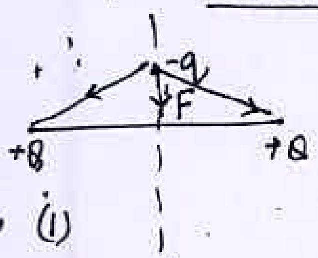
Electrostats (main) MDP

Solution

1-7 are based on theory.

8. $q = Ne = 10^{19} \times 1.6 \times 10^{-19} = 1.6C$ (2)

Coulomb's Law

9. 

There is a restoring force on charge $-q$, which makes it oscillate. (3)

10. Theory (1)

11. The coulomb's force becomes $\frac{1}{K}$ times its value. (1)

12. $F = \frac{Kq(Q-q)}{r^2}$

F is maximum $\Rightarrow \frac{d}{dq} q(Q-q) = 0$

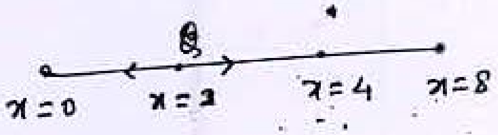
$Q - 2q = 0$

$\frac{Q}{q} = \frac{2}{1}$ (2)

13. $F_{12} = F_{21}$ (2)

14. 

The Net force of Q will be zero. (2)

15. 

$F_{net} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 2 \times 10^{-6}}{(6 \times 10^{-2})^2} = 10\,N$ (2)

16. $F_{net} = \frac{Kq^2}{L^2}$ (1)

17. Theory (3)

18. 

$\frac{KQq}{a^2} + \frac{KQ^2}{(2a)^2} = 0$

$q + \frac{Q}{4} = 0$

$q = -\frac{Q}{4}$ (2)

19.

$$qE = mg$$

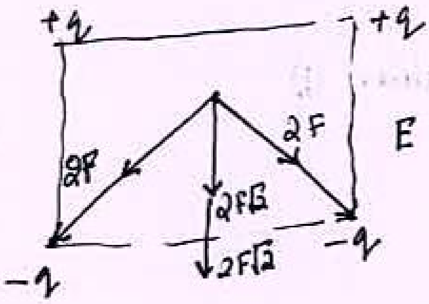
$$E = \frac{mg}{q}$$

$$= \frac{9.1 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}}$$

$$E = 56 \times 10^{-12}$$

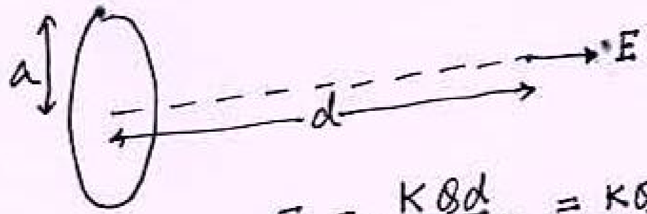
$$E = 5.6 \times 10^{-11} \quad (2)$$

20.



$$E = 4F\sqrt{2} = \frac{4KQ\sqrt{2}}{a^2} \quad (2)$$

21.



$$E = \frac{KQd}{(a^2+d^2)^{3/2}} = \frac{KQd}{d^3} = \frac{KQ}{d^2}$$

$$\boxed{E \propto d^{-2}} \quad (3)$$

22.

Field is uniform

(4)

23. Theory Based.

24.

24.

Distance will be minimum when $V_1 = V_2 = V$

Conserving Energy :- $\dfrac{mv'^2}{2} + \dfrac{mv'^2}{2} + \dfrac{Kq_1q_2}{r} = \dfrac{mv^2}{2}$

Conserving momentum :- $v' = \dfrac{v}{2}$

$$\frac{mv^2}{8} + \frac{mv^2}{8} + \frac{Kq^2}{r} = \frac{mv^2}{2}$$

$$\boxed{\frac{Kq^2}{r} = \frac{mv^2}{4}}$$

Put the values. $r = 10^{-12} m$ —(1).

25.

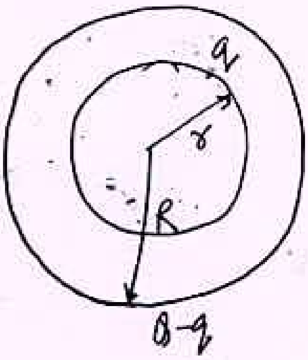
Distance will be minimum when $V_{rel} = 0$.

Conserving Momentum.
$$4v' + 50v' = 4v$$
$$54v' = 4v$$
$$v' = \frac{2v}{27}$$

Conserving Energy
$$\frac{mv'^2}{2} + \frac{Mv'^2}{2} + \frac{Kq_1q_2}{r} = \frac{mv^2}{2}$$

Put the values. $r = 1.44 \times 10^{-14} m$.

