EXERCISE 1

1, If A contains 10 elements then total number of functions defined from A to A is

(a) 10 (b) 2^{10} (c) 10^{10} (d) $2^{10}-1$

Sol. (c)

According to formula, total number of functions $= n^n$

Here, n = 10. So, total number of functions = 10^{10}.

2 If $f(x)=\frac{x-|x|}{|x|}$, then $f(-1)=$

(a) 1 (b) – 2 (c) 0 (d) 2

Sol. (b)

$f(-1)=\frac{-1-|-1|}{|-1|}=\frac{-1-1}{1}=-2$.

3 If $f(y)=\log y$, then $f(y)+f\left(\frac{1}{y}\right)$ is equal to

(a) 2 (b) 1 (c) 0 (d) – 1

Sol. (c)

Given $f(y)=\log y \Rightarrow f(1/y)=\log(1/y)$, then $f(y)+f\left(\frac{1}{y}\right)=\log y+\log(1/y)=\log 1=0$.

4 If $f(x)=\log\left[\frac{1+x}{1-x}\right]$, then $f\left[\frac{2x}{1+x^2}\right]$ is equal to

(a) $[f(x)]^2$ (b) $[f(x)]^3$ (c) $2f(x)$ (d) $3f(x)$

Sol. (c)

$f(x)=\log\left(\frac{1+x}{1-x}\right)$

$$\therefore f\left(\frac{2x}{1+x^2}\right)=\log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right]=\log\left[\frac{x^2+1+2x}{x^2+1-2x}\right]=\log\left[\frac{1+x}{1-x}\right]^2=2\log\left[\frac{1+x}{1-x}\right]=2f(x)$$

5 If $f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$, then

(a) $f\left(\frac{\pi}{4}\right)=2$ (b) $f(-\pi)=2$ (c) $f(\pi)=1$ (d) $f\left(\frac{\pi}{2}\right)=-1$

Sol. (d)

$f(x)=\cos\,[\pi^2]x+\cos\,[-\pi^2]\,x$

$f(x)=\cos(9x)+\cos(-10x)=\cos(9x)+\cos(10x)=2\cos\left(\frac{19x}{2}\right)\cos\left(\frac{x}{2}\right)$

$f\left(\frac{\pi}{2}\right)=2\cos\left(\frac{19\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$; $f\left(\frac{\pi}{2}\right)=2\times\frac{-1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=-1$.

6 If $f:R\to R$ satisfies $f(x+y)=f(x)+f(y)$, for all $x,y\in R$ and $f(1)=7$, then $\sum_{r=1}^{n}f(r)$ is

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $7n(n+1)$ (d) $\frac{7n(n+1)}{2}$

Sol. (d)

$f(x+y) = f(x) + f(y)$

put $x = 1, y = 0 \quad \Rightarrow f(1) = f(1) + f(0) = 7$

put $x = 1, y = 1 \quad \Rightarrow f(2) = 2.f(1) = 2.7$; similarly $f(3) = 3.7$ and so on

$\therefore \sum_{r=1}^{n} f(r) = 7(1+2+3+.....+n) = \frac{7n(n+1)}{2}$.

7 If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for $x > 2$, then $f(11) =$

(a) $\frac{7}{6}$ (b) $\frac{5}{6}$ (c) $\frac{6}{7}$ (d) $\frac{5}{7}$

Sol. (c)

$f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

$f(11) = \frac{1}{\sqrt{11+2\sqrt{18}}} + \frac{1}{\sqrt{11-2\sqrt{18}}} = \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{7} + \frac{3+\sqrt{2}}{7} = \frac{6}{7}$.

8 Domain of the function $\frac{1}{\sqrt{x^2-1}}$ is

(a) $(-\infty,-1)\cup(1,\infty)$ (b) $(-\infty,-1]\cup(1,\infty)$ (c) $(-\infty,-1)\cup[1,\infty)$ (d) None of these

Sol. (a)

For domain, $x^2 - 1 > 0 \quad \Rightarrow (x-1)(x+1) > 0$

$\Rightarrow x < -1$ or $x > 1 \quad \Rightarrow x \in (-\infty,-1)\cup(1,\infty)$.

9 The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is

(a) R^+ (b) R^- (c) R_0 (d) R

Sol. (b)

For domain, $|x| - x > 0 \quad \Rightarrow |x| > x$. This is possible, only when $x \in R^-$.

10 Find the domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$

(a) $(-3,\infty)$ (b) $\{-1,-2\}$ (c) $(-3,\infty)-\{-1,-2\}$ (d) $(-\infty,\infty)$

Sol. (c)

Here $f(x) = \frac{\log_2(x+3)}{x^2+3x+2} = \frac{\log_2(x+3)}{(x+1)(x+2)}$ exists if,

Numerator $x + 3 > 0 \quad \Rightarrow x > -3$ …... (i)

and denominator $(x+1)(x+2) \neq 0 \Rightarrow x \neq -1, -2$ …… (ii)

Thus, from (i) and (ii); we have domain of $f(x)$ is $(-3,\infty)-\{-1,-2\}$.

11 The domain of the function $f(x) = \sqrt{(2-2x-x^2)}$ is

(a) $-3 \le x \le \sqrt{3}$ (b) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$

(c) $-2 \le x \le 2$ (d) None of these

Sol. (b)

The quantity square root is positive, when $-1-\sqrt{3} \le x \le -1+\sqrt{3}$.

12 If the domain of function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is

(a) $(-\infty, \infty)$ (b) $[-2, \infty)$ (c) $(-2, 3)$ (d) $(-\infty, -2)$

Sol. (b)

$x^2 - 6x + 7 = (x-3)^2 - 2$ Obviously, minimum value is –2 and maximum ∞.

13 The domain of the function $f(x) = \sqrt{x - x^2} + \sqrt{4+x} + \sqrt{4-x}$ is

(a) $[-4, \infty)$ (b) $[-4, 4]$ (c) $[0, 4]$ (d) $[0, 1]$

Sol. (d)

$f(x) = \sqrt{x - x^2} + \sqrt{4+x} + \sqrt{4-x}$

clearly $f(x)$ is defined if

$4 + x \ge 0 \Rightarrow x \ge -4$

$4 - x \ge 0 \Rightarrow x \le 4$

$x(1-x) \ge 0 \Rightarrow x \ge 0$ and $x \le 1$

$\therefore$ Domain of $f = (-\infty, 4] \cap [-4, \infty) \cap [0, 1] = [0, 1]$.

14 The domain of the function $\sqrt{\log(x^2 - 6x + 6)}$ is

(a) $(-\infty, \infty)$ (b) $(-\infty, 3-\sqrt{3}) \cup (3+\sqrt{3}, \infty)$

(c) $(-\infty, 1] \cup [5, \infty)$ (d) $[0, \infty)$

Sol. (c)

The function $f(x) = \sqrt{\log(x^2 - 6x + 6)}$ is defined when $\log(x^2 - 6x + 6) \ge 0$

$\Rightarrow x^2 - 6x + 6 \ge 1 \Rightarrow (x-5)(x-1) \ge 0$

This inequality hold if $x \le 1$ or $x \ge 5$. Hence, the domain of the function will be $(-\infty, 1] \cup [5, \infty)$.

15 The domain of definition of the function $y(x)$ given by $3^x + 3^y = 3$ is

(a) (0, 1] (b) [0, 1] (c) $(-\infty, 0]$ (d) $(-\infty, 1)$

Sol. (d)

$3^y = 3 - 3^x$

y is real if $3 - 3^x \ge 0 \Rightarrow 3 > 3^x \Rightarrow 1 > x$

$x \in (-\infty, 1)$

16 The domain of the function $f(x) = \cos^{-1}[\log_2(x/2)]$ is

(a) [1, 4] (b) [– 4, 1] (c) [– 1, 4] (d) None of these

Sol. (a)

$f(x) = \sin^{-1}[\log_2(x/2)]$

Domain of $\cos^{-1} x$ is $x \in [-1, 1]$

$\Rightarrow -1 \le \log_2(x/2) \le 1 \Rightarrow \frac{1}{2} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4$

$\therefore x \in [1, 4]$.

17 If $f(x)=x^2+1,$ then $f^{-1}(17)$ and $f^{-1}(-3)$ will be

(a) 4, 1 (b) 4, 0 (c) 3, 2 (d) None of these

Sol. (d)

Let $y=x^2+1 \Rightarrow x=\pm\sqrt{y-1}$

$\Rightarrow f^{-1}(y)=\pm\sqrt{y-1} \Rightarrow f^{-1}(x)=\pm\sqrt{x-1}$

$\Rightarrow f^{-1}(17)=\pm\sqrt{17-1}=\pm 4$

and $f^{-1}(-3)=\pm\sqrt{-3-1}=\pm\sqrt{-4}$, which is not possible.

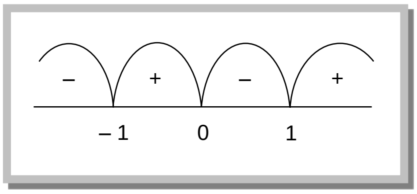
18 Domain of definition of the function $f(x)=\frac{3}{4-x^2}+\log_2(x^5-x^3),$ is

(a) (1, 2) (b) (–1, 0) ∪ (1, 2)

(c) (1, 2) ∪ (2, ∞) (d) (–1, 0) ∪ (1, 2) ∪ (2, ∞)

Sol. (d)

$f(x)=\frac{3}{4-x^2}+\log_{10}(x^3-x)$



So, $4-x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{4} \Rightarrow x \neq \pm 2$

and $x^5-x^3>0 \Rightarrow x^3(x^2-1)>0 \Rightarrow x>0, |x|>1$

$\therefore D=(-1,0)\cup(1,\infty)-\{2\}$

$D=(-1,0)\cup(1,2)\cup(2,\infty)$.

19 The domain of the function $f(x)=\log_{3+x}(x^2-1)$ is

(a) $(-3,-1)\cup(1,\infty)$ (b) $[-3,-1)\cup[1,\infty)$

(c) $(-3,-2)\cup(-2,-1)\cup(1,\infty)$ (d) $[-3,-2)\cup(-2,-1)\cup[1,\infty]$

Sol. (c)

$f(x)$ is to be defined when $x^2-1>0$

$\Rightarrow x^2>1, \Rightarrow x<-1$ or $x>1$ and $3+x>0$

$\therefore x>-3$ and $x \neq -2$

$\therefore D_r=(-3,-2)\cup(-2,-1)\cup(1,\infty)$.

20 Domain of definition of the function $f(x)=\sqrt{2\sin^{-1}(2x)+\frac{\pi}{3}}$, for real value $x,$ is

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$ (b) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (c) $\left(-\frac{1}{2},\frac{1}{9}\right)$ (d) $\left[-\frac{1}{4},\frac{1}{4}\right]$

Sol. (a)

$-\frac{\pi}{6}\leq \sin^{-1}(2x)\leq\frac{\pi}{2} \Rightarrow -\frac{1}{2}\leq 2x \leq 1 \Rightarrow x\in\left[-\frac{1}{4},\frac{1}{2}\right]$.

21 The range of $f(x)=\cos x-\sin x,$ is

(a) $(-1,1)$ (b) $[-1,1)$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (d) $[-\sqrt{2},\sqrt{2}]$

Sol. (d)

Let, $f(x)=\cos x-\sin x \Rightarrow f(x)=\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right) \Rightarrow f(x)=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

Now since, $-1\le\cos\left(x+\frac{\pi}{4}\right)\le 1 \Rightarrow -\sqrt{2}\le f(x)\le\sqrt{2} \Rightarrow f(x)\in[-\sqrt{2},\sqrt{2}]$

Trick : $\because$ Maximum value of $\cos x-\sin x$ is $\sqrt{2}$ and minimum value of $\cos x-\sin x$ is $-\sqrt{2}$. Hence, range of $f(x)=[-\sqrt{2},\sqrt{2}]$.

