GEOMETRICAL OPTICS

Ex-1

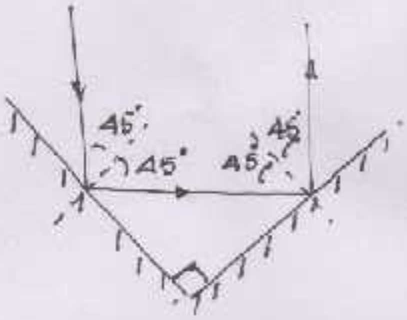
1. (D)

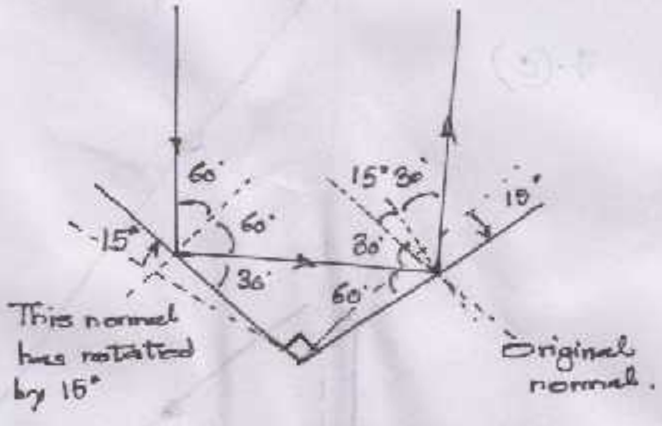
$v_{OM} = v_O - v_M = u - v$

$v_{IM} = -v_{OM} = -(u-v)$

$v_{OI} = v_{OM} - v_{IM} = 2(u-v)$

2. (D)

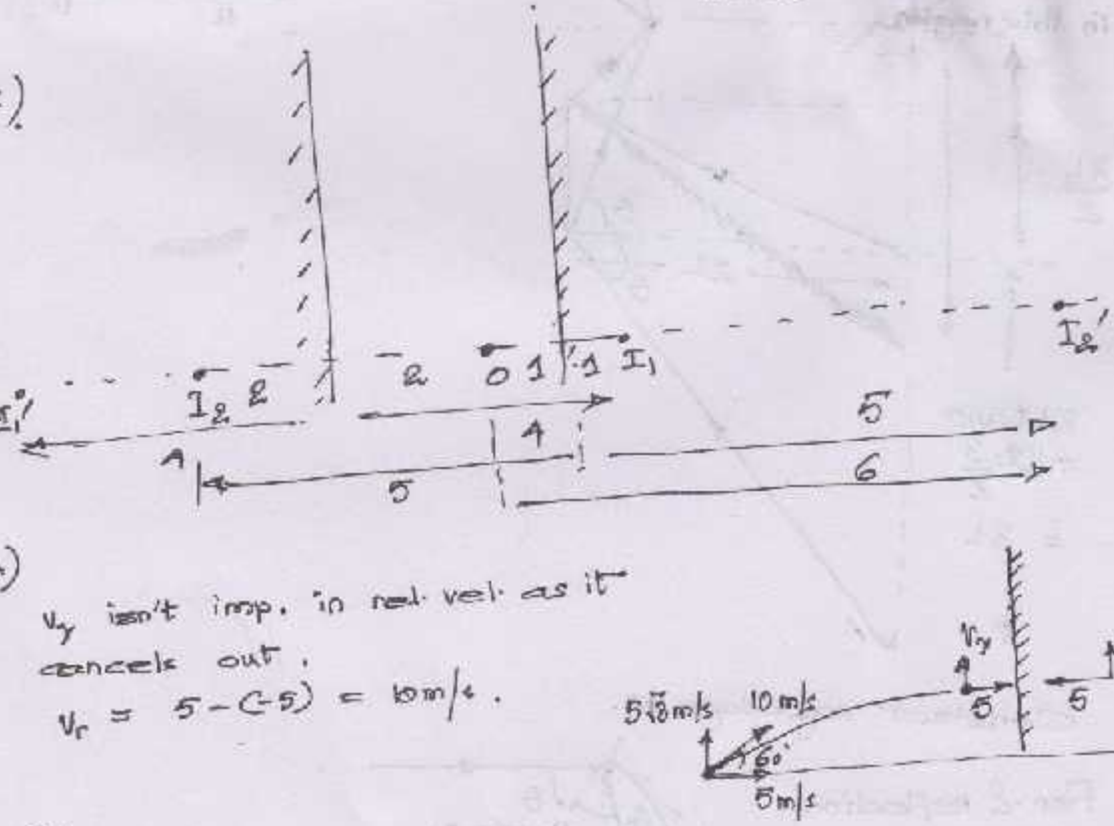




The emergent ray still makes 45° with the original normal.

∴ It is still parallel to the incident beam.

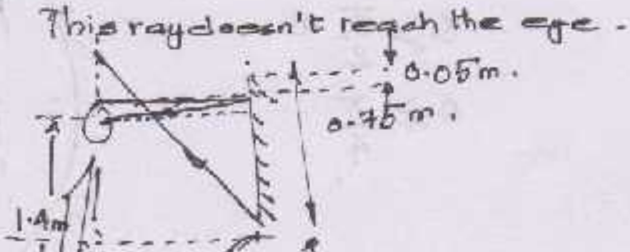
3. (C)



4. (A)

v_y isn't imp. in rel. vel. as it cancels out.

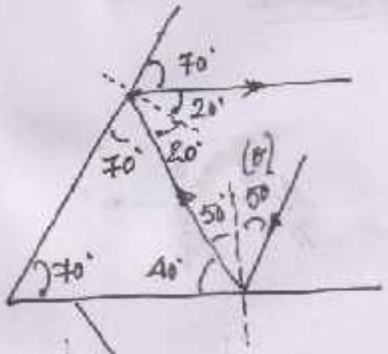
$v_r = 5 - (-5) = 10 \text{ m/s}$.



5. (C)

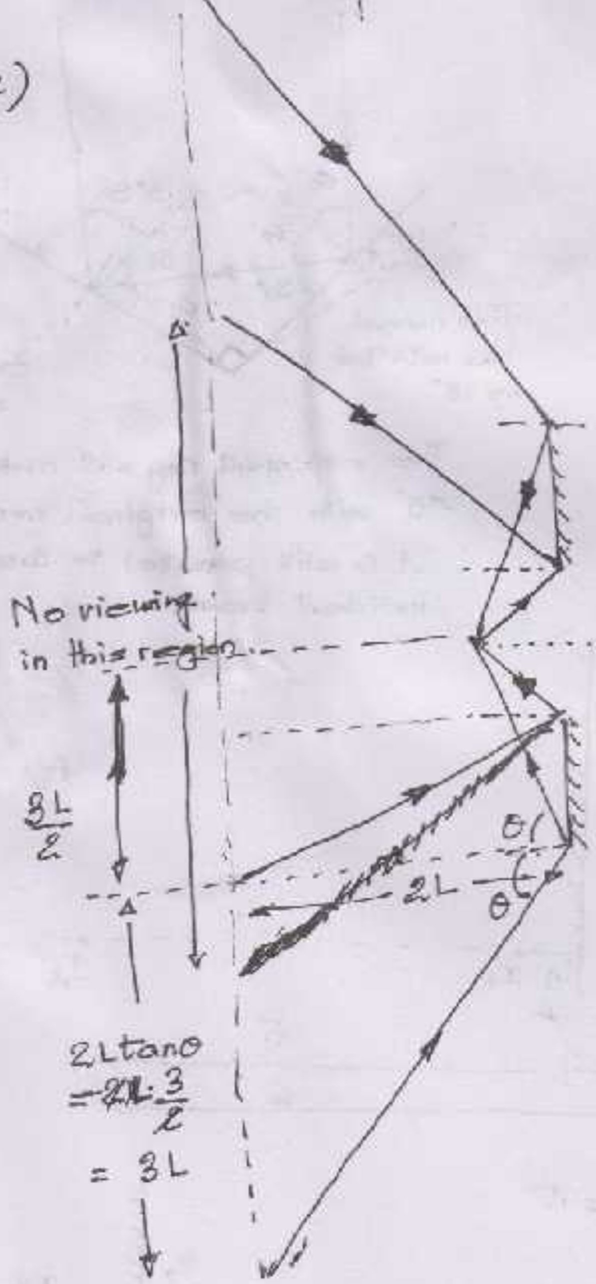
This ray doesn't reach the eye.

6(A)



Start from the reflected ray.

7.(c)



Time taken

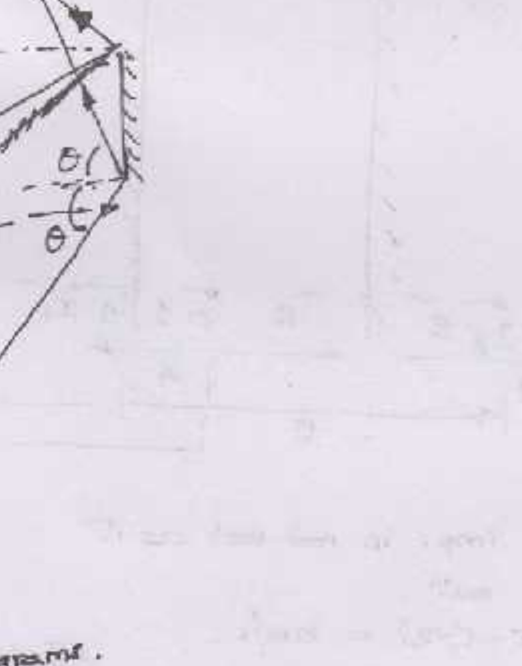
$$= \frac{9L - 3L}{u} = \frac{6L}{u}.$$

8(b) Construct ray diagrams.

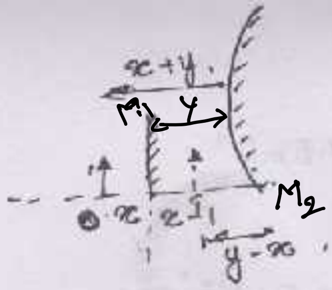
9(c) For 2 reflections

$$\theta \geq \frac{\pi}{2} - \theta$$

$$\theta \geq \frac{\pi}{4}$$



10. (A)



For the mirror.

$$\frac{-1}{f} = \frac{1}{-(y-x)} + \frac{1}{-(x+y)}$$

$$f = \frac{x^2 - y^2}{2y}$$

11. (A) Discussed in theory.

12. (C) Apply mirror formula or Newton's formula $x_1 x_2 = f^2$, $b x_2 = \frac{a^2}{4} \Rightarrow x_2 = \frac{a^2}{4b}$.

13. (B) Lesser rays for the image

Intensity decreases.

14. (C) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \dot{v} = -\frac{v^2}{u^2}\dot{u} \Rightarrow v_{IM} = -m^2 v_{OM} = -\frac{9}{4} \times 4 = -9\,cm/s$.

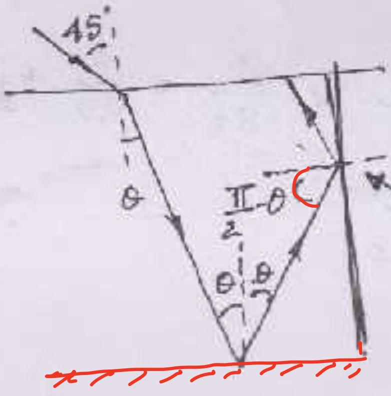
$\left[m = \frac{f}{f-u} = -\frac{3}{2} \right]$

15. ~~(A)~~ D For plane mirrors, $v_{IM} = -v_{OM}$

" curved ", $v_{IM} = -m^2 v_{OM}$

16. (B) $\sqrt{3} \sin 30^\circ = 1 \cdot \sin \theta$

17. (C)



$\frac{\sin 45^\circ}{\sin\theta} = \frac{2}{1} \Rightarrow \sin\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\theta = \sqrt{7}/2\sqrt{2}$

Also, $\sin i_c = \sin^{-1}\frac{1}{2} = 30^\circ$

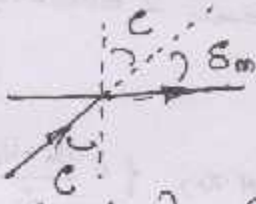
$\cos\theta > \sin i_c$

$\Rightarrow \frac{\pi}{2} - \theta > i_c$

$\Rightarrow$ TIR occurs here

$\therefore$ Deviation $- 180^\circ$

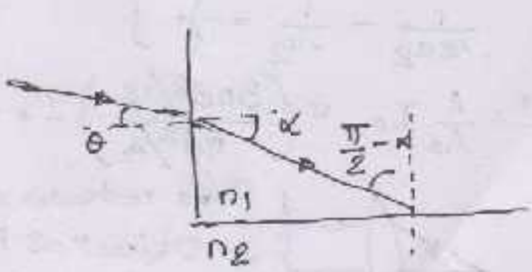
18. (D)



$\delta_{max} = \frac{\pi}{2} - C$

19. (A) $t\left(1 - \frac{1}{3/2}\right) = 2$

20. (A) $\frac{\sin\theta}{\sin\alpha} = n_1$



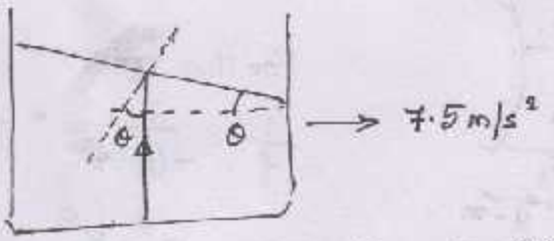
For TIR,

$\frac{\pi}{2} - \alpha > i_c$

$\Rightarrow \cos\alpha > \sin i_c$

$\Rightarrow \sqrt{1 - \frac{\sin^2\theta}{n_1^2}} > \frac{n_2}{n_1}$

21. (B)

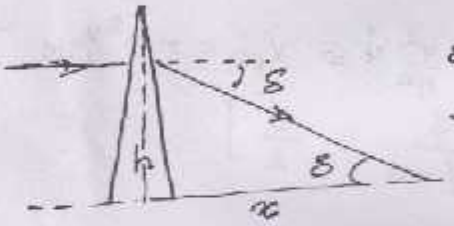


$$\tan\theta = \frac{a}{g} = \frac{7.5}{10} = \frac{3}{4} \Rightarrow \theta = 37^\circ$$

For TIR at water surface,

$$\theta > i_c$$
$$\Rightarrow \sin\theta > \sin i_c$$
$$\Rightarrow \frac{3}{5} > \frac{1}{\mu}.$$

22. (C)



$$\delta = (\mu - 1)A$$
$$\tan\delta = \frac{h}{x}$$
As δ is small, $\tan\delta \approx \delta$
$$x = \frac{h}{\delta} \Rightarrow x = \frac{h}{(\mu-1)A}.$$

23. (C) $$\frac{3}{2v} - \frac{4}{3u} = \frac{3/2 - 4/3}{-10} \Rightarrow \frac{3}{2v} = \frac{4}{3u} - \frac{1}{60}$$

$\because u < 0 \Rightarrow v < 0$

24. (D) $$\frac{4}{3v} - \frac{3}{2u} = \frac{4/3 - 3/2}{-10} \Rightarrow \frac{4}{3v} = \frac{3}{2u} + \frac{1}{60}$$

For real image, $v > 0$
$$\Rightarrow \frac{3}{2u} + \frac{1}{60} > 0$$
$$\Rightarrow u > -90$$
$$\Rightarrow |u| < 90.$$

25. (B) In one case the image is real & other virtual.

