EXERCISE 1(A)

1, If A contains 10 elements then total number of functions defined from A to A is

(a) 10 (b) 2^{10} (c) 10^{10} (d) $2^{10}-1$

Sol. (c)

According to formula, total number of functions $= n^n$

Here, n = 10. So, total number of functions = 10^{10}.

2 If $f(x)=\frac{x-|x|}{|x|}$, then $f(-1)=$

(a) 1 (b) – 2 (c) 0 (d) 2

Sol. (b)

$f(-1)=\frac{-1-|-1|}{|-1|}=\frac{-1-1}{1}=-2$.

3 If $f(y)=\log y$, then $f(y)+f\left(\frac{1}{y}\right)$ is equal to

(a) 2 (b) 1 (c) 0 (d) – 1

Sol. (c)

Given $f(y)=\log y \Rightarrow f(1/y)=\log(1/y)$, then $f(y)+f\left(\frac{1}{y}\right)=\log y+\log(1/y)=\log 1=0$.

4 If $f(x)=\log\left[\frac{1+x}{1-x}\right]$, then $f\left[\frac{2x}{1+x^2}\right]$ is equal to

(a) $[f(x)]^2$ (b) $[f(x)]^3$ (c) $2f(x)$ (d) $3f(x)$

Sol. (c)

$f(x)=\log\left(\frac{1+x}{1-x}\right)$

$$\therefore f\left(\frac{2x}{1+x^2}\right)=\log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right]=\log\left[\frac{x^2+1+2x}{x^2+1-2x}\right]=\log\left[\frac{1+x}{1-x}\right]^2=2\log\left[\frac{1+x}{1-x}\right]=2f(x)$$

5 If $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$, then

(a) $f\left(\frac{\pi}{4}\right)=2$ (b) $f(-\pi)=2$ (c) $f(\pi)=1$ (d) $f\left(\frac{\pi}{2}\right)=-1$

Sol. (d)

$f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$

$f(x)=\cos(9x)+\cos(-10x)=\cos(9x)+\cos(10x)=2\cos\left(\frac{19x}{2}\right)\cos\left(\frac{x}{2}\right)$

$f\left(\frac{\pi}{2}\right)=2\cos\left(\frac{19\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$; $f\left(\frac{\pi}{2}\right)=2\times\frac{-1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=-1$.

6 If $f:R\to R$ satisfies $f(x+y)=f(x)+f(y)$, for all $x,y\in R$ and $f(1)=7$, then $\sum_{r=1}^{n}f(r)$ is

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $7n(n+1)$ (d) $\frac{7n(n+1)}{2}$

Sol. (d)

$f(x+y)=f(x)+f(y)$

put $x=1, y=0 \Rightarrow f(1)=f(1)+f(0)=7$

put $x=1, y=1 \Rightarrow f(2)=2.f(1)=2.7$; similarly $f(3)=3.7$ and so on

$\therefore \sum_{r=1}^{n} f(r)=7(1+2+3+.....+n) = \frac{7n(n+1)}{2}$.

7 If $f(x)=\frac{1}{\sqrt{x+2\sqrt{2x-4}}}+\frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for $x>2$, then $f(11)=$

(a) $\frac{7}{6}$ (b) $\frac{5}{6}$ (c) $\frac{6}{7}$ (d) $\frac{5}{7}$

Sol. (c)

$f(x)=\frac{1}{\sqrt{x+2\sqrt{2x-4}}}+\frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

$f(11)=\frac{1}{\sqrt{11+2\sqrt{18}}}+\frac{1}{\sqrt{11-2\sqrt{18}}}=\frac{1}{3+\sqrt{2}}+\frac{1}{3-\sqrt{2}}=\frac{3-\sqrt{2}}{7}+\frac{3+\sqrt{2}}{7}=\frac{6}{7}$.

8 Domain of the function $\frac{1}{\sqrt{x^2-1}}$ is

(a) $(-\infty,-1)\cup(1,\infty)$ (b) $(-\infty,-1]\cup(1,\infty)$ (c) $(-\infty,-1)\cup[1,\infty)$ (d) None of these

Sol. (a)

For domain, $x^2-1>0 \Rightarrow (x-1)(x+1)>0$

$\Rightarrow x<-1$ or $x>1 \Rightarrow x\in(-\infty,-1)\cup(1,\infty)$.

9 The domain of the function $f(x)=\frac{1}{\sqrt{|x|-x}}$ is

(a) R^+ (b) R^- (c) R_0 (d) R

Sol. (b)

For domain, $|x|-x>0 \Rightarrow |x|>x$. This is possible, only when $x\in R^-$.

10 Find the domain of definition of $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}$

(a) $(-3,\infty)$ (b) $\{-1,-2\}$ (c) $(-3,\infty)-\{-1,-2\}$ (d) $(-\infty,\infty)$

Sol. (c)

Here $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}=\frac{\log_2(x+3)}{(x+1)(x+2)}$ exists if,

Numerator $x+3>0 \Rightarrow x>-3$ ...... (i)

and denominator $(x+1)(x+2)\neq 0 \Rightarrow x\neq -1,-2$ ...... (ii)

Thus, from (i) and (ii); we have domain of $f(x)$ is $(-3,\infty)-\{-1,-2\}$.

11 The domain of the function $f(x)=\sqrt{(2-2x-x^2)}$ is

(a) $-3\le x\le\sqrt{3}$ (b) $-1-\sqrt{3}\le x\le -1+\sqrt{3}$

(c) $-2\le x\le 2$ (d) None of these

Sol. (b)

The quantity square root is positive, when $-1-\sqrt{3} \le x \le -1+\sqrt{3}$.

12 If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is

(a) $(-\infty, \infty)$ (b) $[-2, \infty)$ (c) $(-2, 3)$ (d) $(-\infty, -2)$

Sol. (b)

$x^2 - 6x + 7 = (x-3)^2 - 2$ Obviously, minimum value is –2 and maximum ∞.

13 The domain of the function $f(x) = \sqrt{x - x^2} + \sqrt{4+x} + \sqrt{4-x}$ is

(a) $[-4, \infty)$ (b) $[-4, 4]$ (c) $[0, 4]$ (d) $[0, 1]$

Sol. (d)

$f(x) = \sqrt{x - x^2} + \sqrt{4+x} + \sqrt{4-x}$

clearly $f(x)$ is defined if

$4 + x \ge 0 \Rightarrow x \ge -4$

$4 - x \ge 0 \Rightarrow x \le 4$

$x(1-x) \ge 0 \Rightarrow x \ge 0$ and $x \le 1$

$\therefore$ Domain of $f = (-\infty, 4] \cap [-4, \infty) \cap [0, 1] = [0, 1]$.

14 The domain of the function $\sqrt{\log(x^2 - 6x + 6)}$ is

(a) $(-\infty, \infty)$ (b) $(-\infty, 3-\sqrt{3}) \cup (3+\sqrt{3}, \infty)$

(c) $(-\infty, 1] \cup [5, \infty)$ (d) $[0, \infty)$

Sol. (c)

The function $f(x) = \sqrt{\log(x^2 - 6x + 6)}$ is defined when $\log(x^2 - 6x + 6) \ge 0$

$\Rightarrow x^2 - 6x + 6 \ge 1 \quad \Rightarrow (x-5)(x-1) \ge 0$

This inequality hold if $x \le 1$ or $x \ge 5$. Hence, the domain of the function will be $(-\infty, 1] \cup [5, \infty)$.

15 The domain of definition of the function $y(x)$ given by $3^x + 3^y = 3$ is

(a) (0, 1] (b) [0, 1] (c) $(-\infty, 0]$ (d) $(-\infty, 1)$

Sol. (d)

$3^y = 3 - 3^x$

y is real if $3 - 3^x \ge 0 \Rightarrow 3 > 3^x \Rightarrow 1 > x$

$x \in (-\infty, 1)$

16 The domain of the function $f(x) = \cos^{-1}[\log_2(x/2)]$ is

(a) [1, 4] (b) [– 4, 1] (c) [– 1, 4] (d) None of these

Sol. (a)

$f(x) = \sin^{-1}[\log_2(x/2)]$

Domain of $\cos^{-1} x$ is $x \in [-1, 1]$

$\Rightarrow \quad -1 \le \log_2(x/2) \le 1 \Rightarrow \frac{1}{2} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4$

$\therefore x \in [1, 4]$.

17 If $f(x) = x^2 + 1,$ then $f^{-1}(17)$ and $f^{-1}(-3)$ will be

(a) 4, 1 (b) 4, 0 (c) 3, 2 (d) None of these

Sol. (d)

Let $y = x^2 + 1 \Rightarrow x = \pm\sqrt{y-1}$

$\Rightarrow f^{-1}(y) = \pm\sqrt{y-1} \Rightarrow f^{-1}(x) = \pm\sqrt{x-1}$

$\Rightarrow f^{-1}(17) = \pm\sqrt{17-1} = \pm 4$

and $f^{-1}(-3) = \pm\sqrt{-3-1} = \pm\sqrt{-4}$, which is not possible.

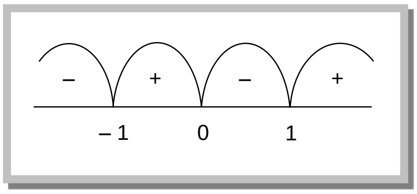
18 Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_2(x^5 - x^3),$ is

(a) (1, 2) (b) $(-1, 0) \cup (1, 2)$

(c) $(1, 2) \cup (2, \infty)$ (d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

Sol. (d)

$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$



So, $4 - x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{4} \Rightarrow x \neq \pm 2$

and $x^5 - x^3 > 0 \Rightarrow x^3(x^2 - 1) > 0 \Rightarrow x > 0, |x| > 1$

$\therefore D = (-1, 0) \cup (1, \infty) - \{2\}$

$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$.

19 The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

(a) $(-3, -1) \cup (1, \infty)$ (b) $[-3, -1) \cup [1, \infty)$

(c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ (d) $[-3, -2) \cup (-2, -1) \cup [1, \infty]$

Sol. (c)

$f(x)$ is to be defined when $x^2 - 1 > 0$

$\Rightarrow x^2 > 1, \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\therefore x > -3$ and $x \neq -2$

$\therefore D_r = (-3, -2) \cup (-2, -1) \cup (1, \infty)$.

20 Domain of definition of the function $f(x) = \sqrt{2\sin^{-1}(2x) + \frac{\pi}{3}}$, for real value $x,$ is

(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

Sol. (a)

$-\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \frac{\pi}{2} \Rightarrow -\frac{1}{2} \leq 2x \leq 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$.

21 The range of $f(x) = \cos x - \sin x,$ is

(a) $(-1, 1)$ (b) $[-1, 1)$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

Sol. (d)

Let, $f(x) = \cos x - \sin x \Rightarrow f(x) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right) \Rightarrow f(x) = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$

Now since, $-1 \le \cos\left(x + \frac{\pi}{4}\right) \le 1 \Rightarrow -\sqrt{2} \le f(x) \le \sqrt{2} \Rightarrow f(x) \in [-\sqrt{2}, \sqrt{2}]$

Trick : $\because$ Maximum value of $\cos x - \sin x$ is $\sqrt{2}$ and minimum value of $\cos x - \sin x$ is $-\sqrt{2}$. Hence, range of $f(x) = [-\sqrt{2}, \sqrt{2}]$.

22 The range of $\frac{1+x^2}{x^2}$ is

(a) $(0, 1)$ (b) $(1, \infty)$ (c) [0, 1] (d) $[1, \infty)$

Sol. (b)

Let $y = \frac{1+x^2}{x^2}$ $\Rightarrow x^2 y = 1 + x^2 \Rightarrow x^2(y-1) = 1 \Rightarrow x^2 = \frac{1}{y-1}$

Now since, $x^2 > 0 \Rightarrow \frac{1}{y-1} > 0 \Rightarrow (y-1) > 0 \Rightarrow y > 1 \Rightarrow y \in (1, \infty)$

Trick : $y = \frac{1+x^2}{x^2} = 1 + \frac{1}{x^2}$. Now since, $\frac{1}{x^2}$ is always > 0 $\Rightarrow y > 1 \Rightarrow y \in (1, \infty)$.

23 For real values of *x*, range of the function $y = \frac{1}{2 - \sin 3x}$ is

(a) $\frac{1}{3} \le y \le 1$ (b) $-\frac{1}{3} \le y < 1$ (c) $-\frac{1}{3} > y > -1$ (d) $\frac{1}{3} > y > 1$

Sol. (a)

$\because y = \frac{1}{2 - \sin 3x}$, $\therefore 2 - \sin 3x = \frac{1}{y} \Rightarrow \sin 3x = 2 - \frac{1}{y}$

Now since,

$-1 \le \sin 3x \le 1 \Rightarrow -1 \le 2 - \frac{1}{y} \le 1 \Rightarrow -3 \le -\frac{1}{y} \le -1 \Rightarrow 1 \le \frac{1}{y} \le 3 \Rightarrow \frac{1}{3} \le y \le 1$.

24 If $f(x) = a\cos(bx + c) + d$, then range of $f(x)$ is

(a) $[d + a, d + 2a]$ (b) $[a - d, a + d]$ (c) $[d + a, a - d]$ (d) $[d - a, d + a]$

Sol. (d)

$f(x) = a\cos(bx + c) + d$ ….. (i)

For minimum $\cos(bx + c) = -1$

from (i), $f(x) = -a + d = (d - a)$,

for maximum $\cos(bx + c) = 1$

from (i), $f(x) = a + d = (d + a)$

$\therefore$ Range of $f(x) = [d - a, d + a]$.

25 The range of the function $f(x) = \frac{x+2}{|x+2|}$ is

(a) {0, 1} (b) {– 1, 1} (c) *R* (d) $R - \{-2\}$

Sol. (b)

$f(x) = \frac{x+2}{|x+2|} = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$

$\therefore$ Range of $f(x)$ is $\{-1, 1\}$.

26 The range of $f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right),\ -\infty < x < \infty$ is

(a) $[1,\sqrt{2}]$ (b) $[1,\infty)$ (c) $[-\sqrt{2},-1]\cup[1,\sqrt{2}]$ (d) $(-\infty,-1]\cup[1,\infty)$

Sol. (a)

$f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right)$

We know that, $0\le\cos^2 x\le 1$ at $\cos x=0,\quad f(x)=1$ and at $\cos x=1,\quad f(x)=\sqrt{2}$

$\therefore\ 1\le x\le\sqrt{2}\ \Rightarrow\ x\in[1,\ \sqrt{2}]$.

27 Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1};x\in R$ is

(a) $(1,\infty)$ (b) $(1,11/7)$ (c) $(1,7/3]$ (d) $(1,7/5]$

Sol. (c)

$f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$ $\Rightarrow$ Range $=(1,7/3]$.

28 Function $f:N\to N, f(x)=2x+3$ is

(a) One-one onto (b) One-one into (c) Many-one onto (d) Many –one into

Sol. (b)

f is one-one because $f(x_1)=f(x_2)\ \Rightarrow\ 2x_1+3=2x_2+3\ \Rightarrow x_1=x_2$

Further $f^{-1}(x)=\frac{x-3}{2}\notin N$ (domain) when x = 1, 2, 3 etc.

$\therefore$ *f* is into which shows that *f* is one-one into.

29 The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

(a) One-one but not onto (b) Onto but not one-one

(c) Both one-one and onto (d) Neither one-one nor onto

Sol. (b)

We have $f(x)=(x-1)(x-2)(x-3)\ \Rightarrow\ f(1)=f(2)=f(3)=0\ \Rightarrow\ f(x)$ is not one-one

For each $y\in R,$ there exists $x\in R$ such that $f(x)=y$. Therefore *f* is onto.

Hence, $f:R\to R$ is onto but not one-one.

30 Find number of surjection from *A* to *B* where $A=\{1,2,3,4\},\ B=\{a,b\}$

(a) 13 (b) 14 (c) 15 (d) 16

Sol. (b)

Number of surjection from *A* to *B* $=\sum_{r=1}^{2}(-1)^{2-r}\ {}^2C_r(r)^4$

$=(-1)^{2-1}\ {}^2C_1(1)^4+(-1)^{2-2}\ {}^2C_2(2)^4\ =-2+16\ =14$

Therefore, number of surjection from *A* to *B* = 14.

Trick : Total number of functions from A to *B* is 2^4 of which two function $f(x)=a$ for all $x\in A$ and $g(x)=b$ for all $x\in A$ are not surjective. Thus, total number of surjection from *A* to *B* $=2^4-2=14$.

31 If $A = \{a, b, c\}$, then total number of one-one onto functions which can be defined from *A* to *A* is

(a) 3 (b) 4 (c) 9 (d) 6

Sol. (d)

Total number of one-one onto functions $= 3!$

32 If $f : R \to R$, then $f(x) = |x|$ is

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto (d) None of these

Sol. (d)

$f(-1) = f(1) = 1$ $\therefore$ function is many-one function.

Obviously, *f* is not onto so *f* is neither one-one nor onto.

33 Let $f : R \to R$ be a function defined by $f(x) = \frac{x-m}{x-n}$, where $m \neq n$. Then

(a) *f* is one-one onto (b) *f* is one-one into

(c) *f* is many one onto (d) *f* is many one into

Sol. (b)

For any $x, y \in R$, we have

$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$

$\therefore f$ is one-one

Let $\alpha \in R$ such that $f(x) = \alpha \Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m-n\alpha}{1-\alpha}$

Clearly $x \notin R$ for $\alpha = 1$. So, *f* is not onto.

34 The function $f : R \to R$ defined by $f(x) = e^x$ is

(a) Onto (b) Many-one

(c) One-one and into (d) Many one and onto

Sol. (c)

Function $f : R \to R$ is defined by $f(x) = e^x$. Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ or $e^{x_1} = e^{x_2}$ $x_1 = x_2$. Therefore *f* is one-one. Let $f(x) = e^x = y$. Taking log on both sides, we get $x = \log y$. We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number. Therefore function *f* is into.

35 A function *f* from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$, is

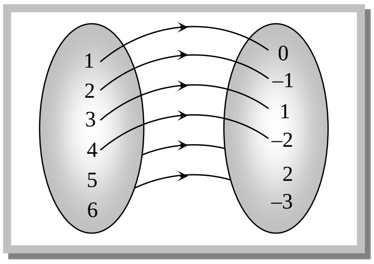
(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto both (d) Neither one-one nor onto

Sol. (c)

$f : N \to I$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

36 Which of the following is an even function

(a) $x\left(\frac{a^x-1}{a^x+1}\right)$ (b) $\tan x$ (c) $\frac{a^x-a^{-x}}{2}$ (d) $\frac{a^x+1}{a^x-1}$

Sol. (a)

We have : $f(x)=x\left(\frac{a^x-1}{a^x+1}\right)$

$$f(-x)=-x\left(\frac{a^{-x}-1}{a^{-x}+1}\right)=-x\left(\frac{\frac{1}{a^x}-1}{\frac{1}{a^x}+1}\right)=-x\left(\frac{1-a^x}{1+a^x}\right)=x\left(\frac{a^x-1}{a^x+1}\right)=f(x)$$

So, $f(x)$ is an even function.

37 Let $f(x)=\sqrt{x^4+15},$ then the graph of the function $y=f(x)$ is symmetrical about

(a) The x-axis (b) The y-axis (c) The origin (d) The line $x=y$

Sol. (b)

$$f(x)=\sqrt{x^4+15} \Rightarrow f(-x)=\sqrt{(-x)^4+15}=\sqrt{x^4+15}=f(x)$$

$\Rightarrow f(-x)=f(x) \Rightarrow f(x)$ is an even function $\Rightarrow f(x)$ is symmetric about y-axis.

38 The function $f(x)=\log(x+\sqrt{x^2+1})$ is

(a) An even function (b) An odd function

(c) Periodic function (d) None of these

Sol. (b)

$f(x)=\log(x+\sqrt{x^2+1})$ and $f(-x)=-\log(x+\sqrt{x^2+1})=-f(x)$, so $f(x)$ is an odd function.

39 Which of the following is an even function

(a) $f(x)=\frac{a^x+1}{a^x-1}$ (b) $f(x)=x\left(\frac{a^x-1}{a^x+1}\right)$ (c) $f(x)=\frac{a^x-a^{-x}}{a^x+a^{-x}}$ (d) $f(x)=\sin x$

Sol. (b)

In option (a), $f(-x)=\frac{a^{-x}+1}{a^{-x}-1}=\frac{1+a^x}{1-a^x}=-\frac{a^x+1}{a^x-1}=-f(x)$ So, It is an odd function.

In option (b), $f(-x)=(-x)\frac{a^{-x}-1}{a^{-x}+1}=-x\frac{(1-a^x)}{1+a^x}=x\frac{(a^x-1)}{(a^x+1)}=f(x)$ So, It is an even function.

In option (c), $f(-x)=\dfrac{a^{-x}-a^{x}}{a^{-x}+a^{x}}=-f(x)$ So, It is an odd function.

In option (d), $f(-x)=\sin(-x)=-\sin x=-f(x)$ So, It is an odd function.

40 The function $f(x)=\sin\left(\log(x+\sqrt{x^2+1})\right)$ is

(a) Even function (b) Odd function

(c) Neither even nor odd (d) Periodic function

Sol. (b)

$f(x)=\sin\left(\log(x+\sqrt{1+x^2})\right)$

$\Rightarrow\quad f(-x)=\sin[\log(-x+\sqrt{1+x^2})]\quad\Rightarrow\quad f(-x)=\sin\log\left((\sqrt{1+x^2}-x)\dfrac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right)$

$\Rightarrow\quad f(-x)=\sin\log\left[\dfrac{1}{(x+\sqrt{1+x^2})}\right]\quad\Rightarrow\quad f(-x)=\sin\left[\log(x+\sqrt{1+x^2})^{-1}\right]$

$\Rightarrow\quad f(-x)=\sin\left[-\log(x+\sqrt{1+x^2})\right]\quad\Rightarrow\quad f(-x)=-\sin\left[\log(x+\sqrt{1+x^2})\right]\Rightarrow\ f(-x)=-f(x)$

$\therefore$ $f(x)$ is odd function.

41 The period of the function $f(x)=2\cos\dfrac{1}{3}(x-\pi)$ is

(a) 6π (b) 4π (c) 2π (d) π

Sol. (a)

$f(x)=2\cos\dfrac{1}{3}(x-\pi)\ =2\cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$

Now, since $\cos x$ has period $2\pi\Rightarrow$ $\cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$ has period $\dfrac{2\pi}{\frac{1}{3}}=6\pi$

$\Rightarrow$ $2\cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$ has period $=6\pi$.

42 The function $f(x)=\sin\dfrac{\pi x}{2}+2\cos\dfrac{\pi x}{3}-\tan\dfrac{\pi x}{4}$ is periodic with period

(a) 6 (b) 3 (c) 4 (d) 12

Sol. (d)

$\because$ $\sin x$ has period $=2\pi\Rightarrow$ $\sin\dfrac{\pi x}{2}$ has period $=\dfrac{2\pi}{\frac{\pi}{2}}=4$

$\because$ $\cos x$ has period $=2\pi\Rightarrow$ $\cos\dfrac{\pi x}{3}$ has period $=\dfrac{2\pi}{\frac{\pi}{3}}=6\Rightarrow 2\cos\dfrac{\pi x}{3}$ has period = 6

$\because$ $\tan x$ has period $=\pi\ \Rightarrow$ $\tan\dfrac{\pi x}{4}$ has period $=\dfrac{\pi}{\frac{\pi}{4}}=4$.

L.C.M. of 4, 6 and 4 =12, period of $f(x)$ = 12.

43 The period of $|\sin 2x|$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

Sol. (b)

Here $|\sin 2x| = \sqrt{\sin^2 2x} = \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of $\cos 4x$ is $\frac{\pi}{2}$. Hence, period of $|\sin 2x|$ will be $\frac{\pi}{2}$

Trick : $\because$ $\sin x$ has period $= 2\pi \Rightarrow$ $\sin 2x$ has period $= \frac{2\pi}{2} = \pi$. Now, if $f(x)$ has period p then $|f(x)|$ has period $\frac{p}{2} \Rightarrow |\sin 2x|$ has period $= \frac{\pi}{2}$.

44 If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals

(a) 0 (b) 2 (c) 4 (d) – 4

Sol. (a)

Given, $f(x)$ is an odd periodic function. We can take $\sin x$, which is odd and periodic.

Now since, $\sin x$ has period = 2 and $f(x)$ has period = 2.

So, $f(x) = \sin(\pi x) \Rightarrow f(4) = \sin(4\pi) = 0$.

45 The period of the function $f(x) = \sin^2 x$ is

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) None of these

Sol. (b)

$\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow$ Period $= \frac{2\pi}{2} = \pi.$

46 The period of $f(x) = x - [x]$, if it is periodic, is

(a) $f(x)$ is not periodic (b) $\frac{1}{2}$ (c) 1 (d) 2

Sol. (c)

Let $f(x)$ be periodic with period *T*. Then,

$f(x+T) = f(x)$ for all $x \in R$ $\Rightarrow$ $x + T - [x+T] = x - [x]$ for all $x \in R$ $\Rightarrow$ $x + T - x = [x+T] - [x]$

$\Rightarrow$ $[x+T] - [x] = T$ for all $x \in R$ $\Rightarrow$ $T = 1, 2, 3, 4, ...$

The smallest value of *T* satisfying,

$f(x+T) = f(x)$ for all $x \in R$ is 1.

Hence $f(x) = x - [x]$ has period 1.

47 The period of $f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$, $n \in Z$, $n > 2$ is

(a) $2\pi n(n-1)$ (b) $4n(n-1)$ (c) $2n(n-1)$ (d) None of these

Sol. (c)

$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$

Period of $\sin\left(\frac{\pi x}{n-1}\right) = \frac{2\pi}{\left(\frac{\pi}{n-1}\right)} = 2(n-1)$ and period of $\cos\left(\frac{\pi x}{n}\right) = \frac{2\pi}{\left(\frac{\pi}{n}\right)} = 2n$

Hence period of $f(x)$ is LCM of $2n$ and $2(n-1) \Rightarrow 2n(n-1)$.

