LEVEL 1

1. $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

$\Rightarrow \sqrt[4]{3^4}-8\sqrt[3]{6^3}+15\sqrt[5]{2^5}+\sqrt{15^2}$

$\Rightarrow 3-48+30+15$

$\Rightarrow$ 0

2. $\sqrt{8}\cdot\sqrt[3]{4}+\sqrt[3]{625}\cdot\sqrt[4]{125}$

$\Rightarrow (2^{3/2}.2^{2/3}+5^{4/3}.5^{3/4}$

$\Rightarrow 2^{13/6}+5^{25/12}$

3. $x+3|x|=10$

When x>0	when x<0
$\Rightarrow$x+3x=10	$\Rightarrow$ x-3x=10
$\Rightarrow$x=5/2	$\Rightarrow$x=-5

4. let the no. be (x) , (x + 1) ,(x + 2).

when a number is divided by 3, the remainder obtained is 0 or 1 or 2.

therefore,

x = 3n or (3n + 1) or (3n + 2)

if x = 3n, then x is divisible by 3

if x = 3n + 1 ,then x + 2 = 3n + 1 + 2 = 3n + 3

=> x = 3(n + 1) is divisible by 3

if x = 3n + 2, then x + 1 = 3n + 2 + 1

=> 3n + 3 = 3(n + 1)

so, we can say that one of the numbers n, n + 1 and n + 2 is always divisible by 3.

n (n + 1) (n + 2) is divisible by 3.

now,

similarly, when a no. is divisible by 2 remainder obtained is 0 or 1.

therefore,

x = 2r or (2r + 1)

if x = 2r, then x = 2r and (x + 2) then,2r + 2

=> 2(r + 1) are divisible by 2

if x = (2r + 1), then x + 1 = 2r + 1 + 1 = 2(r + 1)is divisible by 2.

So,we can say that one of the numbers among x, x + 1 and x + 2 is always divisible by 2.

x (x + 1) (x + 2) is divisible by 2.

n (n + 1) (n + 2) is divisible by 2 and 3.

Therefore, n (n + 1) (n + 2) is divisible by 6.

5. $\dfrac{1}{1+x^{a-b}}+\dfrac{1}{1+x^{b-a}}$

$\Rightarrow \frac{1}{1+\frac{x^b}{x^a}}+\frac{1}{1+\frac{x^b}{x^a}}$

$\Rightarrow \frac{x^a}{x^b+x^a}+\frac{x^a}{x^b+x^a}$

$\Rightarrow 1$

6. $2^x = 3^y = 6^{-z}$ =k

$\Rightarrow 2 = k^{1/x}, 3 = k^{1/y}, 6 = k^{1/z}$

$\Rightarrow 2\ X\ 3 = k^{1/x}.k^{1/y} = k^{(1/x+1/y)}$

$\Rightarrow 6 = k^{1/z} = k^{1/x+1/y}$

$\Rightarrow \frac{1}{x} = \frac{1}{y} + \frac{1}{z}$

7. $25^{x-1} = 5^{2x-1} - 100$

$\Rightarrow 5^{2x-2} = 5^{2x-1} - 100$

$\Rightarrow 100 = \frac{a}{5} - \frac{a}{25}$ $\quad [a = 5^{2x}]$

$\Rightarrow 100 = \frac{4a}{25}$

$\Rightarrow a = 625 = 5^4$

$\Rightarrow 5^{2x} = 5^4$

$\Rightarrow x = 2$

8. $\frac{100\sqrt{25}}{\sqrt{25}+x} = 50$

$\Rightarrow \frac{500}{5+x} = 50$

$\Rightarrow 10 = x + 5\ ,\ x = 5$

9. $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$

$\Rightarrow (x^{(b-c)\cdot\frac{1}{bc}}\ x^{(c-a)\frac{1}{ca}}\ x^{(a-b)\frac{1}{ab}}$

$\Rightarrow x^{\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}+\frac{1}{b}-\frac{1}{a}}$