$v_1 = x u_1$ & $v_2 = -x u_2$

$$\left.\begin{aligned} \frac{1}{x u_1} - \frac{1}{-u_1} &= \frac{1}{f} \\ \frac{1}{-x u_2} - \frac{1}{-u_2} &= \frac{1}{f} \end{aligned}\right\} \text{Eliminating } x,\ f = \frac{u_1 + u_2}{2}$$

26. (D) $$I = \frac{A}{A_0} I_0 = \left(\frac{3\pi d^2/16}{\pi d^2/4}\right) I_0 = \frac{3}{4} I_0$$

27. (B)

Rays retrace path ⇒ Normal incidence on mirror ⇒ Ray ∥ P.A after refraction ⇒ Object at F

28. (D) Entire image formed, brightness halved.

29. (A) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v^2} dv + \frac{1}{u^2} du = \frac{1}{f^2} df$

$f = \frac{R}{2(\mu - 1)}$. If squeezed, $R \downarrow \Rightarrow f \downarrow \Rightarrow df$ -ve

But $du = 0 \Rightarrow dv$ -ve $\Rightarrow$ Moves towards lens.

30. (B) $h_i = \frac{v}{u} h_0$

$v \downarrow \Rightarrow h_i \downarrow$

31. (B) For $dv = 0$, $\frac{1}{u^2} du = \frac{1}{f^2} df$

du -ve

$\Rightarrow$ df -ve

$\Rightarrow f \downarrow \Rightarrow R \downarrow$

$\Rightarrow$ squeeze.

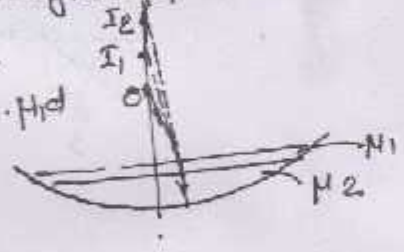
32. (C) Apply snell's Law with small angle approximation.

33. (D) Neglect thickness of layer

$AO = d$, $AI_1 = \mu d$, $AI_2 = \frac{\mu_2}{\mu_1} \cdot \mu_1 d$

$= \mu_2 d$

$= R$



34. (A) $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$.

35. (C)

36. (C)

37. (B) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

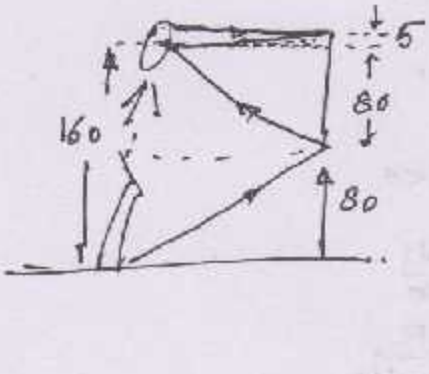
$\frac{1}{v} + \frac{1}{10} = \frac{1}{30}$

$v = -15$ cm

38. A.

Ex-II.

1. A, B
2. B
3. B, C



4. A, C, D
5. C, D
6. C, D.
7. A, C

$\sin i_c = \frac{4/3}{5/3} = 4/5 \Rightarrow \sin i_c = \frac{4}{5}$

$\sin 60° = \frac{\sqrt{3}}{2} > \frac{4}{5} \Rightarrow 60° > i_c$

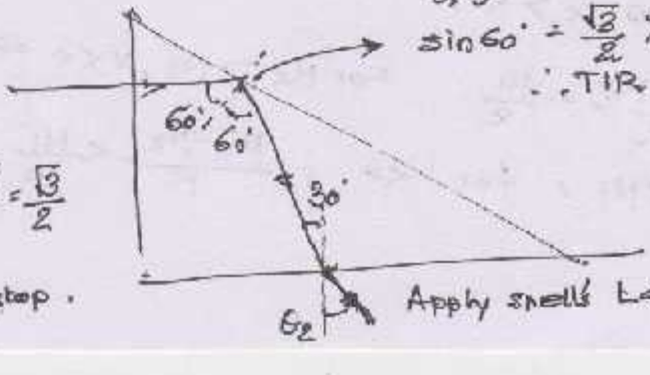
$\therefore$ TIR occurs here.

If $\mu_4 = \frac{5}{2\sqrt{3}}$

$\sin i_c = \frac{5/2\sqrt{3}}{5/3} = \frac{\sqrt{3}}{2}$

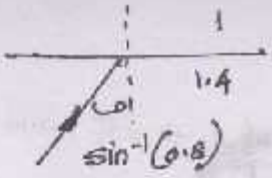
$i_c = 60°$

TIR will stop.



Apply snell's Law.

8. (C)



$\sin i_c = \frac{1}{1.4} = \frac{5}{7}$

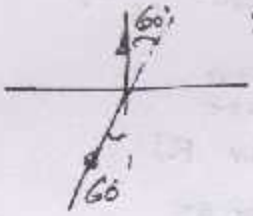
$0.8 > 5/7$

$\therefore$ TIR.

9. B, C, D At R_2 $\delta = 0$, $\mu_1 = \mu_2 \Rightarrow R_2 = 1$.

θ_2 - Max deviation For $\mu_1 > \mu_2$,

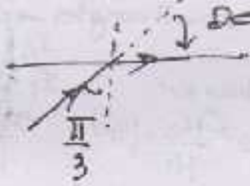
Ray bends towards normal.



$\delta_{max} = 60° = \theta_2$

θ_1 corresponds to critical angle

Deviation $\pi/6$ (θ_1).



10. (A)

11. A, D.

12. (D) $\frac{\mu_2}{v} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{\mu_2}{v} = \frac{\mu_1}{-x} + \frac{\mu_2 - \mu_1}{R}$.

If $\mu_2 > \mu_1$, then even v can be $-ve$.

For $v > 0$ $\frac{\mu_2 - \mu_1}{R} > \frac{\mu_1}{x} \Rightarrow x > \frac{\mu_1 R}{\mu_2 - \mu_1}$ provided $\mu_2 > \mu_1$.

Real image depends on x, R, μ_1, μ_2, not on R only.

13. A, B $\frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R} - \frac{\mu_1}{x} < 0$

if $\mu_2 < \mu_1$

$v < 0$.

If $\mu_2 < \mu_1$, for $v < 0$

$-\frac{(\mu_2 - \mu_1)}{R} < \frac{\mu_1}{x}$

$x > -\frac{\mu_1 R}{\mu_2 - \mu_1}$

$\Rightarrow x > R$.

14. A, B $\frac{\mu_2}{v} = \frac{\mu_1 - \mu_2}{R} - \frac{\mu_1}{x}$ For $\mu_2 > \mu_1$, $v < 0 \Rightarrow$ (A)

For $\mu_2 < \mu_1$, for $v < 0$, $\frac{\mu_1 - \mu_2}{R} < \frac{\mu_1}{x}$.

15. B, C, D. Apply displacement method.

16. (A, B, C, D)

17. (A, C, D)

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

then apply lens formula and mirror formula.

18. (C, D)

$$\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin 45^\circ} = \frac{1}{1/\sqrt{2}} = \sqrt{2} = 1.41$$

possible values are 1.5 & 1.6.

Ex - III.

1(C)

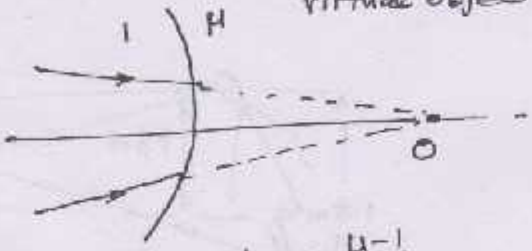
Real object.

$$\frac{\mu}{v} = -\frac{1}{x} - \frac{\mu-1}{R}$$

⇒ $v<0 \ \forall x$

⇒ virtual image

virtual object.

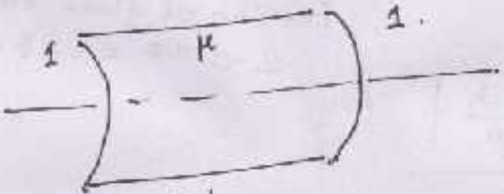


$$\frac{\mu}{v} = \frac{1}{x} - \frac{\mu-1}{-R}$$

v isn't −ve for all x.

2. (D)
$$\frac{\mu}{v_1} - \frac{1}{-x} = \frac{\mu-1}{-R}$$
$$\frac{1}{v} - \frac{\mu}{v_1'} = \frac{1-\mu}{R}.$$
$$\mu\left[\frac{1}{v_1} + \frac{1}{v_1'}\right] + \frac{1}{x} - \frac{1}{v} = -\frac{2(\mu-1)}{R}$$

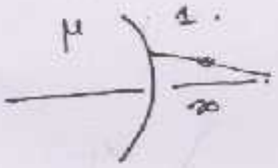
v depends on v_1, v_1'



$v_1 \neq v_1'$

As thickness is unspecified.

3.(A)



$$\frac{\mu}{v} - \frac{1}{-x} = \frac{\mu-1}{-R}$$
$$\frac{\mu}{v} = -\frac{1}{x} - \frac{\mu-1}{R}$$

$v<0 \ \forall x$

⇒ Real image.

4(A) All dimensions of disc are ⊥r to the P.A

⇒ " " " " equally magnified.

∴ Image – circular disc.

5 (B) At t = 1, r = 0.6

$$m = \frac{f}{f-u} = \frac{-10}{-10+15} = -2$$

Radius of image $= 2r = 1.2$ cm.

Area " " $= \pi(.12)^2 = 1.44\pi$ cm^2

$\pi(1.2)^2$

6.(A) $\frac{dR}{dt} = \frac{d}{dt}(2r) = 0.2$ cm/sec

7. A → p, r As μ↑, f↓ ∴ object. dist. > focal length

⇒ Final image – Real & small in size in comparision to image before change.

B → q, r As R is doubled, f↓ ∴ obj. dist. < focal length.

→ Final image – virtual.

Final image becomes smaller as compared to image before change.

∴ Final image becomes

C → q, r Slab introduction shifts the effective object for lens rightward.

Final image – virtual

Final image size ↓

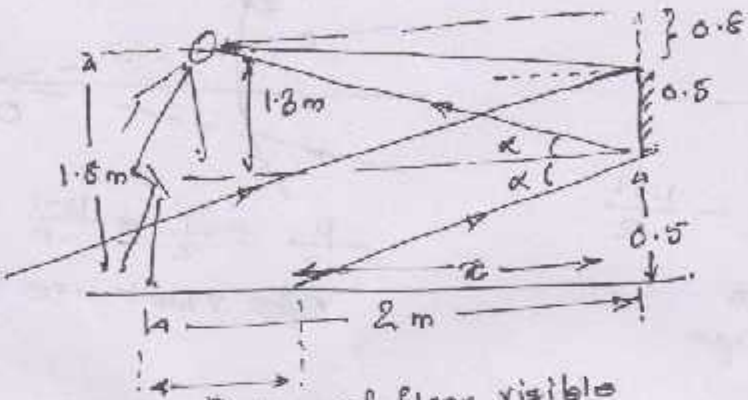
D → q, r. Object is at C ⇒ Final image virtual

" " " size ↓ before change.

