INEQUATIONS & EQUATION
EXERCISE-1-A

1. **(B)**

$\Rightarrow \log(ab) - \log|b|$

We can see that $ab > 0 \Rightarrow a < 0 \& b < 0$ or $a > 0 \& b > 0$

So $\log(ab) - \log|b| = \log|ab| - \log|b|$

$= \log|a|.|b| - \log|b|$

$= \log|a| + \log|b| - \log|b|$

$= \log|a|$

2. **(C)**

$\Rightarrow \sqrt{\log_{0.5}^2 4} = \sqrt{\left[\log_{10} 4\right]^2} = \sqrt{\left[\log_{\left(\frac{1}{2}\right)} 4\right]^2}$

$= \sqrt{\left[\log_{\left(\frac{1}{2}\right)}\left(\frac{1}{2}\right)^{-2}\right]^2} = \sqrt{\left(-2 \times \log_{\frac{1}{2}} \frac{1}{2}\right)^2}$

$= \sqrt{(-2 \times 1)} = \sqrt{4}$

$= 2$ …… (as a square root of a value can't be negative)

3. **(B)**

$\Rightarrow \log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$

By formula $\log_a b = \dfrac{\log_c b}{\log_c a}$

Given value is $= \left(\dfrac{\log 4}{\log 3}\right)\left(\dfrac{\log 5}{\log 4}\right)\left(\dfrac{\log 6}{\log 5}\right)\left(\dfrac{\log 7}{\log 6}\right)\left(\dfrac{\log 8}{\log 7}\right)\left(\dfrac{\log 9}{\log 8}\right)$

$= \dfrac{\log 9}{\log 3} = \log_3 9$

$= 2$

4. **(C)**

$\log_7 \log_7 \left(\sqrt{7\sqrt{7\sqrt{7}}}\right)$

$= \log_7 \log_7 \left(7^{1/2}.7^{1/7}.7^{1/8}\right)$

$= \log_7 \log_7 7^{7/8} = \log_7 (7/8)$

$= 1 - 3\log_7 2$

5. **(D)**

$\Rightarrow 81^{\left(\frac{1}{\log_5 3}\right)} + 27^{(\log_9 36)} + 3^{\frac{4}{\log_7 9}}$

$\Rightarrow (81)^{(\log_3 5)} + (3^3)^{(\log_{3^2} 36)} + 3^4 \log_9 7$

$$\Rightarrow \left(3^4\right)^{\log_3 5} + 3^{3\times\left(\frac{1}{2}\log_3 36\right)} + 3^{4\times\left(\frac{1}{2}\log_3 7\right)}$$

$$\Rightarrow 3^{\log_3 5^4} + 3^{\log_3 (36)^{\frac{3}{2}}} + 3^{\log_3 7^2}$$

$$\Rightarrow 5^4 + \left(36\right)^{\frac{3}{2}} + 7^2$$

$$625 + 36\times 6 + 49 = 890$$

6. **(C)**

$$\Rightarrow \log_x x . \log_5 k = \log_x 5 \qquad\qquad ; \text{ given } k \neq 1, k > 0$$

$$\Rightarrow \frac{\log x}{\log k} . \frac{\log k}{\log 5} = \frac{\log 5}{\log x}$$

$$\Rightarrow \left(\log_5 x\right) = \left(\log_x 5\right)$$

$$\Rightarrow x = 5 \text{ is the only possible solution}$$

7. **(C)**

$$\Rightarrow \log_5 a . \log_a x = 2$$

$$\Rightarrow \frac{\log a}{\log 5}, \frac{\log x}{\log a} = 2$$

$$\Rightarrow \log_5 x = 2$$

$$\Rightarrow x = 5^2$$

$$= 25$$

8. **(C)**

$$\Rightarrow A = \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$$

$$\Rightarrow \log_2 \log_2 \log_4 \left(4\right)^2 + \log_{2^{\frac{1}{2}}} \left(2\right)$$

$$\Rightarrow \log_2 \log_2 4 + 2\times \frac{1}{\left(\frac{1}{2}\right)} \log_2 2$$

$$\Rightarrow \log_2 2 + 4 = 1 + 4$$

$$\Rightarrow 5$$

9. **(D)**

$$\Rightarrow \log_{10} x = y \left(\text{given}\right)$$

$$\Rightarrow \log_{1000} x^2 = 2\log_{10^3} x$$

$$\Rightarrow 2\times \frac{1}{3} \log_{10} x \qquad\qquad (\text{by formula } \log_{a^{k^b}} = \frac{1}{k}\log_a b)$$