(26)



$$\frac{Q-q}{q} = \frac{R^2}{r^2}$$

$$\frac{Q}{q} = \frac{R^2+r^2}{r^2}$$

$$\frac{q}{Q} = \frac{r^2}{R^2+r^2} \quad \text{—} (1)$$

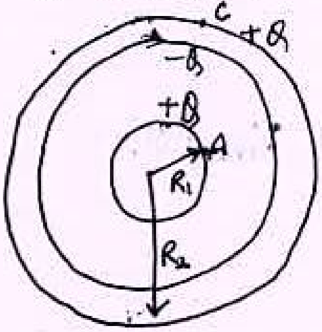
$$V_c = \frac{kq}{r} + \frac{k}{R}(Q-q)$$

$$= kQ\left[\frac{q}{rQ} + \frac{1}{R}\left(1-\frac{q}{Q}\right)\right]$$

$$= kQ\left[\frac{r}{r^2+R^2} + \frac{R}{r^2+R^2}\right]$$

$$V_c = \frac{kQ(r+R)}{(r^2+R^2)} \quad \text{—} (3)$$

(27)

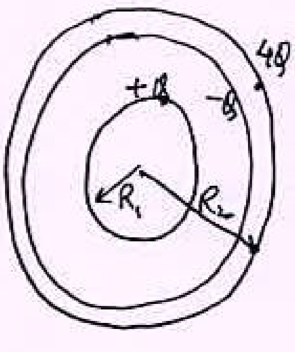


$$V_c = \frac{KQ}{R_2}$$

$$V_A = \frac{KQ}{R_1} - \cancel{\frac{KQ}{R_2}} + \cancel{\frac{KQ}{R_2}}$$

$$V_A = \frac{KQ}{R_1}$$

$$V_c - V_A = V = \frac{KQ}{R_2} - \frac{KQ}{R_1}$$

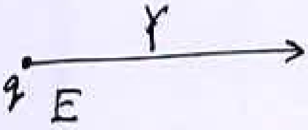


$$V_A = \frac{KQ}{R_1} - \frac{KQ}{R_2} + \cancel{\frac{k(4Q)}{R_2}}$$

$$V_c = \cancel{\frac{4KQ}{R_2}}$$

$$V_c - V_A = \frac{KQ}{R_2} - \frac{KQ}{R_1} = V \quad (1)$$

28.



$W = qEr$ (1).

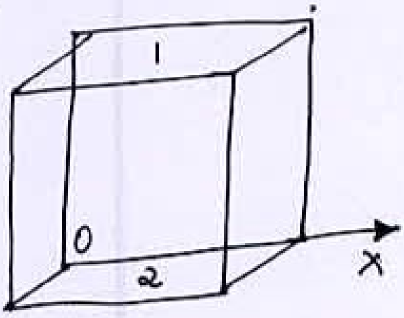
29. As F_c is conservative.

$W = 0$ (4).

30. $K = qV = 1\,eV$ (3)

31. Theory Based (1).

32.



$\phi_1 = \frac{q}{6\epsilon_0}$

$\phi_1 + \phi_2 = \frac{q}{3\epsilon_0}$ (3)

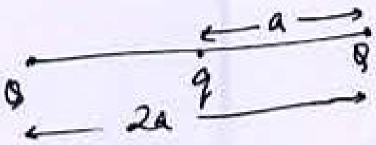
33. $\phi = \vec{E}\cdot\vec{S} = 8\times 20 \cos 60^\circ = 80$

34. $\phi = \frac{q_2+q_3}{\epsilon_0} = 4\pi\times 10^{9}\times 9\times 10^{-6} = 36\pi\times 10^{3}$

WINDOW to JEE MAIN

1. ϕ is equal & opposite. It cancels out. (2)

2.



$\frac{KQq}{a^2} + \frac{KQ^2}{4a^2} = 0$

$q = -\frac{Q}{4}$ (4)

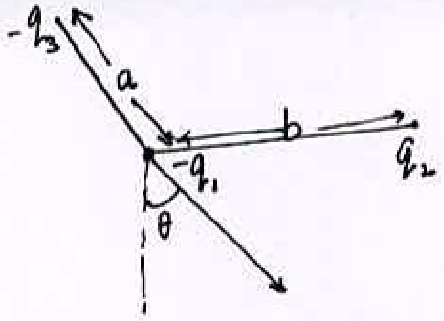
3. $W = qV$

$V = \frac{W}{q} = \frac{2}{20} = 0.1$ volt. (1)

4. $q = \phi\epsilon_0$

$= (\phi_2 - \phi_1)\epsilon_0$ (1)