8. A – S, B – P, S, C – R, T, D – Q, R

Ex–IV

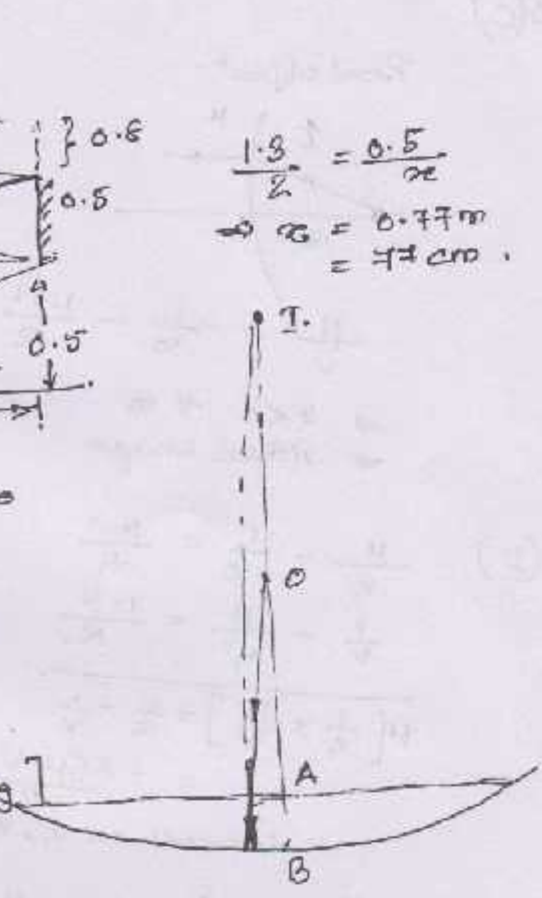
1. **1.28 m**



Portion of floor visible
$2 - 0.77 = 1.23$ m

$$\frac{1.3}{2} = \frac{0.5}{x}$$
$$\Rightarrow x = 0.77 \text{ m} = 77 \text{ cm}.$$

2. $\boxed{\dfrac{R-h}{\mu}}$



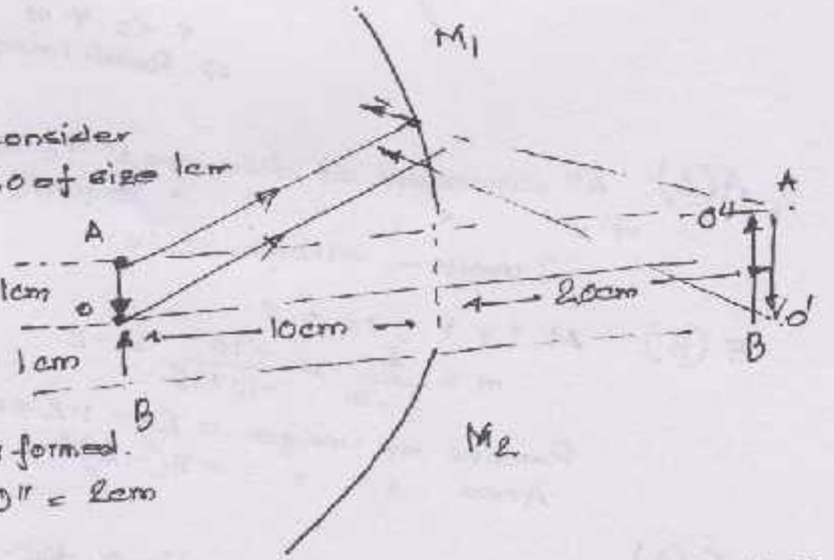
$AO = x$
$AI = \mu x$
$BI = \mu x + h = R$ [For retracing]
$x = \dfrac{R-h}{\mu}$

3. **2cm**

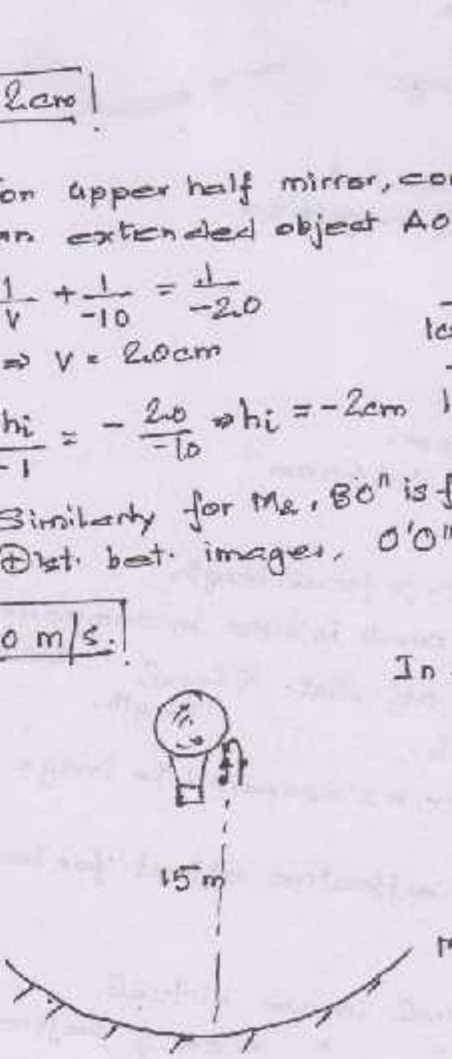
For upper half mirror, consider an extended object AO of size 1cm

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{-20}$$
$$\Rightarrow v = 20 \text{cm}$$
$$\frac{h_i}{-1} = -\frac{20}{-10} \Rightarrow h_i = -2\text{cm}$$

Similarly for M_2, BO″ is formed.
Dist. bet. images, $O'O'' = 2$cm



4. **80 m/s.**



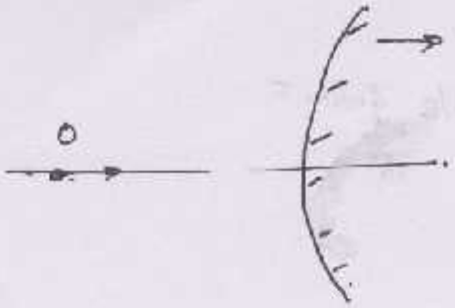
In A_2, the ball reaches the same ht. of 15 m

$$v_{IM} = -m^2 v_{OM} = -(-2)^2(-20) = 80 \text{ m/s}$$

$$m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2.$$

5\.

$v_M = 20\,m/s$ (th = if)



$V_{OI} = 1\,m/s \qquad m = 1/10$

$V_{OI} = V_{OM} - V_{IM} = V_{OM} + m^2 V_{OM}$

$V_{OI} = (1+m^2)\, V_{OM}$

$\therefore\ V_{OM} = \frac{100}{101}\ m/s$

$V_O = V_{OM} + V_M = \frac{2120}{101} \simeq \boxed{21\,m/s}$

$m = \frac{f}{f-u} \Rightarrow \frac{1}{10} = \frac{10}{10-u} \rightarrow u = -90$

$V = -mu = -\frac{1}{10}(-90) = 9\,m$

$m = -\frac{v}{u}$

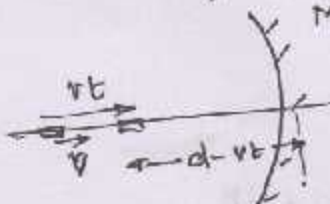
$V_{IM} = -m^2 V_{OM} = -\frac{1}{100}\times 1 = \frac{-1}{100}\ m/s$

$\Rightarrow \frac{dm}{dt} = \frac{1}{u^2}\left[v\cdot V_{OM} - u V_{IM} \right]$

$= \frac{1}{8100}\left[9\times 1 - (-90)\left(-\frac{1}{100}\right) \right]$

$\boxed{\frac{dm}{dt} = 10^{-3}/s}$

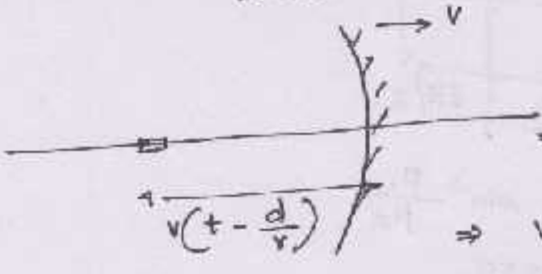
6\. For $t < \frac{d}{v}$, M is stationary



$V_{IM} = -m^2 V_{OM} = -\frac{f^2}{(f-u)^2} v = -\frac{R^2/4\ v}{\left[-R/2 - [-(d-vt)]\right]^2}$

$\boxed{V_{IE} = V_{IM} = \frac{-R^2 v}{\left[2(d-vt) - R\right]^2}}$

For $t > \frac{d}{v}$, M moves with speed v & block is at rest.



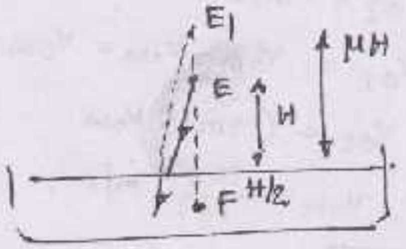
$V_{IM} = -m^2 V_{OM}$

$\Rightarrow V_{IE} = V_{ME} - \left(\frac{f}{f-u}\right)^2 (V_{OE} - V_{ME})$

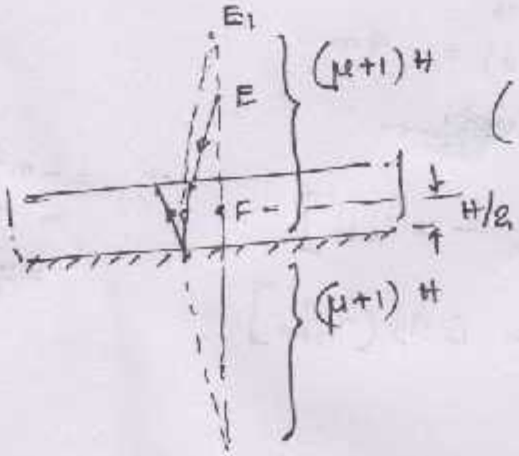
$\Rightarrow V_{IE} = v - \left[\frac{-R/2}{-R/2 - (-(vt-d))}\right]^2 (0 - v)$

$\boxed{V_{IE} = v\left[1 - \frac{R^2}{\left[2(vt-d) - R\right]^2}\right]}$

7 (a)



$\mu H + H/2$ from F



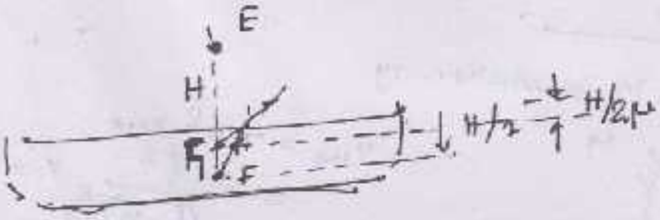
$(\mu+1)H + H/2$

$\left(\mu + \frac{3}{2}\right) H$ below F.

$H + \frac{H}{2\mu}$ from E

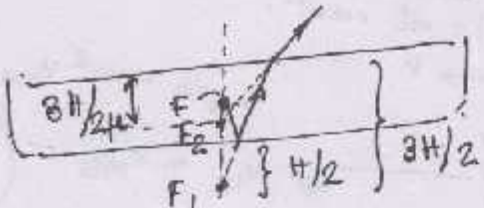
$H\left(1 + \frac{1}{2\mu}\right)$

(b)



$H + \frac{3H}{2\mu}$

$= H\left(1 + \frac{3}{2\mu}\right)$



8. Case I : No slab $\theta \geq i_c = \sin^{-1}\frac{\mu_1}{\mu_2}$

Case II : When slab is there

$\mu_3 \leq \mu_1$

$\sin\theta \geq \frac{\mu_1}{\mu_2} \geq \frac{\mu_3}{\mu_2}$

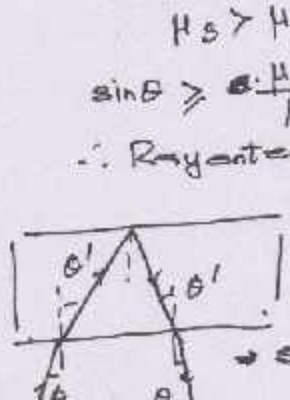
$\Rightarrow$ TIR

$\Rightarrow$ Ray back to med. II

$\mu_3 > \mu_1$

$\sin\theta \geq \frac{\mu_1}{\mu_2} < \frac{\mu_3}{\mu_2}$

$\therefore$ Ray enters med III.

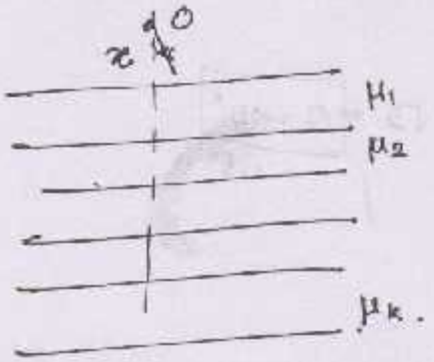


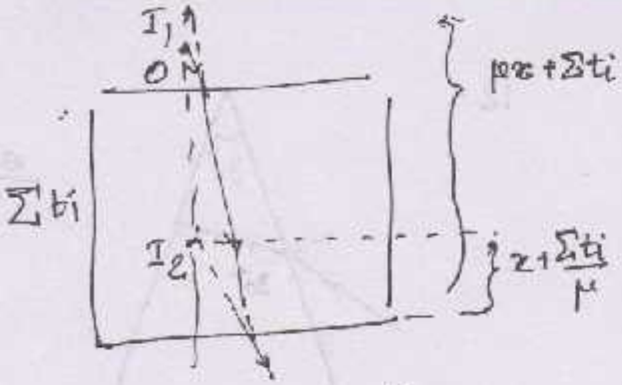
$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2}$

$\Rightarrow \sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta \geq \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$

$\Rightarrow \sin\theta' \geq \frac{\mu_1}{\mu_3} \Rightarrow$ TIR occurs again

9.