$\Rightarrow x^0 = 1$

10. $18^{4x-3} = \left(54\sqrt{2}\right)^{3x-4}$

$\Rightarrow 3^{8x-6}\ X\ 2^{4x-3} = 3^{9x-12} X 2^{3(3x-4)/2}$

When bases are same then powers are equal

$\Rightarrow 8x - 6 = 9x - 12, x = 6$

11. **Step 1**: Divide $(176)_{10}$ successively by 2 until the quotient is 0:

176/2 = 88, remainder is 0

88/2 = 44, remainder is 0

44/2 = 22, remainder is 0

22/2 = 11, remainder is 0

11/2 = 5, remainder is 1

5/2 = 2, remainder is 1

2/2 = 1, remainder is 0

1/2 = 0, remainder is 1

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Step 2: Read from the bottom (MSB) to top (LSB) as 10110000.
So, 10110000 is the binary equivalent of decimal number 176 (Answer).

12. Divide $(472)_{10}$ successively by 8 until the quotient is 0:
472/8 = 59(quotient), remainder is 0
59/8 = 7(quotient) , remainder is 3
7/8 = 0(quotient), remainder = 7
So, 730 is the octal equivalent to the decimal number 472 (Answer).

13. $(523)_{10} = (20B)_{16}$

Divide $(523)_{10}$ successively by 16 until the quotient is 0:
523/16 = 32, remainder is 11(=B in hexadecimal system)
32/16 = 2, remainder is 0
2/16 = 0, remainder is 2
. So, 20B is the hexadecimal equivalent of decimal number 523 (Answer).

14. $(2776)_{10}$

Divide $(2776)_{10}$ successively by 12 until the quotient is 0
2776/12=231, remainder=4
231/12=19, remainder=3
19/12=1, remainder=7
1/12=0, remainder=1
So, $(1734)_{12}$ is the duodecimal equivalent of decimal number 2776.

15. **Step 1**: Look up each octal digit to obtain the equivalent group of three binary digits. You can use the table below to make these conversions.

Octal to Binary Conversion Table

O	0	1	2	3	4	5	6	7
Bin:	000	001	010	011	100	101	110	111

$(7)_8 = (111)_2$

$(4)_8 = (100)_2$

$(6)_8 = (110)_2$

$(4)_8 = (100)_2$

Step 2: Group each value of step 1 to make a binary number.
111 100 110 100

$(7464)_8 = (111100110100)_2$

Step 3: Now convert the binary number from step 2 to hexa by grouping all the digits of the binary in sets of four starting from the LSB (far right).

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1111 0011 0100
Note: add zeros to the left of the last digit if there aren't enough digits to make a set of four.
Step 4: Convert each group of four to the corresponding hexadecimal (use the table below),
1111=F, 0011=3, 0100=4.
So, the octal 7464 is equivalent to F34 in hexadecimal.

16. $(110001110)_2$

Group all the digits in sets of three starting from the right side.. Add zeros to the **left** of the last digit if there aren't enough digits to make a set of three.
110 001 110
. In this case,use binary conversion of grouping
110=6, 001=1, 110=6.
So, the number 616 is the octal equivalent to 110001110 in binary.

17. **Step 1**: Write down the binary number:

0001100111011011
Step 2: Group all the digits in sets of four starting from the LSB (far right). Add zeros to the left of the last digit if there aren't enough digits to make a set of four:
0001 1001 1101 1011
Step 3: Use the table below to convert each set of three into an hexadecimal digit:
0001 = 1, 1001 = 9, 1101 = D, 1011 = B
So, 19DB is the hexadecimal equivalent to the decimal number 1100111011011.

18. Decimal equivalent of(1101.0101)2
1101= $2^3 + 2^2 + 1 = 13$
.0101= $2^{-2} + 2^{-4} = 0.25 + 0.0625 = 0.3125$
Decimal equivalent of(1101.0101)2=13.3125

19.

