Logarithm EXERCISE - 1

LEVEL - 1

1) $\log_{c^2}(a^b) \cdot \log_{a^3}(b^c) \cdot \log_{b^4}(c^a)$

$$\frac{\log a^b}{\log c^2} \cdot \frac{\log b^c}{\log a^3} \cdot \frac{\log c^a}{\log b^4} = \frac{abc}{24}.$$

2) $a^{1-2x} \cdot b^{1+2x} = a^{4+x} \cdot b^{4-x}$

$\log a^{1-2x} + \log b^{1+2x} = \log a^{4+x} + \log b^{4-x}$

$(1-2x)\log a + (1+2x)\log b = (4+x)\log a + (4-x)\log b$

$\log ab - 2x\log(a/b) = 4\log ab + x\log(a/b)$

$3\log ab = -3x\log(a/b) = 3x\log(b/a)$

$$\boxed{x\log(b/a) = \log(ab)}$$

3. $\log_x(3 \cdot 9 \cdot 729) = \cancel{\log}\ 9$

$3^9 = x^9$

$\therefore x = 3$

4. $(x-1)(x+4) = 3x+5$

$\Rightarrow x = 3$

5 $\log_b a \cdot \log_c b \cdot \log_a c = 1$

6. $\log_x 2\cdot4\cdot8 = 3$

$2^6 = x^3$

$x = 4$

7. $\log\frac{a^2}{b} + \log\frac{b^2}{c} + \log\frac{c^2}{a} - \log(abc)$

$= \log\frac{(abc)^2}{abc} - \log abc$

$= 0$

8. $\log\left\{\left(\frac{16}{9}\right)^2\cdot\frac{54}{224}\cdot\frac{21}{26}\right\} = \log\left\{\frac{16\times16\times9\times6\times7\times3}{9\times9\times4\times56\times13\times2}\right\}$

$\log\left(\frac{8}{13}\right)$

9. $\log\left(\frac{a+b}{2}\right) = \frac{\log(ab)}{2}$

$\log\left(\frac{a+b}{2}\right) = \log\sqrt{ab}$

$\frac{a+b}{2} = \sqrt{ab} \Rightarrow a+b = 2\sqrt{ab}$

on squaring, we get

$a^2+b^2+2ab = 4ab$

$a^2+b^2 = 2ab$

10. $\frac{p}{q} = \left(\frac{2}{3}\right)^{0} \Rightarrow \boxed{\frac{p}{q} = 1}$

11. 1

12.

12

$$\frac{5^{0.25} \times (125)^{0.25}}{(256)^{0.10} \times (256)^{0.15}} = \frac{5^{1/4} \cdot 5^{3/4}}{(4^4)^{1/4}} = 5/4$$

13

$$\frac{1}{x^{a-a}+x^{b-a}+x^{c-a}} + \frac{1}{x^{a-b}+x^{a-b}+x^{c-b}} + \frac{1}{x^{c-c}+x^{a-c}+x^{b-c}}$$

$$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c} = 1$$

14

$$\frac{2^{m+3} \cdot 3^{2m-n} \cdot 5^{m+n+3} \cdot 2^{n+1} \cdot 3^{n+1}}{2^{m+1} \cdot 3^{m+1} \times 2^{n+3} \cdot 5^{n+3} \cdot 3^{n} 5^{m}}$$

$$\frac{2^{m+n+4} \cdot 3^{2m+1} \cdot 5^{m+n+3}}{2^{m+n+4} \cdot 3^{2m+1} \cdot 5^{n+3+m}} = 1$$

Level (2)

1) Given
$x_n > x_{n-1} > x_{n-2} > \cdots > x_2 > x_1 > 1,$

$$\log_{x_1} \log_{x_2} \log_{x_3} \log_{x_4} \cdots \log_{x_n} x_n^{x_{n-1}^{x_{n-2}^{\cdots x_1}}} = 1$$

(3) Given $b^2 = ac$

$$\frac{1}{\log_a x} + \frac{1}{\log_c x} = \log_x a + \log_x c$$

$$= \log_x ac = \log_x b^2$$

$$= 2 \log_x b$$

(35) Given

$a = y^2$
$b = z^2$
$c = x^2$

$$8 \cdot \log_a x^3 \cdot \log_b y^3 \cdot \log_c z^3$$

$$8 \cdot 3 \cdot 3 \cdot 3 \cdot \frac{\log x \cdot \log y \cdot \log z}{\log a \cdot \log b \cdot \log c}$$

$$8 \times 27 \cdot \frac{1}{8} = 27.$$

(6) Given $\lambda = \log_{x^2}\left(\sqrt{x\sqrt{x\sqrt{x}}}\right)$

$$= \log_{x^2}\left(x^{7/8}\right) = \frac{7}{16} \cdot \log_x x$$

$$\lambda = \frac{7}{16}$$

$$\therefore 3216\lambda = 3216 \times \frac{7}{16}$$

$$= 201 \times 7 = 1407$$

(5) $A = \log_4(0.25) + \log_2(0.0625)$

$A = \log_4 1/4 + \log_2 1/16$

$A = -1 + (-4) = -5$

$\therefore -5A = 25^0$

(8) $2^{x+1} = 4^{x+7} = 2^{2x+14}$

$x+1 = 2x+14$

$x = -13$

(9) $\left(\left((3^{-2})^{-3}\right)^{-4}\right)^{1/12} = 3^n$

$3^{-\frac{24}{12}} = 3^n$

$-2 = n$

(10)

