IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 28/08/22**

TOPIC: CALCULUS IN PHYSICS

Answer Key

1.	(A)	2.	(B)	3.	(B)	4.	(B)	5.	(C)
6.	(D)	7.	(A)	8.	(A)	9.	(D)	10.	(C)
11.	(C)	12.	(B)	13.	(A)	14.	(B)	15.	(A)
16.	(C)	17.	(D)	18.	(C)	19.	(C)	20.	(D)
21.	(D)	22.	(C)	23.	(D)	24.	(A)	25.	(A)

SOLUTIONS

1. (A)

$$\int_0^\infty e^{-5x}dx = \left[\frac{e^{-5x}}{-5}\right]_0^\infty$$

$$= \frac{0-1}{-5} = \frac{1}{5}$$

2. (B)

$$\int \frac{x^3}{4+x^4}dx$$

$$= \frac{\ln\left(4+x^4\right)}{4} + C$$

3. (B)

$$\int_0^1 \frac{dx}{(ax+b)}$$

$$= \left[\frac{\ln(ax+b)}{a}\right]_0^1$$

$$= \left[\ln(a+b) - \ln b\right]\frac{1}{a}$$

$$= \frac{1}{a}\ln\left(\frac{a+b}{b}\right) = \frac{1}{a}\ln\left(1+\frac{a}{b}\right)$$

4. (B)

$$3y = 2x^3 + 1$$

$$\frac{3dy}{dt} = 6x^2 \frac{dx}{dt}$$

$$dy = 2x^2 \cdot dx$$

$$8 = 2x^2 \Rightarrow x = \pm 2$$

$$y = \frac{2(\pm 2)^3 + 1}{3} = \pm\frac{16+1}{3}$$

$$= \frac{17}{3}, -\frac{15}{3}$$

5. (C)

$$y = x \cdot \cos x$$

$$\frac{dy}{dx} = x \cdot (-\sin x) + \cos x(1)$$

$$= -x\sin x + \cos x$$

6. (D)

$$\frac{dy}{d\theta} = -2\sin 2\theta$$

$$\frac{dx}{d\theta} = \frac{1}{2}\cos\frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{-4\sin 2\theta}{\cos\theta/2} = -16\sin\frac{\theta}{2} \cdot \cos\theta$$

7. (A)

$$\int_2^5 (3x^2 + 4x + 1)\,dx$$

$$= \left[x^3 + 2x^2 + x\right]_2^5$$

$$= 180 - 18 = 162$$

8. (A)

$$\int_0^{\pi/4} \sec^2\theta \cdot d\theta = [\tan\theta]_0^{\pi/4} = 1$$

9. (D)

$$\frac{dy_1}{dx} = \cos x + \sin x = 0$$

$$\tan x = -1 \qquad \therefore\ A_1 = \sqrt{2}$$

$$\frac{dy_2}{dx} = 2\cos x - 4\sin x = 0$$

$$\tan x = \frac{1}{2}$$

$$B = 2\sqrt{5}$$

10. (C)

$$\frac{dp}{dt} = \left(10t + 3t^2\right)$$

$$\Delta p = \int \left(10t + 3t^2\right) dt$$

$$= \left(\frac{10t^2}{2} + \frac{3t^3}{3}\right)_0^2$$

$$= 5(4) + 8$$

$$= 28.$$

11. (C)

$$y = e^x - \ln x + \frac{1}{x}$$

$$\frac{dy}{dx} = e^x - \frac{1}{x} - \frac{1}{x^2}$$

12. (B)

$$\frac{dy}{dx} = x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\frac{d^2y}{dx^2} = 2x + 1$$

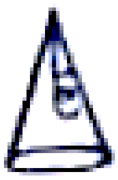
At $x = -4; \frac{d^2y}{dx^2} = -7 < 0$ (maximum)

At $x = 3; \frac{d^2y}{dx^2} = 7 > 0$ (minimum)

