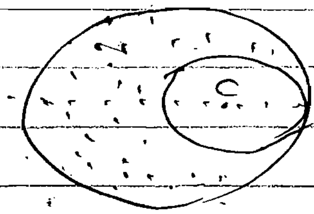
# Gravitation

Only one option is correct

(1)




$$E_C = \frac{G\left(\frac{4}{3}\pi R^3 d\right)\left(\frac{R}{2}\right)}{R^3}$$

$$E_C = \frac{2G\pi d R}{3}$$

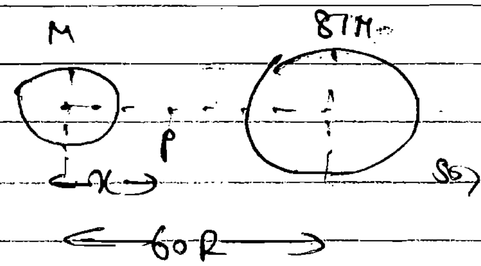
$$F = \left(\frac{2G\pi dR}{3}\right) m = \frac{2\pi R G m d}{3} \quad (A)$$

(2)

$$g' = \frac{g}{4} \Rightarrow \frac{\cancel{G M}}{(R+h)^2} = \frac{1}{4}\frac{\cancel{G M}}{R^2}$$

$$\therefore h = R \quad (B)$$

(3)




$$E_P = 0$$

$$\frac{\cancel{G M}}{x^2} = \frac{\cancel{G\, 81M}\ 81}{(60-x)^2}$$

$$\therefore 60 - x = 9x$$

$$\therefore x = 6R \quad (A)$$

(4) By Conservation of energy

$$\frac{1}{2} m_{reduced} v_{rel}^2 = \frac{G m_1 m_2}{d}$$

$$\frac{1}{2}\frac{\cancel{m_1 m_2}}{m_1+m_2} v_{rel}^2 = \frac{G \cancel{m_1 m_2}}{d}$$

$$\therefore v_{rel} = \sqrt{\frac{2G(m_1+m_2)}{d}} \quad (C)$$

(5) (0,0) (1,0) (2,0) (4,0) ... ∞

$$E_{origin} = \frac{G(1)}{(1)^2} + \frac{G(1)}{2^2} + \frac{G(1)}{4^2} + \dots \infty$$

$$= G\left(1 + \frac{1}{4} + \frac{1}{16} + \dots \infty\right) = \frac{4G}{3}$$

(B)

(6) $$F_A = \frac{G(M_1 + M_2)m}{r^2}$$

$$F_B = \frac{G M_1 m}{r^2}$$

$$F_C = 0$$

(D)

Only one option is correct!

① $V_e = \sqrt{\frac{2GM}{R}} = V_0$ given

$V_{planet(e)} = \sqrt{\frac{2G\,2M}{R/2}} = \sqrt{4\left(\frac{2GM}{R}\right)} = 2V_0$ (B)

② By CoE:

for A: $\frac{1}{2}\cancel{m}V_1^2 = \frac{GM\cancel{m}}{R}$ ∴ $V_1 = \sqrt{\frac{2GM}{R}}$

for B: $\frac{1}{2}\cancel{m}V_2^2 - \frac{GM\cancel{m}}{R} = -\frac{G\cancel{m}M}{7R}$

$\frac{V_2^2}{2} - \frac{GM}{R} - \frac{GM}{7R} = \frac{6GM}{7R}$ ∴ $V_2 = \sqrt{\frac{12GM}{7R}}$

$\frac{V_1}{V_2} = \sqrt{\frac{7}{6}}$ (D)

③ $\frac{\cancel{m}v_0^2}{\cancel{R}} = \frac{GM\cancel{m}}{R_0^{\cancel{2}}}$ ∴ $v_0^2 = \frac{GM}{R_0} \Rightarrow \left(\frac{\sqrt{2gR}}{2}\right)^2 = \frac{GM}{R_0}$

$\Rightarrow \frac{\cancel{2}gR}{\cancel{4}2} = \frac{GM}{R_0} \Rightarrow \frac{\cancel{GM}}{2R} = \frac{\cancel{GM}}{R_0}$ ∴ $h = R$ (B)

④ $\frac{g_1}{g_2} = \frac{GM}{(4R)^2} \Big/ \frac{GM}{(5R)^2} = \frac{25}{16}$ (B)

⑤ By CLM, $2\cancel{M}V_1 = \cancel{M}V_2$ ∴ $V_2 = 2V_1$

$\Rightarrow \int V_2\,dt = 2\int V_1\,dt$

$\Rightarrow d_2 = 2d_1$

Also, $d_1 + d_2 = H \Rightarrow d_1 = H/3 = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2H}{3g}} \quad (C)$$

(6) for $H_{max}$, COE

$$-\frac{GMm}{R} + \frac{1}{2} m \frac{4gR}{3} = -\frac{GMm}{H}$$

$$\Rightarrow -\frac{GMm}{R} + \frac{2m\cancel{R}}{3}\frac{GM}{R^{\cancel{2}}} = -\frac{GMm}{H}$$

$$\Rightarrow -\frac{GMm}{3R} = -\frac{GMm}{H} \quad \therefore H = 3R$$

So $H_{max}$ from surface $= 2R$

<u>Part 2</u> By COE

$$\frac{1}{2} m \frac{4gR}{3} - \frac{GMm}{R} = -\frac{GMm}{2R} + \frac{1}{2}mv^2$$ (9)

$$\Rightarrow \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{GM\cancel{m}}{\cancel{6R}3R} = \frac{1\cancel{m}v^2}{\cancel{2}} \quad \therefore v = \sqrt{\frac{gR}{3}}$$

(B)