$a^x = b$

$b^y = c$

$c^z = a$

$\cancel{a^{xyz}}$

from $c = b^y$

$c = (a^x)^y = a^{xy}$

$c = (c^z)^{xy} = c^{xyz}$

$\therefore \boxed{xyz = 1}$

(11) i) $x^6 = y^2$ and $x^4 = 3y$

$x^3 = y$ and $x^4 = 3y$

$x^4 = 3x^3$

$\therefore \underline{\underline{x = 3}}, y = 27$

Now $2x + y = 2 \times 3 + 27$

$6 + 27$

$= 33$

(12) conceptual problem,

(13) By definition of logarithm,

(14) i) $x^{1/x} = y^{1/y} = k$ (let) $\rightarrow x = k^x$, $y = k^y$

ii) $x^y + y^x = 32$

iii) $xy = 8$

using (2), we get

$2k^{xy} = 32$

$k^{xy} = 16$

$k^8 = 16 = 2^4 = (\sqrt{2})^8$

$\therefore k = \sqrt{2}$

$\therefore$ using (1),

$\boxed{x^{1/x} = \sqrt{2}}$

(15) $a = \log_{2p} p$

$b = \log_{3p} 2p$

$c = \log_{4p} 3p$

$bc = \frac{\log 2p}{\log 3p} \cdot \frac{\log 3p}{\log 4p} = \frac{\log 2p}{\log 4p}$