13. (A)

$$V = \frac{\pi}{3} r^2 h$$

$$\tan\theta = \frac{r}{h} = \frac{5}{20} - \frac{1}{4}$$



$$\Rightarrow h = \frac{r}{\tan\theta} = 4r$$

$$V = \frac{\pi}{3} r^2 (4r)$$

$$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \cdot \frac{dr}{dt}$$

14. (B)

$$\frac{dy}{dx} = \sec x \cdot \tan x + \sec^2 x$$

$$= 2\sqrt{3} + (2)^2$$

$$= 2\left(\sqrt{3} + 2\right)$$

15. (A)

$$W=\int \frac{2}{r^2}\,dr$$

$$=\left[\frac{-2}{r}\right]_{1/2}^{\infty}$$

$$=-\left(-\frac{2}{1/2}\right)=4\text{ J}$$

16. (C)

$$\int_1^2 y\cdot dx=\left(\frac{ax^3}{3}+2x^2-3x\right)_1^2=10$$

$$\Rightarrow\ a=3$$

17. (D)

$$x=y^2+y$$

$$1=2y\cdot\frac{dy}{dx}+\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1}{(2y+1)}$$

18. (C)

$$x=t^2-24t+6$$

$$\frac{dx}{dt}=2t-24=0$$

$$t=12$$

$$\frac{d^2x}{dt^2}=2>0\,(\text{minima})$$

at t = 12

19. (C)

$$y=\ln x^2$$

$$\frac{dy}{dx}=\frac{1}{x^2}\times 2x=\frac{2}{x}=\frac{2}{e}$$

20. (D)

$$v=bs^2$$

$$a=\frac{vdv}{ds}=bs^2(2bs)$$

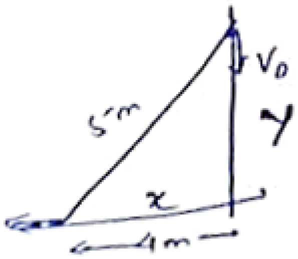
$$=2b^2s^3$$

21. (D)

$$A=4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$
$$= 8\pi(2)(10)$$
$$= 160\pi$$

22. (C)



$$x^2 + + y^2 = 5^2$$
$$2x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$
$$= \frac{4}{3}(3) = 4\,\text{m/s}$$

23. (D)

$$F = \frac{dP}{dt}$$

24. (A)

W = area of $F \cdot x$ curve $= \frac{1}{2} \times 20 \times 10 = 100$ J

25. (A)

$$P \propto t$$
$$F = \frac{dP}{dt} = \text{constant}$$
i.e. $F \propto t^{o}$

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TOPIC: MOLE CONCEPT

SOLUTIONS

26.	(C)	27.	(A)	28.	(C)	29.	(C)	30.	(A)
31.	(C)	32.	(C)	33.	(C)	34.	(D)	35.	(C)
36.	(C)	37.	(D)	38.	(D)	39.	(B)	40.	(B)
41.	(D)	42.	(A)	43.	(D)	44.	(D)	45.	(B)
46.	(D)	47.	(A)	48.	(D)	49.	(C)	50.	(B)

SOLUTIONS

26. (C)

27. (A)

28. (C)

29. (C)

30. (A)

31. (C)

32. (C)

NO_3^-

$= 7 + 24 + 1$

$= 32$

$NO_3^- \longrightarrow 32e^-$

Mole of $NO_3^- = \frac{3.1\times10^{-3}}{62} = \frac{10^{-3}}{20} = \frac{10^{-4}}{2} = 5\times10^{-5}$

$NO_3^- \longrightarrow 32e^-$

$1 \longrightarrow 32$

$5\times10^{-5}\times6\times10^{23}$ $32\times30\times10^{18}$

$=30\times10^{18}$ $=96\times10^{19}$

33. (C)

Mole of $H_2O = \frac{0.45}{18} = \frac{5}{200}$

$H_2O \longrightarrow n = 8$

$1 \longrightarrow 8$

$\frac{5}{200} \qquad \frac{5}{200} \times 8$

$= \frac{40}{200} = 0.2$

No. of neutrons $= 0.2 \times 6 \times 10^{23}$

$= 1.2 \times 10^{23}$

34. (D)

35. (C)

36. (C)

37. (D)

38. (D)

39. (B)

$4Al + 3O_2 \rightarrow 2Al_2O_3$

4 : 3 : 2

$\frac{1.5 \times 4}{3} \quad 1.5$

= 2 mole

Mass of Al $= 2 \times 27 = 54$ gm

40. (B)

41. (D)

42. (A)

43. (D)

44. (D)

45. (B)

46. (D)

47. (A)

48. (D)

49. (C)

50. (B)

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TOPIC: QUADRATIC EQUATIONS

Answer Key

51.	(D)	52.	(A)	53.	(C)	54.	(D)	55.	(A)
56.	(A)	57.	(C)	58.	(D)	59.	(D)	60.	(B)
61.	(B)	62.	(D)	63.	(D)	64.	(B)	65.	(B)
66.	(A)	67.	(C)	68.	(C)	69.	(D)	70.	(B)
71.	(B)	72.	(D)	73.	(D)	74.	(C)	75.	(D)

SOLUTION

51. (D)

$A=1, B=-2(a+b), C=2(a^2+b^2)$

$B^2-4AC=1[2(a+b)]^2-4(1)(2a^2+2b^2)$

$=4a^2+4b^2+8ab-8a^2-8b^2$

$=-4a^2-4b^2+8ab$

$=-4(a-b)^2<0$

So roots are imaginary and different.