Step 1: Write down the hexadecimal number:
$(BAD)_{16}$
Step 2: Show each digit place as an increasing power of 16:
$Bx16^2 + Ax16^1 + Dx16^0$
Step 3: Convert each hexadecimal digits values to decimal values then perform the math:
$11x256 + 10x16 + 13x1 = (2989)_{10}$
So, the number 2989 is the decimal equivalent of hexadecimal number BAD (Answer).

20. $(1101)_2 + (46)_8 + (97)_{10} =$

$(1 * 2^3 + 1 * 2^2 + 1\ ) + (4 * 8 + 6) + 97 = 148$

21. $2^8 = 256$

22. $(256)_{16} - (256)_8 =$

$\Rightarrow 2 * 16^2 + 5 * 16 + 6\ - (2 * 8^2 + 5 * 8 + 6)$

$\Rightarrow 424$

23. $(n)_{n+2} + (n-1)_{n+1} + (n-2)_n + ... + (1)_3 =$

All numbers in their bases are less than their bases in the system

n+(n-1)+(n-2)+.....+1

n(n+1)/2

24. $\dfrac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} \div \dfrac{\sqrt{a^2+b^2}-b}{a-\sqrt{a^2-b^2}}$

$\Rightarrow \frac{\sqrt{a^2-b^2}+a}{\sqrt{a^2+b^2}+b} X \frac{a-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-b}$

$\Rightarrow \frac{a^2-(a^2-b^2)}{(a^2+b^2)-b^2}$

$\Rightarrow \frac{b^2}{a^2}$

25. $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$\Rightarrow \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} X \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \text{ x } \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} X \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$

$\Rightarrow \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 = 1$

26. $\dfrac{1+\sqrt{48}}{5\sqrt{3}+4\sqrt{2}-\sqrt{72}-\sqrt{108}+\sqrt{8}+2} = a + b\sqrt{3}$

$\Rightarrow \frac{1+4\sqrt{3}}{5\sqrt{3}+4\sqrt{2}-6\sqrt{2}-6\sqrt{3}+2\sqrt{2}+2}$

Rationalise the denominator

$\Rightarrow \frac{1+4\sqrt{3}}{2-\sqrt{3}} X \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\Rightarrow \frac{2+\sqrt{3}+8\sqrt{3}+12}{1}$

$\Rightarrow 14 + 9\sqrt{3} = a + b\sqrt{3}$

a=14,b=9

27. $x = 7 + 4\sqrt{3},$

$\sqrt{x} = \sqrt{a} + \sqrt{b} = \sqrt{7+4\sqrt{3}}$

$\Rightarrow a + b + 2\sqrt{ab} = 7 + 2\sqrt{12}$

a+b=7,ab=12

a=4,b=3

$\sqrt{7+4\sqrt{3}} = \sqrt{4}+\sqrt{3} = 2+\sqrt{3}$

$\Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}}$

$\Rightarrow 2+\sqrt{3}+\frac{1}{2+\sqrt{3}}$

Rationalise the denominator

$\Rightarrow 2+\sqrt{3}+\frac{1}{2+\sqrt{3}} X \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow 2+\sqrt{3}+2-\sqrt{3} = 4$

28.

(i) $4\times(81)^{\frac{-1}{2}}\times[81^{\frac{1}{2}}+81^{\frac{3}{2}}]$

$\Rightarrow 4X(9^2)^{-\frac{1}{2}}X[9+729]$

$\Rightarrow 328$

(ii) $\frac{(36)^{7/2}-(36)^{9/2}}{(36)^{5/2}}$

$\Rightarrow 36-36^2 = 1260$

29. Use law of indices

30. $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}.\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}.\left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2.} = 1.$

L.H.S

$\Rightarrow (x)^{(a-b)(a^2+ab+b^2)}(x)^{(b-c)(b^2+bc+c^2)}(x)^{(c-a)(c^2+ca+a^2)}$