10. **(A)**

$$\frac{\left(\log a\right)^2}{\log b / \log c} - 1 + \frac{\left(\log b\right)^2}{\log a \log c} - 1 + \frac{\left(\log c\right)^2}{\log a \log b} - 1 = 0$$

$$\Rightarrow \left(\log a\right)^3 + \left(\log b\right)^3 + \left(\log c\right)^3 = 3\log a \log b \log c$$

$\because$ a, b, c are distinct

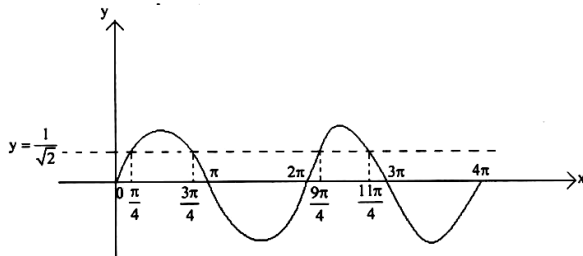$\Rightarrow \log a + \log b + \log c = 0$

$\Rightarrow abc = 1$

11.   (A)

$\Rightarrow \log_{\left(\frac{1}{\sqrt{2}}\right)} \sin x > 0; x \in [0, 4\pi]$

As base $\dfrac{1}{\sqrt{2}}$ lies between 0 to 1 satisfy given inequality, $0 < \sin x < 1$

$\Rightarrow x \in (0, \pi) \cup (2\pi, 3\pi)$

As we can see in this interval

We get $\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}$ as integral

Multiples of $\dfrac{\pi}{4}$

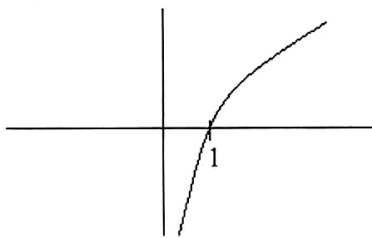12.   (B)

$\Rightarrow \log_{\frac{1}{2}} \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow \log_{2^{-1}} \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow -1 \times \log_2 \left(x^2 - 6x + 12\right) \geq -2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) \leq 2$

$\Rightarrow \log_2 \left(x^2 - 6x + 12\right) - \log_2 4 \leq 0$

$\Rightarrow \log_2 \left(\dfrac{x^2 - 6x + 12}{4}\right) \leq 0$

$\Rightarrow 0 < \dfrac{x^2 - 6x + 12}{4} \leq 1$

Case-1

$\Rightarrow 0 < \dfrac{x^2 - 6x + 12}{4}$

$\Rightarrow x^2 - 6x + 12 > 0$

$\Rightarrow x \in R$ ...... (1) as discriminant if quadratic expression $x^2 - 6x + 12$ is less than zero.

Descriminant $D = (-6)^2 - 4(12)(1)$

$\Rightarrow D = -12$

Case-2
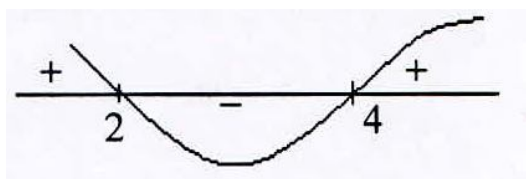
$\Rightarrow \dfrac{x^2 - 6x + 12}{4} \le 4$

$\Rightarrow x^2 - 6x + 12 \le 4$

$\Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x - 4)(x - 2) \le 0$

$\Rightarrow x \in [2, 4]$ ....(ii)

By taking intersection of (i) & (ii) we get $x \in [2, 4]$

13. (B)

$\Rightarrow 2^{\log_{\sqrt{2}}(x-1)} > x + 5$ Here $x - 1 >; x > 1$ ......(1)

$\Rightarrow 2^{\log}(2)^{\frac{1}{2}(x-1)} > x + 5$

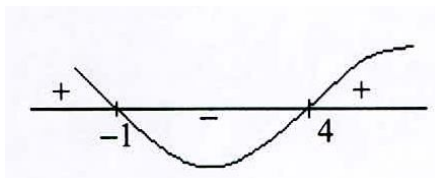$\Rightarrow 2^{2\log_2(x-1)} > x + 5$ (or by formula $\log_{a^k} b = \dfrac{1}{k} \log \dfrac{b}{a}$)

$\Rightarrow 2^{\log_2(x-1)^2} > x + 5$

$\Rightarrow (x - 1)^2 > x + 5$

$\Rightarrow x^2 + 1 - 2x > x + 5$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x - 4)(x + 1) > 0$

So we get $x \in (-\infty, -1) \cup (4, \infty)$ ....(ii)
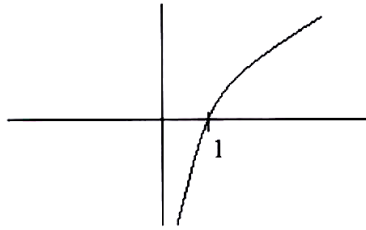
By taking intersection of (i) & (ii)