48 If a, b be two fixed positive integers such that $f(a+x) = b + [b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3]^{\frac{1}{3}}$ for all real x, then $f(x)$ is a periodic function with period

(a) a (b) $2a$ (c) b (d) $2b$

Sol. (b)

$f(a+x) = b + (1 + \{b - f(x)\}^3)^{1/3} \Rightarrow f(a+x) - b = \{1 - \{f(x) - b\}^3\}^{1/3}$

$\Rightarrow \phi(a+x) = \{1 - \{\phi(x)\}^3\}^{1/3}$ $[\phi(x) = f(x) - b]$ $\Rightarrow \phi(x+2a) = \{1 - \{\phi(x+a)\}^3\}^{1/3} = \phi(x)$

$\Rightarrow f(x+2a) - b = f(x) - b \Rightarrow f(x+2a) = f(x)$

$\therefore$ $f(x)$ is periodic with period $2a$.

49 If $f: R \to R, f(x) = 2x - 1$ and $g: R \to R$, $g(x) = x^2$ then $(gof)(x)$ equals

(a) $2x^2 - 1$ (b) $(2x-1)^2$ (c) $4x^2 - 2x + 1$ (d) $x^2 + 2x - 1$

Sol. (b)

$gof(x) = g\{f(x)\} = g(2x-1) = (2x-1)^2$.

50 If $f: R \to R, f(x) = (x+1)^2$ and $g: R \to R, g(x) = x^2 + 1$, then $(fog)(-3)$ is equal to

(a) 121 (b) 144 (c) 112 (d) 11

Sol. (a)

$fog(x) = f\{g(x)\} = f(x^2+1) = (x^2+1+1)^2 = (x^2+2)^2 \Rightarrow fog(-3) = (9+2)^2 = 121$.

51 Which of the following function is invertible

(a) $f(x) = 2^x$ (b) $f(x) = x^3 - x$ (c) $f(x) = x^2$ (d) None of these

Sol. (a)

A function is invertible if it is one-one and onto.

52 If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to

(a) $2x - 3$ (b) $2x + 3$ (c) $2x^2 + 3x + 1$ (d) $2x^2 - 3x - 1$

Sol. (a)

$g(x) = x^2 + x - 2 \Rightarrow (gof)(x) = g[f(x)] = [f(x)]^2 + f(x) - 2$

Given, $\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$ $\therefore$ $\frac{1}{2}[f(x)]^2 + \frac{1}{2}f(x) - 1 = 2x^2 - 5x + 2$

$\Rightarrow [f(x)]^2 + f(x) = 4x^2 - 10x + 6 \Rightarrow f(x)[f(x)+1] = (2x-3)[(2x-3)+1] \Rightarrow f(x) = 2x - 3$.

53 If $f(y) = \frac{y}{\sqrt{1-y^2}}, g(y) = \frac{y}{\sqrt{1+y^2}}$, then $(fog)(y)$ is equal to

(a) $\frac{y}{\sqrt{1-y^2}}$ (b) $\frac{y}{\sqrt{1+y^2}}$ (c) y (d) $\frac{1-y^2}{\sqrt{1+y^2}}$

Sol. (c)

$$f[g(y)] = \frac{y/\sqrt{1+y^2}}{\sqrt{1 - \left(\frac{y}{\sqrt{1+y^2}}\right)^2}} = \frac{y}{\sqrt{1+y^2}} \times \frac{\sqrt{1+y^2}}{\sqrt{1+y^2-y^2}} = y$$

54 If $f(x)=\dfrac{2x-3}{x-2}$, then $[f\{f(x)\}]$ equals

(a) x (b) $-x$ (c) $\dfrac{x}{2}$ (d) $-\dfrac{1}{x}$

Sol. (a)

$$f[f(x)]=\frac{2\left(\frac{2x-3}{x-2}\right)-3}{\left(\frac{2x-3}{x-2}\right)-2}=x$$

55 Suppose that $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$, then $f(x)$ is

(a) $1+2x^2$ (b) $2+x^2$ (c) $1+x$ (d) $2+x$

Sol. (b)

$g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$ ….. (i)

$\Rightarrow$ $f(1+\sqrt{x})=3+2\sqrt{x}+x$

Put $1+\sqrt{x}=y \Rightarrow x=(y-1)^2$

then, $f(y)=3+2(y-1)+(y-1)^2=2+y^2$

therefore, $f(x)=2+x^2$.

56 Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$, then for all x, $f(g(x))$ is equal to

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

Sol. (b)

Here $g(x)=1+n-n=1, x=n\in Z$

$1+n+k-n=1+k$, $x=n+k$ (where $n\in Z,\ 0<k<1$)

Now $f(g(x))=\begin{cases}-1, & g(x)<0\\ 0, & g(x)=0\\ 1, & g(x)>0\end{cases}$

Clearly, $g(x)>0$ for all x. So, $f(g(x))=1$ for all x.

57 If $f(x)=\dfrac{2x+1}{3x-2}$, then $(fof)(2)$ is equal to

(a) 1 (b) 3 (c) 4 (d) 2

Sol. (d)

Here $f(2)=\dfrac{5}{4}$

Hence $(fof)(2)=f(f(2))=f\left(\dfrac{5}{4}\right)=\dfrac{2\times\frac{5}{4}+1}{3\times\frac{5}{4}-2}=2$.

58 If $f:R\to R$ and $g:R\to R$ are given by $f(x)=|x|$ and $g(x)=[x]$ for each $x\in R$, then $\{x\in R: g(f(x))\le f(g(x))\}=$

(a) $Z\cup(-\infty,0)$ (b) $(-\infty,0)$ (c) Z (d) R

Sol. (d)

$g(f(x))\le f(g(x)) \Rightarrow g(|x|)\le f[x] \Rightarrow [|x|]\le|[x]|$. This is true for $x\in R$.

59 If $f:[1,\infty)\to[1,\infty)$ is defined as $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ is equal to

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (d) Not defined

Sol. (b)

Given $f(x) = 2^{x(x-1)} \Rightarrow x(x-1) = \log_2 f(x)$

$\Rightarrow x^2 - x - \log_2 f(x) = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+4\log_2 f(x)}}{2}$

Only $x = \frac{1+\sqrt{1+4\log_2 f(x)}}{2}$ lies in the domain

$\therefore \quad f^{-1}(x) = \frac{1}{2}[1+\sqrt{1+4\log_2 x}]$

60 If the function $f : R \to R$ be such that $f(x) = x - [x]$, where $[y]$ denotes the greatest integer less than or equal to *y*, then $f^{-1}(x)$ is

(a) $\frac{1}{x-[x]}$ (b) $[x]-x$ (c) Not defined (d) None of these

Sol. (c)

$f(x) = x - [x]$ Since, for $x = 0 \Rightarrow f(x) = 0$

For $x = 1 \Rightarrow f(x) = 0$.

For every integer value of x, $f(x) = 0$

$\Rightarrow$ $f(x)$ is not one-one

$\Rightarrow$ So $f^{-1}(x)$ is not defined.

FUNCTIONS

EXERCISE – 1(B)

Q.1 (D)

Domain of $\sqrt{\sec^{-1}\left(\frac{2-|x|}{4}\right)}$

$\sec^{-1}$ x is always positive

So, $\frac{2-|x|}{4} \geq 1$ or $\frac{2-|x|}{4} \leq -1$

$\Rightarrow 2-|x| \geq 4$ or $2-|x| \leq -4$

$\Rightarrow |x| \leq -2$ or $|x| \geq 6$

$\Rightarrow x \in [-\infty, -6] \cup [6, \infty]$

Q.2 (D)

$$f(x) = \log\left(\frac{x^2-5x+6}{x^2+x+1}\right) + \sqrt{\frac{1}{[x^2-1]}}$$

$\frac{x^2-5x+6}{x^2+x+1} > 0$ and $[x^2-1] > 0$

$\Rightarrow (x-2)(x-3) > 0$ and $x^2-1 \geq 1$

$\Rightarrow x \in (-\infty, 2) \cup (3, \infty)$ and $x \in [-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty]$

$\Rightarrow x \in [-\infty, -\sqrt{2}] \cup [\sqrt{2}, 2] \cup (3, \infty)$

Q.3 (A)

$$f(x) = \sin^{-1}\left(\frac{1-|x|}{3}\right) + \cos^{-1}\left(\frac{|x|-3}{5}\right)$$

$-1 \leq \frac{1-|x|}{3} \leq 1$ and $-1 \leq \frac{|x|-3}{5} \leq 5$

$\Rightarrow -3 \le 1 - |x| \le 3$ and $\Rightarrow -5 \le |x| - 3 \le 5$

$\Rightarrow 4 \ge |x| \ge -2$ and $\Rightarrow -2 \le |x| \le 8$

So, $x \in [-4, 4]$ and $x \in [-8, 8]$

Q.4 (B)

$f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$

$2\{x\}^2 - 3\{x\} + 1 \ge 0$

$\Rightarrow (2\{x\} - 1)(\{x\} - 1) \ge 0$

$\Rightarrow \{x\} \in \left(-\infty , \frac{1}{2}\right) \cup (1, \infty)$

But $\{x\}$ has range (0 , 1) only so, $\{x\} \in \left[0, \frac{1}{2}\right]$ and $x = [x] + \{x\}$

in (– 1 , 1), $x \in \left[-1 + 0, -1 + \frac{1}{2}\right] \cup \left[0 + 0, 0 + \frac{1}{2}\right] \cup \{1 + 0\}$

$\Rightarrow x \in \left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$

Q.5 (D)

$f(x) = \sqrt{\cos(\sin x)} + \sqrt{\log_x \{x\}}$

$-1 \le \sin x \le 1$ and $0 \le \{x\} \le 1$

$1 \ge \cos(\sin x) \ge \cos 1$ So, $x \ne n$, $n \in I$

Hence, $x \in R$ $\log_x \{n\} \ge 0$

$\Rightarrow x < 1$, $x > 0$, $x \ne 1$

So, $x \in (0, 1)$

Q.6 (D)

$\sqrt{[x] - 1 + x^2}$

$\Rightarrow x^2 \geq 1-[x]$

Hence for $x \geq 1$ & $x \leq -3$ …………(1)

For, $x \in (-1,1)$, $x^2 \in (0,1)$

And $1-[x] \geq 0$. So, not satisfying inequality.

For, $x \in (-2,-1), [x] = -2$ …………(2)

$\Rightarrow x^2 \geq 3$

So, $x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Hence, $x \in [-\infty, -\sqrt{3}]$ …………..(3)

From (1) , (2) , (3)

$x \in (-\infty, -\sqrt{3}) \cup (1, \infty)$

Q.7 (D)

$f(x) = \sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$ [.] $\rightarrow$ GIF

$-1 \leq \left[\log_2\left(\frac{x^2}{2}\right)\right] \leq 1$

$\Rightarrow -1 \leq \log_2\left(\frac{x^2}{2}\right) < 2$

$\Rightarrow \frac{1}{2} \leq \frac{x^2}{2} < 4$

$\Rightarrow 1 \leq x^2 < 8$

So, $x \in \left(-2\sqrt{2}, -1\right) \cup \left(1, 2\sqrt{2}\right)$

Q.8 (C)

$2^x + 2^{f(x)} = 2$

$\Rightarrow 2^{f(x)} = (2-2^x)$

$\Rightarrow f(x) = \log_2(2-2^x)$

So, $2-2^x > 0$

$\Rightarrow 2^x < 2$

$\Rightarrow \ x < 1$

Solution : $(-\infty, 1)$

Q.9 (B)

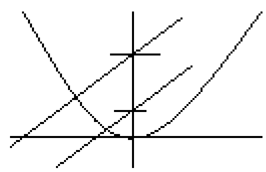
$f(x)$ has domain $[-1, 2]$

For $f([x]-x^2+4)$ to have real value.

$-1 \le [x]-x^2+4 \le 2$

$5 \ge x^2-[x] \ge 2$

$5+[x] \ge x^2 \ge 2+[x]$



$x^2 \ge 2+[x]$ is always true for $x \ge \sqrt{3}$ and $x \le -1$

So, $x \in (-\infty, -1) \cup (\sqrt{3}, \infty)$

$x^2 \le 5+[x]$ is always true for $x \le \sqrt{7}$ and $x \ge -\sqrt{3}$

$x \in (-\sqrt{3}, 7)$

Solution : $x \in [-\sqrt{3}, -1] \cup [\sqrt{3}, \sqrt{7}]$

Q.10 (C)

$f(x) = \dfrac{1}{1-2\cos x}$

$-1 \le \cos x \le 1$

$\Rightarrow -2 \le 2\cos x \le 2$

$\Rightarrow 3 \ge 1-2\cos x \ge -1$

$$\Rightarrow \frac{1}{1-2\cos x} \le -1 \quad \frac{1}{1-2\cos x} \ge \frac{1}{3}$$

So, $x \in (-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$

Q.11 (D)

$$f(x) = \sin^{-1} x + \tan^{-1} x + \cos^{-1} x$$

$$= (\sin^{-1} x + \cos^{-1} x) + \tan^{-1} x$$

$$= \frac{\pi}{2} + \tan^{-1} x$$

But $x \in [-1, 1]$

So, $\frac{-\pi}{4} \le \tan^{-1} x \le \frac{\pi}{4}$

$$\frac{\pi}{4} \le \frac{\pi}{2} + \tan^{-1} x \le \frac{3\pi}{4}$$

Hence $x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Q.12. (B)

$$f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$$

$$= \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left(\left[x^2 + \frac{1}{2}\right] - 1\right)$$

Now, $-1 \le \left[x^2 + \frac{1}{2}\right] \le 1$

And $1 \le \left[x^2 + \frac{1}{2}\right] - 1 \le 1$

So, $0 \le \left[x^2 + \frac{1}{2}\right] \le 1$

Hence, $\left[x^2+\frac{1}{2}\right]=\{0\ ,\ 1\}$

Hence, $f(x)\in\{\pi\}$

Q.13 (C)

$$f(x)=\sin^{-1}\left(\sqrt{x^2+x+1}\right)$$

$$x^2+x+1\geq\frac{3}{4}$$

$$\Rightarrow\sqrt{x^2+x+1}\ \geq\frac{\sqrt{3}}{2}$$

For $\sin^{-1}\sqrt{x^2+x+1}$ to be defined

$$\frac{\sqrt{3}}{2}\leq\sqrt{x^2+x+1}\leq 1$$

$$\Rightarrow\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\leq\sin^{-1}\sqrt{x^2+x+1}\leq\sin^{-1}1$$

$$\Rightarrow\frac{\pi}{3}\leq f(x)\leq\frac{\pi}{2}$$

So, $\left[\frac{\pi}{3}\ ,\ \frac{\pi}{2}\right]$

Q.14 (C)

$$f(x)=\cos^{-1}\left(\frac{x^2}{\sqrt{1+x^2}}\right)$$

Range of $\frac{x^2}{\sqrt{1+x^2}}$: $[0,1]$

hence range of f (x) : $\left[0,\frac{\pi}{2}\right]$

Q.15 (D)

$f(x)=\sqrt{\ln(\cos(\sin x))}$

$-1 \le \sin x \le 1$

$\cos 1 \le \cos\ (\sin x) \le 1$

or $\ln\cos(\sin x) \le \ln 1$

For square root to be defined, $\ln\cos(\sin x) \ge 0$

hence $\ln\cos(\sin x) = 0$.

Range of f (x) : {0}

Q.16 (D)

$f(x) = \dfrac{x-1}{x^2-2x+3}$

Discriminant of x^2-2x+3 is negative so x^2-2x+3 is always positive

$$f(x) = \frac{x-1}{(x-1)^2+2} = \frac{1}{(x-1)+\frac{2}{x-1}}$$

So, for x > 1, $(x-1)+\left(\dfrac{2}{x-1}\right) \ge 2\sqrt{2}$

So, $\dfrac{1}{(x-1)+\frac{2}{x-1}} \le \dfrac{1}{2\sqrt{2}}$

Similarly, $x<1$, $\dfrac{1}{(x-1)+\frac{2}{x-1}} \ge \dfrac{1}{-2\sqrt{2}}$

So, $f(x) \in \left[\dfrac{1}{-2\sqrt{2}}, \dfrac{1}{2\sqrt{2}}\right]$

Q.17 (D)

$$f(x) = \cos^{-1}\left(\frac{1+x^2}{2x}\right) + \sqrt{2-x^2}$$

$$\frac{1+x^2}{2x} \geq 1 \quad \text{or} \quad \frac{1+x^2}{2x} \leq -1$$

So, x = 1 and $x = -1$ are the 2 points in domain.

So, $f(1) = 0 + 1$

$f(-1) = \pi + 1$

So, Range $= \{1 , 1 + \pi\}$

Q.18 (D)

$$f(x) = \frac{\tan(\pi[x^2 - x])}{1 + \sin(\cos x)}$$

Domain is R

$\because$ $[\cdot]$ is an integral value so, $\pi[x^2 - x]$ is an integral multiple of π .

hence $\tan(\pi[\cdot]) = 0 \quad \forall \ x \in R$

Range $= \{0\}$

Q.19 (D)

$$f(x) = \frac{e^x}{[x+1]} , \ x \geq 0$$

$\because$ e^x is an increasing continuous function and [x + 1] ≥ 1

Hence, Range will be $[1 , \infty]$

Q.20 (C)

$$f(x) = \frac{1}{1-x} , \ x \neq 1 \Rightarrow f(f(x)) = \frac{1}{1 - f(x)}$$

$$= \frac{1}{1-\frac{1}{1-x}}$$

$$= \frac{x-1}{x}, \ x \neq 0, \ x \neq 1$$

Further $f(f(f(x))) = \frac{f(x)-1}{f(x)}$

$$= \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}}, \ x \neq 0, \ x \neq 1$$

$$= x, \quad x \neq 0, \ x \neq 1$$

Q.21 (A)

$f(g(x)) = |\sin x|$

$g(f(x)) = \sin^2 \sqrt{x}$

So, $f(x) = \sqrt{x}$ & $g(x) = \sin^2 x$

Q.22 (A)

Given $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$

So, $f(f(x)) = \begin{cases} f(x) & \text{if } f(x) \text{ is rational} \\ 1-f(x) & \text{if } f(x) \text{ is irrational} \end{cases}$

$\Rightarrow f(f(x)) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-(1-x) & \text{if } x \text{ is irrational} \end{cases}$

Hence *f* (*f* (x)) = x.

Q.23 (D)

$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$

$$f(f(x)) = \begin{cases} (f(x))^2 & \text{if} \quad f(x) \le 0 \\ f(x) & \text{if} \quad f(x) > 0 \end{cases}$$

Now f (x) can't be less than 0 hence

f (f (x)) = f (x) for all values of x.

Q.24 (A)

$f(x) = \sin^{-1}(\sin x) + e^{\tan x}$

$\sin^{-1}(\sin x)$ has a period of 2π

and $e^{\tan x}$ has a period of π

So, period of f (x) = LCM{π, 2π} i.e. 2π

Q.25 (D)

(A) $\dfrac{2^x}{2^{[x]}} = 2^{x-[x]} = 2^{\{x\}}$.

Now $2^{\{x+T\}} = 2^{\{x\}} \Rightarrow \{x+T\} = \{x\} \Rightarrow T = 1$.

(B) $\sin^{-1}\{x+T\} = \sin^{-1}\{x\} \Rightarrow \{x+T\} = \{x\} \Rightarrow T = 1$

(C) $\sin^{-1}\left(\sqrt{\sin(x+T)}\right) = \sin^{-1}\left(\sqrt{\sin x}\right) \Rightarrow \sin(x+T) = \sin x \Rightarrow T = 2\pi$

(D) $\sin^{-1}\left(\cos(x+T)^2\right) = \sin^{-1}\left(\cos(x)^2\right) \Rightarrow \cos(x+T)^2 = \cos(x)^2$

Function is non periodic.

Q.26 (C)

$f(x) = \dfrac{\cos(\sin(nx))}{\tan\left(\dfrac{x}{n}\right)}$, $n \in \mathrm{N}$ has period 6π

Numerator has a period of $\left(\dfrac{\pi}{n}\right)$

Denominator has period of $n\pi$, where $n \in \mathrm{I}$

So, period of f (x) = LCM $\left\{\frac{\pi}{n}, n\pi\right\} = n\pi = 6\pi$

$\Rightarrow n = 6$

Q.27 (A)

$f(x) = \sin 3\pi\{x\} + \tan \pi[x]$

As $[x]$ is an integer hence $\tan \pi[x]$ is always equal to 0.

$f(x+\mathrm{T}) = f(x) \Rightarrow \sin\left(3\pi\{x+\mathrm{T}\}\right) = \sin\left(3\pi\{x\}\right) \Rightarrow 3\pi\{x+\mathrm{T}\} = n\pi + (-1)^n\left(3\pi\{x\}\right)$

or $3\{x+\mathrm{T}\} = n + (-1)^n\left(3\{x\}\right)$

(i) $n = 2m \Rightarrow \{x+\mathrm{T}\} - \{x\} = \frac{2m}{3}$

As $0 \le \{x+\mathrm{T}\} - \{x\} < 1$ hence $m = 0$

$\Rightarrow \{x+\mathrm{T}\} = \{x\} \Rightarrow \mathrm{T} = 1$

(ii) $n = 2m+1 \Rightarrow \{x+\mathrm{T}\} + \{x\} = \frac{2m+1}{3}$

As $0 \le \{x+\mathrm{T}\} + \{x\} < 2$ hence $m = 1$

$\Rightarrow \{x+\mathrm{T}\} + \{x\} = 1 \Rightarrow \mathrm{T} = 1$

Therefore period is 1.

Q.28 (B)

$f(x) = \sin(\cos x) - x + \tan(\sin x)\ \forall\ x \in (0, \infty)$.

If f (x) is defined in (0, a), then odd extension of $f(x) = -f(-x)$ in (–a, 0).

So, odd extension of $f(x) = \sin(\cos x) + x - \tan(\sin x)\quad \forall\ x \in (-\infty, 0)$.

Q.29 (C)

(A) $g(x) - g(-x) = f(x)$

$f(-x) = g(-x) - (g(x))$

$= -(g(x) - g(-x))$

Therefore it's an Odd function.

(B) Similar as (A) odd function.

(C) $f(x)=\log\left(\frac{x^4+x^2+1}{x^2+x+1}\right)$

$f(-x)=\log\left(\frac{x^4+x^2+1}{x^2-x+1}\right)$

$=-\log\left(\frac{x^2-x+1}{x^4+x^2+1}\right)\neq -f(x)$

So it's not an odd function

(D) $xg(x)\cdot g(-x)+\tan(\sin x)=f(x)$

$f(-x)=-xg(-x)\cdot g(x)+\tan(\sin x)$

$=-(xg(x)\cdot g(-x)+\tan(\sin x))$

$0\ =-f(x)$

It's an odd function.

Q.30 (B)

$f:[-4,4]\to\{\pi,0,\pi\}\to R$

$f(x)=\cot(\sin x)+\left[\frac{x^2}{|a|}\right]$ is an odd function

Then $f(-x)=-f(x)$

$\Rightarrow -\cot(\sin x)=\left[\frac{x^2}{|a|}\right]=-\cot\sin f\left[\frac{x^2}{|a|}\right]$

$\Rightarrow 2\left[\frac{x^2}{|a|}\right]=0$

$\Rightarrow |a|\ <\ x^2$

$\Rightarrow |a|>(x^2)_{\max}$

$\Rightarrow |a|>16$

$a \in (-\infty, -16) \cup (16, \infty)$

Q.31 (B)

$f : (2, \infty) \to (-\infty, 4)$

$f(x) = x(4 - x)$

$= x^2 - 4x$

$f'(x) = -2x + 4 = 0$

$\Rightarrow x = 2 \;,\; f(2) = 4$

Hence, function is bijective in $(2\;,\;\infty) \to (-\infty,\;4)$

$y = x(y - x)$

$\Rightarrow x^2 - 4x + y = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{46 - 4y}}{2}$$

$\Rightarrow x = 2 \pm \sqrt{4 - y}$

Hence, $f^{-1}(x) = 2 + \sqrt{4 - x}$

Q.32 (C)

$A = \{1\;,\;2, 3, 4\}$

$f : A \to A$

$f(2) \neq 2, f(4) \neq 4\;,\; f(1) = 1$

If $f(3) = 3$, then $f(2) = 4\;,\; f(4) = 2, f(1) = 1$

If $f(3) = 2$, then $f(2) = 4\;,\; f(4) = 3,\; f(1) = 1$

If $f(3) = 4$, then $f(2) = 3\;,\; f(4) = 2\;,\; f(1) = 1$

Q.33 (A)

$f:(-\infty,1) \rightarrow \left(\frac{1}{2}, \infty\right)$

$f(x)=2^{x(x-2)}$

$g(x)=x(x-2)=x^2-2x$

$\forall\, x \in (-\infty,1)\; g(x)$ is one – one

and for, $\forall\, x \in (-\infty,+1), g(x) \in (-1,\infty)$

Hence, $f(x) \in \left(\frac{1}{2}, \infty\right)$

Hence, $f(x)$ is invertible.

