IX Polynomials

Exercise Solutions

Level -1

1. The degree of $\sqrt{3}x^3 - 5x + \sqrt{7}x^5$ is
(a) 2 (b) 1 (C) 5 (d) 3

Answer:- as the highest power of x is 5 in the given polynomial will be equal to 5

2. The degree of $\sqrt{3}x^3 - 5x + \sqrt{7}x^{-5}$ is
(a) 3 (b) 1 (c) -5 (d) none

Given expression is not a polynomial as it contains term with negative power of x

3. The coefficient form of $x^3 + 3x - 1$ is
(a) 1, 3, -1 (b) 1, 0, 3, -1 (c) -1, 3, 1 (d) 1, 0, 0, -1

Coefficient of x cube is 1 and coefficient of X is 3 and coefficient of X raised to 0 is -1 so answer will be option A

4. The value of the polynomial $p(x) = x^3 - 3x^2 + 4x - 5$ at $x = -1$
(a) -1 (b) -13 (c) 3 (d) 1

If we put value of x as -1 in the given polynomial then the value will be is equals to - 13

5. Express the following in index form taking x as a variable. (-2, 3, -5, 6)
(a) $-2x^4 + 3x - 5x + 6$ (b) $-2x + 3x^2 - 5x^3 + 6$
(c) $-2x^2 - 3x - 5 + 6$ (d) $-2x^3 + 3x^2 - 5x + 6$

If we put given coefficients with respective index of x then we will get $-2x^3 + 3x^2 - 5x + 6$

6. The polynomial $p(x) = x^2 + 4x - nx - 5$ is
(a) Binomial (b) Monomial (c) Trinomial (d) None
Answer - None

7. The G.C.D. of $x^3 - x^2 - 4x - 6$ and $x^2 - 2x - 3$ is
(a) 3x + 2 (b) x – 3 (c) – x – 3 (d) – 2x – 3

Let f (x) = x3 – x2 – 4x – 6 = (x – 3) (x2 + 2x + 2)
and g (x) = x2 – 2x – 3 = (x + 1) (x – 3)
Hence GCD of f (x) and g (x) is (x – 3)

8. If x + y = a and xy = b, then the value of $\frac{1}{x^3}+\frac{1}{y^3}$ is

(a) $\frac{a^3-3ab}{b^3}$ (b) $\frac{a^3-3a}{b^3}$ (c) $\frac{a^3-3}{b}$ (d) $\frac{a^3-3}{b^2}$

$x+y = a \qquad ---\left(1\right)$

$xy = b \qquad ---\left(2\right)$

Dividing equation $\left(1\right)$ *by equation* $\left(2\right)$ *we get*

$\frac{x+y}{xy} = \frac{a}{b}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{a}{b}$

Now

$\frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{xy} = \left(\frac{1}{x}\right)^2 + \left(\frac{1}{y}\right)^2 + \frac{2}{xy} - \frac{3}{xy} = \left(\frac{1}{x}+\frac{1}{y}\right)^2 - \frac{3}{xy} = \frac{a^2}{b^2} - \frac{3}{b} = \frac{a^2-3b}{b^2}$

Now

$\frac{1}{x^3} + \frac{1}{y^3} = \left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x^2}+\frac{1}{y^2}-\frac{1}{xy}\right) = \frac{a}{b} \times \frac{(a^2-3b)}{b^2} = \frac{a(a^2-3b)}{b^3}$

9. If x = 3, y = 4 and z = -7, then $x^3 + y^3 + z^3 - 3xyz$ is equal to

(a) 6 (b) 0 (c) 2 (d) 8

Put the values x = 3, y = 4 and z = -7 in given expression and get the answer

10. Factors of $2(a + b)^2 - 9(a + b) - 5$ are

(a) a + b + 5, 2a + 2b – 1 (b) a + b – 5, 2a + 2b + 1
(c) a – b + 5, 2a – 2b + 5 (d) none of these

