IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 09/06/23**

TOPIC: CALCULU IN PHYSICS

SOLUTION

1. (B)

$y=\sqrt{x}\ln x$

$\frac{dy}{dx}=\frac{\ln x}{2\sqrt{x}}+\frac{\sqrt{x}}{x}=\frac{\ln x+2}{2\sqrt{x}}$

2. (D)

$\int_0^{\pi/4}\sin x\,dx-\int_0^{\pi/4}\cos x\,dx+\int_0^{\pi/4}\sec^2 x\,dx$

$=\left[-\cos x\right]_0^{\pi/4}-\left[\sin x\right]_0^{\pi/4}+\left[\tan x\right]_0^{\pi/4}$

$=\frac{-1}{\sqrt{2}}+1-\frac{1}{\sqrt{2}}+1=2-\frac{2}{\sqrt{2}}$

$=2-\sqrt{2}$

3. (A)

$y=\ln(x)^{3/4}=\frac{3}{4}\ln x$

$\Rightarrow \frac{dy}{dx}=\left(\frac{3}{4}\right)\left(\frac{1}{x}\right)=\frac{3}{4}\times\frac{3}{4}$

$\frac{dy}{dx}=\frac{9}{16}$

4. (C)

$\int\tan x\,dx-\int\sec^2 x\,dx-\int\text{cosec}^2 x\,dx$

$=\int\frac{\sin x}{\cos x}dx-\int\sec^2 x\,dx-\int\text{cosec}^2 x\,dx$

$=-\ln\cos x-\tan x+\cot x+C$

$=\ln|\sec x|-\tan x+\cot x+C$

5. (A)

$y=\frac{x^3}{3}-\ln x+4x$

$$\frac{dy}{dx} = x^2 - \frac{1}{x} + 4$$

$$\frac{d^2y}{dx^2} = 2x + \frac{1}{x^2} = 2(2) + \frac{1}{(2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{17}{4}$$

6. (B)

$$y = \frac{x^3}{3} + \frac{5x^2}{n} + 8$$

$$\frac{dy}{dx} = x^2 + \frac{10x}{n}$$

$$\frac{d^2y}{dx^2} = 2x + \frac{10}{n}$$

At $x = 1$, $\frac{d^2y}{dx^2} = 2(1) + \frac{10}{n} = 3$

$\Rightarrow \quad n = 10$

7. (A)

$$\int \frac{3x^2+2}{x^3+2x+8}\,dx$$

Let, $P = x^3 + 2x + 8$

$$\frac{dP}{dx} = 3x^2 + 2$$

$$\Rightarrow \quad \frac{3x^2+2}{x^3+2x+8}\,dx = \int \frac{dP}{P} = \ln P + C$$

$$= \ln\left(x^3 + 2x + 8\right) + C$$

8. (C)

$y = \ln P^2 = 1 \ln P$

$$\Rightarrow \quad \frac{dy}{dP} = \frac{2}{P}$$

$$x = \frac{P^3}{3} \;\Rightarrow\; \frac{dx}{dP} = P^2 \text{ x}$$

$$\frac{dy}{dx} = \frac{dy}{dP} \times \frac{dP}{dx} = \frac{2}{P} \times \frac{1}{P^2} = \frac{2}{P^3}$$

9. (C)

$$\int_0^1 3x^2\,dx - \int_0^1 2x\,dx + \int_0^1 4\,dx$$

$$= \left[3\left(\frac{x^3}{3}\right)\right]_0^1 - \left[2\left(\frac{x^2}{2}\right)\right]_0^1 + \left[4x\right]_0^1$$

$$= 1 - 1 + 4 = 4$$

10. (B)

$$\int y\,dx=\int 3ax^2\,dx+\int 2x\,dx-\int 3dx$$

$$\int_1^2 y\,dx=\left[\frac{3ax^3}{3}\right]_1^2+\left[\frac{2x^2}{2}\right]_1^2-\left[3x\right]_1^2$$

$$14=a(8-1)+(4-1)-3(2-1)$$

$$14=7a \Rightarrow a=2$$

11. (A)

$$y=\frac{x^2+\sec x}{\ln x}$$

$$\frac{dy}{dx}=\frac{(\ln x)(2x+\sec x\tan x)-\dfrac{(x^2+\sec x)}{x}}{(\ln x)^2}$$

$$\frac{dy}{dx}=\frac{(x\ln x)(2x+\sec x\tan x)-(x^2+\sec x)}{x\ln^2 x}$$

12. (C)

$$y=\frac{x^3}{3}-2x^2-32x$$

Step-1 : $\frac{dy}{dx}=x^2-4x-32$

Step-2 : $\frac{dy}{dx}=x^2-4x-32=0$

$\Rightarrow \quad x=8$ or $x=-4$

Step-3 : $\frac{d^2y}{dx^2}=2x-4$

Step-4 : $\frac{d^2y}{dx^2}=2(-4)-4=-12<0$

At $\quad x=-4$

$\Rightarrow \quad x=-4$ is maxima.