$\Rightarrow x \in (4, \infty)$

14. (C)

$\Rightarrow \log_{10}(x^2 - 2x - 2) \le 0$

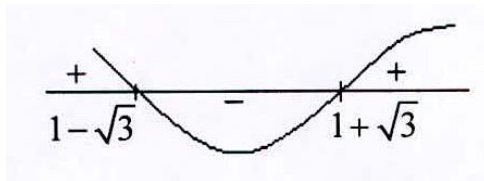As base is greater than 1 so to hold the inequality true

$$\Rightarrow 0 < x^2 - 2x - 2 \le 1$$

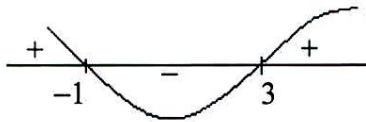So, $0 < x^2 - 2x - 2$ and $x^2 - 2x - 2 \le 1$

Case-I

$$\Rightarrow x^2 - 2x - 2 > 0$$
$$\Rightarrow \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right] > 0$$

So, $x \in \left(-\infty, 1 - \sqrt{3}\right) \cup \left(1 + \sqrt{3}, \infty\right)$ ....(i)

Case-2

$$\Rightarrow x^2 - 2x - 2 \le 1$$

$$\Rightarrow x^2 - 2x - 3 \le 0$$
$$\Rightarrow (x - 3)(x + 1) \le 0$$

So we get $x \in [-1, 3]$

By taking intersection of (i) & (ii) we get,

$$\Rightarrow x \in \left[-1, 1 - \sqrt{3}\right) \cup \left(1 + \sqrt{3}, 3\right]$$

15. (A)
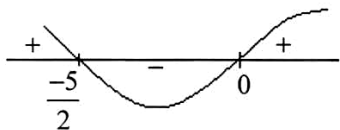$$\Rightarrow \log_{0.2} \frac{x + 2}{x} \le 1$$

As base of log is less than 1 so hold the inequality true

$$\Rightarrow \frac{x + 2}{x} \ge 0.2$$

$$\Rightarrow \frac{x + 2}{x} - 0.2 \ge 0$$

$$\Rightarrow \frac{x + 2 - 0.2x}{x} \ge 0$$

$$\Rightarrow \frac{0.8x + 2}{x} \ge 0$$

So, $x \in \left(-\infty, \dfrac{-5}{2}\right] \cup [0, \infty)$

$\Rightarrow a^{m\log_a n} \Rightarrow a^{\log_a n^m} \Rightarrow n^m$

16. (C)

$\Rightarrow a^{mn} = a^{m^n}$

Take log both side

$\Rightarrow \log_a a^{mn} = \log_a a^{m^n}$

$\Rightarrow mn = m^n$

$\Rightarrow m^{n-1} = n$

$\Rightarrow m = \left(n^{\frac{1}{n-1}}\right)$

18. (B)

$\Rightarrow \left(x^5\right)^{\frac{1}{3}} \left(16x^3\right)^{\frac{2}{3}} \left(\dfrac{1}{4} x^{\frac{4}{9}}\right)^{-\frac{3}{2}} \cdot (4)^{\frac{1}{6}}$

$\Rightarrow x^{\frac{5}{3}} \cdot \left(4^2\right)^{\frac{2}{2}} \cdot \left(x^3\right)^{\frac{2}{2}} \cdot \left(4^{-1}\right)^{\frac{-3}{2}} \cdot \left(x^{\frac{4}{9}}\right)^{\frac{-3}{2}}$

$\Rightarrow x^{\frac{5}{3}} \times 4^{\frac{4}{3}} \times x^2 \times 4^{\frac{3}{2}} \times x^{\frac{4}{9} \times \frac{-3}{2}}$

$x^{\frac{5}{3} - \frac{2}{3} + 2} \cdot 4^{\left(\frac{4}{3} + \frac{3}{2} + \frac{1}{6}\right)}$

$\Rightarrow 4^3 \cdot x^3$

19. (D)

$\Rightarrow \dfrac{\left(2^{n+1}\right)^m \left(2^{2n}\right) 2^n}{\left(2^{m+1}\right)^m 2^{2m}} = 1$

$\Rightarrow \dfrac{2^{nm+m} \times 2^{2n+n}}{2^{mn+m} 2^{2m}} = 1$

$\Rightarrow 2^{(nm+m+2n+n)-(mn+n-2m)} = 1$

$\Rightarrow 2^{2n-m} = 1$

$\Rightarrow 2n - m = 0$

$\Rightarrow m = 2n$

20. (D)

