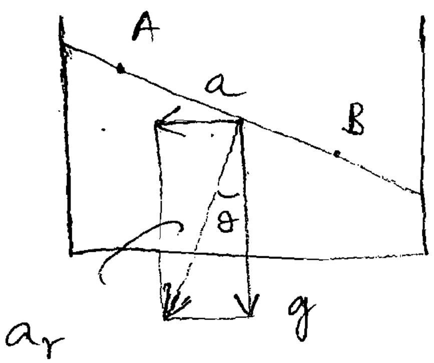
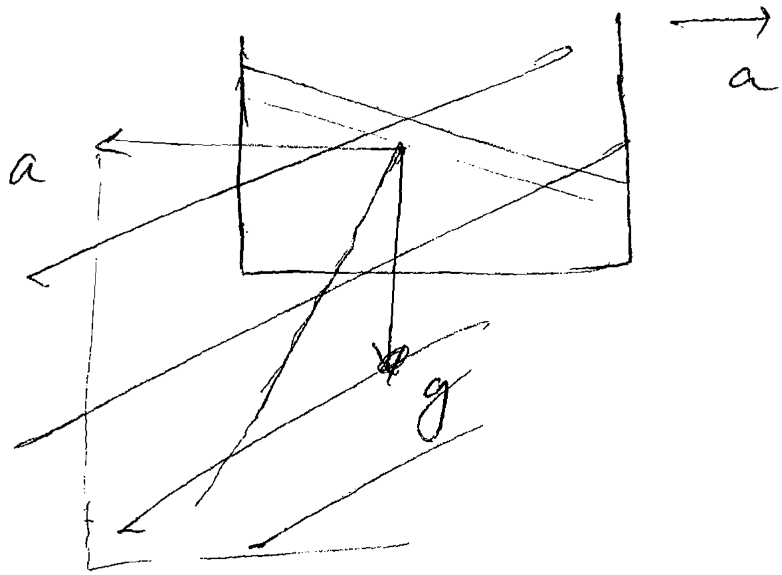
Fluid Mechanics

Inchapter Exercise - I

Q1. (b) If container is ~~not~~ accelerating horizontally then, pressure at different levels can be same





$$\tan\theta = \frac{a}{g}$$

$$P_A = P_B$$

Q2 (b) (c) (d)

$$P_N + H\rho g = P_M \quad , \quad \therefore P_N \neq P_M$$

$$P_N = P_0 = \text{atm. pressure}$$

$\therefore$ Gauge Pressure at $N = 0$

Gauge Pressure at $M = H\rho g$

$$= 0.05 \times 13.6 \times 10^3 \times 9.8 =$$

$$= 6.66 \times 10^3\ N/m^2$$

$P_{\text{at mouth bulb}} = P_M = P_0 + P_{gauge}$

Q3. (B) , Before melting, buoyant force

$B =$ wt. of cube + wt. of glass ball (floating)

~~B' = wt. of glass ball~~

$B' =$ Volume of fluid displaced by glass ball

Now, density of glass is greater than water. So, it will sink and its weight is balanced by buoyant force and reaction from bottom.

So, $B' < B$.

Hence, water level will fall.

Q4 (B)

$B = V \rho_l g$ $\quad \rho_l =$ fluid density

Q5 (A) When ice melts, it gives volume of water equal to that displaced by it. So, water level will remain unchanged.

Q6 (A) (B) (C)

Apparent wt. $= mg - B = V(\sigma - \rho) g$

If $\sigma > \rho$, then Apparent wt. > 0

$\sigma = \rho$, Apparent wt $= 0$

Q7. (C) Earlier wt. of balls was supported by buoyant force.

Now, wt. of balls is supported by buoyant force + Reaction at bottom.

So, $B' < B$.

$\therefore$ Volume displaced decreases and hence level of liquid in tank.

Q8. (B)

Force = Pressure x Area

$= h\rho g \times A$

$= 0.4 \times 900 \times 10 \times 2 \times 10^{-3}$

$= 7.2\ N.$

Q9 (A)

$$(m_1 + m_2) g = B_1 + B_2$$

$$\cancel{(V_1\rho_1 + V_2\rho_2) g = (V_1 + V_2)}$$

$$(V_1\sigma_1 + V_2\sigma_2) g = (V_1 + V_2)\rho g.$$

$$V_1 \frac{\sigma_1}{\rho} + V_2\left(\frac{\sigma_2}{\rho}\right) = V_1 + V_2$$

$$V_1(2) + V_2(1/2) = V_1 + V_2$$

$$3V_1 = V_2.$$

$$\frac{m_1}{} = \frac{\sigma_1 V_1}{} = \cancel{(\sigma_1/\rho_1) V_1} \left(\sigma_1/\rho\right) V_1 = 2$$

Q10 (A) From pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = (1500\,g)\left(\frac{\pi r^2}{\pi R^2}\right) = \cancel{1500\,g \times \left(\frac{1}{20}\right)^2}$$

$$F_1 = 1500\,g\left(\frac{1}{20}\right)^2$$

$$F_1 = \frac{1500\,g}{400} \qquad m = 3.75\text{ kg}.$$

Q11 (A)

$g_{eff} = 0$, $B = V\rho g_{eff}$

$\therefore$ $B = 0$

Q12 (C)

When passengers drink water, weight of boat increases and hence it displaces more water equal to wt of water ~~taken~~ ~~by~~ ~~drunk~~ drunk.

Hence, no change in water level of tank.

8

Fluid - Mechanics

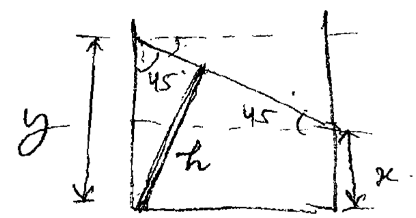
Inchapter - 2

Q1. (C).

Velocity increases with decrease in cross - section, whereas Pressure and hence force decreases with increase in velocity.

Q2 (B).

$\theta = 45^\circ$ = angle made by surface with horizontal.



$$\frac{y-x}{1} = \tan 45^\circ = 1$$

$$y - x = 1$$

$$y = x + 1 \Rightarrow$$

~~Also~~. Volume of water = const.

$$1^3 = 1 \times \left[x \times 1 + \frac{1}{2} \times (y - x) \times 1\right]$$

$$\cancel{2 = 2x + y - x \Rightarrow \quad y = 2m \quad x + y = 2}$$

~~x = 1m.~~ $x + y = 2$.

$$x = \frac{1}{2}\ m, \quad y = \frac{3}{2}\ m$$

~~tan~~ $\sin 45^\circ = \frac{h}{y} \Rightarrow \quad h = \frac{y}{\sqrt{2}} = \frac{3}{2\sqrt{2}}\ m.$

$\therefore\ V = \sqrt{2 \times 10\sqrt{2} \times \frac{3}{2\sqrt{2}}} = 5.48\ m/s.$

Q3 (A)(B)(C)(D)

Equation of continuity is principle of

$\cancel{\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}}$

conservation of mass while Bernoulli's theorem is principle of conservation of energy for ideal fluid.

Q4. (B). As cross-section increases, velocity decreases and pressure increases.

Q5 (A) (D)

$V_B > V_A$ so, $P_B < P_A$

Fluid ~~particle~~ particle at B has to cover more distance as compared to A. in given time.

Q6 (C) $A_1V_1 = A_2V_2$

$V_2^2 = V_1^2 + 2gh$ $\quad V_1 = 1\ m/s$ $\quad h = 0.15\ m$

solving, $A_2 = 5 \times 10^{-5}\ m^2$

Q7 (C) $V = A_1V_1 = A_2V_2 + A_3V_3$.

solving, we get.

$V_3 = 1$ m/s.

Q8. (C). $V_A < V_B$

so, initially level of water decreases in vessel. As a result V_B decreases and hence level starts rising.

Water performs periodic oscillatory motion.

Q9. (B)

loss of weight = Buoyant force

$= V \rho g$.

V = volume of fluid displaced.

Q10 (C)

$$\frac{1}{2}\rho_1 V_1^2 = h_1\rho_1 g + h_2\rho_2 g.$$

$$V_1 = \sqrt{2g\left[h_1 + h_2\left(\frac{\rho_2}{\rho_1}\right)\right]}$$

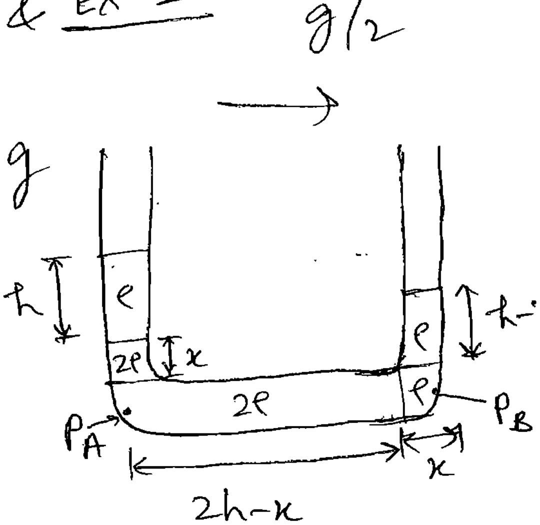
Fluid Mechanics - Ex - I (Solutions)

& Ex - II

Q1 (B) $P_A - P_B =$

$= h\rho g + x(2\rho)g - (h-x)\rho g$

$= 3x\rho g$



Force on horizontal fluid element $= (P_A - P_B) \times A = F$

$A =$ cross - sectional Area.

$F = ma$ $\quad a = g/2$

$m =$ mass of horizontal fluid element.

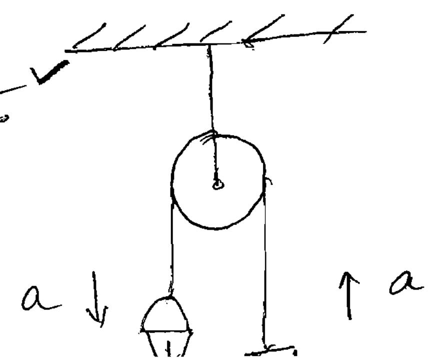
$m = (2h-x)\,2\rho A + x\rho A$

$\therefore (3x\rho g)A = \left((2h-x)2\rho A + x\rho A\right)\frac{g}{2}$

Solving, $x = \frac{4h}{7}$ $\quad$ Difference in level $= 2x = \frac{8h}{7}$

Q2 (B) acc^n of bucket $= \dfrac{mg - mg/2}{3m/2}$

$= \dfrac{g}{3}$



(c) $P_{bucket} = h\rho g'$ $\qquad g' =$ effective gravity

$= 0.15 \times 10^3 \times \frac{2}{3} \times g \qquad = g - a = \frac{2g}{3}$

$= 1\ \text{kPa}$

13 (D) Total force by fluid on cone

$= \text{Buoyant Force} = V\rho g = \frac{\pi}{3}R^2 H\rho g$

Force at bottom of cone $=$ Pressure $\times$ ~~Ar~~ Area

$= H\rho g \times \pi R^2 = \pi R^2 H \rho g$

$\therefore$ Force on slant surface due to liquid

$= \pi R^2 H\rho g - \frac{\pi}{3}R^2 H\rho g = \frac{2\pi}{3} H R^2 \rho g$

(Vertically downward).

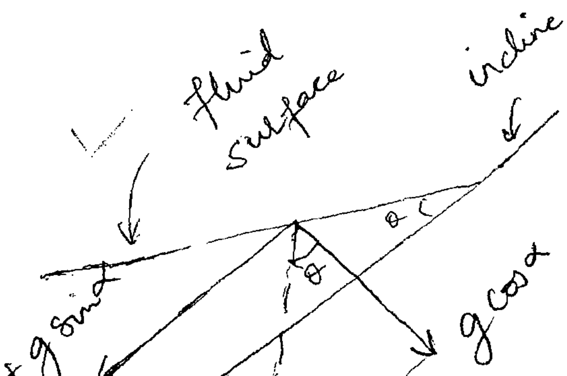
14 ~~(B)~~ (C)

$P_{wider} = P_{narrow}$ (at same horizontal level).

$\frac{12g}{800 \times 10^{-4}}\ N/m^2 = h\rho g$

solving, $h = 15\ \text{cm}$.

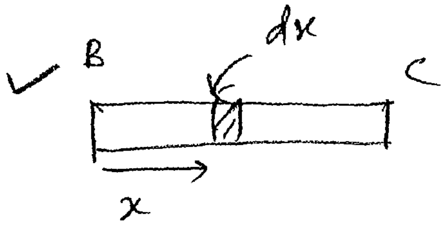
15 (B) $\tan\theta = \frac{a + g\sin\alpha}{\ldots}$



06. (C) New level of liquid in each arm $= l + \frac{l}{2} = \frac{3l}{2}$ (3)

Pressure difference at bottom of B & C

$$= \frac{3l}{2}\rho g.$$



$$dF = (\rho A\, dx)\, x\omega^2$$

$$dP = \frac{dF}{A} = \rho x \omega^2 dx$$

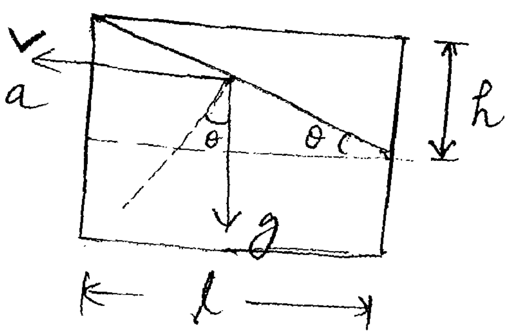
$$P_C - P_B = \int_{x=0}^{l} \rho x \omega^2 dx = \rho\omega^2\left[\frac{x^2}{2}\right]_0^l = \frac{\rho\omega^2 l^2}{2}$$

$$\therefore \quad \frac{\rho\omega^2 l^2}{2} = \frac{3l}{2}\rho g$$

$$\omega = \sqrt{\frac{3g}{l}}$$

07 (B)

Volume spilled $= \frac{l^3}{3}$



$= \frac{1}{2} \times h \times l \times l$

$h = \frac{2l}{3}$

$$\tan\theta = \frac{a}{g} = \frac{h}{l} = \frac{2}{3} \Rightarrow a = \frac{2g}{3}$$

8(D) Volume of liquid in two containers

$$V_1 = V_2$$

$$\pi R^2 H_1 = R^2 H_2$$

H_1, H_2 are heights of liquid in two containers

$$F_1 = H_1 \rho g \pi R^2$$

$$F_2 = H_2 \rho g R^2$$

$$\frac{F_1}{F_2} = \frac{H_1 \rho g \pi R^2}{H_2 \rho g R^2} = 1:1$$

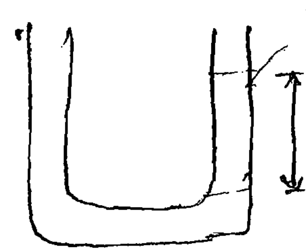
$$F_1 = F_2$$

Q9 (A)

$\because$ liquid is in equilibrium. so, Total force exerted by flask should be equal ~~to~~ and opposite to weight of liquid.

Q10 (D)

length of water in arm



$$= \frac{60\ cc}{1\ cm^2} = 60\ cm.$$

Volume of liquid in horizontal arm $= 20\ cm \times 1\ cm^2 = 20\ cc$

If h be height of liquid in other arm.

then, from ~~p~~ Pascal's law

$$h \rho_L g = 60 \rho_w g$$

$$h = \frac{60}{4} = 15\ cm.$$

F_B = Force exerted by bottom of cone on water

F_W = Force by walls on water.

$W = V\rho g = \frac{\pi}{3} R^2 h \rho g$

$F_B = P_B \times A = h\rho g \times \pi R^2 = \pi R^2 h \rho g$

$\therefore$ For equilibrium of water

$F_B = F_W + W \Rightarrow F_W = \frac{2\pi}{3} R^2 h \rho g$

From Newton's third law, force exerted by water on cone walls is $\frac{2\pi}{3} R^2 h \rho g$ upward.

Q14 (B)

Under equilibrium condition

Buoyant Force = weight of cubes

~~$(1^2 \times 1 \times 1.15 \times 10^3 + 1^2 \times h \times 0.6 \times 10^3) g$~~

$(1^2 \times 1 \times 10^3 + 1^2 \times h \times 10^3) g =$

$= (1^2 \times 1 \times 1.15 \times 10^3 + 1^2 \times 1 \times 0.6 \times 10^3) g$

solving,

$h = 0.75 \text{ m} = 75 \text{ cm}.$

$\therefore$ height above water surface $= 25$ cm.