5

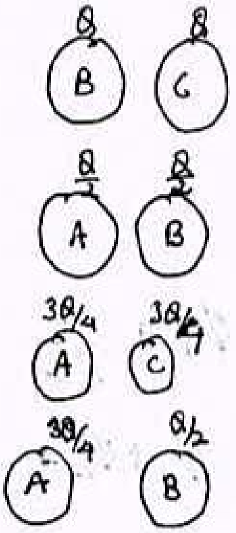


$$F_2 \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta \quad (2)$$

6 After the whole process.

$$Q_A = \frac{3Q}{4}$$

$$Q_B = \frac{Q}{2}$$



$$F \propto Q_A Q_B$$

$$F' = \frac{3}{8} K Q^2$$

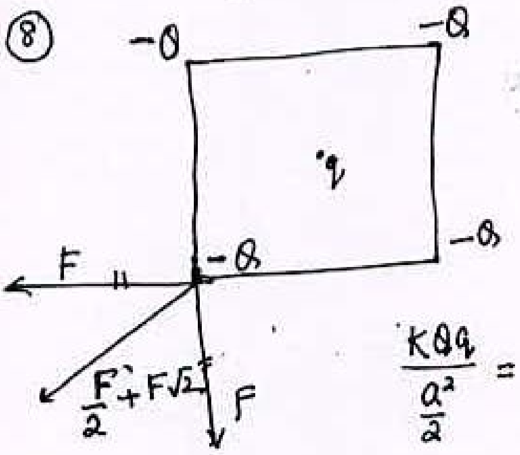
$$\boxed{F' = \frac{3}{8}F}$$

7 (4) $V^2 R = \text{constant}$

$$V^2 r = (2V)^2 r'$$

$$\boxed{r' = \frac{r}{4}}$$

8

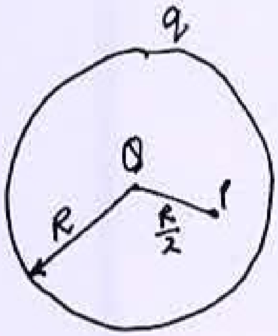


$$\frac{KQq}{\frac{a^2}{2}} = \frac{F(2\sqrt{2}+1)}{2} = \frac{KQ^2}{2a^2}(1+2\sqrt{2})$$

$$2q = \frac{Q(1+2\sqrt{2})}{2}$$

$$q = \frac{Q}{4}(1+2\sqrt{2})$$

9



$V = V_{shell} + V_Q$

$= \frac{Kq}{R} + \frac{2KQ}{R}$ (3)

10

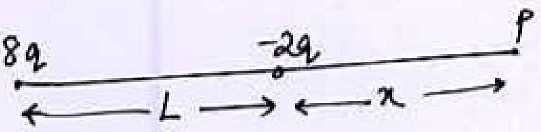
$T\cos\theta = mg$

$T\sin\theta = \frac{\sigma}{2\epsilon_0}$

$mg\tan\theta = \frac{\sigma}{2\epsilon_0}$

$\sigma \propto \tan\theta$ (4)

11



$\frac{8qK}{(L+x)^2} - \frac{2Kq}{x^2} = 0$

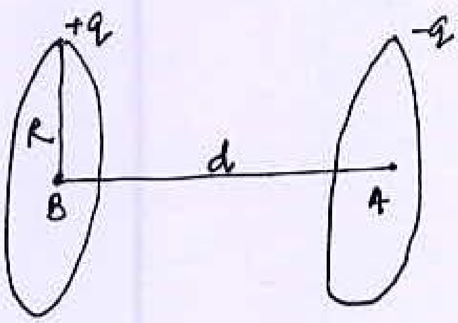
$4 = \left(\frac{L+x}{x}\right)^2$

$\frac{L+x}{x} = 2$

$x = L$

Position is 2L (1)

12



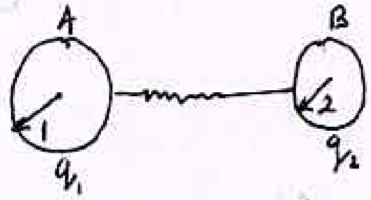
$V_A = \frac{Kq}{\sqrt{d^2+R^2}} - \frac{Kq}{R}$

$V_B = \frac{Kq}{R} - \frac{Kq}{\sqrt{d^2+R^2}}$

$V_B - V_A = 2\left(\frac{Kq}{R} - \frac{Kq}{\sqrt{d^2+R^2}}\right)$

$\frac{q}{2\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

13.



$$\frac{Kq_1}{1} = \frac{Kq_2}{2}$$

$$\frac{q_1}{q_2} = \frac{1}{2} \quad -①$$

$$\frac{E_1}{E_2} = \frac{q_1}{q_2}\,\frac{2^2}{1} = \frac{1}{2}\times 2^2 = \frac{2}{1} \quad (3).$$

