atoms.

Each atom has 26 electrons.

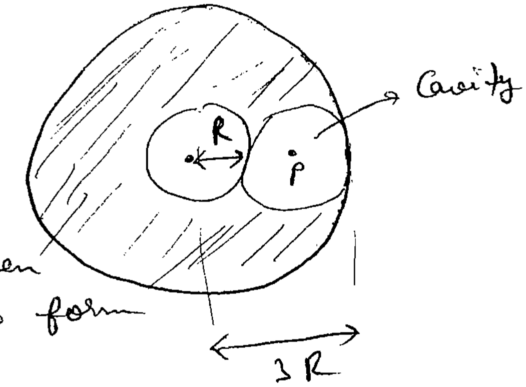
$\therefore$ Total electrons removed

$$= \frac{.02}{100} \times 26 \times 6 \times 10^{23} \times 10^{-5} \times 4$$

Total charge $= 1.6 \times 10^{-19} \times \frac{.02}{100} \times 26 \times 6 \times 10^{23} \times 10^{-5}$

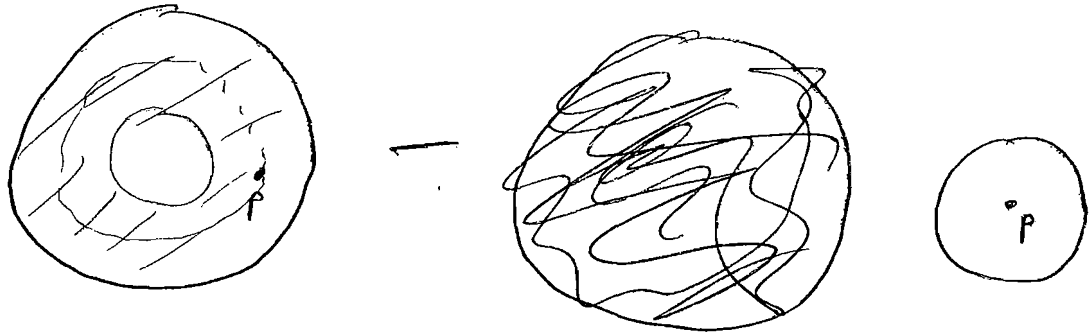
(57) To find:

(39) $E_{\text{cavity centre}} \equiv E_{cc}$



$$\vec{E}_{C.C} = \vec{E}_{\text{no cavity}} - \vec{E}_{\text{solid sphere}}$$ which has been removed to form cavity

$\equiv$



$$= \frac{q_{\text{enclosed}}}{\epsilon_0 \cdot 4\pi (2R)^2} - 0$$

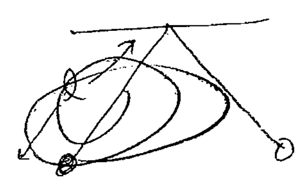
$$\frac{\rho}{3\epsilon_0}$$

$$= \frac{\left[\frac{4}{3}\pi (2R)^3 - \frac{4}{3}\pi R^3\right]\sigma}{\epsilon_0 \, 4\pi (2R)^2}$$

$$= \frac{1}{3}\,\frac{7R^3\sigma}{4\epsilon_0 R^2} = \frac{7\sigma R}{12\epsilon_0}$$ (d)

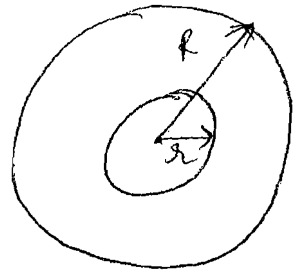
(58) (40) The electric field at any point is due to all the charges inside or outside the gaussian surface

(59)



(60)

(41)



q on inner surface
$= 4\pi r^2 \sigma$

q on outer surface
$= 4\pi R^2 \sigma$

Electric potential at centre.

$$K\left[\frac{4\pi R^2\sigma}{R} + \frac{4\pi r^2\sigma}{r}\right]$$

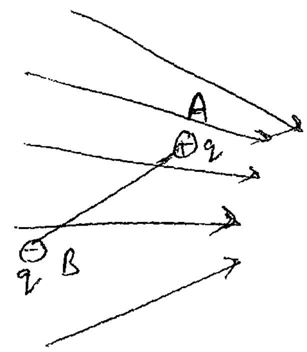
$$= \frac{1}{\cancel{4\pi}\,\varepsilon_0}\cdot\cancel{4\pi}\,\sigma\,(R+r)$$

$$= \frac{\sigma}{\varepsilon_0}(R+r)$$

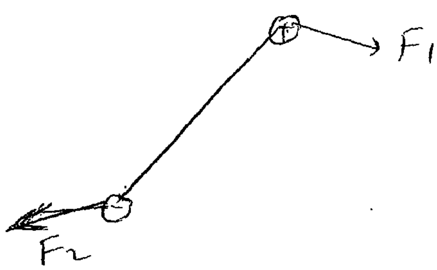
(61)

(42)

Field at A is more than field at B.



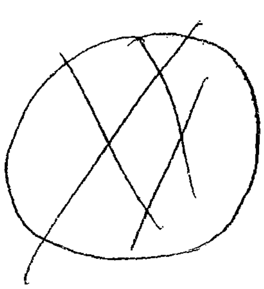
So, $F_1 > F_2$

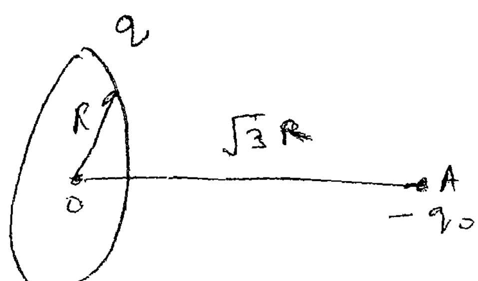


Net force towards right.
Net torque clockwise
(into the plane).

(62)

(43)





$$\text{P.E. at A} = \frac{-kq\,q_0}{\sqrt{r^2+(\sqrt{3}r)^2}} = \frac{-kq\,q_0}{2r}$$

$\cancel{\Delta KE = \frac{1}{2}mv^2}$

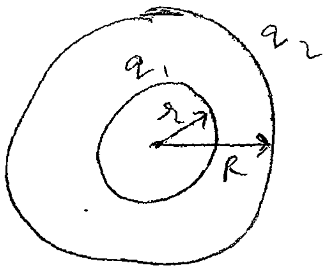
$\Delta KE + \Delta PE = 0$

$\therefore \left(\frac{1}{2}mv^2 - 0\right) + \left(-\frac{kqq_0}{r} - \frac{-kqq_0}{2r}\right) = 0$

$\Rightarrow \cancel{\frac{1}{2}mv^2 - \frac{kqq_0}{2r}}$

$KE = \frac{kqq_0}{2r}$

~~63~~ 44



$q_1 + q_2 = Q$

$\frac{q_1}{q_2} = \frac{4\pi r^2 \sigma}{4\pi R^2 \sigma}$

$\frac{q_1}{q_2} = \frac{r^2}{R^2}$

$q_1 + \frac{q_1 R^2}{r^2} = Q$

$q_1\left(1 + \frac{R^2}{r^2}\right) = Q$

$q_1 = \frac{Q r^2}{R^2 + r^2}$, $q_2 = \frac{Q R^2}{r^2 + R^2}$

Potential at centre

$= \frac{kq_1}{r} + \frac{kq_2}{R}$

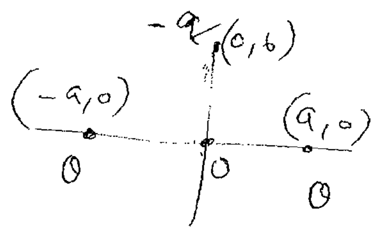
$= k\left[\frac{Qr}{R^2 + r^2} + \frac{QR}{R^2 + r^2}\right]$

$= \frac{kQ}{R^2 + r^2}(r + R)$

~~64~~

17

Ex# 2



Distance b/w $-q$ and O initially $= \sqrt{a^2+b^2}$.

Initial P.E. $= \dfrac{-kQq}{\sqrt{a^2+b^2}} \times 2$

P.E. when $-q$ reaches O

$$= \frac{-kQq}{a} \times 2$$

$$\Delta KE + \Delta PE = 0$$

$$\therefore \left(\frac{1}{2} m v_0^2 - 0\right) + \left(\left(\frac{-kQq}{a} \times 2\right) - \left(-\frac{kQq\,2}{\sqrt{a^2+b^2}}\right)\right) = 0$$

$$\Rightarrow \quad \frac{1}{2} m v_0^2 = 2kQq\left[\frac{1}{a} - \frac{1}{\sqrt{a^2+b^2}}\right].$$

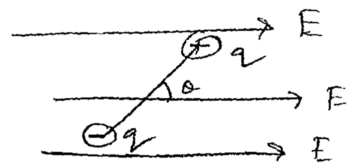
$$V_0 = \sqrt{\frac{4kQq}{m}\left[\frac{1}{a} - \frac{1}{\sqrt{a^2+b^2}}\right]}.$$

~~65~~

18

Ex# 2

given θ is small
(slightly rotated)



$$|\vec{\tau}| = |\vec{p} \times \vec{E}|$$

$$= (qL)\sin\theta\, E$$

$$= qLE\sin\theta.$$

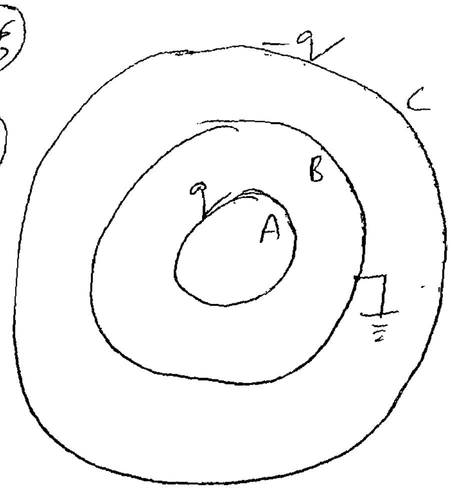
If θ is small $\sin\theta \approx \theta$

$$\therefore \quad |\vec{\tau}| = qLE\theta.$$

$\therefore$ S.H.M. + oscillatory

(19)

Ex#2



Let the charge on B be Q

Then,

$V_B = 0$

$\therefore \frac{Kq}{b} + \frac{KQ}{b} - \frac{Kq}{c} = 0$

$\Rightarrow K\left[\frac{q}{b} + \frac{Q}{b} - \frac{q}{c}\right] = 0$

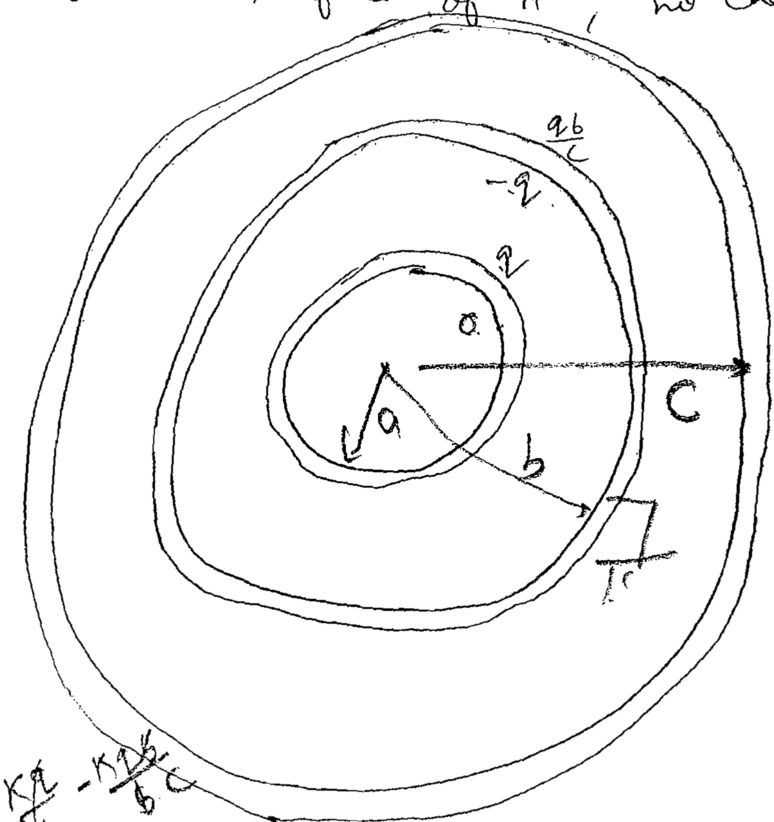
$\Rightarrow \frac{Q}{b} = q\left[\frac{1}{c} - \frac{1}{b}\right]$

$\frac{Q}{\cancel{b}} = q\left[\frac{b-c}{c\cancel{b}}\right]$

$Q = \frac{q(b-c)}{c}$

$\boxed{Q = q\frac{b}{c} - q}$

Inner surface of A, no charge (If we consider the guassian surface in the material of A, the total inside charge must be 0)



Outer surface of A $\rightarrow q$.

Inner surface of B $\rightarrow -q$

Outer surface of B $\rightarrow$

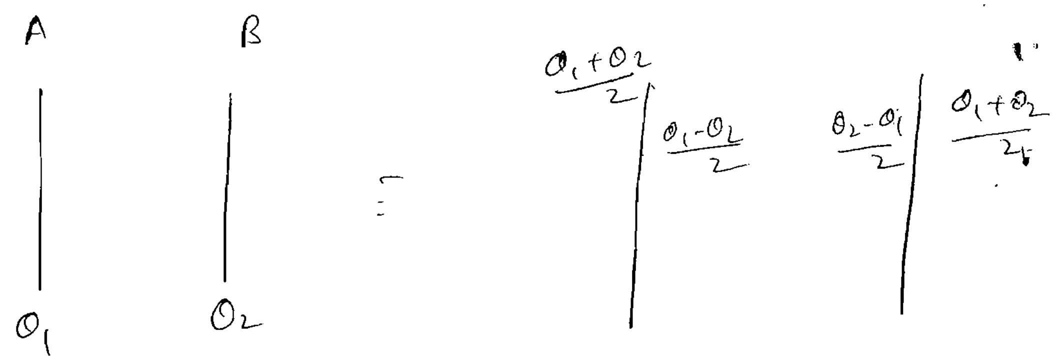
$\left(\frac{qb}{c} - q\right) - (-q)$

$= \frac{qb}{c}$.

67

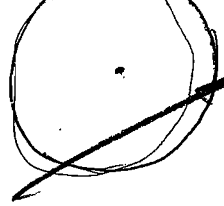
20

Ex#2



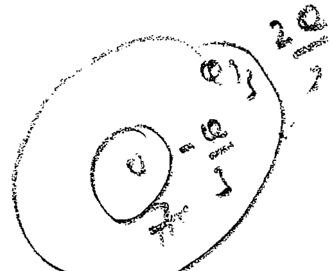
Hence, all options correct

~~Balloon is non-conducting.~~



remove

Rough



$\frac{\sigma}{2\epsilon_0} d = \frac{10^{-7}}{8.85 \times 10^{-12}}$ 5

$5 = \frac{\sigma}{2\epsilon_0} \cdot d$

$d = \frac{10^{10}}{6}$

$q = \frac{4\pi R^2 \sigma}{2} + 4\pi R^2 \sigma$

$= 6\pi R^2 \sigma$

$= 18$

Exercise #1 (Electrostatics)

(26) $\vec{E}$ inside cavity $\propto \vec{l}$

$\vec{F} \propto \vec{l}$.

(27) Potential must be equal at every point.

(28) only potential will be zero at all points on its axis

(45) $\vec{E}_x = \frac{(16-4)}{4}\hat{i} = 3\hat{i}$ $\qquad \vec{E} = 3\hat{i} - 4\hat{j}$

$\vec{E}_y = -\frac{(12-4)}{2}\hat{j} = -4\hat{j}$

(46) $r = \sqrt{(8-2)^2 + (-5-3)^2} = 10$

$E = \frac{9\times10^{9} \times 50\times10^{-6}}{(10)^2} = 4500 \frac{V}{m}$.

(47) charge within the surface

$= \frac{(\sigma.\pi r^2)}{\epsilon_0} = \frac{\sigma.\pi(R^2-x^2)}{\epsilon_0}$

(48) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$= \frac{\lambda}{2\pi\epsilon_0 r}\hat{i} + \frac{\sigma R}{\epsilon_0 r}\hat{i}$

$\vec{E} = 0$

$\boxed{\lambda = -2\pi\sigma R}$

(49) $$\vec{E} = -\left[\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial z}{\partial z}\hat{k}\right)$$

$$= -2K\left[2x\hat{i} - y\hat{j} + z\hat{k}\right)$$

$$\vec{E}(1,1,1) = 2K\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

$$|\vec{E}| = 2K\sqrt{6},$$

(50) $$r = \sqrt{1+1+4} = \sqrt{6} < 5$$

inside point $\Rightarrow \boxed{E = 0}$

Exercise # 2 : Electrostatics

(21) $\vec{E}_A$ is along $\vec{OA}$ & $\vec{E}_B$ is along $\vec{OB}$

$$\left.\begin{aligned}\vec{OA} &= \hat{i} + 2\hat{j} + 3\hat{k}\\ \vec{OB} &= \hat{i} + \hat{j} - \hat{k}\end{aligned}\right]$$

$\vec{OA} \cdot \vec{OB} = 0$

$\Rightarrow \vec{OA} \perp \vec{OB}$

$\Rightarrow \vec{E}_A \perp \vec{E}_B$

$$|\vec{E}| \propto \frac{1}{r^2}$$

$$|\vec{OC}| = 2|\vec{OB}| \Rightarrow |\vec{E}_B| = 4|\vec{E}_C|$$

(22) $AC = 5\,m$

$$V = \frac{Kq}{AC} = 1.8\ kV$$

$$V_B = (V_B)_{\text{due to } q} + (V_B)_i$$

$$1.8 \times 10^3 = \frac{Kq}{AB} + (V_B)_i = 2.25 \times 10^3 + (V_B)_i$$

(23) $\frac{Kq_A}{R} + \frac{Kq_B}{2R} = 2V$ & $\frac{Kq_A}{2R} + \frac{Kq_B}{2R} = \frac{3V}{2}$

$\Rightarrow \boxed{q_A/q_B = \frac{1}{2}}$

$V_B = 0 \Rightarrow q_B' = -q_A' = -q_A$

After ~~[illegible]~~ earthing

$V_A - V_B = Kq_A\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{Kq_A}{2R}$

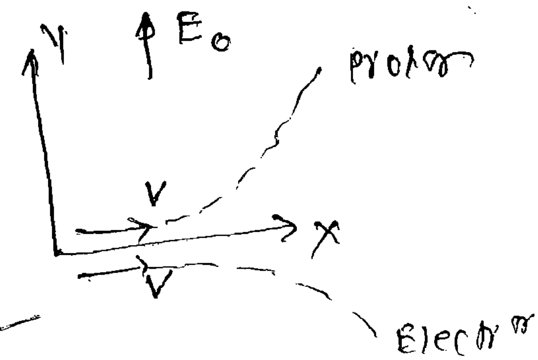
$= V/2$

$V_B = 0 \Rightarrow \boxed{V_A = V/2}$

(24) $\vec{E}$ must be continuous & defined

(25) use property of conductor & symmetry.

Exercise #3 Electrostatics



passage-1

- Trajectory will be parabola.