22 The range of $\frac{1+x^2}{x^2}$ is

(a) $(0,1)$ (b) $(1,\infty)$ (c) [0, 1] (d) $[1,\infty)$

Sol. (b)

Let $y=\frac{1+x^2}{x^2}$ $\Rightarrow x^2y=1+x^2 \Rightarrow x^2(y-1)=1 \Rightarrow x^2=\frac{1}{y-1}$

Now since, $x^2>0 \Rightarrow \frac{1}{y-1}>0 \Rightarrow (y-1)>0 \Rightarrow y>1 \Rightarrow y\in(1,\infty)$

Trick : $y=\frac{1+x^2}{x^2}=1+\frac{1}{x^2}$. Now since, $\frac{1}{x^2}$ is always > 0 $\Rightarrow y>1 \Rightarrow y\in(1,\infty)$.

23 For real values of *x*, range of the function $y=\frac{1}{2-\sin 3x}$ is

(a) $\frac{1}{3}\le y\le 1$ (b) $-\frac{1}{3}\le y<1$ (c) $-\frac{1}{3}>y>-1$ (d) $\frac{1}{3}>y>1$

Sol. (a)

$\because y=\frac{1}{2-\sin 3x}$, $\therefore 2-\sin 3x=\frac{1}{y} \Rightarrow \sin 3x=2-\frac{1}{y}$

Now since,

$-1\le\sin 3x\le 1 \Rightarrow -1\le 2-\frac{1}{y}\le 1 \Rightarrow -3\le-\frac{1}{y}\le-1 \Rightarrow 1\le\frac{1}{y}\le 3 \Rightarrow \frac{1}{3}\le y\le 1$.

24 If $f(x)=a\cos(bx+c)+d$, then range of $f(x)$ is

(a) $[d+a,d+2a]$ (b) $[a-d,a+d]$ (c) $[d+a,a-d]$ (d) $[d-a,d+a]$

Sol. (d)

$f(x)=a\cos(bx+c)+d$ ….. (i)

For minimum $\cos(bx+c)=-1$

from (i), $f(x)=-a+d=(d-a)$,

for maximum $\cos(bx+c)=1$

from (i), $f(x)=a+d=(d+a)$

$\therefore$ Range of $f(x)=[d-a,d+a]$.

25 The range of the function $f(x)=\frac{x+2}{|x+2|}$ is

(a) {0, 1} (b) {– 1, 1} (c) *R* (d) $R-\{-2\}$

Sol. (b)

$$f(x)=\frac{x+2}{|x+2|}=\begin{cases}-1, & x<-2\\ 1, & x>-2\end{cases}$$

$\therefore$ Range of $f(x)$ is $\{-1,1\}$.

26 The range of $f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right),\ -\infty<x<\infty$ is

(a) $[1,\sqrt{2}]$ (b) $[1,\infty)$ (c) $[-\sqrt{2},-1]\cup[1,\sqrt{2}]$ (d) $(-\infty,-1]\cup[1,\infty)$

Sol. (a)

$f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right)$

We know that, $0\le\cos^2 x\le 1$ at $\cos x=0,\quad f(x)=1$ and at $\cos x=1,\quad f(x)=\sqrt{2}$

$\therefore\ 1\le x\le\sqrt{2}\ \Rightarrow\ x\in[1,\ \sqrt{2}]$.

27 Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1};x\in R$ is

(a) $(1,\infty)$ (b) $(1,11/7)$ (c) $(1,7/3]$ (d) $(1,7/5]$

Sol. (c)

$f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$ $\Rightarrow$ Range $=(1,7/3]$.

28 Function $f:N\to N, f(x)=2x+3$ is

(a) One-one onto (b) One-one into (c) Many-one onto (d) Many –one into

Sol. (b)

f is one-one because $f(x_1)=f(x_2)\ \Rightarrow\ 2x_1+3=2x_2+3\ \Rightarrow x_1=x_2$

Further $f^{-1}(x)=\frac{x-3}{2}\notin N$ (domain) when x = 1, 2, 3 etc.

$\therefore$ *f* is into which shows that *f* is one-one into.

29 The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

(a) One-one but not onto (b) Onto but not one-one

(c) Both one-one and onto (d) Neither one-one nor onto

Sol. (b)

We have $f(x)=(x-1)(x-2)(x-3)\ \Rightarrow\ f(1)=f(2)=f(3)=0\ \Rightarrow\ f(x)$ is not one-one

For each $y\in R,$ there exists $x\in R$ such that $f(x)=y$. Therefore *f* is onto.

Hence, $f:R\to R$ is onto but not one-one.

30 Find number of surjection from *A* to *B* where $A=\{1,2,3,4\},\ B=\{a,b\}$

(a) 13 (b) 14 (c) 15 (d) 16

Sol. (b)

Number of surjection from *A* to *B* $=\sum_{r=1}^{2}(-1)^{2-r}\ {}^2C_r(r)^4$

$=(-1)^{2-1}\ {}^2C_1(1)^4+(-1)^{2-2}\ {}^2C_2(2)^4\ =-2+16\ =14$

Therefore, number of surjection from *A* to *B* = 14.

Trick : Total number of functions from A to *B* is 2^4 of which two function $f(x)=a$ for all $x\in A$ and $g(x)=b$ for all $x\in A$ are not surjective. Thus, total number of surjection from *A* to *B* $=2^4-2=14$.

31 If $A=\{a,b,c\}$, then total number of one-one onto functions which can be defined from A to A is

(a) 3 (b) 4 (c) 9 (d) 6

Sol. (d)

Total number of one-one onto functions $= 3!$

32 If $f:R\to R$, then $f(x)=|x|$ is

(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto (d) None of these

Sol. (d)

$f(-1)=f(1)=1$ $\therefore$ function is many-one function.

Obviously, f is not onto so f is neither one-one nor onto.

33 Let $f:R\to R$ be a function defined by $f(x)=\frac{x-m}{x-n}$, where $m\neq n$. Then

(a) f is one-one onto (b) f is one-one into

(c) f is many one onto (d) f is many one into

Sol. (b)

For any $x,y\in R$, we have

$f(x)=f(y)\Rightarrow \frac{x-m}{x-n}=\frac{y-m}{y-n}\Rightarrow x=y$

$\therefore f$ is one-one

Let $\alpha\in R$ such that $f(x)=\alpha\Rightarrow \frac{x-m}{x-n}=\alpha \Rightarrow x=\frac{m-n\alpha}{1-\alpha}$

Clearly $x\notin R$ for $\alpha=1$. So, f is not onto.

34 The function $f:R\to R$ defined by $f(x)=e^x$ is

(a) Onto (b) Many-one

(c) One-one and into (d) Many one and onto

Sol. (c)

Function $f:R\to R$ is defined by $f(x)=e^x$. Let $x_1,x_2\in R$ and $f(x_1)=f(x_2)$ or $e^{x_1}=e^{x_2}$ $x_1=x_2$. Therefore f is one-one. Let $f(x)=e^x=y$. Taking log on both sides, we get $x=\log y$. We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number. Therefore function f is into.

35 A function f from the set of natural numbers to integers defined by $f(n)=\begin{cases}\frac{n-1}{2}, & \text{when } n \text{ is odd}\\ -\frac{n}{2}, & \text{when } n \text{ is even}\end{cases}$, is

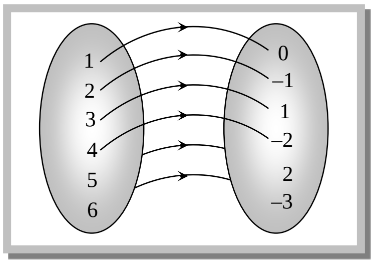
(a) One-one but not onto (b) Onto but not one-one

(c) One-one and onto both (d) Neither one-one nor onto

Sol. (c)

$f:N\to I$

$f(1)=0, f(2)=-1, f(3)=1, f(4)=-2, f(5)=2$ and $f(6)=-3$ so on.



In this type of function every element of set *A* has unique image in set *B* and there is no element left in set *B.* Hence *f* is one-one and onto function.

36 Which of the following is an even function

(a) $x\left(\frac{a^x-1}{a^x+1}\right)$ (b) $\tan x$ (c) $\frac{a^x-a^{-x}}{2}$ (d) $\frac{a^x+1}{a^x-1}$

Sol. (a)

We have : $f(x)=x\left(\frac{a^x-1}{a^x+1}\right)$

$$f(-x)=-x\left(\frac{a^{-x}-1}{a^{-x}+1}\right)=-x\left(\frac{\frac{1}{a^x}-1}{\frac{1}{a^x}+1}\right)=-x\left(\frac{1-a^x}{1+a^x}\right)=x\left(\frac{a^x-1}{a^x+1}\right)=f(x)$$

So, $f(x)$ is an even function.

37 Let $f(x)=\sqrt{x^4+15},$ then the graph of the function $y=f(x)$ is symmetrical about

(a) The *x*-axis (b) The *y*-axis (c) The origin (d) The line $x=y$

Sol. (b)

$f(x)=\sqrt{x^4+15} \Rightarrow f(-x)=\sqrt{(-x)^4+15}=\sqrt{x^4+15}=f(x)$

$\Rightarrow f(-x)=f(x) \Rightarrow f(x)$ is an even function $\Rightarrow f(x)$ is symmetric about y-axis.

38 The function $f(x)=\log(x+\sqrt{x^2+1})$ is

(a) An even function (b) An odd function

(c) Periodic function (d) None of these

Sol. (b)

$f(x)=\log(x+\sqrt{x^2+1})$ and $f(-x)=-\log(x+\sqrt{x^2+1})=-f(x)$, so $f(x)$ is an odd function.

39 Which of the following is an even function

(a) $f(x)=\frac{a^x+1}{a^x-1}$ (b) $f(x)=x\left(\frac{a^x-1}{a^x+1}\right)$ (c) $f(x)=\frac{a^x-a^{-x}}{a^x+a^{-x}}$ (d) $f(x)=\sin x$

Sol. (b)

In option (a), $f(-x)=\frac{a^{-x}+1}{a^{-x}-1}=\frac{1+a^x}{1-a^x}=-\frac{a^x+1}{a^x-1}=-f(x)$ So, It is an odd function.

In option (b), $f(-x)=(-x)\frac{a^{-x}-1}{a^{-x}+1}=-x\frac{(1-a^x)}{1+a^x}=x\frac{(a^x-1)}{(a^x+1)}=f(x)$ So, It is an even function.

In option (c), $f(-x)=\dfrac{a^{-x}-a^{x}}{a^{-x}+a^{x}}=-f(x)$ So, It is an odd function.

In option (d), $f(-x)=\sin(-x)=-\sin x=-f(x)$ So, It is an odd function.

40 The function $f(x)=\sin\left(\log(x+\sqrt{x^2+1})\right)$ is

(a) Even function (b) Odd function

(c) Neither even nor odd (d) Periodic function

Sol. (b)

$f(x)=\sin\left(\log(x+\sqrt{1+x^2})\right)$

$\Rightarrow \quad f(-x)=\sin[\log(-x+\sqrt{1+x^2})] \quad \Rightarrow \quad f(-x)=\sin\log\left((\sqrt{1+x^2}-x)\dfrac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right)$

$\Rightarrow \quad f(-x)=\sin\log\left[\dfrac{1}{(x+\sqrt{1+x^2})}\right] \quad \Rightarrow \quad f(-x)=\sin\left[\log(x+\sqrt{1+x^2})^{-1}\right]$

$\Rightarrow \quad f(-x)=\sin\left[-\log(x+\sqrt{1+x^2})\right] \quad \Rightarrow \quad f(-x)=-\sin\left[\log(x+\sqrt{1+x^2})\right] \Rightarrow f(-x)=-f(x)$

$\therefore$ $f(x)$ is odd function.

41 The period of the function $f(x)=2\cos\dfrac{1}{3}(x-\pi)$ is

(a) 6π (b) 4π (c) 2π (d) π

Sol. (a)

$f(x)=2\cos\dfrac{1}{3}(x-\pi) \;=2\cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$

Now, since $\cos x$ has period $2\pi \Rightarrow \cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$ has period $\dfrac{2\pi}{\frac{1}{3}}=6\pi$

$\Rightarrow 2\cos\left(\dfrac{x}{3}-\dfrac{\pi}{3}\right)$ has period $=6\pi$.