Q11 (B) If H is height of water

(6)

$$2P = H\rho g$$

Now, height of water remaining $= H - \frac{H}{5} = \frac{4H}{5}$

$\therefore$ New pressure at bottom $= P + \frac{4}{5}H\rho g$

$$= P + \frac{4}{5}(2P) = \frac{13}{5}P$$

Q12 (C)

$$h_1 - 5 + h_2 = 40$$

$$h_1 + h_2 = 45 \quad \text{———} \quad (1)$$

From Pascal's law

$$5\rho g + (h_1 - 5)\rho_w g = h_2 \rho_w g$$

$$5 \times 4 + (h_1 - 5) = h_2$$

$$h_2 - h_1 = 15 \quad \text{———} \quad (2)$$

solving (1) & (2) $\quad h_2 = 30\text{ cm}, \; h_1 = 15\text{ cm}$

$$\frac{h_2}{h_1} = \frac{30}{15} = 2:1$$

Q13 (A) <u>FBD of water</u>

Q15 (D)

under equilibrium

$mg = B$

$(abc)dg = (lbc)(1)g$ l = length immersed in water

$\therefore \quad l = ad$ ——— (1).

If cube is further pushed down by x then restoring force is given by

$F = B' - mg = (l+x)bc(1)g - abc\,dg$

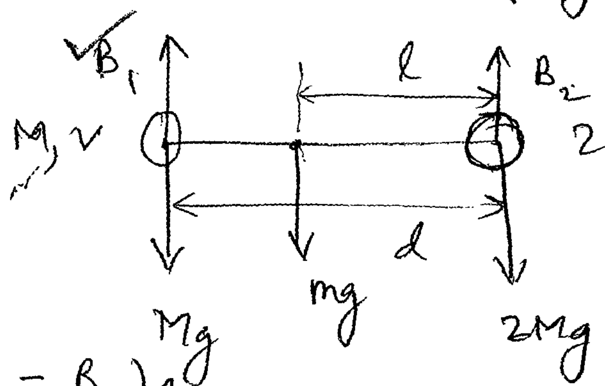
$(abc)dA = x(bc)g$

$A = \frac{x}{ad}g = \frac{g}{ad}x.$

$\therefore \quad \omega^2 = \frac{g}{ad}.$ or $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ad}{g}}$

Q16 (B)

Taking torque about point mass (m).



$(Mg - B_1)(d-l) = (2Mg - B_2)l$

or

$\left(Mg - \frac{V}{2}\rho g\right)(d-l) = \left(2Mg - \frac{V}{2}\rho g\right)l$

$$\frac{d-l}{l} = \frac{2M - V\rho/2}{M - V\rho/2}$$

Q17. (B) Let ρ_1, ρ_2 be the densities of two masses.

$$\therefore \quad V\rho_1 g = 2V\rho_2 g$$

$$\therefore \quad \rho_1 = 2\rho_2 \quad \text{———} \quad (1)$$

When ~~corner~~ immersed in liquid

$$V\rho_1 g - \cancel{V\rho g}\ V\sigma g = 2V\rho_2 g - 2V(0.9)g$$

σ = density of unknown liquid

$$\therefore \quad V\rho_1 g = 2V\rho_2 g.$$

$$\therefore \quad V\sigma g = 2V(0.9)g$$

$$\sigma = 1.8\ gm/cm^3$$

Q18. (C)

$$Mg = \left(a^2\,\frac{4a}{5}\right)\rho g = \frac{4a^3}{5}\rho g \quad \text{———} \quad (1)$$

$$(M+m)g = a^3\rho g \quad \text{———} \quad (2)$$

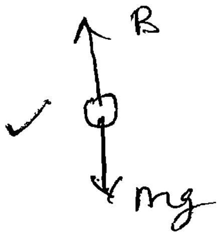
Solving (1) & (2)

$$ma - a^3\rho g$$

$\therefore \; mg = \frac{Mg}{4}$ or $M = 4m$

Q19. (C)

When ~~ship~~ fish is placed in water of bucket then resultant force acting on ~~ship~~ fish is



$F_R = mg - B$ = Effective weight of fish

So, Effective weight of fish decreases.

This buoyant force is applied by water of bucket on fish.

From Newton's third law, the same buoyant force is acting on water and hence bucket downward.

So, net weight carried by boy remains same.

~~Q20. Let V be volume of cork. In equilibrium~~

~~$V'(1)\,g = V^{*}(0.5)\,g$~~

Q20. (D)

10

Let V be volume of ~~water~~ cork and

V' be volume of cork submerged.

In equilibrium,

$$V(0.5)g = V'(1)g$$

$$\frac{V'}{V} = 0.5 = \frac{1}{2} = \text{fraction submerge}$$

or %. $\frac{V'}{V} = \frac{1}{2} \times 100 = 50\%$.

Q21. (A).

Let (A) be cross-section area of cylinders

∴ In equilibrium.

$$(ALd_1)g + (ALd_2)g = \left(A\frac{3L}{2}d\right)g$$

$$d_1 + d_2 = \frac{3}{2}d.$$

$$d_1 > d_2$$

$$d_1 + d_1 > d_2 + d_1 = \frac{3}{2}d.$$

$$2d_1 > \frac{3}{2}d.$$

Q22 (B)

$$W_1 = W - \frac{W \rho_w}{\rho} \quad \text{——— (1)}$$

ρ = density of steel

$$W_2 = W - \frac{W \sigma}{\rho} \quad \text{——— (2)}$$

σ = density of liquid

$\therefore$ from (1) & (2)

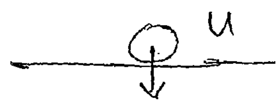
$$\frac{\rho_w}{\rho} = 1 - \frac{W_1}{W} \, , \quad \frac{\sigma}{\rho} = 1 - \frac{W_2}{W}$$

$$\therefore \frac{\sigma}{\rho_w} = \frac{1 - W_2/W}{1 - W_1/W} = \frac{W - W_2}{W - W_1}$$

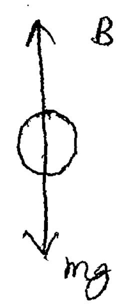
= Relative density

Q23 (A)

velocity of ball just before entering liquid $= \sqrt{2gh} = \sqrt{2g(2)}$,



$u = \sqrt{4g}$



$$ma = B - mg = V(1)g - V(0.8)g$$

$$V(0.8)a = V(0.2)g$$

$$a = g/4$$

V = volume of ball

(12) At maximum depth velocity of ball is zero.

$u = \sqrt{4g}$ = initial velocity (downward)

$a = g/4$ (upward)

$\therefore$ using $v^2 = u^2 + 2as$.

$$0^2 = (\sqrt{4g})^2 + 2(g/4)\,d.$$

$$d = 8\,m$$

Q24 (B)

Net ~~F~~ Force acting on ball

$$= \cancel{V\rho} \; V\sigma g - V\rho g \quad \text{(upward)}.$$

$$= V(\sigma - \rho)g = ma.$$

$$\cancel{V\rho g} \quad V(\sigma - \rho)g = V\rho a$$

$$a = \left(\frac{\sigma}{\rho} - 1\right)g \quad \text{upward}.$$

$\therefore$ velocity of ball when it reaches ~~top face~~ surface of water.

$$= \sqrt{2ah} = \sqrt{2\left(\frac{\sigma}{\rho} - 1\right)gh}$$

When ball comes out of water then, only gravity acts on it.

$\therefore$ initial velocity $= \sqrt{2\left(\frac{\sigma}{\rho} - 1\right)gh}$

Final velocity at height (H) will be zero (13)

$V^2 = u^2 + 2as$

$0^2 = \left(\sqrt{2\left(\frac{\sigma}{\rho}-1\right)gh}\right)^2 - 2gH.$

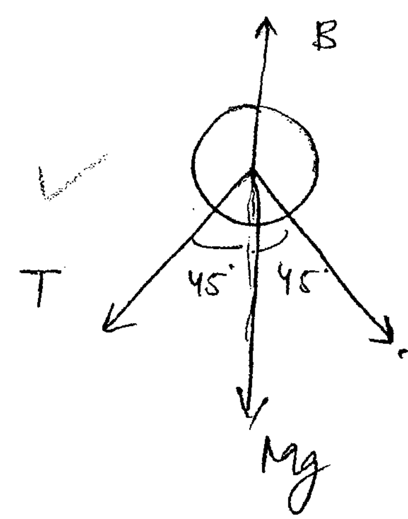
$\therefore \quad H = \left(\frac{\sigma}{\rho}-1\right)h$

Q25 (A)

In equilibrium

$B = Mg + 2T\cos 45°$

$T = \dfrac{\frac{4\pi}{3}R^3 \rho_w g - Mg}{\sqrt{2}}$



FBD

26. (C)

Let V, V_c be volume of ball and cavities in it.

$\therefore \quad 9.8\,kg = 7800\,kg/m^3\,(V - V_c)$

$1.5\,kg = 1000\,kg/m^3\,V.$ (due to buoyant force)

$\frac{V - V_c}{V} = \frac{9.8 \times 1000}{1.5 \times 7800} = \frac{98}{...}$

(14)

$$\frac{V_c}{V} = \frac{19}{117} = 0.1624 = 16.24\,\%$$

§27 (A)

Under condition of floating

$$B = mg.$$

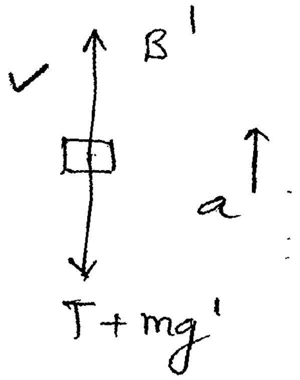
$$\frac{4\pi}{3}R^3(1) = \frac{4\pi}{3}(R^3 - r^3)\,\sigma\, g$$

$$\frac{R^3 - r^3}{R^3} = \frac{1}{\sigma}$$

$$\left(\frac{r}{R}\right)^3 = \frac{\sigma - 1}{\sigma} \quad \text{or} \quad \frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$$

§28 (B)

$$g_{\text{effective}} = g_{eff} = g + a$$



$\therefore$ B' = Buoyant force

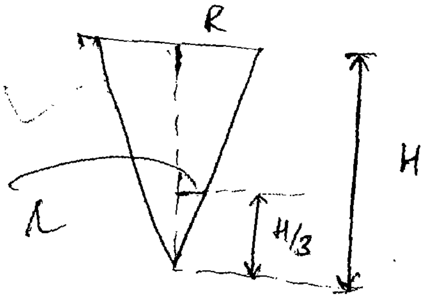
$$= Vd\,(g_{eff}) = Vd(g+a)$$

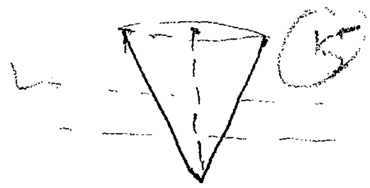
$$mg' = mg_{eff} = V\rho(g+a)$$

$$\therefore \quad T = B' - mg' = V(g+a)(d-\rho)$$

Q29 (D)

$$\frac{r}{R} = \frac{H/3}{H} = \frac{1}{3}$$





$$r = \frac{R}{3}$$

In equilibrium for condition 1.

$$mg = B. \qquad m = \text{mass of cone}$$

$$\frac{\pi}{3} R^2 H \sigma g = \frac{\pi}{3}\left(\frac{R}{3}\right)^2\left(\frac{H}{3}\right)(0.8)\, g$$

$$\therefore \quad \sigma = \frac{0.8}{3^3} = \frac{0.8}{27} = \text{Relative density of material of cone}$$

In equilibrium for condition 2.

$$mg + m_l g = \cancel{B}\ \cancel{\#}\ B'$$

m_l = mass of liquid

B' = New ~~Extra~~ buoyant force acting on cone.

$$\therefore \quad m_l g = B' - B.$$

or. $$\frac{\pi}{3}\left(\frac{R}{3}\right)^2\left(\frac{H}{3}\right) \cancel{\#}\, \rho g = \frac{\pi}{3}\left(\frac{R}{2}\right)^2\left(\frac{H}{2}\right) \overset{(0.8)}{\cancel{\sigma}}\, g$$

or

$$\rho = (0.8)\left[\left(\frac{3}{2}\right)^3 - (1)^3\right]$$

solving,

$$\rho = 1.9$$

Q30. (B) Buoyant force acting on stone by water

$$B = V\rho_w g \cancel{= \frac{0.5}{50}}$$

V = volume of stone $= \frac{\text{mass}}{\text{density}} = \frac{0.5}{5000}\ m^3$

$\therefore$

$$B = \frac{0.5}{500} \times 10^3 \times g = 1g. \text{ (vertically upward)}$$

~~The~~ ~~same~~ From Newton's third law stone will apply same amount of force on water but vertically downward.

$\therefore$ Reading of balance = wt. of water + Force acting on it

$= 1.5\ \text{kg} + 1\ \text{kg}$

$= 2.5\ \text{kg}.$

Q31 (B)

Initially, weight of ice + metal were (1) balanced by buoyant force.

$$\therefore \quad (m_i + m_m) g = V_1 \rho_w g \qquad \text{--- (1)}$$

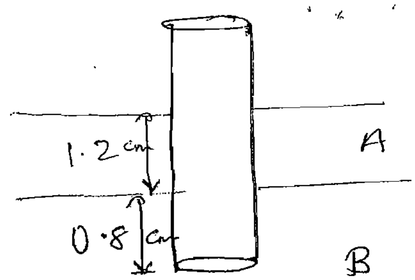
We know that when ice melts it forms volume of water equal to that displaced by it. so, due to melting of ice there is no change in volume ~~of~~ and hence level of liquid.

But ice cube was displacing more volum of water than it ~~was displacing~~ forms, to support the weight of metal cube.

Q32 (B) If H is total height of cylinder

(18)

In equilibrium of cylinder



$$B = mg$$

$$\pi R^2 (1.2)(0.7) g + \pi R^2 (0.8)(1.2) g$$

$$= \pi R^2 H (0.8) g.$$

$$\therefore \quad H = \frac{9}{4} = 2.25 \text{ cm}.$$

$\therefore$ length outside liquids = ~~2.25~~

$$= 2.25 - 1.2 - 0.8 = 0.25 \text{ cm}.$$

Q33. (B)

In equilibrium of block

$$mg = B + Kx.$$

H = Height of block

ρ = density of block.

$$AH\rho g = AH\frac{\rho}{3} g + K\frac{H}{3}$$

$$\therefore \quad \frac{2}{3} AH\rho g = K\frac{H}{3}$$

$$K = 2A\rho g$$

Q34 (A) velocity of efflux $= \sqrt{2gh}$

(19) Time taken (t) to hit ground:

$$S = ut + \frac{1}{2}at^2$$

$$H-D = 0 \times t + \frac{1}{2}gt^2$$

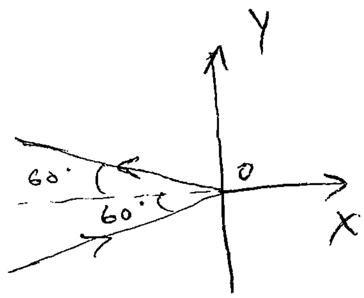
$$t = \sqrt{\frac{2(H-D)}{g}}.$$

$$\therefore \text{Range} = x = \sqrt{2gD} \times t = \sqrt{2gD}\sqrt{\frac{2(H-D)}{g}}$$

$$x = 2\sqrt{D(H-D)}$$

Q35 (B)

$$\because F = \frac{\Delta P}{\Delta t}$$



if $\Delta t = 1$ sec.

$$F = \Delta P$$

$\therefore$ Force acting on wall = Momentum transferred to wall per second

$F = \Delta P$ = change in momentum ~~in~~ ~~1 sec~~ of water in one ~~sec~~ second.

Since collision is elastic, so change in

... is only along x-axis.

20 . $\Delta V = V_f - V_i = -V\cos 60^\circ - V\cos 60^\circ$

$= -2V\cos 60^\circ = -V$

$\therefore \Delta P = m(-V) = -mV$

m = mass of water striking wall in one second.

~~V = velocity of~~

ΔP = change in momentum in one second.

$\therefore$ ~~A~~ $m = \rho(AV) = \rho AV.$

$\therefore \Delta P = -\rho AV^2 = F$ = Force acting on ~~wall~~ water

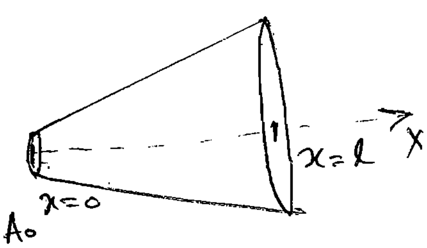
$\therefore$ Force acting on wall $= \rho AV^2$ (along X-axis)

$= 10^3 \times 6 \times 10^{-4} \times (12)^2$

$= 86.4$ N

Q36. (A)

$A - A_0 = m(x - 0)$



$A = A_0 + mx.$ where m = constant

From continuity, $A_0 V_0 = AV.$

$$V = \frac{A_0 V_0}{A_0 + mx}$$

~~Herd~~ Hence, P

From Bernoulli's equation,

(21)

$$P_0 + \frac{1}{2}\rho V_0^2 = P + \frac{1}{2}\rho V^2$$

$$P = P_0 + \frac{1}{2}\rho(V_0^2 - V^2)$$

$$P = P_0 + \frac{\rho}{2}\left[V_0^2 - \left(\frac{A_0 V_0}{A_0 + mx}\right)^2\right]$$

$$P = P_0 + \frac{\rho V_0^2}{2}\left[1 - \frac{A_0^2}{(A_0+mx)^2}\right]$$

$$P = P_0 + \frac{\rho V_0^2}{2}\left[\frac{(mx)^2 + 2A_0 mx}{(A_0+mx)^2}\right]$$

which is equation of curve.