$Y = \frac{1}{2}\left(\frac{E_0 q}{m}\right)\left(\frac{x}{v}\right)^2$

$Y = \frac{1}{2}\cdot\left(\frac{E_0 q}{mv^2}\right)x^2 = \frac{E_0 q}{4K}\cdot x^2$

- $y \ldots$ if $K_1 = K_2$

Passage #2

- $F = F_{repulsion} = \dfrac{2K\varphi\cdot\varphi}{d^2} = \dfrac{2K\varphi^2}{d^2}$,

- $\frac{1}{2}(2m)v_0^2 = \frac{1}{2}\cdot 2m v^2 + \dfrac{2K\varphi^2}{d}$

$$v = \sqrt{v_0^2 - \frac{2K\varphi^2}{md}}$$

Impulse $= mv - mv_0$

- $\dfrac{2K\varphi^2}{d_{min}} = \frac{1}{2}\cdot 2m v_0^2 \Rightarrow d_{min} = \dfrac{2K\varphi^2}{m v_0^2}$

- $F_{2m} = \dfrac{K\cdot 2\varphi^2}{d_{min}^2} \Rightarrow a_{2m} = \dfrac{F_{2m}}{2m}$

- use energy conservation & linear momentum conservation

$$2mv_0 = (2m+m)v \quad (I)$$

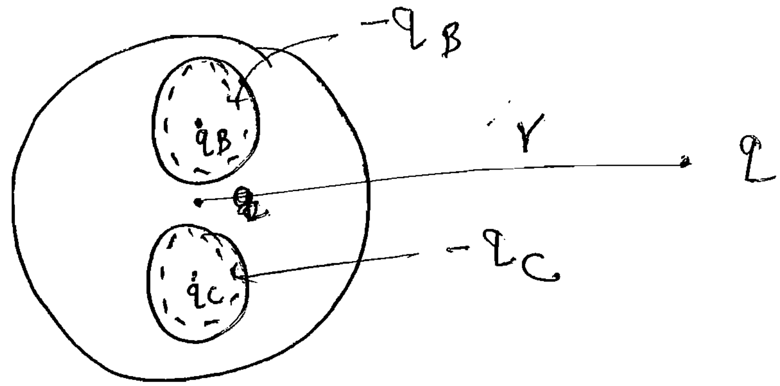
$$\frac{1}{2}\cdot 2m v_0^2 = \frac{1}{2}(2m+m)v^2 + \frac{K\cdot 2\varphi\cdot\varphi}{d_{min}} \quad (II)$$

solve for d_{min}.

Matching type problem

1 → use property of $\vec{E}$ in shell

2 →



$\therefore F_q = \dfrac{K \cdot q\,(q_B + q_C)}{r^2}$

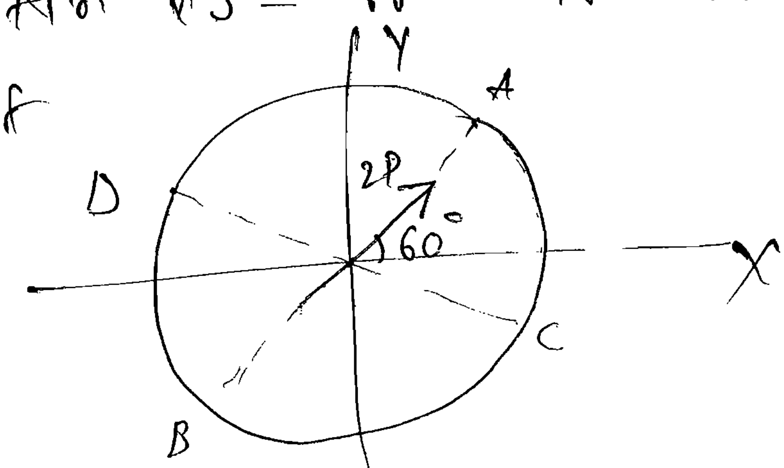
. use the concept of contribution of induced charge on $\vec{E}$ calculation i.e

$\vec{E}$ at any outside point of conducting shell due to all inside charges is zero and vice versa.

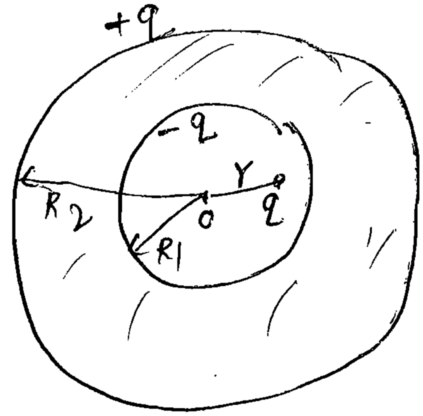
3 → Resultant dipole moment has magnitude $\sqrt{(\sqrt{3}p)^2 + p^2} = 2p$ at an angle

$\theta = \tan^{-1}\sqrt{3} = 60^\circ$ with +ve x-axis

use property of dipole.

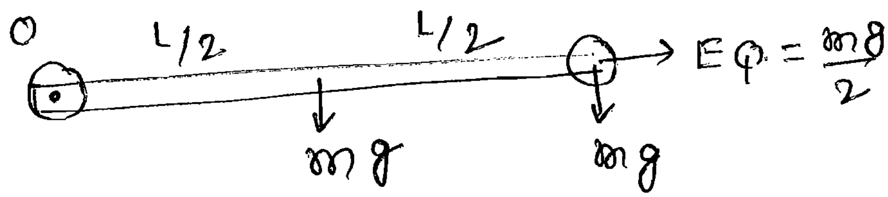


Passage #3



- $V_0 = \frac{Kq}{r} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$
- charge distribution on inner surface unknown.
- $V_{conductor} = \frac{Kq}{R_2}$.

Passage #4



- $\tau_0 = I_0 \alpha$

$$\alpha = \frac{\tau_0}{I_0} = \frac{mg \cdot \frac{L}{2} + mgL}{\frac{mL^2}{3} + mL^2} = \frac{9g}{8L}$$

$$a_t = \alpha r = \alpha L = 9g/8$$

- use energy conservation.

$$\frac{1}{2} I \omega^2 = mg\frac{L}{2} + mgL - \left(\frac{mg}{2}\right) L$$

$$\omega = \sqrt{\frac{3g}{2L}} \Rightarrow v = \omega L = \sqrt{\frac{3gL}{2}}$$

- $a = \sqrt{a_t^2 + a_c^2}$

(4) use property of conductor

(5) use property of conductor & charge induced on surfaces of conductor.

Exercise # 4
(Electrostatics)

(20)

$$U_{in} = \frac{q^2}{8\pi\epsilon_0 R_1} + \frac{q q_0}{4\pi\epsilon_0 R_1}$$

$$U_f = \frac{q^2}{8\pi\epsilon_0 R_2} + \frac{q q_0}{4\pi\epsilon_0 R_2}$$

$$W = -\Delta U = -(U_f - U_{in})$$

$$= \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\epsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

(21) (a) $V_R = K\left[\frac{Q}{R} - \frac{2Q}{2R} + \frac{3Q}{3R}\right] = \frac{KQ}{R}$

$V_{3R} = K\left[\frac{Q}{3R} - \frac{2Q}{3R} + \frac{3Q}{3R}\right] = K\cdot\frac{2Q}{3R}$

(b) using Gauss's Theorem

$\epsilon_0 \oint \vec{E}\cdot\vec{ds} = q_{enclosed} = -Q$

$E\left[4\pi\left(\frac{5R}{2}\right)^2\right] = -Q/\epsilon_0$

$E = -\frac{Q^2}{4\pi\epsilon_0}\cdot\frac{1}{25R^2}$

(c) u = energy density $= \frac{1}{2}\epsilon_0 E^2$

$U = \frac{K^2}{2}\epsilon_0\left[\int_R^{2R}\frac{Q^2}{r^4}(4\pi r^2)\,dr + \int_{2R}^{3R}(4\pi r^2)\,dr + \int_{3R}^{4R}\left(\frac{4Q^2}{r^4}\right)(4\pi r^2)\,dr\right]$

$= \frac{KQ^2}{R}$

(22) (i) $\frac{1}{2}Kx_{max}^2 = (qE)x_{max}$

$x_{max} = \frac{2qE}{K}$

(ii) In eqn. position $F_R = 0$

$Kx_0 = qE$

$x_0 = qE/K = \frac{1}{2}x_{max}$

(iii) $F = -K(x+x_0) = qE = -Kx \Rightarrow$ SHM

(23) since $\frac{Kq^2}{L^2} > mg \Rightarrow$ string will not slack.

$$\frac{1}{2} m u_{min}^2 = mg(2L)$$

$$u_{min} = \sqrt{4gL} = 6 \text{ m/s}$$

(2.4) $V(x, y) = 0$

$$\frac{Kq}{\left[(x-b)^2 + y^2\right]^{1/2}} + \frac{K(-cq/b)}{\left[\left(x - \frac{c^2}{b}\right)^2 + y^2\right]^{1/2}} = 0$$

solving: $\boxed{x^2 + y^2 = c^2}$ circle

(25) $V_0 = (V_+) + (V_-)$

$$V_+ = \frac{3}{2} \cdot \frac{Q}{4\pi\epsilon_0 R} = \frac{3}{2} \cdot \frac{\rho \cdot \left(\frac{4}{3}\pi R^3\right)}{4\pi\epsilon_0 R} = \frac{\rho R^2}{2\epsilon_0}$$

$$V_- = \frac{-Q}{4\pi\epsilon_0 R/2} = \frac{-\rho \cdot \frac{4}{3}\pi (R/2)^3}{4\pi\epsilon_0 (R/2)} = -\frac{\rho R^2}{4\epsilon_0}$$

$$\boxed{V_0 = \rho R^2 / 4\epsilon_0}$$

$V_{0_1} = (V_+) - (V_-)$

$$V_+ = \frac{Q}{8\pi\epsilon_0 R^3}(3R^2 - r^2) = \frac{\rho \cdot (4/3 \pi R^3)}{8\pi\epsilon_0 R^3}\left[3R^2 - (R/2)^2\right] = \frac{11\rho R^2}{24\epsilon_0}$$

$$V_- = -\frac{3}{2} \cdot \frac{Q}{4\pi\epsilon_0 (R/2)} = -\frac{3}{2} \cdot \frac{\rho \cdot \frac{4}{3}\pi (R/2)^3}{4\pi\epsilon_0 (R/2)} = -\frac{\rho R^2}{8\epsilon_0}$$

PHYSICS SOLUTIONS

Electrostatics Booklet **Exercise #2 (Subjective)**

1

1. By application of Gauss' Law, the net electrostatic flux associated with a closed surface and the net electrostatic charge enclosed within it are related by the expression $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$

Therefore when the position of a charged particle is moved from the geometric center to some asymmetric point within the same spherical Gaussian surface, the LHS of the above equation is unchanged, therefore the RHS which represents the total flux through it remains **UNCHANGED**.

2

2. From the definition of Electric Potential at a given point, $V = \sum\left(\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}\right)$, where q_i, r_i are the charges and respective distances of charges from the given point in the system, the presence of a proton (positively charged) additionally in a system will INCREASE the Electric Potential at every point in the vicinity of it.

When a proton (of charge +e) is released inside a uniform Electric Field $\vec{E}$, the workdone by the constant electrostatic force $\vec{F} = e\vec{E}$ acting on it is given by $W = \int e\vec{E}.d\vec{r} = eEs$ where s is the displacement of the proton and can be shown to be equal to $W = \frac{1}{2}mv^2$ where v and m are instantaneous speed and mass of the proton, by application of Work-Energy theorem. Therefore, the workdone by the Electrostatic Force (internal conservative force considering the proton and the uniform Elec field to be a 'system') is positive for any given interval. Therefore, by definition, the change in Electrostatic Potential Energy $\Delta U = -W_{\text{int}\,C} = -\int e\vec{E}.d\vec{r} = -\frac{1}{2}mv^2$ Or simply stated the Electrostatic Potential Energy DECREASES.

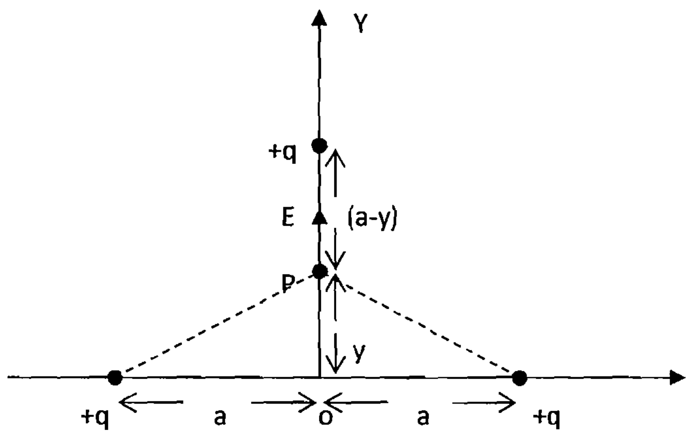
3

3. The electric field at a point on the 'Axial' line for an electric dipole at a distance of 'r' from the center of the dipole is given by the expression $E \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ when r is very large compared to the geometrical size of the dipole. Hence when the distance is doubled, the Electric Field will reduce by a factor of (1/8), therefore the force acting on the particle at the new position $F' = \frac{F}{8}$

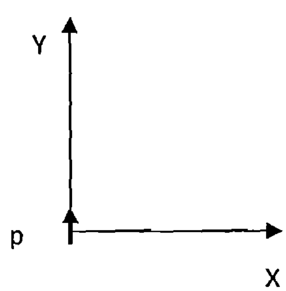
4

4. (a) Assuming the geometrical size of an electric dipole to be 'small', when placed in a non-uniform Electric filed $\vec{E}$, the net force acting on it can be shown to be $\vec{F} = \vec{\nabla}E.\vec{p}$, where $\vec{\nabla}E = \frac{\delta E}{\delta x}\hat{i} + \frac{\delta E}{\delta x}\hat{j} + \frac{\delta E}{\delta z}\hat{k}$ is the 'gradient' of the Electric field $\vec{E}$ and $\vec{p}$ is the electric dipole moment . Now the above equation can be simplified

to $F = \dfrac{dE}{dx} \times p$ for a one-dimensional situation where $\vec{E}$ and $\vec{p}$ are both along the same direction (X-axis). This is the case in the given problem. Let us consider the dipole to be along the Y-axis and it's bottom to be the origin. From symmetry, the electric field at any point on the y-axis due to the three charged particles q_1, q_2 and q_3 will also be along the Y-axis. To calculate the same consider the following diagram



The electric Potential at the point P (0,y) is given by $V = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{2q}{\sqrt{a^2+y^2}} + \dfrac{q}{(a-y)}\right]$ Therefore the Electric Filed at P, $E = -\dfrac{dV}{dy} = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{2qy}{\left(a^2+y^2\right)^{3/2}} - \dfrac{q}{(a-y)^2}\right]$ (along the Y-axis) as shown. Now, for a small Electric dipole placed at 'O'



The net Force acting on it $F = \dfrac{dE}{dy} \times p = \dfrac{1}{4\pi\varepsilon_0}\left[2q\dfrac{\left(a^2+y^2\right)^{3/2} - 3y^2\left(a^2+y^2\right)^{1/2}}{\left(a^2+y^2\right)^3} - q\dfrac{2}{(a-y)^3}\right] \times p$,

also since the 'position' of the dipole is y=0, substituting in the above equation,

$$F == \frac{1}{4\pi\varepsilon_0}\left[2q\frac{\left(a^2\right)^{3/2}}{\left(a^2\right)^3} - q\frac{2}{(a)^3}\right] \times p = 0$$

(b) Electrostatic Potential Energyof the system $U = U_{q_1,q_2} + U_{q_1,q_3} + U_{q_2,q_3} + U_{p,(q_1+q_2+q_3)}$ i.e the total energy being the sum of the Potential energies for all 'pairs' of components in the given system.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a\sqrt{2}} + \frac{q^2}{2a} + \frac{q^2}{a\sqrt{2}}\right] + \frac{1}{4\pi\varepsilon_0}\left[0 + \frac{pq}{a^2} + 0\right]$$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a}\left(\sqrt{2}+2\right) + \frac{pq}{a^2}\right]$$

5

5. Workdone to remove each of them to infinity is $W = \Delta U = 0 - U$ where U is the Potential Energy of the system in the given configuration whereas when removed to infinity the PE will be 0.

$$U = \sum\left[\frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\left(\frac{-1}{1}+\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{-1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+..\right.$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\left(\frac{-3}{1}+\frac{3}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{3}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{2}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-1}{1}+\frac{2}{\sqrt{2}}+\frac{-1}{\sqrt{3}}\right)+\left(\frac{-2}{1}+\frac{1}{\sqrt{2}}\right)+\left(\frac{-1}{1}+\frac{1}{\sqrt{2}}\right)+\left(\frac{-1}{1}\right)\right.$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[(-1\times 12)+\left(\frac{1}{\sqrt{2}}\times 12\right)+\left(\frac{-1}{\sqrt{3}}\times 4\right)\right]$$

Note: The above expression can be most simply deduced from the fact that the given cube has 12 sides, 12 face diagonals and 4 body diagonals (geometrically) and any side chosen has opposite sigh charges at both ends, any face diagonal chosen has same sign charges and any body diagonal opposite charges (from the charge distribution)

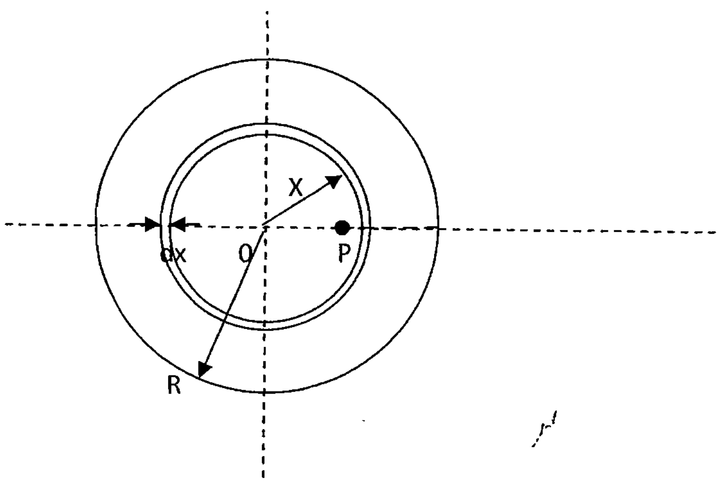
Therefore, $U = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{a}\left[3\times\left(\frac{1}{\sqrt{2}}-1\right)-\left(\frac{1}{\sqrt{3}}\right)\right]$, $W = -U = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{a}\left[3\times\left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{3}}\right)\right]$

6

6. If the particle is projected with a speed v, the condition for it to penetrate to a point P at $r = (R/2) = 20cms$ from the center is given by $\frac{1}{2}mv^2 \geq +q(V_P - V_A)$ where V_A and V_P are the electric potentials at the initial point of projection A (r>R) and at P. (For minimum value of v take limiting condition in the inequality)

$V_A \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (since A is a point 'external' to the dielectric shell)

Whereas for the Point P which is 'inside' the shell, potential can be calculated by integrating over differential spherical shells of radii 'x' and thickness 'dx' ($0 < x < R$)



The distance OP= (R/2). Now, for a shell with x<(R/2), the potential

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R/2)}, \text{ where } dq = \rho \times 4\pi x^2 dx \text{ and } \rho = \frac{3Q}{4\pi R^3} \text{ or } dq = \frac{3Q}{R^3}x^2 dx$$

Whereas for a shell with (R/2) <x<R, the potential

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x} \text{ where. } dq = \frac{3Q}{R^3}x^2 dx$$

Therefore $V_P = \frac{1}{4\pi\varepsilon_0}\int_{x=0}^{x=R/2}\frac{dq}{(R/2)} + \frac{1}{4\pi\varepsilon_0}\int_{x=R/2}^{x=R}\frac{dq}{x}$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{6Q}{R^4}\int_{x=0}^{x=R/2} x^2 dx + \frac{1}{4\pi\varepsilon_0}\frac{3Q}{R^3}\int_{x=R/2}^{x=R} x dx$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{6}{3}\left(\frac{1}{2^3} - 0\right) + \frac{3}{2}\left(1 - \frac{1}{2^2}\right)\right]$$

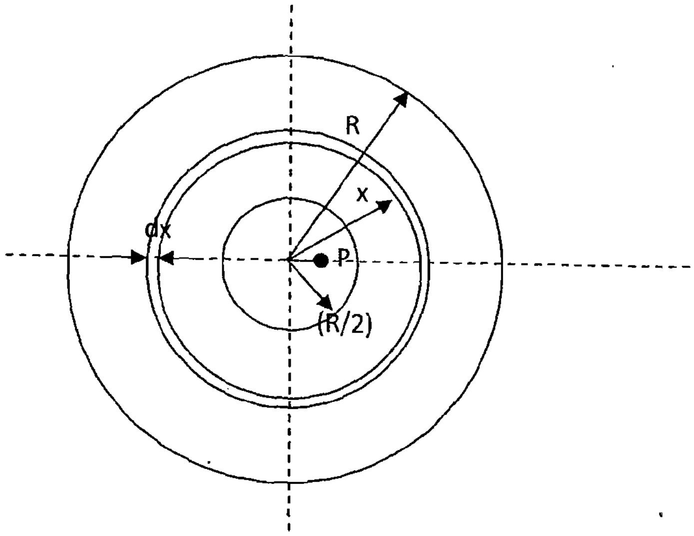
$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{11}{8}\right]$$

Therefore from the equation $\frac{1}{2}mv^2 \geq +q(V_P - V_A)$

$$v = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{m}\left[\frac{11}{8R} - \frac{1}{r}\right]}$$

7

7.