$y=2^{x(x-2)}$

$\Rightarrow x^2-2x=\log y$

$\Rightarrow x^2-2x-\log y=0$

So, $x=\dfrac{2\pm\sqrt{4+4\log y}}{2}$

Hence, $f^{-1}(x)=1-\sqrt{1+\log_2 x}$

Q.34 (C)

$f:R \rightarrow R$

$f(x)=ax+\cos x$ is invertible function

So, $f(x)$ should be injective

for $a \neq 0$, Range is R

So, $f(x)$ to be one – one

$f'(x) \geq 0 \Rightarrow a-\sin x \geq 0$

$\Rightarrow a \geq \sin \Rightarrow a \geq 1$

or, $f'(x) \le 0 \Rightarrow a - \sin x \le 0$

$$\Rightarrow a \le \sin x \Rightarrow a \le -1$$

So, $a \in (-\infty, -1) \cup (1, \infty)$

Q.35 (C)

$f(x) = \cot^{-1} \log_{\frac{1}{2}}(x^4 - 2x^2 + 3)$

$x^4 - 2x^2 + 3 = (x^2 - 1)^2 + 2$

Hence, $(x^2 - 1)^2 + 2 \ge 2$

$\Rightarrow g(x) = \log_{\frac{1}{2}}\left[(x^2 - 1) + 2\right] \le -1$

Hence, $-\infty < g(x) \le -1$

So, $\cot^{-1}(-1) \le \cot^{-1}(g(x)) < \cot^{-1}(-\infty)$

$\Rightarrow \frac{3\pi}{4} \le f(x) < \pi$

Q.36 (C)

$f(x) = \sin(x + 3 - [x + 3])$

$= \sin(\{x + 3\})$

$= \sin(\{x\}$

Hence, period of f (x) = 1.

Q.37 (B)

$f(x) = x^2 + bx + c$

if $f(2+t) = f(2=t) \Rightarrow f(x)$ is symmetric about x = 2

Hence, $f(x)$ is minimum at x = 2

Hence, $f(1) = f(3) > f(2)$

$$f(0) = f(4) > f(1) = f(3) > f(2)$$

Hence, $f(4) > f(1) > f(2)$

Q.38 (A)

$f(x+ay, x-ay) = axy$

Let, $x+ay = u$

$x-ay = w$

So, $x = \frac{u+w}{2}$, $y = \left(\frac{u-w}{2a}\right)$

Hence, $f(u,w) = a\cdot\left(\frac{u+w}{2}\right)\left(\frac{u-w}{2a}\right) = \frac{u^2-w^2}{4}$

So, $f(x,y) = \frac{x^2-y^2}{4}$

Q.39 (D)

$[x]\{x\} = 1 \Rightarrow \{x\} = \frac{1}{[x]}$

$0 \le \{x\} < 1$, hence , $[x] \ge 2$

So, for $[x] = \mathrm{I}$; $\mathrm{I} \ge 2$

$x = [x] + \frac{1}{[x]} = \mathrm{I} + \frac{1}{\mathrm{I}}$

So, solution $= \left\{ m + \frac{1}{m} \mid m \in N - \{1\} \right\}$

Q.40 (A)

$f(x) = 2\tan 3x + 5\sqrt{1-\cos 6x}$

$= 2\tan 3x + 5|\sin 3x|\sqrt{2}$

Period of $\tan 3x$ is $\frac{\pi}{3}$ and period of $|\sin 3x|$ is $\frac{\pi}{3}$.

So, period of f (x) = $\frac{\pi}{3}$.

Hence, $g(x)$ has a period $= \frac{\pi}{3}$

(A) $(\sec^2 3x + \cos ec^2 3x)\tan^2 3x$

$= 3 + \tan^4 3x$ has period $\frac{\pi}{3}$

(B) $2\cos 3x + 3\cos 3x = \sqrt{13}\cos(3x + \phi)$

period $= \frac{2\pi}{3}$

(C) $2\sqrt{1 - \cos^2 3x} + \cos ec 3x$

$= 2 \left| \sin \frac{3x}{2} \right| + \cos ec 3x$

period $= \frac{2\pi}{3}$

(D) g(x) = 3 cosec 3x + 2 tan 3x

Period of cosec 3x = $\frac{2\pi}{3}$ and period of tan 3x = $\frac{\pi}{3}$.

Hence period of g (x) = $\frac{2\pi}{3}$.

Exercise 1 (C)

Q.1 (B)

$\Rightarrow \log_{\frac{1}{2}}\left(x^2 - 5x + 7\right) > 0$

$\Rightarrow 0 < x^2 - 5x + 7 < 1$

$\Rightarrow x \in (2,3)$

Q.2 (B)

$\Rightarrow \log_3\left(x^2 - 6x + 11\right) < 1$

$\Rightarrow 0 < x^2 - 6x + 11 < 3$

$\Rightarrow x \in (2,4)$

Q.3 (D)

In this case base is variable. Thus we must take two separate cases:

(i) $|x| \in (0,1)$. In this case we have to ensure that $0 < x^2 + x + 1 \leq 1$

$\Rightarrow x \in [-1,0]$.

$\Rightarrow$ Common part of $|x| \in (0,1)$

$\Rightarrow$ And $x \in [-1,0]$ is $x \in (-1,0)$.

(ii) $|x| > 1$. In this case we must have $x^2 + x + 1 \geq 1$

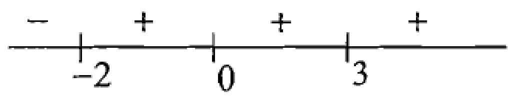
$\Rightarrow x \in (-\infty,1) \cup (0,\infty)$.

$\Rightarrow$ Common part of $|x| > 1$ and $x \in (-\infty,-1) \cup (0,\infty)$ is $(-\infty,-1)$ is $(-\infty,-1) \cup (1,\infty)$

$\Rightarrow$ Thus, the final solution is $x \in (-\infty,-1) \cup (-1,0) \cup (1,\infty)$

Q.4 (C)

$\Rightarrow$ Using wavy curve method and the fact that x = 0 and 3 are the repeated roots of x $(e^x - 1)(x + 2)(x - 3)^2$ we get the sign scheme of the given expression as



$\Rightarrow$ Thus complete solution is $x \in (-\infty, -2] \cup (0, 3)$.

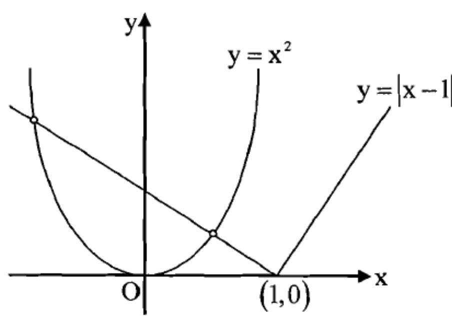
Q.5 (B)

$\Rightarrow \left|\frac{x^2}{x-1}\right| \le 1$

$\Rightarrow x^2 \le |x-1|, x \ne 1$

$\Rightarrow$ Adjacent figure represents the graphs of $y = x^2$ and $y = |x-1|$

$\Rightarrow$ Solving $x^2 = 1 - x$, we get



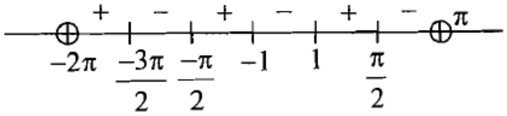
$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow$ Thus solution is $\left[\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right]$

Q.6 (D)

$\Rightarrow |x^2 - 1 + \cos x| = |x^2 + 1| + |\cos x|$. It implies that $(x^2 - 1)\cos \ge 0$ bacause $|x + y| = |x| + |y|$ if $y \ge 0$. Sign

scheme of $(x^2 - 1)\cos x$ is



$\Rightarrow$ Thus solution is $\left[-\frac{\pi}{2},-1\right]\cup\left[1,\frac{\pi}{2}\right]\cup\left(-2\pi,\frac{3\pi}{2}\right]$

Q.7 (D)

$\Rightarrow [x]^2 - 5[x] + 6 = 0$

$\Rightarrow [x] = 2, 3$

$\Rightarrow x \in [2,4)$

Q.8 **(D)**

$$\Rightarrow \left[\log_2\left(\frac{x}{|x|}\right)\right] \geq 0$$

$$\Rightarrow \log_2\left(\frac{x}{[x]}\right) \geq 0$$

$$\Rightarrow \frac{x}{[x]} \geq 1$$

$$\Rightarrow \frac{x-[x]}{[x]} \geq 0$$

$$\Rightarrow \frac{\{x\}}{[x]} \geq 0$$

$\Rightarrow$ It implies that ‘x’ is any positive real number greater than of equal to one or ‘x’ is nay non zero integer.

Q.9 **(B)**

$\Rightarrow 2\,[x] = x + \{x\}$

$\Rightarrow 2\,[x] = [x] + 2\{x\}$

$$\Rightarrow \{x\} = \frac{[x]}{2}$$

$$\Rightarrow 0 \leq \frac{[x]}{2} < 1$$

$$\Rightarrow 0 \leq [x] < 2$$

$\Rightarrow [x] = 0, 1$

$\Rightarrow$ For [x] = 0, we get [x] = 0

$\Rightarrow x = 0$

$\Rightarrow$ For [x] = 1, we get $\{x\} = \frac{1}{2}$

$$\Rightarrow x = \frac{3}{2}$$

Q.10 (B)

$\Rightarrow [x]^2 = x + 2\{x\}$

$\Rightarrow [x]^2 = [x] + 3\{x\}$

$$\Rightarrow \{x\} = \frac{[x]^2 - [x]}{3}$$

$$\Rightarrow 0 \le \frac{[x]^2 - [x]}{3}$$

$$\Rightarrow 0 \le \frac{[x]^2 - [x]}{3} < 1$$

$$\Rightarrow 0 \le [x]^2 - [x] < 3$$

$$\Rightarrow [x] \in \left(\frac{1-\sqrt{13}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{13}}{2}\right)$$

$\Rightarrow [x] = -1, 0, 1, 2$

$$\Rightarrow \{x\} = \frac{2}{3}, 0, 0, \frac{2}{3}$$

$$\Rightarrow x = \frac{2}{3}, 0, 0, \frac{2}{3}$$

$$\Rightarrow x = -\frac{1}{3}, 0, 1, \frac{8}{3}$$

Q.11 (C)

$\Rightarrow$ $[x^2] + x - a = 0$

$\Rightarrow$ ‘x’ i has to be an integer

$\Rightarrow$ $a = x^2 + x = x (x + 1)$

$\Rightarrow$ Thus ‘a’ can be 2, 6, 12, 20.

Q.12 (D)

$\Rightarrow$ $[x + [2x]] < 3$

$\Rightarrow$ $[x]+[2x]\le 2$

$\Rightarrow$ Any non-positive real number will satisfy this inequality.

$\Rightarrow$ Now if $x \in \left(0, \frac{1}{2}\right)$

$\Rightarrow$ $[x] = 0, [2x] = 1$

$\Rightarrow$ inequality is still satisfied

$\Rightarrow$ For $x \in \left(1, \frac{3}{2}\right), [x] = 1, [2x] = 2$

$\Rightarrow$ inequality does not hold true.

$\Rightarrow$ Thus, $x \in (-\infty, 1)$.

Q.13 (B)

$$\Rightarrow \text{We get, } f(x) = \begin{cases} 6-3x & , \quad x<1 \\ 4-x & , \quad 1\le x<2 \\ x & , \quad 2\le x<3 \\ 3x-6 & , \quad x>2 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -3 & , \quad x<1 \\ -1 & , \quad 1<x<2 \\ 1 & , \quad 2<x<3 \\ 3 & , \quad x>3 \end{cases}$$

$\Rightarrow$ Thus f(x) decreasing for x < 2 and increasing for x > 2.

$\Rightarrow$ Hence, $f(x)|_{\min} = f(2) = 2.$

Q.14 (D)

$\Rightarrow$ [5 sin x] + [cos x] = – 6

$\Rightarrow$[5 sin x] = – 5, [cos x] = – 1

$\Rightarrow -5 \le 5\sin x < -4, -1 \le \cos x < 0$

$\Rightarrow -1 \le \sin x < -\frac{4}{5}, -1 \le \cos x < 0$

$\Rightarrow x + \sin^{-1}\left(\frac{4}{5}\right) < x < \frac{3\pi}{2}$

$\Rightarrow$ Now $\quad f(x) = \sin x + \sqrt{3}\cos x = 2\sin\left(x + \frac{\pi}{3}\right)$

$\Rightarrow$we have, we have, $\pi + \frac{\pi}{2} + \sin^{-1}\left(\frac{4}{5}\right) < x + \frac{\pi}{6} < \frac{3\pi}{2} + \frac{\pi}{3}$

$\Rightarrow -1 \le \sin\left(x + \frac{\pi}{6}\right) < -\frac{\sqrt{3}}{2}$

Q.15 (C)

$\Rightarrow y = |\sin x| + |\cos x|$

$\Rightarrow y^2 = 1 + |\sin 2x|$

$\Rightarrow 1 \le y^2 \le 2$

$\Rightarrow y \in \left[1, \sqrt{2}\right]$

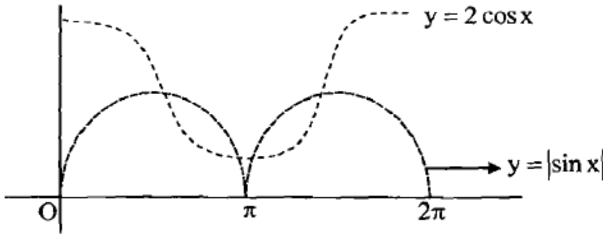
$\Rightarrow f(x) = 1 \ \forall \ x \in R$

Q.16 (B)

$\Rightarrow$Graph of $y = 2^{\cos x}$ and $y = |\sin x|$ meet four times in $[0, 2\pi]$.

$\Rightarrow$Thus, total number of solutions

$\Rightarrow$ 4 + 4 + 4 + 2 = 14.



Q.17 (A)

$\Rightarrow$For function to be one-one, each element of set A must have different image in st B. We first of all choose any 'm' elements in st B. This can be done in ^nC_m ways. Now one-one correspondence of elements of set A with these selected elements can be done in m! ways. Thus total number of one-one functions will be equal to $^nC_m.m!$ i.e. ^nP_m.

Q.18 (A)

$\Rightarrow 2^x + 3^x + 4^x - 5^x = 0$

$\Rightarrow \left(\left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1\right) = 0$

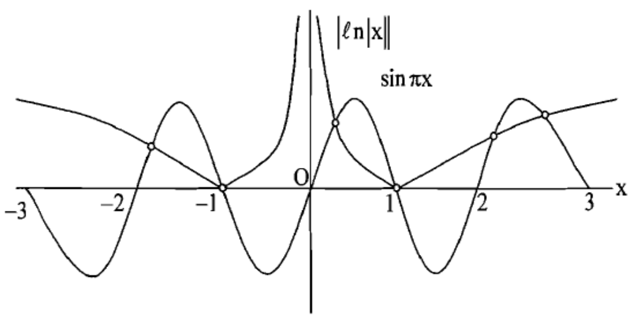
$\Rightarrow$Clearly $g(x) = \left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$

$\Rightarrow$is a decreasing function and Also g (0) = 1.

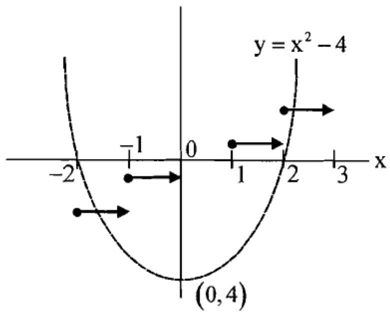
$\Rightarrow$Thus, f (x) = 0 has exactly one root.

Q.19 (D)

$\Rightarrow$There are exactly six solutions.



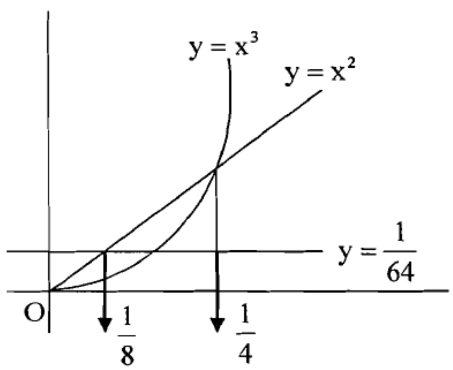
Q.20 (B)



There are exactly 2 solutions.

Q.21 (C)

$$\Rightarrow \text{Clearly, } f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{8} \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & x > 1 \end{cases}$$



22. (C)

Standard fact : Domain of (f + g)(x) = Domain of f(x) $\cap$ Domain of g(x)

23. (D)

If f(x) = f(y) implies only & only x = y, then f(x) is injective.

Hence f(f(f(x))) = f(f(f(y))) will imply only x = y when f(x) is injective.

24. (C)

If f(x) is even, then

$$f(-x) = f(x) \Rightarrow (-ax+b)\cos x - (-cx+d)\sin x = (ax+b)\cos x + (cx+d)\sin x$$

$$\Rightarrow 2ax\cos x + 2d\sin x = 0$$

$\Rightarrow a = d = 0$.

25. **(A)**

$3\sin x - 4\cos x + 6 = 5\sin(x - \alpha) + 6$, where $\alpha = \tan^{-1}\frac{4}{3}$

Now $y = \dfrac{1}{3\sin x - 4\cos x + 6} \Rightarrow 5\sin(x-\alpha) + 6 = \dfrac{1}{y}$

$\Rightarrow \sin(x - \alpha) = \dfrac{1-6y}{5y}$

$\Rightarrow -1 \le \dfrac{1-6y}{5y} \le 1$

$\Rightarrow y \in \left[\dfrac{1}{11}, 1\right]$

26. **(B)**

$f(a_1) \ne b_4$ & $f(a_2) = b_1 \Rightarrow f(a_1)$ can be chosen in 3 ways

Now $f(a_3)$ & $f(a_4)$ can be chosen in 3×2 ways

Hence total number of injective functions = $3\times3\times2 = 18$.

27. **(C)**

Domain of f(x) is [–1, 2]

Now $\sqrt{2-x} + \sqrt{1+x} = y \Rightarrow 2\sqrt{2+x-x^2} = y^2 - 3$

$\Rightarrow y^2 \ge 3$ & $(2x-1)^2 = 6y - y^2$

As x lies in [–1, 2], hence range of $(2x-1)^2$ is [0, 9]

Hence $0 \le 6y^2 - y^4 \le 9$ or $y^4 - 6y^2 + 9 \ge 0$ & $y^4 - 6y^2 \le 0$

$\Rightarrow y \le \sqrt{6}$

$\therefore y \in \left[\sqrt{3}, \sqrt{6}\right]$.

28. **(D)**

f(x) = log(2 + cos 3x)

(A) Domain : $(-\infty,\infty)$ as 2 + cos 3x is always greater than 0.

(B) Range : $\log(2+\cos 3x) = y \Rightarrow \cos 3x = e^y - 2$

Hence $-1 \le e^y - 2 \le 1$

$\Rightarrow 0 \le y \le \ln 3$.

(C) $f(-x) = \log(2+\cos 3x) = f(x)$, hence f(x) is even.

(D) As cos 3x is periodic hence f(x) is periodic.

29. **(C)**

Let ax + b = y .

Interchanging x & y gives ay + b = x

$$\Rightarrow y = \frac{x-b}{a}$$

Now $ax + b = \frac{x}{a} - \frac{b}{a} \Rightarrow a = \frac{1}{a}$ & $b = -\frac{b}{a}$

or $a = 1, b = 0$ & $a = -1, b \in R$.

30. **(D)**

Given $f(x) = \log_{10}\frac{1+x}{1-x}$

(I) $f(-x) = \log_{10}\frac{1-x}{1+x} \Rightarrow f(-x) = -\log_{10}\frac{1+x}{1-x} = -f(x)$

f(x) is odd, hence graph is not symmetric about y – axis.

(II) Domain of f(x) is (–1, 1).

Now $2 \le \frac{1+x}{1-x} < \infty \Rightarrow \log_{10} 2 \le \log_{10}\left(\frac{1+x}{1-x}\right) < \infty$.

Hence graph can’t lie in IV quadrant.

(III) As f(x) is odd hence graph is symmetric about the origin.

(IV) Clearly f(0) = 0 hence graph passes through the origin and lies in I & III quadrant.

31. (B)

$$2^x + 2^y = 1 \Rightarrow y = \log_2\left(1 - 2^x\right)$$

Hence for domain, $1 - 2^x \geq 0$

$\Rightarrow 2^x \leq 1$ or $x \in \left(-\infty, 0\right]$.

32. (D)

f is even hence f(–x) = f(x)

g is odd hence g(–x) = – g(x)

Now f(x) + g(x) = $e^x \Rightarrow f\left(-x\right) + g\left(-x\right) = e^{-x}$ or $f\left(x\right) - g\left(x\right) = e^{-x}$

Hence $\left(f\left(x\right) + g\left(x\right)\right)\left(f\left(x\right) - g\left(x\right)\right) = e^x e^{-x}$

$\Rightarrow f^2\left(x\right) - g^2\left(x\right) = 1$.

33. (B)

$3 < \pi < 4 \Rightarrow \left[\pi\right] = 3 \ \& \left[-\pi\right] = -4$

Hence f(x) = cos 3x – sin 4x.

Period of cos 3x = $\frac{2\pi}{3}$ & period of sin 4x = $\frac{\pi}{2}$.

Therefore period of f(x) = LCM$\left\{\frac{2\pi}{3}, \frac{\pi}{2}\right\}$ = 2π.

34. (C)

Number of ONTO functions from domain containing n elements to a codomain containing r elements is

$$r^n - {}^rC_1\left(r-1\right)^n - {}^rC_2\left(r-2\right)^n + \ldots + \left(-1\right)^{r-1}\,{}^rC_{r-1}\left(r-1\right)^n$$

Hence for the given data, number of ONTO functions is

$$3^6 - {}^3C_1 \times 2^6 + {}^3C_2 \times 1^6 = 540.$$

35. **(A)**

Image of (5, k) in x = y is B(k, 5).

As B lies on y = f(x) hence k = 2.

Reflection of B(2, 5) in origin will be (–2, –5).

36. **(D)**

$P(x^2+1)=(P(x))^2+1 \Rightarrow P(x)>0$

$P(0)=1 \Rightarrow P(1)=2, P(2)=5, P(-1)=2...etc$

Clearly $P(x)=x^2+1$.

37. **(A)**

$f(x)=\cos(\sqrt{2}\,x)+\cos(\sqrt{3}\,x)$

Period of $\cos(\sqrt{2}\,x)=\frac{2\pi}{\sqrt{2}}$ & period of $\cos(\sqrt{3}\,x)=\frac{2\pi}{\sqrt{3}}$

As LCM of $\frac{2\pi}{\sqrt{2}}$ & $\frac{2\pi}{\sqrt{3}}$ doesn't exist hence f(x) is not periodic.

Also at x = 0 f(x) = 2 which is clearly the greatest value of f(x) as cosine has a greatest value 1.

$$\cos(\sqrt{2}\,x)+\cos(\sqrt{3}\,x)=0 \Rightarrow 2\cos\left(\frac{\sqrt{2}+\sqrt{3}}{2}x\right)\cos\left(\frac{\sqrt{2}-\sqrt{3}}{2}x\right)=0$$

$$\Rightarrow x=\left(\frac{2n-1}{\sqrt{2}+\sqrt{3}}\right)\pi \quad \text{or} \quad x=\left(\frac{2n-1}{\sqrt{2}+\sqrt{3}}\right)\pi$$

Hence y = f(x) cuts the x – axis.

As f(–x) = f(x) hence f(x) is even.

38. **(D)**

Let $n \le x < n+\frac{1}{2}$, then $[x]+\left[x+\frac{1}{2}\right]=2004 \Rightarrow 2n=2004$ or $n=1002$

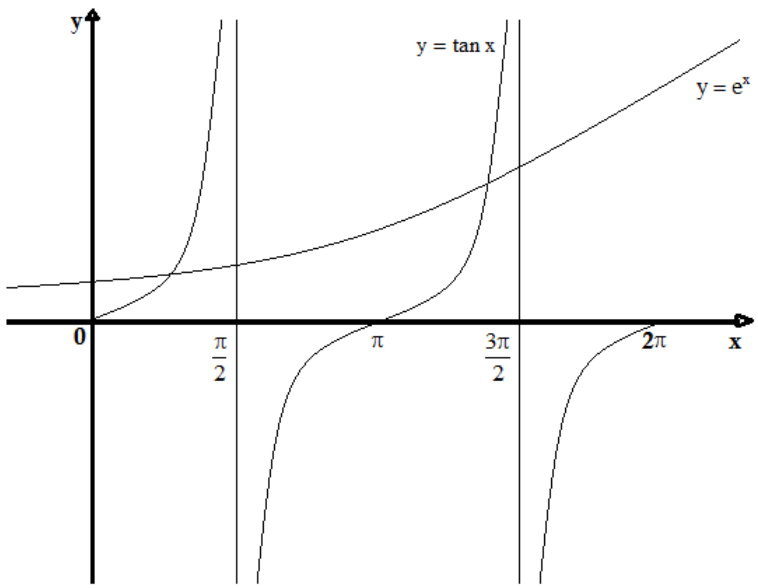
If $n+\frac{1}{2}\le x<n+1$, then $[x]+\left[x+\frac{1}{2}\right]=2004\Rightarrow 2n+1=2004,$

but n can't be non integral.

Hence $1002\le x<1002+\frac{1}{2}$.