$\Rightarrow x^{a^3-b^3}x^{b^3-c^3}x^{c^3-a^3}$

$\Rightarrow x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$\Rightarrow 1$

31. $\sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x} = 1$

L.H.S

$\sqrt{\frac{y}{x}}\sqrt{\frac{z}{y}}\sqrt{\frac{x}{z}} = 1$

32. Use law of indices

33. (a) $\left(\frac{1}{7}\right)^{4-2x} = \sqrt{7}.$

$\Rightarrow 7^{2x-4} = 7^{1/2}$,x=9/4

(b) $(0.2)^{2x-1} = 1$

$\Rightarrow 2x-1 = 0, x = 1/2$

34. Irrational numbers between 0.5 & 0.55 are 0.5010010012398734……,0.546724356254378….

35. (a) $\sqrt[3]{2x-3}-4=0$

2x-3=64,x=67/2

(b) $\sqrt[5]{6x+2}=2$

6x+2=32

x=5

36. (a) $\sqrt[5]{2}$ or $\sqrt[15]{5}$

$\Rightarrow 2^{3/15} \quad 5^{1/15}$

$\Rightarrow 8^{1/15} > 5^{1/15}$

$\Rightarrow 2^{3/15} > 5^{1/15}$

(b) $\sqrt[3]{6}$ or $\sqrt[4]{7}$

$6^{4/12}, \quad 7^{3/12}$

$1296^{4/12} > 343^{3/12}$

$6^{4/12} > 7^{3/12}$

(c) $\sqrt[3]{31}$ or $\sqrt{10}$

$31^{2/6}, \quad 10^{3/6}$

$961^{1/6} < 1000^{1/6}$

$31^{2/6} < 10^{3/6}$

37. (a) $\frac{2}{5}$ and $\frac{3}{4}$

$\Rightarrow \frac{2}{5} = \frac{8}{20}, \frac{3}{4} = \frac{15}{20}$

Rational numbers are $\frac{9}{20}, \frac{10}{20}, \frac{11}{20}, \frac{12}{20}$

(b) $-\frac{2}{5}$ and $-\frac{1}{5}$

$\Rightarrow -\frac{2}{5} = -\frac{8}{20}, \frac{-1}{5} = \frac{-4}{20}$

Rational numbers are $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}$

(c) 0.1 and 0.2

Rational numbers are 0.11,0.12,0.13

38. (a) $\frac{6}{3\sqrt{2}-2\sqrt{3}}$

Rationalise the denominator

$\frac{6}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$\Rightarrow \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$

$\Rightarrow (3\sqrt{2}+2\sqrt{3})$

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(b) $\frac{3}{\sqrt{3}-\sqrt{2}}$

Rationalise the denominator

$\Rightarrow \frac{3}{\sqrt{3}-\sqrt{2}} X \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\Rightarrow \frac{3(\sqrt{3}+\sqrt{2}}{1}$

c) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$

$\Rightarrow \frac{3\sqrt{2}+1}{2\sqrt{5}-3} X \frac{2\sqrt{5}+3}{2\sqrt{5}+3}$

$\Rightarrow \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{17}$

39. (a) $\sqrt{5}, \sqrt[3]{11}, 2\sqrt[6]{3}$

$5^{3/6}, 11^{2/6}, 192^{1/6}$

$125^{1/6}, 121^{1/6}, 192^{1/6}$

$121^{1/6} < 125^{1/6} < 192^{1/6}$

$\sqrt[3]{11} < \sqrt{5} < 2\sqrt[6]{3}$

(b) $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$

$2^{6/18}, 3^{3/18}, 4^{2/18}$

$64^{1/18}, 27^{1/18}, 16^{1/18}$

$16^{1/18} < 27^{1/18} < 64^{1/18}$

$\sqrt[9]{4} < \sqrt[6]{3} < \sqrt[3]{2}$

40. $4\sqrt{48}$

$\Rightarrow 16\sqrt{3}$

$\sqrt{3}$ *is the rationalising factor of* $4\sqrt{48}$

41. $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$\Rightarrow \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} X \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} X \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} X \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$\Rightarrow 3\sqrt{2} - 2\sqrt{3} + 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6}$