42 The function $f(x)=\sin\dfrac{\pi x}{2}+2\cos\dfrac{\pi x}{3}-\tan\dfrac{\pi x}{4}$ is periodic with period

(a) 6 (b) 3 (c) 4 (d) 12

Sol. (d)

$\because$ $\sin x$ has period $=2\pi \Rightarrow \sin\dfrac{\pi x}{2}$ has period $=\dfrac{2\pi}{\frac{\pi}{2}}=4$

$\because$ $\cos x$ has period $=2\pi \Rightarrow \cos\dfrac{\pi x}{3}$ has period $=\dfrac{2\pi}{\frac{\pi}{3}}=6 \Rightarrow 2\cos\dfrac{\pi x}{3}$ has period = 6

$\because$ $\tan x$ has period $=\pi \Rightarrow \tan\dfrac{\pi x}{4}$ has period $=\dfrac{\pi}{\frac{\pi}{4}}=4$.

L.C.M. of 4, 6 and 4 =12, period of $f(x)$ = 12.

43 The period of $|\sin 2x|$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

Sol. (b)

Here $|\sin 2x| = \sqrt{\sin^2 2x} = \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of $\cos 4x$ is $\frac{\pi}{2}$. Hence, period of $|\sin 2x|$ will be $\frac{\pi}{2}$

Trick : $\because$ $\sin x$ has period $= 2\pi \Rightarrow$ $\sin 2x$ has period $= \frac{2\pi}{2} = \pi$. Now, if $f(x)$ has period p then $|f(x)|$ has period $\frac{p}{2} \Rightarrow$ $|\sin 2x|$ has period $= \frac{\pi}{2}$.

44 If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals

(a) 0 (b) 2 (c) 4 (d) – 4

Sol. (a)

Given, $f(x)$ is an odd periodic function. We can take $\sin x$, which is odd and periodic.

Now since, $\sin x$ has period = 2 and $f(x)$ has period = 2.

So, $f(x) = \sin(\pi x) \Rightarrow$ $f(4) = \sin(4\pi) = 0$.

45 The period of the function $f(x) = \sin^2 x$ is

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) None of these

Sol. (b)

$\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow$ Period $= \frac{2\pi}{2} = \pi.$

46 The period of $f(x) = x - [x]$, if it is periodic, is

(a) $f(x)$ is not periodic (b) $\frac{1}{2}$ (c) 1 (d) 2

Sol. (c)

Let $f(x)$ be periodic with period *T*. Then,

$f(x+T) = f(x)$ for all $x \in R$ $\Rightarrow$ $x + T - [x+T] = x - [x]$ for all $x \in R$ $\Rightarrow$ $x + T - x = [x+T] - [x]$

$\Rightarrow$ $[x+T] - [x] = T$ for all $x \in R$ $\Rightarrow$ $T = 1, 2, 3, 4, ...$

The smallest value of *T* satisfying,

$f(x+T) = f(x)$ for all $x \in R$ is 1.

Hence $f(x) = x - [x]$ has period 1.

47 The period of $f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$, $n \in Z$, $n > 2$ is

(a) $2\pi n(n-1)$ (b) $4n(n-1)$ (c) $2n(n-1)$ (d) None of these

Sol. (c)

$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right)$

Period of $\sin\left(\frac{\pi x}{n-1}\right) = \frac{2\pi}{\left(\frac{\pi}{n-1}\right)} = 2(n-1)$ and period of $\cos\left(\frac{\pi x}{n}\right) = \frac{2\pi}{\left(\frac{\pi}{n}\right)} = 2n$

Hence period of $f(x)$ is LCM of $2n$ and $2(n-1) \Rightarrow 2n(n-1)$.

48 If a,b be two fixed positive integers such that $f(a+x)=b+[b^3+1-3b^2f(x)+3b\{f(x)\}^2-\{f(x)\}^3]^{\frac{1}{3}}$ for all real x, then $f(x)$ is a periodic function with period

(a) a (b) $2a$ (c) b (d) $2b$

Sol. (b)

$f(a+x)=b+(1+\{b-f(x)\}^3)^{1/3} \Rightarrow f(a+x)-b=\{1-\{f(x)-b\}^3\}^{1/3}$

$\Rightarrow \phi(a+x)=\{1-\{\phi(x)\}^3\}^{1/3}$ $[\phi(x)=f(x)-b]$ $\Rightarrow \phi(x+2a)=\{1-\{\phi(x+a)\}^3\}^{1/3}=\phi(x)$

$\Rightarrow f(x+2a)-b=f(x)-b \Rightarrow f(x+2a)=f(x)$

$\therefore$ $f(x)$ is periodic with period $2a$.

49 If $f:R\to R, f(x)=2x-1$ and $g:R\to R$, $g(x)=x^2$ then $(gof)(x)$ equals

(a) $2x^2-1$ (b) $(2x-1)^2$ (c) $4x^2-2x+1$ (d) x^2+2x-1

Sol. (b)

$gof(x)=g\{f(x)\}=g(2x-1)=(2x-1)^2$.

50 If $f:R\to R, f(x)=(x+1)^2$ and $g:R\to R, g(x)=x^2+1$, then $(fog)(-3)$ is equal to

(a) 121 (b) 144 (c) 112 (d) 11

Sol. (a)

$fog(x)=f\{g(x)\}=f(x^2+1)=(x^2+1+1)^2=(x^2+2)^2 \Rightarrow fog(-3)=(9+2)^2=121$.

51 Which of the following function is invertible

(a) $f(x)=2^x$ (b) $f(x)=x^3-x$ (c) $f(x)=x^2$ (d) None of these

Sol. (a)

A function is invertible if it is one-one and onto.

52 If $g(x)=x^2+x-2$ and $\frac{1}{2}(gof)(x)=2x^2-5x+2$, then $f(x)$ is equal to

(a) $2x-3$ (b) $2x+3$ (c) $2x^2+3x+1$ (d) $2x^2-3x-1$

Sol. (a)

$g(x)=x^2+x-2 \Rightarrow (gof)(x)=g[f(x)]=[f(x)]^2+f(x)-2$

Given, $\frac{1}{2}(gof)(x)=2x^2-5x+2$ $\therefore$ $\frac{1}{2}[f(x)]^2+\frac{1}{2}f(x)-1=2x^2-5x+2$

$\Rightarrow [f(x)]^2+f(x)=4x^2-10x+6 \Rightarrow f(x)[f(x)+1]=(2x-3)[(2x-3)+1] \Rightarrow f(x)=2x-3$.

53 If $f(y)=\frac{y}{\sqrt{1-y^2}}, g(y)=\frac{y}{\sqrt{1+y^2}}$, then $(fog)(y)$ is equal to

(a) $\frac{y}{\sqrt{1-y^2}}$ (b) $\frac{y}{\sqrt{1+y^2}}$ (c) y (d) $\frac{1-y^2}{\sqrt{1+y^2}}$

Sol. (c)

$$f[g(y)]=\frac{y/\sqrt{1+y^2}}{\sqrt{1-\left(\frac{y}{\sqrt{1+y^2}}\right)^2}}=\frac{y}{\sqrt{1+y^2}}\times\frac{\sqrt{1+y^2}}{\sqrt{1+y^2-y^2}}=y$$

54 If $f(x)=\dfrac{2x-3}{x-2}$, then $[f\{f(x)\}]$ equals

(a) x (b) $-x$ (c) $\dfrac{x}{2}$ (d) $-\dfrac{1}{x}$

Sol. (a)

$$f[f(x)]=\dfrac{2\left(\dfrac{2x-3}{x-2}\right)-3}{\left(\dfrac{2x-3}{x-2}\right)-2}=x$$

55 Suppose that $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$, then $f(x)$ is

(a) $1+2x^2$ (b) $2+x^2$ (c) $1+x$ (d) $2+x$

Sol. (b)

$g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$ ….. (i)

$\Rightarrow$ $f(1+\sqrt{x})=3+2\sqrt{x}+x$

Put $1+\sqrt{x}=y \Rightarrow x=(y-1)^2$

then, $f(y)=3+2(y-1)+(y-1)^2=2+y^2$

therefore, $f(x)=2+x^2$.

56 Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$, then for all x, $f(g(x))$ is equal to

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

Sol. (b)

Here $g(x)=1+n-n=1, x=n\in Z$

$1+n+k-n=1+k$, $x=n+k$ (where $n\in Z,\ 0<k<1$)

Now $f(g(x))=\begin{cases}-1, & g(x)<0\\ 0, & g(x)=0\\ 1, & g(x)>0\end{cases}$

Clearly, $g(x)>0$ for all x. So, $f(g(x))=1$ for all x.

57 If $f(x)=\dfrac{2x+1}{3x-2}$, then $(fof)(2)$ is equal to

(a) 1 (b) 3 (c) 4 (d) 2

Sol. (d)

Here $f(2)=\dfrac{5}{4}$

Hence $(fof)(2)=f(f(2))=f\left(\dfrac{5}{4}\right)=\dfrac{2\times\frac{5}{4}+1}{3\times\frac{5}{4}-2}=2$.

58 If $f:R\to R$ and $g:R\to R$ are given by $f(x)=|x|$ and $g(x)=[x]$ for each $x\in R$, then $\{x\in R: g(f(x))\le f(g(x))\}=$

(a) $Z\cup(-\infty,0)$ (b) $(-\infty,0)$ (c) Z (d) R

Sol. (d)

$g(f(x))\le f(g(x)) \Rightarrow g(|x|)\le f[x] \Rightarrow [|x|]\le|[x]|$. This is true for $x\in R$.

59 If $f:[1,\infty)\to[1,\infty)$ is defined as $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ is equal to

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (d) Not defined

Sol. (b)

Given $f(x) = 2^{x(x-1)} \Rightarrow x(x-1) = \log_2 f(x)$

$\Rightarrow x^2 - x - \log_2 f(x) = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+4\log_2 f(x)}}{2}$

Only $x = \frac{1+\sqrt{1+4\log_2 f(x)}}{2}$ lies in the domain

$\therefore \quad f^{-1}(x) = \frac{1}{2}[1+\sqrt{1+4\log_2 x}]$

60 If the function $f : R \to R$ be such that $f(x) = x - [x]$, where $[y]$ denotes the greatest integer less than or equal to *y*, then $f^{-1}(x)$ is

(a) $\frac{1}{x-[x]}$ (b) $[x]-x$ (c) Not defined (d) None of these

Sol. (c)

$f(x) = x - [x]$ Since, for $x = 0 \Rightarrow f(x) = 0$

For $x = 1 \Rightarrow f(x) = 0$.

For every integer value of x, $f(x) = 0$

$\Rightarrow$ $f(x)$ is not one-one

$\Rightarrow$ So $f^{-1}(x)$ is not defined.