Hence, $y_{\max}=\frac{(-4)^3}{3}-2(-4)^2-32(-4)$

$$=-\frac{64}{3}-32+128$$

$$=\frac{-64-96+384}{3}$$

$$y_{\max}=\frac{224}{3}$$

13. (A)

$$v=3t^2-2t+5$$

$\Rightarrow \quad dS=3\int t^2dt-2\int tdt+5\int dt$

$$[S]_0^S = 3\left[\frac{t^3}{3}\right]_2^3 - 2\left[\frac{t^2}{2}\right]_2^3 + 5[t]_2^3$$

$S = (27-8)-(9-4)+5(3-2)$

$S = 19-5+5 = 19$

14. (C)

$a = t^2 - 4t - 32 = 0$

$\Rightarrow \quad (t-8)(t+4) = 0$

$\Rightarrow \quad t = 8 \text{ or } t = -4$

As time can't be nagative. Hence $t = 8$.

Now, $\int dv = \int t^2\, dt - 4\int t dt - 32\int dt$

$$[v]_0^v = \left[\frac{t^3}{3}\right]_0^8 - 4\left[\frac{t^2}{2}\right]_0^8 - 32[t]_0^8$$

$$\Rightarrow \quad v = -\frac{640}{3}$$

15. (B)

$$P = \frac{t^4}{4} + \frac{t^2}{2} + 1$$

$$F = \frac{dP}{dt} = \frac{4t^3}{4} + \frac{2t}{2} = t^3 + t$$

At, $t = 2,\ F = (2)^3 + 2 = 10$

16. (D)

$$\int (x)^{3/2}\left(x^2 - 1\right)dx = \int x^{7/4}dx - \int x^{3/2}dx$$

$$= \frac{(x)^{\frac{7}{4}+1}}{\frac{7}{4}+1} - \frac{(x)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{4}{11}(x)^{\frac{11}{4}} - \frac{2}{5}(x)^{\frac{5}{2}} + C$$

17. (C)

$$y = \frac{2x^3}{3} + x + 7$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x^2 + 1$$

At $x = 0,\ \frac{dy}{dx} = 1$

18. (B)

$$y = \frac{x^3}{3} + \frac{x^2}{2} - 12x + 6$$

Step-1 : $\frac{dy}{dx} = x^2 + x - 12$

Step-2 : $\frac{dy}{dx} = (x+4)(x-3) = 0$

$\Rightarrow \quad x = -4$ or $x = 3$

Step-3 : $\frac{d^2y}{dx^2} = 2x + 1$

Step-4 : At $x = 3$ $\qquad \frac{d^2y}{dx^2} = 2(3) + 1 = 7 > 0$

$\Rightarrow \quad x = 3 = \text{minima}.$

19. (D)

$$a = \frac{dv}{dt} = 2t + 1$$

$$\Rightarrow \quad \int dv = \int 2t\,dt + \int dt$$

$$[v]_0^v = 2\left[\frac{t^2}{2}\right]_0^t + [t]_0^t$$

$$v = t^2 + t$$

$$\Rightarrow \quad \frac{ds}{dt} = t^2 + t$$

$$\int ds = \int t^2\,dt + \int t\,dt$$

$$S = \frac{t^3}{3} + \frac{t^2}{2} + C$$

20. (B)

$$v = \pi R^2 H, \quad R = \frac{H}{2}$$

$$v = \frac{\pi H^3}{4}$$

$$\Rightarrow \quad \frac{dv}{dH} = \frac{3\pi H^2}{4} \Rightarrow dv = \frac{3\pi H^2 dH}{4}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{3\pi}{4} H^2 \frac{dH}{dt}$$

$$\frac{\pi}{2} = \frac{3\pi}{4}(2)^2 \frac{dH}{dt} \Rightarrow \frac{dH}{dt} = \frac{1}{6}\text{cm/s}$$

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TOPIC: QUADRATIC EQUATION

Solution

21. (C)

In order that the quadratic equation may have two roots with opposite signs, it must have real roots with their product negative, *i.e.* if the discriminant,

$$4(a^2+1)^2-12(a^2-3a+2)>0$$

and $\frac{1}{3}(a^2-3a+2)<0$

Both of these conditions get satisfied if $a^2-3a+2<0$

i.e., if $(a-1)(a-2)<0$ or if $1<a<2$.

