Measurement, Units & Dimensions (IXth)- Exercicse Solutions

Objective Questions:

Level-I

1. (c) Leap year, millisecond and lunar month are the units of time, while light year is the unit of distance, which defined as the distance travelled by light in vacuum in one year.
2. (d) Mass, length and time are the basic or fundamental physical quantities, while density is a derived quantity, which defined as mass per unit volume.
3. (d) $100m^3 = 100(100cm)^3 = 10^8 cm^3$.
4. (c) There are seven basic or fundamental units present in S.I. system which are, mass, length, time, temperature, amount of substance, electric current, luminous intensity.
5. (c) The S.I. units of mass and volume are kg and m^3 respectively hence, the S.I. unit of density is kg/ m^3.
6. (c) Given that,
$$1\frac{g}{cm^3} = x\frac{kg}{m^3}$$
$$1\frac{g}{cm^3} = x\frac{(1000g)}{(100cm)^3} \Rightarrow 1\frac{g}{cm^3} = x\frac{(1000)g}{10^6 cm^3} \Rightarrow x = 10^3 = 1000$$
7. (a) The dimensional formula for velocity can be find out as, we know that v=d/t
Hence, $[v] = \frac{[d]}{[t]} = \frac{L}{T} = [M^0L^1T^{-1}]$
8. (a) The dimensional formula for acceleration can be find out as, we know that a=v/t
Hence, $[a] = \frac{[v]}{[t]} = \frac{[M^0L^1T^{-1}]}{[T]} = [M^0L^1T^{-2}]$. The dimensions of acceleration in mass, length and time are 0, 1, -2.
9. (d) We know that refractive index is a unit less quantity hence its dimensional formula is given by $[M^0L^0T^0]$. For the given equation to be dimensionally correct, each term(additive) of the equation has the same dimensional formula as of refractive index i.e., the term B/λ^2 has dimensional formula $[M^0L^0T^0]$. So, the dimension of B would be
$[B] = [M^0L^0T^0][\lambda^2] = [M^0L^0T^0][L^2] = [M^0L^2T^0]$. The dimensional formula for B is similar to the dimensional formula for area.
10. (b) $1\frac{km}{hr} = 1\frac{(1000m)}{3600\,sec} = \frac{5}{18}m/s$.
11. (a) 7.60 m = 7.60(1000mm) =7600 mm.
12. (d) Mass, density and electric current are not the vector quantities. Impulse is a vector quantity which defined as change in momentum.
13. (d) Displacement is a vector quantity which cannot be added or subtracted algebraically, there are some special methods and laws for vector addition like triangle law and parallelogram law.
14. (b) Given that force, F=40 N, and its one component (say F_1) = $20\sqrt{3}$ N hence , the other component (say F_2) can be find out as $F^2 = F_1^2 + F_2^2$ on solving we get F_2 = 20 N.
15. (c) $\vec{A} + \vec{B} = 3\hat{\imath} + 4\hat{\jmath}$ hence, $|\vec{A} + \vec{B}| = |3\hat{\imath} + 4\hat{\jmath}| = \sqrt{3^2 + 4^2} = 5$

Level-II

1. (d) Given that, $F \propto v \Rightarrow F = kv$, where k is the constant of proportionality.

   The dimensional formula of k can be find as, $[k] = \frac{[F]}{[v]} = \frac{[M^1L^1T^{-2}]}{[M^0L^1T^{-1}]} = [M^1L^0T^{-1}]$

2. (d) For the given equation to be dimensionally correct, each term (additively) of the equation has the same dimensions and hence same unit i.e. the term containing 'c' has unit m hence, unit of 'c' would be $ct^2 \rightarrow m \Rightarrow c \rightarrow \frac{m}{t^2} \rightarrow \frac{m}{s^2}$

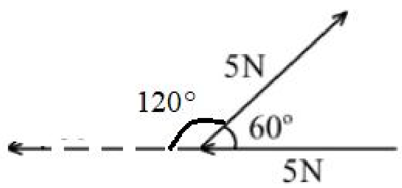
3. (c) The given equation can be written as, $P = \frac{a}{bx} - \frac{t^2}{bx}$, for the given equation to be dimensionally each term (additively) of the equation has the same dimensions and hence the dimensional formula of the term $\frac{a}{bx}$ would be same as of pressure so,

$$[P] = \frac{[a]}{[b][x]} \Rightarrow \frac{[a]}{[b]} = [P][x] = [M^1L^{-1}T^{-2}][M^0L^1T^0] = [M^1L^0T^{-2}]$$