$2(a + b)^2 - 9(a + b) -5 = 0$

$\Rightarrow 2(a + b)^2 - 10 (a + b +1(a + b) -5 = 0$

$\Rightarrow 2(a + b)(a + b - 5) + 1(a + b -5) = 0$

taking (a + b -5) common ,

$\Rightarrow (a + b -5)[2(a + b) + 1] =0$

$\Rightarrow (a + b -5)(2a + 2b +1) =0$

11. If $x^2 - \frac{1}{(x-2)^2} = 4 - \frac{1}{(x-2)^2}$, then x is equal to

(a) 1 (b) 4 (c) – 1 (d) none of these

Answer:- $x^2 = 4$

x = +2 or -2

so (d) none of these

12. If $x+\frac{1}{x}=2$, then $x^2+\frac{1}{x^2}$ is equal to

(a) 2 (b) 0 (c) 4 (d) 6

$x+\frac{1}{x}=2$

Squaring both sides we get

$x^2 + 2 + 1/x^2 = 4$

$x^2+\frac{1}{x^2} = 4 - 2 = 2$

13. If the expression $(125 - x^3) = (5 - x)(x^2 + ax + b)$, then the value of a is

(a) 4 (b) 2 (c) – 7 (d) 5

We know that,

$125 - x^3$

$= (5)^3 - (x)^3$

$= (5 - x)\left[(5)^2 + 5x + (x)^2\right]$

$= (5 - x)\left(x^2 + 5x + 25\right) \quad \ldots.(1)$

but it is given that,

$125 - x^3 = (5 - x)\left(x^2 + ax + b\right) \quad \ldots..(2)$

From (1) and (2), we get

$a = 5 \quad and\, b = 25$

14. If one root of the equation $3x^2 - 9x = kx - k$ is 2, then the value of k is

(a) 4 (b) 3 (c) – 6 (d) – 8

Let the two roots be $(2, \alpha)$ *of the given equation.*

The equation is

$3x^2 - 9x = kx - k$

$\Rightarrow 3x^2 - (9 + k)x + k = 0$

$\therefore \quad 2 + \alpha = \frac{9 + k}{3}$

$\Rightarrow \quad \alpha = \frac{3 + k}{3} ----(1)$

Again $2 \times \alpha = \frac{k}{3}$

$$\therefore \quad \alpha = \frac{k}{6} ----(2)$$

$$from \quad (1) \quad and \quad (2) \quad we \quad get$$

$$\frac{3+k}{3} = \frac{k}{6}$$

$$\Rightarrow \quad k = -6 \quad (Ans)$$

15. If x and y are positive with x – y = 2 and xy = 24, then $\frac{1}{x} + \frac{1}{y}$ is equal to

(a) $\frac{5}{12}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{25}{6}$

x=6, y=4
then x-y=2
6-4=2
x*y=24
6*4=24
=1/6+1/4
=10/24
=5/12
therefore 1/X+1/y=5/12

16. The H.C.F of two expressions is x and their L.C.M. is $x^3 - 9x$. If one of the expressions is $x^2 + 3x$, then the other expression is

(a) $x^2 - 3x$ (b) $x^3 - 3x$ (c) $x^2 + 9x$ (d) $x^2 - 9x$

$$\text{We knopw that HCF} \times \text{LCM} = \text{product of numbers}$$

$$x \times (x^3 - 9x) = [x^2 + 3x] \times \text{second expression}$$

$$\text{second expression} = \frac{x \times (x^3 - 9x)}{[x^2 + 3x]} = \frac{x^2(x^2 - 9)}{x(x+3)}$$

$$= \frac{x(x+3)(x-3)}{(x+3)} = x^2 - 3x$$

17. The value of k for which x + k is a factor of $x^3 + kx^2 - 2x + k + 4$ is

(a) $\frac{-4}{3}$ (b) – 5 (c) 2 (d) $\frac{6}{7}$

Since (x+k) is a factor of the polynomial $x^3 + kx^2 - 2x + k + 4$
Hence, x = -k must be the zero of this polynomial.
Hence, from factor theorem, we have

$$(-k)^3 + k(-k)^2 - 2(-k) + k + 4 = 0$$

$$-k^3 + k^3 + 2k + k + 4 = 0$$

$$3k + 4 = 0$$

$$k = -\frac{4}{3}$$

Therefore, the value of k is -4/3

18. $(x + y)^3 – (x – y)^3$ can be factorized as
(a) $2y (3y^2 + x^2)$ (b) $2y (3x^2 + y^2)$ (c) $2x (3x^2 + y^2)$ (d) $2x (x^2 + 3y^2)$
Use the formula $a^3 - b^3 = (a - b)^3 + 3ab (a- b)$
Put $a = (x + y)$ $b = (x – y)$
$(x + y)^3 – (x – y)^3$
$8y^3 + 3(x + y)(x – y)(2y)$
=> $2y (3x^2 + y^2)$