$\Rightarrow x^{x\sqrt[3]{x}} = \left(x \cdot \sqrt[3]{x}\right)^x$ Here $x \neq 0$

$$\Rightarrow x^{x+\frac{1}{3}} = \left(x^{1+\frac{1}{3}}\right)^x$$

$$\Rightarrow x^{x^{\frac{4}{3}}} = x^{\frac{4}{3}x}$$

Take log both side

$$\Rightarrow x^{\frac{4}{3}} \log_x x = \frac{4}{3} x \log_x x$$

$$\Rightarrow x^{\frac{4}{3}} = \frac{4}{2} x$$

$$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3} x$$

$$\Rightarrow x^{\frac{1}{3}} = \frac{4}{3}$$

$$\Rightarrow x = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

21.   (C)

If $x = 2^{\frac{1}{3}} - 2^{-\frac{1}{3}}$

Then $2x^3 + 6x = 2\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]^3 + 6\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]$

$= 2\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)\left[\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)^2 + 3\right]$

$= 2\left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)\left[2^{\frac{2}{3}} + 2^{\frac{-2}{3}} - 2 + 3\right]$

$= 2\left[2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right]\left[2^{\frac{2}{3}} + 2^{\frac{-2}{3}} + 1\right]$

$= 2\left[\left(2^{\frac{1}{3}}\right)^3 - \left(2^{\frac{-1}{3}}\right)^3\right]$ $\qquad \left(\because \text{by } a^3 - b^3 = (a-b)\left(a^2 + b^2 + ab\right)\right)$

$= 2\left[2 - 2^{-1}\right]$

$= 3$

22.   (A)

$$\Rightarrow (x)^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x \qquad\qquad (\text{here } x \neq 0)$$

$$\Rightarrow x^{x^{\frac{3}{2}}} = \left(x^{\frac{3}{2}}\right)^x$$

$$\Rightarrow x^{x^{\frac{3}{2}}} = x^{\frac{3}{2}x}$$

Take log both sides

$$\Rightarrow x^{\frac{3}{2}} \log_x x = \frac{3}{2} x \log_x x$$

$$\Rightarrow x^{\frac{3}{2}} = \frac{3}{2}x$$

$$\Rightarrow x^{\frac{1}{2}} = \frac{3}{2}$$

$$\Rightarrow x = \frac{9}{4}$$

23. **(C)**

$$\Rightarrow 5^{x-1} + 5(0.2)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5\left(\frac{1}{5}\right)^{x-2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{1-x+2} = 26$$

$$\Rightarrow 5^{x-1} + 5^{3-x} = 26$$

At $x = 1, 3$ above equation satisfy

24. **(D)**

$$\Rightarrow x - \frac{1}{x} = 2$$

$$\Rightarrow x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$$

$$= \left[\left(x - \frac{1}{x}\right)^2 + 2\right]^2 - 2$$

$$= \left[2^2 + 2\right]^2 - 2$$

$$= 36 - 2 = 24$$

25. **(C)**

$$\Rightarrow \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$$

$$\Rightarrow x^{\left(\frac{1}{c} - \frac{1}{b}\right)} \times x^{\left(\frac{1}{a} - \frac{1}{c}\right)} \times x^{\left(\frac{1}{b} - \frac{1}{a}\right)}$$

$$\Rightarrow x^{\frac{1}{c} - \frac{1}{b} + \frac{1}{a} - \frac{1}{c} + \frac{1}{b} - \frac{1}{a}}$$

$$\Rightarrow x^0 = 1$$

26. **(C)**

$$\Rightarrow a^m . a^n = a^{mn}$$

$$\Rightarrow a^{m+n} = a^{nm}$$

$$\Rightarrow m + n = mn \qquad \qquad \ldots\ldots(1)$$

Then $m(n-2) + n(m-2) = ?$

$$\Rightarrow 2mn - 2m - 2n$$

$$\Rightarrow 2(m+n) - 2(m+n)$$

27.   (B)

$\Rightarrow \log(x+1) + \log(x-1) = \log 3$

$\Rightarrow \log(x+1)(x-1) = \log 3$

$\Rightarrow x^2 - 1 = 3$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

But $x+1 > 0$ & $x-1 > 0$

$\Rightarrow x > -1$ & $x > 1$

So, $x \in (1, \infty)$ is our feasible region

Only $x = 2$ lies in the feasible region.