The **Electric Field** at the point P will be **ZERO** (By application of Gauss's Law and spherical symmetry)

To calculate the Potential at the point 'P' as shown, consider differential spherical shells of radii 'x' and thickness 'dx' ($(R/2) < x < R$), potential due to such a shell at P,

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x}$$, where $dq = \rho \times 4\pi x^2 dx$ and $\rho = \frac{3Q}{4\pi\left(R^3 - \frac{R^3}{8}\right)}$ or $dq = \frac{24Q}{7R^3}x^2 dx$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\int_{x=R/2}^{x=R}\frac{dq}{x}$$

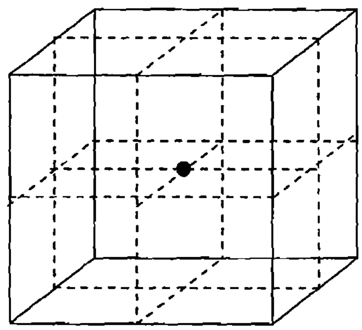
$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{24Q}{7R^3}\int_{x=R/2}^{x=R} x\,dx$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\frac{24Q}{7R}\left[\frac{1}{2}\left(1 - \frac{1}{2^2}\right)\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{24Q}{7R}\cdot\frac{1}{2}\cdot\frac{2}{4}$$

$$\Rightarrow V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{9Q}{7R}\right)$$

8. The flux here can be calculated by application of superposition and symmetry in Gauss's Law. Consider a cubical Gaussian surface of dimension ($2L \times 2L \times 2L$), with the particle +q at the geometrical center as shown

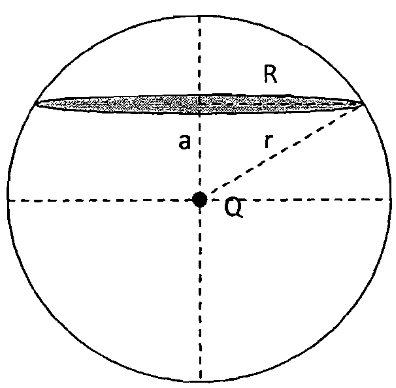
8



Now, the total flux through the entire cubical Gaussian surface is $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$ with the flux through each of the sides of dimensions ($2L \times 2L$) being equal (by symmetry) and therefore equivalent to $\frac{q_{enc}}{6\varepsilon_0}$, now further subdividing each side into four symmetrical squares of dimensions ($L \times L$), the flux through each again being equal (by symmetry) and therefore equivalent to $\frac{1}{4} \times \frac{q_{enc}}{6\varepsilon_0} = \frac{q}{24\varepsilon_0}$

9

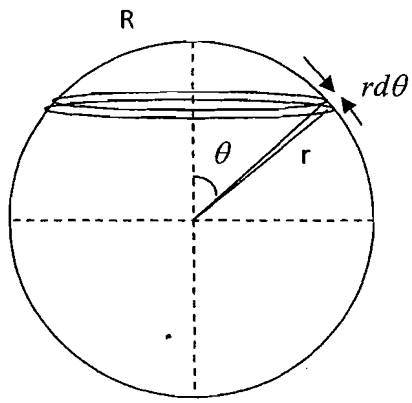
9.



The total flux associated with a spherical Gaussian surface of radius $r = \sqrt{R^2 + a^2}$ as shown in the figure is $\oint \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_0}$ Now, this flux is distributed symmetrically in 3D, therefore the fraction of it passing through the shaded disc shown will be proportionate to the solid angle subtended by the disc at the center (where the charge Q is located). Therefore if this is (1/4th) of the total flux, the solid angle subtended by the portion of the spherical surface 'sliced' out by the disc must be π (total solid angle for a closed surface being 4 π steradians). Hence the area of the portion of the spherical surface

$$S = \pi r^2 \text{ (solid angle } = \frac{S}{r^2})$$

Therefore from the figure below



Taking an elemental ring of radius $rSin\theta$ and thickness $rd\theta$, the area of the sliced portion,

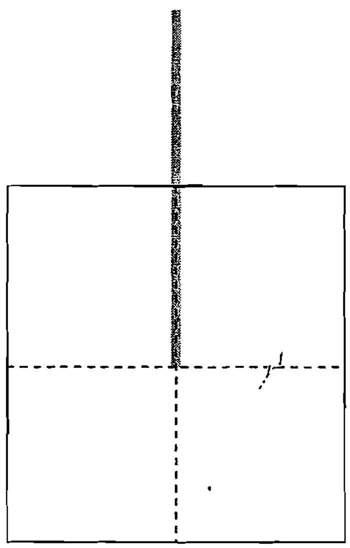
$$S = \int_{\theta=0}^{\theta=Cos^{-1}(a/r)} 2\pi r^2 Sin\theta d\theta = \pi r^2$$

$$\Rightarrow \left[1 - \frac{a}{r}\right] = \frac{1}{2} \Rightarrow r = 2a \Rightarrow \sqrt{a^2 + R^2} = 2a$$

$$\Rightarrow R = a\sqrt{3}$$

10

10. For the given situation, in order to minimize the flux, the portion of the rod enclosed within the cubical surface has to be minimized which will be for the config shown below giving Flux $\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{2\varepsilon_0}$



11

11. The Electric Field at E at a any point on the axis of the disc shaped hole which is at a distance 'x' from the sheet can be calculated by application of superposition, $E = E_{InfiniteSheet} - E_{Disc}$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) = \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{R^2 + x^2}}$$

Therefore the potential difference between the two points P ($x = R\sqrt{3}$) and O (x=0) can be calculated from the relation $\Delta V = -\int \vec{E}.d\vec{r}$

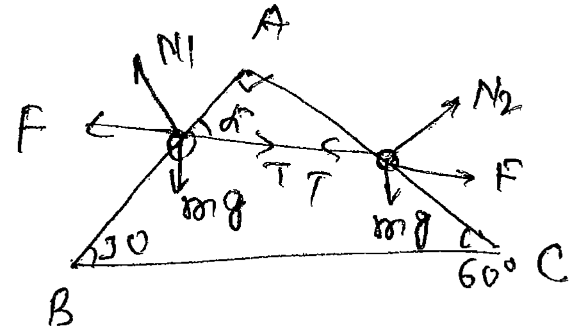
$$\Delta V = -\int_{x=R\sqrt{3}}^{x=0} \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{R^2 + x^2}} dx = \left| -\frac{\sigma}{2\varepsilon_0}\sqrt{R^2 + x^2} \right|_{x=R\sqrt{3}}^{x=R} = \frac{\sigma R}{2\varepsilon_0}.$$

Now, by application of Work Energy to the electron released at P,

$$\frac{1}{2}mv^2 = e\Delta V = \frac{e\sigma R}{2\varepsilon_0}$$

$$\Rightarrow v = \sqrt{\frac{e\sigma R}{m\varepsilon_0}}$$

(12) (a)



(i) $T\cos\alpha = F\cos\alpha + mg\sin 30^\circ$

(ii) $F\sin\alpha + N_1 = mg\cos 30^\circ + T\sin\alpha$

(iii) $T\sin\alpha = F\sin\alpha + mg\cos 30^\circ$

(iv) $N_2 + F\cos\alpha = T\cos\alpha + mg\cos 60^\circ$

solving $\Rightarrow \boxed{\alpha = 60^\circ}$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{l^2}$$

$$\Rightarrow T = mg + \frac{q_1 q_2}{4\pi\epsilon_0 l^2}$$

$N_1 = \sqrt{3}\, mg$, $N_2 = mg$

(b) $T = 0$

$\boxed{F = -mg}$

$$q_1 q_2 = (4\pi\epsilon_0 l^2)\, mg$$

12. ~~use concept of equilibrium by drawing FBD.~~

13

13. Consider a section of the rod of differential length 'dy' at a distance 'y' above the surface of the ring. The force experienced by this differential element would be $dF = dqE$, where $dq = \lambda dy$ and $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qy}{\left(R^2+y^2\right)^{3/2}}$ is the Electric Field due to the charged ring at the location of the diff' element.

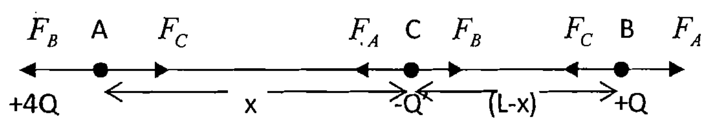
$$\Rightarrow F = \int_{y=0}^{y\to\infty} \frac{1}{4\pi\varepsilon_0}\frac{\lambda Q y\,dy}{\left(R^2+y^2\right)^{3/2}}$$

$$\Rightarrow F = \frac{1}{4\pi\varepsilon_0}\lambda Q\left[\frac{-1}{\sqrt{R^2+y^2}}\right]_{y=0}^{y\to\infty}$$

$$\Rightarrow F = \frac{\lambda Q}{4\pi\varepsilon_0 R}$$

14

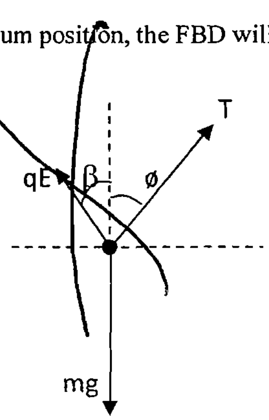
14. Since all the three charges will have to be in equilibrium, the third charge has to be a negative one placed somewhere between the other two positive charges as shown in the figure below



Balancing forces,

$\dfrac{4Q^2}{L^2} = \dfrac{4QQ'}{x^2}$ and $\dfrac{4QQ'}{x^2} = \dfrac{QQ'}{(L-x)^2}$ solving these, $-Q' = -\dfrac{4}{9}Q$ and $x = \dfrac{2}{3}L$ (distance from +4Q)

15. At the equilibrium position, the FBD will be as follows



Therefore at equilibrium $\vec{T} = m\vec{g} + q\vec{E}$ (T: Tension in the string, E is Electric Field)

$\Rightarrow |\vec{T}| = \sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)\text{Cos}\beta} = mg'$ where the time period can now be expressed as

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{ml}{\sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)\text{Cos}\beta}}}$$

and the angle ø is given by $\phi = \text{Tan}^{-1}\left(\frac{qE\text{Sin}\beta}{mg - qE\text{Cos}\beta}\right)$

15 ~~16.~~ The force acting on the particle is given by

$\vec{F} = -q\vec{E}$ where E is the electric field due to the positively charged ring given by $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$

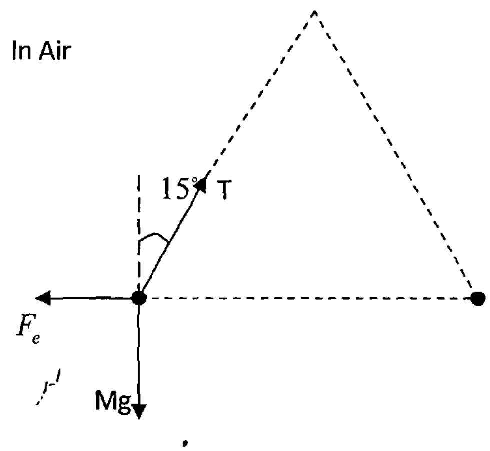
where x is the distance from the center of the ring and the direction of the field is directly away from the center along the axis.

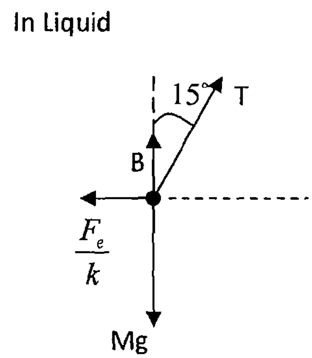
Since the particle is negatively charged, it experiences a force $F = \frac{1}{4\pi\varepsilon_0}\frac{qQx}{(R^2 + x^2)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{qQx}{R^3}$ for x<< R

directed towards the origin (center of the ring). Hence if released from such a position, it will experience an acceleration $a = -\frac{d^2x}{dt^2} \approx \frac{1}{4\pi\varepsilon_0}\frac{qQx}{mR^3}$ (for x<<R). Comparing with the standard equation of SHM

$\frac{d^2x}{dt^2} = -\omega^2 x$, time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{qQ}}$

16 ~~17~~. The FBDs in 'air' and inside the liquid are as follows





From these it is evident that if the angle remains the same 15°, B= Buoyant Force = Wt of liquid displaced, F_e: Electrostatic force and k is the dielectric constant for the medium,

$\frac{F_e}{Mg} = \frac{(F_e / k)}{(Mg - B)}$ and also for a completely submerged object $B = \frac{Mg}{(\rho_m / \rho_l)}$ where ρ_m and ρ_l are densities of the material of the object and the liquid respectively.

$$k = \frac{(Mg)}{(Mg - B)} = \frac{1}{1 - (\rho_l / \rho_m)} = 2$$

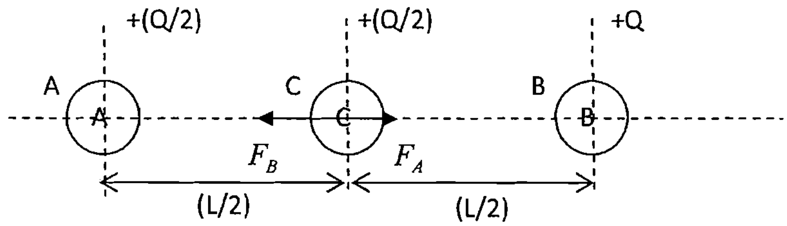
17 ~~18~~. As is evident from the question, the second particle might be either attracted or repulsed by the first due to electrostatic forces such that the maximum value of this Electric Force is equal to the limiting value of static friction $F_e \le \mu mg \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q \times 1.0 \times 10^{-6}}{(0.1)^2} \le 0.1 \times 80 \times 10^{-3} \times 10$

$\Rightarrow |q| \le 8.7 \times 10^{-8}\,C$. Therefore the charge q can range between $-(8.7 \times 10^{-8})C$ and $+(8.7 \times 10^{-8})C$

18 ~~19~~. Let the charge on spheres A and B be '+Q' each and the distance between them is 'L'

Therefore the electrostatic repulsion $F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{L^2} = 2 \times 10^{-5} N$

Now, when a third sphere C is made to touch C and placed at the mid-point, the charge on A gets 'shared' equally between them (they have identical capacitance)



Therefore, the net force on C,

$$F = F_B - F_A = \frac{1}{4\pi\varepsilon_0}\frac{Q\times(Q/2)}{(L/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)^2}{(L/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{L^2} = 2\times10^{-5}\,N$$

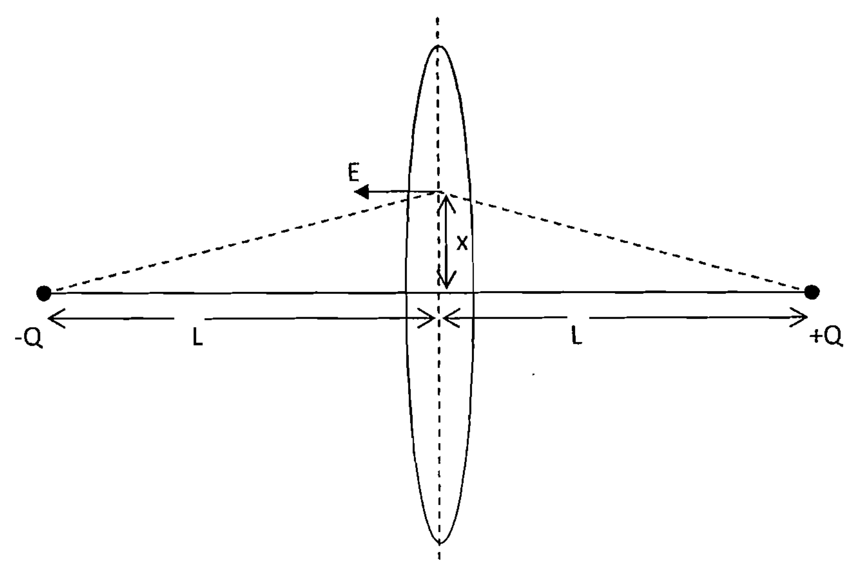
19

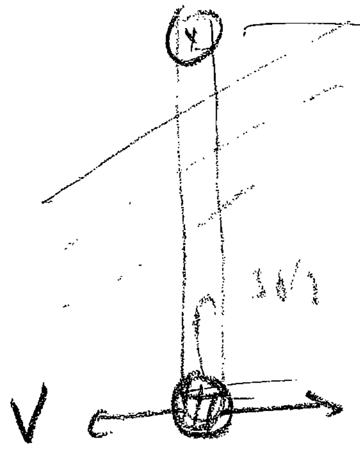
20. The electric field at any point on the disc at a distance of 'r' from the center can be calculated from the formula (E at an equatorial point for an electric dipole)

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q\times 2L}{(r^2+L^2)^{3/2}}$ with direction as shown in the figure (normal to the plane of the disc). Therefore the electric flux through a differential 'ring' sliced out on the disc of radius 'r' and thickness 'dr' would be

$d\phi_E = E\times dS = \frac{1}{4\pi\varepsilon_0}\frac{2QL}{(r^2+L^2)^{3/2}}\times 2\pi r dr = \frac{QL}{\varepsilon_0}\frac{rdr}{(r^2+L^2)^{3/2}}$, therefore the total flux through the

disc $\phi_E = \frac{QL}{\varepsilon_0}\int_{r=0}^{r=R}\frac{rdr}{(r^2+L^2)^{3/2}} = \frac{QL}{\varepsilon_0}\left[\frac{-1}{\sqrt{(r^2+L^2)}}\right]_{r=0}^{r=R} = \frac{Q}{\varepsilon_0}\left(1-\frac{L}{\sqrt{R^2+L^2}}\right)$