19. If the G.C.D. of $x^2 – ax – 6$ and $x^2 – 2x + b$ is $x – 6$, then the value of a and b is
(a) a = 6, b = -24 (b) a = 5, b = -24 (c) a = -5, b = -24 (d) a = -5, b = 24

As x-6 is factor the we will put x-6 = 0
x=6 in both expression and equating to zero
$6^2 – 6a – 6 = 0$
36 - 6a - 6 = 0
a = 5
$6^2 – 2x6 + b = 0$
36 - 12 + b = 0
b = -24

20. The H.C.F. of $8x^3 – 32x^2 + 40x – 16$ and $4x^3 – 24x^2 + 36x – 16$, is
(a) $4 (x - 1)^2$ (b) $4 (x + 1)^2$ (c) $(x - 1)^2$ (d) $(x + 1)^2$
Factorize both given expressions and we can clearly see option c & d are not possible options
And x+1 will not be the factor of both expressions
So A will be correct option

Level – 2

SINGLE CORRECT ANSWER TYPE

1. The polynomials $ax^3 + 3x^2 – 3$ and $2x^3 – 5x + a$ when divided by $(x – 4)$ leaves remainders R_1 and R_2 respectively then value of a if $2R_1 – R_2 = 0$
(a) $-\frac{18}{127}$ (b) $\frac{18}{127}$ (c) $\frac{17}{127}$ (d) $-\frac{17}{127}$

Let the remainder be R1 and R2 as given :
R1 = ax^3 + 3x^2 - 3
Now, x - 4 => x = 4
R1 = a(4)^3 + 3(4)^2 - 3
R1 = 64a + 48 - 3
R1 = 64a + 45

R2 = 2x^2 - 5x + a
R2 = 2(4)^2 - 5(4) + a
R2 = 32 - 20 + a
R2 = 12 + a
R1 + R2 = 0
64a + 45 + 12 + a = 0
65a = - 57
a = - 57/65
2R1 - R2 = 0
2(64a + 45) - 12 - a = 0
128a + 90 - 12 - a = 0
127a = - 78
a = - 78/127

2. If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$, then the values of a and b are
(a) 11 and 5 (b) 1 and –5 (c) –1 and –5 (d) –11 and 5

$2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$
$\Rightarrow 2x^2 + xy - 3y^2 + x + ay - 10 = 2x^2 - 2xy - 4x + 3xy - 3y^2 - 6y + bx - by - 2b$
$\Rightarrow 2x^2 + xy - 3y^2 + x + ay - 10 = 2x^2 + xy - 3y^2 + x(-4 + b) + y(-6 - b) - 2b$
Compairing both side the coefficient of x, y, we have,
$-4 + b = 1 \Rightarrow b = 5$
$-6 - b = a$
$\Rightarrow -6 - 5 = a \Rightarrow a = -11$
Hence, $a = -11$ *and* $b = 5$

3. The value of $\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 1.5 \times 3.8 - 4.7 \times 3.8}$ is
(a) 8 (b) 9 (c) 10 (d) 11

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= \frac{\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]}{\frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]} = (a + b + c)$$

For $a = 1.5, b = 4.7, c = 3.8$ value is $1.5 + 4.7 + 3.8 = 10$

4. If (x+y+z) = , xy+yz+zx = −1, xyz = −1 then value of $x^3 + y^3 + z^3$ is
(a) −1 (b) 1 (c) 2 (d) −2
Identity: $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$
Since x + y + z = 1, xy + yz + zx = -1and xyz= -1,

Putting values, we get:

So $x^3 + y^3 + z^3 - 3 \times (-1) = 1(x^2 + y^2 + z^2 - (-1))$

$x^3 + y^3 + z^3 = x^2 + y^2 + z^2 - 2$ ____(i)

Now

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$1^2 = x^2 + y^2 + z^2 + 2(-1)$

$x^2 + y^2 + z^2 = 3$

Put in eq (i)

$x^3 + y^3 + z^3 = 3 - 2 = 1$

5. What property makes this equation true ? $6a^2 - 2a = 2a(3a - 1)$

(a) The commutative property (b) The reflexive property

(c) The associative property (d) The distributive property

$\Rightarrow$ D

MULTIPLE CORRECT ANSWER TYPE

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c), (d), out of which ONE or MORE is correct. Choose the correct options.