EXERCISE – 2

Q.1 (D)

Domain of $\sqrt{\sec^{-1}\left(\frac{2-|x|}{4}\right)}$

$\sec^{-1}$ x is always positive

So, $\frac{2-|x|}{4} \geq 1$ or $\frac{2-|x|}{4} \leq -1$

$\Rightarrow 2-|x| \geq 4$ or $2-|x| \leq -4$

$\Rightarrow |x| \leq -2$ or $|x| \geq 6$

$\Rightarrow x \in [-\infty, -6] \cup [6, \infty]$

Q.2 (D)

$$f(x) = \log\left(\frac{x^2-5x+6}{x^2+x+1}\right) + \sqrt{\frac{1}{[x^2-1]}}$$

$\frac{x^2-5x+6}{x^2+x+1} > 0$ and $[x^2-1] > 0$

$\Rightarrow (x-2)(x-3) > 0$ and $x^2-1 \geq 1$

$\Rightarrow x \in (-\infty, 2) \cup (3, \infty)$ and $x \in [-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty]$

$\Rightarrow x \in [-\infty, -\sqrt{2}] \cup [\sqrt{2}, 2] \cup (3, \infty)$

Q.3 (A)

$$f(x) = \sin^{-1}\left(\frac{1-|x|}{3}\right) + \cos^{-1}\left(\frac{|x|-3}{5}\right)$$

$-1 \leq \frac{1-|x|}{3} \leq 1$ and $-1 \leq \frac{|x|-3}{5} \leq 5$

$\Rightarrow -3 \leq 1-|x| \leq 3$ and $\Rightarrow -5 \leq |x|-3 \leq 5$

$\Rightarrow 4 \geq |x| \geq -2$ and $\Rightarrow -2 \leq |x| \leq 8$

So, $x \in [-4, 4]$ and $x \in [-8, 8]$

Q.4 (B)

$f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$

$2\{x\}^2 - 3\{x\} + 1 \geq 0$

$\Rightarrow (2\{x\} - 1)(\{x\} - 1) \geq 0$

$\Rightarrow \{x\} \in \left(-\infty, \frac{1}{2}\right) \cup (1, \infty)$

But $\{x\}$ has range (0 , 1) only so, $\{x\} \in \left[0, \frac{1}{2}\right]$ and $x = [x] + \{x\}$

in (– 1 , 1), $x \in \left[-1+0, -1+\frac{1}{2}\right] \cup \left[0+0, 0+\frac{1}{2}\right] \cup \{1+0\}$

$\Rightarrow x \in \left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$

Q.5 (D)

$f(x) = \sqrt{\cos(\sin x)} + \sqrt{\log_x \{x\}}$

$-1 \leq \sin x \leq 1$ and $0 \leq \{x\} \leq 1$

$1 \geq \cos(\sin x) \geq \cos 1$ So, $x \neq n$, $n \in \mathrm{I}$

Hence, $x \in R$ $\log_x \{n\} \geq 0$

$\Rightarrow x < 1$, $x > 0$, $x \neq 1$

So, $x \in (0, 1)$

Q.6 (D)

$\sqrt{[x] - 1 + x^2}$

$\Rightarrow x^2 \geq 1 - [x]$

Hence for $x \geq 1$ & $x \leq -3$ …………(1)

For, $x \in (-1,1)$, $x^2 \in (0,1)$

And $1-[x] \geq 0$. So, not satisfying inequality.

For, $x \in (-2,-1), [x] = -2$ …………(2)

$\Rightarrow x^2 \geq 3$

So, $x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Hence, $x \in [-\infty, -\sqrt{3}]$ …………..(3)

From (1) , (2) , (3)

$x \in (-\infty, -\sqrt{3}) \cup (1, \infty)$

Q.7 (D)

$f(x) = \sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$ [.] $\rightarrow$ GIF

$-1 \leq \left[\log_2\left(\frac{x^2}{2}\right)\right] \leq 1$

$\Rightarrow -1 \leq \log_2\left(\frac{x^2}{2}\right) < 2$

$\Rightarrow \frac{1}{2} \leq \frac{x^2}{2} < 4$

$\Rightarrow 1 \leq x^2 < 8$

So, $x \in \left(-2\sqrt{2}, -1\right) \cup \left(1, 2\sqrt{2}\right)$

Q.8 (C)

$2^x + 2^{f(x)} = 2$

$\Rightarrow 2^{f(x)} = (2 - 2^x)$

$\Rightarrow f(x)=\log_2(2-2^x)$

So, $2-2^x > 0$

$\Rightarrow 2^x < 2$

$\Rightarrow x<1$

Solution : $(-\infty,1)$

Q.9 (B)

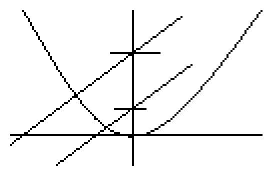
$f(x)$ has domain $[-1,2]$

For $f([x]-x^2+4)$ to have real value.

$-1\le[x]-x^2+4\le 2$

$5\ge x^2-[x]\ge 2$

$5+[x]\ge x^2\ge 2+[x]$



$x^2\ge 2+[x]$ is always true for $x\ge\sqrt{3}$ and $x\le -1$

So, $x\in(-\infty,-1)\cup(\sqrt{3},\infty)$

$x^2\le 5+[x]$ is always true for $x\le\sqrt{7}$ and $x\ge -\sqrt{3}$

$x\in(-\sqrt{3},7)$

Solution : $x\in[-\sqrt{3},-1]\cup[\sqrt{3},\sqrt{7}]$

Q.10 (C)

$f(x)=\dfrac{1}{1-2\cos x}$

$-1\le\cos x\le 1$

$\Rightarrow -2\le 2\cos x\le 2$

$\Rightarrow 3\ge 1-2\cos x\ge -1$

$\Rightarrow \dfrac{1}{1-2\cos x}\le -1 \quad \dfrac{1}{1-2\cos x}\ge\dfrac{1}{3}$

So, $x \in (-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$

Q.11 (D)

$$f(x) = \sin^{-1} x + \tan^{-1} x + \cos^{-1} x$$

$$= (\sin^{-1} x + \cos^{-1} x) + \tan^{-1} x$$

$$= \frac{\pi}{2} + \tan^{-1} x$$

But $x \in [-1, 1]$

So, $\frac{-\pi}{4} \le \tan^{-1} x \le \frac{\pi}{4}$

$$\frac{\pi}{4} \le \frac{\pi}{2} + \tan^{-1} x \le \frac{3\pi}{4}$$

Hence $x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Q.12. (B)

$$f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$$

$$= \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left(\left[x^2 + \frac{1}{2}\right] - 1\right)$$

Now, $-1 \le \left[x^2 + \frac{1}{2}\right] \le 1$

And $1 \le \left[x^2 + \frac{1}{2}\right] - 1 \le 1$

So, $0 \le \left[x^2 + \frac{1}{2}\right] \le 1$

Hence, $\left[x^2 + \frac{1}{2}\right] = \{0, 1\}$

Hence, $f(x) \in \{\pi\}$

Q.13 (C)

$$f(x) = \sin^{-1}\left(\sqrt{x^2+x+1}\right)$$

$$x^2+x+1 \geq \frac{3}{4}$$

$$\Rightarrow \sqrt{x^2+x+1} \geq \frac{\sqrt{3}}{2}$$

For $\sin^{-1}\sqrt{x^2+x+1}$ to be defined

$$\frac{\sqrt{3}}{2} \leq \sqrt{x^2+x+1} \leq 1$$

$$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \leq \sin^{-1}\sqrt{x^2+x+1} \leq \sin^{-1}1$$

$$\Rightarrow \frac{\pi}{3} \leq f(x) \leq \frac{\pi}{2}$$

So, $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

Q.14 (C)

$$f(x) = \cos^{-1}\left(\frac{x^2}{\sqrt{1+x^2}}\right)$$

Range of $\frac{x^2}{\sqrt{1+x^2}}$: $[0,1]$

hence range of f (x) : $\left[0, \frac{\pi}{2}\right]$

Q.15 (D)

$$f(x) = \sqrt{\ln(\cos(\sin x))}$$

$-1 \le \sin x \le 1$

$\cos 1 \le \cos\ (\sin x) \le 1$

or $\ln \cos(\sin x) \le \ln 1$

For square root to be defined, $\ln \cos(\sin x) \ge 0$

hence $\ln \cos(\sin x) = 0$.

Range of f (x) : {0}

Q.16 (D)

$$f(x) = \frac{x-1}{x^2-2x+3}$$

Discriminant of x^2-2x+3 is negative so x^2-2x+3 is always positive

$$f(x) = \frac{x-1}{(x-1)^2+2} = \frac{1}{(x-1)+\frac{2}{x-1}}$$

So, for x > 1, $(x-1)+\left(\frac{2}{x-1}\right) \ge 2\sqrt{2}$

So, $\frac{1}{(x-1)+\frac{2}{x-1}} \le \frac{1}{2\sqrt{2}}$

Similarly, $x<1$, $\frac{1}{(x-1)+\frac{2}{x-1}} \ge \frac{1}{-2\sqrt{2}}$

So, $f(x) \in \left[\frac{1}{-2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Q.17 (D)

$f(x) = \cos^{-1}\left(\frac{1+x^2}{2x}\right) + \sqrt{2-x^2}$

$\frac{1+x^2}{2x} \geq 1$ or $\frac{1+x^2}{2x} \leq -1$

So, x = 1 and $x = -1$ are the 2 points in domain.

So, $f(1) = 0 + 1$

$f(-1) = \pi + 1$

So, Range $= \{1, 1+\pi\}$

Q.18 (D)

$f(x) = \frac{\tan(\pi[x^2 - x])}{1 + \sin(\cos x)}$

Domain is R

$\because$ $[\cdot]$ is an integral value so, $\pi[x^2 - x]$ is an integral multiple of π.

hence $\tan(\pi[\cdot]) = 0 \quad \forall\ x \in R$

Range $= \{0\}$

Q.19 (D)

$f(x) = \frac{e^x}{[x+1]}$, $x \geq 0$

$\because$ e^x is an increasing continuous function and $[x + 1] \geq 1$

Hence, Range will be $[1, \infty]$

Q.20 (C)

$f(x) = \frac{1}{1-x}$, $x \neq 1 \Rightarrow f(f(x)) = \frac{1}{1-f(x)}$

$$= \frac{1}{1 - \frac{1}{1-x}}$$

$$=\frac{x-1}{x}\ ,\ x\neq 0\ ,\ \ x\neq 1$$

Further $f(f(f(x)))=\dfrac{f(x)-1}{f(x)}$

$$=\frac{\frac{1}{1-x}-1}{\frac{1}{1-x}},\ \ x\neq 0\ ,\ x\neq 1$$

$$=x\ ,\quad x\neq 0\ ,\ x\neq 1$$

Q.21 (A)

$f(g(x))=|\sin x|$

$g(f(x))=\sin^2\sqrt{x}$

So, $f(x)=\sqrt{x}$ & $g(x)=\sin^2 x$

Q.22 (A)

Given $f(x)=\begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$

So, $f(f(x))=\begin{cases} f(x) & \text{if } f(x) \text{ is rational} \\ 1-f(x) & \text{if } f(x) \text{ is irrational} \end{cases}$

$\Rightarrow f(f(x))=\begin{cases} x & \text{if } x \text{ is rational} \\ 1-(1-x) & \text{if } x \text{ is irrational} \end{cases}$

Hence *f* (*f* (x)) = x.

Q.23 (D)

$f(x)=\begin{cases} x^2 & \text{if } x\leq 0 \\ x & \text{if } x>0 \end{cases}$

$f(f(x))=\begin{cases} (f(x))^2 & \text{if } f(x)\leq 0 \\ f(x) & \text{if } f(x)>0 \end{cases}$

Now *f* (x) can’t be less than 0 hence

$f(f(x)) = f(x)$ for all values of x.

Q.24 (A)

$f(x) = \sin^{-1}(\sin x) + e^{\tan x}$

$\sin^{-1}(\sin x)$ has a period of 2π

and $e^{\tan x}$ has a period of π

So, period of $f(x)$ = LCM{π, 2π} i.e. 2π

Q.25 (D)

(A) $\frac{2^x}{2^{[x]}} = 2^{x-[x]} = 2^{\{x\}}$.

Now $2^{\{x+T\}} = 2^{\{x\}} \Rightarrow \{x+T\} = \{x\} \Rightarrow T = 1$.

(B) $\sin^{-1}\{x+T\} = \sin^{-1}\{x\} \Rightarrow \{x+T\} = \{x\} \Rightarrow T = 1$

(C) $\sin^{-1}\left(\sqrt{\sin(x+T)}\right) = \sin^{-1}\left(\sqrt{\sin x}\right) \Rightarrow \sin(x+T) = \sin x \Rightarrow T = 2\pi$

(D) $\sin^{-1}\left(\cos(x+T)^2\right) = \sin^{-1}\left(\cos(x)^2\right) \Rightarrow \cos(x+T)^2 = \cos(x)^2$

Function is non periodic.

Q.26 (C)

$f(x) = \frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}$, $n \in N$ has period 6π

Numerator has a period of $\left(\frac{\pi}{n}\right)$

Denominator has period of $n\pi$, where $n \in I$

So, period of $f(x)$ = LCM$\left\{\frac{\pi}{n}, n\pi\right\} = n\pi = 6\pi$

$\Rightarrow n = 6$

Q.27 (A)

$f(x)=\sin 3\pi\{x\}+\tan \pi[x]$

As $[x]$ is an integer hence $\tan \pi[x]$ is always equal to 0.