$\Rightarrow 0$

LEVEL 2

1. $a=\frac{1}{3-2\sqrt{2}}, b=\frac{1}{3+2\sqrt{2}}$

ab= $\frac{1}{(3-2\sqrt{2})(3+2\sqrt{2})}$ = 1 ,a+b= $\frac{1}{3-2\sqrt{2}} X \frac{3+2\sqrt{2}}{3+2\sqrt{2}} + \frac{1}{3+2\sqrt{2}} X \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ =3+2 $\sqrt{2}$ + 3 – 2$\sqrt{2}$ = 6

a^2b+ab^2 .

=ab(a+b)

=6

2. Let x be a rational number = $\sqrt{3}-\sqrt{2}$

x= $\sqrt{3}-\sqrt{2}$

S.B.S

$\Rightarrow x^2 = 5 - 2\sqrt{6}$

L.H.S is a rational number but R.H.S is not a rational number.So, our assumption is wrong and hence, it contradicts the fact that x is a rational number.

$\Rightarrow$ *x is an irrational number*

3. (a) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

L.H.S

Rationalise the denominator

$\Rightarrow \frac{(3+\sqrt{2})(3+\sqrt{2})}{3^2-(\sqrt{2})^2}$

$\Rightarrow \frac{11+6\sqrt{2}}{7}$

$\Rightarrow a = 11/7, b = 6/7$

(b) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

L.H.S

Rationalise the denominator

$\Rightarrow \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{7^3-(4\sqrt{3})^2}$

$\Rightarrow \frac{35-20\sqrt{3}+14\sqrt{3}-24}{1}$

$\Rightarrow$ 11-6 $\sqrt{3}$

4.

Rationalise the denominator

$\Rightarrow a = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}, \quad b = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$

$\Rightarrow a = 5 + 2\sqrt{6},\ b = 5 - 2\sqrt{6}$

C.B.S

$\Rightarrow a^3 = 485 + 198\sqrt{6}\ ,\ b^3 = 485 - 198\sqrt{6}$

$a^3 + b^3 = 970$

5. $x = 3 + \sqrt{8}$

S.B.S

$\Rightarrow x^2 = 17 + 12\sqrt{2}$

$\Rightarrow x^4 = 587 + 408\sqrt{2}$ $\quad , \frac{1}{x^2} = \frac{1}{17+12\sqrt{2}} X \frac{17-12\sqrt{2}}{17-12\sqrt{2}} = \frac{17-12\sqrt{2}}{1}$

$\Rightarrow x^2 + \frac{1}{x^2} = 34$

S.B.S

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 1156$

$\Rightarrow x^4 + \frac{1}{x^4} = 1154$

6. $x = \dfrac{1}{2-\sqrt{3}}$

$\Rightarrow x = \frac{2+\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$

S.B.S

$\Rightarrow x^2 - 4x = -1$

now,

$x^3 - 2x^2 - 7x + 5 = 3$

L.H.S

$\Rightarrow x(x^2 - 4x) + 2x^2 - 8x + x + 5$

$\Rightarrow -x + 2(x^2 - 4x) + x + 5$

$\Rightarrow -x - 2 + x + 5 = 3$

L.H.S=R.H.S

7. $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots + \dfrac{1}{\sqrt{8}+\sqrt{9}}$

$\Rightarrow$ *Rationalise the denominator*

$\Rightarrow \frac{\sqrt{2}-1}{(\sqrt{2})^2-1} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \ldots\ldots. + \frac{\sqrt{9}-\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}$

$\Rightarrow \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \ldots\ldots + \sqrt{9} - \sqrt{8}$