14. $K = \Delta V q$

$$\frac{9.11\times 10^{-31}}{2}v^2 = 20\times 1.6\times 10^{-19}$$

$$v = 2.65\times 10^{6}\ m/s \quad (1)$$

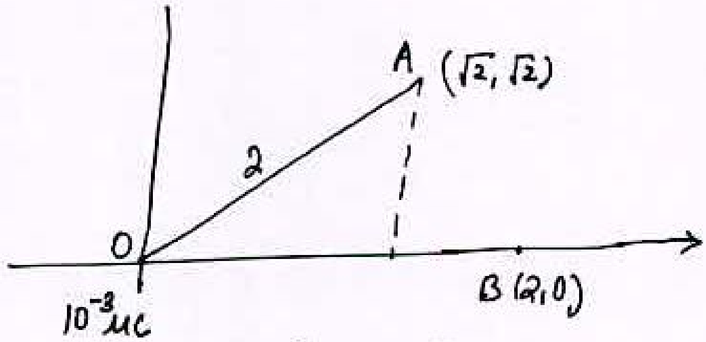
15 Theory Based (3)

16 $E = -\frac{dV}{dx} = \frac{20\,(2x)}{(x^2-4)^2} = \frac{40x}{(x^2-4)^2}$

$$E(x=4) = \frac{40\times 4}{(4^2-4)^2} = \frac{160}{144} = \frac{10}{9}\ V/\mu m \quad (4).$$

17. Theory Based (4)

18.



$$V_A = \frac{KQ}{2}$$

$$V_B = \frac{KQ}{2}$$

$$\Delta V = 0 \quad (2).$$

19. See 22.

20. Theory Based (1)

21 & 22. $\rho = \frac{J}{E}$

$$\rho = \frac{RA}{L}$$

$$\rho = \frac{V}{l}\frac{A}{I}$$

$$\rho = \frac{E}{I}$$

$$J(r) = \frac{i}{2\pi r^2}$$

$$\frac{E(r)}{\rho} = \frac{i}{2\pi r^2}$$

$$E(r) = \frac{i\rho}{2\pi r^2}$$

$$\int_B^C dv = -\int_a^{a+b} E\,dr$$

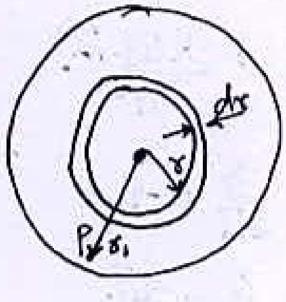
$$V_C - V_B = \left.\frac{i\rho}{2\pi r}\right]_a^{a+b}$$

$$= \frac{i\rho}{2\pi}\left[\frac{1}{a+b} - \frac{1}{a}\right]$$

$$V_B - V_C = \frac{i\rho}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

23. Theory Based.

24. $$dQ = \int_0^{r_1} \rho(r)\, 4\pi r^2 dr$$



$$= \int_0^{r_1} \frac{Qr}{\pi R^4}(4\pi r^2)dr$$

$$= \frac{4Q}{R^4}\int_0^{r_1} r^3 dr$$

$$= \frac{Q r_1^4}{R^4}$$

$$\phi = \frac{Q r_1^4}{\epsilon_0 R^4}$$

$$\phi = \frac{Q_{in}}{\epsilon_0} = \vec{E} \cdot \vec{A}$$

$$\frac{Q \cancel{r_1^4}}{\epsilon_0 R^4} r_1^2 = E(4\pi \cancel{r_1^2})$$

$$E = \frac{Q r_1^2}{4\pi\epsilon_0 R^4}$$

$$E = \frac{KQ r_1^2}{R^4} = \frac{1}{4\pi\epsilon_0} \frac{Q r_1^2}{R^4} \quad (3).$$

25.

$$W = -100 e \Delta V$$

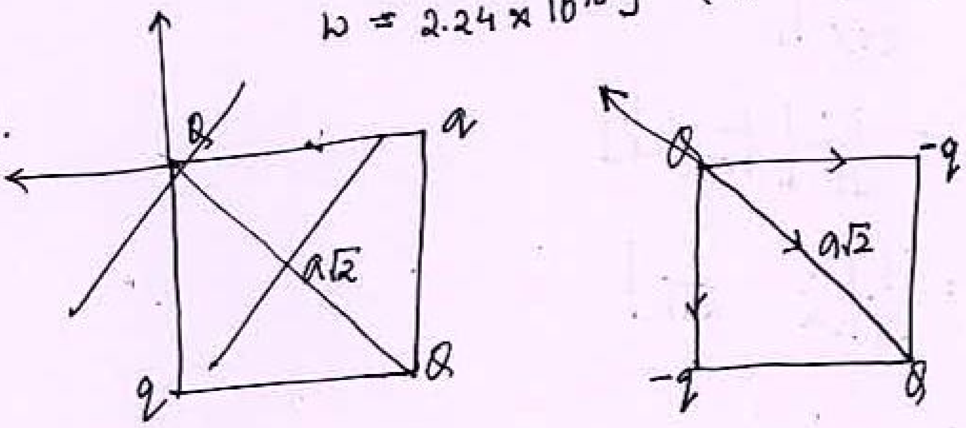
$$= -100 e (-14)$$

$$= 1400 \times e$$

$$W = 1400 \times 1.6 \times 10^{-19}$$

$$W = 2.24 \times 10^{-16}\ J \quad (4)$$

26.