4. (c) Let in this new system of unit the unit is defined as 1X and unit length is defined as 1Y. Now given that, 1X = 100gm and 1Y = 10cm. Hence,

$$4\frac{gm}{cm^3} = 4\frac{10^{-2}(X)}{[10^{-1}(Y)]^3} = 4\frac{10^{-2}(X)}{10^{-3}[Y]^3} = 40\frac{X}{Y^3}\ or\ 40\frac{(100gm)}{(10\ cm)^3}$$

5. (a) $10N = 10\ \frac{kg.m}{s^2} = 10\frac{(1000g)(100cm)}{s^2} = 10^6\ \frac{g.cm}{s^2} = 10^6 dyne$
6. (a) $10^6 dyne.cm^{-2} = 10^6\ (10^{-5}N).(10^{-2}m)^{-2} = 10^5\ N.m^{-2}$
7. (a) $kg^{-1}m^3s^{-2} = (kg.ms^{-2})(kg^{-2}m^2) = N.m^2kg^{-2}$
8. (b) The angle between the given forces is 120° hence, their resultant can be given by,



   $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

   So, $R = \sqrt{5^2 + 5^2 + 2(5)(5)\cos 120} = \sqrt{25 + 25 + 2(25)(-1/2)} = 5N$

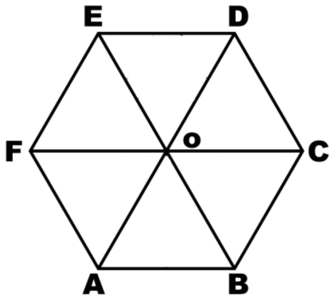
9. (c) The resultant of $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$ is given by,

   $R = \sqrt{(OA)^2 + (OB)^2 + 2(OA)(OB)\cos 90} = \sqrt{r^2 + r^2} = \sqrt{2}r$

   This resultant R will pass through from the angle bisector of OA and OB as both are of equal magnitude r. So, $\vec{R}$ and $\overrightarrow{OC}$ are parallel to each other hence the resultant of these two can be find by simply add their magnitudes hence,

$$|\vec{R} + \overrightarrow{OC}| = |\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}| = \sqrt{2}r + r = r(\sqrt{2} + 1)$$

10. (c)



We have to find the value of $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$. Now,

$\{\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}\} = \overrightarrow{AB} + \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right) + \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}\right) + \left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF}\right)$

$\Rightarrow 5\overrightarrow{AB} + 4\overrightarrow{BC} + 3\overrightarrow{CD} + 2\overrightarrow{DE} + \overrightarrow{EF}$……i)

Now, as it is a regular hexagon hence, its all sides are equal,

So, $\overrightarrow{DE} = -\overrightarrow{AB}$ and $\overrightarrow{EF} = -\overrightarrow{BC}$ (from the figure)

On putting this in equation (i),

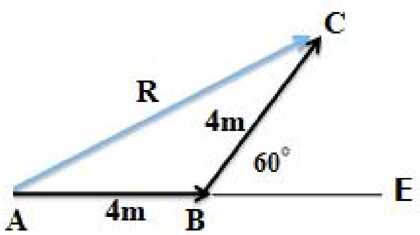
$$\Rightarrow 5\overrightarrow{AB} + 4\overrightarrow{BC} + 3\overrightarrow{CD} + 2\left(-\overrightarrow{AB}\right) + \left(-\overrightarrow{BC}\right)$$

$$\Rightarrow 3\overrightarrow{AB} + 3\overrightarrow{BC} + 3\overrightarrow{CD} = 3\left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right) = 3\overrightarrow{AD}$$

Now from the figure, $\overrightarrow{AD} = 2\overrightarrow{AO}$

Hence, $\left\{\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}\right\} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$

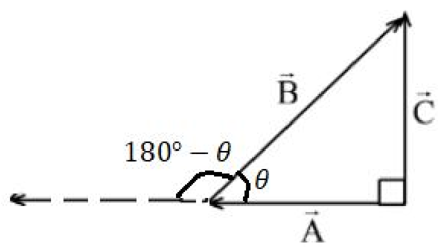
11. (c) The man walks from A to B then, B to C, his net displacement can be given by R as shown in figure.