$\Rightarrow \sqrt{9} - 1 = 2$

8. $x = \dfrac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

$\Rightarrow x = \frac{(\sqrt{a+2b}+\sqrt{a-2b})}{\sqrt{a+2b}-\sqrt{a-2b})} X \frac{(\sqrt{a+2b})+\sqrt{a-2b})}{(\sqrt{a+2b})+\sqrt{a-2b})}$

$\Rightarrow x = \frac{a+2b+a-2b+2\sqrt{a^2-4b^2}}{a+2b-a+2b}$

$\Rightarrow 4bx = 2a + 2\sqrt{a^2 - 4b^2}$

$\Rightarrow (2bx - a) = \sqrt{a^2 - 4b^2}$

$\Rightarrow 4b^2x^2 + a^2 - 4bxa = a^2 - 4b^2$
$\Rightarrow bx^2 - ax + b = 0$

9. On dividing,133098 by 11, we get 9 as remainder. So, 2 is the smallest number must be added to133098 to make it a multiple of 11.

10. $\Rightarrow y = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}, \quad x = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$
$\Rightarrow y = 5 + 2\sqrt{6},\ x = 5 - 2\sqrt{6}$
$\Rightarrow x^2 + xy + y^2.$
$\Rightarrow x(x+y) + (5+2\sqrt{6})^2$
$\Rightarrow (5-2\sqrt{6})10 + 49 + 20\sqrt{6}$
$\Rightarrow 99$

11. $a = 1+\sqrt{2}+\sqrt{3}$ *and* $b = 1+\sqrt{2}-\sqrt{3}$
a-1= $\sqrt{3}+\sqrt{2}$, b-1= $\sqrt{3}-\sqrt{2}$
S.B.S
$\Rightarrow a^2 - 2a + 1 = 5 + 2\sqrt{6}, b^2 - 2b + 1 = 5 - 2\sqrt{6}$
$\Rightarrow a^2 - 2a = 4 + 2\sqrt{6}, b^2 - 2b = 4 - 2\sqrt{6}$
$\Rightarrow a^2 - 2a + b^2 - 2b = 8$

12. $x = 2\sqrt{3}+\sqrt{2}$, $y = 2\sqrt{3}-\sqrt{2},$
$\Rightarrow x^2 = 14 + 4\sqrt{6}$, $y^2 = 14 - 4\sqrt{6}$
$\Rightarrow x^4 = 292 + 112\sqrt{6}$, $y^4 = 292 - 112\sqrt{6}$
$\Rightarrow (x^4 - y^4)^2$
$\Rightarrow (224\sqrt{6})^2$

13. $\sqrt{x^2+2y\sqrt{x^2-y^2}} + \sqrt{x^2-2y\sqrt{x^2-y^2}}$
$\Rightarrow \sqrt{(\sqrt{x^2-y^2})^2 + y^2 + 2y\sqrt{x^2-y^2}} + \sqrt{(\sqrt{x^2-y^2})^2 + y^2 - 2y\sqrt{x^2-y^2}}$
$\Rightarrow \sqrt{x^2-y^2} + y + (y - \sqrt{x^2-y^2})$ $or \sqrt{x^2-y^2} + y + (-y + \sqrt{x^2-y^2})$
$\Rightarrow 2y$ *or* $2\sqrt{x^2-y^2}$

14. $x = 4 + 4^{1/3} + 4^{2/3}$
$\Rightarrow x - 4 = 4^{1/3} + 4^{2/3}$
$\Rightarrow x^3 - 64 - 12x(x-4) = 4 + 16 + 12(x-4)$
$\Rightarrow x^3 - 12x^2 + 36x = 36$
Value of $x^3 - 12x^2 + 36x + 8 = 36 + 8 = 44$

15. $(1.761)^x = (0.1761)^y = 10^z = k$
$1.761 = k^{1/x}, 0.1761 = k^{1/y}, 10 = k^{1/z}$
$\Rightarrow (0.1761)10 = k^{1/y+1/z}$
$\Rightarrow 1.761 = k^{1/y+1/z} = k^{1/x}$
$\Rightarrow 1/y + 1/z = 1/x$