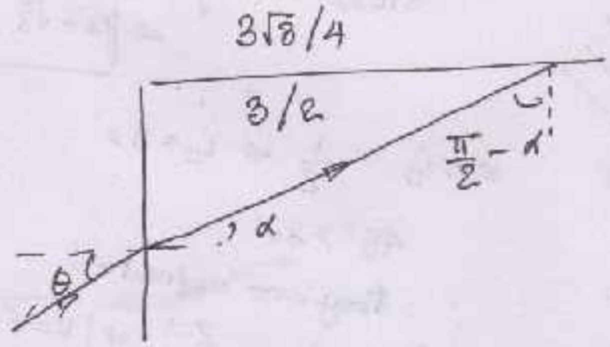


$$\frac{1}{\mu_k}\left[\frac{\mu_k}{\mu_{k-1}}\left[\cdots \frac{\mu_3}{\mu_2}\left(\frac{\mu_2}{\mu_1}(\mu_1 x + t_1) + t_2\right) + \cdots\right] + t_k\right] = x + \frac{\Sigma t_i}{\mu}.$$

$$\Rightarrow \quad x + \sum \frac{t_i}{\mu_i} = x + \frac{\Sigma t_i}{\mu}.$$

$$\Rightarrow \boxed{\mu = \frac{\Sigma t_i}{\Sigma t_i/\mu_i}}$$

10.



$$\sin i_c = \frac{3\sqrt{3}/4}{3/2}$$

$$\Rightarrow i_c = 60^\circ$$

For TIR,

$$\frac{\pi}{2} - \alpha \geq i_c$$

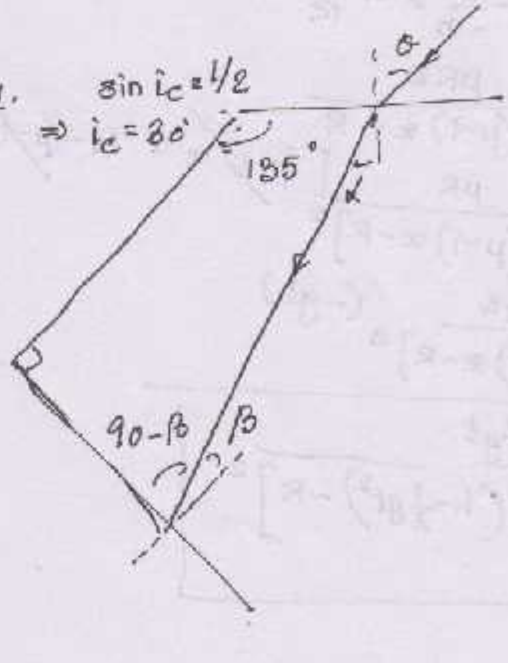
$$\sin\alpha \leq \frac{1}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin\theta \leq \frac{1}{2} \quad \left[\frac{\sin\theta}{\sin\alpha} = \frac{3/2}{3\sqrt{3}/4}\right]$$

$$\Rightarrow \boxed{\theta \leq \sin^{-1}\frac{1}{\sqrt{3}}}.$$

11. $\sin i_c = 1/2$

$\Rightarrow i_c = 30^\circ$



$$90 - \alpha + 90 - \beta + 135^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \alpha + \beta = 45^\circ.$$

For TIR, $\beta \geq i_c$

$$\Rightarrow 45 - \alpha \geq 30^\circ$$

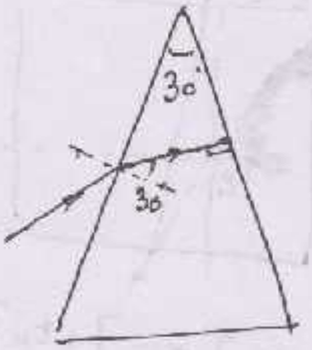
$$\Rightarrow \alpha \leq 15^\circ$$

$$\sin\alpha \leq \frac{\sqrt{3}-1}{2\sqrt{2}} \quad \left[\frac{\sin\theta}{\sin\alpha} = \frac{2}{1}\right]$$

$$\frac{\sin\theta}{2} \leq \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\theta \leq \sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

12.

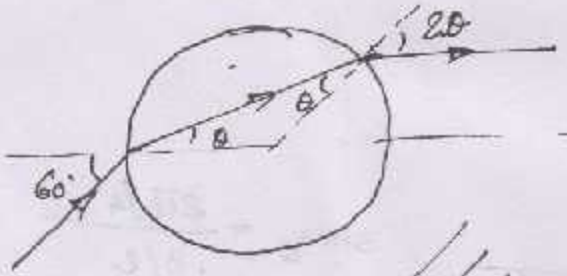


$$\frac{\sin\theta}{1/2} = \sqrt{2} \Rightarrow \boxed{\theta = 45^\circ}$$

13. $$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} \leq \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(A/2\right)} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}.$$

$$\therefore \boxed{\mu \leq \sqrt{2}}$$

14.



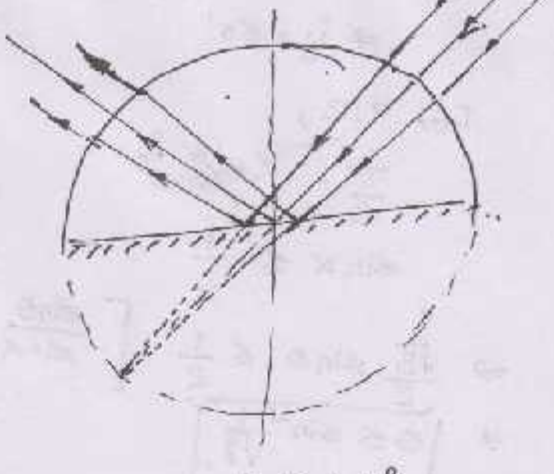
$$\left.\begin{aligned} \frac{\sqrt{3}/2}{\sin\theta} &= \mu \\ \frac{\sin\theta}{\sin 2\theta} &= \frac{1}{\mu} \end{aligned}\right\} \Rightarrow \sin 2\theta = \sqrt{3}/2$$

$$2\theta = 60^\circ$$

$$\boxed{\theta = 30^\circ}$$

$$\Rightarrow \boxed{\mu = \sqrt{3}.}$$

15.



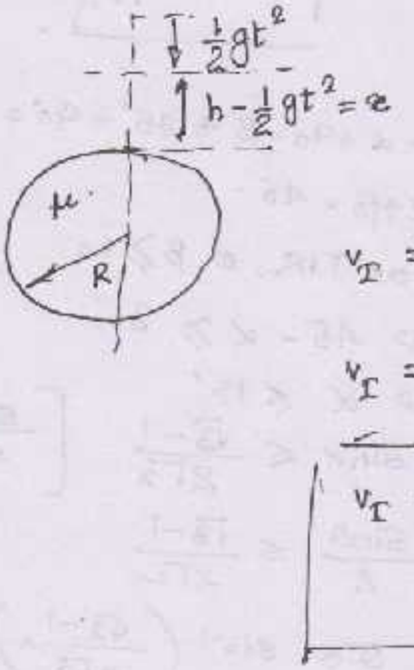
$$\sin i_c = \frac{1}{2} \Rightarrow i_c = 30^\circ$$

$$45^\circ > 30^\circ$$

$\therefore$ They are refracted.

$$\frac{2}{v} - \frac{1}{-\infty} = \frac{2-1}{3} \Rightarrow \boxed{v = 6.}$$

16.



$$\frac{\mu}{v} - \frac{1}{-x} = \frac{\mu - 1}{R}$$

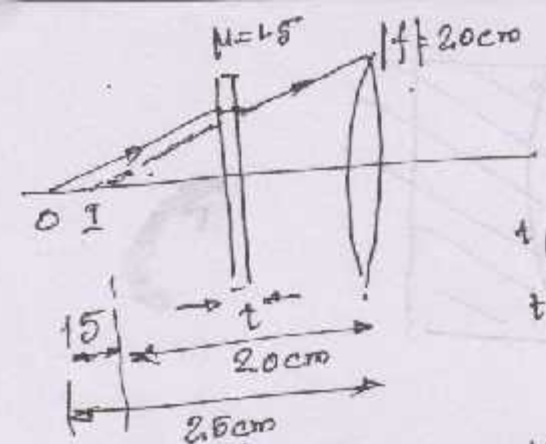
$$v = \frac{\mu R x}{(\mu-1)x - R}$$

$$v_I = \dot{v} = \frac{\mu R}{[(\mu-1)x - R]^2}\left[(\mu-1)x - R - (\mu-1)x\right]\dot{x}$$

$$v_I = -\frac{\mu R^2}{[(\mu-1)x - R]^2}(-gt)$$

$$\boxed{v_I = \frac{\mu R^2 g t}{\left[(\mu-1)\left(h - \frac{1}{2}gt^2\right) - R\right]^2}}$$

17.

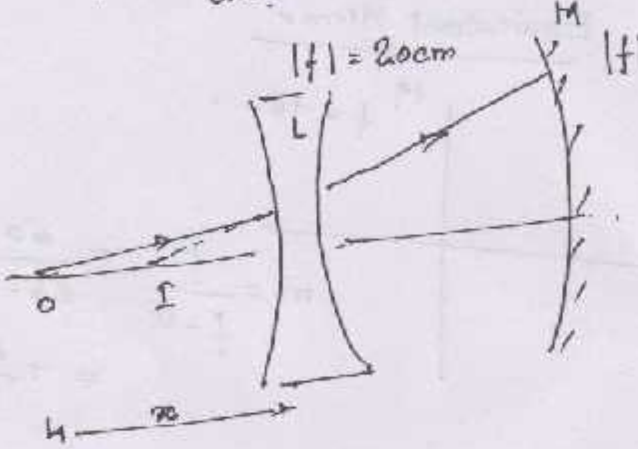


The image by slab must be at the focus of the lens.

$$t\left(1-\frac{1}{\mu}\right) = 5$$

$$t\left(1-\frac{1}{3/2}\right) = 5 \Rightarrow \boxed{t = 15\text{cm}}$$

18.



For L,

$$\frac{1}{v} - \frac{1}{-x} = \frac{1}{-20}$$

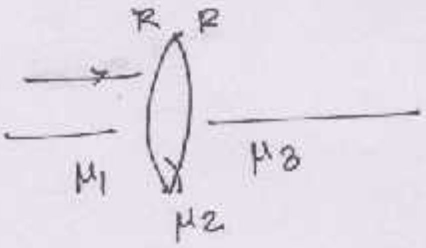
$$\Rightarrow v = -\frac{20x}{20+x}$$

Must be virtual. & must be at C of M.

$$\frac{20x}{x+20} + 5 = 20$$

$$\Rightarrow \boxed{x = 60\text{ cm}}$$

19. (a)

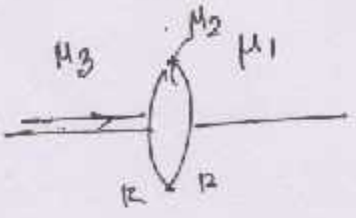


$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2-\mu_1}{R}$$

$$\frac{\mu_3}{v} - \frac{\mu_2}{v_1} = \frac{\mu_3-\mu_2}{-R}.$$

$$v = \frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$$

(b)

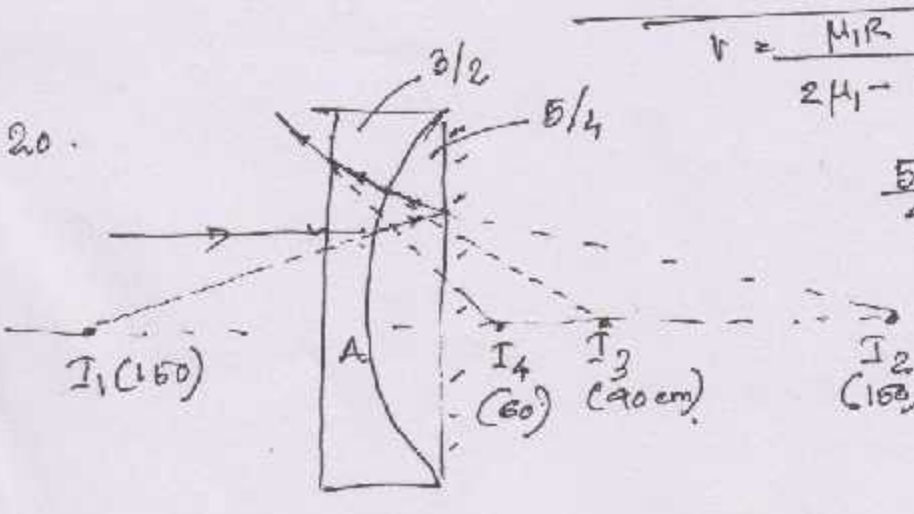


$$\frac{\mu_2}{v_1} - \frac{\mu_3}{-\infty} = \frac{\mu_2-\mu_3}{R}$$

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1-\mu_2}{-R}.$$

$$v = \frac{\mu_1 R}{2\mu_1 - \mu_1 - \mu_3}.$$

20.



$$\frac{5}{4v_1} - \frac{1}{-\infty} = \frac{5/4 - 3/2}{30}$$

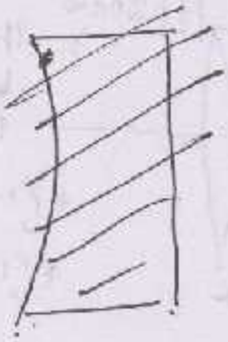
$$\Rightarrow v_1 = -150$$

$$\frac{3}{2v_3} - \frac{5}{4(-150)} = \frac{3/2 - 5/4}{-30}$$

$$\Rightarrow v_3 = -90\text{cm}$$

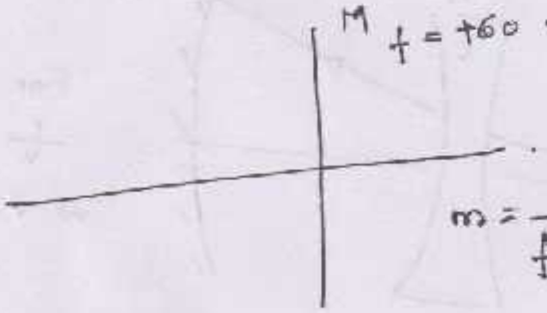
$$AI_3 = 90\text{cm} \Rightarrow AI_4 = \frac{90}{3/2} = 60\text{cm}.$$

(b)



(b)

Equivalent Mirror



$$m = \frac{f}{f-u} = \frac{60}{60-(-15)} = +\frac{4}{5}.$$

Exercise V

1.(D) Apply Snell's law

2.(C)

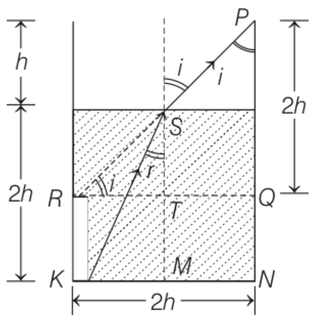
3. B

$$PQ = QR = 2h \Rightarrow \angle i = 45Υ$$

$\therefore$ $$ST = RT = h = KM = MN$$

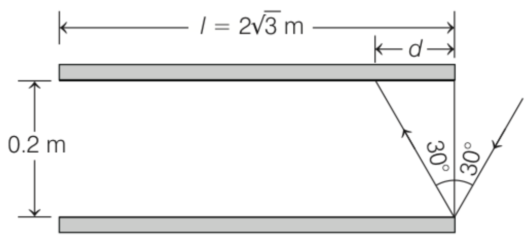
So, $$KS = \sqrt{h^2 + (2h)^2} = h\sqrt{5}$$

$\therefore$ $$\sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$$



$\therefore$ $$∝ = \frac{\sin i}{\sin r} = \frac{\sin 45Υ}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$$

4.B $$d = 0.2 \tan 30Υ = \frac{0.2}{\sqrt{3}}$$



$$\frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$

Therefore, maximum number of reflections are 30.