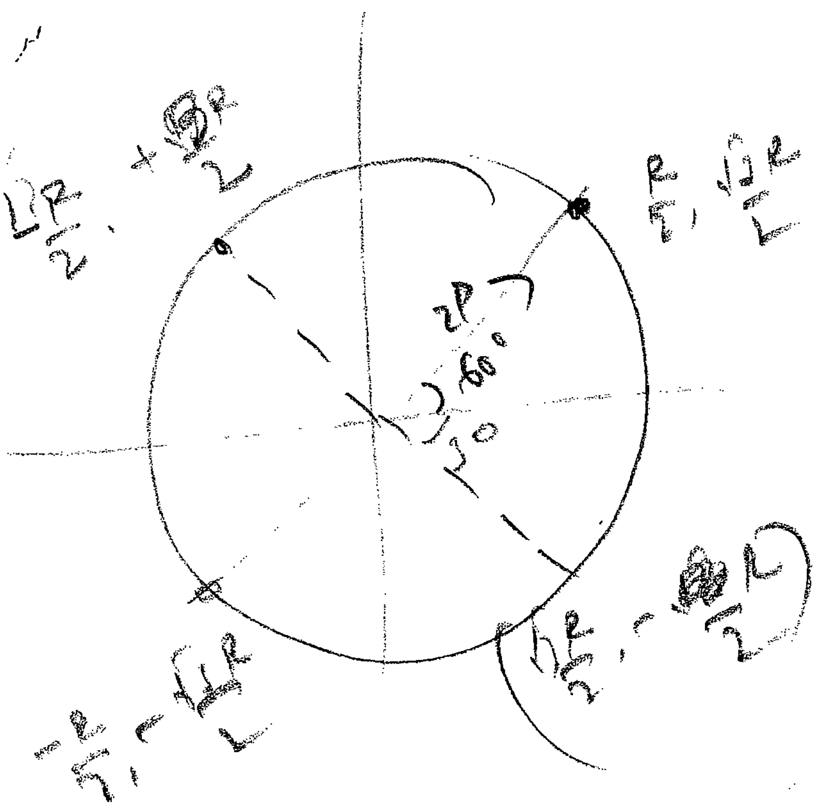
$v^2 = \frac{3g}{2}$

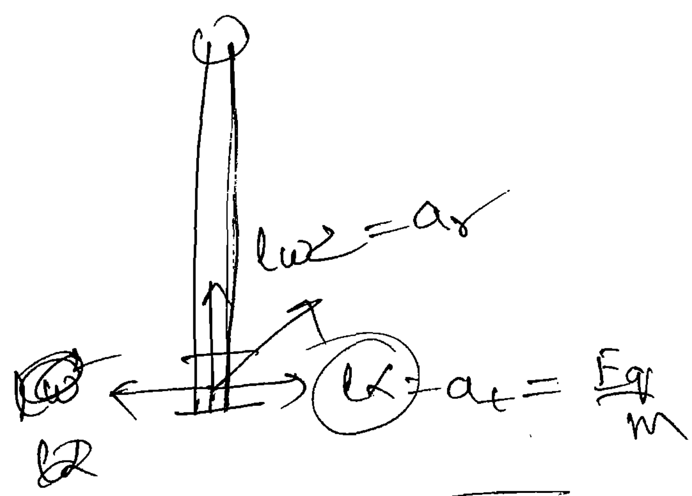
$a_t = \frac{Eq}{m} = g/2$

$\sqrt{\left(\frac{3g}{2}\right)^2 + \left(\frac{g}{2}\right)^2}$

$= \sqrt{\frac{g^2}{4}}$

$\frac{g}{2}\sqrt{5}$





$$\sqrt{\left(\frac{g}{2}\right)^2 + (l\omega^2)^2}$$

$\bar{p} = 2p\left(\frac{1}{2} \quad \frac{\sqrt{3}}{2}\right)$

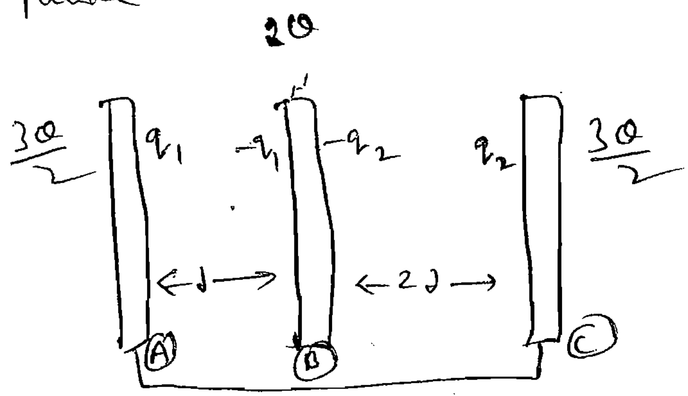
$= 2p\left(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}\right)$

capictance Ex#1

Q. 20.

Capacitance Ex-1

1 Final



$$\frac{3Q}{2} + \frac{3Q}{2} + q_1 + q_2 = Q.$$

$$\boxed{q_1 + q_2 = -2Q}$$

$$V_A = V_C$$

$$\therefore\ V_A - V_B = V_C - V_B$$

$$\Rightarrow \left(\frac{q_1}{\cancel{\varepsilon_0 A}}\right)\cancel{d} = \left(\frac{q_2}{\cancel{\varepsilon_0 A}}\right) 2\cancel{d}$$

$$\boxed{q_1 = 2q_2}$$

$$q_1 + \frac{q_1}{2} = -2Q$$

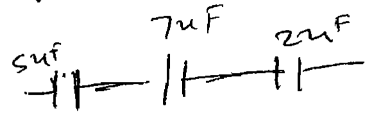
Plate

Charge flown

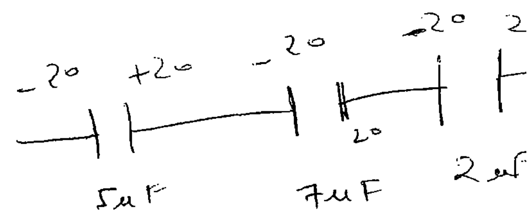
$$= Q - \frac{Q}{6} = \frac{5Q}{6}.$$

(2)

(2) Eq. Capacitance.



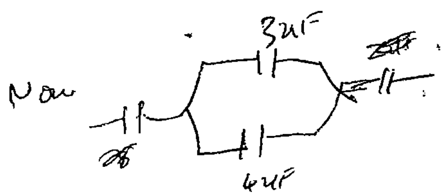
$\Rightarrow$



Charge on 7uF = 20uC.

$\therefore V = \frac{Q}{C} = \frac{20\mu C}{7\mu F}$ (V across 7uF)

Now



V is same,

$\therefore$ Charge on 3uF

$$Q = CV$$
$$= 3\mu F \times \frac{20\mu C}{7\mu F}$$
$$= \frac{60}{7}\ \mu C$$
$$= 8.57\ \mu C.$$

On right plate,

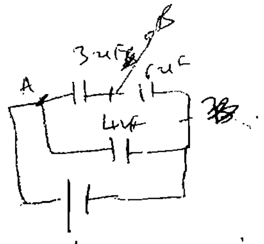
$$Q = +8.57\ \mu C.$$

Ans $\rightarrow$ (A)

(3) 5uF is short-circuited,

(D)

4. Eq. Circuit =



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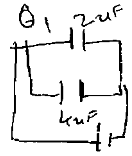
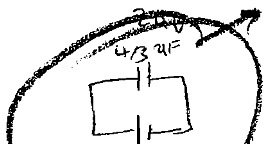
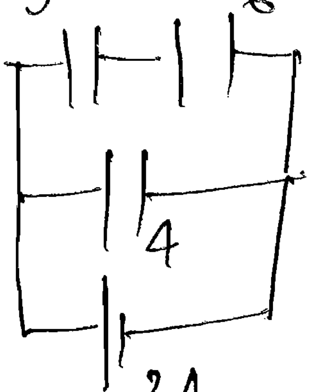




→ 6

(4)



$V_6 = 24 \times \frac{3}{9} = 8$

$U = \frac{1}{2} C V^2$

$= 32$

→ ~~Total Q = CV~~

$= \cancel{24 \times \frac{4}{3}}$ μF

$= \cancel{32}$ μC.

$Q_1 = CV$

$= 2\mu F \times 24V$

$= 48\mu C.$

P.D across A.B $= \frac{48\mu C}{3\mu F}$

$= 16$ V.

∴ P.D across 1μF capacitor = 8V.

$Q = CV$

$= 1\mu F \times 8.$

$= 8\mu C.$

∴ energy $= \frac{Q^2}{2C}$

$= \frac{64}{2 \times (1)}$

$= 32\mu J$

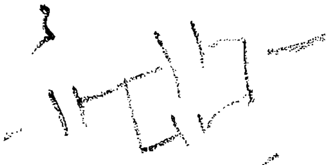
6

24

16 ... = 8

6

3



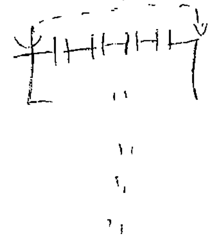
20

$\frac{1}{2} \cdot 1 \cdot 64$

(c)

5. Each capacitor $= \dashv\vdash$ $8\mu F, 250V$.

5



P.D = 1000V.

8 times.

$\therefore 8 \times 4 = 32$.

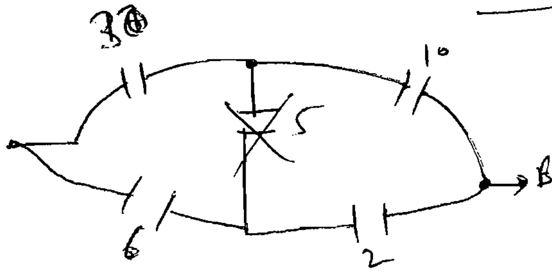
1000V will be produced by 4 in series (250 each)

But then, series → $2\mu F$ ∴

So, 8 in parallel rows.

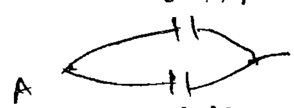
$\therefore 8 \times 4 = 32$.

6



Balanced wheat stone.

∴ 5 can be neglected



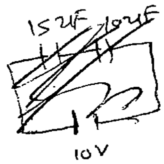
$30 \times 10 / 40$

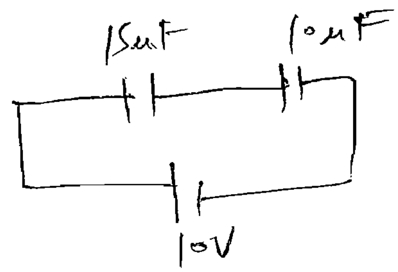
$\frac{6 \times 2}{8}$

$= 9\mu F$.

7. (D) System is not changing in any way.

8 Eq. Capacitance





$= \frac{15 \times 10}{15+10} \mu F$

(B) $= 6\mu F$

9 Total $Q = CV$

$= \cancel{6\mu F} \times 10 = 6 \times 10 \mu C = 60 \mu C$

divided in $5\mu F$ and $10\mu F$

$Q = CV$

V same in $5\mu F$ and $10\mu F$

So charge divided in ratio of capacitance

∴ 1 : 2

$\frac{60}{15} = 4V \to$ P.D.

$\therefore Q = CV$

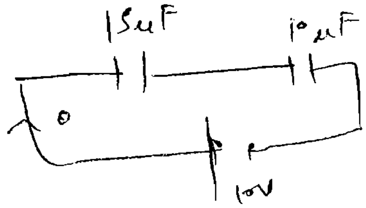
$= 5\mu F \times 4$

$= 20\mu C$.

10 (10) P.D across 5uF is ~~60~~ 4V.

$\therefore$ across 6uF is 6V.

(A)



Charge flown same
$Q = CV$ So Potential divides inverse of capacitance
$15\,\mu F \rightarrow 4V$
$10\,\mu F \rightarrow 6V \rightarrow$ same in $6\,\mu F$

(11) (11) P-D across AB is 4V, BC is 6V.

$\therefore$ Energy of 10uF $= \frac{1}{2}CV^2$
$= 5u \times (4)^2$
$= ~~20~~80$

Energy of 5uF $= ~~10~~40$

Energy of 4uF $= \frac{1}{2}(4)(6)^2$
$= 72$

Energy of 6uF $= \frac{1}{2}(6)(6)^2$
$= 108.$

(B) $\therefore$ Max. Energy is of 6uF Capacitor.

12. $W = \frac{Q^2}{2C}$

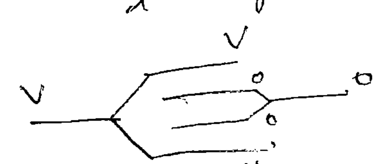
$= \frac{64 \times 10^{-36}}{2 \times 100 \times 10^{-6}}$

$= 32 \times 10^{-32}$

(b)

(12) It is effectively two Capacitor of Capacitance $\frac{\varepsilon_0 A}{d}$ in parallel arrangement.

$\therefore$ Net Capacitance $= \frac{2\varepsilon_0 A}{d}$.

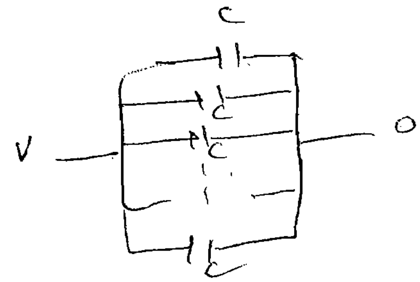


(14) 13

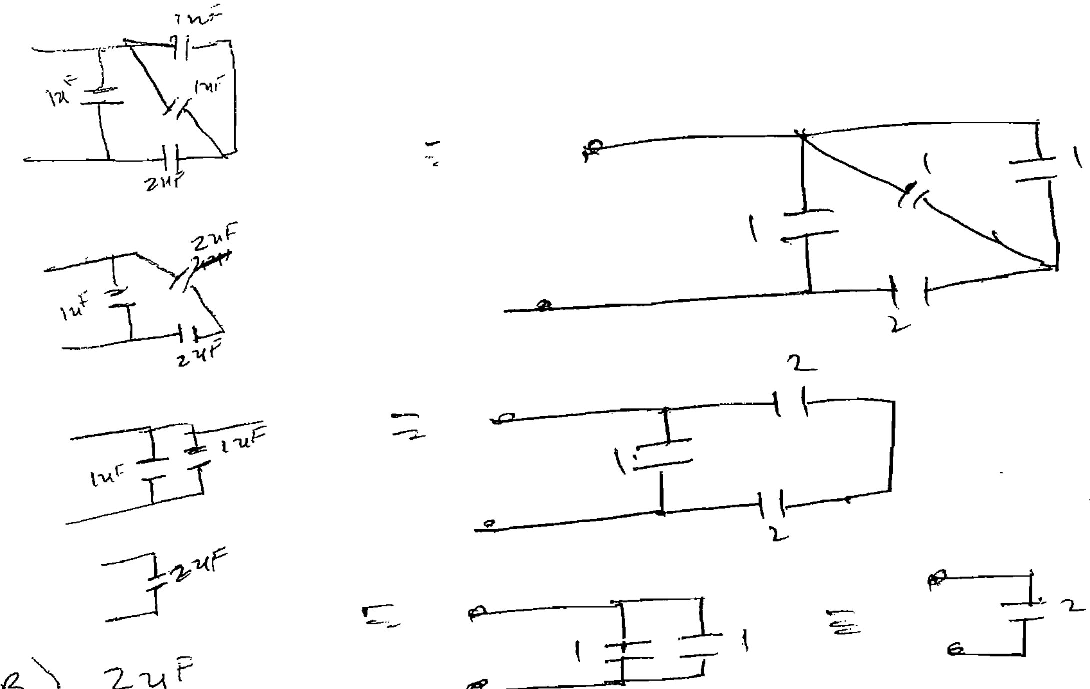
Energy in each Capacitor $= \frac{1}{2}CV^2$.

$\therefore$ Total $= \frac{1}{2}nCV^2$

$\therefore$ (D) (D)



(15) 14



(B) $2\mu F$

(16) 15

Effectively, ~~First~~ There are ~~three~~ four capacitors in series, and

the resultant is ~~parallel with the other~~

$\therefore$ net $= \frac{C}{4}$.

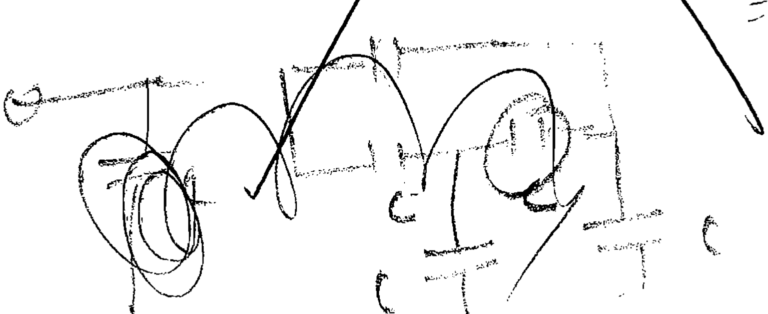
$\therefore$ P.D across each C is $-2.5V$.

$\therefore$ at B its $-2.5V$.

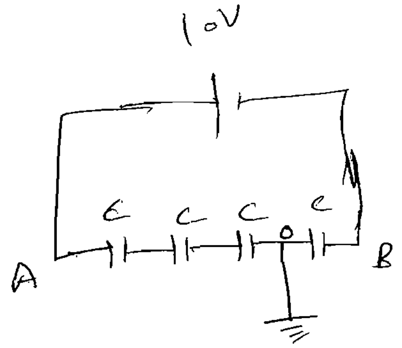
at A ; P.D $= -2.5V + 10V$

$= 7.5V$.

(B)



~~18~~ 15



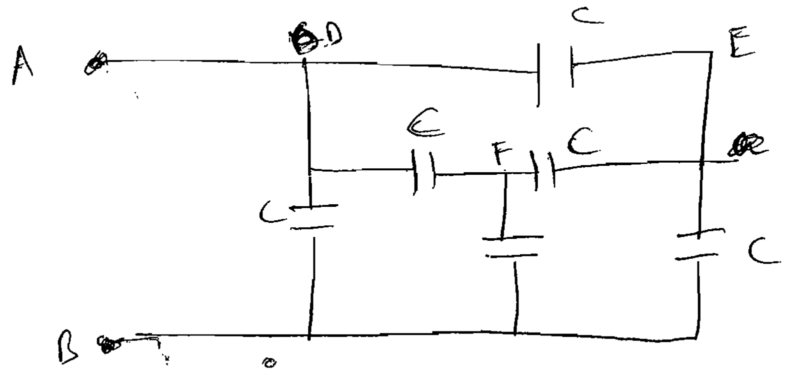
Potential will be equally divided among 4 capacitors in series such that sum = 0.

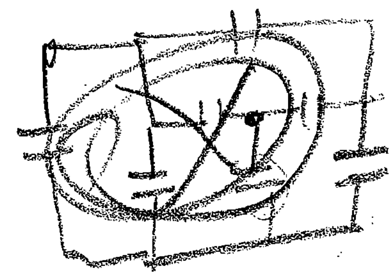
$\therefore$ B → −2.5 V

A → 7.5 V

(because potential across each capacitor = 2.5 V)

~~15~~ 16





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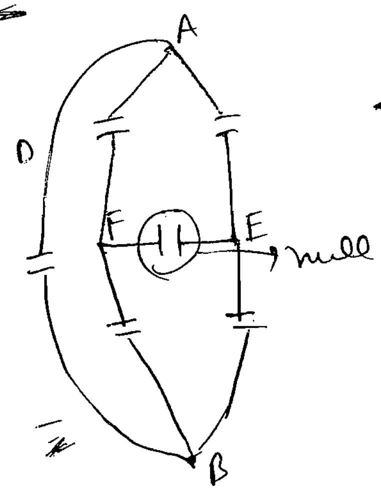
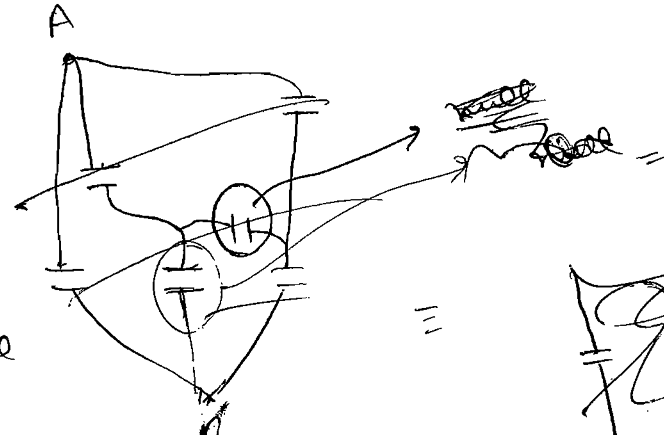
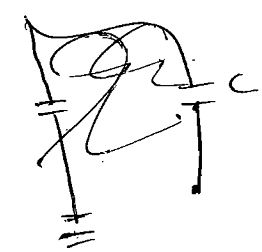




~~null~~ ~~null~~ =

≡



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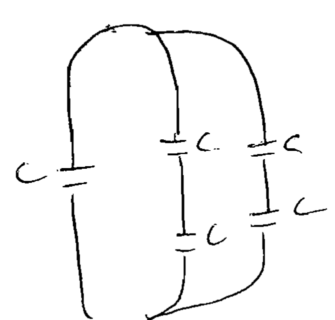

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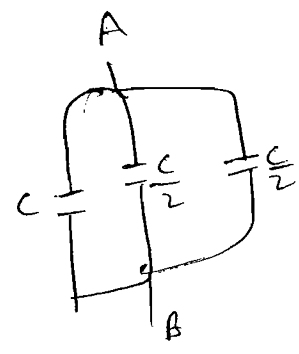

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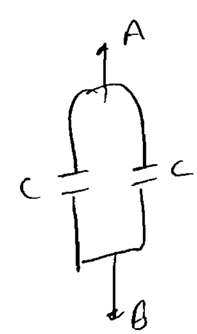

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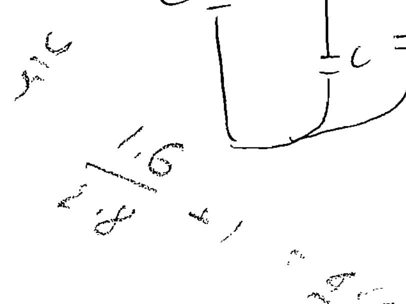
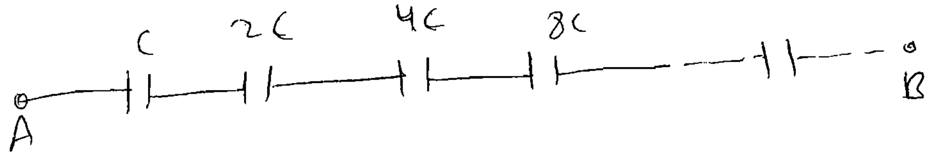


~~18~~

17



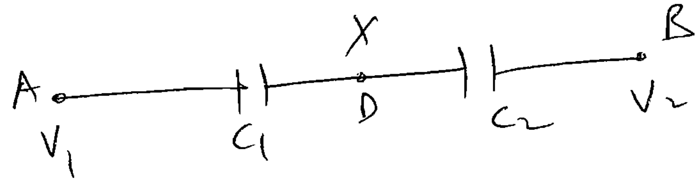
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} + \frac{1}{8C} \text{ --- } \infty$$

$$= \frac{1}{C}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \text{ --- } \infty\right]$$

$$\frac{1}{C_{eq}} = \frac{1}{C}[2]$$

$$\boxed{C_{eq} = \frac{C}{2}}$$

~~18~~



18

Let $V_D = X$

Then $(X - V_1)C_1 + (X - V_2)C_2 = 0$ (K.J.L.)