22. (C)

If D is the discriminant of the equation $x^2-4qx+2q^2-r=0$, then

$$D=16q^2-4(2q^2-r)=8q^2+4r$$

$$=8\alpha^2\beta^2+4(\alpha^4+\beta^4)=4(\alpha^2+\beta^2)^2\geq 0$$

Hence the equation $x^2-4qx+2q^2-r=0$ has always two real roots *i.e.* both real roots.

23. (B)

24. (A)

Let roots of $lx^2+nx+n=0$ are α and β, given that $\frac{\alpha}{\beta}=\frac{p}{q}\Rightarrow\alpha=\beta\left(\frac{p}{q}\right)$ .....(i)

$$\alpha+\beta=\frac{-n}{l} \text{ and } \alpha\beta=\frac{n}{l}$$

Now $\sqrt{\left(\frac{p}{q}\right)}+\sqrt{\left(\frac{q}{p}\right)}+\sqrt{\left(\frac{n}{l}\right)}=\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\alpha\beta}$

$$=\frac{(\alpha+\beta)-(\alpha+\beta)}{\sqrt{\alpha\beta}}=0.$$

25. (D)

Let the correct equation be $x^2+px+q=0$ .....(i)

Roots found by the first student are 6 and 2.

Their sum $=6+2=8=-p$

and the product $=6\times 2=12=q$

So, (i) reduces to $x^2 - 8x + 12 = 0$ .....(ii)

But he has committed mistake only in the coefficient of *x* *i.e.* in *p*. So *q* remains equal to 12 while *p* in the actual equation has been taken wrongly by the first student.

Now roots found by the second student are 2 and –9.

Their sum $= -9 + 2 = -7 = -p$ and the product

$= -9 \times 2 = -18 = q$ *i.e.*, $p = 7$ and $q = 18$ in (i)

But he has committed mistake only in the constant term *i.e.* in *q*. So *p* remains equal to 7.

Hence correct equation from (i) is $x^2 + 7x + 12 = 0$

$\Rightarrow \quad (x+4)(x+3) = 0 \Rightarrow x = -4, -3$.

26. (A)

$\alpha + \beta = p, \alpha\beta = q$

If the given roots be *A* and *B*, then

$A = (\alpha - \beta)^2 (\alpha + \beta)(\alpha^2 + \beta^2 + \alpha\beta)$

$= (p^2 - 4q)p(p^2 - q) = p[p^4 - 5p^2 q + 4q^2]$

$B = \alpha^2 \beta^2 (\alpha + \beta) = q^2 p$

$\therefore \quad S = A + B = p[p^4 - 5p^2 q + 5q^2]$

$P = p^2 q^2 (p^4 - 5p^2 q + 4q^2)$

The required equation is $x^2 - Sx + P = 0$.

Trick : Check by putting $p = 3, q = 2$ so that $\alpha = 2, \beta = 1$.

Now roots of required equation will be $21, 12$.

Therefore $S = 33$ and $P = 252$ which is given by the option (A).

27. (D)

Let the correct equation be $ax^2 + bx + c = 0$ and the correct roots α and β.

Taking *c* wrong, the roots are 3 and 2.

$\therefore \quad \alpha + \beta = 3 + 2 = 5$ .....(i)

Also $a = 1$ and $c = -6$

$\therefore \quad \alpha\beta = c/a = -6$ .....(ii)

Solving (i) and (ii), the correct roots are 6 and – 1.

28. (A)

Let the roots are α and β

so, $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (a-2)^3 - 3(a-3)(a-2)$

$= a^3 - 9a^2 + 27a - 26 = (a-3)^3 + 1$

It assumes the least value, if $(a-3)^3 = 0$.

$\therefore \quad a = 3$.

29. (A)

As the coefficients are real and one root is $2+i,$ therefore, another root is $2-i$ (conjugate of $2+i$).

Let the third root be α then sum of the roots $=2+i+2-i+\alpha$

$\Rightarrow \quad -(-5)=4+\alpha \Rightarrow \alpha=1$

So, the other two roots are $2-i$ and 1.

30. (C)

We have $x^2+px+(1-p)=0$ .....(i)

$(1-p)^2+p(1-p)+(1-p)=0$

$(1-p)[1-p+p+1]=0; \qquad p=1$

Put $p-1$ in equation (i),

$x^2+x=0 \Rightarrow x(x+1)=0$ *i.e.,* $x=0=-1$.