Also,

$$\frac{dP}{dx} = \frac{\rho V_0^2}{2}\left[\frac{(A_0+mx)^2(2m^2x + 2A_0 m) - 2(A_0+mx)m \times ((mx)^2 + 2A_0 mx)}{(A_0+mx)^4}\right]$$

at $x = 0$

$$\frac{dP}{dx} = \frac{\rho V_0^2}{2}\left[\frac{A_0^2(2A_0 m)}{A_0^4}\right] > 0.$$

$\therefore$ slope at $x = 0$ is positive

Hence, option (A).

37 (c) In equilibrium, ~~water~~

Rate of water flowing in = Rate of water flowing out.

(22) ∴ $1\times10^{-4} \times 2\ m^3/s = 0.5\times10^{-4}\times\sqrt{2gh}$

∴ $4 = \sqrt{2\times10\times h}$ ⇒ $\cancel{h = 0.8\ m}$. $h_{eq} = 0.8\ m$.

From equation of continuity,

$$\cancel{A_1V_1 + A_2 \frac{dh}{dt}}$$

$$A_1V_1 - A_2V_2 = A_{tank}\frac{dh}{dt}$$

$$2\times10^{-4}\ m^3/s - 0.5\times10^{-4}\sqrt{2gh} = 4000\times10^{-4}\ \frac{dh}{dt}$$

$$2 - \frac{\sqrt{2gh}}{2} = 4000\ \frac{dh}{dt}$$

$$\cancel{\frac{dh}{4-\sqrt{2gh}} = \frac{1}{8000}\,dt}$$

$$\cancel{\frac{dh}{2-\sqrt{5h}} = \frac{1}{24000}\,dt}$$

$$\frac{dh}{dt} = \frac{2-\sqrt{2gh}/2}{400} \neq \text{constant}$$

So, h-t graph is non linear.

∴ (D)

$V_0 = \sqrt{2gh}$.

$V' = \sqrt{2gh'}$ $\cancel{h' = \frac{h}{\sqrt{2}}\ h\ \ldots}$ $h' = h\cos 45°$

$h' = h$

Q39 (B) From continuity

$$A_1 V_1 = A_2 V_2$$

$$0.02 \times 2 = 0.01 \times V_2$$

$$V_2 = 4 \text{ m/s}.$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$4 \times 10^4 + \frac{1}{2}\rho(V_1^2 - V_2^2) = P_2$$

$$P_2 = 4 \times 10^4 + \frac{1}{2} \times 10^3 \times (2^2 - 4^2)$$

$$P_2 = 4 \times 10^4 + \frac{1}{2} \times 10^3 \times (-12)$$

$$P_2 = 3.4 \times 10^4 \text{ N/m}^2$$

Q40 (C) time taken to reduce the level of water from H to H'.

$$t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right) = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$$

$$t_1 = t_2$$

$$\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right] = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - 0\right].$$

Solving, we get $\sqrt{H} = 2\sqrt{\frac{H}{\eta}}$

$$\therefore \eta = 4$$

Q41 (B) (2y)

Bernoulli's theorem is based on Principle of conservation of Energy.

P = Pressure Energy per unit volume

$h\rho g$ = gravitational potential energy per unit volume.

$\frac{1}{2}\rho V^2$ = kinetic energy per unit volume.

~~Q42~~ From continuity equation

$$\pi\left(\frac{2\times 10^{-2}}{2}\right)^2 \times 3 = 100 \times \pi\left(\frac{0.05\times 10^{-2}}{2}\right)^2 \times V$$

$$V = \frac{3^2 \times 2/2}{100 \times (0.05)^2} = \frac{3^2 \times 2}{(1/2)^2}$$

$V =$

Q42 (D) From equation of continuity

$$A_1 V_1 = n A_2 V_2$$

$$\frac{\pi d_1^2}{4} \times V_1 = n \times \frac{\pi d_2^2}{4} \times V_2$$

$$\frac{\pi}{4}\left(2\times 10^{-2}\right)^2 \times 3 = 100 \times \frac{\pi}{4}\left(0.05\times 10^{-2}\right)^2 \times V_2$$

$$V_2 = \frac{3\times}{100}\left(\frac{2}{0.05}\right)^2$$

Q43 (A).

From continuity equation (25)

$$A_1V_1 = A_2V_2$$

velocity increases, if cross-section decreases.

From Bernoulli's equation

Pressure decreases as velocity increases.

Q44. (D)

time taken to empty the vessel is

$$t_0 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right] = \frac{A}{a}\sqrt{\frac{2H_0}{g}}$$

$H' = 0$.

~~$t' = \frac{A}{a}\sqrt{\frac{2H'}{g}}$~~

$$t' = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{4H} - \sqrt{0}\right]$$

$$t' = \frac{A}{a}\sqrt{\frac{2}{g}}\left(2\sqrt{H}\right) = 2\frac{A}{a}\sqrt{\frac{2H}{g}}$$

$$t' = 2t_0$$

~~Q45~~
Q45 (D) In steady state

water flowing in = water flowing out

$$100\ cm^3/s = \sqrt{2gh} \times A_2 = \sqrt{2gh} \times 1\ cm^2$$

Q46 (D) change in momentum of water

(2E) per second $(\Delta P) = mv_2 - mv_1$

m = mass flowing per second

$\overrightarrow{\Delta P} = m(\Delta \vec{V})$

$m = \rho L$ $\quad \vec{V_2} = v\hat{i} \quad \vec{V_1} = v\hat{j}$

$\Delta \vec{V} = \vec{V_2} - \vec{V_1} = v\hat{i} - v\hat{j}$

$|\Delta \vec{V}| = v\sqrt{2}$

$\therefore |\overrightarrow{\Delta P}| = \rho L V\sqrt{2}$ = change in momentum per second.

$\sqrt{2}\rho L V$ = Force acting on water

Q47. (C)

Force acting on tank $= -V_r \dfrac{dm}{dt}$

V_r = relative velocity of fluid w.r.t. tank.

$V_r = \sqrt{2gh.} = V$

$\therefore$ ~~$F = \frac{dm}{dt} = \rho(NA)V$~~ .

$$\frac{dm}{dt} = \rho(A)V$$

$$\therefore\ F = -\rho A V^2 = -\rho A(2gh)$$

$h = \frac{H}{2}$. $\therefore$ ~~$F = -\rho A(2gh)$~~

$$F = -\rho A\left(2g\frac{H}{2}\right) = Ma$$

M = Total mass of fluid in tank

$$= \rho(NAH)\,a = -\rho A\left(2g\frac{H}{2}\right).$$

$$\therefore\ a = \frac{g}{N}.$$

Q48 (A).

Volume flow rate in P = Volume flow rate in Q.

$$\therefore\ A_P V_P = A_Q V_Q$$

$$\pi\left(\frac{2\times10^{-2}}{2}\right)^2 V_P = \pi\left(\frac{4\times10^{-2}}{2}\right)^2 \times V_Q$$

$$V_P = 4V_Q$$

Q49. (B) In steady state

Volume flowing = volume flowing out.

(28) $10^{-4}\, m^3/s = 10^{-4}\, m^2 \times V$

$V = \sqrt{2gh}$ for small hole.

$\therefore \quad V = \sqrt{2gh} = 1\, m/s.$

$2 \times 9.8 \times h = 1 \Rightarrow h = 0.051\, m$

Q50 (C)

~~Time velocity of efflux = $\sqrt{2gx}$~~

Velocity of efflux = $\sqrt{2g(3H-x)}$

time taken by fluid to reach ground

$$\cancel{= \sqrt{\frac{2(3H-x)}{g}}} = \sqrt{\frac{2x}{g}}$$

$\therefore$ ~~Range = $\sqrt{2gx} \times \sqrt{\frac{2(3H-x)}{g}}$~~

$$R. = 2\sqrt{x(3H-x)} = 2\sqrt{3Hx - x^2}$$

$$\frac{dR}{dx} = 2 \cdot \frac{1}{2\sqrt{3Hx - x^2}} \cdot (3H - 2x) = 0.$$

$$\therefore \quad x = \frac{3}{2}H = 1.5H.$$

1. (D)

(29)

$$R_1 = R_2$$

$$\cancel{\sqrt{2g\,h_1\;2h_2}}$$

$$\sqrt{2g(H-h_1)}\;\sqrt{\frac{2h_1}{g}} = \sqrt{2g(H-h_2)}\;\sqrt{\frac{2h_2}{g}}$$

$$\therefore\quad (H-h_1)(h_1) = (H-h_2)\,h_2$$

$$Hh_1 - Hh_2 = h_1^2 - h_2^2$$

$$H(h_1-h_2) = h_1^2 - h_2^2$$

$$H = h_1 + h_2$$

For maximum range, $h = \frac{H}{2} = \frac{h_1+h_2}{2}$

Q52. (D)

$$V = \sqrt{2gh} = \sqrt{2\times10\times10} = 10\sqrt{2}\ m/s.$$

For Range to be twice, the velocity of efflux must be twice.

$$\therefore\quad V' = 2V = 20\sqrt{2}\ m/s$$

From Bernoulli's equation,

$$P + h\rho g = \frac{1}{2}\rho V'^2$$

$$P + 10\times10^3\times10 = \frac{1}{2}\times10^3\times400\times2$$

$5 \ldots /m^2 = 3$ atm.

Q53. (B)
(30) Let d be depth of water in barrel

so, velocity of efflux. is given by

$$V = \sqrt{2gd}$$

time taken to reach ground $= \sqrt{\frac{2h}{g}}$

$$\therefore \; R = V \times t = \sqrt{2gd} \times \sqrt{\frac{2h}{g}}$$

$$d = \frac{R^2}{4h}$$

Q54. (B). From Bernoulli's theorem

$$P_0 + \frac{mg}{A} + h\rho g = P_0 + \frac{1}{2}\rho V^2$$

$$\frac{10 \times 9.8}{10^{-1} m^2} + \frac{1}{2} \times 10^3 \times 9.8 = \frac{1}{2} \times 10^3 \times V^2$$

$$V^2 = 11.76 \quad \text{or} \quad V = 3.4 \text{ m/s}.$$

Q55. From continuity equation,
(B)

$$A_1 V_1 = A_2 V_2$$

$$1 \times V_1 = \frac{1}{2} \times V_2 \Rightarrow 2V_1 = V_2$$

From 3rd equation of motion

$$v^2 = u^2 + 2as$$

$$\therefore \quad (2V_1)^2 = V_1^2 + 2gh$$

$$3V_1^2 = 2 \times 980 \times 10$$

$$V_1^2 = \frac{19600}{3} \Rightarrow V_1 = \sqrt{\frac{19600}{3}} \text{ cm/s.}$$

$$V_1 = \sqrt{\frac{196}{3}} \times 10 \text{ cm/s.} = \frac{140}{\sqrt{3}} \text{ cm/s.}$$

volume rate of flow $= A_1 V_1$

$$= 1 \text{ cm}^2 \times \frac{140}{\sqrt{3}} \text{ cm/s} = \frac{140}{\sqrt{3}} \text{ cm}^3/\text{s.}$$

$$= \frac{140}{\sqrt{3} \times 1000} \times 60 \ \frac{\text{lit}}{\text{min}} = 4.8496 \ \frac{\text{lit}}{\text{min}}$$

356. (A)
Force acting on pipe = momentum transfered by water to pipe per second.

$$F = \frac{\Delta P}{\Delta t}, \quad \Delta t = 1 \text{ sec.}$$

$$\therefore \quad F = \frac{\Delta P}{\Delta t} = \frac{m}{\Delta t}(\Delta V)$$

ΔV = change in velocity of water

$\frac{m}{\Delta t}$ = mass of water flowing per second.

$$\frac{m}{\Delta t} = (AV)\rho \qquad \Delta V = V\sqrt{2}$$

(37) $\therefore$ $F = (\rho A V)\, V\sqrt{2} = \sqrt{2}\, \rho A V^2$

$F = \sqrt{2} \times 10^3 \times 10 \times 10^{-4} \times (20)^2$

$F = 4\sqrt{2} \times 100 = 565.6$ N.

Q57 (B)

Force acting on plate = Momentum transfered to plate per second

= − change in momentum of ~~plate~~ water per second.

$= -\dfrac{\Delta P}{\Delta t}$.

Initial velocity of water (u) = 10 m/s.

Final velocity of water (v) = 0

$\therefore$ Momentum transfered $= \dfrac{m}{\Delta t} \cdot (\Delta v)$

Δv = change in velocity $= v - u = 0 - 10$

$= -10$ m/s.

$\dfrac{m}{\Delta t}$ = mass of water striking per second.

$\therefore$ ~~Force~~ $= \rho A V$.

Force acting on plate $= (\rho A V)\, V = \rho A V^2$

$= 10^3 \times 2 \times 10^{-4} \times (10)^2 = 20$ N

Q58. (D) From Bernoulli's theorem

$$P_0 + \frac{h}{2}\rho g = P_0 + \frac{1}{2}\rho V_1^2$$

$$\therefore \quad V_1 = \sqrt{gh}.$$

$$P_0 + h\rho g + \frac{h}{2}(2\rho)g = P_0 + \frac{1}{2}(2\rho)V_2^2$$

$$V_2 = \sqrt{2gh}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{1}{\sqrt{2}}$$

Q59 (C) $A_1V_1 = A_2V_2$

$$\therefore \quad 10^{-2} \times 2 = 0.5 \times 10^{-2} \times V_2$$

$$V_2 = 4 \text{ m/s}.$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2.$$

$$8000 + \frac{1}{2} \times \rho(V_1^2 - V_2^2) = P_2$$

$$P_2 = 8000 + \frac{1}{2} \times 10^3 (2^2 - 4^2).$$

$$= 8000 + \frac{1}{2} \times 10^3 (-12)$$

$$P_2 = 2000 \text{ Pa}$$

$\frac{Q60}{34}$. (D)

same as Q 45.

Q61. (C)

V = volume rate of flow = $A_1 V_1$

$10^{-1}\ m^3/s = (10^{-2}\ m^2)\ V_1$

$V_1 = 10\ m/s.$

mass of water flowing per second = ρV

$(m) = \rho A_1 V_1 = 10^3 \times 10^{-1} = 100\ kg/sec.$

~~Kinetic~~ ~~ex~~ From Work - Energy theorem

$W_g + W_{ext} = \Delta K.$

$(-mgh) + W_{ext} = \frac{1}{2} \times m \times V_1^2$

$W_{ext} = mgh + \frac{1}{2} m V_1^2$

$= 100 \frac{kg}{sec} \times \cancel{9.8}\ 10 \frac{m}{s^2} \times 10\ m +$

$\frac{1}{2} \times 100 \frac{kg}{sec} \times \left(10 \frac{m}{s}\right)^2$

$W_{ext} = \cancel{15}\ 15000\ \cancel{Watt}\ \frac{J}{sec}$

... agent

Q62. (A)

(5)

Divide the containers into two equal halves.

The first container will have greater than half of volume in upper half while the second one will have ~~least to~~ less than half of total volume of fluid.

For (i) more than half of volume will come out with greater speed as compared to the other two.

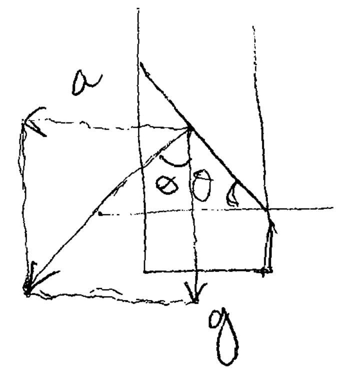
So, (i) will take the least time to empty.

36 ~~Q63~~ Q1. (A) (C)

Ex - II Multiple choice

$$\tan\theta = \frac{a}{g} = \frac{1}{\cot\theta}$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) = \cot^{-1}\left(\frac{g}{a}\right)$$



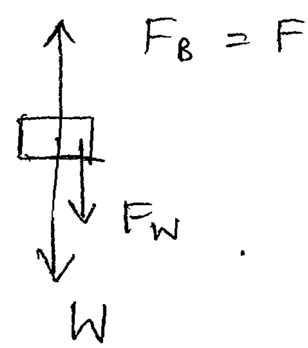
backwards

~~Q64~~ Q2. (D)

Pressure at bottom of vessel = $2h\rho g$

Force exerted by fluid at bottom = $(2h\rho g)A$

FBD fluid



F_B = Force exerted by bottom of vessel on fluid = F

F_W = Force exerted ~~by~~ by wall of vessel on fluid

= ~~F_W~~ =

Force exerted by fluid on wall is due to pressure of fluid acting normal

Since fluid is in equilibrium, so 37

$$F_B = F_W + W$$

or $$W = F_B - F_W$$

$$W < F_B. = F$$

$$F_W = F_B - W = F - W$$

Q65 (A) (C)

38

$W_1 = m_B g$ m_b = mass of ~~balloom~~ empty balloon

$W_2 = m_B g - B + m_a g$ B = Buoyant force on filled balloon

Now, B = weight of air displaced

wt. of Air displaced = wt. of ~~air~~ air filled in balloon

= W (given)

~~Q.3~~ (A) (C)

Q3 :

$W_1 = m_B g$ = wt. of empty balloon.