39. **(B)**

Refer the following graph :



Q.40 **(D)**

$\Rightarrow$ Only in D, the graph has a symmetry w.r.t. origin

Q.41 **(C)**

$f(x)=|\sin^3 2x|+|\cos^3 2x|$

$\Rightarrow f(x)=\sin^6 2x+\cos^6 2x+\frac{1}{4}|\sin^3 4x|$

$\Rightarrow f(x)=1-\frac{3}{4}\sin^2 4x+\frac{1}{4}|\sin^3 4x|$

Now periods of both $\sin^2 4x$ & $|\sin^3 4x|$ **are** $\frac{\pi}{4}$ **hence the period of** $f(x)=|\sin^3 2x|+|\cos^3 2x|$ **is** $\frac{\pi}{4}$**.**

Q.42 **(C)**

We have for $\cos^{-1}(1-x)\ge 0$

$\Rightarrow -1 \le (1-x) \le 1$

$\Rightarrow -2 \le -x \le 0$

$\Rightarrow 0 \le x \le 2$ ......(1)

also, $10 \cdot 3^{x-2} - 9^{x-1} - 1 > 0$

$\Rightarrow 10 \cdot 3^x - 9^x - 9 > 0$

$\Rightarrow 10 \cdot 3^x - 3^{2x} - 9 > 0$

$\Rightarrow 3^{2x} - 10 \cdot 3^x + 9 < 0$

$\Rightarrow (3^x - 1)(3^x - 9) < 0$

$\Rightarrow 1 < 3^x < 9$

$\Rightarrow 0 < x < 2$ ……. (2)

from (1) and (2)

$\Rightarrow 0 < x < 2$

Q.43 (A)

Note that f is bijective hence f^{-1} exist

$\Rightarrow$ when y = 4

$\Rightarrow 2x^3 + 7x - 9 = 0$

$\Rightarrow 2x^2 (x-1) + 2x(x-1) + 9(x-1) = 0$

$\Rightarrow (x-1)(2x^2 + 2x + 9) = 0$

$\Rightarrow$ x = 1 only $\Rightarrow$ A; as $2x^2 + 2x + 9 = 0$ has no other roots

Q.44 (A)

$\Rightarrow f(x) = \frac{4}{\sqrt{1-x^2}}$; $f(\sin x) = \frac{4}{|\cos x|}$ and $f(\cos x) = \frac{4}{|\sin x|}$;

$\Rightarrow$ hence $g(x) = |\sin x| + |\cos x|$

Q.45 (C)

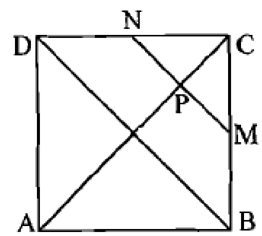
$\Rightarrow$ when $p = \frac{\pi}{2}$ then $D^r \to \cos x + \sin x \Rightarrow \frac{\pi}{2}$ cannot be the period]

Q.46 (B)

$\Rightarrow$ AP = x; MN = y ; $BD = 2\sqrt{2}$

$\Rightarrow$ Hence, $\frac{y}{2\sqrt{2}} = \frac{2\sqrt{2} - x}{\sqrt{2}} \Rightarrow \Delta$'s CNM and CDB are similar $y = 2\left(2\sqrt{2} - x\right)$

$\Rightarrow f(x) = \frac{xy}{2} = x\left(2\sqrt{2} - x\right) = 2 - \left(x - \sqrt{2}\right)^2$



$$\Rightarrow \begin{array}{ll} f(x)_{max} = 2 & \text{when } x = \sqrt{2} \\ f(x)_{max} = 0 & \text{when } x = 2\sqrt{2} \end{array} \Bigg]$$

Q.47 (A)

(A) $\Rightarrow \frac{1}{g(x)} = \frac{1}{\frac{\ln x}{x}}; f(x) = \frac{x}{\ln x}$ $\quad x > 0,\ x \neq 1$ for both

(B) $\Rightarrow \frac{1}{f(x)} = \frac{1}{\frac{x}{1nx}}; g(x) = \frac{\ln x}{x}$ $\quad \frac{1}{f(x)}$ is not defined at x = 1 but g (1) = 0

(C) $\Rightarrow f(x) \cdot g(x) = \frac{x}{\ln x} \cdot \frac{\ln x}{x} = 1$ $\quad$ if x > 0, $x \neq 1 \Rightarrow$ N.I.

(D) $\Rightarrow \frac{1}{f(x) \cdot g(x)} = \frac{1}{\frac{x}{\ln x} \cdot \frac{\ln x}{x}} = 1$ $\quad$ only for x > 0 and $x \neq 1$]

Q.48 (A)

$\Rightarrow$ An equation of this kind is called a functional equation, and can often be solved by choosing particular values for the variables. In this case, by choosing x = 1, we see that $f(y) = \frac{f(1)}{y}$ for all y. put y = 30; f (1) = $30 \cdot f(30) = 30 \cdot 20 = 600$. Now $f(40) = \frac{f(1)}{40} = \frac{600}{40} = 15$

Q.49 (C)

$\Rightarrow f(x) = \sin^2 x + (1 - \sin^2 x)^2 + 2$

$\Rightarrow 3 - \sin^2 x + \sin^4 x$

$\Rightarrow 3 - \sin^2 x \cos^2 x$

$\Rightarrow 3 - \dfrac{\sin^2 2x}{4}$

$\Rightarrow T_1 = \dfrac{\pi}{2}$, and $T_2 = \dfrac{\pi}{2}$

Q.50 (D)

$\Rightarrow D_2$ means range of the function
$\Rightarrow$ let $y = \sqrt{1-2x} + x$
$\Rightarrow (y - x)^2 = 1 - 2x$
$\Rightarrow y^2 - 2xy + x^2 = 1 - 2x$
$\Rightarrow x^2 + 2x(1 - y) + y^2 - 1 = 0$
$\Rightarrow$ as, $x \in R$, $D \geq 0$
$\Rightarrow 4(1-y)^2 \geq 4(y^2 - 1)$
$\Rightarrow 1 + y^2 - 2y \geq y^2 - 1$
$\Rightarrow -2y \geq -2$
$\Rightarrow y \leq 1$
$\Rightarrow y \in (-\infty, 1]$
$\Rightarrow$ Alternatively: $f'(x) = 1 - \dfrac{1}{\sqrt{1-2x}}$; $f'(x) = 0$
$\Rightarrow 1 - 2x = 1$
$\Rightarrow x = 0$
$\Rightarrow f(-\infty) = \rightarrow -\infty$

Q.51 (A)

$\Rightarrow h(x) = \ln(f(x) \cdot g(x)) = \ln e^{\{y\}+[y]} = \{y\} + [y] = y = e^{|x|} \operatorname{sgn} x$

$$\Rightarrow \therefore \quad h(x) = e^{|x|} \operatorname{sgn} x = \begin{bmatrix} e^{x} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -e^{-x} & \text{if } x < 0 \end{bmatrix}$$

$$\Rightarrow h(-x) \begin{bmatrix} e^{x} & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ -e^{-x} & \text{if } x > 0 \end{bmatrix}$$

$\Rightarrow h(x) + h(-x) = 0$ for all x

Q.52 (D)

(A) $f(x) = x^4 + 2x^3 - x^2 + 1 \rightarrow$ A polynomial of degree even will always be into

$\Rightarrow$ say, $f(x) a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + ... + a_{2n}$

$$\Rightarrow \underset{x \to \pm\infty}{\text{Limit}} f(x) = \underset{x \to \pm\infty}{\text{Limit}} \left[x^{2n} \left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ... + \frac{a_{2n}}{x^{2n}} \right) \right] = \begin{bmatrix} \infty & \text{if } a_0 > 0 \\ -\infty & \text{if } a_0 < 0 \end{bmatrix}$$

Hence it will never approach $\frac{\infty}{-\infty}$

(B) f (x) = $x^3 + x + 1$

$\Rightarrow$ f' (x) = $3x^2 + 1$ $\Rightarrow$ injective as well as surjective

(C) $f(x) = \sqrt{1 + x^2}$

$\Rightarrow$ neither injective nor surjective (minimum value = 1)

$\Rightarrow$ f (x) = $x^3 + 2x^2 - x + 1$

$\Rightarrow$ f' (x) = $3x^2 + 4x - 1$

$\Rightarrow$ D > 0

$\Rightarrow$ Hence f (x) is surjective but not injective.

Q.53 (D)

Let f (x_1) = n and f (x_2) = m, x_1, x_2 $\in (a, b)$ with n > m (say). According to the intermediate value theorem, between x_1 and x_2 there must be some value x for which $f(x) = m + \frac{1}{2}$ which is impossible since $m + \frac{1}{2}$ is not an integer.

Q.54 (D)

$$\Rightarrow g\left(-1, -\frac{3}{2}\right) = \max\left(-1, -\frac{3}{2}\right) - \min\left(-1, -\frac{3}{2}\right) = -1 - \left(-\frac{3}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \text{and } g(-4, -1.75) = \max(-4, -1.75) - \min(-4, -1.75) = -1.75 - (-4) = 2.25 = \frac{9}{4}$$

$$\Rightarrow \text{then } f\left(\frac{1}{2}, \frac{9}{4}\right) = \left(\max\left(\frac{1}{2}, \frac{9}{4}\right)\right)^{\min\left(\frac{1}{2}, \frac{9}{4}\right)} = \left(\frac{9}{4}\right)^{\frac{1}{2}} = \frac{3}{2}$$

Q.55 (A)

$$\Rightarrow f(x) = \frac{e^x \ln x \;\; 5^{(x^2+2)} \cdot (x-2)(x-5)}{(2x-3)(x-4)}$$

$\Rightarrow$ Note that at $x = \frac{3}{2}$ & $x = 4$ function is not defined and in open interval $\left(\frac{3}{2}, 4\right)$ function is continuous.

$\Rightarrow \lim_{x\to \frac{3}{2}^{+}} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \to -\infty$

$\Rightarrow \lim_{x\to 4^{-}} = \frac{(+ve)(+ve)(-ve)}{(+ve)(-ve)} \to -\infty$

$\Rightarrow$ In the open interval $\left(\frac{3}{2}, 4\right)$ the function is continuous & takes up all real values from $(-\infty, \infty)$

$\Rightarrow$ Hence range of the function is $(-\infty, \infty)$ or R

Q.56 (D)

$f^2(x) - f(x) - 6 \geq 0$

$\Rightarrow (f(x) - 3)(f(x) + 2) \geq 0$

$\Rightarrow f(x) \geq 3$ or $f(x) \leq -2$

$\Rightarrow$ given $x \in (0, \infty)$ $\Rightarrow$ $x \in (0, \infty)$

$\Rightarrow \therefore f(x) \geq 3$ $\Rightarrow$ $x \in (-\infty, 0]$

$\Rightarrow f(x) > -2$ $\Rightarrow$ $x \in (-\infty, 5)$

$\Rightarrow \therefore f(x) \leq -2$ $\Rightarrow$ $x \in [5, \infty)$

Q.57 (C)

$\Rightarrow f(x) = f^{-1}(x)$
$\Rightarrow f(x) = x$
$\Rightarrow (x + 1)^2 - 1 = x$
$\Rightarrow x^2 + x = 0$
$\Rightarrow x = 0$ or -1

Q.58 (D)

$x\, f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

$\Rightarrow g(x) = \sin x$ near. $x \to \pi$ though rational then $x\, f(x) \to \pi$ but $g(x) \to 0 \Rightarrow x\, f(x) > g(x)$
$\Rightarrow g(x) = x$ is negative for negative irrational x while $x\, f(x)$ is 0 ; $x\, f(x) > g(x)$
$\Rightarrow g(x) = x^2$ is smaller than x for $0 < x < 1$ and rational; so $x\, f(x) > g(x)$
$\Rightarrow g(x) = |x|$ equals $x\, f(x)$ for x positive and rational, is larger than $x\, f(x)$ for x irrational.

Q.59 (D)

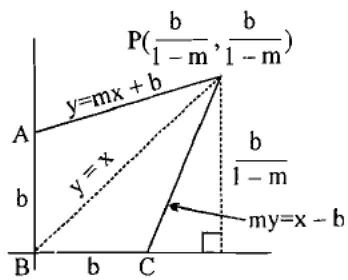
$h(x) = {}^{x+1}C_{2x-8} \cdot {}^{2x-8}C_{x+1};\ x + 1 \geq 2x - 8$
$\Rightarrow x \leq 9;\ 2x - 8 \geq x + 1 \Rightarrow x \geq 9$

$\Rightarrow$ Hence x = 9
$\Rightarrow$ Domain of h (x) = {9}
$\Rightarrow$ Range of h (x) = 1

Q.60 (C)

If f (x) = mx + b, then $f^{-1}(x) = \frac{x-b}{m}$ and their point of intersection

$\Rightarrow$ Can be found by setting x = mx + b since they intersect on y = x. Thus $x = \frac{b}{1-m}$ and the point of intersection is $\left(\frac{b}{1-m}, \frac{b}{1-m}\right)$.



$\Rightarrow$ Region R can be broken up into congruent triangles PAB and PCB which both have a base of b and a height of $\frac{b}{1-m}$.

$\Rightarrow$ The area of R is $\left(\frac{2b}{2}\right)\left(\frac{b}{1-m}\right) = \frac{b^2}{1-m} = 49.$ For $m = \frac{9}{25}, b^2 = \frac{16}{25} \cdot 49$

$\Rightarrow b = \frac{28}{5}$

Q.61 (A)

$\Rightarrow 9 - x^2 \geq 0$
$\Rightarrow -3 \leq x \leq 3$
$\Rightarrow$ Also $9 - |2x+5| > 0$
$\Rightarrow$ – 9 < 2x + 5 < 9
$\Rightarrow$ – 7 < x < 1
Hence domain of f (x) is $[-3, 2)$

$\Rightarrow \% = \frac{2}{5} = 40\%$

Q.62 (D)

$\Rightarrow$ **I** f(x) = x and g(x) =1 – x or f(x) = x and g(x) = $-x^3$
$\Rightarrow$ **II** f (x) = x and g (x) = x^3
$\Rightarrow$ **III** f (x) = sin x which is odd but not one-one

Q.63 (D)

$\Rightarrow x + xe^{f(x)} = 1 - e^{f(x)}$
$\Rightarrow (x+1)e^{f(x)} = 1 - x$
$\Rightarrow f(x) = \ln\left(\frac{1-x}{1+x}\right)$

Q.64 (D)

$\Rightarrow$ Replacing x by $\frac{\pi}{2} - x$; $f\left(\cos\left(\frac{\pi}{2} - x\right)\right) = \cos 17\left(\frac{\pi}{2} - x\right)$
$\Rightarrow$ f (sin x) = sin 17x = g (sin x)
$\Rightarrow$ hence f = g

Q.65 (A)

$y = 2 \log_a x$
$\Rightarrow \log_a x = \frac{y}{2}$
$\Rightarrow x = a^{\frac{y}{2}}$
$\Rightarrow f^{-1}(b+c) a^{\frac{b+c}{2}} = f^{-1}(b) \cdot f^{-1}(c)$

Q.66 (D)

$\Rightarrow p = \frac{2\pi}{\sqrt{[a]}} = \pi,$
Hence $\sqrt{[a]} = 2$
$\Rightarrow$ (A) = 4
$\Rightarrow 4 \le a < 5$

Q.67 (C)

$$2f(x) + f(1-x) = x^2 \quad ....(1)$$
$$f(x) + 2f(1-x) = (1-x)^2 \quad ....(2)$$
$$\Rightarrow \underset{\substack{\text{multiply}(1)\text{ by }(2)\\(3)-(2)}}{x \to 1-x} \qquad 4f(x) + 2f(1-x) = 2x^2 \quad ....(3)$$
$$3f(x) = 2x^2 - (1-x)^2$$
$$3f(4) = 32 - 9 = 23$$
$$f(4) = \frac{23}{3}$$

Q.68 (B)

$$f(x)=\frac{a^x+a^{-x}}{2} \ \& \ f(x+y)+f(x-y)=k\,f(x)f(y)$$

$$\Rightarrow \frac{a^{x+y}+a^{-x-y}}{2}+\frac{a^{x-y}+a^{y-x}}{2}=k\left(\frac{a^x+a^{-x}}{2}\right)\left(\frac{a^y+a^{-y}}{2}\right)$$

$$\Rightarrow 2\left(a^xa^y+\frac{1}{a^xa^y}+\frac{a^x}{a^y}+\frac{a^y}{a^x}\right)=k\left(a^xa^y+\frac{1}{a^xa^y}+\frac{a^x}{a^y}+\frac{a^y}{a^x}\right)$$

$\Rightarrow k=2.$

Q.69 (A)

$\Rightarrow$ A one to one function and its inverse are symmetric across the line y = x. Thus x and y intercept are interchanged and the sum is the same i.e. 5.

Q.70 (C)

$\Rightarrow x(x+3)\geq 0$

$\Rightarrow x\geq 0$ or $x\leq -3$

and $-1\leq x^2+3x+1\leq 1$

$\Rightarrow x(x+3)\leq 0$ and $2^2 + 3x + 2^3$ which is always true.

Hence $-3\leq x\leq 0$

Hence x = 0 or – 3

$\Rightarrow$ x = {0, – 3}

FUNCTIONS

EXERCISE – 2(A)

Q.1 (A, B, C, D)

(A) $f(x)=\log_{x-1}(2-[x]-[x]^2) \Rightarrow 2-[x]-[x]^2 > 0$

$\Rightarrow [x] \in (-2,1)$ So, $[x] = -1, 0 \Rightarrow x \in (-1,1)$

but, $x-1 \neq 0$, $x-1 > 0 \Rightarrow$ x > 1

So f (x) has empty domain.

(B) $g(x) = \cos^{-1}(2-\{x\})$

Now $0 \leq \{x\} < 1 \Rightarrow 1 < 2-\{x\} \leq 2$

but, $\cos^{-1}$x is defined in $[-1, 1]$

So g (x) has empty domain.

(C) $h(x) = \ln\ln(\cos x)$

Now $\ln(\cos x) > 0 \Rightarrow \cos x > 1$

So h (x) has empty domain.

(D) $f(x) = \dfrac{1}{\sec^{-1}(\text{sgn}(e^{-x}))}$

Now $e^{-x} > 0$ for $x \in R$

$\Rightarrow Sgn(e^{-x}) = 1$ for $x \in R$ and thus $\sec^{-1}(\text{sgn}(e^{-x})) = 0$ for $x \in R$.
So h (x) has empty domain.

Q.2 (A, B, D)

A transcendental function is one that cannot be expressed in terms of an algebraic polynomial.

e.g. trigonometric function, exponential, logarithmic function.

So, (A) , (B) , (D) are transcendental function.

But, $f(x)=\sqrt{x^2+2x+1} = \sqrt{(x+1)^2}$

$$= |x+1|$$

$$= x+1\ ;\ x \geq -1$$

$$= -x-1\ ;\ x < -1$$

Q.3 (A, B, C)

(A) $$y = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$$

$$= \frac{\sin x}{|\sec x|} + \frac{\cos x}{|\cos ec x|}$$

$$= \sin x|\cos x| + \cos x|\sin x|$$

$$= 0 \ \ \forall \ \ x \in \left[(4n+1)\frac{\pi}{2}\ ,\ (2n+1)\pi\right] \cup \left[(4n+3)\frac{\pi}{2},\ (2n+2)\right]$$

$$= \sin 2x \ \forall \ x \in \left[2n\pi\ ,(4n+1)\frac{\pi}{2}\right]$$

$$= -\sin 2x \ \forall \ x \in \left((2n+1)\pi\ ,\ (4n+3)\frac{\pi}{2}\right)$$

Hence graph of $y = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$ is dissimilar from $y = \sin 2x$

(B) $$y = \tan x \cdot \cot x = 1 \ \ \forall \ \ x \in (-\infty, \infty) - \frac{x\pi}{2},\ x \in \mathrm{I}$$

$$y = \sin x \cdot \cos ec x = 1 \ \forall \ x \in (-\infty, \infty) - x\pi\ ,\ x \in \mathrm{I}$$

Functions are not identical as domains are not same, hence graphs are dissimilar.

(C) $$y = \frac{|\sec x| + |\cos ec\, x|}{|\sec x||\cos ec\, x|} \Rightarrow y = \frac{1}{|\sec x|} + \frac{1}{|\cos ec x|} \text{ or } y = |\cos x| + |\sin x|,\ x \neq \frac{n\pi}{2}$$

$$y = |\cos x| + |\sin x| \ \ \forall \ x \in (-\infty, \infty)$$

Functions are not identical as domains are not same, hence graphs are dissimilar.

Q.4 (A, B, D)

(A) $\left[x+1+\text{T}\right]=\left[x+1\right]\Rightarrow\left[x+\text{T}\right]=\left[x\right]$

$x+\text{T}-1\le\left[x+\text{T}\right]<x+\text{T}$ & $x-1\le\left[x\right]<x\Rightarrow\text{T}$ is not fixed.

Function is non periodic.

(B) $\sin\left(x+\text{T}\right)^2=\sin x^2\Rightarrow 2\cos\left(\dfrac{\left(x+\text{T}\right)^2+x^2}{2}\right)\sin\left(\dfrac{\left(x+\text{T}\right)^2-x^2}{2}\right)=0.$

$\Rightarrow\dfrac{\left(x+\text{T}\right)^2+x^2}{2}=\dfrac{\left(2n-1\right)\pi}{2}$ or $\dfrac{\left(x+\text{T}\right)^2-x^2}{2}=0$

As value of T is not constant but dependent of x hence $\sin x^2$ is non periodic.

(C) $\sin^2\left(x+\text{T}\right)=\sin^2 x\Rightarrow x+\text{T}=n\pi\pm x\Rightarrow\text{T}=n\pi$

Periodic with period $'\pi'$

$y=\sin^{-1}x\rightarrow$ not periodic as $\text{D}=[-1,1]$ & Range $=\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

Q.5 (A, C, D)

(A) $f(x)=x+1$, $x\ge-1$ is one – one as linear function are one – one

(B) $f(x)=x+\dfrac{1}{x}$ $(x>0)$ has minima at $x=1$

$(g'(x)=0\ \Rightarrow\ x^2-1=0\ \Rightarrow\ x=\pm1)$

So, not one – one in $(0,\infty)$

(C) $h(x)=x^2+4x-5$, $x>0$

$h'(x)=0$ at $x=-2$

So, one – one in $x\in(0,\infty)$

(D) $f(x)=e^{-x}$

$f'(x)<0$ for all $x\in R$

So, one – one in $x \in [0\ ,\ \infty]$

Q.6 (B, C)

A homogenous function is such that if substitution $y = vx$ is made it should come out to be

$x\ f(v)$.

(A) $x\sin y + y\sin x = v\sin\left(\frac{v}{x}\right) + vx - \sin x$

$$= v\left(\sin\left(\frac{v}{x}\right) + x\sin v\right) \quad \rightarrow \text{ not homogeneous.}$$

(B) $xe^{\frac{y}{x}} + ye^{\frac{x}{y}} = xe^{v} + vx \cdot e^{\frac{1}{v}}$

$$= x\left(e^{v} + ve^{\frac{1}{v}}\right) \quad \rightarrow \text{ homogeneous.}$$

(C) $x^2 - xy = x^2 - vx^2 = x^2(1 - v) \quad \rightarrow$ homogeneous.

(D) $\sin^{-1}(xy) = \sin^{-1}(vx^2) \quad \rightarrow$ not homogeneous.

Q.7 (B, C)

Given $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

Hence, $f(x)$ is a polynomial of degree n.

$$f(x) \cdot f\left(\frac{1}{x}\right) - f(x) - f\left(\frac{1}{x}\right) + 1 = 1$$

$$\Rightarrow \left(f(x) - 1\right)\left(f\left(\frac{1}{x}\right) - 1\right) = 1$$

$$\Rightarrow f(x) = 1 + \frac{1}{f\left(\frac{1}{x}\right) - 1}$$

$$\because\ f\left(\frac{1}{x}\right) = \frac{P(x)}{x^n}$$

$\Rightarrow f(x)=1+\dfrac{x^n}{P(x)-x^n}=1+\dfrac{x^n}{k}$ ....(I)

Hence, $P(x)-x^n=k$ (constant) for $f(x)$ to be polynomial

$\Rightarrow P(x)=k+x^n$

$\Rightarrow f\left(\dfrac{1}{x}\right)=1+\dfrac{k}{x^n} \Rightarrow f(x)=1+k^n$ ....(II)

From (I) , (II)

$k=1$

$\because f(2)=9 \Rightarrow 2^n+1=9 \Rightarrow n=3$

Hence, $f(x)=x^3+1$

$f(4)=65$, $f(6)=216 \Rightarrow 3f(6)\neq 2f(4)$

$f(1)=2$, $f(3)=28 \Rightarrow 14f(1)=f(3)$

$f(3)=28$, $f(5)=126 \Rightarrow 9f(3)=2f(5)$

$f(10)=1001, f(11)=1332 \Rightarrow f(10)\neq f(11)$

Q.8 (B, D)

$f(x)=x^2$ is may – one in $[-1,1]$

So, can't be inverted

$g(x)=x^3$ is bijective in $[-1,1]$

So, inverse is possible.

$h(x)=\sin 2x$ is may – one in $[-1,1]$

So, not invertible.

$k(x)=\sin\left(\dfrac{\pi x}{2}\right)$ is one – one in $[-1,1]$

So, invertible.

Q.9 (B, C)

$f(x)=\frac{1}{1+x}$ has the range $(-\infty,\infty)-\{0\}$

$f(x)=\frac{1}{1+x^2}$ has the range $(0,1)$

$f(x)=\frac{1}{1+\sqrt{x}}$ has the range $(0,1)$

$f(x)=\frac{1}{\sqrt{3-x}}$ has the range $(0\ ,\ \infty)$

Q.10 (A, B, C)

(A) $f(x)=\cos(2\tan^{-1}x)=\cos\left(\tan^{-1}\frac{2x}{1-x^2}\right)$

$=\cos\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)=\frac{1-x^2}{1+x^2}$: Domain – R & Range $\in[-1,1]$

$g(x)=\frac{1-x^2}{1+x^2}$: Domain – R, Range $\in[-1,1]$

(B) $f(x)=\frac{2x}{1+x^2}$: Domain – R, Range $\in[-1,1]$

$g(x)=\sin(2\cot^{-1}x)=\frac{2x}{1+x^2}$: Domain – R, Range $\in[-1,1]$

(C) $g\left(x\right)=e^{\ln\left(\operatorname{sgn}\left(\cot^{-1}x\right)\right)}$

$\cot^{-1}x$ must be positive hence domain $\left(0,\infty\right)$.