$f(x+T)=f(x) \Rightarrow \sin\left(3\pi\{x+T\}\right)=\sin\left(3\pi\{x\}\right) \Rightarrow 3\pi\{x+T\}=n\pi+\left(-1\right)^n\left(3\pi\{x\}\right)$

or $3\{x+T\}=n+\left(-1\right)^n\left(3\{x\}\right)$

(i) $n=2m \Rightarrow \{x+T\}-\{x\}=\dfrac{2m}{3}$

As $0\le\{x+T\}-\{x\}<1$ hence $m=0$

$\Rightarrow \{x+T\}=\{x\} \Rightarrow T=1$

(ii) $n=2m+1 \Rightarrow \{x+T\}+\{x\}=\dfrac{2m+1}{3}$

As $0\le\{x+T\}+\{x\}<2$ hence $m=1$

$\Rightarrow \{x+T\}+\{x\}=1 \Rightarrow T=1$

Therefore period is 1.

Q.28 (B)

$f(x)=\sin(\cos x)-x+\tan(\sin x)\ \forall\ x\in(0,\infty)$.

If $f(x)$ is defined in (0, a), then odd extension of $f\left(x\right)=-f\left(-x\right)$ in (–a, 0).

So, odd extension of $f(x)=\sin(\cos x)+x-\tan(\sin x)\ \ \forall\ x\in(-\infty,0)$.

Q.29 (C)

(A) $g(x)-g(-x)=f(x)$

$f(-x)=g(-x)-(g(x))$

$=-(g(x)-g(-x))$

Therefore it’s an Odd function.

(B) Similar as (A) odd function.

(C) $f(x)=\log\left(\dfrac{x^4+x^2+1}{x^2+x+1}\right)$

$$f(-x) = \log\left(\frac{x^4 + x^2 + 1}{x^2 - x + 1}\right)$$

$$= -\log\left(\frac{x^2 - x + 1}{x^4 + x^2 + 1}\right) \neq -f(x)$$

So it's not an odd function

(D) $xg(x) \cdot g(-x) + \tan(\sin x) = f(x)$

$$f(-x) = -xg(-x) \cdot g(x) + \tan(\sin x)$$

$$= -(xg(x) \cdot g(-x) + \tan(\sin x))$$

$$0 = -f(x)$$

It's an odd function.

Q.30 (B)

$f:[-4, 4] \rightarrow \{\pi, 0, \pi\} \rightarrow R$

$f(x) = \cot(\sin x) + \left[\frac{x^2}{|a|}\right]$ is an odd function

Then $f(-x) = -f(x)$

$$\Rightarrow -\cot(\sin x) = \left[\frac{x^2}{|a|}\right] = -\cot \sin f\left[\frac{x^2}{|a|}\right]$$

$$\Rightarrow 2\left[\frac{x^2}{|a|}\right] = 0$$

$\Rightarrow |a| < x^2$

$\Rightarrow |a| > (x^2)_{\max}$

$\Rightarrow |a| > 16$

$a \in (-\infty, -16) \cup (16, \infty)$

Q.31 (B)

$f:(2,\infty)\ \rightarrow (-\infty,4)$

$f(x)=x\left(4-x\right)$

$=x^2-4x$

$f'(x)=-2x+4=0$

$\Rightarrow x=2\ ,\quad f(2)=4$

Hence, function is bijective in $(2\ ,\ \infty)\ \rightarrow\ (-\infty,\ 4)$

$y=x\left(y-x\right)$

$\Rightarrow x^2-4x+y=0$

$\Rightarrow x=\dfrac{4\pm\sqrt{46-4y}}{2}$

$\Rightarrow x=2\pm\sqrt{4-y}$

Hence, $f^{-1}(x)=2+\sqrt{4-x}$

Q.32 (C)

$\text{A}=\{1\ ,\ 2,3,4\}$

$f:A\rightarrow A$

$f(2)\neq 2, f(4)\neq 4\ ,\ f(1)=1$

If $f(3)=3$, then $f(2)=4\ ,\ f(4)=2, f(1)=1$

If $f(3)=2$, then $f(2)=4\ ,\ f(4)=3,\ f(1)=1$

If $f(3)=4$, then $f(2)=3\ ,\ f(4)=2\ ,\ f(1)=1$

Q.33 (A)

$f:(-\infty,1)\ \rightarrow \left(\dfrac{1}{2}\ ,\ \infty\right)$

$f(x)=2^{x(x-2)}$

$g(x)=x(x-2)=x^2-2x$

$\forall\, x\in(-\infty,1)\, g(x)$ is one – one

and for, $\forall\, x\in(-\infty,+1), g(x)\in(-1,\infty)$

Hence, $f(x)\in\left(\frac{1}{2}\,,\,\infty\right)$

Hence, $f(x)$ is invertible.

$y=2^{x(x-2)}$

$\Rightarrow x^2-2x=\log y$

$\Rightarrow x^2-2x-\log y=0$

So, $x=\dfrac{2\pm\sqrt{4+4\log y}}{2}$

Hence, $f^{-1}(x)=1-\sqrt{1+\log_2 x}$

Q.34 (C)

$f:R\to R$

$f(x)=ax+\cos x$ is invertible function

So, $f(x)$ should be injective

for $a\neq 0$, Range is R

So, $f(x)$ to be one – one

$f'(x)\geq 0 \Rightarrow a-\sin x\geq 0$

$\Rightarrow a\geq \sin \Rightarrow a\geq 1$

or, $f'(x)\leq 0\Rightarrow a-\sin x\leq 0$

$\Rightarrow a\leq \sin x \Rightarrow a\leq -1$

So, $a \in (-\infty, -1) \cup (1, \infty)$

Q.35 (C)

$f(x) = \cot^{-1} \log_{\frac{1}{2}} (x^4 - 2x^2 + 3)$

$x^4 - 2x^2 + 3 = (x^2 - 1)^2 + 2$

Hence, $(x^2 - 1)^2 + 2 \geq 2$

$\Rightarrow g(x) = \log_{\frac{1}{2}} \left[(x^2 - 1) + 2\right] \leq -1$

Hence, $-\infty < g(x) \leq -1$

So, $\cot^{-1}(-1) \leq \cot^{-1}(g(x)) < \cot^{-1}(-\infty)$

$\Rightarrow \frac{3\pi}{4} \leq f(x) < \pi$

Q.36 (C)

$f(x) = \sin(x + 3 - [x + 3])$

$= \sin(\{x + 3\})$

$= \sin(\{x\}$

Hence, period of f (x) = 1.

Q.37 (B)

$f(x) = x^2 + bx + c$

if $f(2+t) = f(2 = t) \Rightarrow f(x)$ is symmetric about x = 2

Hence, $f(x)$ is minimum at x = 2

Hence, $f(1) = f(3) > f(2)$

$f(0) = f(4) > f(1) = f(3) > f(2)$

Hence, $f(4) > f(1) > f(2)$

Q.38 (A)

$f(x+ay, x-ay) = axy$

Let, $x+ay = u$

$x-ay = w$

So, $x = \frac{u+w}{2}$, $y = \left(\frac{u-w}{2a}\right)$

Hence, $f(u,w) = a\cdot\left(\frac{u+w}{2}\right)\left(\frac{u-w}{2a}\right) = \frac{u^2-w^2}{4}$

So, $f(x,y) = \frac{x^2-y^2}{4}$

Q.39 (D)

$[x]\{x\} = 1 \Rightarrow \{x\} = \frac{1}{[x]}$

$0 \le \{x\} < 1$, hence , $[x] \ge 2$

So, for $[x] = \text{I}$; $\text{I} \ge 2$

$x = [x] + \frac{1}{[x]} = \text{I} + \frac{1}{\text{I}}$

So, solution $= \left\{ m + \frac{1}{m} \mid m \in N - \{1\} \right\}$

Q.40 (A)

$f(x) = 2\tan 3x + 5\sqrt{1-\cos 6x}$

$= 2\tan 3x + 5|\sin 3x|\sqrt{2}$

Period of $\tan 3x$ is $\frac{\pi}{3}$ and period of $|\sin 3x|$ is $\frac{\pi}{3}$.

So, period of $f(\text{x}) = \frac{\pi}{3}$.

Hence, $g(x)$ has a period $= \frac{\pi}{3}$

(A) $(\sec^2 3x + \operatorname{cos}ec^2 3x)\tan^2 3x$

$= 3 + \tan^4 3x$ has period $\dfrac{\pi}{3}$

(B) $2\cos 3x + 3\cos 3x = \sqrt{13}\cos(3x+\phi)$

period $= \dfrac{2\pi}{3}$

(C) $2\sqrt{1-\cos^2 3x} + \operatorname{cos}ec 3x$

$= 2\left|\sin\dfrac{3x}{2}\right| + \operatorname{cos}ec 3x$

period $= \dfrac{2\pi}{3}$

(D) g(x) = 3 cosec 3x + 2 tan 3x

Period of cosec 3x = $\dfrac{2\pi}{3}$ and period of tan 3x = $\dfrac{\pi}{3}$.

Hence period of g (x) = $\dfrac{2\pi}{3}$.

Exercise 3

Q.1 (B)

$\Rightarrow \log_{\frac{1}{2}}\left(x^2-5x+7\right)>0$

$\Rightarrow 0 < x^2 - 5x + 7 < 1$

$\Rightarrow x \in (2,3)$

Q.2 (B)

$\Rightarrow \log_3\left(x^2-6x+11\right)<1$

$\Rightarrow 0 < x^2 - 6x + 11 < 3$

$\Rightarrow x \in (2,4)$

Q.3 (D)

In this case base is variable. Thus we must take two separate cases:

(i) $|x| \in (0,1)$. In this case we have to ensure that $0 < x^2 + x + 1 \leq 1$

$\Rightarrow x \in [-1,0]$.

$\Rightarrow$ Common part of $|x| \in (0,1)$

$\Rightarrow$ And $x \in [-1,0]$ is $x \in (-1,0)$.

(ii) $|x| > 1$. In this case we must have $x^2 + x + 1 \geq 1$

$\Rightarrow x \in (-\infty,1) \cup (0,\infty)$.

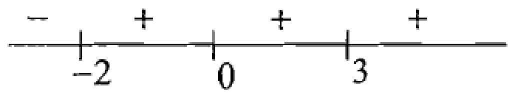
$\Rightarrow$ Common part of $|x| > 1$ and $x \in (-\infty,-1) \cup (0,\infty)$ is $(-\infty,-1)$ is $(-\infty,-1) \cup (1,\infty)$

$\Rightarrow$ Thus, the final solution is $x \in (-\infty,-1) \cup (-1,0) \cup (1,\infty)$

Q.4 (C)

$\Rightarrow$ Using wavy curve method and the fact that x = 0 and 3 are the repeated roots of

$x\,(e^x - 1)\,(x + 2)\,(x - 3)^2$ we get the sign scheme of the given expression as



$\Rightarrow$ Thus complete solution is $x \in (-\infty, -2] \cup (0, 3)$.