28.   (D)

$\Rightarrow f(x) = 5 - |x-3|$

$\Rightarrow f(x) = 5 - (x-3); x \in [3, \infty)$

$\Rightarrow 5 + (x-3); x \in (-\infty, 3)$

$\Rightarrow f(x) = 8 - x; x \in [3, \infty)$

$= 2 + x; x \in (\infty, 3)$

So, greatest value of function occur at $x = 3$

So $f(3) = 8 - 3 = 5$

29.   (B)

$\Rightarrow \dfrac{2^{m+3} \times 3^{2m-2n} \times 5^{m+3+n} \times 6^{n+1}}{6^{m+1} \times 10^{n+3} \times 15^m}$

$\Rightarrow \dfrac{(2^{m+3})(3^{2m-2})(5^{m+n+3}) \times 2^{n+1} \times 3^{n+1}}{(2^{m+1}3^{m+1})(2^{n+3}5^{n+3})(3^m)(5^m)}$

$\Rightarrow 2^{m+3+n+1-(m+1)} 3^{2m-n+n+1-(m+1)-m} 5^{m+n+3-(n+3)-m}$

$\Rightarrow 2^0 3^0 5^0$

$\Rightarrow 1$

30.   (A)

$\Rightarrow x^{\frac{2}{3}} + x^{\frac{1}{3}} = 2$

Take cube both side

$\Rightarrow x^2 + x + 3x^{\frac{2}{3}} x^{\frac{1}{3}} \left(x^{\frac{2}{3}} + x^{\frac{1}{3}} \right) = 8$

$\Rightarrow x^2 + x + 3x(2) = 8$

$\Rightarrow x^2 + 7x - 8 = 0$

$\Rightarrow (x+8)(x-1) = 0$

$\Rightarrow x = 1, -8$

31. **(D)**

If $a + b + c = 0$

$\Rightarrow \dfrac{a^2}{bc} + \dfrac{b^2}{ca} + \dfrac{c^2}{ab} \Rightarrow \dfrac{a^3 + b^3 + c^3}{abc}$

$\Rightarrow \dfrac{\left\{(a+b)^3 - 3ab(a+b)\right\} + c^3}{abc} \Rightarrow \dfrac{\left\{(-c)^3 - 3ab(-c)\right\} + c^3}{abc}$

$\Rightarrow \dfrac{3abc}{abc} = 3$

32. **(C)**

$\Rightarrow 2^x - 2^{x-1} = 4$

$\Rightarrow 2^x - \dfrac{2^x}{2} = 4$

$\Rightarrow 2.2^x - 2^x = 8$

$\Rightarrow 2^x = 8$

$\Rightarrow x = 3$

So, $x^x = 3^x = 27$

33. **(B)**

$\Rightarrow \dfrac{(\log x - \log y)(\log x^2 + \log y^2)}{(\log x^2 - \log y^2)(\log x + \log y)}$

$\Rightarrow \dfrac{(\log x - \log y)2(\log x + \log y)}{2(\log x - \log y)x(\log x + \log y)}$

$= 1$

34. **(B)**

$\Rightarrow \log_{10}(2x^2 + 7x + 16) = 1$

$\Rightarrow 2x^2 + 7x + 16 = 10^1$

$\Rightarrow 2x^2 + 7x + 6 = 0$

$\Rightarrow 2x^2 + 4x + 3x + 6 = 0$

$\Rightarrow (2x + 3)(x + 2) = 0$

$\Rightarrow x = -\dfrac{3}{2}, -2$

35. **(C)**

$\Rightarrow a > 0, b > 0, c > 0$

$\Rightarrow \log(a^a b^b c^c) + \log\left(\dfrac{1}{abc}\right)$

$\Rightarrow \log\left(\dfrac{a^a b^b c^c}{abc}\right)$

$\Rightarrow \log(a^{a-1} b^{b-1} c^{c-1})$

36. (B)

$$\Rightarrow \log_{10}\left[\log_{10}\left(\log_{10} x\right)\right]=0$$

$$\Rightarrow \log_{10}\left(\log_{10} x\right)=10^{0}=1$$

$$\Rightarrow \log_{10} x=10^{1}=10$$

$$\Rightarrow x=10^{10}$$

37. (C)

$$\Rightarrow \left(25\right)^{x-2}=\left(125\right)^{2x-4}$$

$$\Rightarrow \left(5^{2}\right)^{x-2}=\left(5^{3}\right)^{2x-4}$$

$$\Rightarrow 5^{2x-4}=5^{6x-12}$$

$$\Rightarrow 5^{6x-12-(2x-4)}=1$$

$$\Rightarrow 5^{4x-8}=1=5^{\circ}$$

So, $4x-8=0$

$$\Rightarrow x=2$$

38. (C)

$$\Rightarrow \sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}=\frac{13}{6} \text{ here } \frac{x}{1-x}\geq 0;\frac{x}{1-x}\geq 0;\frac{x}{x-1}\leq 0$$