Now $\cot^{-1}x>0\Rightarrow\operatorname{sgn}\left(\cot^{-1}x\right)=1\Rightarrow e^{\ln\left(\operatorname{sgn}\left(\cot^{-1}x\right)\right)}=1.$

Range : $\{1\}$

$g(x)=e^{\ln[1+\{x\}]}\quad x\in R$

$=[\{x\}]+1=1\ \ \forall\ x\in R$

(D) $f(x)=(a)^{\frac{1}{x}}$, $a>0$

$f(x)=\sqrt[x]{a}$, $a>0$

For x being even, there exist 2 value of $g(x)=\pm\sqrt[x]{a}$, $a>0$

Q.11 (A, B)

$f:R\to R,\ \ f(x)=|x|\operatorname{sgn}(x),\ \ x>0$

$=(-x)(-1);\ x<0$

$=0\ ;\ x=0$

$=(x)(1);\ x>0.$

$\Rightarrow f(x)=x\ ,\ x\in R\,.$

$g:R\to R,\ \ f(x)=x^{\frac{3}{5}}$ is monotonic.

$h:R\to R,\ \ h(x)=x^4+3x^2+1$ is many – one

$$k:R\to R, k(x)=\frac{3x^2-7x+6}{x-x^2-2}$$

Denominator is always Negative so, Domain – R

Numerator has $D>0$, $k(x)=0$ at 2 points thus $k\,(x)$ is many – one.

Q.12 (A, B)

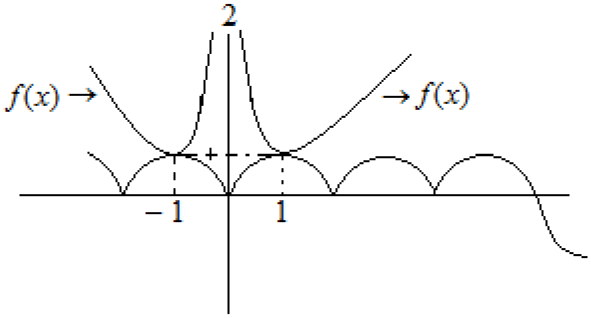
$$f(x)=ax+b=y\Rightarrow x=\left(\frac{y-b}{a}\right)\Rightarrow f^{-1}(x)=\frac{x}{a}-b\,.$$

Now $ax+b=\frac{x}{a}-b\Rightarrow a=\frac{1}{a}$ & $b=-b$

Hence $(a,b)\to(1,0)\ or\ (-1,0)$.

Q.13 (B, C)

(A) $x^4-2x^2\sin^2\frac{\pi x}{2}+1=0\Rightarrow\left(\frac{x^4+1}{2x^2}\right)=\sin^2\left(\frac{\pi x}{2}\right)$



$$\Rightarrow \frac{1}{2}\left(x^2+\frac{1}{x^2}\right)=\sin^2\frac{\pi x}{2}$$

Let, $f(x)=\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)$, $g(x)=\sin^2\left(\frac{\pi x}{2}\right)$

Has 2 solutions.

(B) $x^2-2x+5+\pi^x=0 \Rightarrow x^2-2x+5=-\pi^x$

$f(x)=x^2-2x+5=(x-1)^2+4>0 \;\forall\; x\in R$

$g(x)=-(\pi^x)<0 \;\forall\; x\in R$

Hence, no solution

(C) $\log_{\frac{3}{2}}(\cot^{-1}x-\text{sgn}(e^x))=2$

As $e^x>0$ thus $\text{sgn}\left(e^x\right)=1.$

$$\Rightarrow \cot^{-1}x-1=\left(\frac{9}{4}\right)$$

$\because \cot^{-1}x\in(0,\pi)$ hence, $\cot^{-1}x-1\in(-1\,,\,\pi-1)$

Hence, no solution.

(D) $\tan\left(x+\frac{\pi}{6}\right)=2\tan x$

$$\Rightarrow \frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}=2\tan x\,.$$

$$\Rightarrow 2\tan^2x+\sqrt{3}\tan x-2\sqrt{3}=0\,.$$

Hence infinitely many solutions.

Q.14 (A, B, C)

$g(x)$ & $g^{-1}(x)$ is symmetric about line $y=x$

Hence the point P & Q may lie on the line $y = x$ but not necessarily.

(Ex. $g(x) = \dfrac{15 - x^3}{7}$ & $g^{-1}(x) = (15 - 7x)^{1/3}$ intersect in (1, 2) & (2, 1) which do not lie on y = x)

Also there can be more than 1 points of intersection so P & Q need not coincide.

Slope of line joining points of intersections of y = g(x) & y = g^{-1}(x) may be 1 or –1 as either these points will lie on y = x or will be image of each other in y = x.

Q.15 (A, B, C, D)

$$f(2x)\left(1 - f\left(\frac{1}{2x}\right)\right) + f(16x^2 y) = f(-2) - f(4xy) \quad x, y \in R - \{0\}$$

$f(4) = -255$, $f(0) = 1$

Put $y = \dfrac{1}{8x^2}$ to get $f(2x)\left(1 - f\left(\dfrac{1}{2x}\right)\right) + f(2x) = f(-2) - f\left(\dfrac{1}{2x}\right)$

$\because$ $f(x)$ is even function $f(2) = f(-2)$

Replacing 2x by t

$$\Rightarrow f(t)\cdot\left(1 - f\left(\frac{1}{t}\right)\right) + f\left(\frac{1}{t}\right) = 0$$

$$\Rightarrow f(t) - f(t)\cdot f\left(\frac{1}{t}\right) + f\left(\frac{1}{t}\right) = 0$$

$$\Rightarrow f(t)\cdot f\left(\frac{1}{t}\right) - f(t) - f\left(\frac{1}{t}\right) + 1 = 1$$

$$\Rightarrow f(t) = 1 + \frac{1}{\left(f\left(\frac{1}{t}\right) - 1\right)}$$

Now, $f(t)$ is a polynomial, So, $f\left(\dfrac{1}{t}\right) = \dfrac{P(t)}{t^n}$

$$\Rightarrow f(t) = 1 + \frac{t^n}{P(t) - t^n}$$

For, $f(t)$ to be polynomial

$P(t)-t^n = k \Rightarrow P(t) = k+t^n$

$\Rightarrow f\left(\frac{1}{t}\right) = \frac{k}{t^n}+1$

$\Rightarrow f(t) = 1+k\,t^n$

Hence, $k = \frac{1}{k} \Rightarrow k = \pm 1$

So, $f(x) = \pm x^n + 1$

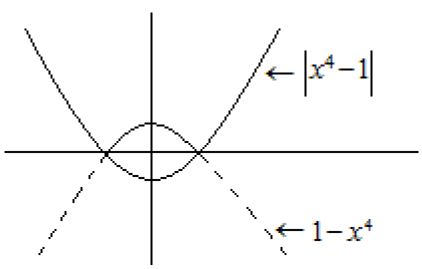
Given $f(4) = -255 \Rightarrow -x^n + 1 = -255 \Rightarrow n = 4$

So, $f(x) = 1-x^4$

(A) $f(3) = -80$

(B) $f(x)\cdot f\left(\frac{1}{x}\right) = \frac{(1-x^4)(x^4-1)}{x^4} = \frac{(x^4-1)^2}{x^4} \le 0$

(C) $\left|f(x)\right| = k-2$



For 4 different solutions. $k-2 \in (0,1)$

$\Rightarrow k \in (2\,,\,3)$

(D) $g(x) = 9-2\sqrt{3+f\left(\sqrt{|x|}\right)}$

$f(x) = 1-x^4$

$f\left(\sqrt{|x|}\right) = 1-\left(\sqrt{|x|}\right)^4 = 1-x^2$

$g(x)=9-2\sqrt{3+1-x^2}$

$=9-2\sqrt{4-x^2}$

Hence, $g(x)\in[5,9]$

So, $p^2+4q=25+36=61$

Q.16 (A, C, D)

$f(x)=\dfrac{x+2}{x-1}\Rightarrow x=\dfrac{y+2}{y-1}$

$\Rightarrow x=f(y)$

Range of $f(x)=\text{R}-\{1\}$

Domain of $f(x)=\text{R}-\{1\}$

Q.17 (B, C)

$f:N\to N,\ f(x)=x+(-1)^{x-1}$

For, $x\in$ set of even number , $f(x)=x-1,\ x=2m$.

For, $x\in$ set of odd number , $f(x)=x+1,\ x=2m+1$.

Now $y=\begin{cases}x-1, & x=2m\Rightarrow y=2m-1,\ (\text{odd})\\ x+1, & x=2m+1\Rightarrow y=2m+2,\ (\text{even})\end{cases}$

$\Rightarrow x=\begin{cases}y+1, & y=2m+1\\ y-1, & y=2m+2\end{cases}$

$\Rightarrow f^{-1}(x)=\begin{cases}x-1, & x=2m-1\\ x+1, & x=2m\end{cases}$

Hence $f^{-1}(x)=x-(-1)^x;x\in N$

Q.18 (A, B, C)

$f(x)=\cos[\pi^2]x+\cos[-\pi]x$

$=\cos 9x+\cos 4x$

$f\left(\frac{\pi}{2}\right)=1,\ f(\pi)=0,\ f\left(\frac{-\pi}{2}\right)=1\ \&\ f\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}-1$.

Q.19 (A, B, D)

$f(x)=\sin x+\tan x+\text{sgn}(x^2-6x+10)$

$x^2-6x+10>0$ for all $x\in R$ as $D<0$, hence, $\text{sgn}(x^2-6x+10)=1$

$\Rightarrow f(x)=\sin x+\tan x+1$

Hence f (x) is periodic with fundamental period 2π.

Also $4\pi \& 8\pi$ can be the periods.

Q.20 (A, C)

$$f(x)=\log_2\left(\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)$$

Now $-\sqrt{2}\le \sin x-\cos x\le \sqrt{2}$

$$\Rightarrow \frac{2\sqrt{2}}{\sqrt{2}}\le \frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\le \frac{4\sqrt{2}}{\sqrt{2}}$$

$$\log_2 2\le \log_2\left(\frac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)\le \log_2 4$$

Hence, $f(x)\in[1\ ,\ 2]$

Domain $\rightarrow$ R & Range $\rightarrow [1\ ,\ 2]$

PASSAGE – 1

Q.21 (B)

$f(x)=1-e^{\frac{1}{x}-1}$

$f(x)>0\Rightarrow 1-e^{\frac{1}{x}-1}>0$

$\Rightarrow e^{\frac{1}{x}-1}<0$

$\Rightarrow \frac{1}{x} - 1 < 0$

$\Rightarrow \frac{x-1}{x} > 0$

$\Rightarrow x < 0$ or $x > 1$..

Q.22 (A)

$f(x_1) = f(x_2) \Rightarrow 1 - e^{\frac{1}{x_1} - 1} = 1 - e^{\frac{1}{x_2} - 1}$ or $\frac{1}{x_1} = \frac{1}{x_2}$.

Hence *f* (x) is one – one.

$1 - e^{\frac{1}{x} - 1} = y \Rightarrow x = \frac{1}{1 + \ln(1 - y)}$

now for x to be real $1 - y > 0$ & $\ln(1 - y) \neq -1$

Hence $y < 1$ & $y \neq 1 - \frac{1}{e}$

Range of $f(x) : (-\infty, 1) - \left\{1 - \frac{1}{e}\right\}$

Hence $f(x)$ is INTO.

Q.23 (B)

Range $= (-\infty, 1) - \left\{1 - \frac{1}{e}\right\}$

PASSAGE – 2

Q.24 (B)

$]x[= \begin{cases} -x, & x > 0 \\ x, & x \leq 0 \end{cases}$

For, $x > 1$, $]x - 1[= 2x + 3 \Rightarrow 1 - x = 2x + 3$

or $x = -\frac{2}{3}$ (not possible)

For, $x \leq 1$, $]x - 1[= 2x + 3 \Rightarrow x - 1 = 2x + 3$

or $x = -4$.

Q.25 (A)

$x^2 + kx + 5 = 0$

For, $\alpha = -4$

$16 - 4k + 5 = 0 \Rightarrow k = \frac{21}{4}$

Q.26 (D)

$x^2 + kx + 5 = 0$

Product of the roots = 5

one root $= -4$, hence other root $= -\frac{5}{4}$

PASSAGE – 3

(i) $\sqrt{x^2 - 6x + 5} \geq x - 4$

Domain : $x^2 - 6x + 5 \geq 0 \Rightarrow x \in (-\infty, 1] \cup [5, \infty)$

For $x > 5$, $\sqrt{x^2 - 6x + 5} \geq x - 4 \Rightarrow (x^2 - 6x + 5) \geq (x-4)^2 \Rightarrow x \geq \frac{11}{2}$

For $x < 1$, always true as LHS > 0 & RHS < 0.

Hence solution set is $(-\infty, 1] \cup \left[\frac{11}{2}, \infty\right)$

(ii) $\left(\frac{1}{3}\right)^{x^2 - 6x - 7} > 1 \Rightarrow x^2 - 6x - 7 < 0$

$\Rightarrow (x-7)(x+1) < 0$

$x \in (-1, 7)$

Q.27 (A)

$[p + q] = \left[1 + \frac{11}{2}\right] = 6$

Q.28 (B)

Common solution is $(-1, 1] \cup [\frac{11}{2}, 7)$

So, integral values are $0 , 1 , 6$

Q.29 (D)

$3(p+2q+a+b) = 3(1+11+(-1)+7)$

$=54$

$=2\times 3^3$

No of factor $=2\times 4=8$

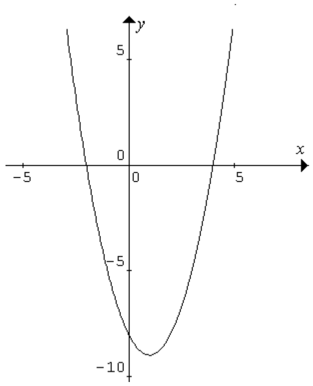
$[x] = 8 \Rightarrow x \in [8 , 9)$

PASSAGE – 4

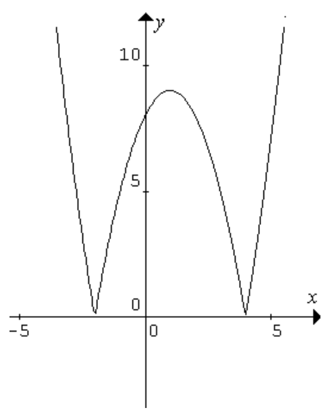
Q.30 (B)

$y=\left|x^2-2x-8\right|$

$f(x)=x^2-2x-8$



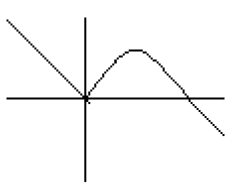
$\rightarrow$



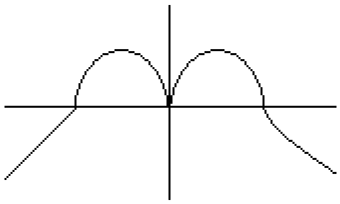
Q.31 (C)

$y=f(x)$

$y=f(|x|)$

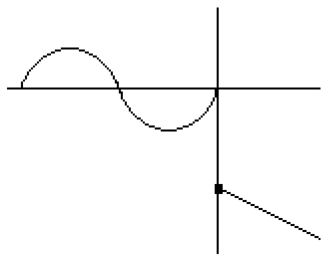


Has the graph same in II & III quad as in I & IV quad.

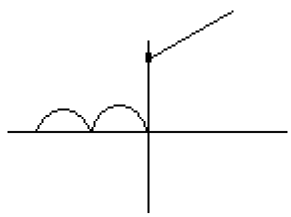


Q.32 (A)

if $y = f(x)$ has graph



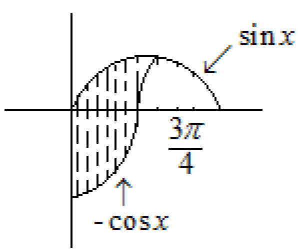
then $y = |f(x)|$ has graph



MATRIX MATCH TYPE

Q.33

(A) for $x \in (0, \pi)$

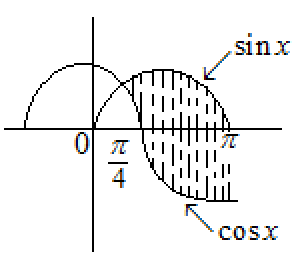


$\sin x + \cos x > 0$

$\Rightarrow \sin x > -\cos x$

$x\left(0, \frac{3\pi}{4}\right)$ ....(R)

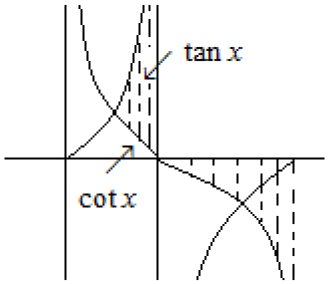
(B)



$\sin x > \cos x$

$x \in \left(\frac{\pi}{4}, \pi\right)$ ....(S)

(C)



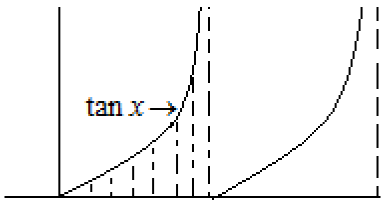
$\tan x - \cot x > 0$

$\tan x > \cot x$

$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

.... (θ)

(D)



$\tan x + \cot x > 0$

$\tan x > -\cot x$

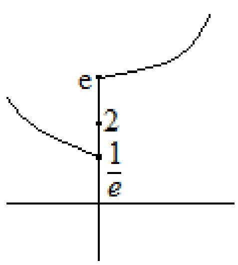
Q.34

(A) $f : R \rightarrow R,\ f(x) = e^{\text{sgn}\, x} + e^{x^2}$

$= \frac{1}{e} + e^{x^2}\ ;\ x < 0$

$f(x) \quad = 2\ ;\ x = 0$

$= e + e^{x^2}\ ;\ x > 0$



Neither either nor odd

many – one

(R,T)

(B) $f : (-1,1) \rightarrow R,\ f(x) = x\,[x^4] + \frac{1}{\sqrt{1-x^2}}$

$= 0 + \frac{1}{\sqrt{1-x^2}}$

$\because f(-x) = f(x)$

$\because$ even function So, may – one.

$(Q\ ,\ T)$

(C) $f:R\to\ R,\ f(x)=\dfrac{x(x+1)(x^4+1)+2x^4+x^2+2}{x^2+x+1}$

$=\dfrac{(x^4+1)(x(x+1)+2)+x^2}{x^2+x+1}$

$=\dfrac{(x^4+1)(x^2+x+1)+x^4+x^2+1}{x^2+x+1}$

$=x^4+1+x^2-x+1$

$=x^4+x^2-x+2$

$f(-x)=x^4+x^2+x+2$

So, neither odd nor even.

$f'(x)$ is a degree equation so at least 1 root.

Hence, not monotonic.

So, (R ,T)

(D) $f:R\to R,\ f(x)=x+3x^3+5x^5+.............101\times 101$

$f'(x)=1+9x^2+25x^4+.............101^2\times 100\ \ >0$ for $x\in R$

Hence, one – one and odd functions.

$\because\ f(-x)=-f(x)$

Q.35

(A) $f:[-1\ ,\ \infty)\to\ (0,\infty)$

$f'(x)=e^{x^2-x}\ \ ;\ x\in[-1,0]$

$=e^{x^2+x}\ ;\ x>0$

$f'(x)=0$ at $x=\dfrac{1}{2}$ for $x<0$

$f'(x) = 0$ at $x = -\frac{1}{2}$ for $x > 0$

(B) $f : (1, \infty) \to [3, \infty)$

$f(x) = \sqrt{10 - 2x + x^2}$

$= \sqrt{(x-1)^2 + 9}$

For, $x \geq 1$, $f(x) > 3$

Hence, $f(x)$ is never equal to 3 in $(1, \infty)$

So, into, one – one, non – periodic.

(P , Q)

(C) $f : R \to \text{I}$

$f(x) = \tan^5 \pi[x^2 + 2x + 3]$

$= \tan^5 \pi[(x+1)^2 + 2]$

For, $x \in R$, $[(x+1)^2 + 2]\pi$ is a multiple of π

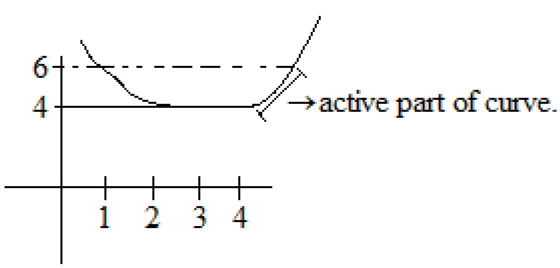
So, $f(x) = 0 \ \forall \ x \in R$

Hence, periodic, many – one into

(Q, R , T)

(D)

$f : [3, 4] \to [4, 6]$



So, one – one, onto.

FUNCTIONS

EXERCISE - 2(B)

Q.1 [03]

$\sin\frac{2x}{3}+\cos 4x+|\tan 3x|+\text{sgn}(x^2+4x+15)$ has period as LCM of $\left(\frac{2\pi\times 3}{2}\ ,\ \frac{2\pi}{4}\ ,\ \frac{\pi}{3}\right)$

$\because \text{sgn}(x^2+4x+15)=1$ as $x^2+4x+15>0$ for all $x,$ so period can be any real number.

LCM of $\left(3\pi,\frac{2\pi}{2},\frac{\pi}{3}\right)$ is 3π.

So, k = 3.

Q.2 [05]

$[x]-\{x\}=\frac{x}{3} \Rightarrow 3\left([x]-\{x\}\right)=[x]+\{x\}$

$\Rightarrow \{x\}=\frac{[x]}{2}$

$\because 0 \le \{x\} < 1 \Rightarrow 0\le\frac{[x]}{2}<1$

$\Rightarrow [x]=0,1$ & $\{x\}=0,\ \frac{1}{2}$

So, $x=\{x\}+[x]$ gives $x=0,\ \frac{3}{2}$

So, sum of values of x, $\lambda=0+\frac{3}{2}$

Hence, value of $\frac{10\lambda}{3}=\frac{10}{3}\times\frac{3}{2}=5$

Q.3 [02]

$f(x)+f(y)+f(xy)=2+f(x)\cdot f(y)$

at x = 1 , y = 1, $3f(1)=2+f(1)^2$

$\Rightarrow f(1)^2 - 3f(1) + 2 = 0$

$\Rightarrow f(1) = 2$ or $f(1) = 1$.

Now at y = 1, $f(x) + f(1) + f(x) = 2 + f(x) \cdot f(1)$

$\Rightarrow f(x)(2 - f(1)) = 2 - f(1)$

$$\Rightarrow f(x) = \frac{2 - f(1)}{2 - f(1)}$$

Hence if f (1) = 1, then f (x) = 1.

If f (x) = 2, then substitute, y = 1/x to get $f(x) + f\left(\frac{1}{x}\right) + f(1) = 2 + f(x) \cdot f\left(\frac{1}{x}\right)$

$$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$$

$$\Rightarrow f(x) = 1 + \frac{1}{f\left(\frac{1}{x}\right) - 1}$$

Solution of such polynomial is, $f(x) = 1 \pm x^n$ but, $f(1) = 2 \Rightarrow f(x) = 1 + x^4$

but $f(4) = 17 \Rightarrow 1 + 4^n = 17 \Rightarrow n = 2$

$$f(5) = \frac{5^2 + 1}{13} = \frac{26}{13} = 2.$$

Q.4 [01]

$$\left(\frac{x}{1+x^2}\right)^2 + a\left(\frac{x}{1+x^2}\right) + 3 = 0 \Rightarrow \frac{1}{\left(x + \frac{1}{x}\right)^2} + \frac{a}{\left(x + \frac{1}{x}\right)} + 3 = 0$$

$$\Rightarrow 3\left(x + \frac{1}{x}\right)^2 + a\left(x + \frac{1}{x}\right) + 1 = 0.$$

Let $x + \frac{1}{x} = t$, then $\Rightarrow 3t^2 + at + 1 = 0$.

Now range of $x+\frac{1}{x}$ is $(-\infty,-2]\cup[2,\infty)$

Every root of $f(t)=3t^2+at+1=0$ which lies in $(-\infty,-2)\cup(2,\infty)$ gives two values of x and t = 2 or –2 gives one value of x.

Hence exactly two distinct roots are possible when exactly one root lies in (-2, 2) and other root is not equal to -2 or 2.

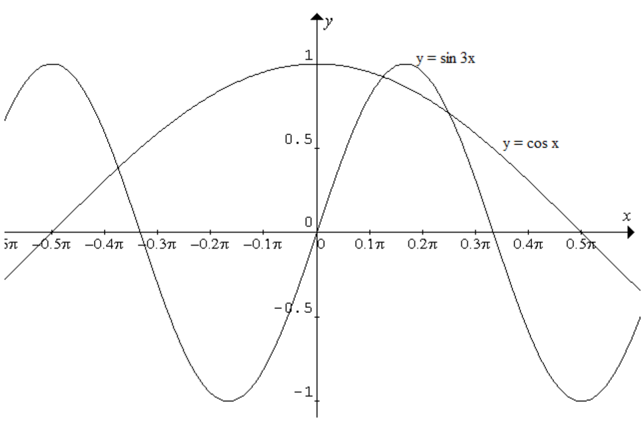
Thus $f(-2)\,f(2)<0$ & $f(\pm 2)\neq 0$

$\Rightarrow (13-2a)(13+2a)<0$

$\Rightarrow a<-\frac{13}{2}$ or $a>\frac{13}{2}$

Hence $\lambda=\mu=\frac{13}{2}\Rightarrow\frac{\lambda+\mu}{13}=1$.