5. B

Image formed by convex lens at I_1 will act as a virtual object for concave lens. For concave lens

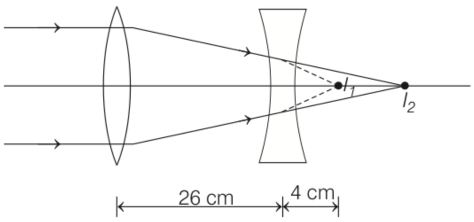
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20}$$

or $$v = 5\text{ cm}$$

Magnification for concave lens

$$m = \frac{v}{u} = \frac{5}{4} = 1.25$$



As size of the image at I_1 is 2 cm. Therefore, size of image at I_2 will be $2 \times 1.25 = 2.5$ cm.

6. B

7. C

8. A

9. None of the option is correct.

Distance of object from mirror

$$= 15 + \frac{33.25}{1.33} = 40\text{ cm}$$

Distance of image from mirror $= 15 + \dfrac{25}{1.33}$

$$= 33.8\text{ cm}$$

For the mirror, $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

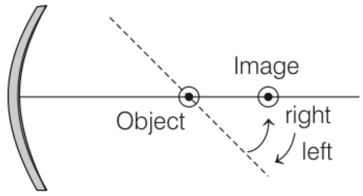
$\therefore$ $$\frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$$

$\therefore$ $$f = -18.3\text{ cm}$$

$\therefore$ Most suitable answer is (c).

10. B

Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$.



11. C

12. A

At minimum deviation ($\delta = \delta_m$):

$$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ \quad \text{(For both colours)}$$

13. C

Refraction from lens : $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$

$\therefore \quad v = 60\,\text{cm} \qquad \xrightarrow{\hspace{1cm}}$ + ve direction

i.e. first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \qquad \longleftarrow \text{+ ve direction}$$

or $\qquad v_3 = 12$ cm

Therefore, the final image is formed at 12 cm to the left of the lens system.

14. C

15. C

From the lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \text{we have,}$$

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

or $\qquad f = +5$

Further, $\qquad \Delta u = 0.1$

and $\qquad \Delta v = 0.1 \qquad$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

or $\qquad \Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f^2$

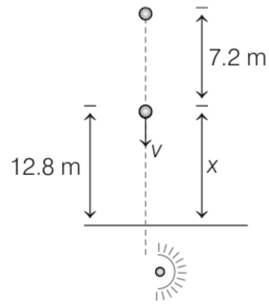
Substituting the values, we have

$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05$$

$\therefore \qquad f \pm \Delta f = 5 \pm 0.05$

16.C

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 7} = 12\,\text{ms}^{-1}$$



In this case when eye is inside water,

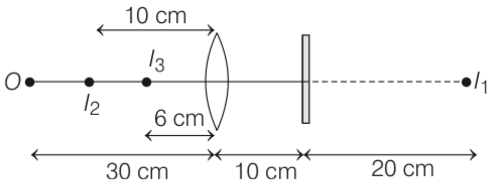
$$x_{\text{app.}} = \propto x$$

$$\therefore \quad \frac{dx_{\text{app.}}}{dt} = \propto \cdot \frac{dx}{dt}$$

or $$v_{\text{app.}} = \propto v = \frac{4}{3} \times 12 = 16\text{ms}^{-1}$$

17.B

Object is placed at distance $2f$ from the lens. So first image I_1 will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

or $$v = 6\text{ cm}$$

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

18. C

After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at θ = 0Y, which is not true.

20. B

Using the lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \text{ or } \frac{1}{v}=\frac{1}{u}+\frac{1}{f_1}+\frac{1}{f_2}$$

$$=\frac{1}{u}+(n_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+(n_2-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

Substituting the values, we get

$$\frac{1}{v}=\frac{1}{-40}+(1.5-1)\left(\frac{1}{14}-\frac{1}{\infty}\right)+(1.2-1)\left(\frac{1}{\infty}-\frac{1}{-14}\right)$$

Solving this equations, we get

$$v=+40\,\text{cm}$$

21. C

$$\propto=\frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}}=\frac{1}{(2/3)}=\frac{3}{2}$$

Further, $|m|=\frac{1}{3}=\left|\frac{v}{u}\right|$

$\therefore$ $|v|=\frac{|u|}{3}$

$\Rightarrow$ $u=-24$ m (Real object)

$\therefore$ $v=+8$ m (Real image)

Now, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=(\propto-1)\left(\frac{1}{+R}-\frac{1}{\infty}\right)$

$\therefore$ $\frac{1}{8}+\frac{1}{24}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}\right)$

$\therefore$ $R=3$ m

22. D

For $e \rightarrow i$

$\Rightarrow$ 45Υ> θ_c

$\Rightarrow$ sin 45Υ> $\sin\theta_c$

$\Rightarrow$ $\frac{1}{\sqrt{2}}>\frac{\propto_2}{\propto_1}$

$\Rightarrow$ $\propto_1>\sqrt{2}\propto_2$

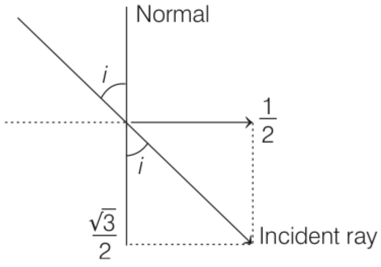
For $e \rightarrow f$

angle of refraction is lesser than angle of incidence, so $\propto_2>\propto_1$ and then $\propto_2>\propto_3$

For $e \rightarrow g$, $\propto_1=\propto_2$

for $e \rightarrow h$, $\propto_2<\propto_1<\sqrt{2}\propto_2$ and $\propto_2>\propto_3$

23. A



$$\tan i = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad i = 30Υ$$

25. B

(P) $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{r} + \frac{1}{r}\right) = \frac{1}{r} \Rightarrow f = r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = \frac{2}{r} \Rightarrow f_{eq} = \frac{r}{2}$

(Q) $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{r}\right) \Rightarrow f = 2r$

$\Rightarrow \frac{1}{\phi} + \frac{1}{\phi} = \frac{2}{\phi} = \frac{1}{\rho} \Rightarrow f_{eq} = r$

(R) $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r} \Rightarrow f = -2r$

$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = -\frac{2}{2r} \Rightarrow f_{eq} = -r$

(S) $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow f_{eq} = 2r$

26. B

$R = 10\text{ cm}$

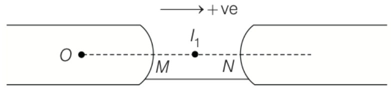
Applying $\frac{\alpha_2}{v} - \frac{\alpha_1}{u} = \frac{\alpha_2 - \alpha_1}{R}$ two times

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$$

$$\frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$$

$$\frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50} = \frac{2.5 - 1.5}{50} \Rightarrow v = 50$$

$$MN = d, MI_1 = 50\text{ cm},$$

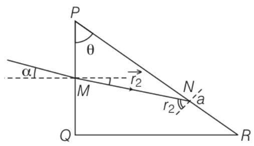


Again, $\frac{1.5}{\infty} - \frac{1}{-(d - 50)} = \frac{1.5 - 1}{10}$

$$\frac{1}{d - 50} = \frac{1}{20}$$

$$d = 70$$

27. A



Applying Snell's law at M,

$$n = \frac{\sin\alpha}{\sin r_1} \Rightarrow \sqrt{2} = \frac{\sin 45^\circ}{\sin r_1}$$

$$\Rightarrow \quad \sin r_1 = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$$

$$r_1 = 30^\circ$$

$$\sin\theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45^\circ$$

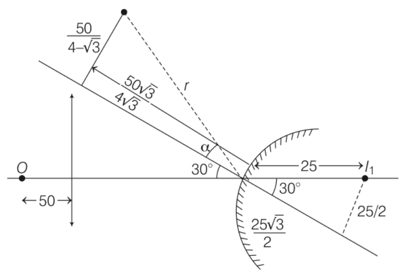
Let us take $r_2 = \theta_c = 45^\circ$ for just satisfying the condition of TIR.

In ΔPNM,

$$\theta + 90 + r_1 + 90 - r_2 = 180^\circ$$

or $$\theta = r_2 - r_1 = 45^\circ - 30^\circ = 15^\circ$$

30. A



For Lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u+f}$$

$$\Rightarrow \quad v = \frac{(-50)(30)}{-50+30} = 75$$

For Mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u-f}$$

$$\Rightarrow \quad v = \frac{\left(\frac{25\sqrt{3}}{2}\right)(50)}{25\sqrt{3}} = \frac{-50\sqrt{3}}{4-\sqrt{3}}$$

$$\Rightarrow \quad m = -\frac{v}{u} = \frac{h_2}{h_1} \Rightarrow h_2 = -\left(\frac{\frac{-50\sqrt{3}}{4-\sqrt{3}}}{\frac{25\sqrt{3}}{2}}\right)\cdot\frac{25}{2}$$

$$\Rightarrow \quad h_2 = \frac{+50}{4-\sqrt{3}}$$

The x-coordinate of the images

$= 50 - v\cos 30 + h_2\cos 60 \approx 25$

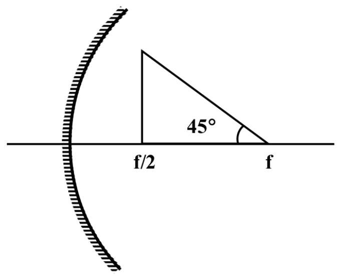
The y-coordinate of the images

$$= v\sin 30 + h_2 \sin 60 \approx 25\sqrt{3}$$

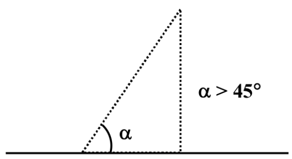
31.

A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length *f*, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)

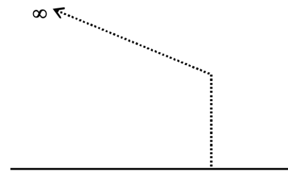
(JEE - 2018)



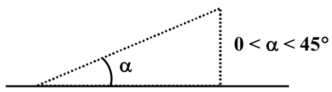
(A)



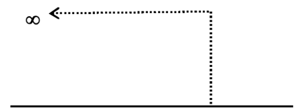
(B)



(C)



(D)

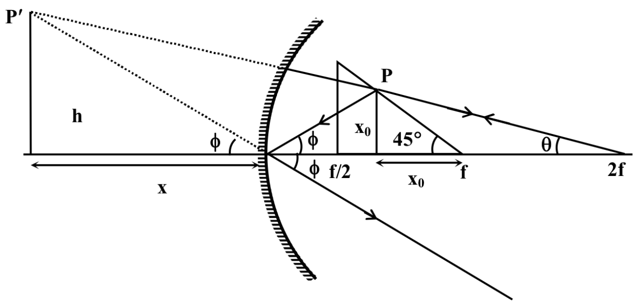


Ans. D

D

$$\tan\phi = \frac{x_0}{f - x_0}$$

$$\frac{h}{x} = \frac{x_0}{f - x_0} \Rightarrow x = \frac{h}{x_0}(f - x_0)$$



$$\tan\theta = \frac{x_0}{f + x_0}$$

$$\frac{h}{x + 2f} = \frac{x_0}{f + x_0}$$

$$\Rightarrow \frac{h}{x_0}(f + x_0) = x + 2f$$

$$\Rightarrow \frac{h}{x_0}(f + x_0) = \frac{h}{x_0}(f - x_0) + 2f$$

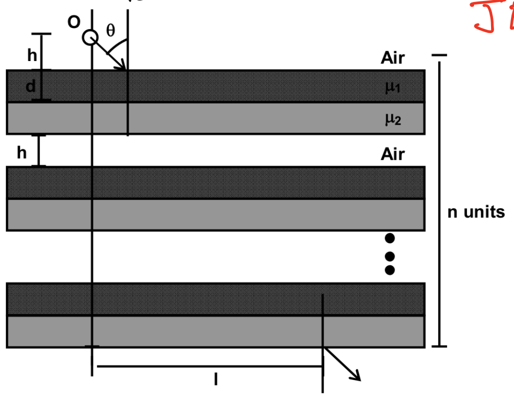
$$\Rightarrow 2hx_0 = 2fx_0$$

$$\therefore h = f$$

32.

Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2} cm$, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of n is__________.

JEE-2022



Ans. 4.