$W_2 = m_B g + m_a g - B$. = wt. of filled balloon

$W_2 = W_1 + W - B$.

m_a = mass of air inside balloon.

B = Buoyant Force = Weight of air displaced.

Air inside balloon is identical to outside air.

$\therefore \quad B = W$ = weight of ~~air~~ air inside balloon.

$\therefore \quad W_2 = W_1 + W - W$

$W_2 = W_1$ Hence (A)

$W_2 < W_1 + W$ (C)

~~Q.6~~ Q4.

(C) (D) If x cm of block is below surface common to oil and water. In equilibrium,

$ma - B$

(40) $0.92\,g = B_o + B_w$

B_o = Buoyant force due to oil

B_w = Buoyant force due to water

$$B_o = (0.1-x)\rho_o g = (0.1-x) \times 0.6 \times 10^3 g$$

$$B_w = x \rho_w g = x \times 1 \times 10^3 g.$$

$$\therefore \quad 0.92 g = (0.1-x) \times 0.6 \times 10^3 g + x \times 1 \times 10^3 g$$

Solving,

$$x = 0.08 \text{ m} = 8 \text{ cm}.$$

$\therefore$ 8 cm below oil-water interface.

Q5.
(B) (C)

Due to ~~bouy~~ buoyant force acting on mass m, its effective ~~wef~~ weight decreases.

$$W = mg - B.$$

Hence, A will read less than 2kg.

This buoyant force (B) was exerted by liquid

block

buoyant force is applied by ~~block~~ mass (m) ~~of~~ liquid.

Hence, net force on liquid acting downwar increases. so, reading of spring balance B increas

~~Q6~~ Q6. (A)(C)

Velocity of efflux $= V = \cancel{\sqrt{2gh}}\ \sqrt{2gy}$

~~For~~ If t is the time taken by liquid to reach the ~~bottom of the~~ ground, then,

$$\cancel{2h = 0\times t}$$

$$2h - y = 0\times t + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(2h-y)}{g}}$$

$\therefore$ Range $= x = V\times t = \sqrt{2gy}\sqrt{\frac{2(2h-y)}{g}}$

$$= 2\sqrt{y(2h-y)} = 2\sqrt{2hy - y^2}$$

$$x = \cancel{4}\ 2\sqrt{h^2-(h-y)^2}$$

x will be maximum when

(112) $\therefore$ Maximum Range $(x_m) = 2\sqrt{h(h)} = 2\sqrt{h^2}$

$= 2h$

Q69 (B)(C)

Initial velocity of water $= V$

Final velocity of water $= 0$

Change in velocity $(\Delta V) = 0 - V = -V$

change in momentum $=$ mass $\times (\Delta V) = -mV$.

~~change in momentum (per second)~~

Force acting on water $= \dfrac{dP}{dt}$

$= \dfrac{d(mV)}{dt} = V\dfrac{dm}{dt}$

$\dfrac{dm}{dt} =$ mass of water striking per second.

44 $\therefore$ Maximum Range $(x_m) = 2\sqrt{h(h)} = 2h$

Q7

(B) (~~c~~)(d)

Initial velocity of water = V

Final velocity of water = 0

ΔV = change in velocity = $0 - V = -V$

ΔP = change in momentum = mass $\times \Delta V$

change in momentum (per second)

= mass of striking per second $\times$ change in velocity

mass of water striking per second

$= \rho A V$ = mass of water in element having length = V and area of cross-section ~~of~~ of tube (say A)

$\therefore \Delta P$ (per second) $= \cancel{\rho A}(\rho A V)(-V)$

$= -\rho A V^2$

... water = F = dP

If $dt = 1$ second

(48)

$$F = \frac{\text{change in momentum}}{1 \text{ sec}}$$

F = change in momentum per second

$\therefore$ $F = -\rho A V^2$

$\therefore$ From Newton's ~~that~~ third law

$$F_{wall} = -F = \rho A V^2$$

$\therefore$ If velocity ~~becoming~~ becomes ~~four~~ two times, force acting on ~~to~~ wall becomes four times.

Energy lost per second = ~~KE~~ kinetic energy lost by water in 1 second.

$$= \frac{1}{2} \times (\rho A V) \times V^2 = \frac{1}{2} \rho A V^3$$

$\therefore$ Energy lost becomes eight times.

~~370~~ (B)

Q8. Velocity of efflux $= V = \sqrt{2gy}$

(46)

$$= \sqrt{\frac{2(H-y)}{g}}$$

$\therefore$ Range $(x) = \sqrt{2gy} \times \sqrt{\frac{2(H-y)}{g}}$

$$= 2\sqrt{y(H-y)} = 2\sqrt{\left(\frac{H}{2}\right)^2 - \left(y-\frac{H}{2}\right)^2}$$

$\therefore$ ~~For~~ $x_{max} = H$ at $y = \frac{H}{2}$

If y is increased from 0 to H, then, x first increases and becomes maximum at $y = \frac{H}{2}$ and then decreases.

& x ~~depends~~ is independent of density of the liquid.

Q9. (D)

$A_P V_P = A_Q V_Q$ (Equation of continuity)

$A_P > A_Q$

$\therefore V_P < V_Q$

From Bernoulli's theorem, greater velocity implies less pressure.

~~$P_p + \frac{1}{2}\rho V_p^2 = P_y + \frac{1}{2}$~~

$$P_P + \frac{1}{2}\rho V_P^2 = P_Q + \frac{1}{2}\rho V_Q^2$$ 47

$$V_P < V_Q$$

$$\therefore \quad P_P > P_Q$$

$$\text{K.E at } X = \frac{1}{2}\rho V_P^2$$

$$\text{K.E at } Y = \frac{1}{2}\rho V_Q^2$$

$$(\text{K.E.})_X < (\text{K.E.})_Y$$

Ex-3 - ~~comprehension~~

Passage

Fluid Mechanics

Q1. (C) Pressure at P is less than atmos-pheric pressure. So, mercury will not ~~flow~~ come out from hole.

Q2 (B) Pressure at P is atmospheric pressure. Pressure at P' is less than atmospheric pressure. ~~due~~ So, mercury may come out after some time.

Q3 (C) $P_0 = h\rho g$

$\rho_m > \rho_w$ so, $h_m < h_w$

Q4 (C) ~~B~~

If all of air is pulled out.

$P_{inside} = 0 = h\rho_m g \Rightarrow h = 0$

Q5 (B)

Total length = 1 m

Free length = 100 − 76 = 24 cm.

Q6 (A)

Buoyant force arises due to the pressure difference.

Q7 (B)

Mass flowing per second at every cross-section ~~is~~ constant, is basis for equation of continuity.

Q8 (A)

Buoyant force arises due to pressure difference.

When container is accelerated horizontally Pressure ~~varies~~ ~~as~~ varies horizontally.

$$\therefore \quad \text{Buoyant force} = V\rho g_{eff.}$$

$$\text{where, } g_{eff.} = \sqrt{g^2 + a_H^2}$$

a_H = horizontal acceleration.

~~Q10~~

Q9. 2

If y be height of water at any time above orifice. then,

velocity of efflux $= V = \sqrt{2gy}$

$R = V \times t = \sqrt{2gy} \times 2$

Speed of block $= -\frac{dR}{dt} = -\frac{d}{dt}\left(2\sqrt{2gy}\right)$

$= -2\sqrt{2g}\ \frac{1}{2\sqrt{y}}\ \frac{dy}{dt} = -\sqrt{\frac{2g}{y}}\ \frac{dy}{dt}$

From continuity eqn:

$\cancel{A\,dx} \quad -A\frac{dy}{dt} = a\sqrt{2gy}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{a}{A}\sqrt{2gy}$

So, speed of block

$= -\sqrt{\frac{2g}{y}} \times \frac{a}{A}\sqrt{2gy} = -\frac{a}{A}(2g)$

$= -20\ \frac{a}{A}\ m/s = -0.02\ m/s = -2\ cm/s.$

Q10 $\quad K = 9$

Net Force acting on ball at any instant can be given as

$F = 6\pi\eta R V_t - 6\pi\eta R V$

or $$a = \frac{6\pi\eta R(V_t - V)}{\cancel{m\rho/2}\ V(\rho/2)}$$

$$a = V\frac{dV}{ds} = \frac{9\eta}{R^2\rho}(V_t - V)$$

$$\frac{V\,dV}{V_t - V} = \frac{9\eta}{R^2\rho}\,ds.$$

$$\int_{V=0}^{V=V_t/2} \frac{V}{V_t - V}\,dV = \frac{9\eta}{R^2\rho}\int_0^s ds.$$

Solving,

$$\cancel{S = \rho R^2}$$

$$\boxed{S = \frac{\rho R^2 V_t}{9\eta}\left[\ln 2 - \frac{1}{2}\right]}$$

$\therefore \quad k = 9$

Q1 Let m_1, m_2 be masses of liquids

$\therefore$ Volume of liquids V_1, V_2 is given by

$$V_1 = \frac{m_1}{\rho_1}, \quad V_2 = \frac{m_2}{\rho_2}$$

Total volume of mixture $= V_1 + V_2 = V$ (given)

$$\therefore \quad \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} = V \quad \text{——— (1)}$$

Total mass of mixture $= m_1 + m_2$

$\therefore$ Density of mixture $= \dfrac{m_1 + m_2}{V} = \sigma$ (given)

$$\therefore \quad m_1 + m_2 = \sigma V \quad \text{——— (2)}$$

Solving equations (1) & (2)

$$m_1 = \frac{(\sigma - \rho_2) V}{(1 - \rho_2/\rho_1)}, \quad m_2 = \frac{(\sigma - \rho_1) V}{(1 - \rho_1/\rho_2)}$$

JP Page 1–15.

92. Let ρ_1, ρ_2 be specific gravity of two metals respectively. 2.

$\therefore$ When equal volumes (say V) of two metals are mixed together

Total volume of mixture $= V + V = 2V$

Total mass of mixture $= \rho_1 V + \rho_2 V = (\rho_1 + \rho_2) V$

$\therefore$ Density of mixture $= \frac{(\rho_1+\rho_2)V}{2V} = \frac{\rho_1+\rho_2}{2} =$

$\cancel{\frac{\rho_1+\rho_2}{2}}$ $= 4$ (given)

$\therefore \frac{\rho_1+\rho_2}{2} = 4 \Rightarrow \rho_1 + \rho_2 = 8$ ——— ①

When equal masses (say m) of two metals mixed together.

Total mass of mixture $= m + m = 2m$

Total volume of mixture $= \frac{m}{\rho_1} + \frac{m}{\rho_2}$

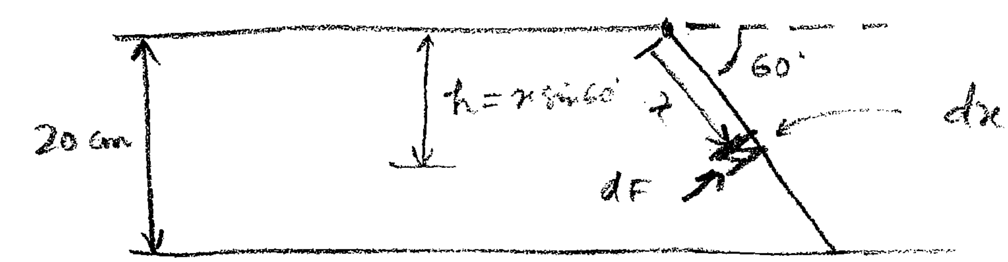
$\therefore$ Density of mixture $= \frac{2m}{\frac{m}{\rho} + \frac{m}{\rho}} = \frac{2}{\frac{1}{\rho_1} + \frac{1}{\rho_2}}$

$$\frac{2\rho_1\rho_2}{\rho_1+\rho_2} = 3 \quad \text{———} \quad (2)$$

solving (1) & (2).

$\rho_1 = 2$ or 6 $\qquad$ $\rho_2 = 6$ or 2

Q3.



Consider a small element of thickness dx at distance x measured along the wall from free surface. The pressure at this element is

$$P = h\rho g = (x \sin 60^\circ)\rho g = x\rho g \sin 60^\circ$$

Force on this element is given by

$$dF = P(dA) = (x\rho g \sin 60^\circ)(b\,dx)$$

where b = width of flap. $= 0.4$ m.

$\therefore$ Total force on flap $= \displaystyle\int_{x=0}^{x=H/\sin 60^\circ} dF$

$$= \int^{x=H/\sin 60^\circ} \rho g x \sin 60^\circ \; b\,dx$$

$$= \rho g b \sin 60^\circ \left[\frac{x^2}{2} \right]_0^{H/\sin 60^\circ} = \rho g b \sin 60^\circ \cdot \frac{H^2}{2 \sin^2 60^\circ}$$ 4.

$$= \rho g b \frac{H^2}{2 \sin 60^\circ} = \frac{10^3 \times 9.8 \times (0.4)(0.2)^2}{2(\sqrt{3}/2)}$$

$F = 90.5312$ N

24.

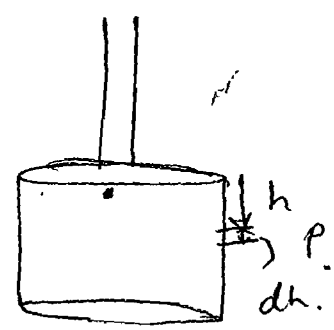
(a) Pressure at bottom = length of water column above it

$= h\rho g = 0.3 \times 10^3 \times 10 = 3000$ N/m²

thrust $= h\rho g A = 3000 \times \frac{22}{7} \times (0.1)^2$

$= 94.29$ N

(b) Pressure at point P



$= (0.2 + h)\rho_w g$

$= (0.2 + h) \times 10^3 \times 10$

$= (0.2 + h) \times 10^4$ N/m²

∴ Force ~~on~~ at Point P due to hydro-static pressure $= P \times dA$

Total force acting on vertical sides

$$= \int_0^{0.1} dF$$

$$= \int_0^{0.1} (0.2+h) \times 10^4 \times 2 \times \frac{22}{7} \times (0.1)\, dh$$

$$= \frac{44}{7} \times 10^3 \left[0.2h + \frac{h^2}{2} \right]_0^{0.1}$$

$$= \frac{44}{7} \times 10^3 \left[(0.2)(0.1) + \frac{(0.1)^2}{2} \right]$$

$$= \frac{1100}{7} \text{ N} = 157.14 \text{ N.}$$

(c). Pressure at top face

$$= h \rho_w g = 0.2 \times 10^3 \times 10 = 2000 \text{ N/m}^2$$

$\therefore$ Force at top face

$$= 2000 \times \frac{22}{7} \left((0.1)^2 - \right.$$

$$= \frac{44000}{7} \left((0.1)^2 - (0.02)^2 \right)$$

$$= 60.34 \text{ N.}$$

Solutions

Q1. Let m_1, m_2 be masses of two liquids

$\therefore$ $m_1 = \rho_1 V_1$ $m_2 = \rho_2 V_2$

Total mass $= m_1 + m_2 = (\rho_1 + \rho_2) V$

of mixture

35.

Pressure at Base due to water column $= h \rho_w g = 0.5 \times 10^3 \times 10$

$P_{BW} = 5 \times 10^3 \ N/m^2$

Force at Bottom $= P_{BW} \times$ Area of cross section $\left(\frac{\pi d^2}{4}\right)$

$= 5 \times 10^3 \times \frac{22}{7} \times \frac{(0.1)^2}{4}$

$= 39.286$ N. (downwards)

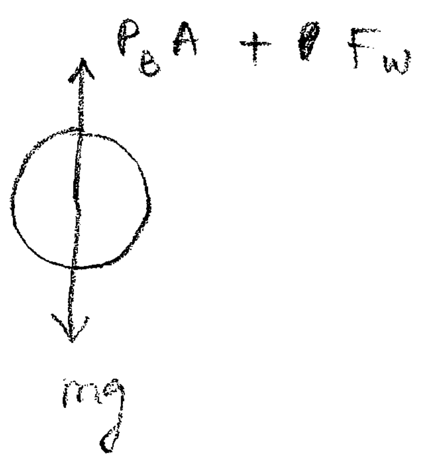
Weight of water $= V \rho g =$ ~~(10 lit) (1 kg/lit) (10 m~~

$=$ ~~(10 lit) (1 kg/lit) (g) =~~ 10

$= (10 \text{ lit}) (1 \text{ kg/lit}) (10 \text{ m/s}^2)$

$= 100$ N.

FBD water



mg = weight of water

$P_B A$ = Pressure force extented by bottom face on water

From Newton's third law

Force by water at bottom of pot

= Force by bottom of pot on water

$\therefore \quad P_B A = P_w A = 39.286$ N upwards

Since, water is in equilibrium. So net force on water must be zero.