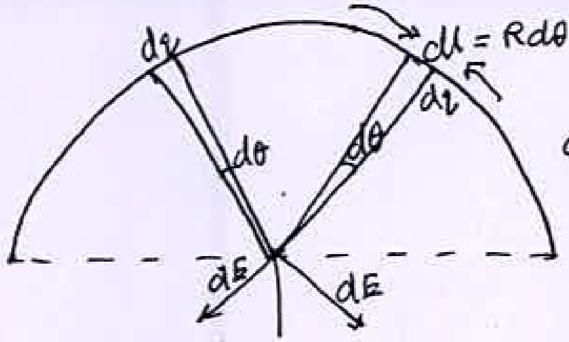


$$\frac{KQ^2}{2a^2} = \frac{KQq\sqrt{2}}{a^2}$$

$$\frac{Q}{2\sqrt{2}} = -q$$

$$\frac{Q}{q} = -2\sqrt{2}$$

27.



(27)

$$dq = \frac{q}{\pi r} r\,d\theta$$

$$dq = \frac{q}{\pi} d\theta$$

$$E = \frac{K\,dq}{r^2}$$

$$E = \frac{2K}{r^2} dq \cos\theta$$

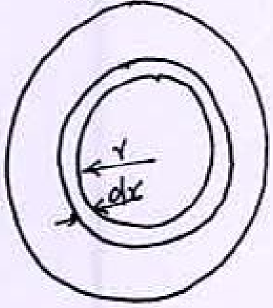
$$= \frac{2Kq}{\pi r^2} \int_0^{\pi/2} \cos\theta$$

$$= \frac{2Kq}{\pi r^2} (\sin\theta)_0^{\pi/2}$$

$$= \frac{2Kq}{\pi r^2}$$

$$E = \frac{-q}{2\pi^2 \epsilon_0 r^2} \hat{j} \quad -(3).$$

(28).



$$Q = \int 4\pi r^2 \rho_0 \left(\frac{5}{4} - \frac{r}{R}\right)$$

$$Q = 4\pi \rho_0 \left(\frac{5}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$E = \frac{KQ}{r^2} = \frac{\cancel{4\pi} \rho_0}{\cancel{4\pi}\epsilon_0 r^2}\left(\frac{5}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right).$$

$$= \frac{\rho_0 r}{4\epsilon_0 \cancel{r^2}}\left[\frac{5}{3} - \frac{r}{R}\right].$$

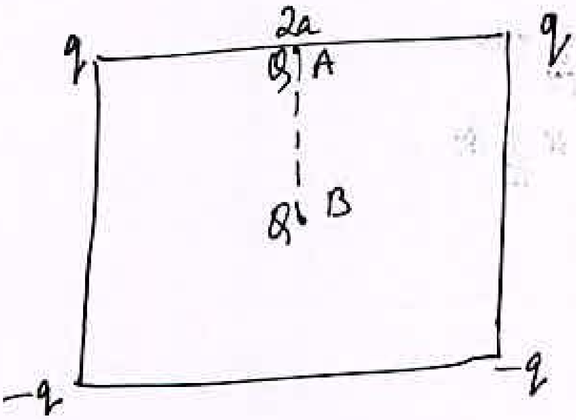
(29).

$$F = q\vec{E} + q\vec{V}\times\vec{B}$$

$$= q(3\hat{i} + \hat{j} + 2\hat{k}) + q(3\hat{i} + 4\hat{j} + \hat{k}) \times (\hat{i} + \hat{j} - 3\hat{k})$$

$$F_y = q + (9q + q) = 11q \quad (2)$$

30.



$$V_A = \frac{2KQq}{a} - \frac{2qQK}{a\sqrt{5}}$$

$$= \frac{2KQq}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2Qq}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

32. $\phi = ar^2 + b$

Electric Field $E = \frac{-d\phi}{dr} = -2ar$.

According to Gauss's Theorem.

$$\oint E \cdot ds = \frac{q_{in}}{\epsilon_0}$$

$$-2ar\,4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = -8ar\pi\epsilon_0 r^2$$

$$\rho_{in} = \frac{q_{in}}{\frac{4}{3}\pi r^3}$$

$$\boxed{\rho_{in} = -6a\epsilon_0} \quad (3).$$

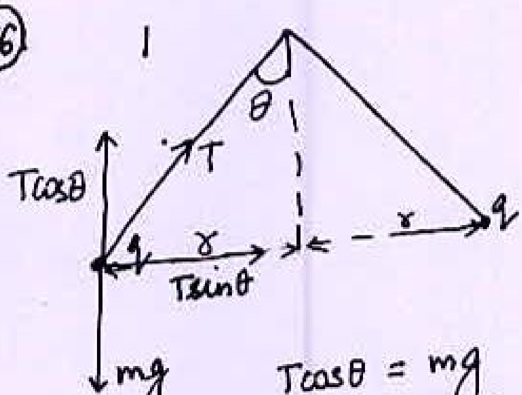
33. Theory Based

34. Theory Based

(35) Theory Based.