$R = \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 60} = \sqrt{4^2 + 4^2 + 2(4)(4)\cos 60} = \sqrt{32+16} = 4\sqrt{3}m = 6.9m.$

12. (d) From the figure, $\sin\theta = \frac{C}{B} = \frac{1}{2} \Rightarrow \theta = 30°$

Hence, the angle between $\vec{A}$ and $\vec{B}$ is $180° - \theta = 150°$



13. (c) The x-component of the resultant force will be equal to summation of the x-components of all the forces i.e.,

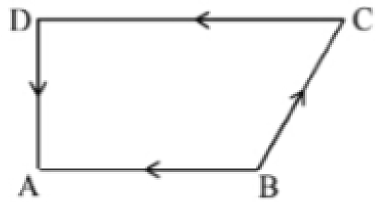
$F_x = 3\cos 0° + 4\cos 90° + 5\cos 217° = 3(1) + 4(0) + 5\left(\frac{-4}{5}\right) = -1\,N$

Similarly the y-component of the resultant force is given by,

$$F_y = 3\sin 0° + 4\sin 90° + 5\sin 217° = 3(0) + 4(1) + 5\left(\frac{-3}{5}\right) = 1\,N$$

Hence, the net force $F^2 = F_x^2 + F_y^2 = (-1)^2 + 1^2 = 2 \Rightarrow F = \sqrt{2}\,N$.

14. (a) The resultant force is given by $\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$



As we know that the vector sum of all the vectors forming a close loop is zero. Hence, from figure

$$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{AB} = \vec{0}$$
$$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = -\overrightarrow{AB}$$
$$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA}$$

On adding $\overrightarrow{BA}$ both side,

$$\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA} + \overrightarrow{BA} = 2\overrightarrow{BA}$$

<u>Subjective Questions:</u>

3. (Ans: $[M^2L^{-2}T^{-2}]$)
Given, force = x/density hence, the dimensional formula of x is given as, $[x] = [Force][Density] = [M^1L^1T^{-2}][M^1L^{-3}T^0] = [M^2L^{-2}T^{-2}]$

4. (Ans:10^{-5})
Given that, $1g.cm.s^{-1} = xN.s$

$$1g.cm.s^{-1} = x\frac{kg.m}{s^2}.s = x\,kg.m.s^{-1}$$
$$1g.cm.s^{-1} = x\,(1000g).(100cm).s^{-1} \Rightarrow x = 10^{-5}$$

5. (Ans: 2.16×10^{12})
Let in this new system of unit the unit is defined as 1X and unit length is defined as 1Y and the unit time is defined as 1Z. Now given that, 1X = 10kg, 1Y = 1dm=0.1m and 1Z = 1min = 60s. Hence, $1MW = 10^6\frac{J}{s} = 10^6\frac{N.m}{s} = 10^6\frac{(kg.m.s^{-2})(m)}{s} = 10^6\,kg.m^2.s^{-3}$

$$1MW = 10^6\,kg.m^2.s^{-3} = 10^6\frac{(10^{-1}X)(10Y)^2}{\left(\frac{1}{60}Z\right)^3} = 10^6 \times 10^{-1} \times 10^2 \times (60)^3\frac{XY^2}{Z^3}$$
$$= 2.16 \times 10^{12}\frac{(10kg)(1dm)^2}{(1min)^3}$$

6. (Ans: km/s^2)
For the given equation to be dimensionally correct, each term (additively) of the equation has the same dimensions and hence same unit i.e. the term containing ‘b’ has unit km hence, unit of ‘b’ would be $bt^2 \to km \Rightarrow b \to \frac{km}{t^2} \to \frac{km}{s^2}$

7. (Ans: (a) FTV^{-1} (b) FTV)

(a) Mass can be related with force(F), velocity (V) and time (T) as, F= M(V/T) hence, the dimensional formula of mass on considering F,V and T as fundamental quantities is given as, $[M] = \frac{[F][T]}{[V]} = [FTV^{-1}]$

(b) Now we know that kinetic energy can be given as, $E = \frac{1}{2}MV^2$, hence, the dimensional formula for energy on considering F,V and T as fundamental quantities is given as, $[E] = [M][V^2] = [FTV^{-1}][V^2] = [FTV]$

8. (Ans: (a) $ML^{-1}T^{-2}$ (b) ML^{-3} (c) ML^2T^{-2})

(a) $[Pressure] = \frac{[Force]}{[Area]} \Rightarrow [P] = \frac{[MLT^{-2}]}{L^2} = [ML^{-1}T^{-2}]$

(b) $[Density] = \frac{[Mass]}{[Volume]} \Rightarrow [D] = \frac{[M]}{[L^3]} = [ML^{-3}T^0]$

(c) $[Work] = [Force][Displacement] \Rightarrow [W] = [MLT^{-2}][L] = [ML^2T^{-2}]$

9. (Ans: $\frac{m^3}{kg.s^2}$ and $\frac{(cm)^3}{g.s^2}$)