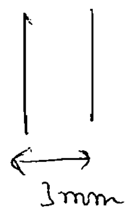
$\Rightarrow XC_1 - V_1C_1 + XC_2 - V_2C_2 = 0$

$\Rightarrow X(C_1 + C_2) = V_1C_1 + V_2C_2$

$$\therefore \boxed{X = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

~~20~~

19



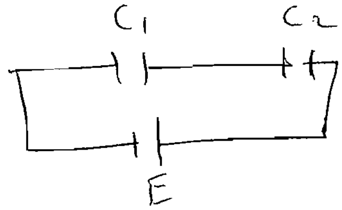
$\Delta V = 3 \times 10^5$ V.

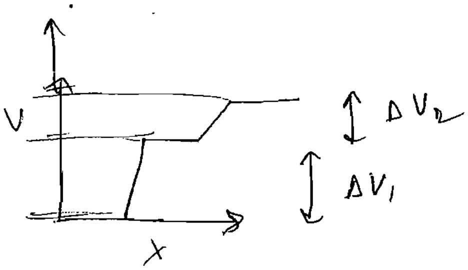
$e\Delta V = 3 \times 10^5$ eV

(a)

(21)

20





q will be same (series)

$$\left(\frac{dV}{dx}\right)_{C_1} > \left(\frac{dV}{dx}\right)_{C_2}$$

$\underline{\Delta V_1 > \Delta V_2}$

$$\therefore \quad E_{C_1} > E_{C_2}.$$

$$C_1 = \frac{q}{\Delta V_1}.$$

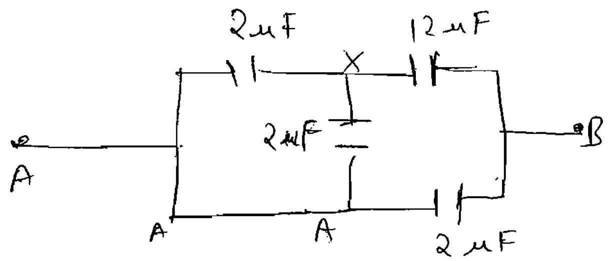
~~we don't know~~

$$\Delta V_1 > \Delta V_2$$

$$C_2 = \frac{q}{\Delta V_2} \qquad \therefore \quad \underline{C_1 < C_2}.$$

(22)

21



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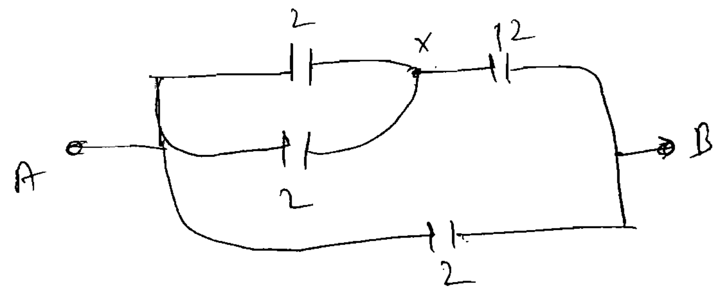

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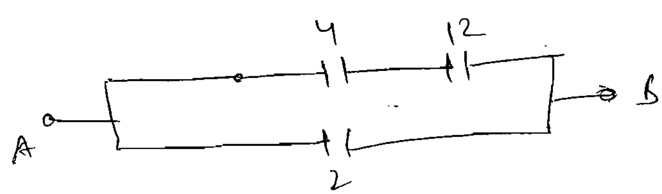
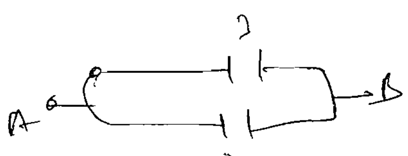


$$\frac{4\times 12}{16} = 3$$

=



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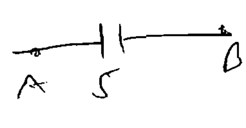
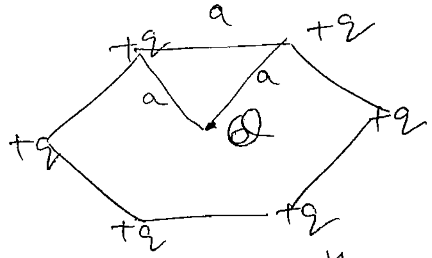


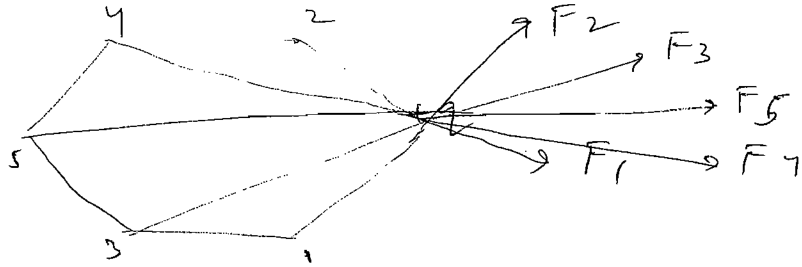
(23)

(22)



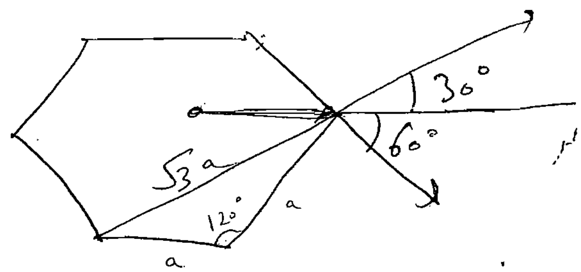
Let charge at centre be Q.

Then, On any charge,



Net force due to all q is

=



$a^2 + a^2 + \cancel{2aa\cos 60}$

$\sqrt{3}a$

$$2F_1 \cos 60° + 2F_2 \cos 30° + F_5$$

$$= 2 \times \frac{kq^2}{a^2} \cdot \frac{1}{2} + \cancel{2\times} \frac{kq^2}{(\sqrt{3}a)^2} \cdot \frac{\sqrt{3}}{\cancel{2}} + \frac{kq^2}{(2a)^2}$$

$$= \frac{kq^2}{a^2}\left[1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right].$$

This should be -ve of $\frac{kQq}{a^2}$.

$$\therefore \frac{kQq}{a^2} = -\frac{kq^2}{a^2}\left[1 + .58 + .25\right]$$

$$= -\frac{kq^2}{a^2}\left[1.83\right]$$

$\therefore$ $\boxed{Q = -1.83q}$

(24)

23

$Q = CV$

$= 3 \times 300 \times 10^{-6}$ $\quad = 5 \times 500 \times 10^{-6}$

$= 9 \times 10^{-4}$ $\quad = 25 \times 10^{-4}$.

$E_1 = \frac{1}{2} C_1 V_1^2$ $\quad E_2 = \frac{1}{2} C_2 V_2^2$

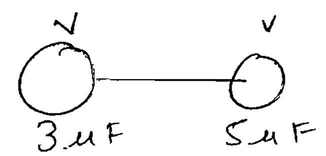
$= \frac{1}{2} (3 \times 10^{-6}) . (300)^2$ $\quad = \frac{1}{2} \times (5 \times 10^{-6}) (500)^2$

$= \frac{1}{2} \times 27 \times 10^{-2}$ $\quad = \frac{1}{2} \times 125 \times 10^{-2}$.

Initial total $= \frac{1}{2} \times [27 + 125] \times 10^{-2}$

$= 76 \times 10^{-2}$. J

Final.



Potential will be same. (Let V)

Also total charge is same.

Then $(3+5).V \times 10^{-6} = (25+9) \times 10^{-4}$

$\therefore 8 V \times 10^{-6} = 34 \times 10^{-4}$

$\therefore V = \frac{34}{8} \times 100 = \frac{1700}{4} = 425$ volt

$U_f = \frac{1}{2} (C_1 + C_2) V^2$

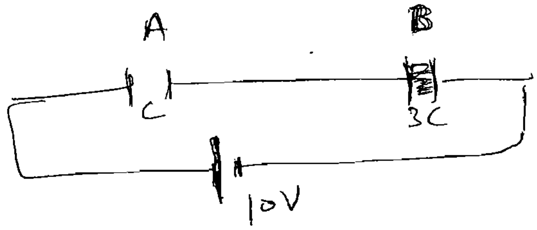
$= \frac{1}{2} (8 \times 10^{-6}) (425)^2$

$= 72.25 \times 10^{-2}$ J

Loss $= 3.75 \times 10^{-2}$ J

~~25~~

24



Charge flow is same.

$$\Delta V_1 = \frac{q}{C_1} = \frac{q}{C}$$

$$\Delta V_2 = \frac{q}{C_2} = \frac{q}{3C}.$$

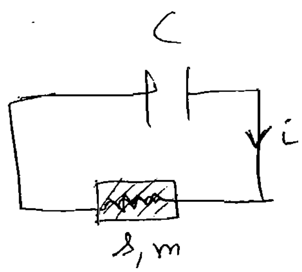
$$\therefore \frac{\Delta V_1}{\Delta V_2} = \frac{3}{1}$$

Also $\Delta V_1 + \Delta V_2 = 10.$

$$\therefore \Delta V_1 = 7.5 \quad , \quad \Delta V_2 = 2.5$$

~~26~~

25



$$\cancel{q = CV\left[1 - e^{-t/RC}\right]}$$

$$\cancel{q = CVe^{-t/RC}}$$

$$i = \frac{CV}{RC} e^{-t/RC}.$$

Heat produced $= m s \Delta T$

But heat produced $= \int_0^{\infty} i^2 R\,dt$

$$= \int_0^{\infty} i_0^2 R\,dt$$

$$= \int_0^{\infty} \frac{V^2}{R^2} e^{-2t/RC} \cdot R \cdot dt.$$

$$= \frac{V^2}{R} \frac{e^{-2t/RC}}{-2/RC}$$

$$= \left[-CV^2 e^{-2t/RC}\right]_0^{\infty}$$

$$= CV^2 \left[1\right] = \frac{1}{2} C V^2.$$

$$m s \Delta T = \frac{1}{2} C V^2$$

$$\boxed{\sqrt{\frac{2 m s \Delta T}{C}} = V}$$

(27)

$V = V$

$q = CV$

where $C = \frac{\varepsilon_0 A}{d}$

$q = \frac{\varepsilon_0 A V}{d}$

Battery disconnected, $q \to$ constant.

$\therefore$ New $C \to CK$

$q \to q$

$V \to V/K$

$V_{final} = \frac{V_0}{K}$

$E_{final} = \frac{V_0}{Kd}$

$W_{all} = \Delta KE$

$W_{ext} + W_{intc} = \Delta KE$

$W_{ext} = - W_{intc}$

$= \Delta PE$

Work done on system $=$ P.E final $-$ P.E. initial

$= \frac{1}{2}(Ck)\left(\frac{V}{k}\right)^2 - \frac{1}{2}CV^2$

$= \frac{1}{2}CV^2\left(\frac{1}{k} - 1\right)$

(28) Total flux $= q/\varepsilon_0$

flux through one wall $= \frac{q}{6\varepsilon_0}$

(29) Uniformly charged. Non-conducting

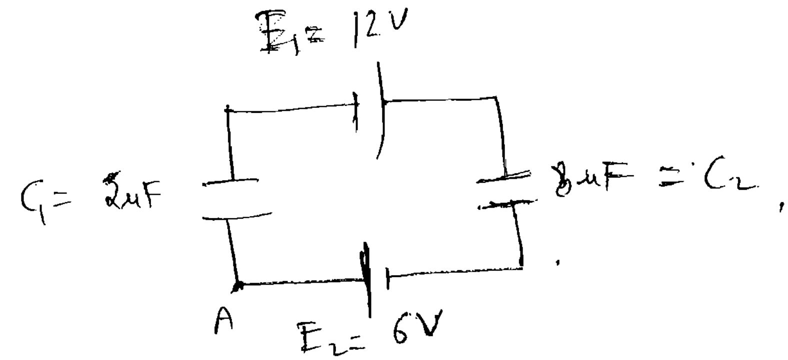
By shell theorem,

~~$E = 0$ (inside),~~

Outside electric field $\to \frac{kQ}{r^2}$ (by gauss law)

... potential constant ...

(30)

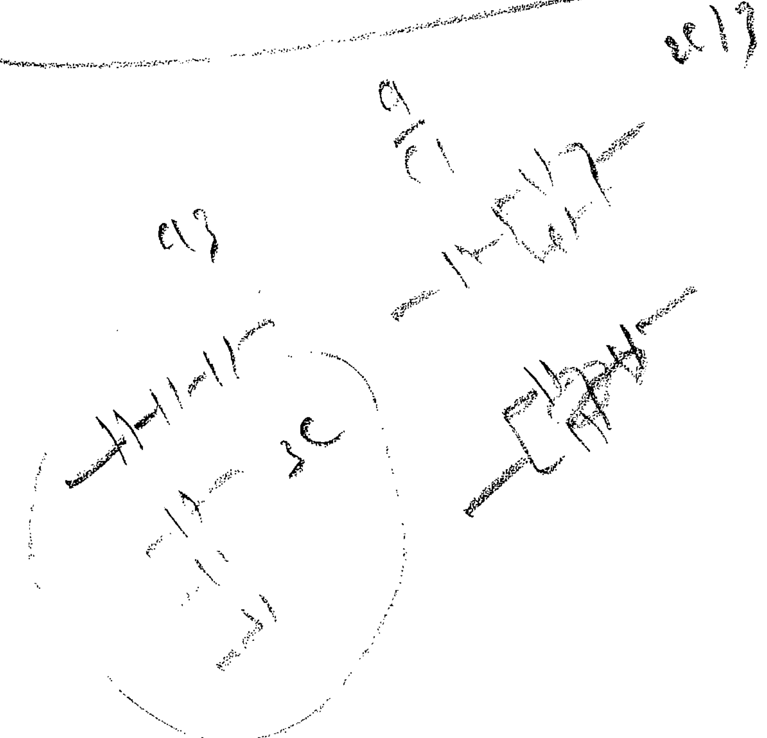


Charge flowing through C_1 and C_2 will be same

$V_{C_1} = \frac{q}{C_1}$, $V_{C_2} = \frac{q}{C_2}$.

$C_1 < C_2$

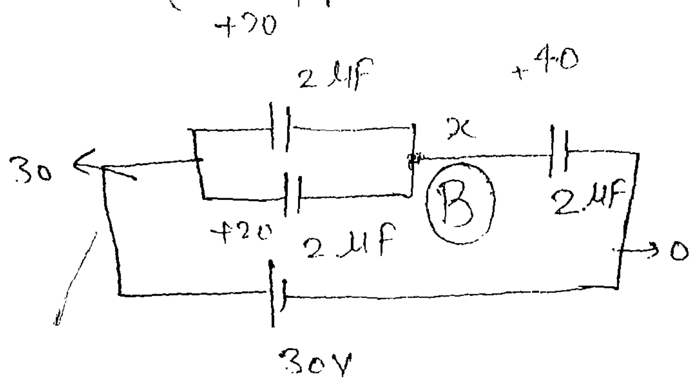
$\therefore V_{C_1} > V_{C_2}$.



Acharya ... 24/3/12 Exercise - I (subjective)

1.

(1)



$(x-0)2 + (x-30)4 = 0$

$3x = 60$

$x = 20V$

So initial charge on (1) & (2) ⇒ 20 μC

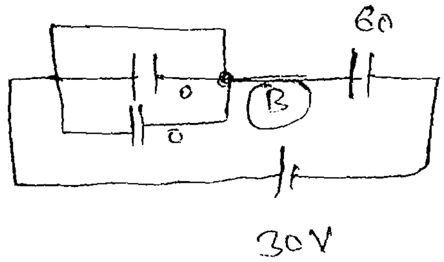
(3) ⇒ 40 μC

(a) when S is closed,

final charge (1) & (2) ⇒ 0

(3) ⇒ 60 μC

So charge flown = 20 μC



(b) initial Energy $= 2\left[\frac{1}{2} \times 2 \times (10)^2\right] + \frac{1}{2} \times 2 \times (20)^2$

$= 200 + 400 = 600\ \mu J$

final Energy $= \frac{1}{2} \times 2 \times (30)^2 = 900\ \mu J$

(c) Work done by battery $= 30 \times 20 = 600\ \mu J$

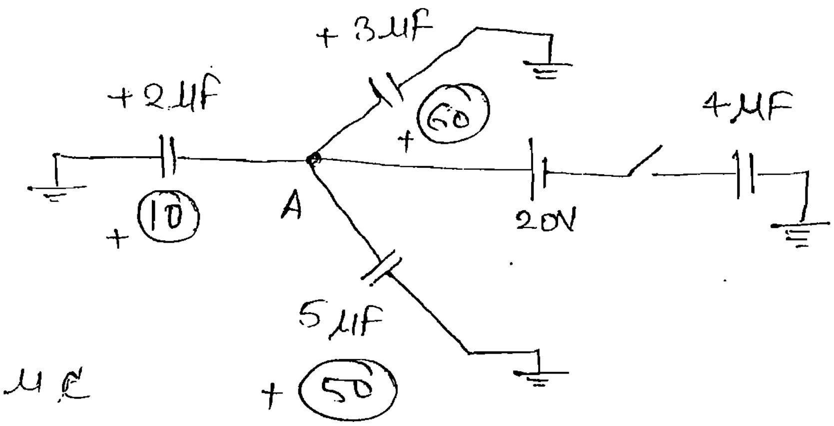
Energy loss $= 600 + 600 - 900 = 300\ \mu J$

(d) charge flown through $S = 60$

initial charge at 'B' $= 0\ \mu C$

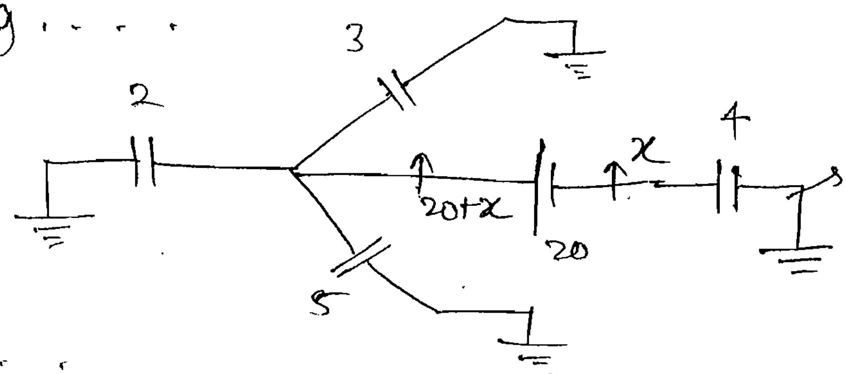
final charge at 'B' $= 60\ \mu C$

2.



initial charge at junction 'A' $= 120\ \mu C$

after Connecting ....