6. If $x = \frac{a}{2}$, which of the following is/are not the value(s) of 4x2 + 8x + 18

(a) $a^2 + 2a + 8$ (b) $a^2 + 3a + 18$ (c) $a^2 + 4a + 18$ (d) $a^2 + 5a + 18$

$\Rightarrow$ ONLY OPTIONS A, B, D SATISFIES $x = \frac{a}{2}$

7. The polynomial $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3, when divided by $x - 3$. Then the values of

(a) $a = -4$ (b) $a = 3$ (c) $b = 1$ (d) $b = 3$

Given that (x-2) is a factor of polynomial

$P(x) = x^3 + ax^2 + bx + 6$

Also when divided by (x - 3) leaves a remainder 3.

we have **to find the value of a and b.**

As 2 is the zero of the polynomial therefore **by remainder theorem**

$P(2) = 0$

$2^3 + a(2)^2 + 2b + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a + 2b + 14 = 0$

$2a + b = -7$ ➡ (1)

Also the polynomial x^3+ax^2+bx+6 when divided by (x - 3) leaves a remainder 3

$\therefore P(3)=3$

$3^3+a(3)^2+3b+6=3$

$27+9a+3b+6=3$

$9a+3b+33=3$

$3a+b=-10$ ➡ (2)

Solving (1) and (2), we get

Subtracting equation (2) from (1)

$2a+b-3a-b=-7-(-10)$

$-a=3$

$a=-3$

$2a+b=-7$

$2(-3)+b=-7$

$b=-7+6=-1$

Hence, the value of a and b are -3 and -1 respectively.

8. Factors of a (x + y + z) + bx + by + bz is/are
(a) ax + ay + az (b) bx + by + bz (c) x + y + z (d) a + b
$ax+by+bx+az+ay+bz$
$=(ax+ay+az)+(bx+by+bz)$
$=a(x+y+z)+b(x+y+z)$
$=(x+y+z)(a+b)$

9. If $x+\frac{1}{x}=7$, then which of the following is not the value of $x^3+\frac{1}{x^3}$ is/are
(a) 318 (b) 325 (c) 343 (d) 322
Given,
$x + 1/x = 7$.
Now,
Cubing on both sides,
$(x + 1/x)^3 = 7^3$
$x^3 + 1/x^3 + 3x(1/x)(x + 1/x) = 343$
$x^3 + 1/x^3 = 343 - 3(7)$
$x^3 + 1/x^3 = 322$

10. If $x - \frac{1}{x} = 3$, find the value of $x^3 - \frac{1}{x^3}$

(a) 16 (b) 36 (c) 48 (d) 32

x - 1 / x = 3
On cubing both sides ;
$(x - 1 / x) = (3)^3$
Using Identity :
$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
$\Rightarrow x^3 - 1 / x^3 - 3(x - 1 / x) = 27$
$\Rightarrow x^3 - 1 / x^3 - 3(3) = 27$
$\Rightarrow x^3 - 1 / x^3 - 9 = 27$
$\Rightarrow x^3 - 1 / x^3 = 27 + 9$
$\Rightarrow x^3 - 1 / x^3 = 36$

INTEGER TYPE

The answer to each of the questions is a single–digit integer, ranging from 0 to 9.

15. The polynomials $kx^3 + 3x^2 - 3$ and $2x^3 - 5x + k$ when divided by $(x - 4)$ leave the same remainder in each case, then the value of k is
Put X=4 in both equation
And both eq would be equal to each other

16. The remainder when $f(x)=3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$ is

We have,

$f(x) = 3x^4+2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ and $g(x) = x + \frac{2}{3}$

Therefore, by remainder theorem when f (x) is divided by g (x) = x – $(-\frac{2}{3})$, the remainder is equal to $f(\frac{-2}{3})$

Now, $f(x) = 3x^4+2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$

$f(\frac{-2}{3}) = 3(\frac{-2}{3})^4 + 2(\frac{-2}{3})^3 - (\frac{\frac{-2}{3} * \frac{-2}{3}}{3}) - \frac{\frac{2}{3}}{9} + \frac{2}{27}$

$= 3 * \frac{16}{81} + 2 * \frac{-8}{27} - \frac{4}{9*3} - \frac{-2}{3*9} + \frac{2}{27}$