52. (A)

The required equation is

$x^2-\left\{\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)\right\}x+\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=0$

Or $x^2-4x+1=0$

53. (C)

Roots are of opposite sign so

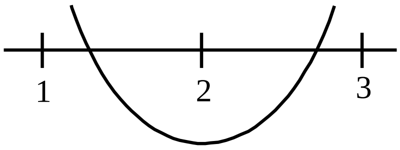
$f(0)<0$

$\lambda^2-5\lambda+6<0$

$(\lambda-2)(\lambda-3)<0$

$\Rightarrow \quad \lambda\in(2,3)$

54. (D)



$f(1) > 0$

$f(2) < 0$

$f(3) > 0$

So $\quad 1+\lambda+1+\lambda^2-3\lambda-6>0 \Rightarrow \quad \lambda^2-2\lambda-4>0$

$\lambda \in \left(-\infty, 1-\sqrt{5}\right) \cup \left(1+\sqrt{5}, \infty\right)$

$f(2) = 4+2\lambda+2+\lambda^2-3\lambda-6<0 \quad \Rightarrow \quad \lambda^2-\lambda<0$

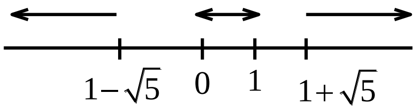
$\lambda(\lambda-1)<0$

$\lambda \in (0,1)$

$f(3) = 9+3\lambda+3+\lambda^2-3\lambda-6>0$

$\lambda^2+6>0$

$\lambda \in R$



$\lambda \in \phi$

55. (A)

α, β are roots of equation $x^2+px+q=0$ so

$\alpha+\beta=-p$

$\alpha\beta=q$

And $\alpha^2+p\alpha+q=0$

$$\alpha+p=\frac{-q}{\alpha}$$

So $\quad (\alpha+p)^{-2}=\dfrac{\alpha^2}{q^2} \qquad$ similarly $(\beta+p)^{-2}=\dfrac{\beta^2}{q^2}$

So required equation

$$x^2-\left(\frac{\alpha^2+\beta^2}{q^2}\right)x+\frac{\alpha^2\beta^2}{q^4}=0 \Rightarrow x^2-\left(\frac{p^2-2q}{q^2}\right)x+\frac{1}{q^2}=0 \Rightarrow q^2x^2-\left(p^2-2q\right)x+1=0$$

56. (A)

One root of equation $x^2+Ax+12=0$ is 4.

Sum of roots $=-A$

Product of roots = 12

So other root is 3.

$A=-7$

Now roots of equation $x^2+2Ax+B=0$ are equal

So $\quad 4A^2-4B=0$

$B=A^2 \Rightarrow B=49$

57. (C)

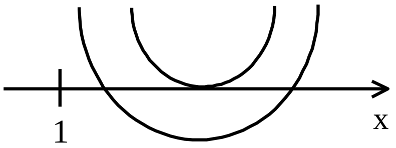
Roots are real so $4q^2 - 4pr \geq 0$

And $4pr - 4q^2 \geq 0$

Both inequations are true if

$$q^2 - pr = 0 \quad \Rightarrow \frac{p}{q} = \frac{q}{r}$$

58. (D)



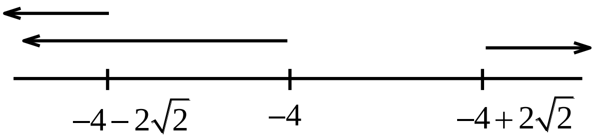
$f(x) = 2x^2 + \lambda x - (\lambda + 1) = 0$

$D = \lambda^2 + 8(\lambda + 1) \geq 0$

$\lambda \in \left[-\infty, -4 - 2\sqrt{2}\right] U \left[-4 + 2\sqrt{2}, \infty\right]$ ………… (1)

$\frac{-b}{2a} = \frac{-\lambda}{4} > 1 \quad \Rightarrow \quad \lambda < -4$ ………… (2)

$f(1) = 2 + \lambda - (\lambda + 1) > 0, \ \forall \lambda \in R$



$\Rightarrow \quad \lambda \in \left(-\infty, -4 - 2\sqrt{2}\right)$

59. (D)

It is identity in x so

$r^2 - 2r + 1 = 0 \quad \Rightarrow \quad r = 1$

$r^2 - 3r + 2 = 0 \quad \Rightarrow \quad r = 1, 2$

$r^2 + 4r + 3 = 0 \quad \Rightarrow \quad r = -1, -3$

No common value of r is possible so it is not possible.