(a) applying Nodal ...

$$(20+x)\,10 + (x-0)\,4 = 120$$

$$5(20+x) + 2x = 60$$

$$7x + 100 = 60$$

$$x = -\frac{40}{7}$$

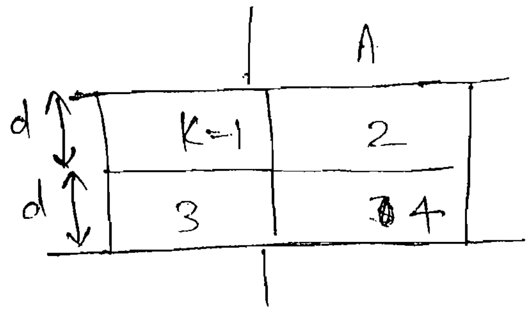
$\therefore$ potential of junction 'A' $= 20 - \frac{40}{7} = \frac{100}{7}$ V

(b) charges ~~of~~ on each Capacitors.

200   300   500   −160

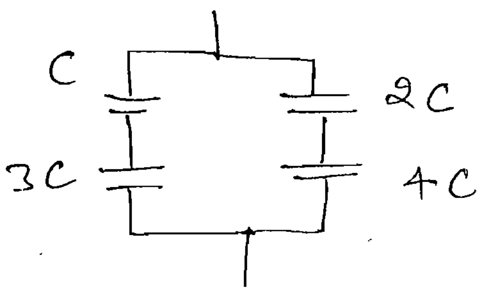
3.

3

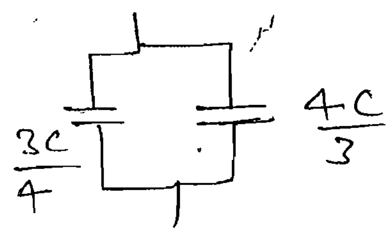


$$C_1 = \frac{K\epsilon_0 A}{d} = \cancel{\frac{K\epsilon_0 A}{2d}} = \frac{\epsilon_0 A}{2d} = C$$

$$C_2 = \frac{2\epsilon_0 A}{2d} = 2C \qquad C_3 = 3C\ ,\ C_4 = 4C$$

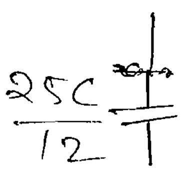


$\Rightarrow$



$$\cancel{\frac{1}{C}+\frac{1}{3C}} = \frac{3C^2}{4C} = \frac{3C}{4}$$

$\Downarrow$

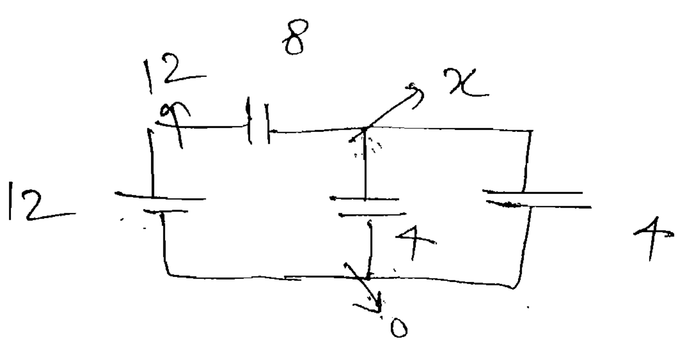


$$\frac{8C^2}{6C} = \frac{4C}{3}$$

$$\frac{3C}{4} + \frac{4C}{3} = \frac{25C}{12}$$

$$C_{eff} = \frac{25C}{12} = \frac{25}{12} \times \frac{\epsilon_0 A}{2d} = \frac{25\epsilon_0 A}{24d}$$

4.



$$(x-12)8 + (x-0)4 + (x-0)4 = 0$$

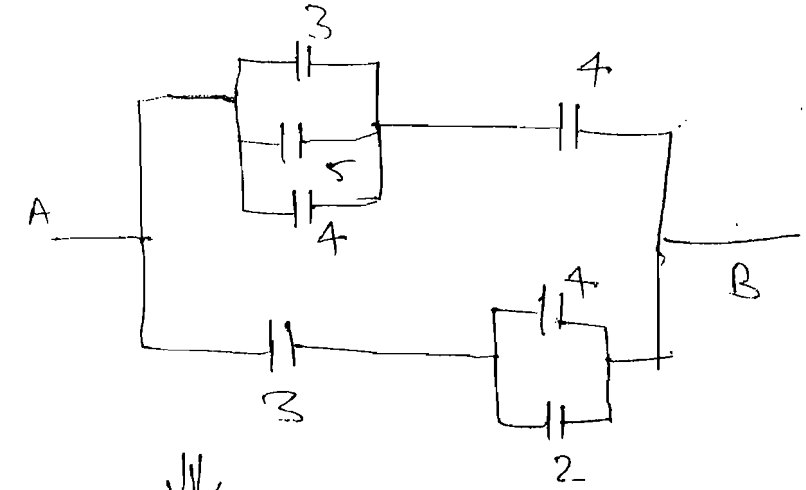
$$4x = 24$$

$$x = 6\ V$$

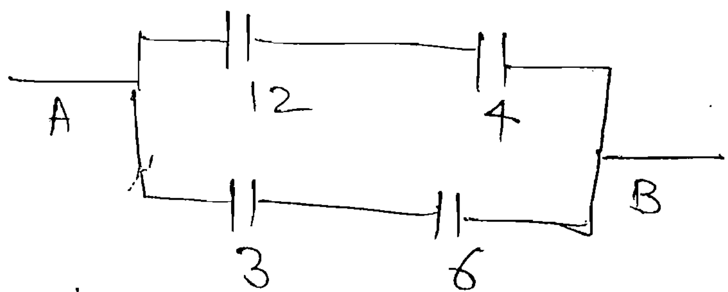
charge on '8' = 48 μC

" " '4' = 24 μC

5.

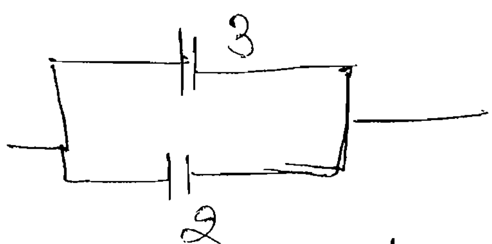


$3+5+4 = 12$



$\frac{12 \times 4}{16} = 3$

$\frac{3 \times 6}{3+6} = 2$



$5\,\mu f = C_{eff}$.

potential drop ~~accor~~ across '5' = $\frac{120}{5} = 24V$

" " '12' = 24V (all in parallel)

'12' & '4' are in series so both should have same charge $\Rightarrow$

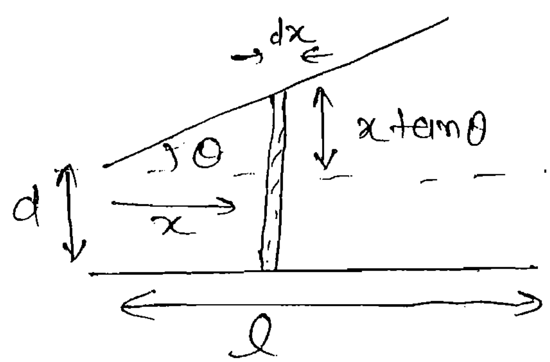
$12 \times 24 = 4 \times V$

P.D across AB = 24 + 72 = 96V

P.D across '3' = $\frac{6}{3+6} \times 96 =$

$= \frac{2}{3} \times 96 = 64\,V$

(8)



Capacitence of small plate is 'dc' = $\frac{\epsilon_0 A}{d}$

$$\Rightarrow \quad dc = \frac{\epsilon_0\, l\, dx}{d + x\tan\theta}$$

as ~~to~~ all plates are in $\parallel^{rl}$

$$C = \int dc = \int_0^l \frac{\epsilon_0\, l\, dx}{d + x\tan\theta}$$

$$= \frac{\epsilon_0 l}{\tan\theta} \ln\left(\frac{d + l\tan\theta}{d}\right)$$

as ~~d≫l~~ θ is very small $\tan\theta \approx \theta$

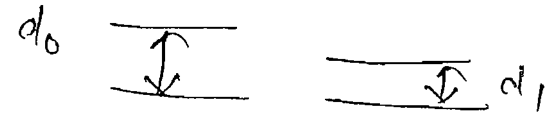
$$C = \frac{\epsilon_0 l}{\theta}\left(\frac{l}{d}\theta - \frac{l^2\theta^2}{2d^2}\right)$$

$$C = \frac{\epsilon_0 l^2}{d}\left(1 - \frac{l\theta}{2d}\right)$$

$\ln(1+x)$

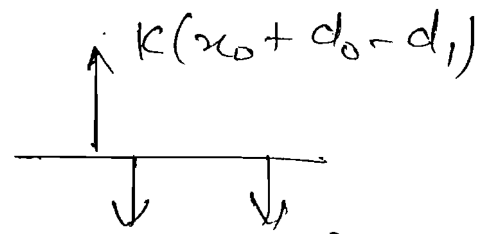
1. Suppose initial extension is $= x_0$

(a) therefore $Kx_0 = mg$



distance decrease b/w plates $= d_0 - d_1$

on upper plate



$$\frac{Q^2}{2A\epsilon_0} = \frac{\epsilon_0 A V^2}{2d_1^2}$$

LOM for New equilibrium

$$mg + \frac{\epsilon_0 A V^2}{2d_1^2} = K(x_0 + d_0 - d_1)$$

$$K = \frac{\epsilon_0 A V^2}{2d_1^2(d_0 - d_1)}$$

(b)
$$V^2 = \frac{2K d_1^2 (d_0 - d_1)}{\epsilon_0 A}$$

$y = d_1^2(d_0 - d_1)$

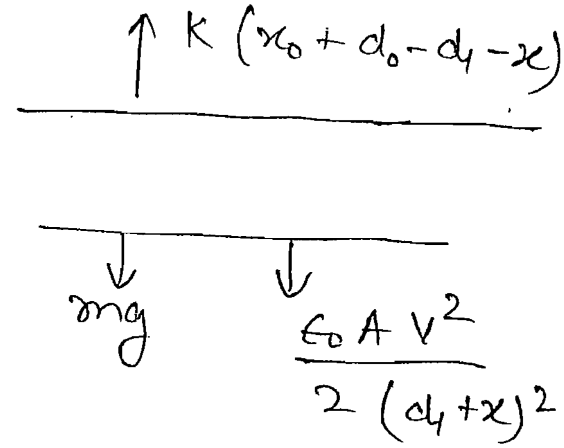
for max. value of $V \Rightarrow$ $2d_1 d_0 - 3d_1^2 = 0$

$d_1 = \frac{2d_0}{3}$

$$V^2 = \frac{2K}{\epsilon_0 A} \frac{4d_0^2}{9} \times \frac{d_0}{3}$$

$$V = \sqrt{\frac{K}{\ldots}} \left(\frac{2}{3}d_0\right)^{3/2}$$

(c) Suppose ~~block~~ plate is displaced by a distance 'x', therefore.



$$F_{net} = ma$$

$$K(x_0 + d_0 - d_1 - x) - mg - \frac{\epsilon_0 A V^2}{2(d_1+x)^2} = ma$$

$$\frac{\epsilon_0 A V^2}{2 d_1^2} \cancel{K(d_0 - d_1)} - Kx - \frac{\epsilon_0 A V^2}{2(d_1+x)^2} = ma$$

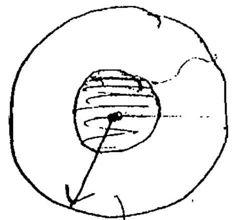
$$-Kx + \frac{\epsilon_0 A V^2}{2}\left[\frac{1}{d_1^2} - \frac{1}{(d_1+x)^2}\right] = ma$$

$$-Kx + \frac{\epsilon_0 A V^2}{\cancel{2}}\left[\frac{\cancel{x^2}^{\,0} + \cancel{2} d_1 x}{d_1^2 (d_1+x)^2}\right] = ma$$

$$ma = -\frac{[K d_1^3 - \epsilon_0 A V^2]}{d_1^3}\,x \qquad \left| \; d_1 + x \approx d_1 \right.$$

$$\omega^{\cancel{2}} = \left[\frac{K d_1^3 - \epsilon_0 A V^2}{m d_1^3}\right]^{1/2}$$

8.



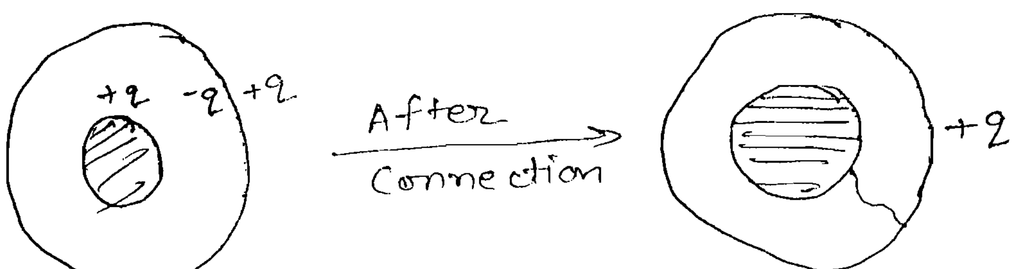
$\to r = 10$ cm

$q = 20\,\mu C$ 20 cm

$$\text{Initial Energy} = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{(20\,\mu C)^2}{\frac{K \times .1 \times .2}{.1}} = \frac{(20)^2}{K \times .4}$$

when wire is connected, all charge will transfer to outer surface.



So we can say, inner capacitor will discharge if we connect both sphere.

$$\text{initial energy} = \frac{(20\times10^{-6})^2 \times 9\times10^9}{.4}$$

$$\boxed{\frac{Q^2}{2C}}$$

$$= 9\ J$$

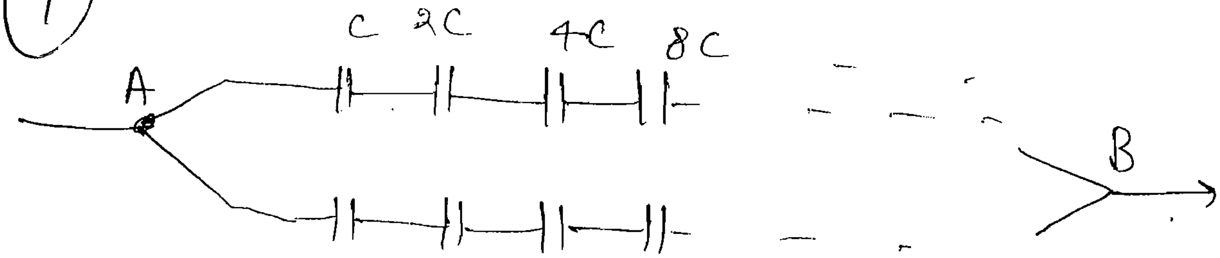
$$\boxed{C = \frac{\epsilon_0 A}{d} \quad C = \epsilon_0 \frac{4\pi R_1 R_2}{R_2 - R_1}}$$

final energy $= 0\ J$

loss $= 9 - 0 = 9\ J$

Ans

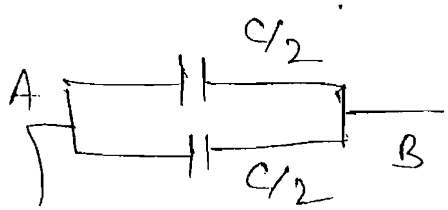
9. (9)



we can remove all middle capacitence (c) as ~~both~~ ~~or~~ p.D across them = 0

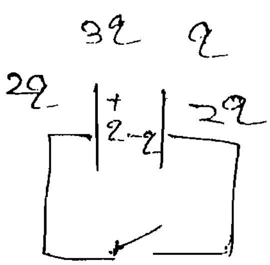
$$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} - - -$$

$$\frac{1}{C_{eff}} = \frac{2}{C} \Rightarrow C_{eff} = C/2$$



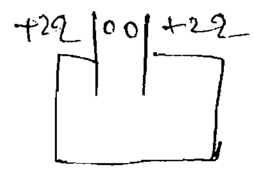
$$\Rightarrow \frac{C}{2} + \frac{C}{2} = C$$

(10.)



$$C = \frac{\epsilon_0 A}{d}$$

⇓



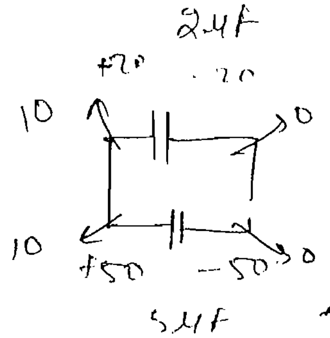
Capacitor would be in discharged condition.

$$\text{initial Energy} = \frac{1}{2}\frac{Q^2}{C} = \frac{q^2 d}{2\epsilon_0 A}$$

$$\text{final energy} = 0$$

$$\text{Heat} = \frac{1}{}\, q^2 d$$

11.



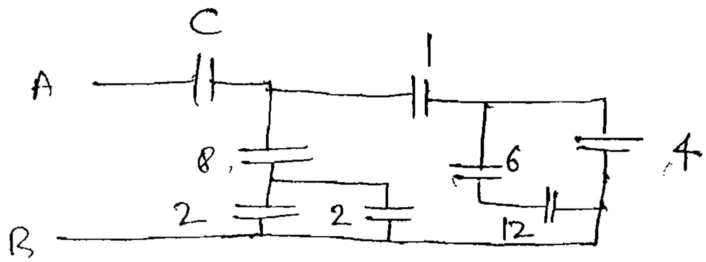
$\Delta V = \frac{20}{2} = 10\ V$

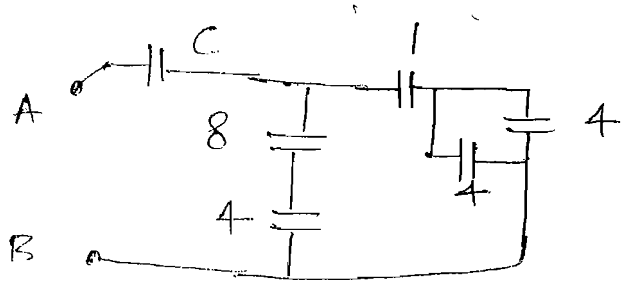
$\Delta V = \frac{50}{5} = 10$

if we ~~join~~ put switch in on condition, then no charge will flow as they are at same potential.

$Q = 0$

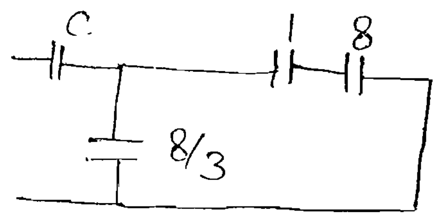
12.





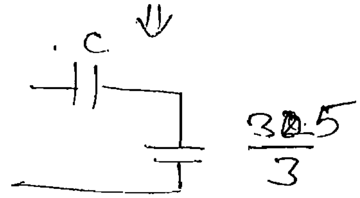
$\frac{6 \times 12}{18} = 4$

$2 + 2 = 4$



$\frac{8 \times 4}{12} = \frac{8}{3}$

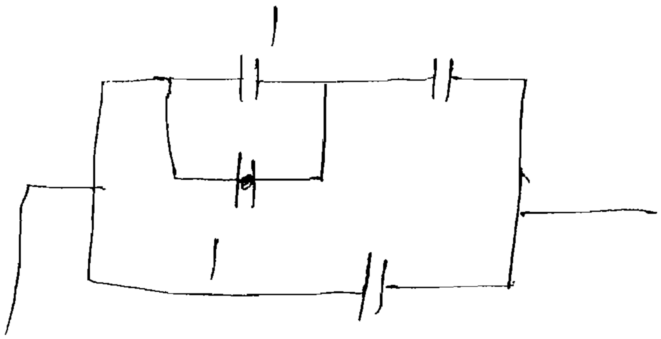
⇓

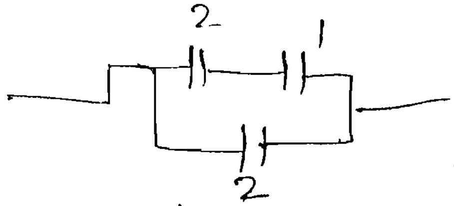


$1 = \frac{C \times 32/3}{C + 32/3}$??