16. $\left(\sqrt[16]{a}+\sqrt[16]{\frac{1}{a}}\right)\left(\sqrt[16]{a}-\sqrt[16]{\frac{1}{a}}\right)\left(\sqrt[8]{a}+\sqrt[8]{\frac{1}{a}}\right)\ \left(\sqrt[4]{a}+\sqrt[4]{\frac{1}{a}}\right)\left(\sqrt{a}+\sqrt{\frac{1}{a}}\right)=1$

L.H.S

$\Rightarrow k\left(\sqrt[16]{a}+\sqrt[16]{\frac{1}{a}}\right)\left(\sqrt[16]{a}-\sqrt[16]{\frac{1}{a}}\right)\left(\sqrt[8]{a}+\sqrt[8]{\frac{1}{a}}\right)\ \left(\sqrt[4]{a}+\sqrt[4]{\frac{1}{a}}\right)\left(\sqrt{a}+\sqrt{\frac{1}{a}}\right)=1$

$\Rightarrow k(\sqrt[8]{a}-\sqrt[8]{\tfrac{1}{a}})(\sqrt[8]{a}-\sqrt[8]{\tfrac{1}{a}})\ (\sqrt[4]{a}+\sqrt[4]{\tfrac{1}{a}})(\sqrt[2]{a}+\sqrt[2]{\tfrac{1}{a}})=1$

$\Rightarrow k(\sqrt[4]{a}-\sqrt[4]{\tfrac{1}{a}}\ (\sqrt[4]{a}+\sqrt[4]{\tfrac{1}{a}})(\sqrt[2]{a}+\sqrt[2]{\tfrac{1}{a}})=1$

$\Rightarrow k(\sqrt[2]{a}-\sqrt[2]{\tfrac{1}{a}})(\sqrt[2]{a}+\sqrt[2]{\tfrac{1}{a}})=1$

$\Rightarrow k(a-1/a)=1$

$\Rightarrow k=a/(a^2-1)$

17. $a^x = b^y = c^z$ and $b^2 = ac$,

$\Rightarrow k=a^x = b^y = c^z$

$\Rightarrow a=k^{1/x}, b=k^{1/y}, c=k^{1/z}$

$b^2=ac$

$(k^{1/y})^2=k^{1/x}k^{1/z}$

$\Rightarrow k^{2/y}=k^{1/x+1/z}$

$\Rightarrow 2/y=1/x\ +1/z$

18. $5^{x-3}3^{2x-8}=225,$

$\Rightarrow 5^{x-3}3^{2x-8}=3^2 5^2$

$\Rightarrow x-3=2, x=5$

19. $\dfrac{9^{n+1}\left(3^{\frac{-n}{2}}\right)^{-2}-(27)^n}{\left(3^m\times 2\right)^3}=\dfrac{1}{729},$

$\Rightarrow 3^{3n-3m}(9-1)=\frac{8}{729}$

$\Rightarrow 3^{3n-3m}=3^{-6}$

$\Rightarrow$ m-n=2

20. abc=1

L.H.S

$\left(1+a+\frac{1}{b}\right)^{-1}+\left(1+b+\frac{1}{c}\right)^{-1}+\left(1+c+\frac{1}{a}\right)^{-1}$

$\Rightarrow \frac{1}{1+a+ac} + \frac{1}{1+\frac{1}{ac}+\frac{1}{c}} + \frac{1}{\frac{a+ac+1}{a}}$

$\Rightarrow \frac{1}{1+a+ac} + \frac{ca}{1+a+ac} + \frac{a}{1+a+ac}$

$\Rightarrow 1$

21. k= $2^x = 7^y = 14^z$

$\Rightarrow 2 = k^{1/x}, 7 = k^{1/y}, 14 = k^{1/z}$

$\Rightarrow 2 * 7 = k^{1/x+1/y}$

$\Rightarrow k^{1/z} = k^{1/x+1/y}$

$\Rightarrow 1/z = 1/x + 1/y$

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