4

$$1 \sin 60° = \frac{\sqrt{3}}{2} r$$

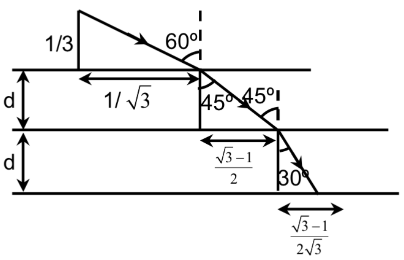
$$r = 45°$$

$$\frac{\sqrt{3}}{2} \sin 45° = \sqrt{3} \sin r_2$$

$$r_2 = 30°$$

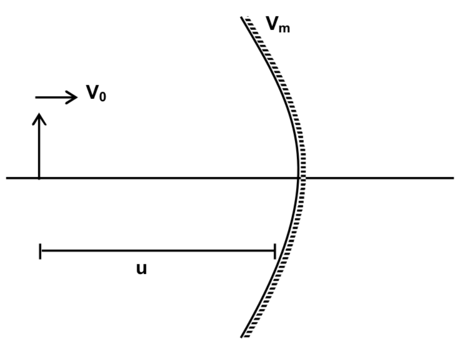
$$\left(\frac{1}{\sqrt{3}} + \frac{(\sqrt{3}-1)}{2} + \frac{(\sqrt{3}-1)}{2\sqrt{3}}\right) \times n = \frac{8}{\sqrt{3}}$$

n = 4.



33. An object and a concave mirror of focal length f=10cm both move along the principal axis of the mirror with constant speeds. The object moves with speed V_0=15 cm s^{-1} towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When u=30 cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is ________ cms^{-1}

(JEE-2022)



Ans. 3

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{-x}+\frac{1}{-30}=\frac{1}{-10}$$

$$x = 15 \text{ cm}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{-x}+\frac{1}{(-u)}=\frac{1}{f}$$

$$\frac{1}{x^2}\frac{dx}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

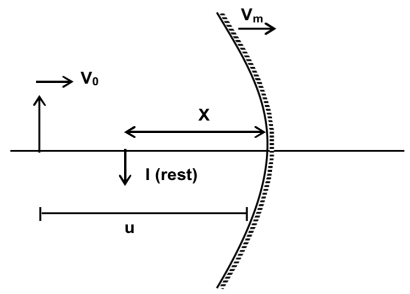
$$\frac{dx}{dt}=-\left(\frac{x}{u}\right)^2\frac{du}{dt}$$

$$(v_m-0)=-\left(\frac{15}{30}\right)^2[v_m-v_0]$$

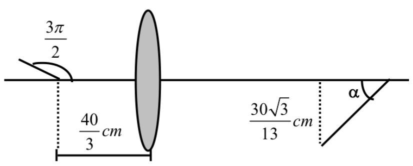
$$v_m=-\frac{1}{4}[v_m-15]$$

$$4v_m+v_m=15$$

$$v_m=3 \text{ cm/s}.$$



34. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of $\frac{40}{3}$ cm from the object as shown in the figure, The height of the image is $\frac{30\sqrt{3}}{13}$ and the angle made by it with respect to the principal axis is α rad. The value of α is $\frac{\pi}{n}rad$, where n is __________



(JEE - 2022)

Ans. **6.00**

$$u_1=-\frac{40}{3}$$

$$f = 10$$

$$\frac{1}{V_1}+\frac{1\times 3}{40}=\frac{1}{10}$$

$$V_1 = 40$$

$$u_2=-\frac{43}{3}$$

$$\frac{1}{V_2}=\frac{1}{10}-\frac{3}{43}=\frac{43-30}{430}$$

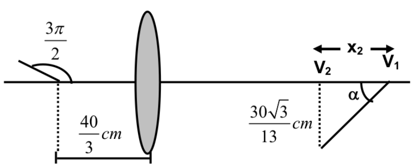
$$V_2=\frac{430}{13}$$

$$x_2=V_1-V_2=40-\frac{430}{13}=\frac{90}{13}$$

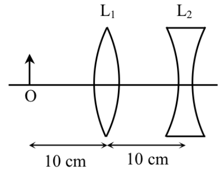
$$\tan\alpha=\frac{30\sqrt{3}}{13\times x}=\frac{30\sqrt{3}\times 13}{13\times 90}=\frac{1}{\sqrt{3}}$$

$$\alpha=\frac{\pi}{6}=\frac{\pi}{n}$$

$$n = 6$$



35. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is (JEE-2021)



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

B

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{1}{20}$$

$$\frac{1}{f_2} = (1.5-1)\left(-\frac{1}{20}-\frac{1}{20}\right) = -\frac{1}{20}$$

So, $\frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$

So, $v = -20$ cm

and $\frac{1}{v'} - \frac{1}{-30} = \frac{1}{-20}$

So, $v' = -12$ cm

So total magnification $= \left(\frac{-20}{-10}\right)\left(\frac{-12}{-30}\right) = 0.8$

More than one option correct

1. A student performed the experiment of determination of focal length of a concave mirror by $u - v$ method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)

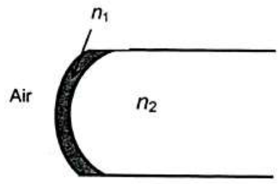
(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39) **[2009]**

Ans. C, D

Values of options (c) and (d) don't match with the mirror formula.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

2. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then **[JEE 2014]**



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

Ans. A, C

$$\frac{1}{f_{\text{film}}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right) \Rightarrow f_{\text{film}} = \infty \qquad \text{(infinite)}$$

$\therefore$ There is no effect of presence of film.

From Air to Glass

Using the equation $\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R} \Rightarrow v = 3R$$

$\therefore$ $f_1 = 3R$

From Glass to Air Again using the same equation

$$\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R} \Rightarrow \frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R} \Rightarrow v = 2R$$

$\therefore$ $f_2 = 2R$

3. A plano-convex lens is made of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance 10 cm away from the lens. Which of the following statement(s) is(are) true? **[JEE 2016]**

(A) The refractive index of the lens is 2.5

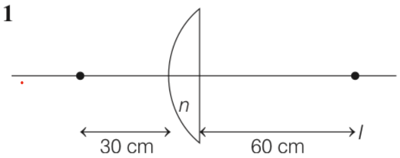
(B) The radius of curvature of the convex surface is 45 cm

(C) The faint images is erect and real

(D) The focal length of the lens is 20 cm

Ans. A, D

Case 1



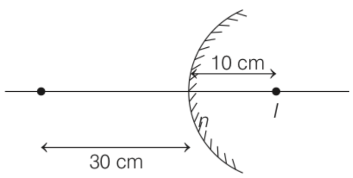
Using lens formula,

$$\frac{1}{60} + \frac{1}{30} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$$

$\Rightarrow$ $f_1 = 20\,\text{cm}$

Further, $\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow f_1 = \frac{R}{n-1} = +20\,\text{cm}$

Case 2



Using mirror formula

$$\frac{1}{10} - \frac{1}{30} = \frac{1}{f_2}$$

$$\frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$$

$$f_2 = 15 = \frac{R}{2} \Rightarrow R = 30$$

$$R = 30\,\text{cm}$$

$$\frac{R}{n-1} + 20\,\text{cm} = \frac{30}{n-1}$$

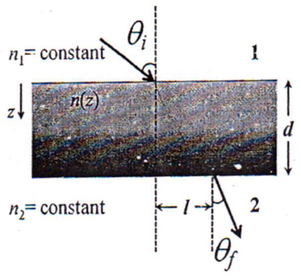
$\Rightarrow$ $= 2n - 2 = 3 \Rightarrow f_1 = +20\,\text{cm}$

Refractive index of lens is 2.5.

Radius of curvature of convex surface is 30 cm.

Faint image is erect and virtual focal length of lens is 20 cm.

4. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l. **[JEE 2016]**



Which of the following statement(s) is(are) true?

(A) l is independent of n_2 (B) l is dependent on $n(z)$

(C) $n_1 \sin\theta_i = n_2 \sin\theta_f$ (D) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

Ans. A, B, C

From Snell's law,

$n \sin\theta = \text{constant}$

$\therefore \quad n_1 \sin\theta_i = n_2 \sin\theta_f$

Further, l will depend on n_1 and $n(z)$. But it will be independent of n_2.

4. For an isosceles prism of angle A and refractive indes μ, it is found that the angle of minimum deviation δ_m = A. which of the following options is/are correct? **[JEE 2017]**

(A) At minimum deviation the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$

(B) For this prism the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(C) For this prism the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

(D) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

Ans. (A, C, D)

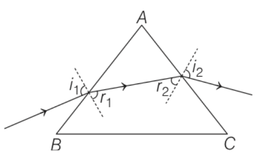
The minimum deviation produced by a prism

$\delta_m = 2i - A = A$

$\therefore \quad i_1 = i_2 = A$ and

$r_1 = r_2 = A/2$

$\therefore \quad r_1 = i_1/2$



Now, using Snell's law

$\sin A = \propto \sin A/2$

$\Rightarrow \quad \propto = 2\cos(A/2)$

For this prism when the emergent ray at the second surface is tangential to the surface

$i_2 = \pi/2 \Rightarrow r_2 = \theta_c \Rightarrow r_1 = A - \theta_c$

so, $\sin i_1 = \propto \sin(A - \theta_c)$

so, $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

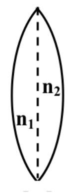
For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

But it is not necessarily parallel to the base if,

$\angle A = \angle B$ or $\angle A = \angle C$

5. [2019]

A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n - 1)$ and $1 < n < 2$. The correct statement(s) is/are.



A. $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

B. If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

C. For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

D. The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

Ans. **B, C, D**

When $n_1 = n_2 = n$

$$\frac{1}{f} = (n-1)\left(\frac{2}{R}\right) \quad \ldots(i)$$

When, $n_1 = n$ and $n_2 = n + \Delta n$

$$\frac{1}{f+\Delta f} = (n-1)\left(\frac{1}{R}\right) + (n+\Delta n - 1)\left(\frac{1}{R}\right) \quad \ldots(ii)$$

So from equation (i) and (ii)

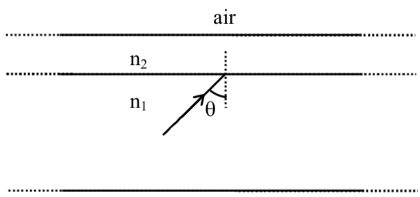
$$\frac{1}{f} - \frac{1}{f+\Delta f} = -(\Delta n)\left(\frac{1}{R}\right)$$

$$\Rightarrow \frac{\Delta f}{f^2} = -(\Delta n)\left(\frac{1}{R}\right)$$

$$\text{So } \frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \approx -\frac{\Delta n}{2n}$$

6. (2021)

A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where $\sin\theta$ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct?



(A) The light ray enters air if $n_2 = n_1$

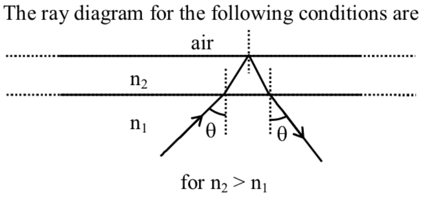
(B) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$

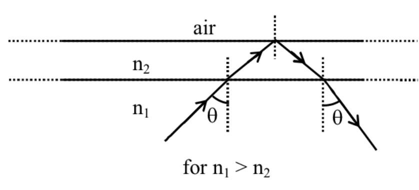
(C) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$

(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

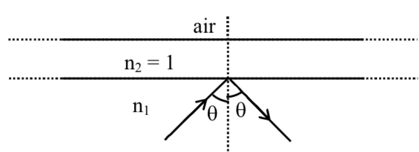
Ans. **B, C, D**

The ray diagram for the following conditions are





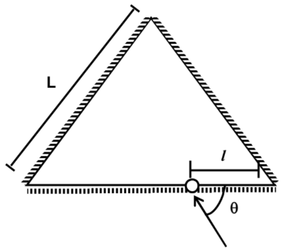
The light finally must reflected back in medium of refractive index n_1 for all values of n_2.



7.

Three plane mirrors form an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane.

2022



Which of the following statement(s) is(are) correct?

(A) The ray of light will come out for θ = 30°, for $0 < l < L$.

(B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

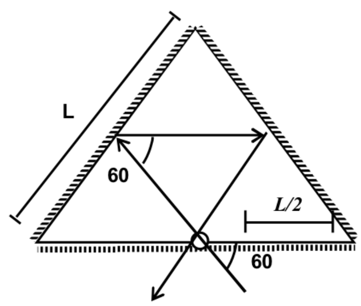
(C) The ray of light will **NEVER** come out for θ = 60°, and $l = \frac{L}{3}$

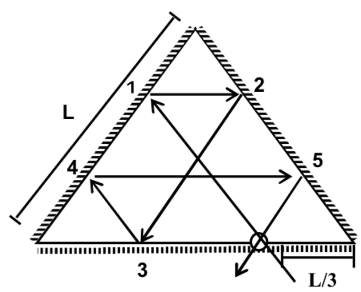
(D) The ray of light will come out for θ= 60°, and $0 < l < \frac{L}{2}$ after six reflections.

Ans.

A, B

For option 'A' there will be normal incidence and ray retrace its path. For option 'B'





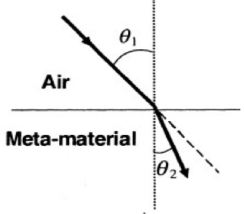
COMPREHENSION TYPE
PARAGRAPH – 1

Most materials have the refraction index, n > 1. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism , the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity ans permeability of the medium respectively.

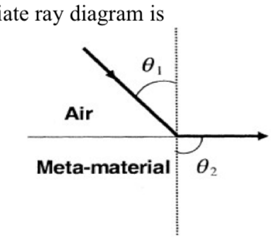
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. such negative refractive index materials can now be artificially prepared and are called meta-materials. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[JEE 2012]**

1. For light incident from air on a meta-material, the appropriate ray diagram is

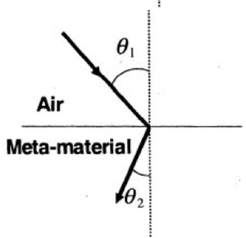
(A)



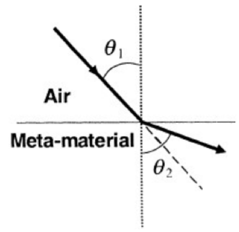
(B)



(C)



(D)



2. Choose the correct statement.

(A) the speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the Wavelength of the light in air

Ans.