This will be possible only when ~~resultant~~ net resultant force exerted by walls on water ~~ball~~ acts vertically upward.

$$\therefore \quad P_B A + F_w = mg$$

$$F_w = 100\text{ N} - 39.286\text{ N}$$

$$F_w = 60.714\text{ N} \quad (\text{vertically upwards}).$$

From Newton's third law

Force by walls on ~~walls~~ water

= Force by water on walls

26/ Let the ~~densities~~ densities of three liquids be $a+d$, a, $a-d$

~~This will be~~
In given situation equilibrium is possible only when liquid c is lightest liquid.

$$\therefore \quad \rho_c = a - d$$

$$\rho_b = a \qquad \rho_a = a + d.$$

Let x be fraction of liquid A in right ~~Arm~~ arm.

If l be length of each arm then, Pressure due to both arms at the bottom will be same.

$$1 \times a \times g = x(a-d)g + (1-x)(a+d)g$$

$$\cancel{\rho_c l g = \rho_a (xl) g + \rho_b (1-x) l g}$$

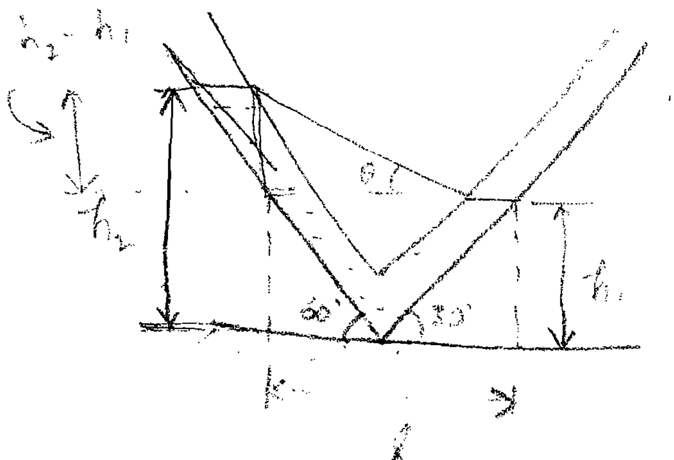
$$\cancel{(a-d) l g = (a+d) x l g + a(1-x) l g}$$

Solving, we get

$x = \frac{1}{2}$ or 0.5

Q7.

$$\tan\theta = \frac{h_2 - h_1}{l}$$ ✗



$$\tan 60° = \frac{h_2}{l_2}$$

$$l_2 = \frac{h_2}{\tan 60°} = \frac{h_2}{\sqrt{3}}$$

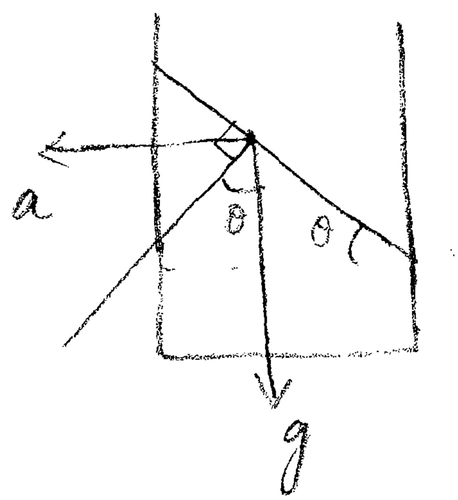
$$\tan 30° = \frac{h_1}{l_1} \Rightarrow l_1 = \frac{h_1}{\tan 30°} = \sqrt{3}\, h_1$$

$$l = l_1 + l_2 = \sqrt{3}\, h_1 + \frac{h_2}{\sqrt{3}}$$

$$\therefore \tan\theta = \frac{h_2 - h_1}{\sqrt{3}\, h_1 + h_2/\sqrt{3}} = \frac{\sqrt{3}\,(h_2 - h_1)}{(3h_1 + h_2)} \quad \text{——— (1)}$$

If tube were continuous container, then

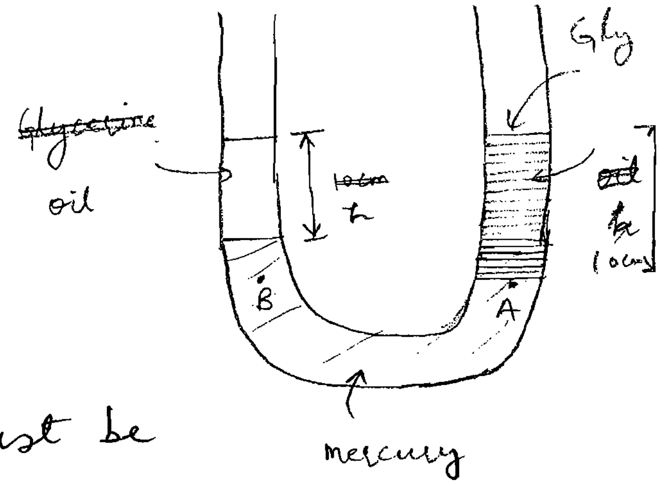
$$\tan\theta = \frac{a}{g} \quad \text{——— (2)}$$



from eqⁿ (1) & (2).

$$a = \frac{\sqrt{3}\,(h_2 - h_1)}{(3h_1 + h_2)}\, g$$

08. since oil has lower density than glycerine, so, in this case to balance pressure in same horizontal level height of oil column must be less than 10 cm.



Taking two points A & B at same horizontal level.

$$P_A = P_B$$

$$P_0 + \frac{h}{100}\rho_0 g + \frac{(10-h)}{100}\rho_m g = P_0 + \frac{10}{100}\rho_{gly} g$$

$$\therefore \quad h\rho_0 + (10-h)\rho_m = \frac{10}{\cancel{100}}\rho_{gly}$$

$$h(\overset{0.8}{\cancel{1.3}}) + (10-h)(13.6) = \frac{10}{\cancel{100}}(1.3)$$

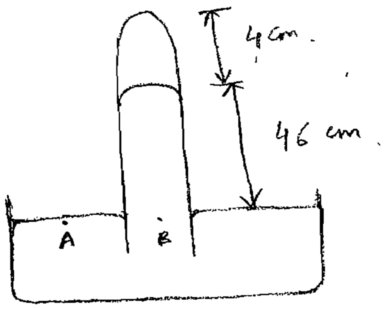
$$0.8\cancel{1.3}h + 136 - 13.6h = 13$$

$$123 = \cancel{12.3h}\ 12.8h$$

$$h = \frac{123}{12.8}\text{ cm} = 9.6094$$

$$h = 9.6094 \text{ cm}$$

Q9 Let P_1 be pressure of air trapped in tube (in cm Hg).



$$\therefore \quad P_{atm} = P_A = P_B = P_1 + 46$$

$$P_{atm} = 76 \text{ cm Hg.}$$

$$\therefore \quad 76 = P_1 + 46 \quad \text{or} \quad P_1 = 30 \text{ cm Hg}$$

Now, when 20 cm of more tube is released then, suppose height of mercury be h.

$\therefore$ Length of air column trapped

$$= 50 + 20 - h = 70 - h.$$

$$P_{atm} = P_2 + h \quad (2)$$

P_2 = pressure of air trapped

$$\text{or} \quad 76 = P_2 + h$$

$$P_2 = (76 - h) \text{ cm Hg.}$$

Let (A) be cross-section of tube then, if temperature of air is constant

$$P_i V_i = P_f V_f$$

$$30 \times (A \times 4) = (76 - h)\,(A \times (70 - h))$$

or

$$120 = (76 - h)(70 - h)$$

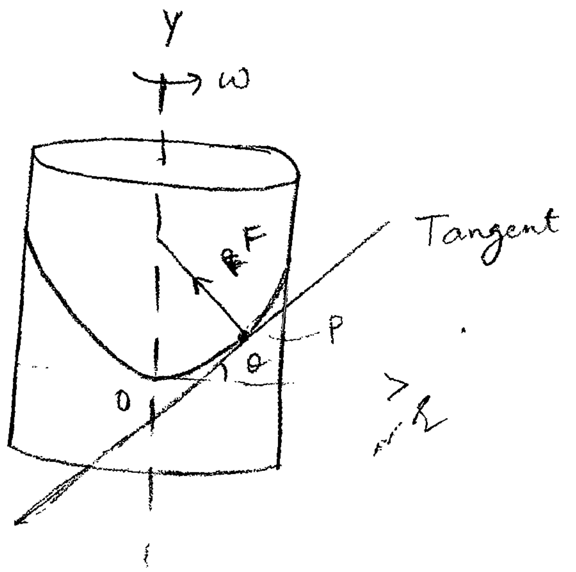
$$h^2 - 146h + 5200 = 0$$

$$h = 61.642 \text{ or } 84.36$$

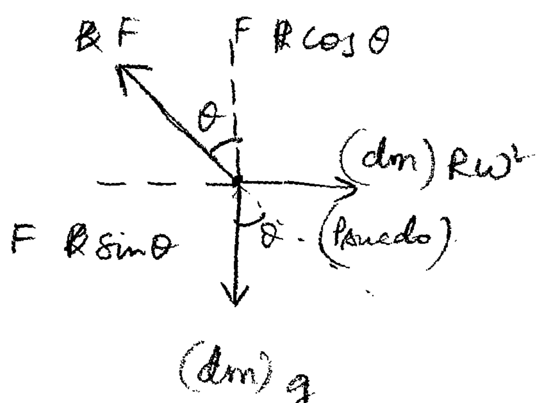
$\because$ $h < 70$ cm.

$\therefore$ $h = 61.642$ cm

Q10.



FBD particle P



Take point P on the surface of rotating liquid.

If F is a force exerted by neighbouring particles on P then,

$$F \sin\theta = (dm)\, \cancel{R\omega^2}\, r\omega^2$$

$$\tan\theta = \frac{r\omega^2}{g} \quad \text{———} \quad (1)$$

Also, $\tan\theta$ = slope of tangent at P

$$\tan\theta = \left(\frac{dy}{dr}\right)_{at\ P} \quad \text{———} \quad (2)$$

from (1) & (2).

$$\frac{dy}{dr} = \frac{r\omega^2}{g}$$

$$\int_{y_{min}}^{y} dy = \int_{r=0}^{r} \frac{r\omega^2}{g}\, dr$$

$$\left[y \right]_{y_{min}}^{y} = \frac{\omega^2}{g}\left[\frac{r^2}{2}\right]_0^r$$

$$y - y_{min} = \frac{\omega^2}{2g}\left[r^2 - 0^2\right]$$

$$y = y_{min} + \frac{\omega^2 r^2}{2g}$$

Given

$d = 10$ cm $= 2R$, $r = 5$ cm.

$y_{max} - y_{min} = ?$ $\quad \omega = 120$ rpm $= 120 \times \frac{2\pi}{60}$

$\omega = 4\pi$ rad/sec

At $r = R$ $\quad y = y_{max}$.

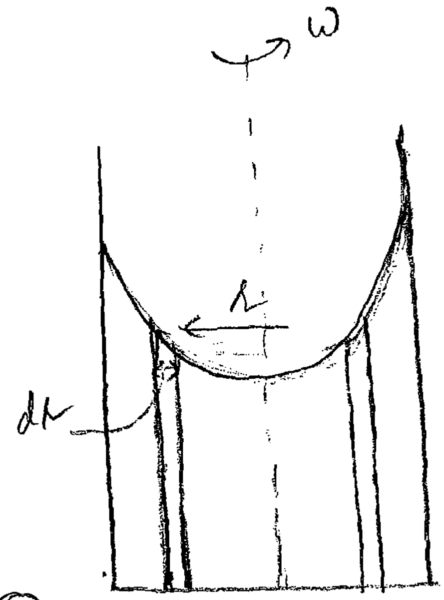
$$\therefore \quad y_{max} - y_{min} = \frac{(R\omega)^2}{2g}$$

$$= \frac{(5 \times 4\pi)^2}{2 \times 980} \text{ cm} = 2.012 \text{ cm}.$$

Q14. Given $\quad y_{min} = 25$ cm.

R ~~r~~ $= 10$ cm

Equation for surface of water



$$y = y_{min} + \frac{(r\omega)^2}{2g}$$

$$y = 25 + \frac{(r\omega)^2}{2g} \quad \text{———} (1)$$

Consider a hollow cylinderical water surface of radius (x) thickness (dx) and height (y).

Volume of water in surface is given by (dV)

$$dV = (2\pi x)(dx)(y)$$

$$V = \int_{r=0}^{R} 2\pi \cancel{x} \left(25 + \frac{(\cancel{x}\omega)^2}{2g}\right) dx$$

$$= 2\pi \left[\frac{25 r^2}{2} + \frac{\omega^2}{2g} \cdot \frac{x^{4}}{4} \right]_0^R$$

$$V = 2\pi \left[\frac{25R^2}{2} + \frac{\omega^2 R^3}{8g} \right]$$

volume of water initially present

$$= \pi R^2 h = \pi R^2 (50)$$

$$50\pi R^2 = \cancel{2\pi} \left[\frac{25}{2} R^2 + \frac{\omega^2 R^4}{8g} \right]$$

$$25 = \frac{25}{2} + \frac{\omega^2 R^2}{8g}$$

$$\omega^2 = \frac{25 \times 8g}{2R^2} = \frac{100 g}{R^2} = 980$$

$$\omega = 31.32 \text{ rad/sec}$$

substituting $\omega^2 = 980$ $R = 10$ cm. $g = 980$ cm/s^2

$$y = y_{max} = 75 \text{ cm}$$

Equation of water surface

$y = 25 + (0.5)\cancel{t}x^2$ in cgs units

$y = 0.25 + 50\cancel{t}x^2$ in S.I. units

Q12 (v) Volume of cube $= (10)^3 = 1000 \text{ cm}^3$

Let ~~(t)~~ v' be volume of mercury displaced by cube.

$\therefore$ In equilibrium

$$v' \rho_m g = v \rho_{iron} g$$

$$v' = \frac{\rho_{iron}}{\rho_m} \times V = \frac{7.8}{13.6} \times 1000$$

$v' = 573.529 \text{ cm}^3$ = volume of mercury displaced.

Q13. Since, block is in equilibrium. So, net force on block is zero.

OR Buoyant Force = Weight of block.

$$(10)^2 (2) \rho_w g + (10)^2 (8) \rho_o g = \cancel{(10)^2 (10)} \rho \; mg$$

$$\therefore \quad m = (10)^2 (2)\rho_w + (10)^2 (8)\rho_o$$

$$= (200) \times (0.8) + (800) \times (1)$$

$$m = 160 + 800 = 960 \text{ gm}$$

~~$M = 960$ gm or 0.96 kg~~

$$m = 960 \text{ gm} \quad \text{or} \quad 0.96 \text{ kg}.$$

(b) Gauge pressure = Pressure excess to atmospheric pressure

or Excess pressure relative to atmospheric pressure

$$\cancel{P_{gauge} = P + P_{atmosp}}$$

$$P_{gauge} = P_{absolute} - P_{atm}$$

$$P_{gauge} = (10)\rho_o g + (2)\rho_w g$$

$$= (10)(0.8)(980) + (2)(1)(980)$$

$$= 9800 \text{ dyne/cm}^2$$

214 (a) ~~P~~ Let ρ_s = density of solid

ρ_o = density of oil

$$\therefore \quad mg = V\rho_s g = 60 \quad \text{———} \quad (1)$$

where, ~~where~~ V = volume of solid

$$mg - B_w = 40$$

$$V\rho_s g - V\rho_w g = 40 \quad \text{———} \quad (2)$$

from (1) & (2)

$$V\rho_w g = 20$$

$$\therefore \quad \frac{V\rho_s g}{V\rho_w g} = \cancel{\frac{2060}{60} = \frac{1}{3}} \quad \frac{60}{20} = 3$$

$$\cancel{\rho_s = \frac{1}{3}\rho_w}$$

$$\rho_s = 3\rho_w = 3 \times 10^3 \ kg/m^3$$

when immersed in oil

$$mg - B_{oil} = 45$$

$$V\rho_s g - V\rho_o g = 45 \quad \text{———} \quad (3)$$

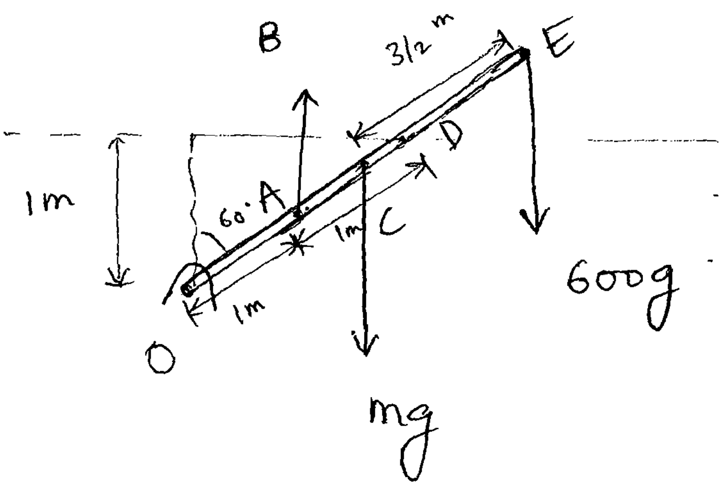
from (1) & (3)

$$\frac{V\rho_o g}{V\rho_w g} = \frac{15}{20} \quad \text{or} \quad \rho_o = \frac{3}{4}\rho_w.$$

$$\rho_o = 0.75 \times 10^3 \text{ kg/m}^3 = 750 \text{ kg/m}^3$$

Q15.