So, $x\in\left[0,1\right)$ is the feasible region for the equation

$$\Rightarrow \frac{\left(x\right)+\left(1-x\right)}{\sqrt{1-x}\sqrt{x}6}=\frac{13}{6}$$

$$\Rightarrow \frac{1}{\sqrt{x\left(1-x\right)}}=\frac{13}{6}$$

Taking square both side

$$\Rightarrow x\left(1-x\right)=\frac{36}{169}$$

$$\Rightarrow x^{2}-x+\frac{36}{169}=0$$

$$\Rightarrow \left(x-\frac{9}{13}\right)\left(x-\frac{4}{13}\right)=0$$

$$\Rightarrow x=\frac{9}{13},\frac{4}{13}$$

Here values lies in the feasible region

So, $x=\frac{9}{13},\frac{4}{13}$

39. (D)

$$\Rightarrow \sqrt{3y+1}=\sqrt{y-1} \qquad\qquad ....(1)$$

$\Rightarrow 3y+1\ge 0\,\&\,y-1\ge 0$

$\Rightarrow y\ge -\dfrac{1}{3}\,\&\,y\ge 0$

$\Rightarrow y\in[0,\infty)$ is our feasible region

By equation (1), taking square of both side,

$\Rightarrow 3y+1=y-1$

$\Rightarrow 2y=-2$

$\Rightarrow y=-1$; which does not lie in feasible range of y.

So no solution of y.

40.　(D)

$\Rightarrow x=7+4\sqrt{3};x=4+4\sqrt{3}+3$

$\Rightarrow x=(2)^2+2.2\sqrt{3}+\left(\sqrt{3}\right)^2$

$\Rightarrow x=\left(2+\sqrt{3}\right)^2$

$\Rightarrow \sqrt{x}=2+\sqrt{3}$　　　　　　　　　　　……..(1)

$\Rightarrow \dfrac{1}{\sqrt{x}}=\dfrac{1}{2+\sqrt{3}}$

$\Rightarrow \dfrac{1}{\sqrt{x}}=\dfrac{1}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}=\dfrac{2-\sqrt{3}}{4-3}$

$\Rightarrow \dfrac{1}{\sqrt{x}}=2-\sqrt{3}$　　　　　　　　　　　………(2)

$\Rightarrow \sqrt{x}+\dfrac{1}{\sqrt{x}}=\left(2+\sqrt{3}\right)+\left(\dfrac{1}{2+\sqrt{3}}\right)$

$=\left(2+\sqrt{3}\right)+\left(2-\sqrt{3}\right)$

$=4$

41.　(A)

$\Rightarrow x^3-3x+9=0$

Let the roots are α,α,β

So, $\alpha+\alpha+\beta=0$

$\Rightarrow 2\alpha+\beta=0$　　　　　　　　　　　……..(1)

$\Rightarrow \alpha.\alpha+\alpha.\beta+\beta.\alpha=-3$

$\Rightarrow \alpha^2+2\alpha\beta=-3$　　　　　　　　　　　………(2)

$\Rightarrow \alpha^2\beta=-q$

$\Rightarrow q=-\alpha^2\beta$　　　　　　　　　　　………(3)

By equations (2), (1)

$\Rightarrow \alpha^2+2\alpha\left(-2\alpha\right)=-3$

$\Rightarrow -3\alpha^2=-3$

$\Rightarrow \alpha^2=1$

$\Rightarrow \alpha=\pm 1$　　　　　　　　　　　………(4)

$\Rightarrow \because q = -\alpha^2\beta$

$= -\alpha^2(-2\alpha)$

$\Rightarrow q = 2\alpha^3$

$\Rightarrow q = \pm 2$

42. **(D)**

$\Rightarrow \log_{16} x + \log_4 x + \log_2 x = 14$

$\Rightarrow \dfrac{1}{4}\log_2 x + \dfrac{1}{2}\log_2 x + \log_2 x = 14 \qquad\qquad \left(\text{by } \log_{a^k b} = \dfrac{1}{k}\log_a b\right)$

$\Rightarrow \dfrac{7}{4}\log_2 x = 14$

$\Rightarrow \log_2 x = 8$

$\Rightarrow x = 2^8 = 256$

43. **(C)**

$\Rightarrow |4 - 3x| \le \dfrac{1}{2}$

Case 1: $4 - 3x \ge 0 \Rightarrow x \le \dfrac{4}{3}$ \qquad\qquad\qquad …….(i)

So $4 - 3x \le \dfrac{1}{2}$

$\Rightarrow -3x \le -\dfrac{7}{2}$

$\Rightarrow x \ge \dfrac{7}{6}$

By intersection of (i) & (ii) $x \in \left[\dfrac{7}{6}, \dfrac{4}{3}\right]$ ……..(A)

Case 2: $4 - 3x \le 0 \Rightarrow x > \dfrac{4}{3}$ \qquad\qquad\qquad ………(iii)

$\Rightarrow$ So $-(4 - 3x) \le \dfrac{1}{2}$

$\Rightarrow -4 + 3x \le \dfrac{1}{2}$

$\Rightarrow 3x \le \dfrac{9}{2}$

$\Rightarrow x \le \dfrac{3}{2}$ \qquad\qquad\qquad ………(iv)

Taking intersection of (iii) & (iv)

$\Rightarrow x \in \left(\dfrac{4}{3}, \dfrac{3}{2}\right]$ \qquad\qquad\qquad ……….(B)

So union of A & B is the solution of the given inequality $x \in \left[\dfrac{7}{6}, \dfrac{3}{2}\right]$ \qquad\qquad ……..(C)

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44. (A)

$\Rightarrow x^2 - |x| - 6 = 0$

Case 1: $x \geq 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x = 3, -2$

But $x \geq 0$ so $x = 3$ is the only root.