$= \frac{16}{27} - \frac{16}{27} - \frac{4}{27} + \frac{2}{27} + \frac{2}{27}$

$= \frac{16-16-4+2+2}{27} = \frac{0}{27}$

$= 0$

17. If $A = -8x^2 - 6x + 10$, then its value when 'x' = $\frac{1}{2}$ is
Put $x = \frac{1}{2}$
In given expression to get value of A

18. Degree of the polynomial $\frac{1}{2}x^5 + 3x^4 + 2x^3 + 3x^2$ is
Degree is 5

19. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, then the value of '$-a$' is
$f(x) = ax^3+4x^2+3x-4$
$g(x)= x^3-4x +a$
$f(3)=A(27)+4(9)+3(3)-4$
$27a+41$
$g(3)=27-4(3)+a$
$15+a$
$f(3)=G(3)$
$27a+41=15+a$
$26a=15-41$
$a=15-41/26$
$a=-26/26$
$a=-1$
$-a = 1$

EXERCISE - 2

Level -1

1. Find the value of each of the following polynomials at the indicated values of variables

   (i) $2x^2 - 5x + 3$ at $x = 2$

   (ii) $p(y) = y^3 - 5y + 5$ at $y = 0$

   (iii) $p(z) = 3z^2 - \frac{1}{2} + z$ at $z = -\frac{1}{2}$.

2. Factorize $(x^2 - 4x)(x^2 - 4x - 1) - 20$.

   let $x^2 - 4x = y$

   then the question will be

   $(y)(y-1)-20$

   $=y^2-y-20$

   $=y^2-5y+4y-20$

   $=y(y-5)+4(y-5)$

   $=(y+4)(y-5)$

   by substituting the value of y

   $=(x^2-4x+4)(x^2-4x-5)$

   $=(x-2)^2(x^2-5x+x-5)$

   $=(x-2)^2(x(x-5)+1(x-5))$

   $=(x-2)^2(x+1)(x-5)$

3. If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$, when divided by $(x - 2)$ leave the same remainder, find the value of a.

   value of x = 2 (simplifying x - 2)

   so ,

   $8a + 12 - 13 = 16 - 10 + a$ (after putting x= 2)

   $8a - a = 16 - 10 - 12 + 13$

   $7a = 6 + 1$

   $7a = 7$

   $a = 1$

4. Using factor theorem, factorize the polynomial $x^4 + 2x^3 - 13x^2 - 14x + 24$.

   $x^4 - 2x^3 - 13x^2 + 14x + 24$

   $x^4 - 2x^3 - 15x^2 + 2x^2 + 14x + 24$

   $x^2(x^2-2x-15) + 2(x^2+7x+12)$

   $x^2(x-5)(x+3) + 2(x+4)(x+3)$

   $(x+3)[x^2(x-5) + 2(x+4)]$

   $(x+3)[x^3-5x^2+2x+8]$

   $(x+3)[x^3-4x^2+4x-x^2-2x+8]$

   $(x+3)[x(x^2-4x+4) - (x^2+2x-8)]$

   $(x+3)[x(x-2)(x-2) - (x+4)(x-2)]$

   $(x+3)(x-2)[x(x-2)-(x+4)]$

   $(x+3)(x-2)[x^2-2x-x-4]$

(x+3)(x-2)[x²-3x-4]
(x+3)(x-2)(x-4)(x+1)

5. Find the remainder when f (x) = $x^3 - 6x^2 + 2x - 4$ is divided by g (x) = 1 – 3x.
G(X) = 3X-1
=> 3X = 1
=> X = 1/3
F(X) = X^3-6X^2+2X+4
F(1/3) = $(1/3)^3 - 6 \times (1/3)^2 + 2 \times 1/3 + 4$
=> 1/27 - 6 × 1/9 + 2/3 + 4
=> 1/27 - 2/3+2/3 +4
=> 1 - 18+18-108 ÷ 27
=> -107/27
Hence, remainder = -107/27

6. Factorize $64a^2 + 112ab + 49b^2$

$64a^2 + 49b^2 + 112ab$

$= 64a^2 + 112ab + 49b^2$

$= (8a)^2 + 2 \times 8a \times 7b + (7b)^2$

$= (8a + 7b)^2$

$= (8a + 7b)(8a + 7b)$

7. Evaluate $(999)^3$ by using suitable identities.
Write (999) = (1000-1) and apply identity

8. Find the value of k, if x + 2 is a factor $x^2 + kx + 6$.
As x+2 is a factor so if we put x=-2 in expression given we will get value of 'k'