For Rotational equilibrium of Rod.



$$mg\,\frac{300}{2}\sin 60^\circ + 600g\,(300)\sin 60^\circ - B\,(100)\sin 60^\circ = 0$$

$$m = (300\,A)(0.4)\text{ gm.}$$

or. $(300\,A)(0.4)\,g\left(\frac{300}{2}\right) + (600g)(300)$

$$= (200\,A)(1)\,g\,(100)$$

$$(3)(0.2)(A)(3) + 18 = 2A.$$

$$18 = 2A - 1.8A = 0.2A$$

$$A = \frac{18}{0.2} = 90 \text{ cm}^2$$

Q16.

Upthrust B acting on stick

$= V \rho_\omega g$

$= A (200)(1)\ 980$

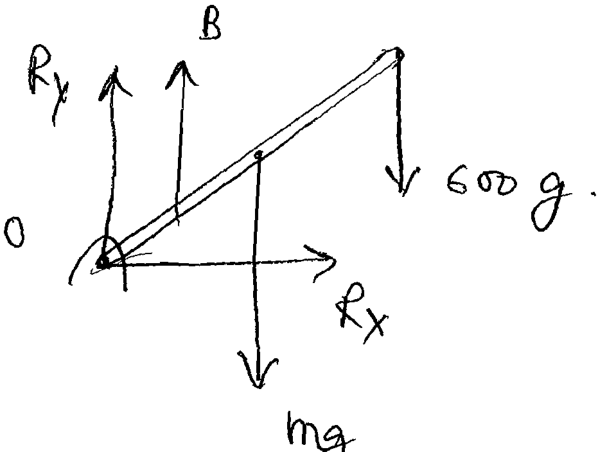
$= 90 \times 200 \times 1 \times 980$ dynes

$= 1.764 \times 10^7$ dynes.

$= 176.4$ N

Reaction at hinge

In vertical direction



$R_y + B - mg - 600g = 0$

$R_y = 600g + mg - B$

$= 600g + 300 \times 0.4\, g A - 1.764 \times 10^7$

$= 5.88 \times 10^5 + 1.0584 \times 10^7 - 1.764 \times 10^7$

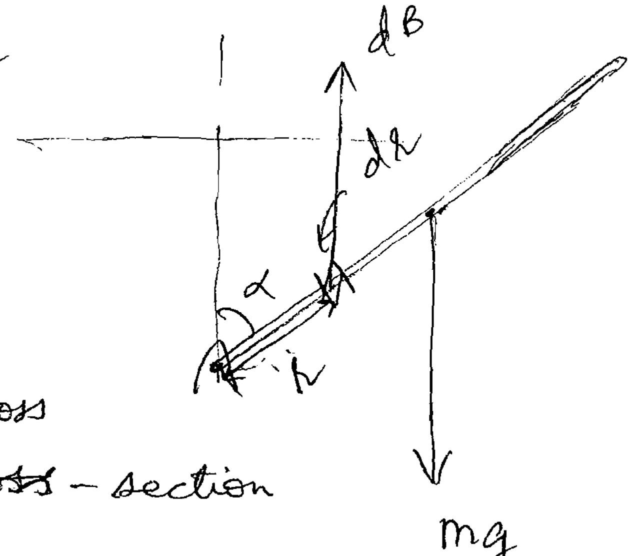
$= -6.468 \times 10^6$ dynes.

$= -64.68$ N

$R_x = 0$

Q17.

Let (σ) be density of material of rod.



$\therefore$ mass of rod

$= \sigma(A l)$

where A = Area of ~~cross~~ cross-section

$\therefore$ Torque due to weight $= mg \frac{l}{2} \cos\alpha$.

$= (\sigma A g l^2/2) \cos\alpha$.

~~Torq~~ Now, ~~the~~ buoyant force will be variable. and hence the torque

Consider a small element ~~at~~ ~~d~~ of length (dr) at distance (r) from hinge along the length of rod.

Small torque on this element due to buoyant force about point O ~~will~~ is given by

$$d\tau = (dB)\, r\cos\alpha.$$

$$dB = \rho_0 (k - r\cos\alpha)\, A\, g\, dr$$

$\therefore$ Net torque about point O is given by

$$= \int d\tau = \int_{r=0}^{r=H/\cos\alpha} \rho_0 A \, (K - r\cos\alpha) \, r\cos\alpha \; g \, dr$$

For rotational equilibrium of rod.

$$\sigma A g \frac{l^2}{2} \cos\alpha = \rho_0 A g \cos\alpha \int_0^{H/\cos\alpha} (K - r\cos\alpha) \, r dr$$

$$\sigma = \frac{2\rho_0}{l^2} \left[\frac{K r^2}{2} - \frac{r^3}{3} \cos\alpha \right]_0^{H/\cos\alpha}$$

$$= \frac{2\rho_0}{l^2} \left[\frac{K H^2}{2\cos^2\alpha} - \frac{H^3}{3\cos^2\alpha} \right]$$

$$\sigma = \frac{2\rho_0 H^2}{l^2 \cos^2\alpha} \left[\frac{K}{2} - \frac{H}{3} \right]$$

$$\sigma = \frac{\rho_0 H^2 (3K - 2H)}{3 l^2 \cos^2\alpha}$$

Q18. Rate of flow of liquid $= 7200 \frac{lit}{min}$

$$V = 7200 \times \frac{10^{-3} m^3}{60 \, sec.} = 0.12 \, m^3/sec.$$

Velocity at $d_1 = 25 \, cm = 0.25 \, m = \frac{1}{4} \, m.$

$$V = A_1 V_1 = \frac{\pi d_1^2}{4} V_1$$

$$\therefore \quad 0.12 = \frac{22}{7} \times \frac{(1/4)^2}{4} \times V_1$$

$$V_1 = 2.44 \, m/s.$$

From Bernoulli's theorem

$$P_1 + \frac{1}{2} \rho_w V_1^2 = \frac{1}{2} \rho_w V_2^2$$

$$6 \rho_w g + \frac{1}{2} \rho_w V_1^2 = \frac{1}{2} \rho_w V_2^2$$

$$12 g + V_1^2 = V_2^2$$

$$V_2 = 11.13 \, m/s$$

$$V = A_2 V_2 = \frac{\pi d_2^2}{4} \times V_2$$

$$d_2^2 = \frac{4 \times 0.12}{22 \times 11.13}$$

~~$d_2 = 11.7$~~

$d_2 = 0.1172\ m = 11.72\ cm$

Q19. Let P_1, P_2 be pressures at entrance and throat.

$\therefore \quad P_1 - P_2 = h\rho_m g = (0.12\ m)(13.6\times10^3)(9.8)$

$= 1.6\times10^4\ N/m^2$

From ~~Berno~~ Bernoulli's theorem

$$P_1 - P_2 = \frac{1}{2}\rho_w (V_2^2 - V_1^2)$$

V_1, V_2 are velocities at entrance and throat.

$$\therefore \quad 1.6\times10^4\ N/m^2 = \frac{1}{2}\times 0.9\times10^3 (V_2^2 - V_1^2)$$

$$V_2^2 - V_1^2 = \frac{320}{9} \quad \text{——} \quad (1)$$

From ~~A~~ equation of continuity

$$A_1 V_1 = A_2 V_2$$

$$\therefore \quad \frac{\pi d_1^2}{4} \cdot V_1 = \frac{\pi d_2^2}{4} V_2$$

$$\cancel{\frac{\pi}{2}} \quad V_1 = \left(\frac{d_2}{d_1}\right)^2 V_2 = \left(\frac{3}{9.5}\right)^2 V_2$$

$$V_1 = \left(\frac{6}{19}\right)^2 V_2 \qquad \text{———} \quad (2)$$

$\therefore$ from (1) & (2)

$$V_2^2 = 35.91 \quad \text{or} \quad V_2 = 5.99 \text{ m/s.}$$

$\therefore$ Volume rate of flow $= A_2 V_2$

$$= \frac{\pi d_2^2}{4} V_2$$

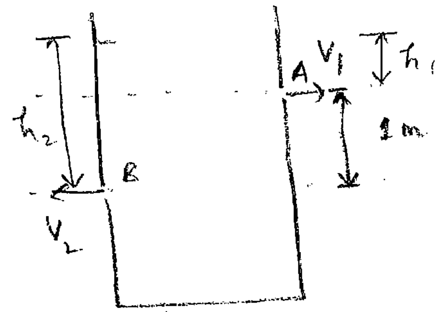
$$= \frac{22}{7} \times \frac{(3 \times 10^{-2})^2}{4} \times V_2$$

$$= 42.36 \times 10^{-4} \text{ m}^3/\text{sec.}$$

Volume rate of flow in lit/sec

$= 4.236$ lit/sec.

Q20 V_1 = velocity of efflux at hole A



$$V_1 = \sqrt{2gh_1}$$

$$V_2 = \sqrt{2gh_2}$$ = velocity of efflux at B.

If a system of mass (m) is releasing mass at the rate $\left(\frac{dm}{dt}\right)$ ~~then~~ with velocity (V_r) relative to it then, force acting on system is given by

$$\vec{F} = -\vec{V}\frac{dm}{dt}.$$ (From ~~reduced~~ variable - mass concept).

If A is cross-section of hole ~~then~~, and (ρ) be density of fluid then, ~~dm~~ rate of reducing mass is given by

$$\therefore \frac{dm}{dt} = \rho AV$$

$$\therefore F = \rho AV^2$$

$$\therefore F_1 = \rho AV_1^2 \qquad F_2 = \rho AV_2^2$$

$$F_{net} = |F_1 - F_2| = \rho A(V_2^2 - V_1^2) = \rho A(2gh_2 - 2gh_1$$

$$F_{net} = \rho A(2g)(h_2 - h_1)$$

$$= 10^3 \times 4 \times 10^{-4} \times 2 \times 10 \times 1$$

$$F_{net} = 8\,N$$

Properties of matter

In chapter - I

Q1 (B) Y is material property.

Q2 (B)

$$F = \frac{YA\,\Delta l}{l}$$

$$\frac{F_1}{F_2} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{3l}{l} \cdot \frac{\pi (3d)^2/4}{\pi d^2/4} = \frac{27}{1}$$

Q3 (A)

For wire, $V = Al$

$dV = A\,dl$ $\quad A =$ constant

$\Delta V = A\,\Delta l.$

$$\therefore \frac{\Delta V}{V} = \frac{\Delta l}{l}.$$

Q4. (B)

$$F = \frac{YA(\Delta l)}{l}$$

$\Delta l = 2\pi(R - r)$ $\quad l = 2\pi r$

$$\frac{\Delta l}{l} = \frac{R-r}{r} \Rightarrow F = YA\left(\frac{R-r}{r}\right)$$

Q5 (C) $\frac{\Delta V}{V} = \frac{0.01}{100} = 10^{-4}$

$B = \frac{-P}{\Delta V/V} = -\frac{100 \times 10^{6} \text{ dynes}}{10^{-4}}$

$B = 1 \times 10^{12} \text{ dynes/cm}^2$

Q6 (A) stress $= \frac{\text{wt.}}{\text{Area}} = \frac{Al\rho g}{A} = l\rho g$

$\sigma = l\rho g$.

Q7 (A) $B_{iso} = P$

$P =$ constant so, $B_{iso} =$ constant.

~~In chapter~~

In-chapter -2

Q1 (C) $U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{(\text{stress})^2}{2Y}$

Q2 (B) Work performed $= \frac{1}{2} \times$ stress $\times$ strain $\times$ volume

= Energy stored

$= \frac{1}{2} \times (Y\alpha t) \times (\alpha t) \times (A \times l)$

$= \frac{1}{2} \times (YA\alpha t) \times (l\alpha t)$

Q.3 (B) $F = \dfrac{YA\,\Delta l}{l}$

$$\frac{1\times10^{10}\times \pi R_B^2 \times (1\times10^{-3})}{L} = \frac{2\times10^{10}\times \pi R_S^2 \times (1\times10^{-3})}{l}$$

$$R_S = \frac{R_B}{\sqrt{2}}$$

Q.4. (A) $k = +\dfrac{PV}{\Delta V} \Rightarrow \cancel{V = \dfrac{k(\Delta V)}{P}}$

$$\Delta V = +\frac{PV}{k}$$

$$\Delta V = V(\alpha\,\Delta T) = +\frac{PV}{k}$$

$$\Delta T = +\frac{P}{\alpha k}$$

Q5 (B) $Y = \dfrac{W/A}{l_a/L} = \dfrac{V\sigma g/A}{l_a/L}$

$$Y = \frac{V(\sigma-\rho)g/A}{l_\omega/L}$$

$$\frac{\sigma}{l_a} = \frac{\sigma-\rho}{l_\omega}$$

$$\frac{\sigma}{\rho} = \frac{l_a}{l_a-l_\omega} = \sigma_r$$

Q6 (B) $B_{iso} = P$.

Q7 (B)

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} \qquad K_1 = K \qquad K_2 = \frac{YA}{L}$$

$$T = 2\pi\sqrt{\frac{m(YA + KL)}{YAK}}$$

Inchapter - Q3

Q1 (a) (b) (c). $\Delta P_{drop} = \frac{2T}{R}$

$$\Delta P_{bubble} = \frac{4T}{R}$$

$$(\Delta P_{cy})(2\pi R)(R) = T(2\pi R) \qquad \text{height of drop} = R$$

$$\Delta P_{cy} = \frac{T}{R}$$

Q2 (b) (c) (d)

force of cohesion > force of adhesion

so, convex meniscus. Angle of contact is obtuse and liquid ~~is~~ will not wet solid.

Q3 (b) (c)

$$h = \frac{2T\cos\theta}{R\rho g}$$

Q4 (a)

Viscous force opposes motion.

Q5 (c) (d) If capillary is broken, radius of meniscus will be adjusted such that

$$hr = h'r'$$

Flow rate will be constant.

Surface energy = $2TS$.

~~surface energy, also~~

surface energy increases by = $T(\Delta S)$

ΔS = increase in surface energy

$$\frac{4\pi}{3} R^3 = 10^6 \frac{4\pi}{3} r^3$$

$$r = \frac{R}{100}$$

$$\Delta S = 4\pi r^2 \times 10^6 - 4\pi R^2$$

$$= 4\pi R^2 \, 10^4 - 4\pi R^2 \approx 4\pi R^2 \times 10^4$$

$$\therefore T\Delta S = 4\pi R^2 \times T \times 10^4$$

Q6 (c)

$$F = -\eta A \frac{dv}{dy}, \quad \frac{F}{A} = -\eta \frac{dv}{dy}$$

$$\frac{F}{A} = -1\times10^{-3} \times \frac{5}{10} = \cancel{3}\, 0.5\times10^{-3} \ N/m^2$$

Q7 (d) surface tension of liquid decreases.

Q8 (b) Work done = Increase in surface energy

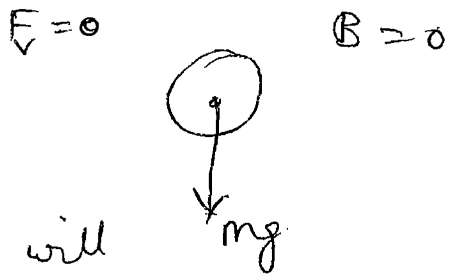
$$= 2S \times \left(4\pi \left(\frac{D}{2}\right)^2 - 4\pi\left(\frac{d}{2}\right)^2\right)$$

$$= 2S\ \pi\,(D^2 - d^2)$$

$$= 2\pi\,(D^2 - d^2)S.$$

Q9. (d) In vacuum, no medium is present. So, no viscous force. Hence, ~~ball~~ body will continue to accelerate downward with acceleration 'g'.

$\therefore\ a = g$



So, terminal velocity will never be attained.

Q10 (a) Energy needed = Increase in surface energy

$$= T \times (\Delta S)$$

$$= T \times (n \times 4\pi r^2 - 4\pi R^2)$$

$$= 4\pi T\,(nr^2 - R^2)$$

Q11. (c)

$$\Delta P = \frac{4T}{R} = P_{in} - P_{out}$$

$$P_{in} = P_{out} + \frac{4T}{R}$$

$$P_{out} = P_0 = \text{atm. pressure}$$

$$P_{in} = P_0 + \frac{4T}{R}$$

$$(P_{in})_{smaller} > (P_{in})_{bigger}$$

Air will flow from ~~the~~ smaller bubble to larger.

JP sir Solutions to Exercise - 1 Objective (single choice) 1.