Magnetic Field = 0

(36)



$T\cos\theta = mg$

$T\sin\theta = \frac{kq^2}{4r^2}$

$\tan\theta = \frac{kq^2}{4r^2 mg}$

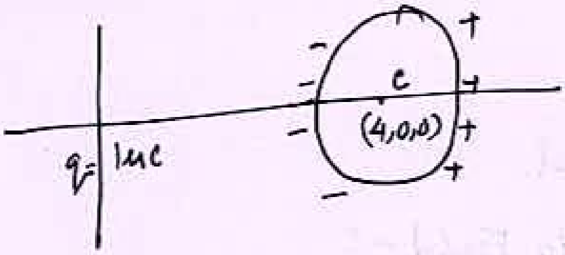
$r = \sqrt{\frac{kq^2}{4mg\tan\theta}}$

$V = \frac{2kq}{r}$

$= \frac{2kq}{q\sqrt{\frac{k}{4mg\tan\theta}}}$

$= 4\sqrt{kmg\tan\theta}$

37

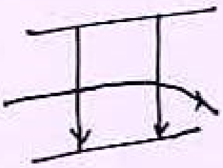


$$V_c = V_{ind} + V_q = 0 + \frac{Kq}{r}$$

$$= \frac{9\times10^9\times10^{-6}}{4} = 2.25\times10^3\,V$$

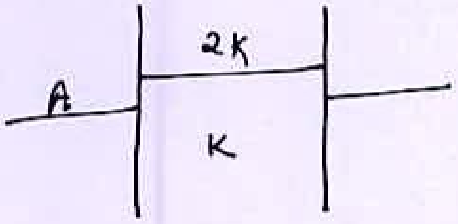
$$E_{ind} = \frac{Kq}{r^2} = \frac{9\times10^9\times10^{-6}}{16} = 5.625\times10^2\,V/m$$

38 Parabola



39 Next Page.

39



$$C_0 = \frac{KA\epsilon_0}{d}.$$

$$C = \frac{2KA\epsilon_0}{3d} + \frac{K\epsilon_0 2A}{3d}$$

$$= \frac{4}{3}\frac{AK\epsilon_0}{d}$$

$$\frac{C}{C_0} = \frac{4}{3}$$

40 electric lines of force are perpendicular to equipotential surface.

41 Field due to Disk.

$$E = \frac{\sigma}{2\epsilon_0}\left[\frac{x}{R} - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$$x = R.$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$E = \frac{0.4}{1.44}\frac{\sigma}{2\epsilon_0} = .29\, E_c$$

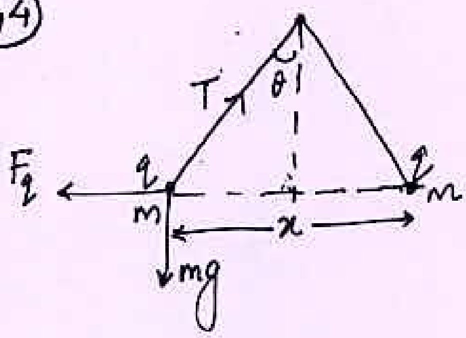
E reduces by 71%

42 (1) Theory based.

43

$$F = \frac{1}{4\pi\epsilon_0}\frac{6p_1p_2}{x^4}$$

44



$$T\cos\theta = mg$$

$$T\sin\theta = F_q = \frac{kq^2}{x^2}$$

$$\tan\theta = \frac{kq^2}{x^2 mg}$$

$\tan\theta \cong \sin\theta$ (θ is small)

$$\frac{Kq^2}{x^2 mg} = \frac{x}{2l}$$

$$x = \sqrt[3]{\frac{2Kq^2 l}{mg}}$$

$$x \propto l^{1/3}$$

45

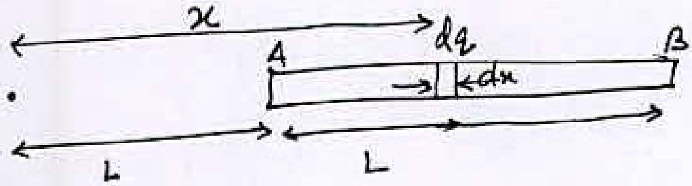
$$Q_1 = Q_2$$

$$120C_1 = 200C_2$$

$$6C_1 = 10C_2$$

$$3C_1 = 5C_2$$

46.