Q.5 [03]



Refer the adjoining graph of

$y=\cos x$ & $y=\sin 3x$

Number of points intersection in

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$,

k = 3

Q.6 [05]

$f(x)=\sqrt{8x-x^2}-\sqrt{14x-x^2-48}$

$=\sqrt{(8-x)x}-\sqrt{(8-x)(x-6)}$

Domain : $6\leq x\leq 8$

Now $f(x)=\sqrt{8-x}\left(\sqrt{x}-\sqrt{x-6}\right)$

$$\Rightarrow f'(x) = -\frac{\sqrt{x} - \sqrt{x-6}}{2\sqrt{8-x}} + \sqrt{8-x}\left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x-6}}\right)$$

$$\Rightarrow f'(x) = \left(\sqrt{x-6} - \sqrt{x}\right)\left(\frac{\sqrt{x-6}\sqrt{x}+8-x}{2\sqrt{8-x}\sqrt{x-6}\sqrt{x}}\right)$$

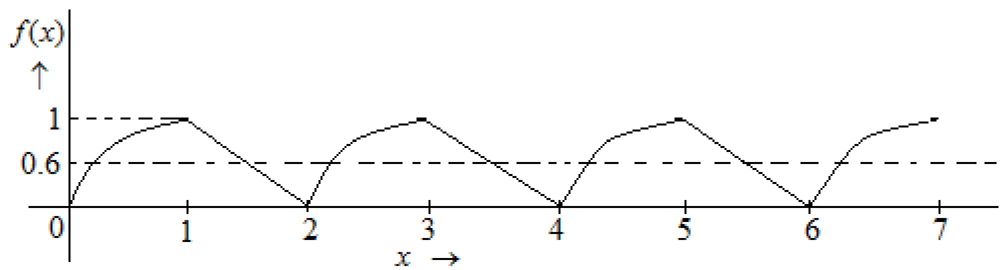
Now $\sqrt{x-6} < \sqrt{x}$ & $\sqrt{x-6}\sqrt{x} > (x-8) \Rightarrow f'(x) < 0$ for $6 \le x \le 8$

Hence $f_{MAX} = f(6) = \sqrt{12}$ & $f_{MIN} = f(8) = 0$.

Thus $m\sqrt{n} = 2\sqrt{3}$.

Q.7 [02]

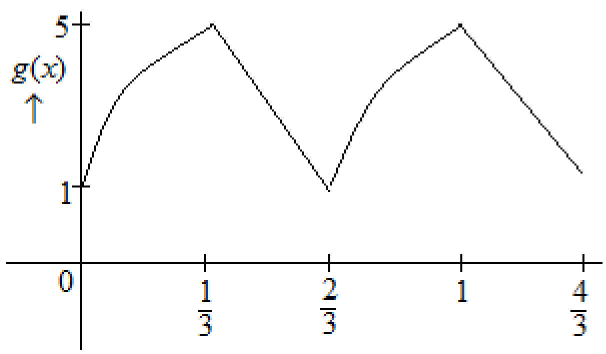
Given $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 2-x & 1 \le x < 2 \\ f(x+2) & \text{for all } x \end{cases}$



f (x) = 0.6 : $\sqrt{x} = 0.6 \quad \Rightarrow x = (0.6)^2 = 0.36$, so sum $= 4+6+2\times 0.36 = 10.72$

& $2-x = 0.6 \Rightarrow x = 0.4$, so sum $= 3+0.4+5+0.4 = 8.8$

A $= 10.72 + 8.8 = 19.52$



Now $g(x) = 4\,f(3x) + 1 \;\forall\; x \in R$

$$\Rightarrow g(x) = \begin{cases} 4\sqrt{3x}+1 & x \in \left[0, \frac{1}{3}\right) \\ 3-4x & x \in \left[\frac{1}{3}, \frac{2}{3}\right) \\ f(3x+2) & x \in \text{all} \end{cases}$$

Fundamental Period $=\left(\frac{2}{3}\right) \Rightarrow$ B $=\frac{2}{3}$.

$g(x)=4f(3x)+1 \Rightarrow g'(x)=12f'(2x)+0$

or $g'\left(\frac{13}{2}\right)=12x\ f'\left(\frac{39}{2}\right)$

g;(6.5) = – 12

So, $|C|=12$

Hence, $\frac{[A]\ B\,|C|}{76}=17\times\frac{2}{3}\times\frac{12}{76}=2$

Q.8 [05]

$x^4-4x^3+6x^2-4x=2008 \Rightarrow (x-1)^4=2009$

$\Rightarrow (x-1)=(2009)^{\frac{1}{4}}, -(2009)^{\frac{1}{4}},\ (2009)^{\frac{1}{4}}i,\ -(2009)^{\frac{1}{4}}i$

So, non-real roots $=1\pm(2009)^{\frac{1}{4}}\cdot i$

product of non-real roots, P $=\left[1+(2009)^{\frac{1}{4}}\cdot i\right]\left[1-(2009)^{\frac{1}{4}}i\right]$

$P=1+(2009)^{\frac{1}{2}}$

So, $[P]=\left[1+(2009)^{\frac{1}{2}}\right]=45$.

Q.9 [03]

Given $f\left(\frac{2x-3}{x-2}\right)=5x-2,\ x\neq 2$

$\Rightarrow$ let , $\frac{2x-3}{x-2}=t$

$\Rightarrow 2x-3=tx-2t$ or $x=\frac{2t-3}{t-2}$

$\Rightarrow f(t) = 5\left(\dfrac{2t-3}{t-2}\right) - 2$

$\Rightarrow f(t) = \dfrac{8t-17}{t-2}$

So, $f(x) = \dfrac{8x-11}{x-2}$

Now let $y = \dfrac{8x-11}{x-2}$

$\Rightarrow x = \left(\dfrac{2y-11}{y-8}\right)$

So, $f^{-1}(x) = \dfrac{2x-11}{x-8}$

$f^{-1'}(13) = \dfrac{26-11}{5} = \dfrac{15}{5} = 3$

Q.10 [04]

$\because P(x)$ has odd degree terms only so $P(-x) = -P(x)$

$P(x)$ divided by $(x-3)$ gives remainder 6 hence $P(3) = 6$

$P(x)$ divided by $(x+3)$ will give remainder $P(-3) = -P(3) = -6$

Now let $P(x) = \left(x^2-9\right)Q\left(x\right) + Ax + B$, where g(x) = Ax + B

So, $P(3)=6 \quad \Rightarrow 3A+B=6$

& $P(-3)=-6 \Rightarrow -3A+B=-6$

Solving simultaneously gives A = 2, B = 0.

g(2) = 4.

Q.11 [04]

$f : R \to \left(0, \dfrac{2\pi}{3}\right], \ f(x) = \cot^{-1}(x^2 - 4x + \alpha)$

For f(x) to be an ONTO function, $0 \le \cot^{-1}(x^2 - 4x + \alpha) \le \dfrac{2\pi}{3}$ for all real x.

or $x^2 - 4x + \alpha \ge \cot\left(\dfrac{2\pi}{3}\right)$.

$\Rightarrow x^2 - 4x + \alpha \ge -\dfrac{1}{\sqrt{3}}$.

$\Rightarrow x^2 - 4x + \left(\alpha + \dfrac{-1}{\sqrt{3}}\right) \ge 0$ for all real x.

So, $D \le 0 \Rightarrow 16 - 4\left(\alpha + \dfrac{1}{\sqrt{3}}\right) \le 0$.

$\Rightarrow \alpha \ge 4 - \dfrac{4}{\sqrt{3}}$.

So, smallest integral value of α is 4.

Q.12 [04]

$f(x) = \sin^{-1} x + \tan^{-1} x + x^2 + 4x + 1 \Rightarrow f(x) = \sin^{-1} x + \tan^{-1} x + (x+2)^2 - 3$

Now for $x \in [-1, 1]$, all of $\sin^{-1} x, \tan^{-1} x$ & $(x+2)^2$ are increasing functions.

Hence p = f(-1) & q = f(1)

Therefore p + q = 4.

Q.13 [00]

$\log_{\sin x} 2^{\tan x} > 0$

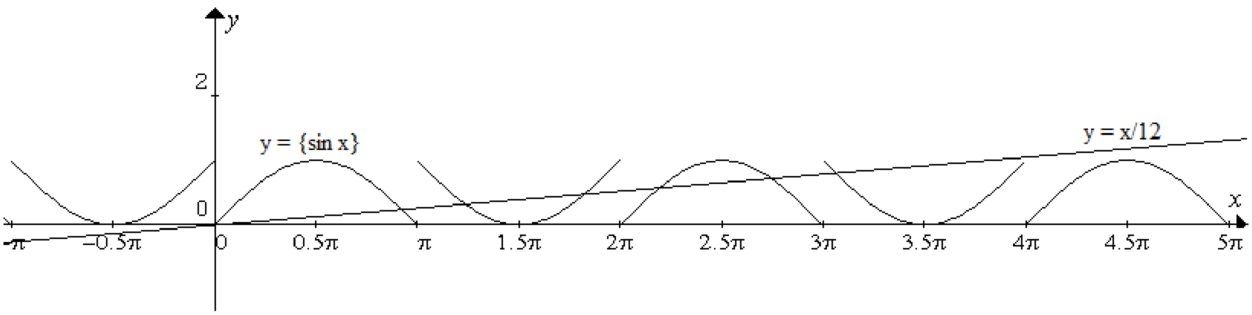
$\Rightarrow (\tan x) \cdot \log_{\sin x} 2 > 0$

$\Rightarrow \dfrac{\tan x}{(\log_2 \sin x)} > 0$

$\tan x > 0$ & $\log_2(\sin x) < 0$ in $\left(0, \dfrac{\pi}{2}\right)$ hence no solution.

{ $\log_a b$ is negative if a > 0 & 0 < a < 1}

Q.14 [07]



$12\{\sin x\} - x = 0$

$\Rightarrow \{\sin x\} = \left(\dfrac{x}{12}\right)$

Refer the adjoining graph.

Q.15 [04]

$[x] + 2\{-x\} = 3x \Rightarrow [x] + 2\{-x\} = 3[x] + 3\{x\}$

Case I : For, $x \in \text{I}$, $\{-x\} = \{x\} = 0$

$\Rightarrow [x] = 3[x]$

$\Rightarrow [x] = 0$

$\Rightarrow x = 0$

Case II : For $x \notin \text{I}$, $[x] + 2(1 - \{x\}) = 3[x] + 3\{x\}$

$\Rightarrow \{x\} = \dfrac{2 - 2[x]}{5}$

Now $0 \le \{x\} < 1$, hence $0 \le \dfrac{2 - 2[x]}{5} < 1$

$\Rightarrow -2 \le -2[x] < 3$

$\Rightarrow -\dfrac{3}{2} < [x] \le -1$

So, $[x] = 1$, $[x] = 0$, $[x] = -1$

$\{x\} = 0, \{x\} = \dfrac{2}{5}$, $\{x\} = \dfrac{4}{5}$

So, $x=1$, $x=\frac{2}{5}, x=-\frac{1}{5}$

Q.16 [02]

$(x)=[x]+1 : x \notin \mathrm{I}$

Hence, $[x]^2+([x]+1)^2<4$

$\Rightarrow 2[x]^2+2[x]-3<0$

So, $[x] \in \left(\frac{-1-\sqrt{7}}{2} , \frac{-1+\sqrt{7}}{2}\right)$

So, $x \in [-1,1)$

Length of interval = 2

Q.17 [02]

$$g(x)=\left(4\cos^4 x-2\cos 2x-\frac{1}{2}\cos 4x-x^7\right)^{\frac{1}{7}}$$

$$\Rightarrow g(x)=\left[4\cos^4 x-4\cos^2 x+2-\frac{1}{2}(2\cos^2 2x-1)-7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x+2-\cos^2 2x+\frac{1}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x-(2\cos^2 x-1)^2+\frac{3}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left[4\cos^4 x-4\cos^2 x-4\cos^4 x+4\cos^2 x-1+\frac{3}{2}-x^7\right]^{\frac{1}{7}}$$

$$=\left(\frac{1}{2}-x^7\right)^{\frac{1}{7}}$$

So, $g(g(x)) = \left[\frac{1}{2} - \left(\frac{1}{2} - x^7\right)^{\frac{1}{7}\times 7}\right]^{\frac{1}{2}}$

$$= \left(\frac{1}{2} - \frac{1}{2} + x^7\right)^{\frac{1}{7}}$$

$$= x$$

So, $\frac{g(g(100))}{50} = \frac{100}{50} = 2$

Q.18 [01]

$f(x) = \frac{3x-2}{x+4} = y \Rightarrow 3x-2 = xy+4y$

$\Rightarrow x = \left[\frac{4y+2}{3-y}\right]$

So, $f^{-1}(x) = \frac{4x+2}{3-x} = \frac{x+\frac{1}{2}}{\frac{3}{4} - \frac{x}{4}}$.

Hence $b = \frac{1}{2}, c = -\frac{1}{4} \;\&\; d = \frac{3}{4} \Rightarrow b+c+d = 1.$

Q.19 [02]

$f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$

$f(x) = -f(x)$

Hence, $f(-5) = -f(5) = -(-28) = 28$

So, $f\frac{(-5)}{14} = \frac{28}{14} = 2$.

Q.20 [01]

$$\log_2(3-x)+\log_{\frac{1}{2}}\left(\frac{\sin\frac{9\pi}{4}}{5-x}\right) = \cos\frac{11\pi}{3} - \log_{\frac{1}{2}}(x+7)$$

Domain : $x < 3$, $x > -7$

Sol : $\log_2(3-x)+\log_{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} - \log_{\frac{1}{2}}(5-x) = \frac{1}{2} - \log_{\frac{1}{2}}(x+7)$

$\Rightarrow \log_2(3-x)+\log_2(5-x)-\log_2(x+7)=0$

$\Rightarrow \frac{(3-x)(5-x)}{x+7}=1$

$\Rightarrow x^2-8x+15=x+7$

$\Rightarrow x^2-9x+8=0$

$\Rightarrow (x-1)\ (x-8)=0$

$\Rightarrow x=1$, $x=8$ but, $x\in(-7, 3)$, hence only one integral value of x is possible.

FUNCTIONS

EXERCISE – 2(C)

Q.1

(i) $f(x) = \sqrt{x^2 - |x| - 2}$

For the function to be defined, $x^2 - |x| - 2 \geq 0$

$\Rightarrow (|x|+1)(|x|-2) \geq 0$

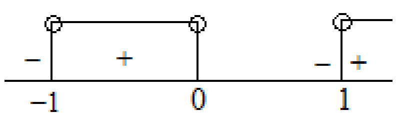
$\Rightarrow |x| - 2 \geq 0$ or $|x| \geq 2$

Hence $x \in (-\infty, -2] \cup [2, \infty)$

(ii) $f(x) = \dfrac{1}{4-x^2} + \log_5(x^3 - x)$

For $f(x)$ to be defined, $4 - x^2 \neq 0$ & $x^3 - x > 0$

$\Rightarrow x \neq \pm 2$ & $x(x-1)(x+1) > 0$



Hence domain is $x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$ \

Q.2

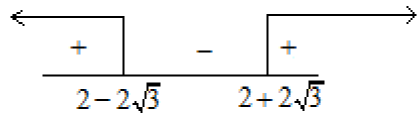
(i) $f(x) = \dfrac{x^2 + 2x + 3}{x}$

Let $y = \dfrac{x^2 + 2x + 3}{x}$

Then $x^2 + x(2 - y) + 3 = 0$

For x to be real $D \geq 0 \Rightarrow (2-y)^2 - 12 \geq 0$

or $y^2 - 4y - 8 \geq 0$



Hence range is $(-\infty, 2-2\sqrt{3}] \cup [2+2\sqrt{3}\ ,\ \infty)$

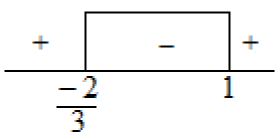
(ii) $f(x) = \dfrac{x^2-2}{x^2+3}$

Let $y = \dfrac{x^2-2}{x^2+3}$

Then $x^2 = \dfrac{-(3y+2)}{y-1}$

Now $\dfrac{-(3y+2)}{y-1} \geq 0$

or $\dfrac{3y+2}{y-1} \leq 0$



Hence range is $\left[-\dfrac{2}{3}, 1\right)$.

(iii) $f(x) = 3\cos x - 4\sin x + 2$

max. & min. value of $3\cos x - 4\sin x$ is 5 & -5 respectively.

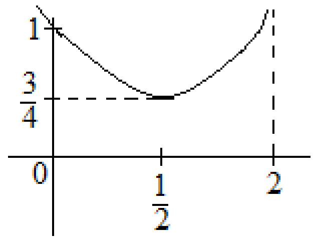
$\left\{-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}\right\}$

$f(x)]_{\max} = 5+2 = 7$

$f(x)]_{\min} = -5+2 = -3$

(iv) $f(x) = [x^2 - x + 1]$ Graph of y = $y = x^2 - x + 1$

min. value at $x=\frac{1}{2}$ & max. value at x = 2 for x^2-x+1 when $0 \le x \le 2$



$f(x)]_{\min} = 0$

$f(x)]_{\max} = 3$

Q.3

(I)
Case I
$x > 0 \Rightarrow \text{sgn}(x) = 1$
& |x| = x
Hence x sgn (x) = |x|
Case II
$x = 0 \Rightarrow \text{sgn}(x) = 0$
$\& \; |x| = 0$
Hence x sgn (x) = |x|
Case III
$x < 0 \Rightarrow \text{sgn}(x) = -1$
& |x| = - x
Hence x sgn (x) = |x|

CORRECT

(II)
Case I
$x > 0 \Rightarrow \text{sgn}(x) = 1$
& |x| = x.
Hence |x| sgn (x) = x
Case II
$x = 0 \Rightarrow \text{sgn}(x) = 0$
$\& \; |x| = 0$
Hence |x| sgn (x) = 0
Case III
$x < 0 \Rightarrow \text{sgn}(x) = -1$
& |x| = - x
Hence | x| sgn (x) = x

CORRECT

(III)
Case I
$x > 0 \Rightarrow \text{sgn}(x) = 1$
Hence
x (sgn (x))^2 = x
Case II
$x = 0 \Rightarrow \text{sgn}(x) = 0$
hence
x (sgn (x))^2 = 0
Case III
$x < 0 \Rightarrow \text{sgn}(x) = -1$
Hence
x (sgn (x))^2 = x

CORRECT

(IV)
Case I
$x > 0 \Rightarrow \text{sgn}(x) = 1$
& |x| = x. Hence
|x| (sgn (x))^3 = x
Case II
$x = 0 \Rightarrow \text{sgn}(x) = 0$
$\& \; |x| = 0$, hence
|x| (sgn (x))^3 = 0
Case III
$x < 0 \Rightarrow \text{sgn}(x) = -1$
& |x| = - x, Hence
|x| (sgn (x))^3 = x

CORRECT

Q.4

(i) $f(x) = \log_{10}\left(\frac{1-x}{1+x}\right) \Rightarrow f(-x) = \log_{10}\left(\frac{1+x}{1-x}\right)$

Now $f(x) + f(-x) = \log 1$

$\Rightarrow f(x) + f(-x) = 0$ or $f(-x) = -f(x)$

Hence $f(x)$ is odd.

(ii) $f(x) = \frac{x(2^x+1)}{2^x-1} \Rightarrow f(-x) = -\frac{x(2^{-x}+1)}{2^{-x}-1}$

$\Rightarrow f\left(x\right)=-\dfrac{x\,(1+2^x)}{1-2^x}$ or $f\left(-x\right)=\dfrac{x\,(1+2^x)}{2^x-1}$

$\Rightarrow f(-x) = f(x)$

Hence $f(x)$ is even.

(iii) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2} \Rightarrow f(-x)= \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$

$\Rightarrow f(x)+f(-x)=0$ or $f\left(-x\right)=-f\left(x\right)$

Hence $f(x)$ is odd.

(iv) $f(x)=(2x^4-5x^2+3)\cos x$

Product of two even function is even only.

Hence $f(x)$ is even.

Q.5

Let $y=\dfrac{x-2}{x+3}$

or $yx+3y=x-2$

$\Rightarrow x=\dfrac{3y+2}{1-y}$

Range : R – {1}

Now $\mathrm{f}\left(\mathrm{x}_1\right)=\mathrm{f}\left(\mathrm{x}_2\right)\Rightarrow\dfrac{\mathrm{x}_1-2}{\mathrm{x}_1-3}=\dfrac{\mathrm{x}_2-2}{\mathrm{x}_2-3}$

$\Rightarrow \mathrm{x}_1\mathrm{x}_2-3\mathrm{x}_1-2\mathrm{x}_2+6=\mathrm{x}_1\mathrm{x}_2-2\mathrm{x}_1-3\mathrm{x}_2+6$

$\Rightarrow \mathrm{x}_1=\mathrm{x}_2$.

Hence f (x) is ONE – ONE & ONTO.

Further $x=\dfrac{-2-3y}{y-1}$ implies $f^{-1}(x)=\dfrac{-2-3x}{x-1}=\dfrac{3x+2}{1-x}$.

Q.6

Let $y = x(2-x)$

$\Rightarrow x^2 - 2x + y = 0$

or $x = 1 \pm \sqrt{1-y}$

Now $x \in (-\infty, 1] \to x = 1 - \sqrt{1-y},\ y \in (-\infty, 1]$

Hence $f(x)$ is ONTO.

Further $x_1(2-x_1) = x_2(2-x_2) \Rightarrow 2(x_1 - x_2) = (x_1^2 - x_2^2)$

$\Rightarrow x_1 = x_2$ or $x_1 + x_2 = 2$

But if $x_1 \neq x_2$, then as $x_1, x_2 \geq 1$ & $x_1 + x_2 \neq 2$.

Hence $f(x)$ is ONE – ONE.

Now $x = 1 - \sqrt{1-y}$ gives

$f^{-1}(x) = 1 - \sqrt{1-x},\ f^{-1} : (-\infty, 1] \to (-\infty, 1]$

Q.7

$$f(x) = \frac{1}{1-x} \Rightarrow f(f(x)) = \frac{1}{1-f(x)} \quad \text{or} \quad f\left(f(x)\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$$

$$f\left(f\left(f(x)\right)\right) = \frac{f(x)-1}{f(x)} \quad \text{or} \quad f\left(f\left(f(x)\right)\right) = \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}} = x$$

$$f\left(f\left(f\left(f(x)\right)\right)\right) = f(x)$$

It is repeating after every interval of 4.

So, $f^{2006}(x) = f^{(4\times 501+2)}(x)$

$$= f^2(x) = \frac{x-1}{x}$$

$$f^{2006}(2005) = \frac{2005-1}{2005} = \frac{2004}{2005}.$$

Q.8

$3x = [\sin x + [\sin x + [\sin x]]] \Rightarrow 3x = [3\sin x] \quad \therefore\ [x+n] = [x] + n$ for $n \in I$

In R.H.S. there can be only integers {-3, -2, -1, 0, 1, 2, 3}.

$\Rightarrow \sin \mathrm{x} = -1, -\frac{2}{3}, -\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$

But none of these values except 0 can occur for 3x being an integer thus,

L.H.S. has to be 0 integer only.

Hence possible solutions are $x = \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}$

Q.9

Period of $|\sin x| + |\cos x|$ is $\frac{\pi}{2}$, because in each quadrant values of |sin x| and |cos x| complement each other.

Now period of $\sin px + \cos px$ is $\frac{2\pi}{p}$.

So $p = 4$

Q.10

$$f(x) = \left[\frac{x^2}{k}\right] \sin x + \cos x$$

$$f(-x) = -\left[\frac{x^2}{k}\right] \sin x + \cos x$$

If f(x) is even, $f(x) = f(-x)$

Hence $\left[\frac{x^2}{k}\right] \sin x = 0$

$$\Rightarrow \left[\frac{x^2}{k}\right] = 0$$

Thus $0 \le \frac{x^2}{k} < 1$

As $-5 \le x \le 5$, hence $25 \le x^2 \le 0$.

Hence k > 25.

Q.11

For $f(x) = \log\log\log\log x$ to be defined $\log\log\log x > 0$

$\Rightarrow \log\log x > 1$

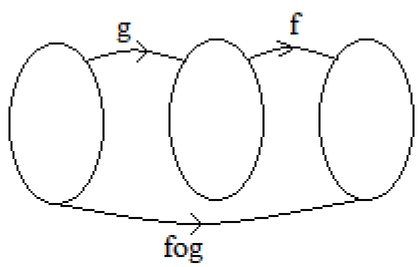
$\Rightarrow \log x > 10$

$x \in (10^{10}, \infty)$

Q.12

$$f(x) = \log_{100_x}\left(\frac{2\log_{10} x + 2}{-x}\right) \qquad g(x) = \{x\}$$

If fog exists, then range of g should come in domain of f.



$$\log_{100x}\left(\frac{2\log_{10} x + 2}{-x}\right)$$

$$\Rightarrow 100x > 0 \ \ \& \ \ 100x \ne 1 \quad \text{as well} \ \frac{2\log_{10} x + 2}{-x} > 0$$

$$\Rightarrow x > 0, \ x \ne \frac{1}{100} \ \ \& \ \ 2\log_{10} x + 2 < 0 \ \text{i.e.} \ x < \frac{1}{10}$$

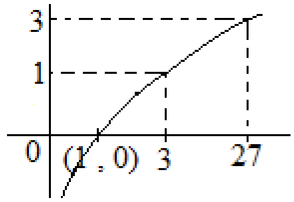
Hence $x \in \left(0, \frac{1}{100}\right) \cup \left(\frac{1}{100}, \frac{1}{10}\right)$

Q.13

(i) $f:[3\,,\,27] \to \text{A}$

$f(x) = \log_3 x$

$\text{A} \in [1\,,\,3]$



(ii) $f(x) = \log_{10}(5x - x^2 - 6)$

For f (x) to be defined $5x - x^2 - 6 > 0$

or $(x-3)(x-2) < 0$

$\Rightarrow x \in (2, 3)$

(iii) $f\left(x+\dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2} \Rightarrow f\left(x+\dfrac{1}{x}\right) = \left(x+\dfrac{1}{x}\right)^2 - 2$

Let $x + \dfrac{1}{x} = t$, then $f(t) = t^2 - 2$

Hence $f(\sqrt{5}) = 3$.