Elasticity

Page 1 to 5

Q1. (A) Radius of bubble increases under Isothermal conditions,

$PV = \text{constant}$

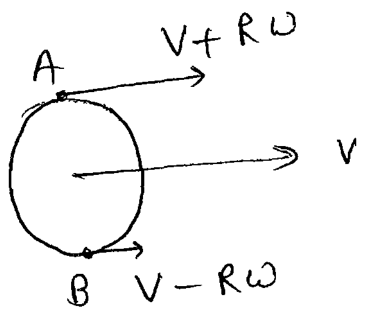
$\therefore$ At surface $P_{surface} < P_{bottom}$

$\therefore$ $V_{surface} > V_{bottom}$

~~(B)~~

Q2. (A)

delete



speed of air at A > speed of air at B.

$P_A < P_B$ (From Bernoulli's Theorem)

$\therefore$ Net force due to pressure difference is upwards.

$$g_{effective} = g - \frac{F}{m}$$

$\therefore$ Time of flight increases.

Q3. (D) Hooke's law is obeyed in region. where, stress $\propto$ strain

It is oa region

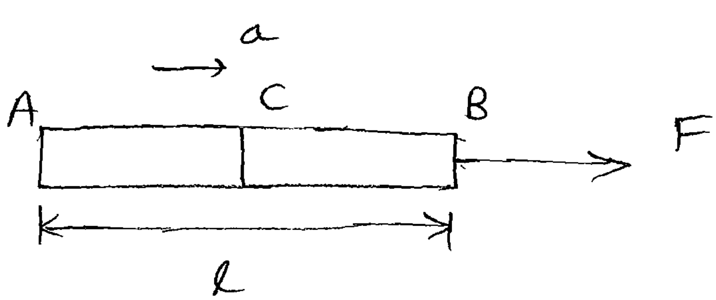
~~bc~~ b is ~~elas~~ elastic limit which comes just after proportional limit.

bc is region in which ~~cont~~ materials yields, ~~me~~ which means material continuously ~~deform~~ deforms ~~with~~ with little or no increase in strain.

bc ~~as~~ is also called region in which material behaves as viscous liquid.

Q4. (C)
If material ~~deforms be~~ deforms beyond elastic limit then, on unloading there remains a net permanent deformation.

5. (A)



A

$F = (\rho A l) a$

Solution to Exercise - 1

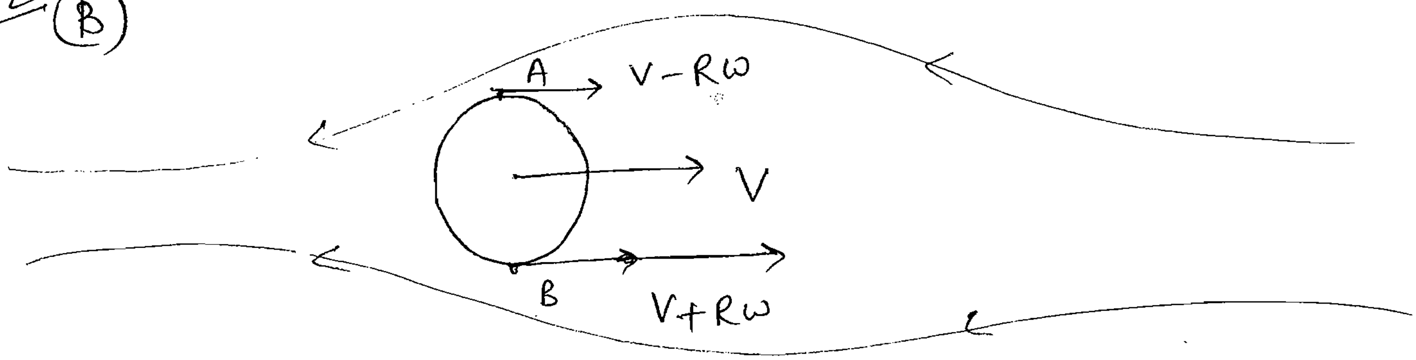
Properties of Matter

Q1 (A) $PV =$ constant if $T =$ constant

At surface $\qquad P_{surface} < P_{bottom}$

$$\therefore V_{surface} > V_{bottom}$$

Q2 (B)



Velocity of air at A > Velocity of air at B.

From Bernoulli's theorem,

$$P_A < P_B$$

$\therefore$ Net force acts upwards due to pressure difference.

$$g_{eff} = g - \frac{F}{m}$$

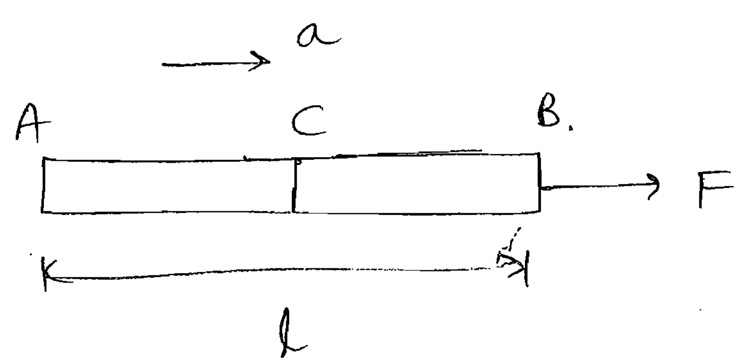
Q3 (D) Hooke's law, stress $\propto$ strain

It is region Oa. a = Proportional limit
b is elastic limit which comes after propor-
-tional limit.

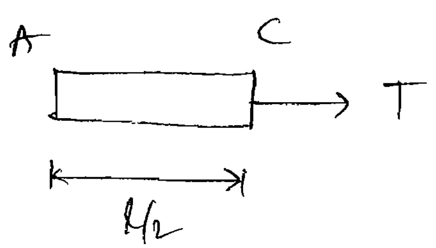
bc → region in which material yields.
Material behaves as Viscous liquid.

Q4 (C)
If material deforms beyond elastic limit then, on unloading there remains a net permanent deformation.

Q5 (A)

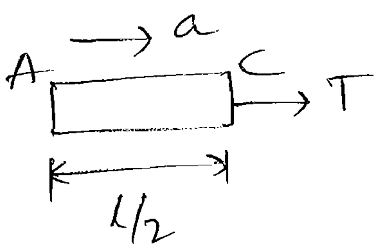


$$F = (\rho A l) a$$



$$T = ma = \rho A \frac{l}{2} a$$

$\frac{T}{A} = \frac{\rho l A}{2}$ = stress at midpoint.



$$T = ma$$

$$T = \rho A \frac{l}{2} a$$

$$\frac{T}{A} = \frac{\rho l a}{2} \qquad \frac{T}{A} = \text{stress}$$

$\therefore$ stress at midpoint $= \frac{1}{2} \rho l a$

06. (C) $\quad \frac{\Delta l}{l} = \frac{\text{stress}}{Y} = \frac{F}{AY}.$

$$\frac{l_1 - l}{l} = \frac{F_1}{AY} \qquad \frac{l_2 - l}{l} = \frac{F_2}{AY}.$$

$$\frac{l_1 - l}{l_2 - l} = \frac{F_1}{F_2}$$

solving, $\quad l = \frac{l_1 F_2 - l_2 F_1}{F_2 - F_1}$

07. (B)
Viscous force $= 6\pi\eta R V$. depends only on viscosity of medium surrounding.

08. (A) $\quad$ Force $= F = \frac{YAl}{\Delta l}$

and hence deforming force is lower at high temperature for given elongation.

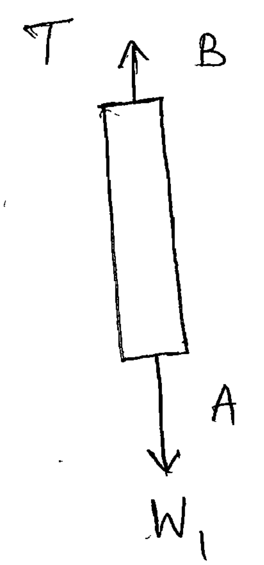
9. (C)

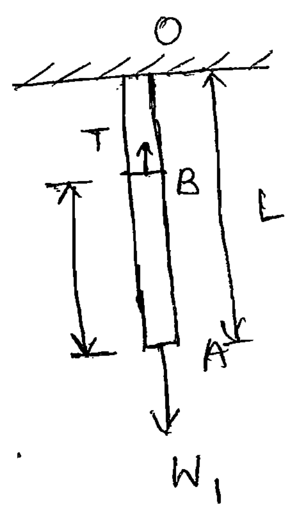
Breaking stress is a material property.

$$\therefore \frac{20 \text{ kg-wt}}{\pi d^2/4} = \frac{F}{\pi (2d)^2/4}$$

$$\therefore F = 80 \text{ kg-wt}$$

Q10. (C)





$$T = W_1 + W_{AB} = W_1 + \frac{W}{l} \times \frac{3l}{4}$$

$$T = W_1 + \frac{3W}{4}$$

$$\therefore \text{stress at } B = \frac{T}{A} = \frac{(W_1 + 3/4\, W)}{S}$$

Q11. (B)

$$Y = \frac{\text{stress}}{\text{strain}} \qquad \text{strain} = \frac{2l - l}{l} = \frac{l}{l} = 1$$

3.

$$Y = \frac{\text{stress}}{1} = \frac{F}{A} = \frac{2 \times 10^5 \text{ dyne}}{2 \text{ cm}^2} = 1 \times 10^5 \frac{\text{dyne}}{\text{cm}^2}$$

Q12 (C)

$$\text{stress} = Y \times \text{strain}$$

$$\text{strain} = \frac{\Delta l}{l} = \frac{\alpha l \Delta \theta}{l} = \alpha \Delta \theta.$$

$$(\sigma) \text{ stress} = Y(\alpha \Delta \theta)$$

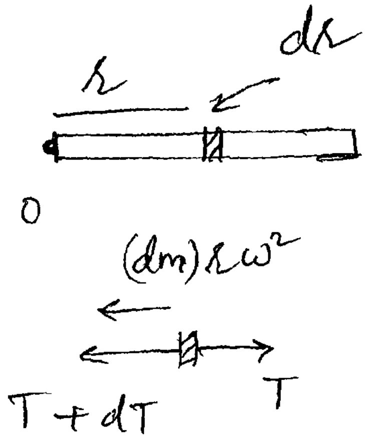
$$\sigma_1 = \sigma_2 \quad \Rightarrow \quad Y_1 \alpha_1 \Delta \theta = Y_2 \alpha_2 \Delta \theta$$

$$\therefore \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2} = 3:2$$

Q13. (C)

$$(T + dT) - T = (dm) r \omega^2$$

$$dT = (dm) r \omega^2$$



$$(\sigma) \text{ stress} = \cancel{\frac{dT}{dA}} \quad \cancel{\frac{dT}{A}}$$

$$\text{strain} = \frac{\text{stress}}{Y} = \cancel{\frac{dT}{YA}} =$$

$dl \qquad dT$

$$dT = dm\, r\, \omega^2 \qquad dm = \frac{m}{L}\, dr$$

$$\int_{0}^{T} dT = \int_{r=L}^{r} \frac{m}{L}\, dr\, r\omega^2$$

$$\Big[T\Big]_0^T = \frac{m\omega^2}{L}\left[\frac{r^2}{2}\right]_L^r$$

$$T = \frac{m\omega^2}{2L}\left(r^2 - L^2\right)$$

$\therefore$ stress at element $= \frac{T}{A}$

strain at element $= \frac{dl}{dr} = \frac{\text{stress}}{Y} = \frac{T}{YA}$

$$dl = \frac{m\omega^2}{2YLA}\left(r^2 - L^2\right) dr$$

$$\int_0^{l} dl = \int_{r=0}^{L} \frac{m\omega^2}{2YLA}\left(r^2 - L^2\right) dr$$

$$l = \frac{m\omega^2}{2YLA}\left[\frac{r^3}{3} - rL^2\right]_0^L$$

$$= \frac{m\omega^2}{2YLA}\left[\frac{L^3}{3} - L^3\right] = -\frac{m\omega^2 L^2}{3YA}.$$

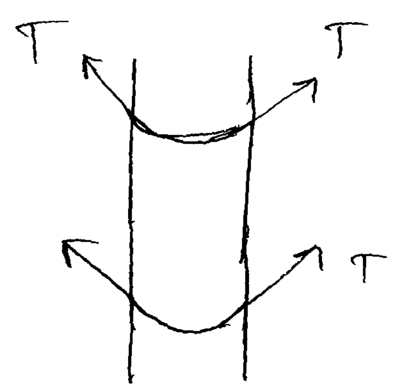
$$\therefore \quad \frac{2T}{R} = h \rho_w g$$

$$R = \frac{2T}{h\rho_w g} = \frac{2(0.07)}{10^{-2} \times 10^3 \times 9.8}$$

$R = 1.4285$ mm.

$R =$ radius of meniscus

Q19 (A)



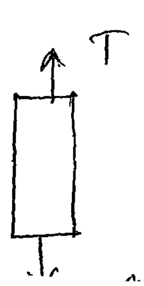
$T(2\pi r) + T(2\pi r)$

$= mg$

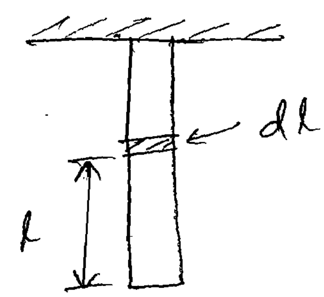
$2T(2\pi r) = \pi r^2 L \rho g$ (Two surfaces)

$\cancel{T=}$ $L = \frac{4T}{r\rho g}$

Q20 (B)



$T = mg$



$$T = \frac{m}{L} l g$$

$$\frac{T}{A} = \frac{mlg}{AL}$$

strain in length (dl) $= \frac{\text{stress}}{Y} = \frac{mlg}{YAL} =$

$$= \frac{\cancel{dx}}{\cancel{dl}} = \frac{dy}{dl}$$

where dy = small increase in length of dl.

$$\therefore \int_0^y dy = \int_0^L \frac{mlg}{YAL}\, dl = \frac{mg}{YAL}\left[\frac{l^2}{2}\right]_0^L$$

$$y = \frac{mg L^2}{2YAL} = \frac{mgL}{2AY}.$$

~~Fluid (Sub)~~

~~4,5, 6, 11, 13, 14(b), 15, 16, 19.~~

~~(obj) → (due)~~

~~Elasticity (obj) 2, 18.~~

~~(Sub)~~

Q14 (B)

$$\cancel{V_t = \frac{2}{g\eta}}$$

At terminal velocity

$$6\pi\eta r V_t = mg.$$

$$V_t = \left(\frac{m}{r}\right)\frac{g}{6\pi\eta}$$

$$\therefore \quad V_t \propto \frac{m}{r}$$

Q15 (C)

Initially velocity increases and then, becomes constant.

Q16 (C)

volume of big drop = volume of smaller drops

$$\frac{4\pi}{3}R^3 = 8 \times \frac{4\pi}{3} r^3$$

$$R^3 = 8r^3$$

$$r = \frac{R}{2}$$

Work done = change in surface energy

$$= T \times \Delta S$$

$$W = T \times (4\pi r^2 \times 8 - 4\pi R^2)$$

$$= T(4\pi R^2) = 4\pi R^2 T$$

Q17 (B)

Pressure at depth (h) $= P_0 + h\rho_w g$

$$P_h = P + hdg$$

$P_{inside} - P_h =$ Excess pressure $= \cancel{\frac{4T}{R}} \; \frac{2T}{R}$

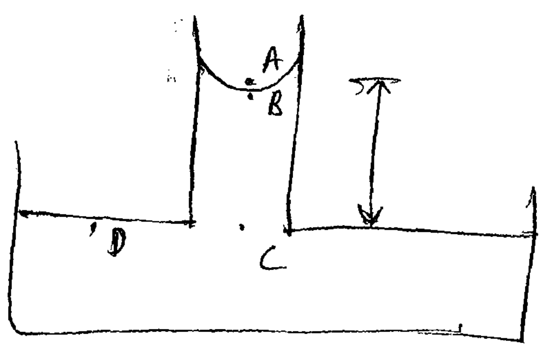
($\because$ Bubble in water has only one surface).

$$\therefore P_{inside} = P_h + \frac{2T}{R}$$

$$= P + hdg + \frac{2T}{R}$$

Q18 (D)

$$P_{atm} = P_D = P_C = P_A$$



$$P_B + h\rho_w g = P_C$$

$$P_B + h\rho_w g = P_A$$

$$P_A - P_B = h\rho_w g$$

Ex - 2 ~~PoM~~

Properties of Matter

Q1. (b)(c)

When rod falls vertically & slides on rough surface, tension in it is zero. so, stress is zero and hence, strain.

Q2 (a) (b)(c)

~~Avel~~ At any point $T_A = \frac{4}{3} mg$.

$T_B = \frac{mg}{3}$.

(a) ∴ A will reach breaking stress before B, if $r_A = r_B$.

(b) if $r_A < 2r_B$.

$\sigma_A = \frac{4/3\, mg}{\pi R_A^2}$ $\sigma_B = \frac{mg/3}{r_B^2}$.