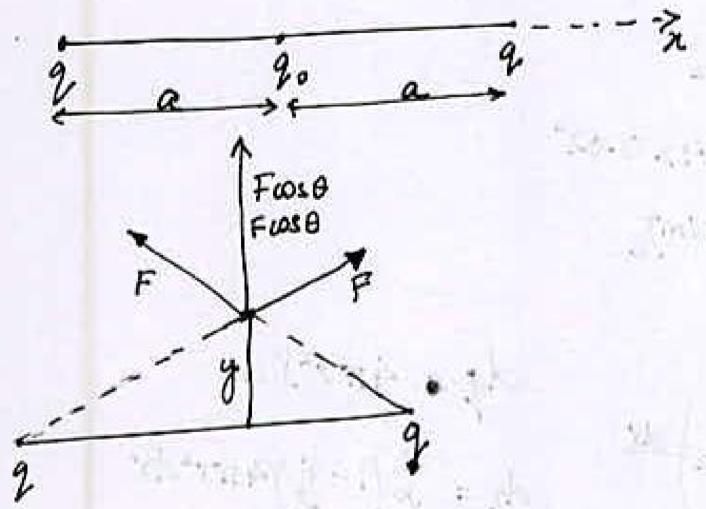
$$dq = \frac{Q}{L}\,dx$$

$$\int dV = \int \frac{k\,dq}{x} = \frac{kQ}{L}\int \frac{dx}{x}$$

$$V = \frac{kQ}{L}\left(\ln x\right)_{L}^{2L}$$

$$V = \frac{kQ}{L}\ln 2 = \frac{1}{4\pi\epsilon_0}\frac{Q\ln 2}{L}$$

47.



$$F_{Net} = 2F\cos\theta = \frac{2\cdot 1}{4\pi\epsilon_0}\frac{q q_0}{\left(\sqrt{y^2+a^2}\right)^2}\frac{y}{\sqrt{y^2+a^2}}$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q\left(\frac{q}{2}\right)}{(y^2+a^2)2}\frac{y}{\sqrt{y^2+a^2}} \quad \left[\because q_0 = \frac{q}{2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{(y^2+a^2)^{3/2}}$$

As $y << a$

$$F_{Net} = \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{a^3}$$

or

$$F_{Net} \propto y$$

(48) $\vec{E} = 30x^2\hat{i}$

$$V = -\int E \cdot dx$$

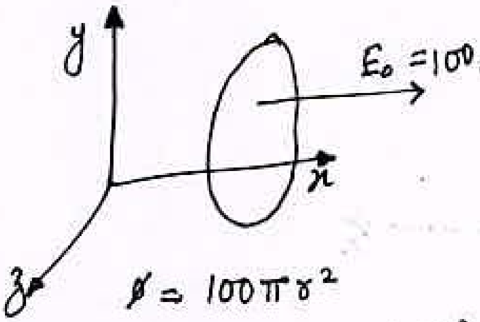
$$= -\int 30x^2 dx$$

$$= \frac{-30x^3}{3}$$

$$V = -10x^3$$

$$V(x=2) = -10\times 2^3 = -80J$$

(49)

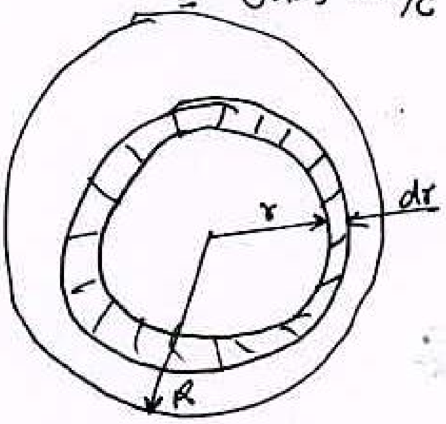


$$\phi = 100\pi r^2$$

$$= 100\times 3.14\times 0.02^2$$

$$= 0.125\ Nm^2/C$$

(50)



$$dq = 4\pi r^2 \rho\, dr$$

$$dq = \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

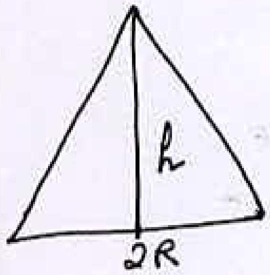
$$Q = \rho_0 4\pi \int r^2 dr - \frac{\rho_0 4\pi}{R}\int r^3 dr$$

$$Q = \frac{4\pi\rho_0 r^3}{3} - \frac{\rho_0 \pi r^4}{R}$$

$$E = \frac{Q}{4\pi r^2 \epsilon_0} = \frac{4\pi\rho_0}{4\pi r^2\epsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$E = \frac{\rho}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$$

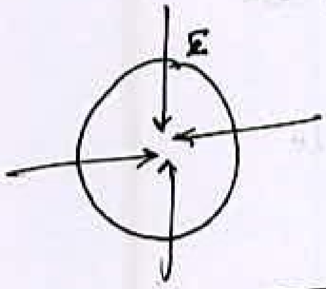
(51)



$\phi = E \times \frac{1}{2} \times 2R \times h$

$\phi = ERh$

(52)



$\int E\,ds = \frac{q}{\epsilon_0}$

$E(4\pi R^2) = \frac{-q}{\epsilon_0}$ (ϕ is entering)