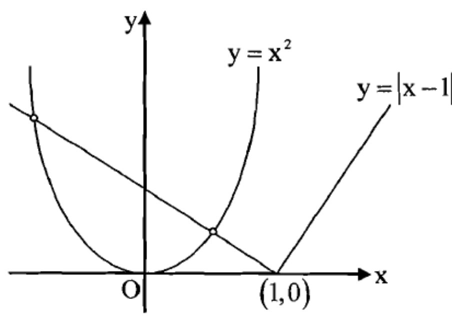
Q.5 (B)

$$\Rightarrow \left|\frac{x^2}{x-1}\right| \le 1$$

$$\Rightarrow x^2 \le |x-1|, x \ne 1$$

$\Rightarrow$ Adjacent figure represents the graphs of $y = x^2$ and $y = |x-1|$

$\Rightarrow$ Solving $x^2 = 1 - x$, we get

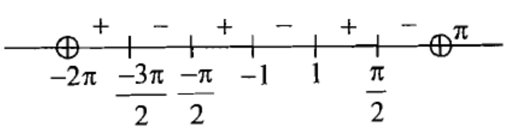


$$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$$

$\Rightarrow$ Thus solution is $\left[\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right]$

Q.6 (D)

$\Rightarrow |x^2 - 1 + \cos x| = |x^2 + 1| + |\cos x|$. It implies that $(x^2 - 1)\cos \ge 0$ bacause $|x + y| = |x| + |y|$ if $y \ge 0$. Sign scheme of $(x^2 - 1) \cos x$ is



$\Rightarrow$ Thus solution is $\left[-\frac{\pi}{2},-1\right]\cup\left[1,\frac{\pi}{2}\right]\cup\left(-2\pi,\frac{3\pi}{2}\right]$

Q.7 (D)

$\Rightarrow [x]^2 - 5[x] + 6 = 0$

$\Rightarrow [x] = 2, 3$

$\Rightarrow x \in [2,4)$

Q.8 **(D)**

$$\Rightarrow \left[\log_2\left(\frac{x}{|x|}\right)\right] \geq 0$$

$$\Rightarrow \log_2\left(\frac{x}{[x]}\right) \geq 0$$

$$\Rightarrow \frac{x}{[x]} \geq 1$$

$$\Rightarrow \frac{x-[x]}{[x]} \geq 0$$

$$\Rightarrow \frac{\{x\}}{[x]} \geq 0$$

$\Rightarrow$ It implies that 'x' is any positive real number greater than of equal to one or 'x' is nay non zero integer.

Q.9 **(B)**

$\Rightarrow 2\,[x] = x + \{x\}$

$\Rightarrow 2\,[x] = [x] + 2\{x\}$

$$\Rightarrow \{x\} = \frac{[x]}{2}$$

$$\Rightarrow 0 \leq \frac{[x]}{2} < 1$$

$$\Rightarrow 0 \leq [x] < 2$$

$\Rightarrow [x] = 0, 1$

$\Rightarrow$ For [x] = 0, we get [x] = 0

$\Rightarrow x = 0$

$\Rightarrow$ For [x] = 1, we get $\{x\} = \frac{1}{2}$

$$\Rightarrow x = \frac{3}{2}$$

Q.10 (B)

$\Rightarrow [x]^2 = x + 2\{x\}$

$\Rightarrow [x]^2 = [x] + 3\{x\}$

$$\Rightarrow \{x\} = \frac{[x]^2 - [x]}{3}$$

$$\Rightarrow 0 \le \frac{[x]^2 - [x]}{3}$$

$$\Rightarrow 0 \le \frac{[x]^2 - [x]}{3} < 1$$

$$\Rightarrow 0 \le [x]^2 - [x] < 3$$

$$\Rightarrow [x] \in \left(\frac{1-\sqrt{13}}{2}, 0\right] \cup \left[1, \frac{1+\sqrt{13}}{2}\right)$$

$\Rightarrow [x] = -1, 0, 1, 2$

$$\Rightarrow \{x\} = \frac{2}{3}, 0, 0, \frac{2}{3}$$

$$\Rightarrow x = \frac{2}{3}, 0, 0, \frac{2}{3}$$

$$\Rightarrow x = -\frac{1}{3}, 0, 1, \frac{8}{3}$$

Q.11 (C)

$\Rightarrow$ $[x^2] + x - a = 0$

$\Rightarrow$ 'x' i has to be an integer

$\Rightarrow$ $a = x^2 + x = x(x + 1)$

$\Rightarrow$ Thus 'a' can be 2, 6, 12, 20.

Q.12 (D)

$\Rightarrow$ $[x + [2x]] < 3$

$\Rightarrow [x]+[2x]\le 2$

$\Rightarrow$ Any non-positive real number will satisfy this inequality.

$\Rightarrow$ Now if $x \in \left(0, \frac{1}{2}\right)$

$\Rightarrow$ $[x] = 0, [2x] = 1$

$\Rightarrow$ inequality is still satisfied

$\Rightarrow$ For $x \in \left(1, \frac{3}{2}\right), [x] = 1, [2x] = 2$

$\Rightarrow$ inequality does not hold true.

$\Rightarrow$ Thus, $x \in (-\infty, 1)$.

Q.13 (B)

$$\Rightarrow \text{We get, } f(x) = \begin{cases} 6-3x & , \quad x<1 \\ 4-x & , \quad 1\le x<2 \\ x & , \quad 2\le x<3 \\ 3x-6 & , \quad x>2 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -3 & , \quad x<1 \\ -1 & , \quad 1<x<2 \\ 1 & , \quad 2<x<3 \\ 3 & , \quad x>3 \end{cases}$$

$\Rightarrow$ Thus f(x) decreasing for $x < 2$ and increasing for $x > 2$.

$\Rightarrow$ Hence, $f(x)|_{min} = f(2) = 2.$

Q.14 (D)

$\Rightarrow$ [5 sin x] + [cos x] = – 6

$\Rightarrow$[5 sin x] = – 5, [cos x] = – 1

$\Rightarrow -5 \le 5\sin x < -4, -1 \le \cos x < 0$

$\Rightarrow -1 \le \sin x < -\frac{4}{5}, -1 \le \cos x < 0$

$\Rightarrow x + \sin^{-1}\left(\frac{4}{5}\right) < x < \frac{3\pi}{2}$

$\Rightarrow$ Now $f(x) = \sin x + \sqrt{3}\cos x = 2\sin\left(x + \frac{\pi}{3}\right)$

$\Rightarrow$we have, we have, $\pi + \frac{\pi}{2} + \sin^{-1}\left(\frac{4}{5}\right) < x + \frac{\pi}{6} < \frac{3\pi}{2} + \frac{\pi}{3}$

$\Rightarrow -1 \le \sin\left(x + \frac{\pi}{6}\right) < -\frac{\sqrt{3}}{2}$

Q.15 (C)

$\Rightarrow y = |\sin x| + |\cos x|$

$\Rightarrow y^2 = 1 + |\sin 2x|$

$\Rightarrow 1 \le y^2 \le 2$

$\Rightarrow y \in \left[1, \sqrt{2}\right]$

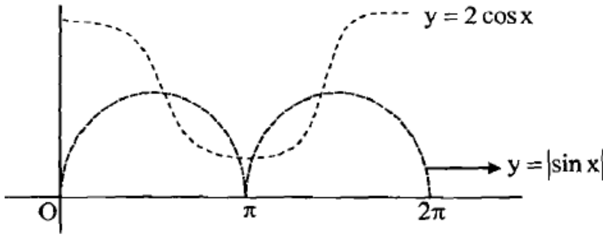
$\Rightarrow f(x) = 1 \ \forall \ x \in R$

Q.16 (B)

$\Rightarrow$Graph of $y = 2^{\cos x}$ and $y = |\sin x|$ meet four times in $[0, 2\pi]$.

$\Rightarrow$Thus, total number of solutions

$\Rightarrow$ 4 + 4 + 4 + 2 = 14.



Q.17 (A)

$\Rightarrow$ For function to be one-one, each element of set A must have different image in st B. We first of all choose any 'm' elements in st B. This can be done in ${}^nC_m$ ways. Now one-one correspondence of elements of set A with these selected elements can be done in m! ways. Thus total number of one-one functions will be equal to ${}^nC_m.m!$ i.e. ${}^nP_m$.

Q.18 (A)

$\Rightarrow 2^x + 3^x + 4^x - 5^x = 0$

$$\Rightarrow \left(\left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1\right) = 0$$

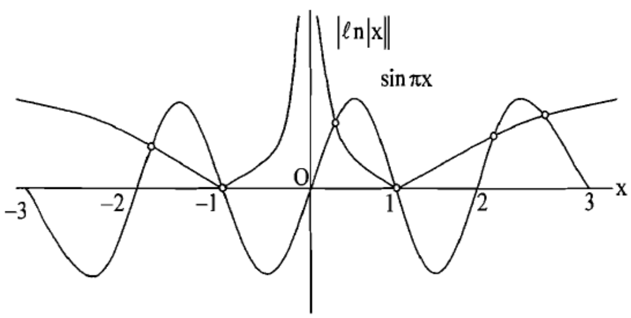
$$\Rightarrow \text{Clearly } g(x) = \left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$$

$\Rightarrow$ is a decreasing function and Also g (0) = 1.

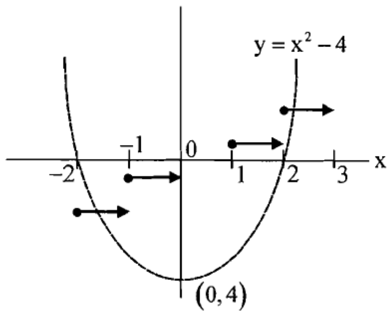
$\Rightarrow$ Thus, f (x) = 0 has exactly one root.

Q.19 (D)

$\Rightarrow$ There are exactly six solutions.



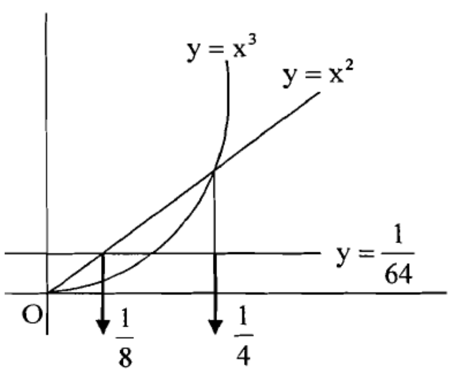
Q.20 (B)



There are exactly 2 solutions.

Q.21 (C)

$$\Rightarrow \text{Clearly, } f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{8} \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & x > 1 \end{cases}$$



22. (C)

Standard fact : Domain of (f + g)(x) = Domain of f(x) $\cap$ Domain of g(x)

23. (D)

If f(x) = f(y) implies only & only x = y, then f(x) is injective.

Hence f(f(f(x))) = f(f(f(y))) will imply only x = y when f(x) is injective.

24. (C)

If f(x) is even, then

$$f(-x) = f(x) \Rightarrow (-ax+b)\cos x - (-cx+d)\sin x = (ax+b)\cos x + (cx+d)\sin x$$

$$\Rightarrow 2ax\cos x + 2d\sin x = 0$$

$\Rightarrow a = d = 0$.

25. **(A)**

$3\sin x - 4\cos x + 6 = 5\sin(x - \alpha) + 6$, where $\alpha = \tan^{-1}\frac{4}{3}$

Now $y = \frac{1}{3\sin x - 4\cos x + 6} \Rightarrow 5\sin(x - \alpha) + 6 = \frac{1}{y}$

$\Rightarrow \sin(x - \alpha) = \frac{1 - 6y}{5y}$

$\Rightarrow -1 \le \frac{1 - 6y}{5y} \le 1$

$\Rightarrow y \in \left[\frac{1}{11}, 1\right]$

26. **(B)**

$f(a_1) \ne b_4 \;\&\; f(a_2) = b_1 \Rightarrow f(a_1)$ can be chosen in 3 ways

Now $f(a_3) \;\&\; f(a_4)$ can be chosen in 3×2 ways

Hence total number of injective functions = $3 \times 3 \times 2 = 18$.

27. **(C)**

Domain of f(x) is [–1, 2]

Now $\sqrt{2 - x} + \sqrt{1 + x} = y \Rightarrow 2\sqrt{2 + x - x^2} = y^2 - 3$

$\Rightarrow y^2 \ge 3 \;\&\; (2x - 1)^2 = 6y - y^2$

As x lies in [–1, 2], hence range of $(2x - 1)^2$ is [0, 9]

Hence $0 \le 6y^2 - y^4 \le 9$ or $y^4 - 6y^2 + 9 \ge 0 \;\&\; y^4 - 6y^2 \le 0$

$\Rightarrow y \le \sqrt{6}$

$\therefore y \in \left[\sqrt{3}, \sqrt{6}\right]$.

28. **(D)**

f(x) = log(2 + cos 3x)

(A) Domain : $(-\infty, \infty)$ as 2 + cos 3x is always greater than 0.

(B) Range : $\log(2+\cos 3x) = y \Rightarrow \cos 3x = e^y - 2$

Hence $-1 \le e^y - 2 \le 1$

$\Rightarrow 0 \le y \le \ln 3$.