$\left(\frac{4}{\sigma_A}\right)^2 < 2\left(\frac{1}{\sigma_B}\right)^2$

$\sigma_A > 4\sqrt{2}\,\sigma_B$

ii) $r_A = 2r_B$.

$$\sigma_A = \frac{4/3\, mg}{\pi r_A^2} \qquad \sigma_B = \frac{mg/3}{\pi r_B^2}$$

(c) $\sigma_A = \sigma_B$. = Breaking stress

Q3. (a) (c) (d)

$$\Delta U_G = -Mgl$$

~~From energy conservation~~

$$\cancel{\Delta U_G + \Delta U_E = 0}$$

$$\Rightarrow \quad \cancel{\Delta U_E = -\Delta U_G = Mgl}$$

$$\Delta U_E = \frac{1}{2} \times \left(\frac{Mg}{A}\right) \times \left(\frac{l}{L}\right) \times (l \times A)$$

$$\Delta U_E = \frac{1}{2} Mgl.$$

From conservation of Energy.

$$\Delta H = \frac{1}{2} Mgl = \text{Heat produced.}$$

Q4 (a) (c) (d)

(a) F = -F internal

$\therefore \quad W_F = -W_{internal} = -(-\Delta U_E)$

$$W_F = \Delta U_E = \frac{YAl^2}{2L}$$

$$\Delta U_E = \frac{YAl^2}{2L}$$

$$\Delta H = 0$$

Q5 (b) (d)

Elastic forces are conservative upto elastic ~~lim~~ limit only.

~~Q6.~~ (a) (b) (c)

~~$F = YA\alpha t$~~

~~$ma = YA\alpha t$~~

~~$a = \frac{YA\alpha t}{Al\rho}$~~

$$F = \frac{YA}{l}(\Delta l)$$

$$a = \frac{YA}{ml}(\Delta l)$$

$$m = \rho A l$$

$$\therefore \quad a = \frac{Y}{\rho l^2}(\Delta l) = \frac{Y}{\rho l^2}x.$$

$$\omega = \sqrt{\frac{Y}{\rho l^2}} = \frac{1}{l}\sqrt{\frac{Y}{\rho}}$$

~~Q6~~ (a) (b) ~~(c)~~ $\theta < 90°$ → Concave meniscus → (wetting liquid)

cohesive < adhesive

$\theta > 90°$ → convex → cohesive > adhesive

(Non - wetting).

$\theta = 0°$ → hemi-spherical ~~Plane meniscus~~ → cohesive < Adhesive (water).

$$h = \frac{2T\cos\theta}{R\rho g} \qquad R' = \frac{R}{2} \Rightarrow h' = 2h.$$

~~Q7~~ (c) (d)

Radius of meniscus will be changed such that

$$hr = h'r' = \frac{2T}{\rho g}$$

h' = new height of tube

r' = new radius of meniscus.

Q8 (a)(b)(d)

Surface tension is independent of area

$$\Delta P = \frac{4T}{R}$$

$$T_m > T_w$$

Q9. (b) (c) (d).

$\theta =$ obtuse, liquid depresses in tube, ~~conve~~ ~~meni~~

$\theta =$ acute, liquid ~~res~~ rises and wets tube.

~~$\theta =$~~ $\theta =$ obtuse, convex meniscus

T decreases with temp.

Q10 (a)(d).

$$P_{in} > P_{out}$$

$$\Delta P = \frac{2T}{R} = P_{in} - P_{out}$$

$$P_{in} = P_{out} + \frac{2T}{R}$$

As bubble comes up P_{out} decreases

Properties of Matter - Ex - 3

Passage 1

Q1 (A)

$$F = -\eta A \frac{dV}{dr}$$

$A = 2\pi r l \qquad V = V_0 \left(1 - \frac{r^2}{R^2}\right)$

$$\frac{dV}{dr} = -\frac{2V_0 r}{R}.$$

$\therefore \quad F \propto r^2$

Q2 (A) $\quad V = V_0 \left(1 - \frac{r^2}{R^2}\right) \qquad V_0 = \frac{PR^2}{4\eta l}$

$r = R/2$

$$V = \frac{3PR^2}{16\eta l}$$

Q3 (B)

$V = \frac{\pi P R^4}{8\eta l} \qquad V_0 = \frac{PR^2}{4\eta l}$

$$\therefore \quad V = \frac{(\pi R^2 V_0)}{2}$$

Q4 (C) $\quad V_0 = \frac{PR^2}{4\eta l} \Rightarrow P = \frac{4\eta V_0 l}{R^2}$

Q5 (C) $F = P \times \pi R^2$. $P = \frac{4\eta V_0 l}{R^2}$

$F = 4\pi\eta l V_0$

Passage 2

Q6 (A) Range of force = $10 A^\circ = 1 nm$.

Q7 (C) Work done against intermolecular forces (Cohesive Force).

Q8 (A)

Concave meniscus → adhesive force > cohesive force

Angle of contact 90° → Plane meniscus

~~Cohes~~ Adhesive forces = Cohesive forces

Pressure below meniscus is greater → Convex meniscus (Adhesive force < ~~cohesive force~~ Cohesive force).

Q9 (C) Depends on both cohesive and adhesive

forces

Q10 (B) Net force on liquid is ~~at~~ normal to liquid surface.

~~Ma~~ Matrix Match type

Q11. Terminal velocity → Resultant of upthrust and viscous force.
(A → S)

Objects of high density can also float → When average density becomes less than liquid
B (B → P)
eg. Iron built ship.

$\rho_{iron} > \rho_{water}$

(C → Q) $B = V \rho g_{eff}$

under free fall $g_{eff} = 0$

(D → R) $F_v = 6\pi\eta R V$

$F_v \propto V$

JP - sir (1 to 5) Elasticity - Ex - IV - solutions (subjective)

Q1 Let Y_s, Y_c be their Young's modulii.

$$Y_s = \frac{\text{stress}}{\text{strain}} = \frac{F/A_s}{\Delta l_s / l_s} = \frac{l_s}{\Delta l_s A_s}$$

$$Y_c = \frac{F/A_c}{\Delta l_c / l_c}$$

$$\frac{Y_s}{Y_c} = \frac{l_s}{\Delta l_s A_s} \cdot \frac{\Delta l_c A_c}{l_c} \qquad \Delta l_s = \Delta l_c \text{ (given)}$$

$$= \frac{(4.7)\text{m}}{\cancel{(\Delta l_s)}(3 \times 10^{-5}\text{m}^2)} \cdot \frac{\cancel{(\Delta l_c)}(4 \times 10^{-5})}{(3.5\text{m})}$$

$$\frac{Y_s}{Y_c} = \frac{47}{35} \cdot \frac{4}{3} = \frac{188}{105} = 1.79 = 1.79$$

Q2. For steel wire

Load = 10 kg $\qquad$ stress $= \dfrac{10g}{A_{steel}} = \dfrac{10g}{\pi\left(\dfrac{0.25 \times 10^{-2}}{2}\right)^2}$

$\therefore$ ~~$\frac{\Delta l_s}{l_s} = \frac{Y}{st}$~~ $\qquad Y = \dfrac{\text{stress}}{\text{strain}}$

~~sta~~ strain $= \dfrac{\Delta l_s}{l_s} = \dfrac{\text{stress}}{Y}$ =

$$\Delta l_c = l_s \times \text{stress} = 1.5\text{m} \times \frac{10g}{\ldots}$$

$$\Delta l_{steel} = 1.5 \times 10^{-4}\ m$$

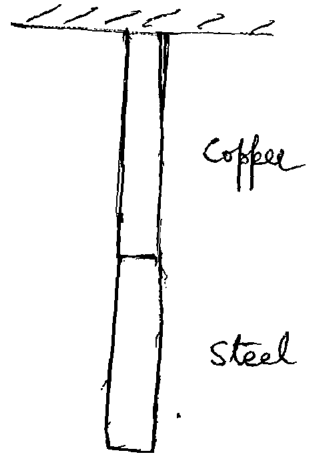
For Brass, load = 6 kg stress = $\dfrac{6g}{\pi\left(\dfrac{0.25\times10^{-2}}{2}\right)^2}$

$$\Delta l_{brass} = \frac{stress}{Y} \times l_{brass}$$

$$= \frac{6g}{0.91\times10^{11} \times \frac{22}{7}\left(\frac{0.75\times10^{-2}}{2}\right)^2} \times (1m)$$

$$= 1.3 \times 10^{-4}\ m$$

93. $\Delta l = \dfrac{stress}{Y} \times l = \dfrac{Fl}{AY}$



Δl = change in length

l = original length

$$\Delta l_s + \Delta l_c = 0.7\ mm = 0.7 \times 10^{-3}\ m$$

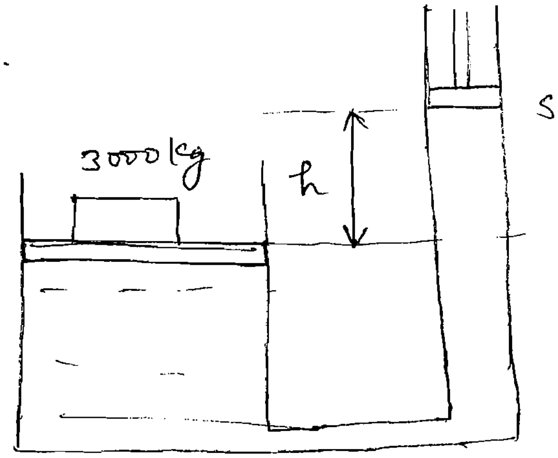
$$\frac{F l_s}{A_s Y_s} + \frac{F l_c}{A_c Y_c} = 0.7\times10^{-3}$$

$$F = \frac{0.7\times10^{-3}\ m}{\dfrac{l_s}{A_s Y_s} + \dfrac{l_c}{A_c Y_c}}$$

$$F = \frac{0.7\times10^{-3}\ m}{\dfrac{1.6\ m}{\ldots} + \dfrac{2.2\ m}{\ldots}}$$

$$F = 1.77 \times 10^{2}\ N$$

74 From Pascal's law.



Hydraulic lift

$$P_s + h\rho g = P_L$$

$$P_s = P_L - h\rho g.$$

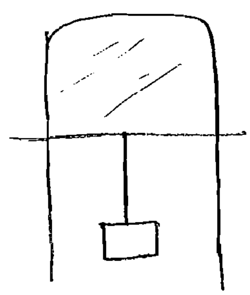
For maximum pressure,

$$h = 0 \qquad P_s = P_L$$

$$\cancel{P_s = P_L = \frac{3000\,g}{(425 \times 10^{-4})^2} =}$$

$$P_s = P_L = \frac{3000\,g\ N}{425 \times 10^{-4}\ m^2} = 6.92 \times 10^{5}\ N/m^2$$

75. There are two surfaces of soap film.



So, force due to surface tension is given by

$$\cancel{T(2l) = mg \text{ (weight)}}$$

$$\cancel{T = \frac{mg}{2l}}$$ since, two surfaces are there.

$$\cancel{F = 2T(2l)}$$

where T = surface

This force supports weight (external + slider).

$$\therefore \quad T(2l) = mg$$

$$T = \frac{mg}{2l} = \frac{1.5 \times 10^{-2}}{2 \times 0.3}$$

$$T = 2.5 \times 10^{-2} \text{ N/m}$$

76. Excess pressure inside drop

$$= \frac{2T}{R} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

$$\text{Excess pressure} = P_{inside} - P_{atm}$$

$$P_{inside} = P_{atm} + \text{Excess pressure}$$

$$= 1.01 \times 10^{5} \text{ Pa} + 310 \text{ Pa}$$

$$P_{inside} = 1.0131 \times 10^{5} \text{ Pa}$$

07. At terminal velocity (V_t)

Viscous force = weight of drop.

$$6\pi\eta R V_t = mg = \frac{4\pi}{3} R^3 \rho g$$

$$V_t = \frac{2}{9}\frac{\rho R^2 g}{\eta}$$

$$V_t = \frac{2 \times (1.2 \times 10^3) \times (2 \times 10^{-5})^2 \times 9.8}{9 \times 1.8 \times 10^{-5}}$$

$$V_t = 5.8 \times 10^{-2}\ m/s = 5.8\ cm/s.$$

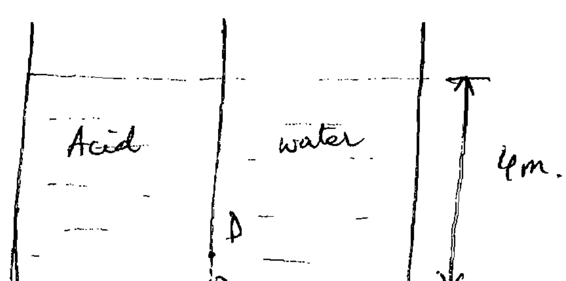
$$\text{Viscous Force} = 6\pi\eta R V_t = \frac{4\pi}{3} R^3 \rho g$$

$$= 6 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 2 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= 3.93 \times 10^{-10}\ N$$

08

Pressure difference at the door



$= h_{acid} \rho_{acid} g - h_{water} \rho_{water} g$

$= 4\,m \times 1.7 \times 10^3 \times 9.8 - (4m) \times (1 \times 10^3) \times 9.8$

$= 4 \times 0.7 \times 10^3 \times 9.8 \ N/m^2$

$= 2.744 \times 10^4 \ N/m^2$

$\therefore$ Force required $= \Delta P \times A$

$= 2.744 \times 10^4 \times 20 \times 10^{-4} \ N.$

$= 54.88$ Newton

79. Pressure inside soap bubble is given by

$$P_{in} = P_{atm} + \frac{4T}{R}$$

P_{atm} = Atmospheric pressure = Pressure in cylinder $= P_{cy}$

T = Surface Tension

R = Radius of bubble

Initially, $\cancel{P_i = P_{atm} + \frac{4T}{R}}$

$$P_i = P_{cy} + \frac{4T}{R}$$

$$P_{cy} = 10^5 \text{ N/m}^2$$

$$P_i = \left(10^5 + \frac{4}{3} \times 10^3\right) \text{ Pa}$$

$$P_f = P + \frac{4T}{R_2} = \left(P + \frac{8}{3} \times 10^3\right) \text{ Pa}$$

$$R_2 = \frac{R}{2}$$

Under Isothermal conditions

$$PV = \text{constant}$$

$$P_i V_i = P_f V_f$$

$$\left(P + \frac{8}{3} \times 10^3\right) \times \frac{4\pi}{3} \left(\frac{R}{2}\right)^3 = \left(10^5 + \frac{4}{3} \times 10^3\right) \times \frac{4\pi}{3} R^3$$

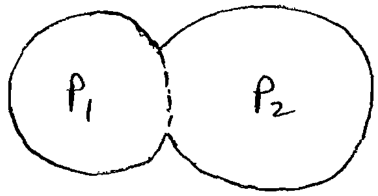
$$8\left(10^5 + \frac{4}{3}\times 10^3\right) = P + \frac{8}{3}\times 10^3$$

$$\therefore \quad P = 8.08 \times 10^5 \text{ Pa}$$

Q10

$$P_{in} = P_{atm} + \frac{4T}{R}$$

$$P_1 = P_0 + \frac{4T}{R_1}$$



$$P_2 = P_0 + \frac{4T}{R_2}$$

$$P_1 - P_2 = 4T\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{——— (1)}$$

$P_1 - P_2$ = Excess pressure for common surface

$$P_1 - P_2 = \frac{4T}{R_{common}} \quad \text{——— (2)}$$

from (1) & (2).

$$\frac{1}{R_{common}} = \frac{1}{R_1} - \frac{1}{R_2}$$

$R_{common} = 0.004\ m$

Q11 Let a & ~~be~~ b be radii of two soap bubbles and P_a, P_b be pressure inside them before they coalesce. If c be radius and P_c be pressure inside combined bubbles. Then

$$\therefore \quad P_a = P_0 + \frac{4T}{a} \qquad P_b = P_0 + \frac{4T}{b}$$

$$P_c = P_0 + \frac{4T}{c}$$

If V_a, V_b, & V_c are corresponding ~~soap~~ volumes then, from Boyle's law under isothermal condition.

$$P_a V_a + P_b V_b = P_c V_c$$

$$\left(P_0 + \frac{4T}{a}\right)\frac{4\pi}{3}a^3 + \left(P_0 + \frac{4T}{b}\right)\frac{4\pi}{3}b^3 = \left(P_0 + \frac{4T}{c}\right)\frac{4\pi}{3}c^3$$

$$P\left(\frac{4\pi}{3}a^3 + \frac{4\pi}{3}b^3 - \frac{4\pi}{3}c^3\right) + \frac{4T}{3}\left(4\pi a^2 + 4\pi b^2 - 4\pi c^2\right) = 0$$

Now, $\frac{4\pi}{3}a^3 + \frac{4\pi}{3}b^3 - \frac{4\pi}{3}c^3 = V$

= change in volume

$4\pi a^2 + 4\pi b^2 - 4\pi c^2 = S$ = change in surface area

$$\therefore \quad PV + \frac{4T}{3}S = 0$$

$$\cancel{3PV + 4TS = 0}$$

$$3PV + 4ST = 0$$