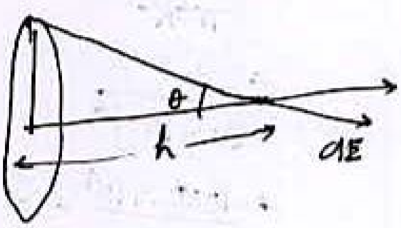
$\boxed{q = -4\pi\epsilon_0 E R^2}$

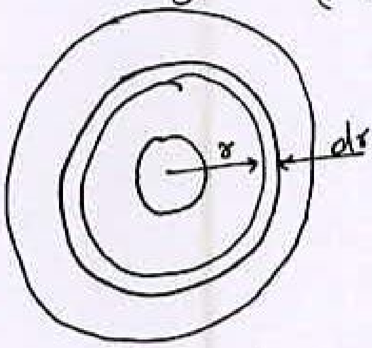
Put the values.

$q = -680\,KC$

(53)

$E_{ring} = \frac{Khq}{(r^2+h^2)^{3/2}}$





$dE = \frac{kh\sigma(2\pi r\,dr)}{(r^2+h^2)^{3/2}}$

$E = K\sigma h 2\pi \int_A^{2A} \frac{r\,dr}{(r^2+h^2)^{3/2}}$

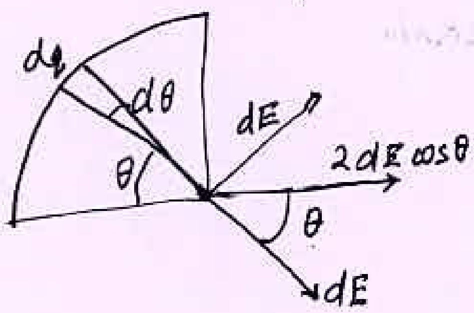
$E = \frac{\sigma}{4\epsilon_0}$

(54)

$\sigma_1 \neq 0 \quad Q_1 = 0$

$\sigma_2 = 0 \quad Q_2 = 0$

(55)



$$dE_x = dE \cos\theta$$

$$= \frac{k\,dq}{R^2} \cos\theta$$

$$= \frac{K}{R^2} \frac{Q}{\frac{\pi R}{2}} R\,d\theta \cos\theta$$

$$= \frac{2KQ}{\pi R^2} \int \cos\theta \, d\theta$$

$$= \frac{2KQ}{\pi R^2}$$

$$2dE_x = \frac{4KQ}{\pi R^2} = \frac{4 \cdot 10^3 \epsilon_0}{4\pi\epsilon_0 \times \pi R^2}$$

$$= \frac{10^3}{\pi^2 R^2}$$

$$= \frac{10^3}{(20 \times 10^{-2})^2}$$

$$= \frac{1000 \times 10^4}{20 \times 20} = 2.5 \times 10^4 \hat{i}$$

(56)

$$\vec{E} = (25\hat{i} + 30\hat{j}) \ NC^{-1}$$

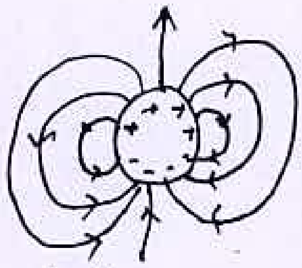
$$V = \int E_x dx + \int E_y dy$$

$$= -(25x + 30y)$$

$$= -(50 + 60)$$

$$\vec{E} = -110$$

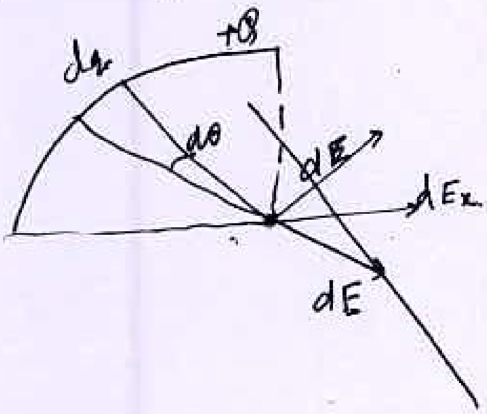
(57)



(1)

Lines are perpendicular to the surface.
& they emerge from +ive charge & enters the -ive charge.

(58) Theory Based.



$dE_x = dE\cos\theta$

$= \frac{k\,dq}{R^2}\cos\theta$

$= \frac{k}{R^2}\,\frac{Q}{\frac{\pi R}{2}}\,R\,d\theta\cos\theta$

$= \frac{2kQ}{\pi R^2}\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta$

$= \frac{2kQ}{\pi R^2}$

$E = \frac{4kQ}{\pi R^2}$

$= \frac{4\times}{4\pi\epsilon_0}\,\frac{10^3\epsilon_0}{\pi R^2} = \frac{10^3}{\pi^2R^2} = \frac{10^3}{(20\times10^{-2})^2} = \frac{1000\times10^4}{20\times20} = 2.5\times10^4\,\hat{i}$