(7) $V_o < V < V_e$ for an orbit

$$V_{net} = n\sqrt{\frac{GM}{R}} < \sqrt{\frac{2GM}{R}}$$

$$n < \sqrt{2} \quad (B)$$

(8) $U_i = -\frac{GMm}{R}$, for $r = 2R$

$$\frac{GM\cancel{m}}{(2R)^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{2R}}$$

$$v = \sqrt{\frac{GM}{2R}}$$

$$U_f = -\frac{GMm}{2R} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{2R} + \frac{1}{2}m\frac{GM}{2R}$$

$$U_f = \frac{GMm}{4R} - \frac{GMm}{2R} = -\frac{GMm}{4R}$$

So $E_{supply} = U_f - U_i = \frac{GMm}{R} - \frac{GMm}{4R}$

$$= \frac{3GMm}{4R} = \frac{3}{4}mRg = \frac{3}{4}mgR$$

(D)

(2) $\frac{(V_e)_e}{(V_e)_m} = \sqrt{\frac{2g_1R_1}{2g_2R_2}} = \sqrt{\frac{g_1R_1}{g_2R_2}}$

$$\cancel{\sqrt{\frac{M_1}{R_1}\frac{R_2}{M_2}}} = \sqrt{10\times 6} = \sqrt{60}$$

(D)

# Exercise # 1

(1) As $\vec{F} = 0$ then $\vec{V} = \text{const}$ (B)

(2) $g_{planet} = \frac{GM_p}{R_p^2} = \frac{G\left(\frac{4}{3}\pi R_p^3\right)(8\rho_e)}{R_p^2}$

$M_e = \frac{4}{3}\pi R_e^3 \rho_e \quad = \left(\frac{\cancel{\frac{4}{3}\pi} R_p^3\, G}{R_p^2}\right)\frac{8\, M_e}{\cancel{\frac{4}{3}\pi} R_e^3}$

$= \frac{(2R_e)^3\, G\, \cancel{4}\cancel{8}^{2} M_e}{\cancel{2}R_e(\cancel{2}R_e)\, R_e^3}$

$= \frac{16 R_e^3\, GM_e}{R_e^5} = 16\left(\frac{GM_e}{R_e^2}\right)$

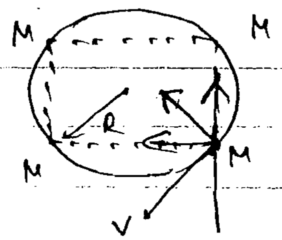
$g_{planet} = 16g$

So, $S = \frac{1}{2}gt^2 = \frac{1}{2}10(1)^2 = 5\text{ m}$

for Planet ! $S = \frac{1}{2}16gt^2 : t = \frac{1}{4}\text{ sec}$

(A)

(3)




$\frac{Mv^2}{R} = \frac{\sqrt{2}GM^2}{(\sqrt{2}R)^2} + \frac{GM^2}{4R^2}$

$\frac{\cancel{M}v^2}{\cancel{R}} = \frac{GM\cancel{^2}}{4R\cancel{^2}}\left(2\sqrt{2}+1\right)$

$v = \sqrt{\frac{GM}{R}\frac{(2\sqrt{2}+1)}{4}}$ (A)

(4) $$\frac{\cancel{G}M}{(R+h)^2} = \frac{1}{100}\frac{\cancel{G}M}{R^2}$$

$$\therefore 10R = R+h \therefore h = 9R \quad \text{(A)}$$

(5)

$$\text{Center of gravity} = \frac{x_1F_1 + x_2F_2}{F_1+F_2}$$

$$= \frac{\frac{GMm}{R^2}R + 0}{\frac{GMm}{R^2}} = R$$

$$\text{distance} = \frac{R}{2} \quad \text{(B)}$$

(6) $$\frac{GM\cancel{m}}{r^2} = \cancel{m}\omega_0^2 r \therefore \omega_0^2 r^3 = GM$$

$$g = \frac{GM}{R^2} = \left(\frac{\omega_0^2 r^3}{R^2}\right) \quad \text{(D)}$$

(7) $$F_1 = \frac{GMm}{9R^2} = \frac{Gm}{9R^2}\frac{4}{3}\pi R^3\rho$$

$$= \frac{4}{27}Gm\rho\pi R$$

$$F_2 = \frac{4}{27}Gm\rho\pi R - \frac{2}{75}G\pi\rho R m$$

$$= \frac{Gm\rho\pi R(100-18)}{27\times 25} = \frac{82(Gm\rho\pi R)}{27\times 25}$$

$$\frac{F_1}{F_2} = \frac{4}{\cancel{27}}\frac{\cancel{27}\times 25}{82} = \frac{50}{41} \therefore \frac{F_2}{F_1} = \frac{41}{50}$$

(B)

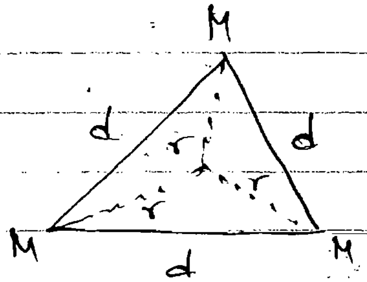
(8) $g_{planet} = \frac{G\,2M}{(2R)^2} = \left(\frac{GM}{R^2}\right)\frac{1}{2} = \frac{g}{2}$

$T = 2\pi\sqrt{\frac{L}{g/2}} = \sqrt{2} \times 2 = 2\sqrt{2}$ seconds (B)

(14)

(9) (D) from basic E/V formulae

(10)




$F_{net} = \frac{2GM^2}{d^2}\cos 30$ (15)

$= \frac{Mv^2}{d/\sqrt{3}}$

$\left(\text{Put } r = \frac{d}{\sqrt{3}}\right)$

$\frac{\cancel{2}GM^{\cancel{2}}\cancel{\sqrt{3}}}{d^{\cancel{2}}\,\cancel{2}} = \frac{\cancel{M}v^2\cancel{\sqrt{3}}}{\cancel{d}}$ $v = \sqrt{\frac{GM}{d}}$

$\therefore d = \frac{GM}{v^2}$ (D)