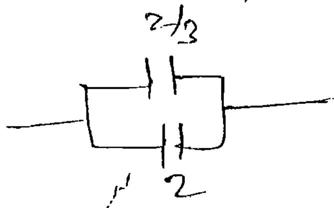
13.

13



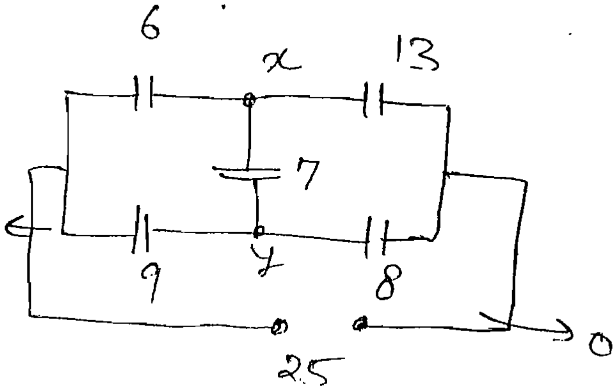


$\Rightarrow$ $\frac{2\times 1}{3} = \frac{2}{3}$



$C_{eff} = 2 + \frac{2}{3} = \frac{8}{3}\,\mu F$

14



$$(x-25)6 + (x-0)13 + (x-y)7 = 0$$
$$(y-25)9 + (y-0)8 + (y-x)7 = 0$$

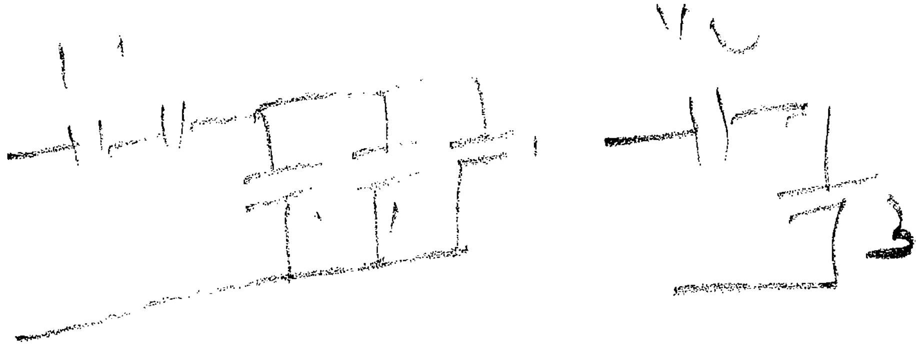
$$26x - 7y = 150$$
$$24y - 7x = 225$$

Solving both eq.

$$\boxed{\begin{aligned} x &= 9V \\ y &= 12V \end{aligned}}$$

Voltage across ...

(b)



$\frac{1}{c} = \frac{1}{5} + 1 = \frac{7}{5}$

15

$k = k_1 \left[1 + \sin \frac{\pi}{d} x \right]$

$dc = \frac{k \epsilon_0 A}{dx}$

all would be in series

$\frac{1}{C_{eff}} = \int \frac{1}{dc} = \int_0^d \frac{dx}{k \epsilon_0 A}$

$= \int_0^d \frac{dx}{k_1 (1 + \sin \frac{\pi}{d} x) \epsilon_0 A}$

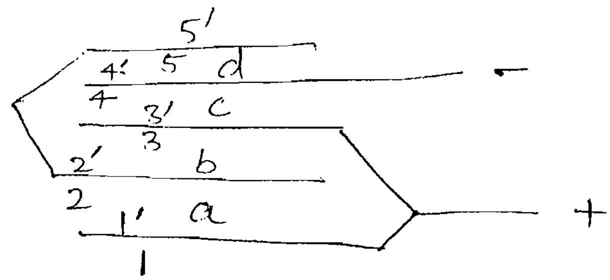
$\frac{1}{C_{eff}} = \frac{1}{k_1 \epsilon_0 A \frac{\pi}{d}} \int \left(\frac{1 - \sin \frac{\pi}{d} x}{1 - \sin^2 \frac{\pi}{d} x} \right) dx$

$= \frac{1}{k_1 \epsilon_0 A \frac{\pi}{d}} \left[\tan \frac{\pi}{d} x + \frac{1}{\cos \frac{\pi}{d} x} \right]_0^d$

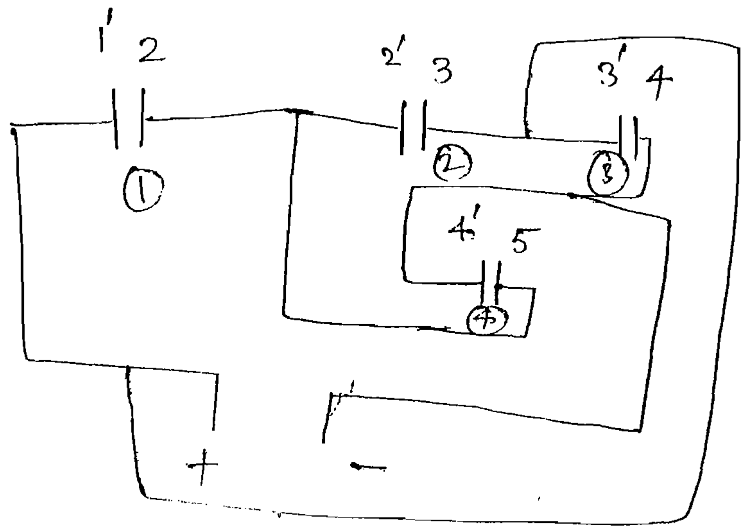
$\frac{1}{} = \frac{2d}{} \quad k \epsilon_0 A \pi$

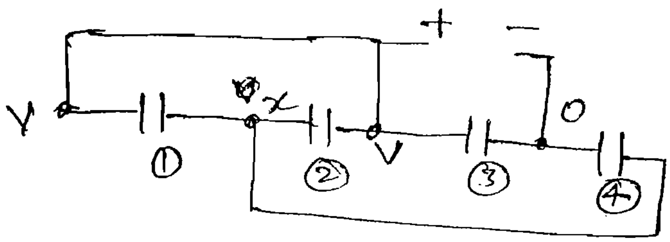
~~16~~ 17

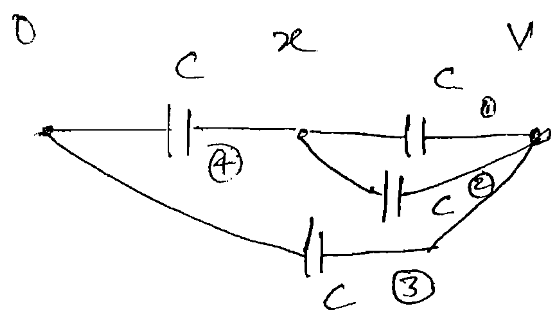
16

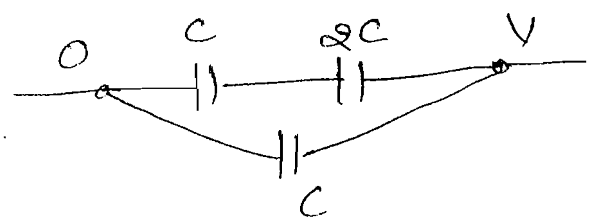


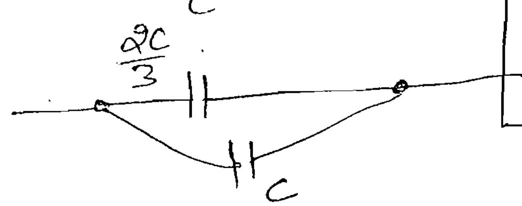
we can form 4 capacitor











$(x-0)C + 2(x-V)C = 0$

$3x = 2V$

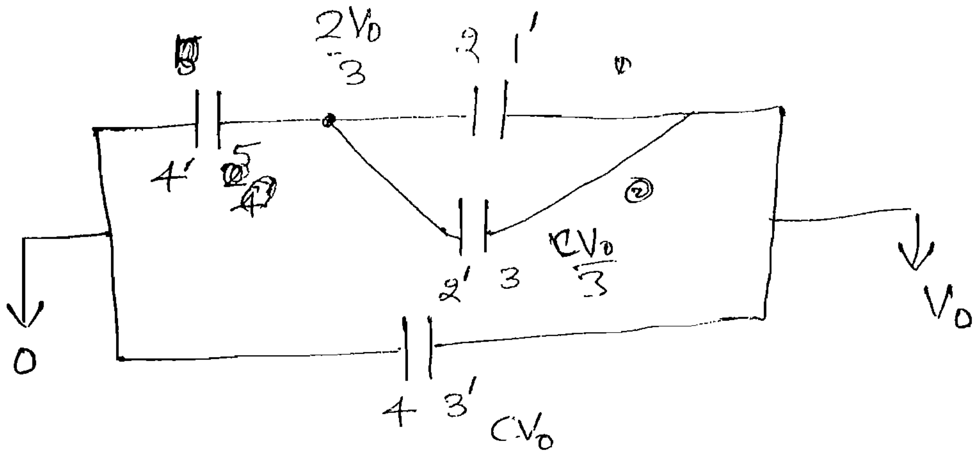
$x = \frac{2V}{3}$

$C_1 = \frac{2CV}{3}$

$C_2 = \frac{CV}{3} = C_3$

$C_4 = CV$

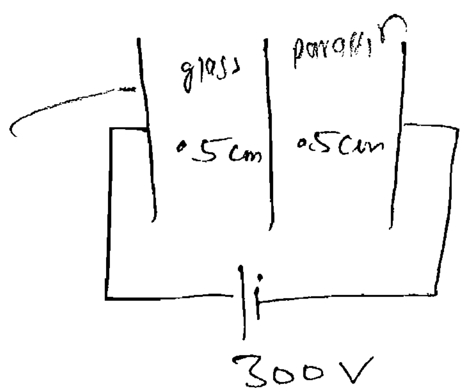
$C_{eff} = C + \frac{2C}{3} = \frac{5C}{3} = \frac{5}{3}\frac{\epsilon_0 A}{d}$

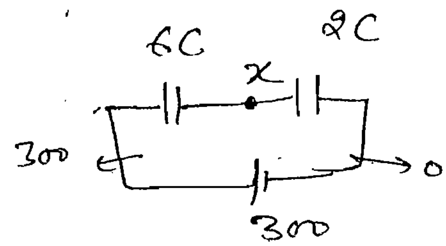


charge on plate 5 $= \frac{2V_0}{3} \times C = \frac{2CV_0}{3}$

charge on plate 3 $= CV_0 + \frac{CV_0}{3} = \frac{4CV_0}{3}$

(17)





$$C = \frac{\epsilon_0 A}{0.5}$$

$$(x - 300)\, 6C + (x - 0)\, 2C = 0$$

$$3x + x = 900$$

$$x = \frac{900}{4} = 225V$$

(i) $E_1 = \frac{V_1}{d} = \frac{75}{.5\,cm} = 150 \times 10^2 = 1.5 \times 10^4\ V/m$

$E_2 = \frac{V_2}{d} = \frac{225}{.5} = 4.5 \times 10^4\ V/m$

(ii) drop $\Rightarrow$ 75V

then $\Rightarrow$ 225 V

(iii)

$Q_1 = C_1 V_1$

$Q_1 = 6C\ 75$

$= 6 \frac{\epsilon_0 A}{d} \times 75$

$\sigma_1 = \frac{Q_1}{A} = \frac{450\ \epsilon_0}{.5\,cm} =$

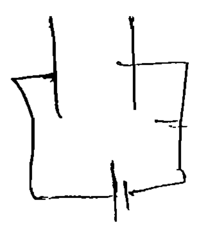
$= 900 \times 10^2 \times 8.85 \times 10^{-12}$

$= 7.965 \times 10^{-7}\ C/m^2$

$\approx 8 \times 10^{-7}\ C/m^2$

~~18.~~ ~~19.~~ 18

$q = CV_0$



$C = \frac{\epsilon_0 A}{d}$

now battery is revomed.

then charge remain Conserved.

$CV_0\ |K|\ CV_0$ new Capacitance $= KC$

initial Energy $= \frac{1}{2} C V_0^2$

final Energy $= \frac{1}{2} \frac{Q^2}{C} = \frac{C^2 V_0^2}{2KC}$

$= \frac{C V_0^2}{2K}$

$W_0 = \cancel{\Delta K} \to 0$

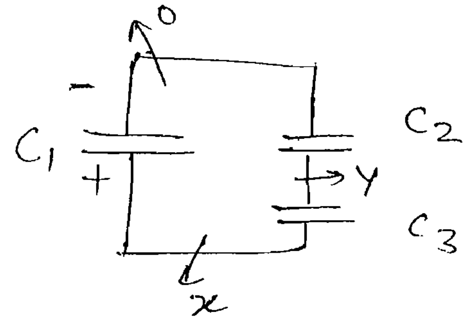
$W_{ext} + W_{cap.} = 0$

$\frac{1}{2} C V_0^2 \left(1 - \frac{1}{K}\right)$ ans

~~19.~~
20

19

initial charge on $C_1 \Rightarrow C_1 V$



$$(x-0)C_1 + (x-y)C_3 = C_1 V$$

$$(y-0)C_2 + (y-x)C_3 = 0$$

$$(C_1 + C_3)x - C_3 y = C_1 V$$

$$(C_2 + C_3)y - C_3 x = 0$$

$$\frac{(C_1 + C_3)(C_2 + C_3)y}{C_3} - C_3 y = C_1 V$$

$$y = \frac{C_1 C_3 V}{C_1 C_2 + (C_1 + C_2)C_3}$$

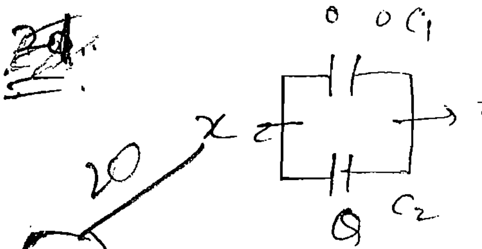
Charge on 'C_2' $= (y-0)C_2$

$$= \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$q_1 = x C_1$$

$$= \frac{(C_2 + C_3)y}{C_3} \cdot C_1 = \frac{(C_2+C_3)C_1}{C_3}$$

20



$$(x-0)\,C_1 + (x-0)\,C_2 = Q$$

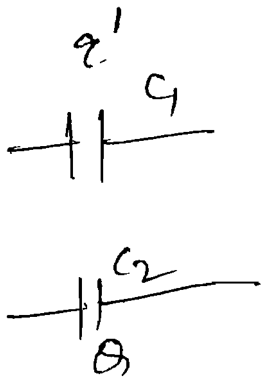
$$x = \frac{Q}{C_1+C_2}$$

charge on ~~plate~~ capacitor ① is

$$C_1 x = \frac{C_1 Q}{C_1+C_2} = q \quad \text{—①}$$

charge transfer will stop between C_1 & C_2 when both will be at same potential.

$$\Rightarrow \quad \frac{q'}{C_1} = \frac{Q}{C_2}$$



$$q_1' = \frac{C_1}{C_2} Q =$$

$$q_1' = \frac{Qq}{Q-q}$$

from eq. ①

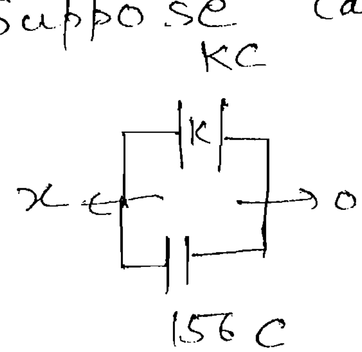
$$\frac{C_1}{C_1+C_2} = \frac{q}{Q}$$

$$\frac{C_1}{C_2} = \frac{q}{Q-q}$$

21

Suppose capacitance $= C$

$K = \alpha V$



$$(x-0)\,KC + (x-0)\,C = 156\,C$$

$$(K+1)\,x = 156$$

$$(1+x)\,x = 156$$

$$x^2 + x = 156$$

$K = \alpha V$
$= 1 \times x$

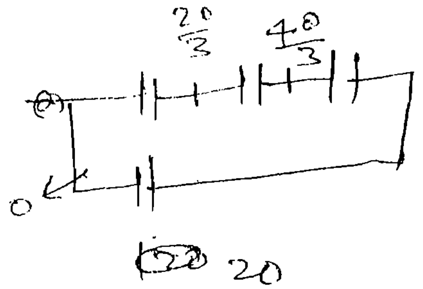
22.

~~29.~~

23

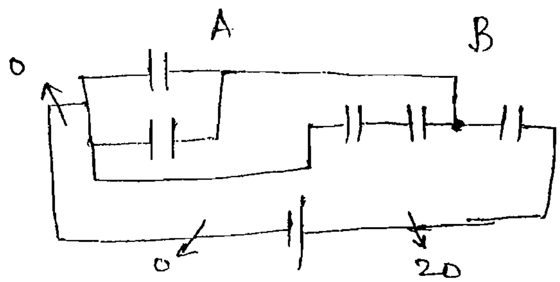
22

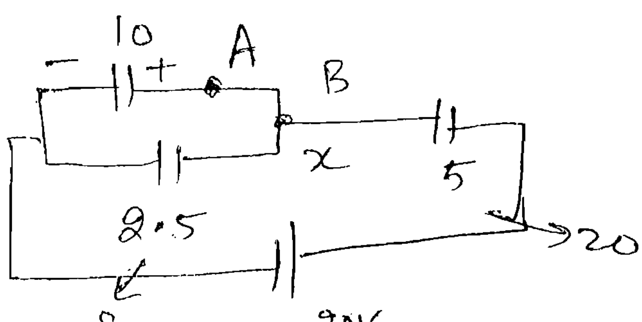
when A& switch is not closed ⇒



initial charge $= \frac{100}{3} \mu C$
on each capacitor

charge at junction 'B' $= 0$





$$(x-0)\,\cancel{12.5}\;2.5 + (x-20)5 = 0$$

$$3.5x = 20$$

$$x = \frac{40}{7}$$

So charge on Capacitor '10' $= \frac{400}{7}$

charge will flow from B → A

$400\ \mu C$

(30) No effect of closing S

Heat dissipated $= 0$

(31) use series, parallel rule for dividing applied potential & ensure voltage less than break down.

(32) $q = C_1 V - C_2 V = (C_1 - C_2) V$

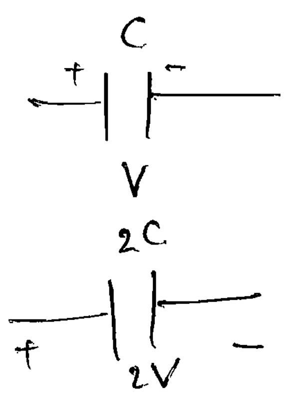
$V_2 = V_1 = \frac{q}{C_1 + C_2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right) V = \frac{100}{3}$

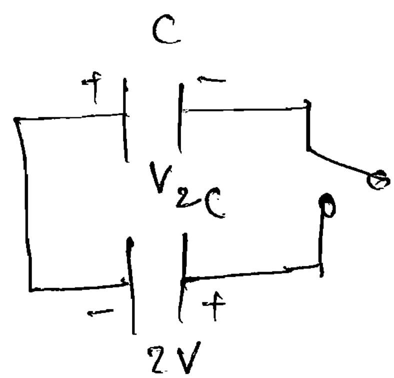
$E_{in} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2}(C_1 + C_2) V^2$

$E_f = \frac{1}{2}(C_1 + C_2) V_1^2$

$\frac{E_f}{E_{in}} = \left(\frac{V_1}{V}\right)^2 = 1/9.$

(33)





Final common potential $V_0 = \frac{4CV - CV}{3C} = V$

$U_f = \frac{1}{2} C V^2 + \frac{1}{2} \cdot 2C \cdot V^2 = \frac{3}{2} C V^2$

(34)

Capacitance

Exercise #1.