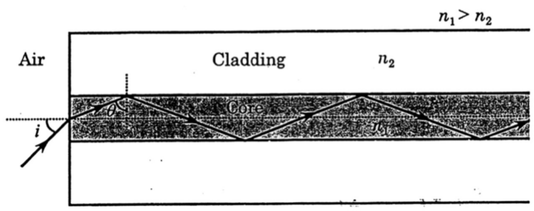
2 Since value of *n* in meta-material is negative. (C)

$$\therefore \qquad v = \frac{c}{|n|}$$

1 According to the paragraph, refracted ray in meta-material should be on same side of normal. (B)

PARAGRAPH – 2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m. **[JEE 2015]**



3. For two structures namely S_1 and $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option (s) is (are).
(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$.
(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same that of S_2 immersed in water.
(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.
(D) NA of S_1 placed in air the same as that S_2 placed in water.

4. If two structures of same cross-sectional area, but different numerical apertures NA_1 and $NA_2\,(NA_2 < NA_1)$ are joined longitudinally, the numerical aperture of the combined structure is
(A) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

Ans: 3- (A, C) 4 - D.

$$\frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\sin(90 - \theta_c) = \frac{\sqrt{45}}{4}\cos\theta_c$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\therefore \quad \cos\theta_c = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$\Rightarrow \quad \frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\frac{3}{\sqrt{45}}$$

$$\sin i = \frac{9}{16}$$

In second case,

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8} \Rightarrow \cos\theta_c = \frac{\sqrt{15}}{8}$$

$$\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90 - \theta_c)$$

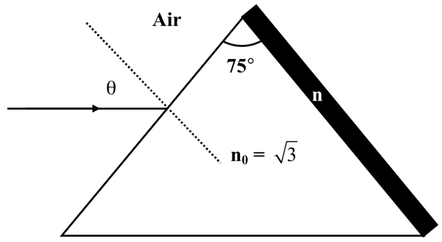
Simplifying we get,

$$\sin i = \frac{9}{16}$$

Same approach can be adopted for other options. Correct answers are (a) and (c).

Done 2019

A monochromatic light is incident from air on a refracting surface of prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is _______.



1.50

When angle of incidence on first face of the prism is 60° the angle of incidence on the other surface of the prism will be slightly greater than critical angle.

For refraction at first surface of the prism

$\sin 60° = \sqrt{3} \sin r_1$

$\Rightarrow r_1 = 30°$

For second surface $r_2 = 75° - 30° = 45°$

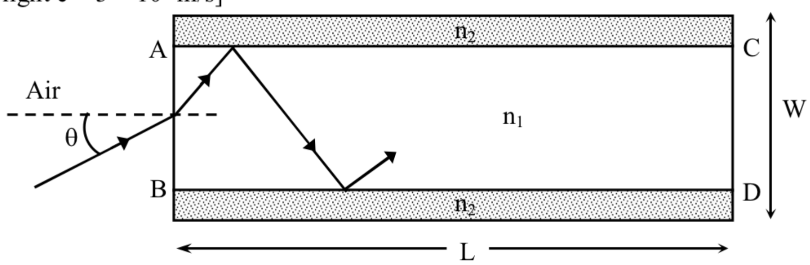
Since $r_2 \approx \theta_C$

$\Rightarrow \sin 45° = \frac{n}{\sqrt{3}}$

$\Rightarrow n^2 = 1.50$

Q 2019

A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is ________.
[Speed of light $c = 3 \times 10^8$ m/s]



Sol. **50.00**

$1.5 \sin \theta_C = 1.44 \sin 90°$

$\sin \theta_C = \frac{24}{25}$

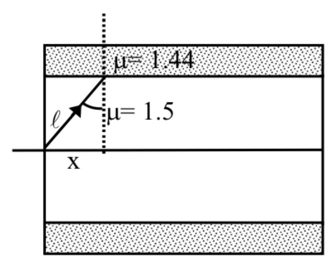
$\ell = \frac{x}{\sin \theta_C} = \frac{25}{4} x$

total length for light to travel

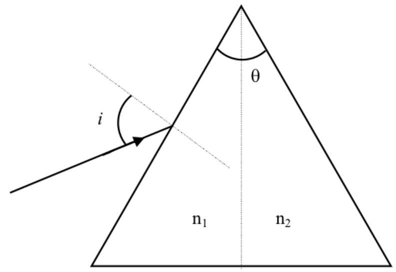
$\ell' = \frac{25}{4} \times 9.6 = 10\text{m}$

$\therefore \text{time} = \frac{\ell'}{c/1.5} = 5 \times 10^{-8}\text{s} \Rightarrow 50 \times 10^{-9}\text{s}$

t = 50.00



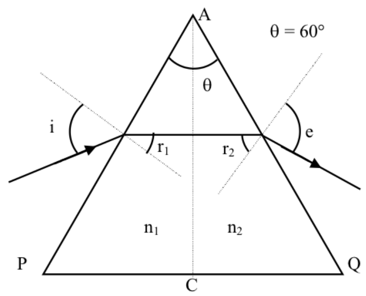
For a prism of prism angle $\theta = 60°$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence *i* is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n \ll n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct?



(A) The value of Δe (in radians) is greater than that of Δn
(B) Δe is proportional to Δn
(C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
(C) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

B, C

Diagram at minimum deviation for $n_1 = n_2 = n$



$n = 1.5$

$r_1 = r_2 = \theta/2 = 30°$

for face AQ

$n \sin r_2 = \sin e$

$1.5 \sin 30° = \frac{3}{2} \times \frac{1}{2} = \sin e$

$\sin e = \frac{3}{4}$, $\quad \cos e = \frac{\sqrt{7}}{4}$

When n_2 is given small variation there will be no change in path of light ray inside prism. As deviation on face AC is zero.

So, $r_2 = 30°$

Now for face AQ

$n_2 \sin 30° = \sin e$

for small change in n_2 change in e is given by

$dn_2 \sin 30° = \cos e \, de$

or $dn_2 = \Delta n \quad de = \Delta e$

$\Delta n \sin 30° = \cos e \, \Delta e$

$\Delta n \frac{1}{2} = \frac{\sqrt{7}}{4} \Delta e$

$\Delta n = \frac{\sqrt{7}}{2} \Delta e \qquad \ldots(i) \; \Delta n > \Delta e$

$\Delta n \propto \Delta e$

Hence, option (B) is correct.

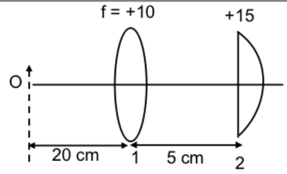
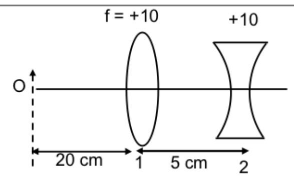
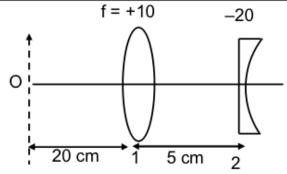
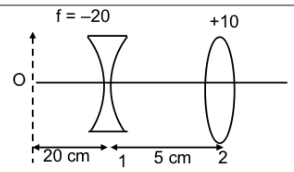
$$\Delta e = \frac{2.8 \times 10^{-3} \times 2}{\sqrt{7}}$$

Hence, option (C) is correct.

Que

2022

List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 cm from the first lens on the left, and the distance between the two lenses is 5 cm. List II contains the positions of the final images.

	List –I		List -II
(I)		(P)	Final image is formed at 7.5 *cm* on the right side of lens 2.
(II)		(Q)	Final image is formed at 60.0 *cm* on the right side of lens 2.
(III)		(R)	Final image is formed at 30.0 *cm* on the left side of lens 2.
(IV)		(S)	Final image is formed at 6.0 *cm* on the right side of lens 2.
		(T)	Final image is formed at 30.0 cm on the right side of lens 2.

Which one of the following options is correct?

(A) (I) → P; (II) → R; (III) → Q; (IV) → T (B) (I) → Q; (II) → P; (III) → T; (IV) → S

(C) (I) → P; (II) → T; (III) → R; (IV) → Q (D) (I) → T; (II) → S; (III) → Q; (IV) → R

Ans.

A

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u} \quad (concave\, lens)$$

$$\Rightarrow \quad v = -10$$

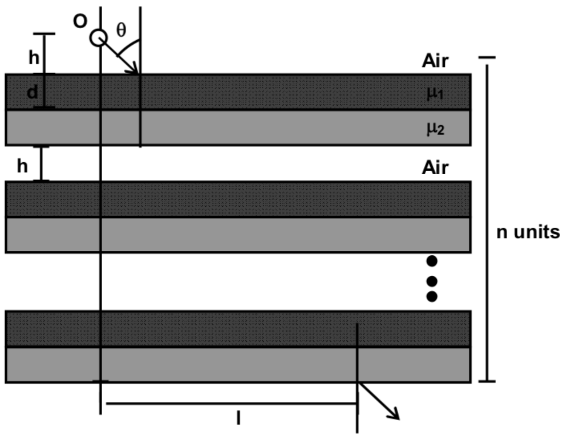
$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u} \quad (convex\, lens)$$

$$\frac{1}{v}=\frac{1}{10}+\frac{1}{-15}$$

$$\Rightarrow \quad v = +30$$

Adv 2022

Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2} cm$, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of n is__________.



Ans.

4

$1 \sin 60° = \frac{\sqrt{3}}{2} r$

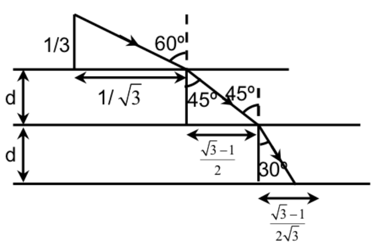
$r = 45°$

$\frac{\sqrt{3}}{2} \sin 45° = \sqrt{3} \sin r_2$

$r_2 = 30°$

$\left(\frac{1}{\sqrt{3}} + \frac{(\sqrt{3}-1)}{2} + \frac{(\sqrt{3}-1)}{2\sqrt{3}}\right) \times n = \frac{8}{\sqrt{3}}$

n = 4.



Subjective

1. Given $i_1 = 60Y, A = 30Y, \delta = 30Y$

From the relation

$$\delta = (i_1 + i_2) - A$$

we have, $i_2 = 0Y$

i.e. the ray is perpendicular to the face from which it emerges.

Further, $i_2 = 0Y$

∴ $r_2 = 0Y$

$$r_1 + r_2 = A$$

$$r_1 = A = 30Y$$

$$\propto = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60Y}{\sin 30Y} = \sqrt{3}$$

2.

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

First on plane surface

$$\frac{1.5}{AI_1} - \frac{1}{-mR} = \frac{1.5 - 1}{\infty} = 0 \qquad (R = \infty)$$

$\therefore$ $$AI_1 = -(1.5\,mR)$$

Then, on curved surface

$$\frac{1}{\infty} - \frac{1.5}{-(1.5\,mR + R)} = \frac{1 - 1.5}{-R}$$

[$v = \infty$, because final image is at infinity]

$\Rightarrow$ $$\frac{1.5}{(1.5\,m + 1)R} = \frac{0.5}{R}$$

$\Rightarrow$ $$3 = 1.5m + 1$$

$\Rightarrow$ $$\frac{3}{2}m = 2$$

or $$m = \frac{4}{3}$$

3.

(a) Rays coming from object AB first refract from the lens and then reflect from the mirror.

Refraction from the lens

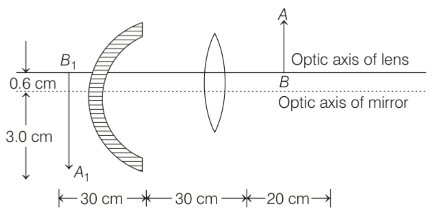
$$u = -20\,\text{cm}, \; f = +15\,\text{cm}$$

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$

$\therefore$ $$v = +60\,\text{cm}$$

and linear magnification, $m_1 = \frac{v}{u} = \frac{+60}{-20} = -3$

i.e. first image formed by the lens will be at 60 cm from it (or 30 cm from mirror) towards left and 3 times magnified but inverted. Length of first image A_1B_1 would be $1.2 \times 3 = 3.6$ cm (inverted).



Reflection from mirror Image formed by lens (A_1B_1) will behave like a virtual object for mirror at a distance of 30 cm from it as shown. Therefore $u = +30$ cm, $f = -30$ cm.

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} + \frac{1}{30} = -\frac{1}{30}$

$\therefore$ $$v = -15\,\text{cm}$$

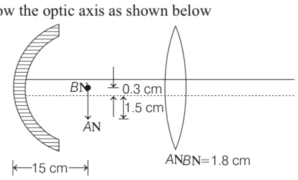
and linear magnification,

$$m_2 = -\frac{v}{u} = -\frac{-15}{+30} = +\frac{1}{2}$$

i.e. final image $A'B'$ will be located at a distance of 15 cm from the mirror (towards right) and since magnification is $+\frac{1}{2}$, length of final image would be $3.6 \times \frac{1}{2} = 1.8$ cm.

$\therefore$ $$A'B' = 1.8\,\text{cm}$$

Point B_1 is 0.6 cm above the optic axis of mirror, therefore, its image B' would be $(0.6)\left(\frac{1}{2}\right) = 0.3$ cm above optic axis. Similarly, point A_1 is 3 cm below the optic axis, therefore, its image A' will be $3 \times \frac{1}{2} = 1.5$ cm below the optic axis as shown below



Total magnification of the image,

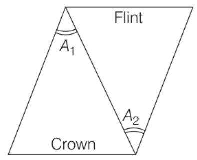
$$m = m_1 \times m_2 = (-3)\left(+\frac{1}{2}\right) = -\frac{3}{2}$$

$\therefore$ $$A'B' = (m)(AB) = \left(-\frac{3}{2}\right)(1.2) = -1.8\,\text{cm}$$

4.

(a) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\propto - 1)A$.