60. (B)

$2x^2 - 5x - 7 = 0$ roots are α, β

So $\alpha + \beta = \frac{5}{2}$

$\alpha\beta = -\frac{7}{2}$

Now roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

$$\text{Sum } = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{\frac{25}{4}+7}{\frac{-7}{2}} = -\frac{53}{14}$$

Product = 1

So equation is $x^2+\frac{53}{14}x+1=0$

$14x^2+53x+14=0$

61. (B)

$ax^2+bx+c=0$ roots are imaginary so $b^2-4ac<0$

$(4c+2b+a)x^2-2(a+b)x+a=0$

$D=4(a+b)^2-4a(4c+2b+a)$

$=4\left[a^2+b^2+2ab-4ac-2ab-a^2\right]$

$=4(b^2-4ac)<0$

So roots are imaginary.

62. (D)

α,β,γ are root of $x^3-5x^2+x-2=0$

Now $y=\frac{\alpha+2}{\alpha-2} \quad \Rightarrow \quad \alpha=\frac{2(y+1)}{y-1}$

It is root of given equation so

$$\frac{8(y+1)^3}{(y-1)^3}-5.4\frac{(y+1)^2}{(y-1)^2}+\frac{2(y+1)}{(y-1)}-2=0$$

$$8(y+1)^3-20(y+1)^2(y-1)+2(y+1)(y-1)^2-2(y-1)^3=0 \Rightarrow 3y^3-2y^2-9y-8=0$$

Roots of this equation are $\frac{\alpha+2}{\alpha-2},\frac{\beta+2}{\beta-2},\frac{\gamma+2}{\gamma-2}$

So product of roots

$$\left(\frac{\alpha+2}{\alpha-2}\right)\left(\frac{\beta+2}{\beta-2}\right)\left(\frac{\gamma+2}{\gamma-2}\right)=\frac{8}{3}$$

63. (D)

Equation $8x^3+1001x+2008=0$ has roots r, s and t.

$$r+s+r=0, rst=-\frac{2008}{8}=-251$$

Now, let $r+s=A, s+t=B, t+r=C$.

$\therefore \quad A+B+C=2(r+s+t)=0$

Hence,

$$A^3+B^3+C^3=3ABC$$

$\therefore \quad (r+s)^3+(s+t)^3+(t+r)^3$

$=3(r+s)(s+t)(t+r)$

$=3(r+s+t-t)(s+t+r-r)(t+r+s-s)$

$=3(251)=753$

64. (B)

Let,

$$\frac{x}{x^2-5x+9}=y$$

$\Rightarrow \quad yx^2-5yx+9y=x$

$\Rightarrow \quad yx^2-(5y+1)x+9y=0$

Now, x is real, so

$D\geq 0$

$\Rightarrow \quad (-(5y+1))^2-4y(9y)\geq 0$

$\Rightarrow \quad -11y^2+10y+1\geq 0$

$\Rightarrow \quad 11y^2-10y-1\leq 0$

$\Rightarrow \quad (11y+1)(y-1)\leq 0$

$\Rightarrow \quad -\frac{1}{11}\leq y\leq 1$

65. (B)

Let,

$$y=\frac{3x^2+9x+17}{3x^2+9x+7}$$

$\Rightarrow \quad 3x^2(y-1)+9x(y-1)+7y-17=0$

Since x is real, so,

$D\geq 0$

$\Rightarrow \quad 81(y-1)^2-4\times 3(y-1)(7y-17)\geq 0$

$\Rightarrow \quad (y-1)(y-41)\leq 0\Rightarrow 1\leq y\leq 41$

Therefore, the maximum value of y is 41.

66. (A)

$D=b^2-4a<0\Rightarrow a>0$

Therefore the graph is concave upwards.