Case 2: $x < 0$

$\Rightarrow x^2 - (-x) - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x = 3, -2$

But $x < 0$ so $x = -3$ is the solution.

So multiplication $= 3(-3) = -9$
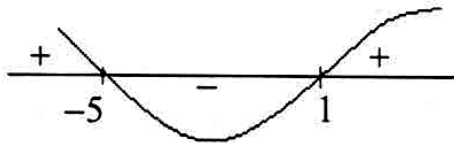
45. (C)

$\Rightarrow \dfrac{|x| - 1}{|x| + 2} > 0$

So, $|x| > -2$ or $|x| > 1$

$\because |x| > -2$ holds true for $x \in R$

Now, $|x| > 1$

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$

46. (A)

$\Rightarrow |x - 1| + |x + 5| = 6$

This is special case as $(x - 1) - (x + 5) = -6$

So the given expression will hold true if $(x - 1)(x + 5) \leq 0$

$\Rightarrow x \in [-5, 1]$

47. (D)

$\Rightarrow \log_{\frac{1}{2}}(x^2 - 1) > 0$

As base of log is less than 1

So, $\log_{\frac{1}{2}}(x^2 - 1) > 0$

$\Rightarrow 0 < x^2 - 1 < 1$

$$\Rightarrow 1 < x^2 < 2$$

$$\Rightarrow x^2 > 1 \text{ and } x^2 < 2$$

$$\Rightarrow x \in (-\infty, -1) \cup (1, \infty) \,\&\, x \in (-\sqrt{2}, \sqrt{2}) \Rightarrow x \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$$

48. **(D)**

$$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$$

$$\Rightarrow \log_2 x + \log_x 2 = \frac{10}{3}$$

$$\Rightarrow \log_2 x + \frac{1}{\log_2} x = \frac{10}{3}$$

Let's take $\log_2 x = a$

So, $a + \dfrac{1}{a} = \dfrac{10}{3}$

$$\Rightarrow a^2 - \frac{10}{3}a + 1 = 0$$

$$\Rightarrow \left(a - \frac{9}{3}\right)\left(a - \frac{1}{3}\right) = 0$$

$$\Rightarrow a = \frac{9}{3}, \frac{1}{3}$$

$$\Rightarrow \log_2 x = \frac{9}{3}, \frac{1}{3} = 3, \frac{1}{3}$$

$$\Rightarrow x = 2^3, 2^{\frac{1}{3}}$$

$$\Rightarrow x = 8, 2^{\frac{1}{3}}$$

Similarly, $y = 8, 2^{\frac{1}{3}}$

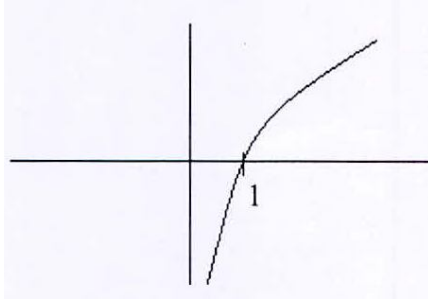If $x \neq y$ then $x + y = 8 + 2^{\frac{1}{3}}$

49. **(A)**

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(x^2 - 3x + 2\right) \geq 2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(x^2 - 3x + 2\right) \geq \log_{\frac{\sqrt{3}}{2}} \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \log_{\frac{\sqrt{3}}{2}} \left(\frac{x^2 + 3x + 2}{\left(\frac{3}{4}\right)}\right) \geq 0$$

$\because$ base is less than 1 so

$$\Rightarrow 0 < \left(\frac{x^2 - 3x + 2}{\frac{3}{4}}\right) \leq 1$$

$$\Rightarrow 0 < x^2 - 3x + 2 \,\&\, \left(x^2 - 3x + 2\right) \le \frac{3}{4}$$

$$\Rightarrow (x-2)(x-1) > 0 \,\&\, x^2 - 3x + \frac{5}{4} \le 0$$

$$\Rightarrow x \in (-\infty, 1) \cup (2, \infty) \qquad \text{.......(1)} \,\&\, x^2 - \frac{5}{2}x - \frac{1}{2}x + \frac{5}{4} \le 0$$