Given, $F = \frac{Gm_1m_2}{d^2} \Rightarrow G = \frac{Fd^2}{m_1m_2}$

Hence, the S.I. unit of G is, $G \rightarrow \frac{Nm^2}{(kg).(kg)} = \left(\frac{kg.m}{s^2}\right)\left(\frac{m^2}{(kg).(kg)}\right) = \frac{m^3}{kg.s^2}$

And the C.G.S. unit is $G \rightarrow \frac{(cm)^3}{g.s^2}$

10. (Ans: (a) MLT^{-2} (b) LT^{-2} (c) ML^{-3})

(a) $Force = (mass)(acceleration) = (mass)\left(\frac{change\ in\ velocity}{time}\right)$

Hence, the dimensional formula of force is given as,

$$[F] = \frac{[M][v]}{[T]} = \frac{[M][LT^{-1}]}{[T]} = [MLT^{-2}]$$

(b) $(acceleration) = \left(\frac{change\ in\ velocity}{time}\right)$

Hence, the dimensional formula of acceleration is given as,

$$[a] = \frac{[v]}{[T]} = \frac{[LT^{-1}]}{[T]} = [M^0LT^{-2}]$$

(c) $(Density) = \left(\frac{Mass}{Volume}\right)$

Hence, the dimensional formula of density is given as,

$$[D] = \frac{[M]}{[V]} = \frac{[M]}{[L^3]} = [ML^{-3}T^0]$$

13. (Ans: (a) 8m (b) 11m)

(a) Both the vectors are parallel to each other hence, their sum of the vectors in this condition can be find out by simply adding their magnitudes and the direction of this vector will be in the direction of the given vectors,

$$|\vec{A} + \vec{B}| = 5m + 3m = 8m$$

(b) Similarly,

$$|\vec{A} + \vec{B}| = 8m + 3m = 11m$$

14. (Ans: 4m)

We have to find the magnitude of$(\vec{A} - \vec{B})$. As both the vectors are parallel hence,

$|\vec{A} - \vec{B}| = |\vec{A}| - |\vec{B}| = 7m - 3m = 4m$. The direction of $(\vec{A} - \vec{B})$ will be along $\vec{A}$.

15. (Ans: (a) 4m (b) 7m)

(c) Both the vectors are anti-parallel hence, their sum of the vectors in this condition can be find out by subtracting their magnitudes and the direction of this vector will be in the direction of the vector having larger magnitude,

$$|\vec{A} + \vec{B}| = 6m - 2m = 4m$$

(d) $|\vec{A} + \vec{B}| = 3m + 4m = 7m$

17. (Ans: F_x=4.33 N and F_y=2.5 N)

The x-component of the given force is, $F_x = 5\cos 30° = 5\left(\frac{\sqrt{3}}{2}\right) = 4.33\ N$

And, the y-component is given by, $F_y = 5\sin 30° = 5\left(\frac{1}{2}\right) = 2.5\ N$

18. (Ans: F_x=5 N and F_y=8.66 N)

The x-component of the given force is, $F_x = 10\sin 30° = 10\left(\frac{1}{2}\right) = 5\ N$

And, the y-component is given by, $F_y = 10\cos 30° = 10\left(\frac{\sqrt{3}}{2}\right) = 8.66\ N$

19. (Ans: Dimensionally incorrect)

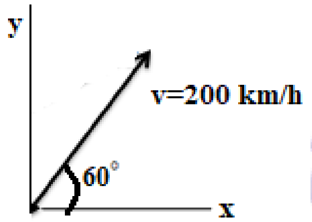
Given equation is, $S_t = u + \frac{1}{2}a(2t - 1)$

$$S_t = u + at - \frac{a}{2}$$

If the equation is dimensional correct then the dimension of each term will be the same so, $[S_t] = [M^0L^1T^0]$, $[u] = [M^0L^1T^{-1}]$, $[at] = [M^0L^1T^{-2}][T] = [M^0L^1T^{-1}]$, and $[a] = [M^0L^1T^{-2}]$.

As we can see the dimensional formula is not the same for each term of the equation and hence the equation is dimensionally incorrect.

20. (Ans: v_x=100 km/hr and v_y=173.2 km/hr)



The X (horizontal)-component of the given velocity is,

$v_x = 200\cos 60° = 200\left(\frac{1}{2}\right) = 100\ kmh^{-1}$

And, the y (vertical)-component is given by,

$v_y = 200\sin 60° = 200\left(\frac{\sqrt{3}}{2}\right) = 173.2\ kmh^{-1}$