$f(x)>0, \forall x\in R$

$\Rightarrow \quad f(-1)>0$

$\Rightarrow \quad a+b+1>0$

67. (C)

$(\alpha-\gamma)(\alpha-\delta)=\alpha^2-(\gamma+\delta)\alpha+\gamma\delta$

$=\alpha^2+p\alpha-4$ (because $\gamma+\delta=-p,\ \gamma\delta=-4$)

α is root of $x^2+px+7=0$

$\Rightarrow \quad \alpha^2+p\alpha+7=0 \Rightarrow \alpha^2+p\alpha=-7$

Now $\quad (\alpha-\gamma)(\alpha-\delta)=\alpha^2+p\alpha-4=-7-4=-7-4=-11$

68. (C)

$2x^2+9x+4a=0$ one root is α then 2α is root at $2x^2+3x+a=0$

So $\quad 2\alpha^2+9\alpha+4a=0 \quad$ ………. (1)

$8\alpha^2+6\alpha+a=0 \quad$ ………. (2)

By solving (1) and (2) $\alpha=-\frac{a}{2}$

From (1)

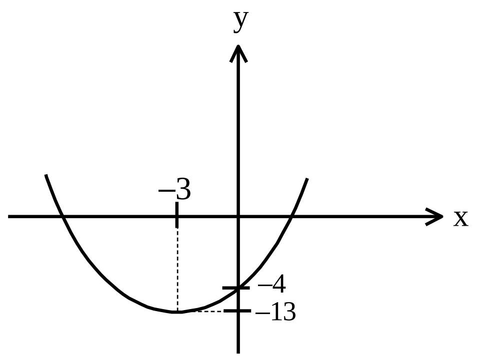
$2.\frac{a^2}{4}-\frac{9a}{2}+4a=0 \quad$ So $a=1$

69. (D)

$y(-4)=16-24-4=-12 \qquad$ min. value = $\frac{-D}{4a}=\frac{-(36+16)}{4}=-13$

$y(3)=9+18-4=23$

So range is $[-13,23)$



70. (B)

Correct equation

$x^2+13x+q=0$ ………. (1)

Incorrect equation is

$x^2+17x+q=0$ ………. (2)

Given that roots of Eq. (1) are –2 and –15.

Therefore, product of the roots of incorrect equation is $q=(-2)(-15)=30$. From (1), the correct equation is

$x^2+13x+30=0$

$\therefore \quad x=-3,-10$

71. (B)

Trick: By inspection, we see that all the values of *x* lying in $(-\infty,1)\cup(2,3)$ satisfy the equations and no other value outside the interval satisfy it.

72. (D)

8, 2 are the roots of $x^2 + ax + \beta = 0$,

$\therefore\ 8 + 2 = 10 = -a,\ 8.2 = 16 = \beta,\ i.e.\ a = -10, \beta = 16$ 3, 3 are the roots of

$x^2 + \alpha x + b = 0$ $\therefore$ 3 + 3 = 6 = $-\alpha$

i.e. $\alpha = -6, b = 9$

Now, $x^2 + ax + b = 0$ becomes $x^2 - 10x + 9 = 0$ or $(x-1)(x-9) = 0 \Rightarrow x = 1, 9$

73. (D)

Here $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

Since, α, β be the roots of $ax^2 + bx + c = 0$.

Also, $\alpha < k < \beta$, so $a(k-\alpha)(k-\beta) < 0$

Also, $a^2k^2 + abk + ac = a(ak^2 + bk + c) = a^2(k-\alpha)(k-\beta) < 0 \Rightarrow a^2k^2 + abk + ac < 0$

74. (C)

$k - 2 > 0$ $k > 2$ and

$64 - 4(k+4)(k-2) < 0$

$16 - (k^2 + 2k - 8) < 0$

$16 - k^2 - 2k + 8 < 0$

$-k^2 - 2k + 24 < 0$

$k^2 + 2k - 24 > 0$

$(k-4)(k+6) > 0 \Rightarrow k \in (-\infty, -6) \cup (4, \infty)$

Hence, $k \in (4, \infty)$

75. (D)

$\alpha + \beta + \gamma = 0$,

$$\frac{\alpha^3 - 3\alpha + \beta^3 - 3\beta + \gamma^3 - 3\gamma}{\alpha\beta\gamma} = \frac{(\alpha^3 + \beta^3 + \gamma^3) - 3(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$$

$$= \frac{3\alpha\beta\gamma - 0}{\alpha\beta\gamma} = 3$$