(11) $\left(\frac{\cancel{GM}x}{R^3}\right)\cancel{m} = \frac{1}{4}\left(\frac{\cancel{GMm}}{4R^2}\right)$

$\frac{x}{R} = \frac{1}{16} \therefore x = \frac{R}{16}$ depth $= R - \frac{R}{16}$ (16)

$= \frac{15R}{16}$ (B)

(12) (C) $t = \pi/\omega = \frac{T}{2} = 12H$

(13) By COE : (17)

$-\frac{GM\cancel{m}}{\sqrt{2}R} = -\frac{GM\cancel{m}}{R} + \frac{1}{2}\cancel{m}v^2$ (18)

$\Rightarrow \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{\sqrt{2}R}$

$$\Rightarrow \frac{v^2}{2} = \frac{GM}{R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$$

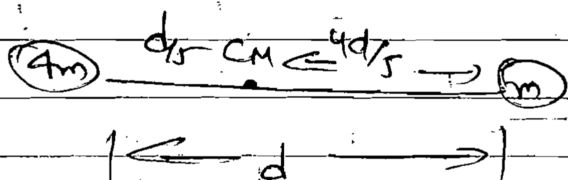
$$\therefore v = \sqrt{\frac{GM(2-\sqrt{2})}{R}} \quad (D)$$

(14) $(g - \omega^2 R_e) = \frac{1}{2}g \therefore \omega^2 R_e = \frac{g}{2}$ —(1)

$$V = \omega R_e \quad -(2)$$

$$V_e = \sqrt{2gR_e} = \sqrt{2 \times 2\omega^2 R_e^2}$$
$$= 2v \quad (A)$$

(15)




$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}I_1\omega_1^2}{\frac{1}{2}I_2\omega_2^2} = \frac{m_1\left(\frac{d}{5}\right)^2}{m_2\left(\frac{4d}{5}\right)^2} \quad (A)$$

(Here $\omega_1 = \omega_2$)

$$= \frac{4m}{m}\frac{1}{16} = \frac{1}{4}$$

(16) $\frac{V_A}{V_B} = \sqrt{\frac{\cancel{2}g_A R_A}{\cancel{2}g_B R_B}} = \sqrt{\frac{\cancel{G}M_A}{R_A}\frac{R_B}{\cancel{G}M_B}} = \sqrt{\frac{R_A^2}{R_B^2}} = \frac{R_A}{R_B}$

$$= 2 \quad (A)$$

(17) (D) Package will move circularly.

(18) By COE!

$$W_{ext} = \Delta K + \Delta U$$

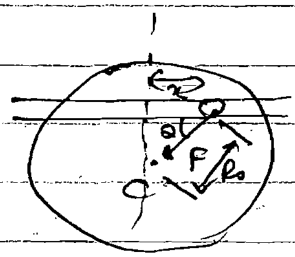
$$-3 = V \times 1 + \frac{1}{2}1(2)^2 \therefore V = -5\,J/kg$$

(C)

(19) $$E = \frac{\frac{GMm}{R}}{m} = \frac{GM}{R}$$ (23

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2E} \quad (A)$$

(20)




$$\cancel{F = F\cos\theta \quad F_{net} = \frac{GM}{R^3}(R_0^2+x^2)^{1/2} \times \frac{x}{R_0}}$$ (24

$$F_{net} = F\cos\theta$$
$$= \frac{GM\cancel{R_0}}{R^3} \times \frac{x}{\cancel{R_0}} = \frac{GMx}{R^3}$$

$$\boxed{F \propto x} \quad (D)$$

(21) $$V_i = -\frac{GM}{R}, \quad V_f = -\frac{2GM}{R}$$

$$V_i - V_f = \frac{GM}{R} > 0 \quad \therefore V_i > V_f \quad (B)$$

(22) $$W = U_f - U_i$$ (25,
$$= \left(-\frac{G2m^2}{a} - \frac{G6m^2}{a} - \frac{G3m^2}{a}\right)$$
$$- \left(-\frac{G2m^2}{a} - \frac{G3m^2}{a} - \frac{G6m^2}{\sqrt{2}a}\right)$$
$$= -\frac{6Gm^2}{a} + \frac{6Gm^2}{\sqrt{2}a}$$
$$= -\frac{6Gm^2}{a}\left(1 - \frac{1}{\sqrt{2}}\right) \quad (C)$$

(23) For circular motion

$$V_C = (GM/R)^{1/2}$$

Due to collision $V_C < \sqrt{\frac{GM}{R}}$

So (C)

(24) By COE (Before st)

$$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$V = \sqrt{\frac{GM}{R}}$$

Next: (After colln)

$$-\frac{GMm}{3R/2} = -\frac{GMm}{R} + \frac{1}{2}mv_0^2$$

$$v_0 = \sqrt{\frac{2}{3}}\sqrt{\frac{GM}{R}} = \sqrt{\frac{2}{3}}\,V$$

So $e = \frac{v_0}{V} = \sqrt{\frac{2}{3}}$ (B)

(25) $\Delta U = \left(-\frac{GMm}{4R}\right) - \left(-\frac{GMm}{R}\right)$

$$= \frac{GMm}{R} - \frac{GMm}{4R} = \frac{3GMm}{4R} \times \frac{R}{R}$$

$$= \frac{3}{4}\left(\frac{GM}{R^2}\right)mR = \frac{3}{4}mgR \quad \text{(B)}$$

(26) $\frac{GMm}{r^2} = m\omega^2 r$ — (I) (30)

$\frac{GMm}{(2r)^2} = m\omega_1^2 2r$ — (II)

(I) / (II) (31)

$4 = \frac{\omega^2}{2\omega_1^2} \therefore \omega_1 = \sqrt{\frac{\omega^2}{8}}$

$\therefore \omega_1 = \frac{\omega}{2\sqrt{2}}$ (A)

$\frac{V_1}{V_2}$ :