9. Evaluate $\frac{(1.2\times1.2\times1.2-0.2\times0.2\times0.2)}{(1.2\times1.2+1.2\times0.2+0.2\times0.2)}$
Apply identity in numerator and denominator

10. The polynomial $p(x) = kx^3 + 9x^2 + 4x - 8$ when divided by x + 3 leaves the remainder -20, find the value of k.

take x=-3 and solve for 'k'

11. (i) If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find $a^3 + b^3$

(ii) If $x = 3-\sqrt{8}$, find $x^4 + \frac{1}{x^4}$

(iii) If $x = \frac{1}{2-\sqrt{3}}$, prove that $x^3 - 2x^2 - 7x + 5 = 3$.

12. If $a^2 + b^2 + c^2 = 13$ and $ab + bc + ca = 6$, then evaluate $a^3 + b^3 + c^3 - 3abc$.

13. Factorize the following:
(a) $a^3 + b^3 + c^3 - 3abc$ (b) $(a + b + c)^3 - (a^3 + b^3 + c^3)$.

14. Find the polynomial which when subtracted from $(6x^4 + 3x^3 + 2x^2 + 1)$ gives $(5x^4 + 2x^3 + x - 2)$.

15. By how much does the sum of $2x^2 - 3xy + 4y^2$ and $5y^2 - 3xy - x^2$ exceed $5x^2 - y^2$.

4. Use factor theorem to verify that $(x + a)$ is a factor of $(x^n + a^n)$ for any odd positive integer n.

Let $p(x) = x^n + a^n$

The zero of x + a is - a. | $x + a = 0 \Rightarrow x = -a$

Now,

$p(-a) = (-a)^n + a^n = (-1)^n a^n + a^n$

$= (-1)a^n + a^n$

∵ n is an odd positive interger

∴ $(-1)^n = -1$

$= -a^n + a^n = 0$

∴ By Factor Theorem, x + a is a factor of $x^n + a^n$ for any odd positive integer n.

6. Find the value of N for which $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = N(b - c)(c - a)(a - b)$ becomes an identity.

$a\left(b^2 - c^2\right) + b\left(c^2 - a^2\right) + c\left(a^2 - b^2\right)$

$= ab^2 - ac^2 + bc^2 - a^2b + c(a - b)(a + b)$

$= ab^2 - a^2b - ac^2 + bc^2 + c(a - b)(a + b)$

$= -ab(a - b) - c^2(a - b) + c(a - b)(a + b)$

$= (a - b)(-ab - c^2 + ca + bc)$

$= (a - b)(-ab + ca - c^2 + bc)$

$= (a - b)[-a(b - c) + c(b - c)]$

$= (a-b)\left[(b-c)(c-a)\right]$

$= (a-b)(b-c)(c-a)$

Given, $a\left(b^2-c^2\right)+b\left(c^2-a^2\right)+c\left(a^2-b^2\right) = \text{N}\ \ (a-b)(b-c)(c-a)$

$\therefore (a-b)(b-c)(c-a) = \text{N}\ \ (a-b)(b-c)(c-a)$

$\Rightarrow \text{N} = 1$

Thus, the value of N is 1.

7. Factorize $x^2 - 11xy - x + 11y$.

$x^2 - 11xy - x + 11y = (x^2 - x) + (11y - 11xy)$[Regrouping the expressions]

$= x(x-1) + 11y(1-x)$

$= x(x-1) - 11y(x-1)[\because (1-x) = -(x-1)]$

$= (x-11y)(x-1)$[Taking out the common factor (x - 1)]

8. Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, then find the value of a.

Given the polynomials $P(x) = x^3+2x^2-5ax-7$ and $P'(x) = x^3+ax^2-12x+6$

when the above polynomials are divided by x - 1 and x + 2 resp.

then R_1 and R_2 are remainders such that

$2R_1 + R_2 = 2$

By remainder theorem

$P(1) = x^3 + 2x^2 - 5ax - 7 = 1^3 + 2(1)^2 - 5a(1) - 7 = 1 + 2 - 5a - 7 = -4 - 5a$

$P'(-2) = x^3 + ax^2 - 12x + 6 = (-2)^3 + a(-2)^2 - 12(-2) + 6 = -8 + 4a + 24 + 6 = 22 + 4a$

As given

$2P(1) + P'(-2) = 2$

$2(-4-5a) + 22 + 4a = 2$

$-8 - 10a + 22 + 4a = 2$

$6a = 12$

$a = 2$

9. If $ax^3 + bx^2 + x – 6$ hs $x + 2$ as a factor and leaves a remainder 4 when divided by $x – 2$, then find the value of a and b.