(C) $f(-x) = \log(2+\cos 3x) = f(x)$, hence f(x) is even.

(D) As cos 3x is periodic hence f(x) is periodic.

29. (C)

Let ax + b = y .

Interchanging x & y gives ay + b = x

$\Rightarrow y = \frac{x-b}{a}$

Now $ax + b = \frac{x}{a} - \frac{b}{a} \Rightarrow a = \frac{1}{a}$ & $b = -\frac{b}{a}$

or $a = 1, b = 0$ & $a = -1, b \in R$.

30. (D)

Given $f(x) = \log_{10} \frac{1+x}{1-x}$

(I) $f(-x) = \log_{10} \frac{1-x}{1+x} \Rightarrow f(-x) = -\log_{10} \frac{1+x}{1-x} = -f(x)$

f(x) is odd, hence graph is not symmetric about y – axis.

(II) Domain of f(x) is (–1, 1).

Now $2 \le \frac{1+x}{1-x} < \infty \Rightarrow \log_{10} 2 \le \log_{10}\left(\frac{1+x}{1-x}\right) < \infty$.

Hence graph can't lie in IV quadrant.

(III) As f(x) is odd hence graph is symmetric about the origin.

(IV) Clearly f(0) = 0 hence graph passes through the origin and lies in I & III quadrant.

31. (B)

$2^x + 2^y = 1 \Rightarrow y = \log_2\left(1 - 2^x\right)$

Hence for domain, $1 - 2^x \geq 0$

$\Rightarrow 2^x \leq 1$ or $x \in \left(-\infty, 0\right]$.

32. (D)

f is even hence f(–x) = f(x)

g is odd hence g(–x) = – g(x)

Now f(x) + g(x) = $e^x \Rightarrow f\left(-x\right) + g\left(-x\right) = e^{-x}$ or $f\left(x\right) - g\left(x\right) = e^{-x}$

Hence $\left(f\left(x\right) + g\left(x\right)\right)\left(f\left(x\right) - g\left(x\right)\right) = e^x e^{-x}$

$\Rightarrow f^2\left(x\right) - g^2\left(x\right) = 1$.

33. (B)

$3 < \pi < 4 \Rightarrow \left[\pi\right] = 3 \ \& \left[-\pi\right] = -4$

Hence f(x) = cos 3x – sin 4x.

Period of cos 3x = $\frac{2\pi}{3}$ & period of sin 4x = $\frac{\pi}{2}$.

Therefore period of f(x) = LCM$\left\{\frac{2\pi}{3}, \frac{\pi}{2}\right\}$ = 2π.

34. (C)

Number of ONTO functions from domain containing n elements to a codomain containing r elements is

$r^n - {}^rC_1\left(r-1\right)^n - {}^rC_2\left(r-2\right)^n + ... + \left(-1\right)^{r-1}\ {}^rC_{r-1}\left(r-1\right)^n$

Hence for the given data, number of ONTO functions is

$3^6 - {}^3C_1 \times 2^6 + {}^3C_2 \times 1^6 = 540$.

35. **(A)**

Image of (5, k) in x = y is B(k, 5).

As B lies on y = f(x) hence k = 2.

Reflection of B(2, 5) in origin will be (–2, –5).

36. **(D)**

$P\left(x^2+1\right)=\left(P\left(x\right)\right)^2+1 \Rightarrow P\left(x\right)>0$

$P\left(0\right)=1 \Rightarrow P\left(1\right)=2, P\left(2\right)=5, P\left(-1\right)=2...etc$

Clearly $P\left(x\right)=x^2+1$.

37. **(A)**

$f\left(x\right)=\cos\left(\sqrt{2}\,x\right)+\cos\left(\sqrt{3}\,x\right)$

Period of $\cos\left(\sqrt{2}\,x\right)=\dfrac{2\pi}{\sqrt{2}}$ & period of $\cos\left(\sqrt{3}\,x\right)=\dfrac{2\pi}{\sqrt{3}}$

As LCM of $\dfrac{2\pi}{\sqrt{2}}$ & $\dfrac{2\pi}{\sqrt{3}}$ doesn't exist hence f(x) is not periodic.

Also at x = 0 f(x) = 2 which is clearly the greatest value of f(x) as cosine has a greatest value 1.

$$\cos\left(\sqrt{2}\,x\right)+\cos\left(\sqrt{3}\,x\right)=0 \Rightarrow 2\cos\left(\frac{\sqrt{2}+\sqrt{3}}{2}x\right)\cos\left(\frac{\sqrt{2}-\sqrt{3}}{2}x\right)=0$$

$$\Rightarrow x=\left(\frac{2n-1}{\sqrt{2}+\sqrt{3}}\right)\pi \quad \text{or} \quad x=\left(\frac{2n-1}{\sqrt{2}+\sqrt{3}}\right)\pi$$

Hence y = f(x) cuts the x – axis.

As f(–x) = f(x) hence f(x) is even.

38. **(D)**

Let $n \le x < n+\frac{1}{2}$, then $\left[x\right]+\left[x+\frac{1}{2}\right]=2004 \Rightarrow 2n=2004$ or $n=1002$

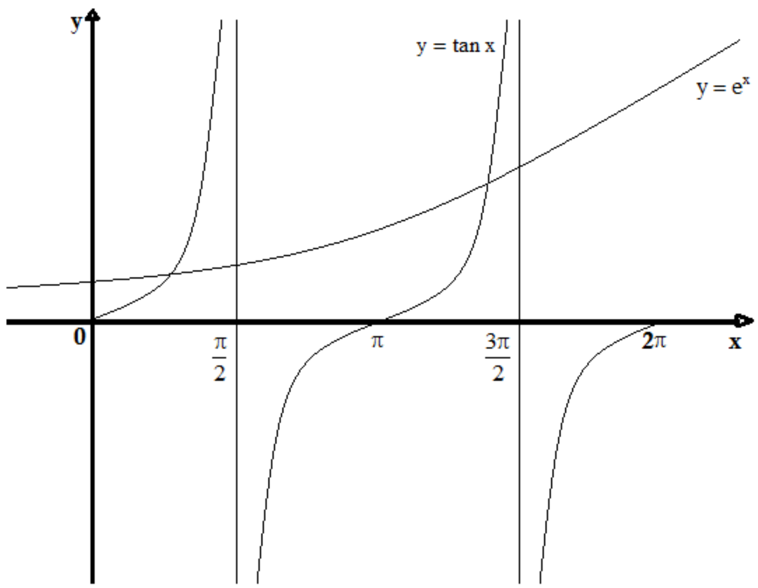
If $n+\frac{1}{2}\le x<n+1,$ then $[x]+\left[x+\frac{1}{2}\right]=2004\Rightarrow 2n+1=2004,$

but n can't be non integral.

Hence $1002\le x<1002+\frac{1}{2}$.

39. (B)

Refer the following graph :



Q.40 (D)

$\Rightarrow$ Only in D, the graph has a symmetry w.r.t. origin

Q.41 (C)

$f(x)=\left|\sin^3 2x\right|+\left|\cos^3 2x\right|$

$\Rightarrow f(x)=\sin^6 2x+\cos^6 2x+\frac{1}{4}\left|\sin^3 4x\right|$

$\Rightarrow f(x)=1-\frac{3}{4}\sin^2 4x+\frac{1}{4}\left|\sin^3 4x\right|$

Now periods of both $\sin^2 4x$ & $\left|\sin^3 4x\right|$ **are** $\frac{\pi}{4}$ **hence the period of** $f(x)=\left|\sin^3 2x\right|+\left|\cos^3 2x\right|$ **is** $\frac{\pi}{4}$**.**

Q.42 (C)

We have for $\cos^{-1}(1-x)\ge 0$

$\Rightarrow -1 \le (1-x) \le 1$

$\Rightarrow -2 \le -x \le 0$

$\Rightarrow 0 \le x \le 2$ ......(1)

also, $10 \cdot 3^{x-2} - 9^{x-1} - 1 > 0$

$\Rightarrow 10 \cdot 3^x - 9^x - 9 > 0$

$\Rightarrow 10 \cdot 3^x - 3^{2x} - 9 > 0$

$\Rightarrow 3^{2x} - 10 \cdot 3^x + 9 < 0$

$\Rightarrow (3^x - 1)(3^x - 9) < 0$

$\Rightarrow 1 < 3^x < 9$

$\Rightarrow 0 < x < 2$ ……. (2)

from (1) and (2)

$\Rightarrow 0 < x < 2$

Q.43 (A)

Note that f is bijective hence f^{-1} exist

$\Rightarrow$ when y = 4

$\Rightarrow 2x^3 + 7x - 9 = 0$

$\Rightarrow 2x^2 (x - 1) + 2x (x - 1) + 9 (x - 1) = 0$

$\Rightarrow (x - 1)(2x^2 + 2x + 9) = 0$

$\Rightarrow x = 1$ only $\Rightarrow$ A; as $2x^2 + 2x + 9 = 0$ has no other roots

Q.44 (A)

$\Rightarrow f(x) = \dfrac{4}{\sqrt{1-x^2}}$; $f(\sin x) = \dfrac{4}{|\cos x|}$ and $f(\cos x) = \dfrac{4}{|\sin x|}$;

$\Rightarrow$ hence $g(x) = |\sin x| + |\cos x|$

Q.45 (C)

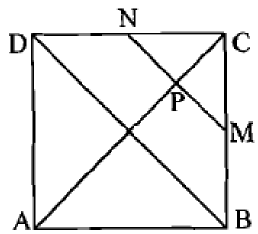
$\Rightarrow$ when $p = \frac{\pi}{2}$ then $D^r \to \cos x + \sin x \Rightarrow \frac{\pi}{2}$ cannot be the period]

Q.46 (B)

$\Rightarrow AP = x$; $MN = y$; $BD = 2\sqrt{2}$

$\Rightarrow$ Hence, $\frac{y}{2\sqrt{2}} = \frac{2\sqrt{2} - x}{\sqrt{2}} \Rightarrow \Delta$'s CNM and CDB are similar $y = 2\left(2\sqrt{2} - x\right)$

$\Rightarrow f(x) = \frac{xy}{2} = x\left(2\sqrt{2} - x\right) = 2 - \left(x - \sqrt{2}\right)^2$



$\Rightarrow \begin{array}{ll} f(x)_{max} = 2 & \text{when } x = \sqrt{2} \\ f(x)_{max} = 0 & \text{when } x = 2\sqrt{2} \end{array}$]

Q.47 (A)

(A) $\Rightarrow \frac{1}{g(x)} = \frac{1}{\frac{\ln x}{x}}$; $f(x) = \frac{x}{\ln x}$ $\quad x > 0, \ x \neq 1$ for both

(B) $\Rightarrow \frac{1}{f(x)} = \frac{1}{\frac{x}{\ln x}}$; $g(x) = \frac{\ln x}{x}$ $\quad \frac{1}{f(x)}$ is not defined at x = 1 but g (1) = 0

(C) $\Rightarrow f(x) \cdot g(x) = \frac{x}{\ln x} \cdot \frac{\ln x}{x} = 1$ $\quad$ if $x > 0, \ x \neq 1 \Rightarrow$ N.I.