(27) (C) Kepler's Second law

(28) $\therefore$

$T^2 \propto R^3$

$\frac{T_1}{T_2} = \left(\frac{R}{1.02R}\right)^{3/2}$ (32)

$T_2 = T_1(1.02)^{3/2} = T_1(1+0.03)$

$\% = \frac{T_2 - T_1}{T_1} \times 100 = 3\%$ (B)

(29) $V_{initial} = \sqrt{\frac{GM_0}{R}}$ $P_i = 5MV_{initial}$

$P_f = -MV_M + 4MV_{4M}$

By CLM $\cancel{4M}V_{4M} = \cancel{M}V_M + 5\cancel{M}V_i$ (33)

$\therefore \cancel{M}V_M + 4MV_{4M} = 5\cancel{M}V_i$

for M: $V_M = \sqrt{\frac{GM_0}{R}} = V_i$ $\therefore V_{4M} = 1.5V_i = 1.5\sqrt{\frac{GM_0}{R}}$

So $V_{4M} > V_{escape}$ $\therefore$ (B)

(30) $L = mvR$

$= m\omega R^2$

$= m\dfrac{2\pi R^2}{T} = \dfrac{2mA}{T}$ (A)

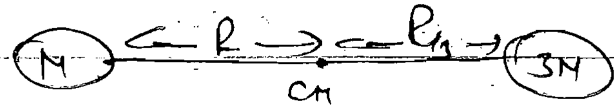
(31) 1st orbit: $\dfrac{\cancel{m}v^2}{\cancel{R_1}} = \dfrac{G\cancel{m}M}{R_1^{\cancel{2}}}$ $v_1 = \sqrt{\dfrac{GM}{R_1}}$

2nd orbit: $\dfrac{\cancel{m}v^2}{\cancel{R_2}} = \dfrac{GM\cancel{m}}{R_2^{\cancel{2}}}$ $\therefore v_2 = \sqrt{\dfrac{GM}{R_2}}$

$\dfrac{v_1}{v_2} = \dfrac{1}{2} = \sqrt{\dfrac{R_2}{R_1}}$ $\dfrac{T_1}{T_2} = \left(\dfrac{\cancel{2\pi}R_1}{v_1}\right)\left(\dfrac{v_2}{\cancel{2\pi}R_2}\right) = \left(\dfrac{R_1}{R_2}\right)\left(\dfrac{v_2}{v_1}\right)$

$\therefore \dfrac{R_1}{R_2} = 4$ $\dfrac{T_1}{T_2} = 4 \times 2 = 8 \quad \therefore T_2 = \dfrac{T}{8}$ (D)

(32)




$$\frac{GM3\cancel{M}}{(4R/3)^2} = \cancel{M}\omega^2 R$$

$$\omega = \sqrt{\frac{9GM\times 3}{16R^3}} = \sqrt{\frac{27GM}{16R^3}} = \frac{2\pi}{T}$$

$$\therefore T = 2\pi\sqrt{\frac{16R^3}{27GM}} \quad \text{(D)}$$

(33) $E_1 = U_1 + K_1$

$= -\dfrac{GMm}{2R_e} + \dfrac{1}{2}mv_1^2$

$= -\dfrac{GMm}{2R_e} + \dfrac{1}{2}\dfrac{GMm}{2R_e}$

$= \dfrac{GMm}{4R_e} - \dfrac{GMm}{2R_e} = -\dfrac{GMm}{4R_e}$

$E_f = U_f + K_f$

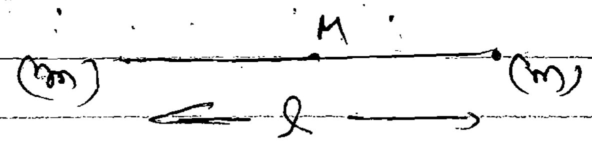
$E_f = -\frac{GMm}{6R_e}$

$\Delta E = E_f - E_i$ ①

$= \frac{GMm}{4R_e} - \frac{GMm}{6R_e} = \frac{GMm}{12R_e}$

(D) ②

(34)



$P.E = -\frac{GMm}{l/2} \times 2 = -\frac{4GMm}{l}$

$\frac{1}{2}\cancel{M}v_e^2 = \frac{4G\cancel{M}m}{l}$

$\therefore v_e = \sqrt{\frac{8Gm}{l}} = 2\sqrt{\frac{2GM}{l}}$

(D)

(35) By COE ③

$-\frac{GMm}{R} = -\frac{3GMm}{2R} + \frac{1}{2}mv^2$ ④

$\Rightarrow \frac{3GMm}{2R} - \frac{2GMm}{2R} = \frac{1}{2}mv^2$

$\Rightarrow \frac{GM\cancel{m}}{\cancel{2}R} = \frac{1}{\cancel{2}}\cancel{m}v^2$

$\therefore v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{2GM}{R}}/\sqrt{2}$

$= v_e/\sqrt{2}$ (B)

# Exercise # II

(1) $g(\text{in}) = \frac{GMr}{R^3}$ $g(\text{out}) = \frac{GM}{r^2}$

(A) (D)

(2) For 1st : $-\frac{GMm}{R} + \frac{1}{\cancel{2}} m \frac{\cancel{2}gR}{3}$

$= -\frac{GMm}{H}$

$\Rightarrow -\frac{GMm}{R} + \frac{mR}{3}\frac{GM}{R^2} = -\frac{GMm}{H}$

$\Rightarrow -\frac{\cancel{GMm}}{R} + \frac{\cancel{GMm}}{3R} = -\frac{\cancel{GMm}}{H}$

$\Rightarrow +\frac{2}{3R} = +\frac{1}{H}$ & $H = \frac{3R}{2}$

$R + h_1 = H$ $\therefore h_1 = \frac{R}{2}$ Similarly $h_2 = R$

$h_3 = 2R$

(C) (D)

(3) $V = -\frac{GM}{R}$ so (A), (B), (C)