Q.14

(i) $f(x) = 2^{-x^2} = f(-x) = 2^{-x^2}$

Hence, f (x) is even.

(ii) $f(x) = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$

$f(-x) = \dfrac{10^{-x} - 10^x}{10^{-x} + 10^x}$

$f(x) + f(-x) = 0$

Hence, f(x) is odd.

(iii) $f(x) = \log \dfrac{(x^2 - x + 1)}{x^2 + x + 1}$

$$f(-x) = \log\left(\frac{x^2+x+1}{x^2-x+1}\right)$$

$$f(x)+f(-x) = \log 1 = 0$$

Hence f(x) is odd.

(iv) $f(x) = x\sin x$

$$f(-x) = (-x)\sin(-x)$$

$$= x\sin x$$

Hence, f (x) is even.

Q.15

$$f(x) = \frac{x^2-x}{x^2+2x}$$

<u>Domain : -</u>

$$x^2+2x \neq 0$$

$$x \neq 0,\ x \neq -2$$

$$x \in R - \{0\ ,\ -2\}$$

<u>Range : -</u>

$$y = \frac{x(x-1)}{x(x+2)} \qquad x \neq 0$$

$$y = \frac{x-1}{x+2} \qquad y \in R - \left\{-\frac{1}{2}, -1\right\}$$

Q.16

$$f(x)+f(x+4) = f(x+2)+f(x+6) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Put $x = k+t$

$$f(x+t)+f(z+4+t) = t(x+2+t)+f(x+6+t)$$

Put $t = 2$

$$f(x+2)+f(x+6)=f(x+4)+f(x+8)$$

$$f(x)+f(x+4)=f(x+4)+f(x+8) \quad \text{.... From (1)}$$

$$f(x)=f(x+8)$$

Hence function is periodic.

Period is 8.

Q.17

$$P(x)\cdot P\left(\frac{1}{x}\right)=P(x)+P\left(\frac{1}{x}\right)$$

$$P(x)=1\pm x^n \quad \text{hence} \quad P(x)=1+x^n$$

$$P(4)=65 \Rightarrow n=3$$

Hence P(x) = 1 + x^3.

Now 1 + x^3 = 344 gives x = 7.

Q.18

$$f(x)=\frac{9^x}{3+9^x} \Rightarrow f(1-x)=\frac{9^{1-x}}{3+9^{1-x}}=\frac{\frac{9}{9^x}}{\frac{3.9^x+9}{9^x}}=\frac{3}{9^x+3}$$

$$f(x)+f(1-x)=\frac{3+9^x}{3+9^x}=1, \text{ Hence,}$$

$$S=f\left(\frac{1}{2003}\right)+f\left(\frac{2}{2003}\right)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots+f\left(\frac{2002}{2003}\right)$$

$$S=f\left(\frac{2002}{2003}\right)+f\left(\frac{2001}{2003}\right)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots+f\left(\frac{1}{2003}\right)$$

$$2S=2002$$

$$S=1001$$

Q.19

$$P(x)P(y)+2=P(x)+P(y)+P(xy)$$
$$x=1, y=2 \to P(1)P(2)+2=P(1)+2P(2)$$
$$P(2)=5 \Rightarrow P(1)=2$$

Now differentiate w.r.to y treating x as an independent variable to get

Now $P(x)P'(y)=P'(y)+xP'(xy)$

$$y=1 \Rightarrow (P(x)-1)P'(1)=xP'(x)$$
$$\Rightarrow \frac{dP(x)}{P(x)-1}=P'(1)\frac{dx}{x}$$

Integrate w.r.t. x to get

$$\Rightarrow \ln|P(x)-1|=P'(1)\ln|x|+C$$
$$P(1)=2 \to C=0$$

$P(2)=5 \to \ln 4=P'(1)\ln 2$ i.e. $P'(1)=2$

$$\Rightarrow \ln|P(x)-1|=2\ln|x| \Rightarrow P(x)=x^2+1$$

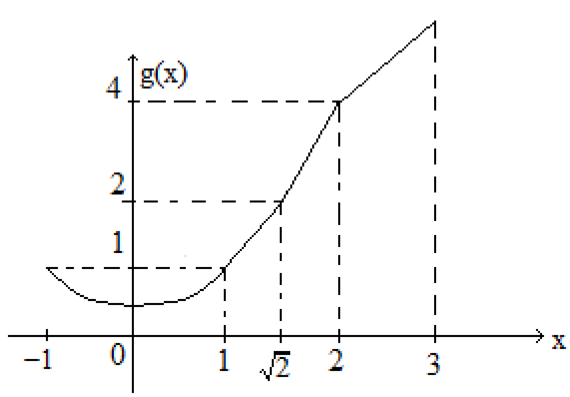
Hence P(5) = 26.

Q.20

$$f(x)=\begin{cases} x+1, & x\le 1 \\ 2x+1, & 1< x\le 2 \end{cases}$$

$$g(x)=\begin{cases} x^2, & -1\le x<2 \\ x+2, & 2\le x<3 \end{cases}$$

$$f(g(x)) = \begin{cases} g(x)+1, & g(x)\le 1 \\ -2g(x)+1, & 1< g(x)\le 2 \end{cases}$$



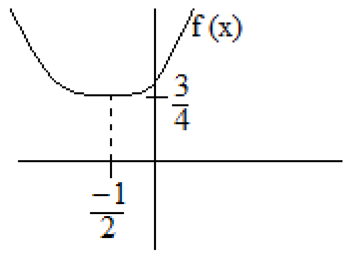
$$f(g(x)) = \begin{cases} g(x), & g(x) \le 1 \\ 2g(x)+1, & 1 < g(x) \le 2 \end{cases}$$

$$= \begin{cases} x^2+1, & -1 \le x \le 1 \\ 2x^2+1, & 1 < x < \sqrt{2} \end{cases}$$

Q.21

$f(x) = x^2 + x + 1$

(i) Range $\equiv [\frac{3}{4}, \infty)$



(ii) Reflection in y − axis

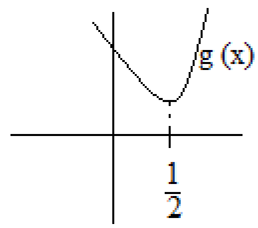
$g(x) = x^2 - x + 1$

$y = x^2 - x + 1$

$x^2 - x + 1 - y = 0$

$$x = \frac{1 \pm \sqrt{1-4(1-y)}}{2}$$

$$x = \frac{1 \pm \sqrt{4y-3}}{2}$$



$$g^{-1}(x) = \frac{1+\sqrt{4x-3}}{2}$$

Q.22

(a) Given $f(f(x))(1+f(x)) = -f(x)$

Let $f(a) = b$, then $f(b)(1+b) = -b$

or $f(b) = -\dfrac{b}{1+b}$

Hence $f(3) = -\dfrac{3}{4}$.

(b) Given $f(x+f(x)) = 4f(x)$

$x = 1 \rightarrow f(1+f(1)) = 4f(1)$ or $f(5) = 16$

$x = 5 \rightarrow f(5+f(5)) = 4f(5)$ or $f(21) = 64$

(c) Given $\left(f\left(xy\right)\right)^2 = x\left(f\left(y\right)\right)^2$.

$x = 25, y = 2 \rightarrow \left(f\left(50\right)\right)^2 = 25\left(f\left(2\right)\right)^2$ *or* $f\left(50\right) = 30$.

(d) Given $f\left(x+y\right) = x + f\left(y\right)$

$x = 1, y = 0 \rightarrow f\left(1\right) = 3$

$x = 1, y = 1 \rightarrow f\left(2\right) = 4$

$x = 1, y = 2 \rightarrow f\left(3\right) = 5$

$\Rightarrow f\left(100\right) = 102$

(e) Given $f\left(3x\right) = x + f\left(3x-3\right)$

$x = 2 \rightarrow f\left(6\right) = 3$

$x = 3 \rightarrow f\left(9\right) = 4$

$\Rightarrow f\left(3x\right) = x + 1$

$\Rightarrow f\left(300\right) = 101$

Q.23

(a) $f(x) + f\left(\frac{1}{x}\right) = x$

Replace x by $\frac{1}{x}$ to get $f\left(\frac{1}{x}\right) + f\left(x\right) = \frac{1}{x}$

$\Rightarrow x = \frac{1}{x}$

Hence $x = \pm 1$.

(b) $f\left(x\right) = \sqrt{ax^2 + bx}$

Domain and range can be same only if f (x) is self-inverse.

y = $\sqrt{ax^2 + bx}$

If a = 0, then $y = \sqrt{bx}$ has domain as well as range $\left[0, \infty\right)$ for all b > 0.

Now $y = \sqrt{ax^2 + bx} \Rightarrow y^2 = x\left(ax + b\right)$

$$\Rightarrow \text{Domain}: \begin{cases} \left(-\infty, -\frac{b}{a}\right] \cup [0, \infty) & \text{if } a > 0 \\ \left[0, -\frac{b}{a}\right] & \text{if } a < 0 \end{cases} \quad \& \text{ Range}: \begin{cases} [0, \infty) & \text{if } a > 0 \\ \left[0, \sqrt{-\frac{b^2}{4a}}\right] & \text{if } a < 0 \end{cases}$$

Clearly for a > 0 interval of x & interval of y can't be same but for a < 0, the two intervals can be same if

$$-\frac{b}{a} = \sqrt{-\frac{b^2}{4a}} \text{ i.e. } \frac{b^2}{a^2} = -\frac{b^2}{4a} \Rightarrow a = -4.$$

Q.24

(i)

(a)

$10^x + 10^y = 10$

$10^y = 10 - 10^x$

$\log 10^y = \log(10 - 10^x)$

$y = \log(10 - 10^x)$

(b)

$x + |y| = 2y$

If $y > 0$

$x + y = 2y$

$y = x$

If $y < 0$

$x - y = 2y$

$y = \frac{x}{3}$

(ii)

(a)

$f(x) \to [0, 1]$

$f(\sin x)$

$0 \le \sin x \le 1$

$x \in [0, \pi]$

$x \in [2n\pi, (2n+1)\pi]$

$\boxed{n \in I}$

(b)

$f(2x+3)$

$0 \le 2x + 3 \le 1$

$-3 \le 2x \le -2$

$\frac{-3}{2} \le x \le -1$

(iii)

(a)

$g(x) = \frac{1}{3} + (x)$

Domain remains same $[4, 7]$

Range is $\left[\frac{-1}{3}, \frac{9}{3}\right]$ i.e. $\left[\frac{-1}{3}, 3\right]$

(b)

$h(x) = f(x-7)$

$4 \le x - 7 \le 7$

Domain is $[11, 14]$

$11 \le x \le 14$

Range will not change i.e. $[-1, a]$

Q.25

(a) $y = \ln\left(x + \sqrt{x^2+1}\right)$

Domain : R, Range : R

Also $x_1 + \sqrt{x_1^{\,2}+1} = x_2 + \sqrt{x_2^{\,2}+1} \Rightarrow x_1 = x_2$, hence f (x) is invertible.

Now $y = \ln\left(x + \sqrt{x^2+1}\right) \Rightarrow e^y = x + \sqrt{x^2+1}$

$\Rightarrow e^{-y} = \sqrt{x^2+1} - x$

$\Rightarrow \frac{e^y - e^{-y}}{2} = x$

Hence $f^{-1}(x) = \frac{e^x - e^{-x}}{2}$, $f^{-1} : \mathrm{R} \to \mathrm{R}$

(b) $f(x) = 2^{\frac{x}{x-1}}$

Domain : R – {1}.

Range of $\frac{x}{x-1}$: $\mathrm{R} - \{1\}$, hence Range of f(x) : (0, ∞) – {2}.

Further $\frac{x_1}{x_1 - 1} = \frac{x_2}{x_2 - 1} \Rightarrow x_1x_2 - x_1 = x_1x_2 - x_2$ or $x_1 = x_2$

Hence f (x) is invertible.

Now let $y = 2^{\frac{x}{x-1}}$

$$\Rightarrow \log_2 y = \frac{x}{x-1}$$

or $x = \dfrac{\log_2 y}{\log_2 y - \log_2 2}$

Hence $f^{-1}(x) = \dfrac{\log_2 x}{\log_2 \frac{x}{2}},\ f^{-1} : R - \{2\} \to R - \{1\}$

(c) $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$

Domain : R, Range : R – {1}

Further $\dfrac{10^{x_1} - 10^{-x_1}}{10^{x_1} + 10^{-x_1}} = \dfrac{10^{x_2} - 10^{-x_2}}{10^{x_2} + 10^{-x_2}} \Rightarrow 10^{x_1 - x_2} - 10^{x_2 - x_1} = 10^{x_2 - x_1} - 10^{x_1 - x_2}$

$\Rightarrow 10^{x_1 - x_2} = 10^{x_2 - x_1}$ or $x_1 = x_2$.

Hence f (x) is invertible.

Now $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}} \Rightarrow y = \dfrac{10^{2x} - 1}{10^{2x} + 1}$

or $10^{2x} = \dfrac{y+1}{1-y}$

or $2x = \log_{10} \dfrac{y+1}{1-y}$

$\Rightarrow f^{-1}(x) = \dfrac{1}{2} \log_{10} \dfrac{x+1}{1-x},\ f^{-1} : R - \{1\} \to R$.

Q.26

Case I : $x - \dfrac{1}{2} > 0$

$\left[x - \dfrac{1}{2}\right]\left[x + \dfrac{1}{2}\right]$ can be a prime number only if one of the two factors is 1 & other is a prime.

Now $\left[x-\frac{1}{2}\right]=1 \Rightarrow 1 \le x-\frac{1}{2}<2$ *i.e.* $\frac{3}{2} \le x < \frac{5}{2}$.

For this interval $2 \le x+\frac{1}{2}<3$, so $\left[x+\frac{1}{2}\right]=2$.

Hence $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]=2$ for $\frac{3}{2} \le x < \frac{5}{2}$

Similarly $\left[x+\frac{1}{2}\right]=1 \Rightarrow 1 \le x+\frac{1}{2}<2$ *i.e.* $\frac{1}{2} \le x < \frac{3}{2}$.

For this interval $0 \le x-\frac{1}{2}<1$, so $\left[x+\frac{1}{2}\right]=0$.

Not possible.

Case II : $x+\frac{1}{2}<0$

$\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]$ can be a prime number only if one of the two factors is -1 & other is negative of a prime.

Now $\left[x+\frac{1}{2}\right]=-1 \Rightarrow -1 \le x+\frac{1}{2}<0$ *i.e.* $-\frac{3}{2} \le x < -\frac{1}{2}$.

For this interval $-2 \le x-\frac{1}{2}<0$, so $\left[x-\frac{1}{2}\right]=-2,-1$.

Hence $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]=2$ for $-\frac{3}{2} \le x < -\frac{1}{2}$

Similarly $\left[x-\frac{1}{2}\right]=-1 \Rightarrow -1 \le x-\frac{1}{2}<0$ *i.e.* $-\frac{1}{2} \le x < \frac{1}{2}$.

For this interval $0 \le x+\frac{1}{2}<1$, so $\left[x+\frac{1}{2}\right]=0$.

Not possible.

Hence $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]=2$ for $\left[-\frac{3}{2},-\frac{1}{2}\right)\cup\left[\frac{3}{2},\frac{5}{2}\right)$

Now $x_1^2+x_2^2+x_3^2+x_4^2=\frac{9+1+9+25}{4}=11$.

Q.27

Let $P(x)=(x-1)(x-4)Q(x)+ax+b$, where r(x) = ax + b.

Now given that P(1) = 1 & P(4) = 10, hence

a + b = 1 & 4a + b = 10.

Thus a = 3 & b = -2.

Now r(x) = 3a – 2.

Hence r(2006) = 6016.

Q.28

(i) Given $2f(x)+x f\left(\frac{1}{x}\right)-2f\left(\left|\sqrt{2}\sin\left(\pi\left(x+\frac{1}{4}\right)\right)\right|\right)=4\cos^2\frac{\pi x}{2}+x\cos\frac{\pi}{x}$

$x=1\rightarrow 2f(1)+f(1)-2f\left(\left|\sqrt{2}\sin\left(\frac{\pi}{4}\right)\right|\right)=4\cos^2\frac{\pi}{2}+\cos\pi$

$\Rightarrow f(1)=-1$

$x=2\rightarrow 2f(2)+2f\left(\frac{1}{2}\right)-2f\left(\left|\sqrt{2}\sin\left(\frac{9\pi}{4}\right)\right|\right)=4\cos^2\pi+2\cos\frac{\pi}{2}$

$\Rightarrow 2f(2)+2f\left(\frac{1}{2}\right)=4+2f(1)$

$\Rightarrow f(2)+f\left(\frac{1}{2}\right)=1$

(ii) $x=\frac{1}{2}\rightarrow 2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)-2f\left(\left|\sqrt{2}\sin\left(\frac{3\pi}{4}\right)\right|\right)=4\cos^2\frac{\pi}{4}+\frac{1}{2}\cos 2\pi$

$\Rightarrow 4f\left(\frac{1}{2}\right)+f(2)-4f(1)=5$

$$\Rightarrow f(2)+4f\left(\frac{1}{2}\right)=1 \Rightarrow f(2)=1$$

$$\Rightarrow f(2)+f(1)=0.$$

Q.29

$4\{x\}=x+[x]$ & $[x]=x+\{x\} \Rightarrow 3\{x\}=2x$

As $0\le\{x\}<1,\ 0\le x<\frac{3}{2}$

Case I : $0\le x<1$

$4\{x\}=x+[x] \Rightarrow 4x=x$ or $x=0$

Case II : $1\le x<\frac{3}{2}$

$4\{x\}=x+[x] \Rightarrow 4(x-1)=x+1$ or $x=\frac{5}{3}$.

Q.30

As a, b, c are natural numbers hence x > 0.

Now $\left[\frac{3}{x}\right]+\left[\frac{4}{x}\right]=5 \Rightarrow \left[\frac{3}{x}\right]=n$ & $\left[\frac{4}{x}\right]=5-n$

$$\Rightarrow n\le\frac{3}{x}<n+1 \quad \& \quad 5-n\le\frac{4}{x}<6-n$$

$$\Rightarrow \frac{3}{n+1}<x\le\frac{3}{n} \quad \& \quad \frac{4}{6-n}<x\le\frac{4}{5-n}$$

$n=1 \to \frac{3}{2}<x\le 3$ & $\frac{4}{5}<x\le 1 \Rightarrow$ no solution

$n=2 \to 1<x\le\frac{3}{2}$ & $1<x\le\frac{4}{3} \Rightarrow 1<x\le\frac{4}{3}$

$n=3 \to \frac{3}{4}<x\le 1$ & $\frac{4}{3}<x\le 2 \Rightarrow$ no solution

Hence $x \in \left(1, \frac{4}{3}\right]$.

FUNCTIONS

EXERCISE – 3

Q.1

$g(x) = f(x) + f(-x)$ is an even function.

$h(x) = f(x) - f(-x)$ is an odd function.

Now $g(x) + h(x) = 2f(x)$ or $f(x) = \frac{1}{2}g(x) + \frac{1}{2}h(x)$.

Hence any function *f* (x) can be represented as sum of one even and one odd function.

Q.2

$f(x+T) = f(x) \Rightarrow \sqrt{|\cos(x+T)|} = \sqrt{|\cos x|}$

$\Rightarrow \cos^2(x+T) = \cos^2 x$

$\Rightarrow x+T = n\pi \pm x$

$\Rightarrow T = n\pi$

Hence Fundamental period is π.

Q.3

$f : \left[\frac{1}{2}, \infty\right) \to \left[\frac{3}{4}, \infty\right), \; f(x) = x^2 - x + 1$

$f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$.

So f (x) is one – one in $\left[\frac{1}{2}, \infty\right)$

Also as range is $\left[\frac{3}{4}, \infty\right)$ so *f* (x) is onto function.

Hence *f* (x) is bijective.

Now let $y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$, then $x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$.

$\Rightarrow f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$.

Now for a quadratic function, $f(x) = f^{-1}(x) \Rightarrow f(x) = x$.

$\Rightarrow x^2 - x + 1 = x$ or x = 1.

Q.4

Number of ways to choose three f – images from {1, 2, …, 5} = $^5C_3$.

These can be associated with elements of domain in just one way.

Hence total number of functions = 10.

Q.5

$$f(x)+f\left(\frac{x-1}{x}\right)=1+x \qquad ...(1)$$

$$f\left(\frac{x-1}{x}\right)+f\left(\frac{\left(\frac{x-1}{x}\right)-1}{\left(\frac{x-1}{x}\right)}\right)=1+\left(\frac{x-1}{x}\right) \qquad \left[\text{puttin } x\to\frac{x-1}{x} \text{ in (1)}\right]$$

$$\text{i.e } f\left(\frac{x-1}{x}\right)+f\left(\frac{1}{1-x}\right)=2-\frac{1}{x} \qquad ...(2)$$

puttin $x\to\frac{1}{1-x}$ **in (1)**

$$f\left(\frac{1}{1-x}\right)+f(x)=1+\frac{1}{1-x}$$

$$f\left(\frac{1}{1-x}\right)=1+\frac{1}{1-x}-f(x) \qquad \text{puttin } f\left(\frac{1}{1-x}\right) \text{ in (2)}$$

$$f\left(\frac{x-1}{x}\right)+1+\frac{1}{1-x}-f(x)=2-\frac{1}{x}$$

$$\therefore f\left(\frac{x-1}{x}\right)=\left(1-\frac{1}{x}\right)-\left(\frac{1}{1-x}\right)+f(x) \qquad \text{puttin } f\left(\frac{x-1}{x}\right) \text{ in (1)}$$

$$f(x)+\frac{x-1}{x}+\frac{1}{x-1}+f(x)=1+x$$

$$\therefore 2f(x)=(1+x)-\frac{(x-1)}{x}-\frac{1}{(x-1)}$$

$$=\frac{x(x+1)(x-1)-(x-1)^2-x}{x(x-1)}=\frac{x^3-x-\left[x^2-2x+1\right]-x}{x(x-1)}$$

$$2f(x)=\frac{x^3-x^2-1}{x(x-1)}$$

$$\therefore f(x)=\frac{x^3-x^2-1}{2x(x-1)}$$

Q.6

$f(x)+f(x+2)=\sqrt{3}f(x+1)$ ...(1)

Replacing x by x – 1 gives $f(x-1)+f(x+1)=\sqrt{3}f(x)$ ...(2)

Replacing x by x + 1 gives $f(x+1)+f(x+3)=\sqrt{3}f(x+2)$ ...(3)

Adding (1) & (2) gives $f(x-1)+2f(x+1)+f(x+3)=\sqrt{3}\left(f(x)+f(x+2)\right)$ ...(4)

From (1), $f(x-1)+f(x+3)=f(x+1)$ (5).

Replacing x by x + 2 gives $f(x+1)+f(x+5)=f(x+3)$ ...(6)

Adding (5) & (6) gives $f(x+5)=-f(x-1)$

Replacing x by x + 6 gives $f(x+11)=-f(x+5)\Rightarrow f(x+11)=f(x-1)$

Hence *f* (x) is periodic with period 12.

Now *f* (5 + 12r) = *f* (5) = 3 thus $\sum_{r=0}^{99} f(5+12r)=100f(5)=300$.

Q.7

$$f(x+T)=\frac{f(x)-5}{f(x)-3}\Rightarrow f(x+2T)=\frac{f(x+T)-5}{f(x+T)-3}$$

$$\Rightarrow f(x+2T)=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}$$

$$\Rightarrow f(x+2T)=\frac{5-2f(x)}{2-f(x)}$$

$$\textit{Further}\ \ f(x+2T)=\frac{5-2f(x)}{2-f(x)}\Rightarrow f(x+4T)=\frac{5-2f(x+2T)}{2-f(x+2T)}$$

$$\Rightarrow f(x+4T)=\frac{5-2\left(\frac{5-2f(x)}{2-f(x)}\right)}{2-\frac{5-2f(x)}{2-f(x)}}=f(x).$$

Hence *f* (x) is periodic with period 4T.

Q.8

(i) for $\log_2\left(\sqrt{x-4}+\sqrt{6-x}\right)$ to be defined x – 4 & 6 – x must be nonnegative.

Hence domain is $4\le x\le 6$.

{Square root function is defined for nonnegative values, log is defined for positive values and square root is a positive valued function}

(ii) For $\sin^{-1}\left(\frac{2-3[x]}{4}\right)$ to be defined $-1 \le \frac{2-3[x]}{4} \le 1$

$\Rightarrow -\frac{2}{3} < [x] \le 2 \Rightarrow 0 < x < 3$.

Hence domain is (0, 3),

(iii) For $\sqrt{2\{x\}^2 - 3\{x\} + 1}$ to be defined $2\{x\}^2 - 3\{x\} + 1 \ge 0$.

$\Rightarrow (2\{x\}-1)(\{x\}-1) \ge 0 \Rightarrow \{x\} \le \frac{1}{2}$

$\Rightarrow n \le x \le n + \frac{1}{2},\ n \in I$.

Hence domain is $\left[n,\ n+\frac{1}{2}\right],\ n \in I$.

Q.9

(i) Let $y = \frac{x+2}{2x^2+3x+6}$, then $2yx^2 + (3y-1)x + 6y - 2 = 0$.