(26) Final charge on (+)ve plate $= CV + Q = Q_1$

Final net charge on (-)ve plate $= -CV = Q_2$

Final charge on cap. $= \dfrac{Q_1 - Q_2}{2} = \dfrac{2CV + Q}{2}$

$$p.d = \frac{2CV + Q}{2C} = V + \frac{Q}{2C}.$$

(27) $d_0 = 3\ mm$

$$C_0 = \frac{(A\epsilon_0)\,4}{d_0} = \frac{4\epsilon_0 A}{3}$$

$$C_1 = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} = \frac{A\epsilon_0}{\frac{3}{4} + \frac{5}{K}}$$

$$\boxed{C_1 = \frac{1}{2} C_0} \Rightarrow \boxed{K = \frac{20}{3}}$$

(28) capacitance of Left section $= \dfrac{2\epsilon_0 A}{d} = 2\,C_0$ (say)

capacitance of right section $= \dfrac{6\epsilon_0 A}{2d} = 3\,C_0$

since charges will be same

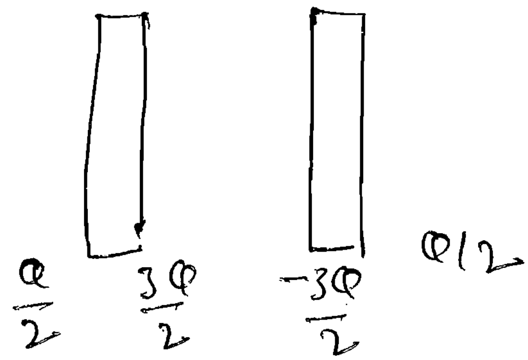
$$\Rightarrow V \propto \frac{1}{C} \Rightarrow \frac{V_1}{V_2} = \frac{C_2}{C_1} = 3/2$$

(29) case I : $C_{eq} = \dfrac{KC}{K+1}$, $Q_1 = Q_2 = \dfrac{KCE}{K+1}$

case II , $Q_1' = Q_2' = \dfrac{CE}{2}$

$$\Rightarrow \frac{Q_1'}{Q_1} = \frac{Q_2'}{Q_2} = \frac{K+1}{}$$

(34)



$$U_{in} = q^2/2C = \frac{(3Q/2)^2}{2\left(\frac{A\epsilon_0}{d}\right)} = \frac{9Q^2 d}{8A\epsilon_0}.$$

Exercise # 2

(1) $\phi_{cube} = \frac{\Sigma q}{\epsilon_0} = q/\epsilon_0$

$\phi_{one\ face} = \frac{1}{6} \cdot \phi_{cube} = \frac{q}{6\epsilon_0}$

(2) use std. expression of $\vec{E}$ inside & outside

(3) Both C_1 & C_2 are in series

$\Rightarrow Q_1 = Q_2$

$\Rightarrow V_1 = \frac{Q_1}{C_1}, \quad V_2 = \frac{Q_2}{C_2} = \frac{Q_1}{C_2}$

(4) slab pulled out $\Rightarrow C_B \downarrow \Rightarrow$ Net cap. $\downarrow$

$\Rightarrow q = CV \Rightarrow$ $q_A \downarrow$

$\Rightarrow$ (+)ve charge move from a to b

but A & B are in series.

(5) Initial $q_0 = C_0 V_0$

1st contact : $q_1 = \left(\frac{C_0}{C+C_0}\right) q_0$ (sharing of charge)

2nd contact : $q_2 = \left(\frac{C_0}{C+C_0}\right) q_1 = \left(\frac{C_0}{C+C_0}\right)^2 q_0$

nth contact : $q_n = \left(\frac{C_0}{C+C_0}\right)^n q_0$

$$\boxed{V = \frac{q_n}{C_0}} \Rightarrow C = C_0\left[\left(\frac{V_0}{V}\right)^{1/n} - 1\right]$$

(6) Initial

$C_0 = A\epsilon_0/d$

$Q_0 = C_0 V$

$V_0 = V$

$E_0 = V/d$

$U_0 = \frac{1}{2} C_0 V^2$

Final

$C = \frac{AK\epsilon_0}{d} = KC_0$

$Q = KC_0 \cdot V$

$V = V$

$E = V/d = E_0$

$U = \frac{1}{2} C V^2 = \frac{1}{2} K C_0 V^2$

$$\Rightarrow \boxed{\begin{matrix} Q > Q_0, & U > U_0 \\ E = E_0, & V = V_0 \end{matrix}}$$

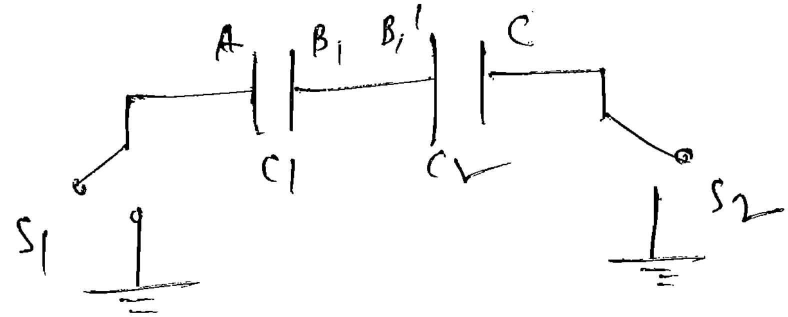
(7) $C_{net} = C + \frac{C}{2} + \frac{C}{4} + \cdots = 2C = 4F$

$U = \frac{1}{2} C_{net} \cdot V^2 = 200J$

charge on each row is same

since cap. in 1st row is max. hence

(8)



$$C_1 = \frac{A\epsilon_0}{2d}$$

$$C_2 = \frac{A\epsilon_0}{d}$$

$$Q_{B_1} + Q_{B_1'} = Q$$

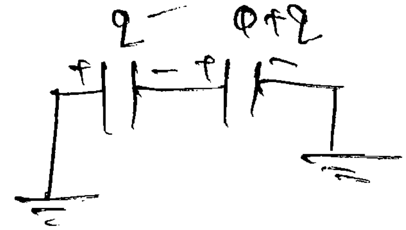
- If S_1 is closed, charge on $C_1 = 0$

$\Rightarrow Q_{B_1} = 0 \Rightarrow Q_{B_1'} = Q$

$\Rightarrow -Q$ will flow thru S_1.

- If S_2 is closed $-Q$ will flow thru S_2.

- If S_1 & S_2 both closed



$$-\frac{q}{C1} + \left(\frac{q+Q}{C2}\right) = 0$$

$$\Rightarrow \boxed{q = -Q/3}$$

$-Q/3$ will flow thru S_1 $\Rightarrow$ $\frac{2Q}{3}$ will flow thru S_2.

(9) use std. expression taking charge on cap. to remain const.

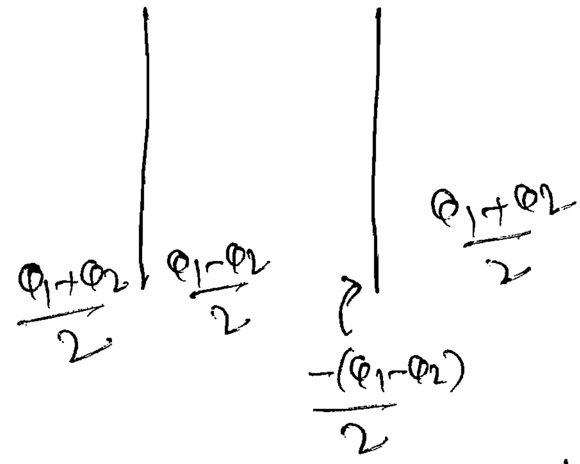
(10) $V_f = \frac{V_0 C_0}{C_0 + KC_0} = \frac{30 C_0}{C_0 + V_f C_0}$ $\Rightarrow \boxed{V_f = 5 \text{ volt}}$

$\boxed{\text{charge} = C \cdot V_f}$

$Q_A = (5C_0)5 = 25C_0$

$Q_B = (C_0)5 = 5C_0$

(11)



Heat produced $= \frac{1}{2}\frac{Q^2}{C_0} = \frac{1}{2C_0}\left(\frac{Q_1-Q_2}{2}\right)^2$

(12) use concept of sharing of charge.

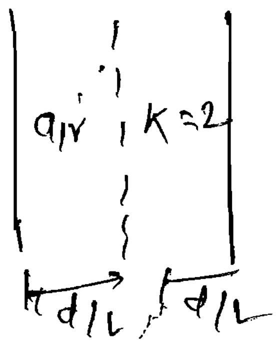
Exercise #3

Passage #1

use std. series, parallel arrangement.

Passage #2

$$C_1 = \frac{\epsilon_0 A}{(d/2)} = \frac{2\epsilon_0 A}{d}.$$



$$C_2 = 2\cdot\left(\frac{\epsilon_0 A}{d/2}\right) = \frac{4\epsilon_0 A}{d},$$

$$\boxed{C_2 = 2C_1}$$

$$C_{in} = \frac{\epsilon_0 A}{d}$$

$$C_f = \frac{\epsilon_0 A}{\frac{d}{2}+\frac{d}{4}} = \frac{4\epsilon_0 A}{3d}$$

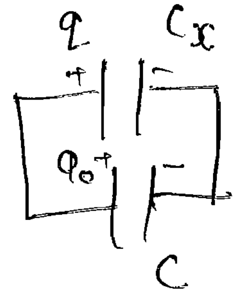
$$V = V_1 + V_2 = 1200$$

$$V_2 = \left(\frac{C_1}{C_1+C_2}\right)V = \frac{C_1}{3C_1}V = V/3 = 400 \text{ volt}$$

$$U_{air} = \frac{1}{2}\frac{Q^2}{C_1} \qquad \frac{U_{air}}{U_{di}} = \frac{C_2}{C_1} = 2$$

$$U_{di} = \frac{Q^2}{2C_2}$$

Passage #3



- $C_x = \dfrac{A\epsilon_0}{x}$, $C = \dfrac{A\epsilon_0}{d}$

charge on C_x, $q = Q_x = \dfrac{C_x \cdot Q_0}{C + C_x}$

$$F_x = \frac{Q_x^2}{2A\epsilon_0} = \frac{C_x^2 Q_0^2}{(C+C_x)^2\, 2\epsilon_0 A}$$

$$W = \int_d^{2d} F_x\, dx = \frac{Q_0^2 d}{12 A\epsilon_0}$$

- $$F_{ext} = F_x = \frac{Q_0^2}{2\left(\frac{x}{d}+1\right)^2 \cdot A\epsilon_0}$$

- $$V = \frac{Q_x}{C_x} = \frac{C_x \cdot Q_0}{(C+C_x)\, C_x} = \frac{Q_0}{C+C_x} = \frac{Q_0 x d}{A\epsilon_0 (x+d)}$$

- $$U_{in} = \frac{1}{2C}\left(\frac{Q_0}{2}\right)^2 = \frac{Q_0^2}{8C}$$

Final charge distribution : $\dfrac{Q-q}{C/2} = \dfrac{q}{C}$

$\boxed{q = Q/3}$

Final energy stored in cap. 2

$$U = \frac{1}{2}(Q/3)^2 \cdot \frac{1}{C} = \frac{Q^2}{18C}$$

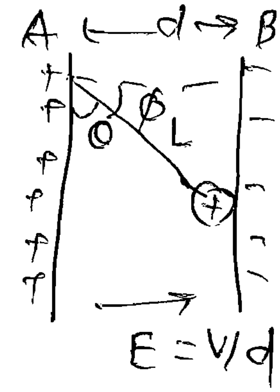
$$\Delta U = U_0 - U = \frac{5Q^2}{72C}$$

Passage # 4

- consider the ball in final position

$\sin\theta = \frac{d}{L} = 1/2 \Rightarrow \theta = 30^0$

$\boxed{\phi = 60^0}$



- when the ball again touches the plate A, then redistribution of charge takes place betn plate A & ball

$Q_1 : q_1 = C_0 : C_1$

$q_1 = \left(\frac{C_1}{C_1+C_0}\right) Q_0 = \left(\frac{C_1}{C_0+C_1}\right) C_0 V_0$

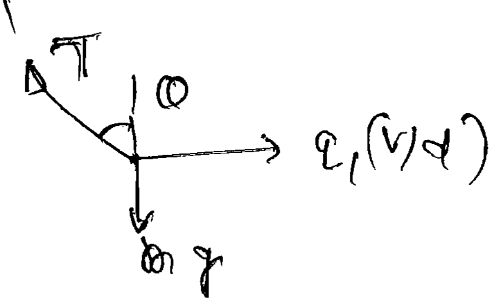
new potential ; $V = \frac{q_1}{C_1} = \frac{C_0 V_0}{C_0+C_1}$

- charge carried by ball in final position

$q_1 = C_1 V = \frac{C_1 C_0 V_0}{C_0+C_1}$

- $Q_1 = C_0 V = \frac{C_0^2 V_0}{C_0+C_1} \Rightarrow Q_1 : q_1 = C_0 : C_1$

$q_1 = C_1 V = \frac{C_1 C_0 V_0}{C_0+C_1}$



- $mg\tan\theta = \frac{q_1 V}{d} = \frac{C_1 V^2}{d}$

$\Rightarrow = \frac{C_1}{d}\left(\frac{C_0 V_0}{C_0+C_1}\right)^2$

$V_0 = \left(\frac{C_0+C_1}{C_0}\right)\sqrt{\frac{mgd\tan\theta}{C_1}}$

Passage #5

Finally both capacitor have charge CE each. The net charge crossing the cell is $= 2CE$

The work done by cell $= W = (2CE)E$

$$= 2CE^2$$

Energy stored = $U = \frac{1}{2}CE^2 + \frac{1}{2}CE^2$

Heat prod. $= W - U$

$$= CE^2$$

Matching

1 →

	Q	C	V
(a)	C_0V_0	C_0	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$V_0\left(\frac{K+1}{2K}\right)$
(b)	0	$\frac{C_0}{2d}$	0
	$V_0 \cdot \frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$V_0 \quad \frac{C_0}{2d}$	$\frac{C_0}{2d}$	V_0
	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$V_0\left(\frac{K+1}{2K}\right)$
(c)	0	$\frac{KA\epsilon_0}{d}$	0
	KC_0V_0	KC_0	V_0
	KC_0V_0	KC_0	V_0
	KC_0V_0	$\frac{KC_0}{K+1}$	$(K+1)V$
(d)	C_0V_0	C_0	V_0
	KC_0V_0	KC_0	"
	$KC_0V_0/K+1$	$\frac{KC_0}{K+1}$	"

(2) $Q_{in} = CV_{in} = 2\mu C$

$Q_f = CV_f = 4\mu C$

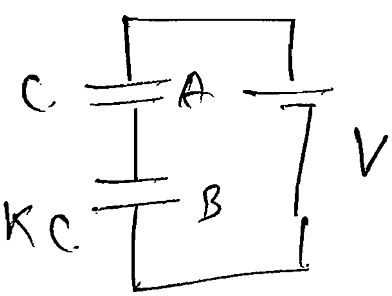
Net Q crossing 4V battery $= Q_f - Q_{in} = 2\mu C$

$|W| = (Q_f - Q_{in})\,4 = 8\mu J$

$\Delta U = \frac{1}{2}C\left(V_f^2 - V_{in}^2\right) = 6\mu J$

(3) As slab is inserted, $C_B \uparrow$, $C_{Total} \downarrow$

$\Rightarrow Q \downarrow$.



$Q = \frac{KC}{K+1} \cdot V$

$V_A = Q/C = \frac{KV}{K+1} > \frac{V}{2}$

$V_B = \frac{Q}{KC} = \frac{V}{1+K} < V/2$

(4) Battery is always connected $\Rightarrow V_C = \text{const}$

$C \uparrow \Rightarrow Q_C \uparrow$

Battery disconnected $\Rightarrow Q_C = \text{const}$.

$C \uparrow \Rightarrow V_C \downarrow$

(5) $C_1 = 3\mu F$, $V_1 = 12V \Rightarrow Q_1 = 36\mu C$

$Q_{Total} = 36\mu C$, $C_{Total} = 9\mu F$

$V \quad = 36/$

(23) • $Q_A = Q_B = Q \Rightarrow V_A = V_B = \frac{Q}{C}$

Subjective Ex # 4

Capacitor

$E = V/d$,

emf of battery $= \mathcal{E} = 2V = \frac{2Q}{C}$ — (I)

• Now $C_A = KC$; $C_B = C$

$V_A' + V_B' = \mathcal{E} \Rightarrow V_A' = \frac{Q'}{C_A} = \frac{Q'}{KC}$

$V_B' = \frac{Q'}{C_B} = Q'/C$

$\Rightarrow \mathcal{E} = \frac{Q'}{C}\left(1 + \frac{1}{K}\right)$ — (II)

$\boxed{Q' = \left(\frac{2K}{1+K}\right) Q}$ $\Rightarrow K > 1 \Rightarrow \boxed{Q' > Q}$

$V_A' = \frac{2V}{K+1}$, $V_B' = \frac{2K}{K+1} \cdot V$

E. field & p.d. in cap. A ↓ by factor $\left(\frac{2}{K+1}\right)$
while in B ↑ by $\frac{2K}{K+1}$

$\Delta Q = Q' - Q = \left(\frac{K-1}{K+1}\right) \frac{C\mathcal{E}}{2}$

$U_{in} = 2 \cdot \frac{1}{2} CV^2 = CV^2$

$U_f = \frac{1}{2} C_A V_A'^2 + \frac{1}{2} C_B V_B'^2 = \left(\frac{4K}{1+K}\right) \frac{1}{2}(CV^2)$

$= \left(\frac{2K}{K+1}\right) U_{in}$

(24) S_1 : closed

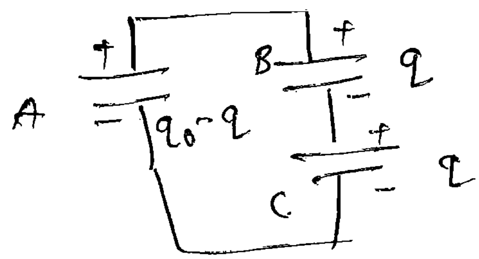
$$q_A = C_1 V = 360\ \mu C$$

Energy supplied by battery $= q_A \cdot \mathcal{E} = 0.0648\ J$

$$U_A = \frac{1}{2} q_A \cdot \mathcal{E} = 0.0324\ J$$

S_1 : opened , S_2 : closed

$$q_0 = q_A = 360\ \mu C$$



$$\frac{q}{C_2} + \frac{q}{C_3} - \left(\frac{q_0 - q}{C_1}\right) = 0$$

$$\boxed{q = 180\ \mu C}$$

$$U_{system} = U_A + U_B + U_C = \frac{(q_0 - q)^2}{2C_1} + \frac{q^2}{2C_2} + \frac{q^2}{2C_3}$$

$$= 0.0162\ J$$

$$E_B = q / \epsilon_0 A$$

S_2 : open dielectric slab of A → removed $C_1' = \frac{C_1}{K}$

$= 1\ \mu F$

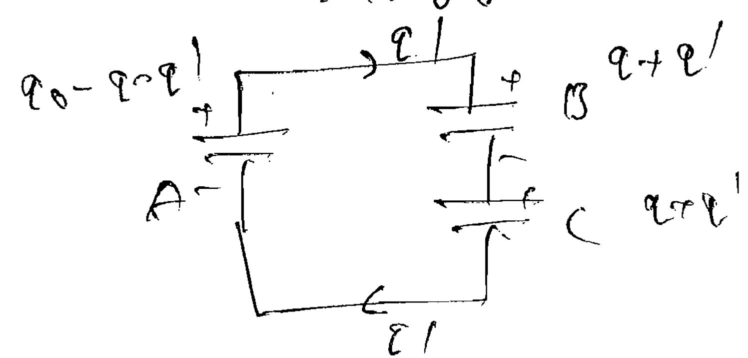
Another slab $K = 2$ is inserted in B

$$C_2' = KC_2 = 6\ \mu F$$

Now S_2 is closed ⟹ redistribution of charge

$$\frac{q + q'}{C_2'} + \frac{q + q'}{C_3'} - \frac{q_0 - q - q'}{C_1'} = 0$$

$$\boxed{q' = 90\ \mu C}$$



Final charge on B = $q + q' = 220\,\mu C$

$$E' = \frac{q+q'}{\epsilon_0 KA}$$

$$\frac{E'}{E} = 0.75$$

$$\text{Energy lost} = \frac{(-q')^2}{2C_1'} + \frac{(q')^2}{2C_2} + \frac{(q')^2}{2C_3'}$$

$$= 5.4\ mJ$$

(25) use Kirchoff's junction & loop rule.

(26) use concept of inverse symmetry.

(27) use Kirchoff's junction & loop rule.