(4) $\frac{V_1}{V_2} = \sqrt{\frac{2g_1R_1}{2g_2R_2}} = \sqrt{\frac{R_2}{R_1}} = \frac{1}{\sqrt{2}}$ (D)

As m same so $\cancel{8}\frac{4}{\cancel{3}}\pi R_1^3 = 8\frac{4}{3}\pi R_2^3$

$\frac{g_1}{g_2} = \left(\frac{R_2}{R_1}\right)^2 = \frac{1}{4}$ (B) $R_1 = 2R_2$

3)

(5) (B) (D)

(6) $U = -\frac{GMm}{r}$

$U_i = -\frac{GMm}{r_1}$, $U_f = -\frac{GMm}{r_2}$

$U_f - U_i = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$

(A)

$V_{orbital} = \sqrt{\frac{GM}{R}}$ As $R\uparrow$ $V_{orbital}\downarrow$

$\omega = \frac{V}{R} \Rightarrow$ as $R\uparrow$ $V\downarrow$ $\omega\downarrow$

$a_c = \frac{V^2}{R}$ as $V\downarrow$ $R\uparrow$ $a_c\downarrow$

so (B) (C) (D) are wrong

(7) $T^2 \propto R^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{32}{256}\right)^2 = \left(\frac{x}{R_2}\right)^3$

$\Rightarrow \frac{1}{64} = \left(\frac{x}{R_2}\right)^3$ ∴ $R_2 = 4x$

(B)

$E = -\frac{GMm}{r}$ so (C)

(8) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{1}{4}\right)^3$

$\frac{T_1}{T_2} = \frac{1}{8}$ (A)

$V = \omega R \propto \frac{R}{T}$

(B)

$\frac{V_1}{V_2} = \left(\frac{R_1}{T_1}\right)\left(\frac{T_2}{R_2}\right) = \frac{1}{4} \times \frac{8}{1} = 2$

$$\frac{L_1}{L_2} = \frac{\cancel{m_1} v_1 R_1}{\cancel{m_2} v_2 R_2} = \frac{2}{1} \times \frac{1}{4} = \frac{1}{2}$$

(C) Wrong

(9) (A)

(10) $v \propto \frac{1}{\sqrt{r}}$ (A) Correct $T^2 \propto R^3$ (B) Correct

$T.E = -\frac{GMm}{2r}$ (C) is Correct

# Exercise III

(1) $t_1 > t_2$ (B)

In region $V_{(ADB)} > V_{(ACB)}$

(2) $\frac{m v_1^2}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}$ $\therefore \frac{1}{2} m v^2 = \left(\frac{GMm}{2r}\right)$

$\therefore K = \frac{|U|}{2}$ $\therefore |U| = 2K$

(C)

(3) $F = \frac{GM\cancel{m}\cancel{x}}{R^3} = \cancel{m}\omega^2\cancel{x}$ : $\omega = \sqrt{\frac{GM}{R^3}} = \frac{2\pi}{T}$

$\therefore T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

$T_{required} = \frac{T}{2} = \pi\sqrt{\frac{R}{g}} = 3.14\sqrt{\frac{6400\,000}{10}}$

$\therefore T_{req} = \frac{3.14 \times 800}{60}$ min

$= 42$ min (B)

(4) By CAM: $v_1 r_1 = v_2 r_2$

$V_p > V_a$ (B)

(5) $V_{orbit} = \sqrt{\frac{GM}{r}}$

As P.E. Cont so circular orbit

$V_{new} = \frac{V_0}{\sqrt{2}}$ (B)

(6) (C)

(7) $TE = -\frac{GMm}{2r} = -\left(\frac{1}{2}\right) mv^2$ (A)

(8) $Ma_1 = ma_2$

$\frac{a_1}{a_2} = \frac{m}{M}$ (B)

(9) $KE = \frac{mv^2}{2} = +\frac{GMm}{2r}$

$PE = -\frac{GMm}{r}$

$TE = K + P = -\frac{GMm}{2r}$

$v_0 = \sqrt{\frac{GM}{r}}$

(A) → (iv) (B) → (iii) (C) → (i)

(D) → (ii)

(10) (A) → (iii) (B) → (i) (C) → (iv) (D) → (ii)

(11) (A)

(12) (C)

(13) (B)

(14) (B) $g_{pole} = g$, $g_{equator} = g - \omega^2 R$ $\left(\omega = \frac{2\pi}{T};\ T^2 \propto R^3\right)$

(15) (D)

## Exercise IV

(1) $$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\frac{G\cancel{m}^2}{4r^2} = \frac{\cancel{m}v^2}{\cancel{r}} \quad \therefore v = \sqrt{\frac{Gm}{4r}}$$

(2) $$F = \int_{r}^{r+l} \frac{GM}{x^2}\,\frac{m}{l}\,dx = \frac{GMm}{l}\left(\frac{1}{r} - \frac{1}{r+l}\right)$$

$$= \frac{GMm}{\cancel{l}}\,\frac{\cancel{l}}{r(r+l)}$$

$$F = \frac{GMm}{r(r+l)}$$

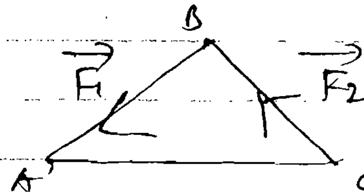
(3) $$U = -\int_{a-l/2}^{a+l/2} \frac{Gm}{x}\,\frac{m}{l}\,dx$$

$$U = -\frac{Gm^2}{l}\ln\left(\frac{a+l/2}{a-l/2}\right)$$

$$U = -\frac{Gm^2}{l}\ln\left(\frac{2a+l}{2a-l}\right)$$

$$U = \frac{Gm^2}{l}\ln\left(\frac{2a-l}{2a+l}\right).$$

(4)




$$\vec{F_1} = \frac{G\left(\frac{4\pi \cancel{R^3}\rho}{3}\right) r}{\cancel{R^3}}$$

$$\vec{F_1} = \frac{4G\rho\,\vec{r}_A}{3}$$

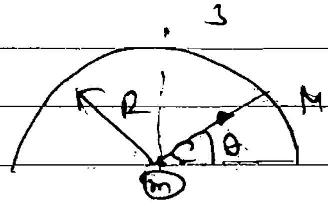
$$\vec{F_2} = \frac{G\frac{4}{3}\pi \cancel{r^3}\rho}{\cancel{r^3}}\,\vec{r_0}$$

$$\vec{F_2} = \frac{4G\pi\vec{r_0}\rho}{3}$$

$$F_{net} = \vec{F_1}+\vec{F_2}$$
$$= \frac{4G\pi l(\vec{r_0}+\vec{r})}{3} = \frac{4G\pi\rho l}{3}$$