For *x* to be real $(3y-1)^2 - 8y(6y-2) \ge 0$

$\Rightarrow 39y^2 - 4y - 4 \le 0$

or $-\frac{1}{13} \le y \le \frac{1}{3}$.

Hence range is $\left[-\frac{1}{13}, \frac{1}{3}\right]$.

(ii) $f(x) = \sqrt{[\sin x] + [\cos x]}$

$$[\sin x] = \begin{cases} 0 & x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right] \cup \{2\pi\} \\ 1 & x = \frac{\pi}{2} \\ -1 & x \in (\pi, 2\pi) \end{cases} \quad \& \quad [\cos x] = \begin{cases} 1 & x = 0, 2\pi \\ 0 & x \in \left(0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right) \cup \{2\pi\} \\ -1 & x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \end{cases}$$

Hence [sin x] + [cos x] = -2, -1, 0, 1.

Now for square root to be defined, [sin x] + [cos x] = 0, 1.

Hence range of f (x) is {0, 1}

Q.10

(i) Number of functions from A to B = Number of ways to distribute n distinct objects in m distinct groups = m^n .

(ii) Number of one – one functions from A to B = Number of ways to permute n distinct objects in at n out of m places = ${}^mC_n \times n!$.

(iii) If m = 2 and function is into then all the elements of A must be associated with one of the two elements in B. Number of such functions = 2.
Number of onto function = $m^n - 2$.

Q.11

Given $f(x) = \dfrac{x+1}{2x-3}$.

Let $y = \dfrac{x+1}{2x-3}$, then $x = \dfrac{3y+1}{2y-1}$

Hence range of f (x) is $R - \left\{\dfrac{1}{2}\right\}$.

Now $\dfrac{x_1+1}{2x_1-3} = \dfrac{x_2+1}{2x_2-3} \Rightarrow x_1 = x_2$, hence f (x) is one – one.

Q.12

Given $f(x+y) = f(xy)$.

For y = 0 we get f (x) = f (0), hence f (x) is a constant function.

Thus $f(2003) = -\dfrac{1}{2}$.

Q.13

$P(x) = x^4 + x^3 + x^2 + x + 1$

clearly tha roots of P(x) are fifth roots of unity.

Let the roots be $\alpha, \alpha^2, \alpha^3, \alpha^4$. (where $\alpha^5 = 1$)

Now, $P(x^5)$ when divided by $(x-\alpha)$ gives $P(\alpha^5)$ as the remainder. but since $\alpha^5 = 1$

$P(x^5)$ gives $P(1) = 5$ as the remainder when divided by $x - \alpha$.

Similarly $P(x^5)$ gives 5 as the remainder when divided by $(x-\alpha^2),(x-\alpha^3),(x-\alpha^5)$ as

$P(\alpha^{10}).p(\alpha^{15})$.

$P(\alpha^{20})$ are all 5

$\therefore P(x^5)-5$ is divided by $(x-\alpha),(x-\alpha^2),$

$(x-\alpha^3),(x-\alpha^4)$.

$\therefore P(x^5)-5=g(x)\{(x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)\}$

or the remainder when $P(x^5)$ is divided by

$P(x)$ is 5.

Q.14

$f:s\to s$

$s=\{4,5,6,7,....\}$

$f(x+y)=f(xy) \quad \forall x,y$

$$\left.\begin{array}{l} f(6)=f(2+4)=f(2\times4)=f(8) \\ f(6)=f(3+3)=f(3\times3)=f(9) \end{array}\right\}$$

$$\because f(x+y)=f(xy)$$

Q.15

(i) $f(x) = |x-1| + |x-2|,\ -1\le x\le 3$

$$\Rightarrow f(x)=\begin{cases} 3-2x, & -1\le x<1 \\ 1, & 1\le x\le 2 \\ 2x-3, & 2<x\le 3 \end{cases}$$

Now range of $3-2x$ in $-1\le x<1$ is (1, 5]

In $1\le x\le 2$, range of $f(x)$ is {1}

Range of 2x – 3 in $2<x\le 3$ is (1, 3].

Hence range of $f(x)$ is [1, 5].

(ii) $f(x)=\log_3(5+4x-x^2)$.

Domain : $5+4x-x^2>0$ or $x^2-4x-5<0 \Rightarrow -1<x<5$.

Now log is an increasing function as base > 1.

Also $g(x)=-x^2+4x+5=-(x-2)^2+9$.

Further g (-1) = 0, g (2) = 9 & g (5) = 0.

Maximum of g (x) = 9 & minimum of g (x) = 0.

Hence maximum of $f(x) = \log_3 9 = 2$.

Hence range of $f(x)$: $(-\infty, 2]$.

(iii) $f(x) = \dfrac{\sin x}{\sqrt{1+\tan^2 x}} - \dfrac{\cos x}{\sqrt{1+\cot^2 x}} \Rightarrow f(x) = |\cos x|\sin x - |\sin x|\cos x$

$$\Rightarrow f(x) = \begin{cases} 0, & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \\ -\sin 2x, & x \in \left(\frac{\pi}{2}, \pi\right) \\ \sin 2x, & x \in \left(\frac{3\pi}{2}, 2\pi\right) \end{cases}$$

Range of $f(x) = -\sin 2x$ in $\left(\frac{\pi}{2}, \pi\right)$ is (0, 1) so range of $f(x) = \sin 2x$ in $\left(\frac{3\pi}{2}, 2\pi\right)$ is (-1, 0)

Range of $f(x)$ in $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$, is {0}.

Hence range of $f(x)$: (-1, 1)

{Values at $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$ & 2π are not included as tan x & cot x are not defined}

Q.16

$$f(m,m) = m$$

$$f(m,n) = f(n,m)$$

$$f(m, m+n) = \left(1 + \frac{m}{n}\right) f(m,n)$$

$$f(14,52) = \left(1 + \frac{14}{38}\right) f(14,38)$$

$$= \left(1 + \frac{7}{19}\right)\left(1 + \frac{14}{24}\right) f(14,24)$$

$$= \frac{26}{19} \times \frac{19}{12} \times \left(1 + \frac{14}{10}\right) f(14,10)$$

$$= \frac{26}{12} \times \frac{12}{5} f(10,14) \quad [\because f(m,n) = f(n,m)]$$

$$= \frac{26}{5} \times \left(1 + \frac{10}{4}\right) f(10,4)$$

$$= \frac{26}{5} \times \frac{7}{2} \times f(4,10)$$

$$= \frac{91}{5} \times \left(1 + \frac{4}{6}\right) f(4,6)$$

$$= \frac{91}{3} \times \left(1 + \frac{4}{2}\right) f(4,2)$$

$$= 91 \times f(2,4)$$

$$= 91 \times \left(1 + \frac{2}{2}\right) f(2,2)$$

$$= 91 \times 2 \times 2 \quad [\because f(m,m) = m]$$

$$= 364.$$

Q.17

$k = 2p+1, p \in z$

$f(k) = f(2p+1) = (2p+1)+3 \quad [\because (2p+1) \text{ is odd}]$

$= 2p+4$

$f(f(k)) = f(2p+4)$

$= \frac{2p+4}{2} \quad [\because (2p+4) \text{ is even}]$

$= p+2$

now $f(f(f(k))) = 27$

$\Rightarrow \quad f(p+2) = 27$

$(p+2)$ it self could be even or odd

case (i) is even or (p is even)

$f(p+2) = \frac{p+2}{2} = 27$

$\therefore p = 52$

case (ii) If $(p+2)$ is odd [i.e p is odd]

$f(p+2) = (p+2)+3 = p+5$

$=$ even number $\quad [\because \text{p is odd}]$

$\therefore f(p+2)$ can never be 27 (odd number) for p odd

$\therefore \quad k = 2p+1$

$= 2\times 52+1$

$\boxed{k = 105}$

Q.18

$f(x) + f(x+4) = f(x+2) + f(x+6) \ ...(i)$

Replace x with $x - 2$ to get $f(x-2) + f(x+2) = f(x) + f(x+4) \ ...(ii)$

Add (i) & (ii) to get $f(x-2) = f(x+6)$

Replace x with $x + 2$ to get $f(x) = f(x+8)$.

Hence $f(x)$ is periodic with period 8.

Now $\sum_{n=0}^{99} f(8n) = 100 f(0) = 500$.

Q.19

Let $f(x) = ax + b$ & $g(x) = cx + d$.

Case I : $f(-1) = 0$ & $f(1) = 3$, $g(-1) = 3$ & $g(1) = 0$

Then $f(x) = \frac{3x+3}{2}$ & $g(x) = \frac{3-3x}{2}$

Case II : $f(-1) = 3$ & $f(1) = 0$, $g(-1) = 0$ & $g(1) = 3$

Then $f(x) = \frac{3-3x}{2}$ & $g(x) = \frac{3x+3}{2}$.

Now $f(x) = g(x)$ gives $x = 0$.

Q.20

Given $x^2 f(x) + f(1-x) = 2x - x^4 \ \ldots$(i)

Replace x by $1 - x$ to get

$$(1-x)^2 f(1-x) + f(x) = 2(1-x) - (1-x)^4 \quad \ldots\text{(ii)}$$

Eliminating $f(1-x)$ between (i) & (ii) gives

$$f(x) = \frac{(2x - x^4)(1-x)^2 - 2(1-x) + (1-x)^4}{(1-x)^2 x^2 - 1} \text{ or } f(x) = 1 - x^2.$$

Q.21

$\text{fog} = \sin(\tan x)$ & $\text{gof} = \tan(\sin x)$

Period of tan x is π and range of tan x is R.

Hence $\sin(\tan(x+T)) = \sin(\tan x) \Rightarrow T = \pi$

& range of fog will be complete range of sine function i.e. [-1, 1].

Period of sin x is π and range of sin x is [-1, 1].

Hence $\tan(\sin(x+T)) = \tan(\sin x) \Rightarrow T = 2\pi$

Now as range of sine function is [-1, 1] which is a subset of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ hence gof will be an increasing function in each period.

Therefore Range of gof will be [-tan 1, tan 1].

Q.22

The domain of f is

$$D = \{x \mid x + 4 \geq 0 \text{ and } x - 5 \neq 0\}$$
$$= \{x \mid x \geq -4 \text{ and } x \neq 5\}$$

The range of f is

$$R = \{y : y = f(x) \text{ and } x \text{ in } D\}$$
$$= \{f(x) : x \geq -4 \text{ and } x \neq 5\}$$

Because $f(x) = \frac{\sqrt{x+4} - 3}{x - 5} = \frac{\sqrt{x+4} - 3}{x - 5} \cdot \frac{\sqrt{x+4} + 3}{\sqrt{x+4} + 3} = \frac{x + 4 - 9}{(x-5)(\sqrt{x+4} + 3)}$

$$= \frac{x - 5}{(x-5)(\sqrt{x+4} + 3)} = \frac{1}{\sqrt{x+4} + 3} \text{ for } x \geq -4,\ x \neq 5$$

$f(x)|_{max} = \frac{1}{3}$ where $x = -4$

$f(x)|_{min} = 0$; and $f(5) = \frac{1}{6}$

Q.25

(A) $f[g(x)] = f(1 - \sqrt{x}) = \sin(1 - \sqrt{x})$

domain is $x \ge 0$; range $[-1, 1]$

(B) $g[f(x)] = 1 - \sqrt{f(x)} = 1 - \sqrt{\sin x}$

domain $2k\pi \le x \le 2k\pi + \pi$; range $[0, 1]$

(C) $(fof)(x) = f[f(x)] = f(\sin x) = \sin(\sin x)$

Domain $x \in R$; range $[-\sin 1, \sin 1]$

(D) $(gog)(x) = g[g(x)] = 1 - \sqrt{g(x)} = 1 - \sqrt{1 - \sqrt{x}}$

domain is $0 \le x \le 1$; range is $[0, 1]$

hence (B) and (D) have the same range]

26.

(a) prove that $f(f(x)) = x$

Let $\boxed{f(x) = \frac{1-x}{1+x} = y}$ ...(2) putting in (1)

we get $\boxed{f(y) = x}$ ...(3)

or $\boxed{f[f(x)] = x}$ **proved.**

(b) let $f(x) = \frac{1-x}{1+x}$ ...(1)

then $f\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}}$

or $f\left(\frac{1}{x}\right) = \frac{x-1}{x+1}$ ...(2)

or $f\left(\frac{1}{x}\right) = -\left(\frac{1-x}{1+x}\right) = -f(x)$ **proved.**

(c) Q $f(x) = \frac{1-x}{1+x}$ ...(1)

$\therefore$ $f(-x-2) = \frac{1-(-x-2)}{1+(-x-2)} = \frac{1+x+2}{1-x-2} = \frac{3+x}{-1-x}$ ...(2)

we have to prove that $f(-x-2) = -f(x) - 2$

$-\left(\frac{3+x}{1+x}\right) = \frac{x-1}{1+x} - 2$

or $-\left(\frac{3+x}{1+x}\right) = \frac{x-1-2-2x}{1+x}$

or $\frac{3+x}{1+x} = \frac{3+x}{1+x}$ **proved.**

27.

$f\left(\frac{1-x}{1+x}\right)=x$ (let $\frac{1-x}{1+x}=t \Rightarrow t+tx=1-x \Rightarrow x(1+t)=1-t \Rightarrow x=\frac{1-t}{1+t}$)

$\therefore \quad f(x)=\frac{1-x}{1+x} \quad ....(1)$

$\therefore \quad f(f(x))=\frac{1-f(x)}{1+f(x)}=\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}=\frac{1+x-1+x}{1+x+1-x}=\frac{2x}{2}$

$\therefore \quad f(f(x))=x \Rightarrow$ **(A) is correct**

again from (1)

$f\left(\frac{1}{x}\right)=\frac{1-(1/x)}{1+(1/x)}=\frac{x-1}{x+1}$

$f\left(\frac{1}{x}\right)=-\left(\frac{1-x}{1+x}\right)=-f(x) \Rightarrow$ **(C) is correct**

Also $f(-x-2)=\frac{1+(x+2)}{1-(x+2)}=-\left(\frac{x+3}{x+1}\right)$ ....(2)

and $-f(x)-2=-\left(\frac{1-x}{1+x}+2\right)=-\left(\frac{1-x+2+2x}{1+x}\right)=-\left(\frac{x+3}{x+1}\right)$ ....(3)

from (2) and (3)

$f(-x-2)=-f(x)-2 \Rightarrow$ **(D) is correct**

hence A, C, D are correct]

28.

(a)

$f(x)=\frac{x}{x+1}$

$g(x)=x^{10}$

$h(x)=x+3$

$f(g(h(x)))=\frac{g(h(x))}{g(h(x))+1}$

$(h(x))^{10}$

$=\frac{(x+3)^{10}}{(x+3)^{10}+1}$

$\Rightarrow (fogoh)(x)=\frac{(x+3)^{10}}{(x+3)^{10}+1}$

$\therefore \ f(g(h(-1)))=\frac{(-1+3)^{10}}{(-1+3)^{10}+1}$

$=\frac{2^{10}}{2^{10}+1}=\frac{1024}{1024+1}=\frac{1024}{1025}$

(b)

Given $F(x)=\cos^2(x+9)$. Find the function f, g, h such that F = fogoh.

$F(x)=\cos^2(x+9)$

$\therefore F(x)=fogoh$

Since, there will be infinitaly many solution exist for $F(x)=(f_0 goh)(x)$.

$f(x)=\cos^2 x$

$g(x)=x$

$h(x)=x+9$

29.

f(x) = man {x, 1/x} for x > 0

$$g(x) = f(x).\ f(1/x) = \begin{cases} x.x & x \geq 1 \\ \frac{1}{x}.\frac{1}{x} & x < 1 \end{cases}$$

the functions which is max. for x will be min for 1/x.

30.

$$\text{Given } f(x) = \begin{cases} 1-x, & x \leq 0 \\ x^2, & x > 0 \end{cases} \quad \& \quad g(x) = \begin{cases} -x, & x < 1 \\ 1-x, & x \geq 1 \end{cases}$$

$$(\text{i})\quad fog(x) = \begin{cases} 1-g(x), & g(x) \leq 0 \\ g^2(x), & g(x) > 0 \end{cases}$$

$$\text{Now } g(x) = \begin{cases} -x, & x < 0 & \to g(x) \in (0, \infty) \\ -x, & 0 \leq x < 1 & \to g(x) \in (-1, 0] \\ 1-x, & x \geq 1 & \to g(x) \in (-\infty, 0] \end{cases}$$

$$\Rightarrow fog(x) = \begin{cases} x^2, & x < 0 \\ 1+x, & 0 \leq x < 1 \\ x, & x \geq 1 \end{cases}$$

$$(\text{ii})\quad gof(x) = \begin{cases} -f(x), & f(x) < 1 \\ 1-f(x), & f(x) \geq 1 \end{cases}$$

$$\text{Now } f(x) = \begin{cases} 1-x, & x \leq 0 & \to f(x) \in [1, \infty) \\ -x, & 0 < x < 1 & \to f(x) \in (0,1) \\ 1-x, & x \geq 1 & \to g(x) \in [1, \infty) \end{cases}$$

$$\Rightarrow gof(x) = \begin{cases} x, & x \leq 0 \\ -x^2, & 0 < x < 1 \\ 1-x^2, & x \geq 1 \end{cases}$$

31.

(a) $$f(-x) = \log\left(-x + \sqrt{1+x^2}\right) \Rightarrow f(-x) = \log\frac{\left(-x+\sqrt{1+x^2}\right)\left(x+\sqrt{1+x^2}\right)}{\left(x+\sqrt{1+x^2}\right)}$$

$$\Rightarrow f(-x) = \log\frac{1}{\left(x+\sqrt{1+x^2}\right)} \text{ or } f(-x) = -\log\left(x+\sqrt{1+x^2}\right) = -f(x)$$

Hence f (x) is an ODD function.

(b) $f(-x) = -x\dfrac{a^{-x}+1}{a^{-x}-1} \Rightarrow f(-x) = -x\dfrac{\frac{1}{a^x}+1}{\frac{1}{a^x}-1}$

$\Rightarrow f(-x) = -x\dfrac{1+a^x}{1-a^x}$ or $f(-x) = x\dfrac{a^x+1}{a^x-1} = f(x)$.

Hence f (x) is an EVEN function.

(c) $f(-x) = \sin(-x)+\cos(-x) \Rightarrow f(x) = -\sin x + \cos x$

Hence f (x) is nether EVEN nor ODD.

(d) $f(-x) = -x\sin^2(-x)-(-x)^3 \Rightarrow f(-x) = -x\sin^2 x + x^3$

$\Rightarrow f(-x) = -\left(x\sin^2 x - x^3\right) = -f(x)$

Hence f (x) is an ODD function.

(e) Same as (c)

(f) $f(-x) = \dfrac{\left(1+2^{-x}\right)^2}{2^{-x}} \Rightarrow f(-x) = \dfrac{\left(1+\frac{1}{2^x}\right)^2}{\frac{1}{2^x}}$

$\Rightarrow f(-x) = \dfrac{\left(1+2^{-x}\right)^2}{2^{-x}} = f(x)$.

Hence f (x) is an EVEN function.

(g) $f(-x) = -\dfrac{x}{e^{-x}-1} - \dfrac{x}{2}+1 \Rightarrow f(-x) = \dfrac{xe^x}{e^x-1} - \dfrac{x}{2}+1$

$\Rightarrow f(-x) = \dfrac{xe^x}{e^x-1} - x + \dfrac{x}{2}+1$

$\Rightarrow f(-x) = \dfrac{xe^x - xe^x + x}{e^x-1} + \dfrac{x}{2}+1$

$\Rightarrow f(-x) = \dfrac{x}{e^x-1} + \dfrac{x}{2}+1 = f(x)$

Hence f (x) is an EVEN function.

(h) Clearly f (x) is an even function.

32.

$y = 2^{\log_{10} x} + 8 \Rightarrow 2^{\log_{10} x} = y-8$

$\Rightarrow \log_{10} x = \log_2(y-8)$

$\Rightarrow x = 10^{\log_2(y-8)}$

$\Rightarrow f^{-1}(x) = 10^{\log_2(x-8)},\ x \in (8, \infty)$.

Now $f^{-1}(x) = f(x) \Rightarrow f(x) = x$

$\Rightarrow 10^{\log_2(x-8)} = x$

$\Rightarrow x = 10.$

33.

Period of $\cos nx$ is $\frac{2\pi}{n}$ & period of $\sin\frac{5x}{n}$ is $\frac{2n\pi}{5}$, thus

period of f (x) is $\text{LCM}\left\{\frac{2\pi}{n}, \frac{2n\pi}{5}\right\}$

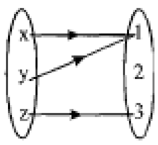
$\Rightarrow \text{LCM}\left\{\frac{2}{n}, \frac{2n}{5}\right\} = 3 \Rightarrow n = \pm1, \pm3, \pm5, \pm15$.

34.

Domain = {x, y, z}, range = {1, 2, 3}

case I- 1st is true i.e.

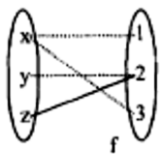
$f(x) = 1,\ f(y) = 1,\ f(z) = 2$



not one –one function

Case II- 2nd is true i.e.

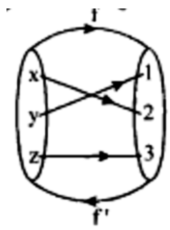
$f(x) \neq 1;\ f(y) \neq 1;\ f(z) = 2$



not one-one function

Case III- 3rd is true i.e.

$f(x) \neq 1,\ f(y) = 1,\ f(z) \neq 2$



one-one function

$f^{-1}(1) = y$ **Ans**

35.

$x = \log_4^{9} + \log_9^{28}$

$= \log_2^{3} + \frac{1}{2}\log_3^{28}$

$x = \log_2^{3} + \log_3^{\sqrt{28}}$

$\log_2^{2\sqrt{2}} < \log_2^{3} < \log_2^{4}$

$1.5 < \log_2^{3} < 2$

$\log_3^{3\sqrt{3}} < \log_3^{\sqrt{28}} < \log_3^{\sqrt{81}}$

$1.5 < \log_3^{\sqrt{28}} < 2$

$3 < x < 4$

then $[x] = 3$ **Ans**

36.

(a)

$4f(2) = f(1) + f(2)$

$f(2) = \frac{f(1)}{3} = \frac{f(1)}{1+2}$

$9f(3) = f(1) + f(2) + f(3)$

$8f(3) = \frac{4f(1)}{3}$

$f(3) = \frac{f(1)}{6} = \frac{f(1)}{1+2+3}$ & so on

$f(2004) = \frac{f(1)}{1+2+3+...+2004} = \frac{(2005)\times 2}{2005\times 2004} = \frac{1}{1002}$

(b)

Let $f(x) = \sqrt{x^2+ax} - \sqrt{x^2+bx}$

$\therefore$ $f(x) = \frac{(a-b)}{\sqrt{1+(a/x)}+\sqrt{1+(b/x)}}$ (on rationalising)

$\therefore$ $f(x)_{max} = \frac{a-b}{2}$ $(x \to \infty)$

$\therefore$ L = 1 Ans.]

(c)

$f(f(x)) = f(x^2+kx) = (x^2+kx)(k+x^2+kx)$

$f(f(x)) = f(x)\,(k+f(x)) = 0$

for f(x) and $f(f(x))$ to have same solution set

$k + f(x) = 0$ should have no solution

$x^2 + kx + k = 0$ ....(1)

$D < 0$ (for no solution)

$k^2 - 4k < 0$

Also for $k = 0$ both $f(x) = 0$ and

$f(f(x)) = 0$ has same solution set

$\therefore$ $k \in [0, 4)$

(d)

Let, $2x + 1 = t$

now, $4x^2 + 14x = (2x+1)^2 + 10x - 1$

$= (2x+1)^2 + 5(2x+1) - 6$

$\therefore$ $f(t) = t^2 + 5t - 6$

$\therefore$ $f(x) = x^2 + 5x - 6 = 0 \Rightarrow$ $x = 1$ or -6

(e)

$f(x) = a\sin x + b\sqrt[3]{x} + 4$

$f\left(\log_{10}\left(\frac{1}{\log_3 10}\right)\right) = 5$

$= f(\log_{10} - \log_{10}(\log_3 10))$

$= f(-\log_{10}(\log_3 10))$

$= -[a\sin x + bx^{1/3}] + 4$

$= -1 + 4 = 3$ **Ans**

37.

f(g(x)) and f(g(x)) are identity function when f(x) and g(x) are inverse to each other

$f(x) = ax + b$ (let linear function)

$f^{-1}(x) = y$

$f(y) = x$

$ay + b = x$

$y = \frac{x-b}{a}$

$g(x) = f^{-1}(x)$

$g(x) = \frac{x-b}{a}$

$g(5) = \frac{5-4}{a}$

&

$f(x) = ax + b$

$f(0) = 6$

$17 = \frac{1}{a} \Rightarrow \boxed{a = \frac{1}{17}}$ $\boxed{4 = b}$

then $f(2006) = a.(2006) + b$

$= \frac{1}{17}(2006) + 4$

$= 122$ **Ans**

38.

$f(x+y) - kxy = f(x) + 2y^2$ ...(1)

$f(1) = 2$

$f(2) = 8$

$y = -x$

$f(0) + kx^2 = f(x) + 2x^2$ ...(2)

$f(x) = (k-2)x^2 + f(0)$

$= f(1) = (k-2) + f(0)$

$k - 2 + f(0) = 2$

$k + f(0) = 4$ ...(3)

$f(2) = (k-2).4 + f(0)$

$4k + f(0) = 16$

$3k = 12$

$k = 4$ $f(0) = 0$

put in (2) $f(x) = 2x^2$ then $f(x+y)\ f\left(\frac{1}{x+y}\right) = k$ **Ans**