Let $p(x) = ax^3 + bx^2 + x - 6$

A/C to question,

$(x + 2)$ is the factor of $p(x)$, and we know this is possible only when $p(-2) = 0$

So, $p(2) = a(-2)^3 + b(-2)^2 - 2 - 6 = 0$

$\Rightarrow -8a + 4b - 8 = 0$

$\Rightarrow 2a - b + 2 = 0$ -------------(1)

again, question said that if we p(x) is divided by (x -2) then it leaves remainder 4.

so, $P(2) = a(2)^3 + b(2)^2 + 2 - 6 = 4$

$\Rightarrow 8a + 4b - 4 = 4$

$2a + b -2 = 0$ -------------(2)

solve equations (1) and (2),

$4a = 0 \Rightarrow a = 0$ and $b = 2$

10. Factorize $x^4 – 5x^2 + 4$.

$$x^4 - 5x^2 + 4 = (x^2)^2 - 5(x^2) + 4$$

$$= (x^2 - 4)(x^2 - 1)$$

$$= (x^2 - 2^2)(x^2 - 1^2)$$

$$= (x - 2)(x + 2)(x - 1)(x + 1)$$

11. What must be subtracted from $4x^4 – 2x^3 – 6x^2 + x – 5$ so that the result is exactly divisible by

$2x^2 + x – 1$

Let's divide $4x^4 - 2x^3 - 6x^2 + x - 5$ by $2x^2 + x - 1$.

We get; (-6)

Hence, (−6) should be subtracted from $(4x^4 - 2x^3 - 6x^2 + x - 5)$ so that the resulting polynomial is exactly divisible by $(2x^2 + x – 1)$.

12. Factorize $2x^4 + x^3 – 14x^2 – 19x – 6$.

$$p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$$

at x =-1,

$$p(-1) = 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0$$

thus (x+1) is a factor of p(x).

now divide p(x) by (x+1):

$$x+1 \overline{)2x^4+x^3-14x^2-19x-6}\quad 2x^3-x^2-13x-6$$

$$\underline{2x^4+2x^3}$$

$$\times \quad -x^3-14x^2$$

$$\underline{-x^3-x^2}$$

$$\times \quad -13x^2-19x$$

$$\underline{-13x^2-13x}$$

$$\times \quad -6x-6$$

$$\underline{-6x-6}$$

$$\times \quad \times$$

now let us factorize the polynomial

$$q(x)=2x^3-x^2-13x-6$$

where

$$p(x)=(x+1)q(x)$$

now at x =-2

$$q(-2)=2*(-8)-4+26-6=26-26=0$$

thus (x+2) is a factor of q(x).
now let us divide q(x) by (x+2).

$$x+2 \overline{)2x^3-x^2-13x-6}\quad 2x^2-5x-3$$

$$\underline{2x^3+4x^2}$$

$$\times \quad -5x^2-13x$$

$$\underline{-5x^2-10x}$$

$$\times \quad -3x-6$$

$$\underline{-3x-6}$$

$$\times \quad \times$$

$$q(x)=(x+2)(2x^2-5x-3)$$

$$=(x+2)[2x^2-6x+x-3]$$

$$=(x+2)[2x(x-3)+1(x-3)]$$

$$=(x+2)(x-3)(2x+1)$$

therefore

$$p(x)=(x+1)(x+2)(x-3)(2x+1)$$

13. Find the product of $\left(1+\frac{1}{a}\right)$, $\left(1+\frac{1}{a+1}\right)$, $\left(1+\frac{1}{a+2}\right)$ and $\left(1+\frac{1}{a+3}\right)$.

14. Write down the square of $x^2 - 2x + 1$ and find the quotient of $\frac{(x^2-2x+1)^2}{1-3x+3x^2-x^3}$.

15. Simplify the expression $(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})(1+x^{32})(1+x^{64})$

Let x = q

Let

$$P = (1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64}).$$

One has

$$(1-q)P = (1-q)(1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64}) = 1-q^{128}.$$

So, one gets

$$P = \frac{1-q^{128}}{1-q}$$