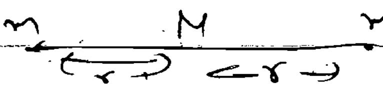
$$F_{net} = \frac{4\pi G\rho l}{3}\ \overleftarrow{r_2}$$

(5)




$$F = \int_0 2\frac{G}{R^2}\left(\frac{M\,d\theta}{\pi}\right) m\sin\theta$$
$$= \frac{2GMm}{\pi R^2}$$

(6)




$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^2} + \frac{Gm^2}{4r^2}$$

$$v^2 = \frac{GM}{r} + \frac{Gm}{4r}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}+\frac{Gm}{4r}}}$$

$$= \frac{2\pi r}{\sqrt{\frac{G(4M+m)}{4r}}} = \frac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$$

(7)

$$F = \int_d^{\infty} \frac{Gm}{x^2}\,\frac{\lambda}{x}\,dx = \frac{G\lambda m}{-2}\left[\frac{1}{x^2}\right]_d^{\infty}$$

$$= \frac{G\lambda m}{2d^2}$$

(8)

$$F_{net} = \frac{2}{3}\pi G m \rho R \quad [Q(4)]$$

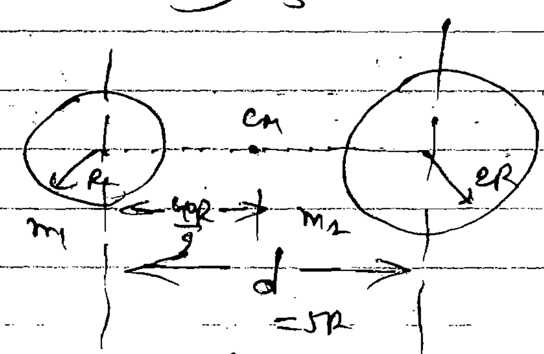
$$a = \frac{2}{3}\pi G \rho R$$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + \cancel{2}\,\frac{2}{3}\pi G\rho R\,\frac{R}{\cancel{2}}$$

$$v = \sqrt{\frac{2\pi G \rho R^2}{3}}$$

(9)




$$m_1 = \frac{4}{3}\pi R^3 \rho = m_0$$

$$m_2 = 8\left(\frac{4}{3}\pi R^3 \rho\right) = 8m_0$$

$$\frac{G \cancel{m_1} m_2}{(5R)^2} = \cancel{m_1}\,\omega^2\,\frac{40R}{9}$$

$$\omega = \left(\frac{9Gm_2}{1000R^3}\right)^{1/2} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{\frac{1000R^3}{9GM_P}}$$

$$= 2\pi\sqrt{\frac{1000\cancel{R^3}}{9G\,8\times\frac{4\pi\cancel{R^3}\rho}{3}}}$$

$$= 2\pi\sqrt{\frac{1000}{3G\rho(32\pi)}}$$

$$= \sqrt{\frac{1000\times\cancel{4}\pi\times\cancel{\pi}}{3G\rho\times\cancel{32\pi}}} = \sqrt{\frac{125\pi}{3G\rho}}$$

$$\therefore T = 5\sqrt{\frac{5\pi}{3G\rho}}$$

(10) $R_{max} = \frac{V_0^2}{g}$ $\therefore V_0 = \sqrt{bg}$

let radius of planet $= R$

$$M_{planet} = \frac{4}{3}\pi R^3 \rho_e$$

$$\therefore M_{planet} = \frac{\cancel{4}\cancel{\pi}R^3}{\cancel{3}}\left(\frac{M_e}{\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}R_e^3}\right) = \frac{2M_eR^3}{R_e^3}$$

$$(V_e)_{planet} = \sqrt{\frac{2GM}{R}} = V_0 = \sqrt{bg}$$

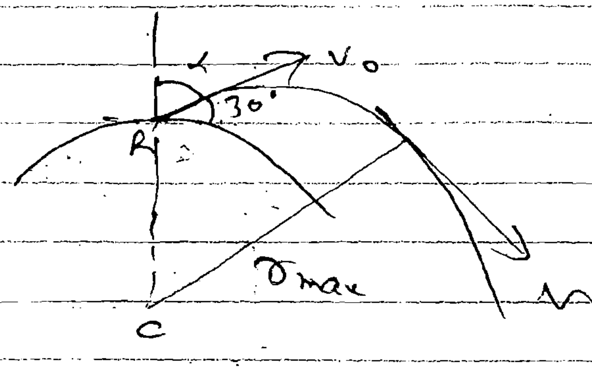
$$\frac{2\cancel{G}M}{R} = b\frac{\cancel{G}M_e}{R_e^2}$$

$$R = \frac{2MR_e^2}{M_eb} = \frac{2\cancel{R_e^2}}{\cancel{M_e}b}\times\frac{2\cancel{M_e}R^3}{\cancel{R_e^2}R_e}$$

$$R = \frac{4R^3}{bR_e} \quad \therefore R = \frac{1}{2}\sqrt{bR_e}$$

$$R = \sqrt{6.4}\ \text{km}$$

(11)