$\therefore \frac{1}{bc} = \frac{\log 4p}{\log 2p}$

$\therefore a + \frac{1}{bc} = \frac{\log p + \log 4p}{2p} = 2$

EXERCISE-2

LEVEL-1

1) conceptual problem

2) $\log(a+b) + \log(a-b) - \log(a^2-b^2)$

$= \log(a^2-b^2) - \log(a^2-b^2)$

$= 0$

3) $\frac{\log x}{\log y} = \frac{\log 343}{\log 7}$

$\log_y x = 3$

$x = y^3$

4) $\log_{10} x - \log_{10}(2x-3) = \log_{10} 10$

$\frac{x}{2x-3} = 10$

$x = \frac{30}{19}$

5) $\log(x+4)^2 = \log 16$

$x^2 + 4^2 + 8x = 16$

$x(x+8) = 0$

$x = 0$, and $x = -8$ rejected

$\therefore x = 0$

6) $\log_x 512 = 3$

$512 = x^3$

$x = 8$

7 $\log_{16} 3 \cdot \log_{18} 4 \cdot \log_{9} 18$

$$\frac{\log 3 \cdot \log 4 \cdot \log 18}{\log 16 \cdot \log 18 \cdot \log 9}$$

$$\frac{\log 3 \cdot 2\log 2}{4\log 2 \cdot 2\log 3} = \frac{1}{4}$$

8 $p = 3$

$q = 2$

$\therefore \ p^q = 3^2 = 9$

9 $\log_2 16 - \log_2 1024$

$4 - 10 = -6$

10 $\log_2 \log_3 \log_5 125 = 0$

11 $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$

$\log_7 \log_7 \sqrt{7\sqrt{7^{3/2}}} = \log_7 \log_7 \sqrt{7^{7/4}}$

$\log_7 \log_7 7^{7/8} \quad = \log_7 7/8$

$= \log_7 7 - \log_7 8$

$= 1 - \log_7 8 \quad = 1 - 3\log_7 2$

12. $\log_{\sqrt{2}} b = \frac{10}{3}$

$b = (\sqrt{2})^{\frac{10}{3}} = 8^{5/3}$

$= 2^5 = 32$

13. $A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$

$= \log_2 \log_2 \log_4 4^4 + 2 \times 2 \log_2 2$

$= \log_2 \log_2 4 + 4 = \log_2 \log_2 2^2 + 4$

$= 1 + 4$ {using $\log_a a = 1$

$= 5$

14. $\log a = p$

$\log b = q$

Now $2\log(ab) \cdot \log(a/b)$

$2[\log a + \log b][\log a - \log b]$

$2(p+q)(p-q)$

$2(p^2 - q^2)$

15. $\frac{3^{5x} \cdot 81 \cdot 6561}{3^{2x}} = 3^3$

$3^{5x+4} \cdot 3^{8-2x} = 3^3$

$5x + 4 + 8 - 2x = 3$

$3x = -9$

$x = -3$

16

$$\log_3\{5+4\log_3(x-2)\}=2$$

$$5+4\log_3(x-2)=3^2=9$$

$$4\log_3(x-2)=4$$

$$\log_3(x-2)=1$$

$$\cancel{\log(x-2)}\quad x-2=3$$

$$x=5$$

17

$$\frac{\log p}{2}=\frac{\log q}{4}=\frac{\log r}{8}=K$$

$$\log p=2K \Rightarrow p=10^{2K}$$

$$\log q=4K \Rightarrow q=10^{4K}$$

$$\log r=8K \Rightarrow r=10^{8K}$$

$$\therefore\ pqr=10^{2K+4K+8K}=100=10^2$$

$$14K=2$$

$$K=1/7$$

LEVEL 2

1. $\log_x y = \frac{\log_a y}{p}$

$\frac{\log y}{\log x} = \frac{\log y}{p \log a}$

$\log x = p \log a$

$p = \frac{\log x}{\log a} = \log_a x$

2. $\log_{a+b}(a^3+b^3) - \log_{a+b}(a^2-ab+b^2)$

$\log_{a+b}\{(a+b)(a^2-ab+b^2)\} - \log_{a+b}(a^2-ab+b^2)$

$= \log_{(a+b)}(a+b) = 1$

Given

3. $\log_y x = 2$

and $a^{\log_a(\log_x y)} = \log_x y = \frac{1}{2}$

4. $\log[4 - 5\log_{32}(x+3)] = 0$

$4 - 5\log_{32}(x+3) = 1$

$-5\log_{32}(x+3) = -3$

$\log_{32}(x+3) = 3/5$

$x+3 = (32)^{3/5} = 2^3$

$x = 5.$

5

$$\log_a\left(\frac{13^2}{\sqrt{2^3}\times 5}\right) = 2\log_a 13 - \log_a 5 - x$$

$$2\log_a 13 - \frac{3}{2}\log_a 2 - \log_a 5 = 2\log_a 13 - \log_a 5 - x$$

$$x = \frac{3}{2}\log_a 2 = \log_a 2^{3/2}$$

$$a^x = 2^{3/2}$$

6

$$(6561)^{0.125} + (3125)^{0.2}$$

$$(3^8)^{1/8} + (5^5)^{1/5}$$

$$3+5 = 8$$

7. $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$

$$\log_{10}\left(\frac{2^3\times 3^2}{5^2}\right) = 3\log_{10} 2 + 2\log_{10} 3 - 2\log_{10} 5$$

$$= 3\log_{10} 2 + 2\log_{10} 3 - 2 + 2\log 2$$

$$= 5\log_{10} 2 + 2\log_{10} 3 - 2$$

$$= 5\times 0.3010 + 2\times 0.4771 - 2$$

$$1.505 + 0.9542 - 2$$

$$= 2.4592 - 2$$

$$= 0.4592$$

8

8

$$x^2+y^2=z^2$$

$$y^2=z^2-x^2=(z+x)(z-x)$$

$$2\log y = \log(z+x)+\log(z-x)$$

$$2=\frac{\log(z+x)}{\log y}+\frac{\log(z-x)}{\log y}$$

$$2=\log_y(z+x)+\log_y(z-x)$$

(9) Given $\log 2 = 0.3010$

let $x = 2^{1024}$

$$\log x = 1024\cdot\log 2$$

$$=1024\times 0.3010$$

$$\log x = 308.224$$

$\therefore$ Number of digits in $\log x$ = $\underline{\underline{309}}$

10

$$x^2-y^2=1$$

$$(x+y)(x-y)=1$$

$$\log_{(x-y)}(x+y)+1=0$$

$$\log_{(x-y)}(x+y)=-1$$

(11)

$$x^2+y^2=3xy$$

$$x^2+y^2-2xy=xy$$

$$(x-y)^2=xy$$

$$\therefore \log_{xy}(x-y)=\frac{1}{2}$$

12 $4^{2^{2^{1/56}}} = 4^{2^2} = 4^4 = 2^8$

13 $5^{n-3} = 625 = 5^4$

$n = 7 \Rightarrow 5^{n+3} = 5^{10}$

14 $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \cdots + \frac{1}{\log_{43} n}$

$\log_n 2 + \log_n 3 + \log_n 4 + \cdots + \log_n 43$

$\log_n 2\cdot3\cdot4\cdot5\cdots43 = \log_n 43!$

$= \frac{1}{\log_{43!} n}$

15 $\log\left(\frac{a+b}{3}\right) = \frac{\log a + \log b}{2} = \frac{\log ab}{2}$

$\log\left(\frac{a+b}{3}\right) = \log\sqrt{ab}$

$\frac{a+b}{3} = \sqrt{ab}$

$(a+b)^2 = 9ab$

$a^2+b^2 = 7ab$

$\therefore \frac{a}{b} + \frac{b}{a} = 7$

16 $\log(x+y) = \log x + \log y$

$x+y = xy$

$x = \frac{y}{y-1}$

17 $\frac{1}{1+\log_{ab} c} + \frac{1}{1+\log_{ac} b} + \frac{1}{1+\log_{bc} a}$

$= \frac{1}{\log_{ab} abc} + \frac{1}{\log_{ac} abc} + \frac{1}{\log_{bc} abc}$

$= 2$

18 $\log_{x^n} y^m = K \cdot \log_x y$

$\frac{m}{n} = K$