(D) $\Rightarrow \frac{1}{f(x) \cdot g(x)} = \frac{1}{\frac{x}{\ln x} \cdot \frac{\ln x}{x}} = 1$ $\quad$ only for $x > 0$ and $x \neq 1$]

Q.48 (A)

$\Rightarrow$ An equation of this kind is called a functional equation, and can often be solved by choosing particular values for the variables. In this case, by choosing x = 1, we see that $f(y) = \frac{f(1)}{y}$ for all y. put y = 30; f (1) = $30 \cdot f(30) = 30 \cdot 20 = 600$. Now $f(40) = \frac{f(1)}{40} = \frac{600}{40} = 15$

Q.49 (C)

$\Rightarrow f(x) = \sin^2 x + (1 - \sin^2 x)^2 + 2$

$\Rightarrow 3 - \sin^2 x + \sin^4 x$

$\Rightarrow 3 - \sin^2 x \cos^2 x$

$\Rightarrow 3 - \dfrac{\sin^2 2x}{4}$

$\Rightarrow T_1 = \dfrac{\pi}{2}$, and $T_2 = \dfrac{\pi}{2}$

Q.50 (D)

$\Rightarrow D_2$ means range of the function
$\Rightarrow$ let $y = \sqrt{1-2x} + x$
$\Rightarrow (y - x)^2 = 1 - 2x$
$\Rightarrow y^2 - 2xy + x^2 = 1 - 2x$
$\Rightarrow x^2 + 2x(1 - y) + y^2 - 1 = 0$
$\Rightarrow$ as, $x \in R$, $D \geq 0$
$\Rightarrow 4(1-y)^2 \geq 4(y^2 - 1)$
$\Rightarrow 1 + y^2 - 2y \geq y^2 - 1$
$\Rightarrow -2y \geq -2$
$\Rightarrow y \leq 1$
$\Rightarrow y \in (-\infty, 1]$
$\Rightarrow$ Alternatively: $f'(x) = 1 - \dfrac{1}{\sqrt{1-2x}}$; $f'(x) = 0$
$\Rightarrow 1 - 2x = 1$
$\Rightarrow x = 0$
$\Rightarrow f(-\infty) = \rightarrow -\infty$

Q.51 (A)

$\Rightarrow h(x) = \ln(f(x) \cdot g(x)) = \ln e^{\{y\}+[y]} = \{y\} + [y] = y = e^{|x|} \operatorname{sgn} x$

$$\Rightarrow \therefore \quad h(x) = e^{|x|} \operatorname{sgn} x = \begin{bmatrix} e^x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -e^{-x} & \text{if } x < 0 \end{bmatrix}$$

$$\Rightarrow h(-x) \begin{bmatrix} e^x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ -e^{-x} & \text{if } x > 0 \end{bmatrix}$$

$\Rightarrow h(x) + h(-x) = 0$ for all x

Q.52 (D)

(A) $f(x) = x^4 + 2x^3 - x^2 + 1 \rightarrow$ A polynomial of degree even will always be into

$\Rightarrow$ say, $f(x) a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + ... + a_{2n}$

$$\Rightarrow \underset{x \to \pm\infty}{\text{Limit}} f(x) = \underset{x \to \pm\infty}{\text{Limit}} \left[x^{2n} \left(a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + ... + \frac{a_{2n}}{x^{2n}} \right) \right] = \begin{bmatrix} \infty & \text{if } a_0 > 0 \\ -\infty & \text{if } a_0 < 0 \end{bmatrix}$$

Hence it will never approach $\frac{\infty}{-\infty}$

(B) f (x) = $x^3 + x + 1$

$\Rightarrow$ f' (x) = $3x^2 + 1 \Rightarrow$ injective as well as surjective

(C) $f(x) = \sqrt{1 + x^2}$

$\Rightarrow$ neither injective nor surjective (minimum value = 1)

$\Rightarrow$ f (x) = $x^3 + 2x^2 - x + 1$

$\Rightarrow$ f' (x) = $3x^2 + 4x - 1$

$\Rightarrow$ D > 0

$\Rightarrow$ Hence f (x) is surjective but not injective.

Q.53 (D)

Let f (x_1) = n and f (x_2) = m, x_1, x_2 $\in (a, b)$ with n > m (say). According to the intermediate value theorem, between x_1 and x_2 there must be some value x for which $f(x) = m + \frac{1}{2}$ which is impossible since $m + \frac{1}{2}$ is not an integer.

Q.54 (D)

$$\Rightarrow g\left(-1, -\frac{3}{2}\right) = \max\left(-1, -\frac{3}{2}\right) - \min\left(-1, -\frac{3}{2}\right) = -1 - \left(-\frac{3}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \text{and } g(-4, -1.75) = \max(-4, -1.75) - \min(-4, -1.75) = -1.75 - (-4) = 2.25 = \frac{9}{4}$$

$$\Rightarrow \text{then } f\left(\frac{1}{2}, \frac{9}{4}\right) = \left(\max\left(\frac{1}{2}, \frac{9}{4}\right)\right)^{\min\left(\frac{1}{2}, \frac{9}{4}\right)} = \left(\frac{9}{4}\right)^{\frac{1}{2}} = \frac{3}{2}$$

Q.55 (A)

$$\Rightarrow f(x) = \frac{e^x \ln x \; 5^{(x^2+2)} \cdot (x-2)(x-5)}{(2x-3)(x-4)}$$

$\Rightarrow$ Note that at $x = \frac{3}{2}$ & $x = 4$ function is not defined and in open interval $\left(\frac{3}{2}, 4\right)$ function is continuous.

$$\Rightarrow \lim_{x \to \frac{3}{2}^+} = \frac{(+ve)(-ve)(-ve)}{(+ve)(-ve)} \to -\infty$$

$$\Rightarrow \lim_{x \to 4^-} = \frac{(+ve)(+ve)(-ve)}{(+ve)(-ve)} \to -\infty$$

$\Rightarrow$ In the open interval $\left(\frac{3}{2}, 4\right)$ the function is continuous & takes up all real values from $(-\infty, \infty)$

$\Rightarrow$ Hence range of the function is $(-\infty, \infty)$ or R

Q.56 (D)

$f^2(x) - f(x) - 6 \geq 0$

$\Rightarrow (f(x) - 3)(f(x) + 2) \geq 0$

$\Rightarrow f(x) \geq 3$ or $f(x) \leq -2$

$\Rightarrow$ given $x \in (0, \infty)$ $\Rightarrow$ $x \in (0, \infty)$

$\Rightarrow \therefore f(x) \geq 3$ $\Rightarrow$ $x \in (-\infty, 0]$

$\Rightarrow f(x) > -2$ $\Rightarrow$ $x \in (-\infty, 5)$

$\Rightarrow \therefore f(x) \leq -2$ $\Rightarrow$ $x \in [5, \infty)$

Q.57 (C)

$\Rightarrow f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x$

$\Rightarrow (x + 1)^2 - 1 = x$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x = 0$ or -1

Q.58 (D)

$$x\, f(x) = \begin{cases} x \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$$

$\Rightarrow g(x) = \sin x$ near. $x \to \pi$ though rational then $x\, f(x) \to \pi$ but $g(x) \to 0 \Rightarrow x\, f(x) > g(x)$

$\Rightarrow g(x) = x$ is negative for negative irrational x while $x\, f(x)$ is 0 ; $x\, f(x) > g(x)$

$\Rightarrow g(x) = x^2$ is smaller than x for $0 < x < 1$ and rational; so $x\, f(x) > g(x)$

$\Rightarrow g(x) = |x|$ equals $x\, f(x)$ for x positive and rational, is larger than $x\, f(x)$ for x irrational.

Q.59 (D)

$h(x) = {}^{x+1}C_{2x-8} \cdot {}^{2x-8}C_{x+1};\ x + 1 \geq 2x - 8$

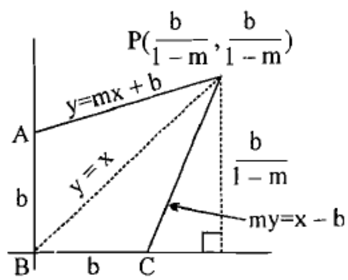
$\Rightarrow x \leq 9;\ 2x - 8 \geq x + 1 \Rightarrow x \geq 9$

$\Rightarrow$ Hence x = 9
$\Rightarrow$ Domain of h (x) = {9}
$\Rightarrow$ Range of h (x) = 1

Q.60 (C)

If f (x) = mx + b, then $f^{-1}(x) = \frac{x-b}{m}$ and their point of intersection

$\Rightarrow$ Can be found by setting x = mx + b since they intersect on y = x. Thus $x = \frac{b}{1-m}$ and the point of intersection is $\left(\frac{b}{1-m}, \frac{b}{1-m}\right)$.



$\Rightarrow$ Region R can be broken up into congruent triangles PAB and PCB which both have a base of b and a height of $\frac{b}{1-m}$.

$\Rightarrow$ The area of R is $\left(\frac{2b}{2}\right)\left(\frac{b}{1-m}\right) = \frac{b^2}{1-m} = 49$. For $m = \frac{9}{25}, b^2 = \frac{16}{25} \cdot 49$

$\Rightarrow b = \frac{28}{5}$

Q.61 (A)

$\Rightarrow 9 - x^2 \geq 0$
$\Rightarrow -3 \leq x \leq 3$
$\Rightarrow$ Also $9 - |2x+5| > 0$
$\Rightarrow -9 < 2x + 5 < 9$
$\Rightarrow -7 < x < 1$
Hence domain of f (x) is $[-3, 2)$

$\Rightarrow \% = \frac{2}{5} = 40\%$

Q.62 (D)

$\Rightarrow$ **I** f(x) = x and g(x) =1 – x or f(x) = x and g(x) = $-x^3$
$\Rightarrow$ **II** f (x) = x and g (x) = x^3
$\Rightarrow$ **III** f (x) = sin x which is odd but not one-one

Q.63 (D)

$\Rightarrow x + xe^{f(x)} = 1 - e^{f(x)}$
$\Rightarrow (x+1)e^{f(x)} = 1 - x$
$\Rightarrow f(x) = \ln\left(\frac{1-x}{1+x}\right)$

Q.64 (D)

$\Rightarrow$ Replacing x by $\frac{\pi}{2} - x$; $f\left(\cos\left(\frac{\pi}{2} - x\right)\right) = \cos 17\left(\frac{\pi}{2} - x\right)$
$\Rightarrow$ f (sin x) = sin 17x = g (sin x)
$\Rightarrow$ hence f = g

Q.65 (A)

$y = 2 \log_a x$
$\Rightarrow \log_a x = \frac{y}{2}$
$\Rightarrow x = a^{\frac{y}{2}}$
$\Rightarrow f^{-1}(b+c)a^{\frac{b+c}{2}} = f^{-1}(b) \cdot f^{-1}(c)$

Q.66 (D)

$\Rightarrow p = \frac{2\pi}{\sqrt{[a]}} = \pi,$
Hence $\sqrt{[a]} = 2$
$\Rightarrow$ (A) = 4
$\Rightarrow 4 \le a < 5$

Q.67 (C)

$2f(x) + f(1-x) = x^2$ ....(1)
$\Rightarrow \underset{\substack{\text{multiply(1) by (2)} \\ (3)-(2)}}{x \to 1-x}$ $f(x) + 2f(1-x) = (1-x)^2$ ....(2)
$4f(x) + 2f(1-x) = 2x^2$ ....(3)

$3f(x) = 2x^2 - (1-x)^2$
$3f(4) = 32 - 9 = 23$
$f(4) = \frac{23}{3}$

Q.68 (B)

$$f(x)=\frac{a^x+a^{-x}}{2} \ \& \ f(x+y)+f(x-y)=k f(x) f(y)$$

$$\Rightarrow \frac{a^{x+y}+a^{-x-y}}{2}+\frac{a^{x-y}+a^{y-x}}{2}=k\left(\frac{a^x+a^{-x}}{2}\right)\left(\frac{a^y+a^{-y}}{2}\right)$$

$$\Rightarrow 2\left(a^x a^y+\frac{1}{a^x a^y}+\frac{a^x}{a^y}+\frac{a^y}{a^x}\right)=k\left(a^x a^y+\frac{1}{a^x a^y}+\frac{a^x}{a^y}+\frac{a^y}{a^x}\right)$$

$$\Rightarrow k=2.$$

Q.69 (A)

$\Rightarrow$ A one to one function and its inverse are symmetric across the line y = x. Thus x and y intercept are interchanged and the sum is the same i.e. 5.

Q.70 (C)

$\Rightarrow x(x+3)\geq 0$
$\Rightarrow x\geq 0$ or $x\leq -3$
and $-1\leq x^2+3x+1\leq 1$
$\Rightarrow x(x+3)\leq 0$ and $2^2 + 3x + 2^3$ which is always true.
Hence $-3\leq x\leq 0$
Hence x = 0 or – 3
$\Rightarrow$ x = {0, – 3}