**By CAM about C;**

$$\cancel{m} v_0 \sin\alpha\, R = \cancel{m} v\, r_{max}$$

$$v_0 \sin\alpha\, R = v\, r_{max} \quad \text{—(1)}$$

**By COE!**

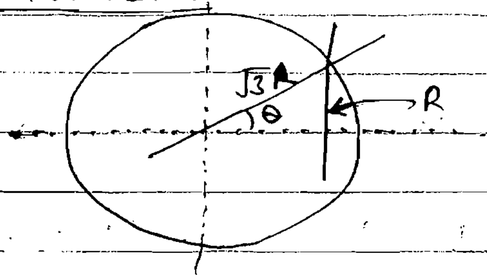
$$\frac{1}{2} m v_0^2 - \frac{GMm}{R} = \frac{1}{2} m v^2 - \frac{GMm}{r_{max}}$$

$$\frac{1}{2} m\left(\frac{1.5\, GM}{R}\right) - \frac{GMm}{R} = \frac{1}{2} m\left(\frac{v_0 \sin 60\, R}{r_{max}}\right)^2 - \frac{GMm}{r_{max}}$$

Solving $r_{max} = \frac{\sqrt{7}R}{2} + 2R$

So $h_{max}$ (from surface) $= \left(\frac{\sqrt{7}}{2} + 1\right) R$

(12) Projection method:




$$V = \omega (A^2 - x^2)^{1/2}$$

$$\frac{2GM}{R} = \omega^2 (A^2 - R^2) \quad \text{— (I)}$$

By COE

$$E_{center} = E_{surface}$$

$$\Rightarrow -\frac{3GMm}{2R} + \frac{1}{2}mv^2 = \frac{1}{2} \times 2gR - \frac{GMm}{R}$$

$$\therefore V = \sqrt{\frac{3GM}{R}} = A\omega \quad \text{— (II)}$$

By (I) & (II)

$$\frac{2\cancel{GM}}{\cancel{R}} = \frac{3\cancel{GM}}{\cancel{R}A^2}(A^2 - R^2)$$

$$2 = \frac{3}{A^2}(A^2 - R^2) \Rightarrow 2A^2 = 3A^2 - 3R^2$$

$$\therefore A = \sqrt{3}R$$

$$A\omega = \sqrt{\frac{3GM}{R}}$$

$$\sqrt{3}R\omega = \sqrt{\frac{3GM}{R}}$$

$$\omega = \sqrt{\frac{\cancel{3}GM}{\cancel{3}R^3}} = \sqrt{g/R}$$

from fig 1 $\sin\theta = \frac{1}{\sqrt{3}}$

$$\omega t = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore t = \sqrt{\frac{R}{g}}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

# IITJEE QUESTIONS

## Objective Question (I)

① $g = \frac{GM}{R^2}$ $\therefore g \propto \frac{1}{R^2}$ (M const)

ⓒ

② $U_f = -\frac{GMm}{2R}$

$U_i = -\frac{GMm}{R}$

$U_f - U_i = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$

$\Delta U = \frac{1}{2}\left(\frac{GM}{R^2}\right)mR = \frac{1}{2}mgR$ ⓐ

③ $\frac{GM\cancel{m}}{R^{5/2}} = \cancel{m}\omega^2 R$

$\omega = \sqrt{\frac{GM}{R^{7/2}}} = \frac{2\pi}{T}$

$T = 2\pi\sqrt{\frac{R^{7/2}}{GM}}$ $\therefore T^2 \propto R^{7/2}$ ⓑ

④ $T^2 \propto r^3$

$T \propto r^{3/2}$

$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$ $\therefore T_2 = \left(\frac{365}{2\sqrt{2}}\right)$

$T_2 = 129$ ⓑ

(5) (a)

(6) (c) $T \propto \frac{1}{\sqrt{g}}$ ∴ $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{R^3}{4R^2}} = \frac{1}{2}$ ∴ $\frac{T_2}{T_1} = 2$ (d)

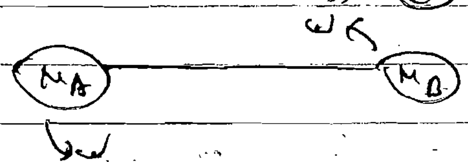
(7) $T_{\text{very close to earth}} = 2\pi\sqrt{\frac{R}{g}} = 84$ min

As particle few km above surface

$T > 84.6$ min

so (c)

(8)




$\omega_A = \omega_B$ so $T_A = T_B$ (d)

(9) $\frac{GM\cancel{m}}{R^3} r = \frac{\cancel{m} v^2}{r}$ ∴ $v^2 \propto r^2$

$v \propto r$ $\quad r \le R$

so (c)

Assertion & Reason:

(1) (A)

## Objective Question II :

(1) By Symmetry $E = 0$ (a)

(c) & (d)

As on yz plane all circle have equal potential.

(2)
$$F_1 = \frac{GM}{r_1^2}$$

$$F_2 = \frac{GM}{r_2^2}$$

$$\frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2 \qquad r_1 > R \quad r_2 > R$$

(b)

Also $f_1 = \frac{GMm\, r_1}{R^3}$

$$f_2 = \frac{GMm\, r_2}{R^3} \qquad \frac{f_1}{f_2} = \frac{r_1}{r_2} \quad (a)$$

Fill in the blanks:

① $g = \omega^2 R$ $\quad \omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6400\times10^3}}$

$\therefore \omega = 1.24\times10^{-3}$ rad/sec

② Angular momentum

③ $T^2 \propto r^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{7R}{3.5R}\right)^3$

$\therefore T_2 = \frac{T_1}{2\sqrt{2}} = \frac{24}{2\sqrt{2}} = 8.48H$

④ $\frac{1}{2}\cancel{m}v_e^2 = \left(\frac{GM_1\cancel{m}}{d/2} + \frac{GM_2 m}{d/2}\right)$