31. (A)

$y=\sqrt{\dfrac{(x+1)(x-3)}{(x-2)}}$

Here x cannot be 2.

$\Rightarrow$ Either both N^r and D^r are positive

$x \geq -1, x \geq 3$ and $x>2 \Rightarrow x \geq 3$ .....(i)

Or N^r is negative and D^r is negative

$x \geq -1$ and $x<2 \Rightarrow -1 \leq x<2$ .....(ii)

From (i) and (ii), $-1 \leq x<2$ or $x \geq 3$.

32. (D)

Domain of definition of the function $y=\sqrt{x(x-3)}$ is $x(x-3) \geq 0$ *i.e.* $x \leq 0$ or $x \geq 3$ .....(i)

Given equation can be re-written as

$9|x|^2-19|x|+2=0$

$\Rightarrow \quad (9|x|-1)(|x|-2)=0$

$\Rightarrow \quad |x|=2$ or $|x|=1/9$

$\therefore$ Solution of the given equation are $\pm 2, \pm 1/9$

In the domain (i), the required solutions are $-2,-1/9$.

33. (C)

Given that $x^2+2ax+a^2$ is a factor of

$x^3-3px+2q=0$

Let $x^3-3px+2q=(x^2+2ax+a^2)(x+\lambda)$,

where λ is a constant.

Then equating the coefficients of like powers of x on both sides,

$x^3-3px+2q=x^3+(2a+\lambda)x^2+(a^2+2a\lambda)x+\lambda a^2$

$\Rightarrow \quad 2a+\lambda=0 \Rightarrow \lambda=-2a$ ..... (i)

and $-3p=a^2+2a\lambda$ .....(ii)

and $2q=\lambda a^2$ .....(iii)

Put the value of λ in (iii),

$\Rightarrow \quad 2q=-2a^3 \Rightarrow q=-a^3$ .....(iv)

Put the value of λ in (ii),

$\Rightarrow \quad -3p=a^2+2a(-2a)=a^2-4a^2=-3a^2$

$\Rightarrow \quad -3p=-3a^2 \Rightarrow p=a^2 \Rightarrow p=(-q)^{2/3} \Rightarrow p^3=q^2$.

34. (A)

Roots α, β lie in the interval $(0,1)(\alpha \neq \beta)$, so

(i) $\Delta > 0$, (ii) $f(0)>0, f(1)>0$, (iii) $0<\alpha+\beta<2$

$\therefore$ Here $\alpha+\beta=3 \not< 2$ 1 for every k

Hence no such k exist.

35. (A)

36. (A)

From $k=\dfrac{x^2-x+1}{x^2+x+1}$

We have $x^2(k-1)+x(k+1)+k-1=0$

As given, x is real $\Rightarrow$ $(k+1)^2-4(k-1)^2 \geq 0$

$\Rightarrow 3k^2-10k+3 \geq 0$

Which is possible only when the value of k lies between the roots of the equation $3k^2-10k+3=0$

That is, when $\dfrac{1}{3} \leq k \leq 3$ {Since roots are $\dfrac{1}{3}$ and 3}

37. (A)

$x=\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$ to ∞

We have $x=\sqrt{1+x}$

$\Rightarrow x^2=1+x \quad \Rightarrow x^2-x-1=0$

$\Rightarrow x=\dfrac{1 \pm \sqrt{1+4}}{2}=\dfrac{1 \pm \sqrt{5}}{2}$

As $x>0$, we get $x=\dfrac{1+\sqrt{5}}{2}$

38. (C)

Given equations are $2x^2+3x+5\lambda=0$ and $x^2+2x+3\lambda=0$ have a common root if

$\dfrac{x^2}{-\lambda}=\dfrac{x}{-\lambda}=\dfrac{1}{1} \Rightarrow x^2=-\lambda, x=-\lambda$ or $\lambda=-1, 0$.

39. (A)

As given, $\sin\alpha+\cos\alpha=-\dfrac{b}{a}$, $\sin\alpha\cos\alpha=\dfrac{c}{a}$

To eliminate α, we have

$1 = \sin^2 \alpha + \cos^2 \alpha = (\sin\alpha + \cos\alpha)^2 - 2\sin\alpha\cos\alpha$

$= \frac{b^2}{a^2} - \frac{2c}{a} \Rightarrow a^2 - b^2 + 2ac = 0$

40. (C)

Let $\alpha = 1$ and the other root is β, then product of roots 1.

$\beta = \frac{c(a-b)}{a(b-c)} \Rightarrow \beta = \frac{c(a-b)}{a(b-c)}$