Net deviation by the two prisms is zero. So,



$$\delta_1 + \delta_2 = 0$$

or $$(\propto_1 - 1)A_1 + (\propto_2 - 1)A_2 = 0 \qquad \ldots \text{(i)}$$

Here, $\propto_1$ and $\propto_2$ are the refractive indices for crown and flint glasses respectively.

Hence, $$\propto_1 = \frac{1.51 + 1.49}{2} = 1.5$$

and $$\propto_2 = \frac{1.77 + 1.73}{2} = 1.75$$

A_1 = Angle of prism for crown glass = 6Υ

Substituting the values in Eq. (i), we get

$$(1.5 - 1)(6\Upsilon) + (1.75 - 1)A_2 = 0$$

This gives $A_2 = -4\Upsilon$

Hence, angle of flint glass prism is 4° (Negative sign shows that flint glass prism is inverted with respect to the crown glass prism.)

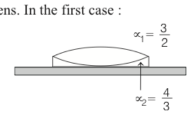
(b) Net dispersion due to the two prisms is

$$= (\propto_{b_1} - \propto_{r_1})A_1 + (\propto_{b_2} - \propto_{r_2})A_2$$
$$= (1.51 - 1.49)(6\Upsilon) + (1.77 - 1.73)(-4\Upsilon) = -0.04\Upsilon$$

$\therefore$ Net dispersion is – 0.04°

5.

Let R be the radius of curvature of both the surfaces of the equi-convex lens. In the first case :



Let f_1 be the focal length of equi-convex lens of refractive index $\propto_1$ and f_2 the focal length of plano-concave lens of refractive index $\propto_2$. The focal length of the combined lens system will be given by

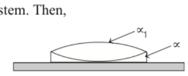
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$
$$= (\propto_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\propto_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$
$$= \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) + \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R}\right)$$
$$= \frac{1}{R} - \frac{1}{3R} = \frac{2}{3R} \quad \text{or} \quad F = \frac{3R}{2}$$

Now, image coincides with the object when ray of light retraces its path or it falls normally on the plane mirror. This is possible only when object is at focus of the lens system.

Hence, $F = 15$ cm (Distance of object = 15 cm)

or $$\frac{3R}{2} = 15\,\text{cm} \quad \text{or} \quad R = 10\,\text{cm}$$

In the second case, let $\propto$ be the refractive index of the liquid filled between lens and mirror and let F' be the focal length of new lens system. Then,



$$\frac{1}{F'} = (\propto_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\propto - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

or $$\frac{1}{F'} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) - \frac{(\propto - 1)}{R}$$

or $$= \frac{1}{R} - \frac{\propto - 1}{R} = \frac{(2 - \propto)}{R}$$

$\therefore$ $$F' = \frac{R}{2 - \propto} = \frac{10}{2 - \propto} \qquad (\because R = 10\,\text{cm})$$

Now, the image coincides with object when it is placed at 25 cm distance.

Hence, $$F' = 25$$

or $$\frac{10}{2 - \propto} = 25$$

or $$50 - 25\propto = 10$$

or $$25\propto = 40$$

$\therefore$ $$\propto = \frac{40}{25} = 1.6$$

or $$\propto = 1.6$$

6.

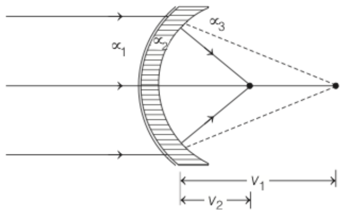
For refraction at first surface,

$$\frac{\propto_2}{v_1} - \frac{\propto_1}{-\infty} = \frac{\propto_2 - \propto_1}{+R} \quad \ldots\text{(i)}$$

For refraction at second surface,

$$\frac{\propto_3}{v_2} - \frac{\propto_2}{v_1} = \frac{\propto_3 - \propto_2}{+R} \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get



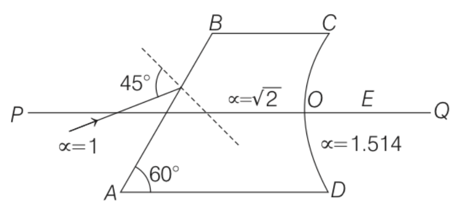
$$\frac{\propto_3}{v_2} = \frac{\propto_3 - \propto_1}{R}$$

or $$v_2 = \frac{\propto_3 R}{\propto_3 - \propto_1}$$

Therefore, focal length of the given lens system is

$$\frac{\propto_3 R}{\propto_3 - \propto_1}.$$

7. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face *AB* as shown in the figure. After refraction it is incident on a spherical surface *CD* of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet *PQ* at *E*. Find the distance *OE* upto two places of decimal. **(2004, 2M)**



......

Ans.

Applying Snell's law on face *AB*,

$$(1) \sin 45Υ = (\sqrt{2}) \sin r$$

$\therefore$ $$\sin r = \frac{1}{2}$$

or $$r = 30Υ$$

i.e. ray becomes parallel to *AD* inside the block.

Now applying,

$$\frac{\propto_2}{v} - \frac{\propto_1}{u} = \frac{\propto_2 - \propto_1}{R} \text{ on face } CD,$$

$$\frac{1.514}{OE} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{0.4}$$

Solving this equation, we get $OE = 6.06$ m

8.

Differentiating the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with respect to time, we get

$$-\frac{1}{v^2}.\frac{dv}{dt} + \frac{1}{u^2}.\frac{du}{dt} = 0 \qquad \text{(as } f = \text{constant)}$$

$$\therefore \qquad \left(\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right).\frac{du}{dt} \qquad \ldots\text{(i)}$$

Further, substituting proper values in lens formula, we have

$$\frac{1}{v} + \frac{1}{0.4} = \frac{1}{0.3} \qquad (u = -\,0.4 \text{ m}, \; f = 0.3 \text{ m})$$

or $$v = 1.2 \text{ m}$$

Putting the values in Eq. (i), we get

Magnitude of rate of change of position of image= 0.09 m/s

Lateral magnification, $m = \frac{v}{u}$

$$\therefore \qquad \frac{dm}{dt} = \frac{u.\frac{dv}{dt} - v.\frac{du}{dt}}{u^2} = \frac{(-0.4)(0.09) - (1.2)(0.01)}{(0.4)^2}$$

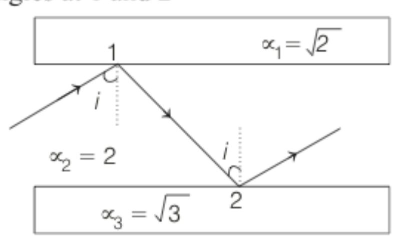
$$= -\,0.3 \text{ /s}$$

∴ Magnitude of rate of change of lateral magnification

$$= 0.3 \text{ /s}$$

9.

Critical angles at 1 and 2



$$\theta_{C_1} = \sin^{-1}\left(\frac{\propto_1}{\propto_2}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45\Upsilon$$

$$\theta_{C_2} = \sin^{-1}\left(\frac{\propto_3}{\propto_2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60\Upsilon$$

Therefore, minimum angle of incidence for total internal reflection to take place on both slabs should be 60°.

$$i_{\min} = 60\Upsilon$$

10.

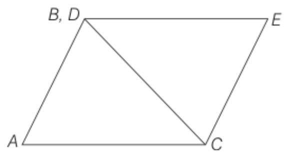
(a) At minimum deviation, $r_1 = r_2 = 30^\circ$

$\therefore$ From Snell's law

$$\propto = \frac{\sin i_1}{\sin r_1} \quad \text{or} \quad \sqrt{3} = \frac{\sin i_1}{\sin 30^\circ}$$

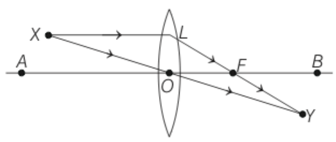
$$\therefore \quad \sin i_1 = \frac{\sqrt{3}}{2} \quad \text{or} \quad i_1 = 60^\circ$$

(b) In the position shown net deviation suffered by the ray of light should be minimum. Therefore, the second prism should be rotated by 60° (anti-clockwise).



11.

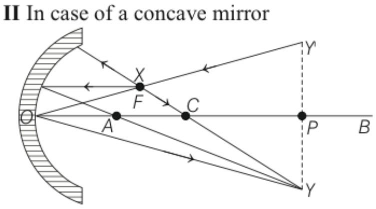
Steps I In case of a lens



(a) Join X and Y. The point O, where the line XY cuts the optic axis AB, is the optical centre of the lens.

(b) Draw a line parallel to AB from point X. Let it cuts the lens at L. Join L and Y. The point F where the line LY cuts the optic axis AB is the focus of the lens F.

Step II In case of a concave mirror



(a) Draw a line YY' perpendicular to AB from point Y. Let it cuts the line AB at point P. Locate a point Y' such that $PY = PY'$.

(b) Extend the line XY'. Let it cuts the line AB at point O. Then O is the pole of the mirror.

(c) Join X and Y. The point C, where the line XY cuts the optic axis AB, is the centre of curvature of the mirror.

(d) The centre point F of OC is the focus of the mirror.

12.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{u}{v} - 1 = \frac{u}{f} \quad \text{or} \quad \frac{u}{v} = \left(\frac{u+f}{f}\right)$$

$$\therefore \quad m = \frac{v}{u} = \left(\frac{f}{u+f}\right) \Rightarrow \frac{m_{25}}{m_{50}} = \frac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$$

$\therefore$ Answer is 6.

13.

Using mirror formula twice,

$$\frac{1}{+25/3} + \frac{1}{-u_1} = \frac{1}{+10}$$

or $$\frac{1}{u_1} = \frac{3}{25} - \frac{1}{10}$$

or $$u_1 = 50 \text{ m and } \frac{1}{(+50/7)} + \frac{1}{-u_2} = \frac{1}{+10}$$

$\therefore$ $$\frac{1}{u_2} = \frac{7}{50} - \frac{1}{10} \text{ or } u_2 = 25 \text{ m}$$

$$\text{Speed of object} = \frac{u_1 - u_2}{\text{time}}$$

$$= \frac{25}{30} \text{ ms}^{-1} = 3 \text{ kmh}^{-1}$$

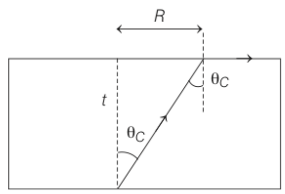
$\therefore$ Answer is 3.

14.

$$\frac{R}{t} = \tan\theta_C \quad \text{or} \quad R = t(\tan\theta_C)$$

But, $$\sin\theta_C = \frac{1}{\propto} = \frac{3}{5}$$

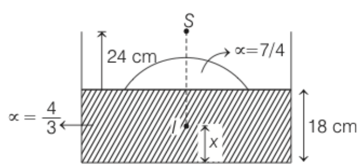
$\therefore$ $$\tan\theta_C = \frac{3}{4}$$



$\therefore$ $$R = \frac{3}{4}t = \frac{3}{4}(8 \text{ cm}) = 6 \text{ cm}$$

Hence the answer is 6.

15.



Two refractions will take place, first from spherical surface and the other from the plane surface.

So, applying

$$\frac{\propto_2}{v} - \frac{\propto_1}{u} = \frac{\propto_2 - \propto_1}{R}$$

two times with proper sign convention.

Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{7/4}{v} - \frac{1.0}{-24} = \frac{7/4 - 1.0}{+6} \quad \text{...(i)}$$

$$\frac{4/3}{(18 - x)} - \frac{7/4}{v} = \frac{4/3 - 7/4}{\propto} \quad \text{...(ii)}$$

Solving these equations, we get, $x = 2$ cm

$\therefore$ Answer is 2.

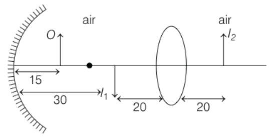
16.

Case I

Reflection from mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15}$$

$\Rightarrow \quad v = -30$



For lens

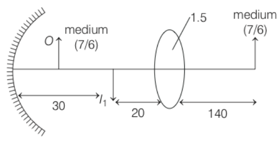
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-20}$$

$$v = 20$$

$$|M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2 \qquad \text{(in air)}$$

Case II For mirror, there is no change.



For lens,
$$\frac{1}{f_{air}} = \left(\frac{3/2}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_{medium}} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

with $f_{air} = 10\,\text{cm}$

We get $\dfrac{1}{f_{medium}} = \dfrac{4}{70}\,\text{cm}^{-1}$

$$\frac{1}{v} - \frac{1}{-20} = \frac{4}{70}$$

$$\frac{1}{v} + \frac{1}{20} = \left(\frac{2}{7}\right)\left(\frac{2}{10}\right) = \frac{4}{70}$$

$$\frac{1}{v} = \frac{4}{70} - \frac{1}{20} \quad \Rightarrow \quad v = 140,$$

$$|M_2| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{140}{20}\right),$$

$$= (2)\left(\frac{140}{20}\right) = 14$$

$$\Rightarrow \quad \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$$

17.

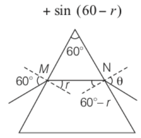
Applying Snell's law at M and N,

$$\sin 60Y = n \sin r \qquad \text{...(i)}$$

$$\sin \theta = n \sin (60 - r) \qquad \text{...(ii)}$$

Differentiating we get

$$\cos\theta \frac{d\theta}{dn} = -n\cos(60-r)\frac{dr}{dn} + \sin(60-r)$$



Differentiating Eq. (i),

$$n\cos r \frac{dr}{dn} + \sin r = 0$$

or
$$\frac{dr}{dn} = -\frac{\sin r}{n\cos r} = \frac{-\tan r}{n}$$

$$\Rightarrow \quad \cos\theta \frac{d\theta}{dn} = -n\cos(60-r)\left(\frac{-\tan r}{n}\right) + \sin(60-r)$$

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60-r)\tan r + \sin(60-r)]$$

Form Eq. (i), $r = 30Y$ for $n = \sqrt{3}$

$$\Rightarrow \frac{d\theta}{dn} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30) = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$