$\frac{v_e^2}{2} = \frac{2G(M_1+M_2)}{d}$ $\therefore v_e = 2\sqrt{\frac{G(M_1+M_2)}{d}}$

⑤ $\frac{dA}{dt} = \frac{L}{2m}$ $\therefore A = \frac{L}{2m}T$

$= \frac{1}{2}\times4.4\times10^{+15}\times365\times86400$

$= 6.94\times10^{22}\,m^2$

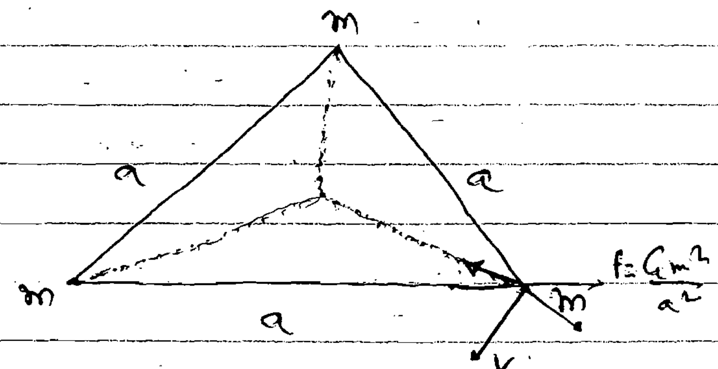
⑥ By COE

$-\frac{GMm}{R} + \frac{1}{2}\left(\frac{1}{2}mv_e^2\right) = -\frac{GMm}{H}$

$\therefore H = 2R$ $\therefore h_{surface} = R$

## Analytical & Descriptive questions

(1)




$\therefore r = a/\sqrt{3}$

$$F_{net} = 2F\cos 30 = \frac{\cancel{2}Gm^2}{a^2}\frac{\sqrt{3}}{\cancel{2}} = \frac{mv^2}{a/\sqrt{3}}$$

$$\frac{Gm^{\cancel{2}}\cancel{\sqrt{3}}}{a^{\cancel{2}}} = \frac{\cancel{m}v^2\cancel{\sqrt{3}}}{\cancel{a}} \quad \therefore v = \sqrt{\frac{Gm}{a}}$$

$$T = \frac{2\pi r}{v} = 2\pi\frac{a}{\sqrt{3}} \Big/ \sqrt{\frac{Gm}{a}}$$

$$= 2\pi\sqrt{\frac{a^3}{3Gm}}$$

(2) (a) $$v_o = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{GM}{R+h}} = \frac{v_e}{2}$$

$$\Rightarrow \sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{2gR}$$

$$\Rightarrow \frac{\cancel{GM}}{R+h} = \frac{1}{\cancel{4}2}\cancel{2}\frac{\cancel{GM}}{R} = \frac{1}{2R}$$

$h = R = 6400\,km$

(b) By COE

$$-\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$$

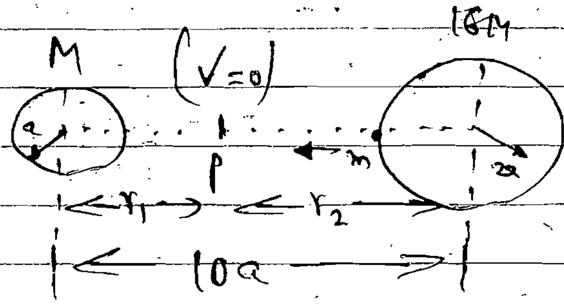
$$\therefore v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

$$= \sqrt{9.8 \times 6400 \times 10^3}$$

$$= 7919 \text{ m/s}$$

$$= 7.9 \text{ km/sec}$$

(3)




$E_p = 0$ ; $\frac{\cancel{G}M}{r_1^2} = \frac{\cancel{G}\,16M}{r_2^2} \Rightarrow r_2 = 4r_1$

$r_1 + r_2 = 10a$

$\therefore r_1 = 2a,\ r_2 = 8a$

If Particle reach P then after P $F_M > F_{16M}$
so particle will reach its destination.

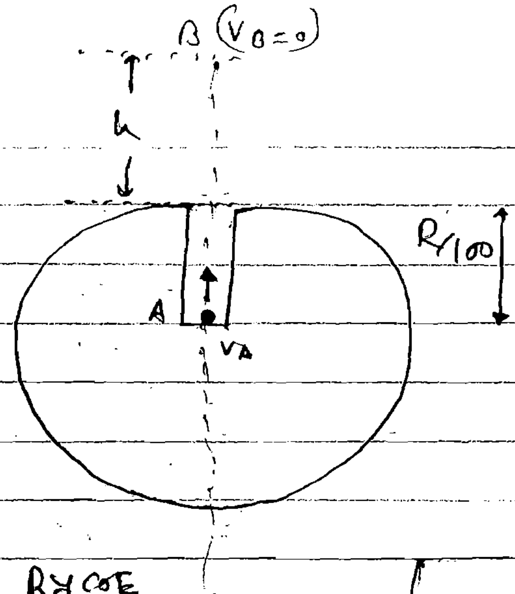
By COE,

$$\frac{1}{2}mv^2_{min} + \left[-\frac{G\,16Mm}{2a} - \frac{GMm}{8a}\right]$$

$$= -\frac{GMm}{r_1} - \frac{16GMm}{r_2}$$

$$\frac{1}{2}mv^2_{min} = \frac{45GMm}{8a} \Rightarrow v_{min} = \sqrt{\frac{45GM}{4a}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

(4)




BY COE

$$v_A = \sqrt{2GM/R}$$

$$\frac{1}{2}\cancel{m}v_A^2 - \frac{GM\cancel{m}}{R^3}\left(1.5R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right) = -\left(\frac{GM\cancel{m}}{R+h}\right)$$

$$\frac{GM}{R} - \frac{GM}{R^3}\left(1.5R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right) = -\frac{GM}{R+h}$$

$$\therefore \boxed{h = 99.5R}$$