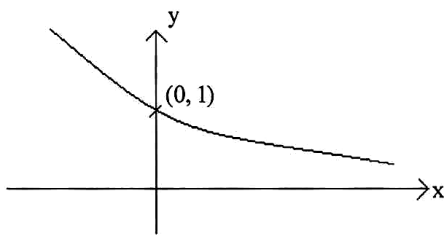$$\&\left(x - \frac{5}{2}\right)\left(x - \frac{1}{2}\right) \le 0$$

$$\& x \in \left(\frac{1}{2}, \frac{5}{2}\right) \qquad \text{.......(2)}$$

Taking intersection of (i) & (ii)

$$\Rightarrow x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$$

50.   (A)

$$\Rightarrow a^{x^2 - x} \ge a^2 ; 0 < 9 < 1$$

$$\Rightarrow \frac{a^{x^2 - x}}{a^2} \ge 1$$

$$\Rightarrow a^{x^2 - x - 2} \ge 1$$

$$\Rightarrow x^2 - x - 2 \le 0$$

$$\Rightarrow (x-2)(x+1) \le 0$$
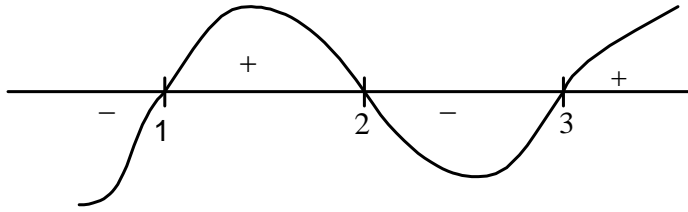
$$\Rightarrow x \in [-1, 2]$$

51.   (C)

$$(x-1)\left(x^2 - 5x + 7\right) < (x-1)$$

$$(x-1)\left[x^2 - 5x + 6\right] < 0$$

$$(x-1)(x-2)(x-3) < 0$$

$x \in (-\infty, 1) \cup (2, 3)$

52. **(D)**

$\left\| x \right| - 1 \right| < \left| 1 - x \right|$

Case I $x \geq 1$

$x - 1 < -(1 - x)$

$x - 1 < -(1 - x)$

$-1 < -1$

No solution

Case II $0 \leq x \leq 1$

$1 - x < 1 - x$

No solution

Case III $-1 \leq x < 0$

$\left| -x - 1 \right| < 1 - x$

$1 + x < 1 - x$

$x < 0$

$\Rightarrow x \in [-1, 0)$

Case IV: $x < -1$

$\left| -x - 1 \right| < \left| 1 - x \right|$

$\left| 1 + x \right| < 1 - x$

$-(1 + x) < 1 - x$

$-1 - x < 1 - x$

$-1 < 1$

True for all x.

$x \in (-\infty, -1)$

$\therefore x \in (-\infty, -1] \cup [-1, 0)$

$\Rightarrow x \in (-\infty, 0)$

53. **(A)**

$4^{-x+0.5} - 7.2^{-x} < 4$

$\dfrac{4^{0.5}}{4^x} - \dfrac{7}{2^x} < 4$
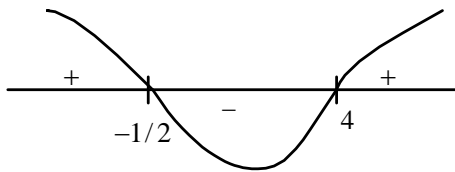
Let $\dfrac{1}{2^x} = k$

$\therefore 2.k^2 - 7k < 4$

$2k^2 - 7k - 4 < 0$

$2k^2 - 7k - 4 < 0$

$2k^2 - 8k + k - 4 < 0$

$2k(k-4) + (k-4) < 0$

$(2k+1)(k-4) < 0$

$k \in \left(-\dfrac{1}{2}, 4\right)$

As $k = \dfrac{1}{2^x}$ it can only be +ve

$\therefore 0 < \dfrac{1}{2^x} < 4$
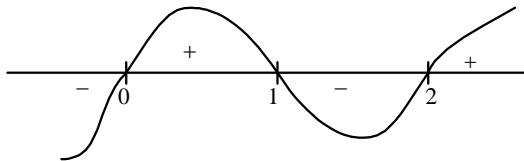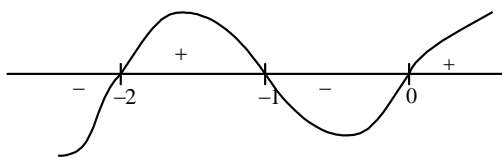
$\Rightarrow x \in (-2, \infty)$

54.   (A)

$\left| x + \dfrac{2}{x} \right| < 3$

$-3 < x + \dfrac{2}{x} < 3$

$x + \dfrac{2}{x} + 3 > 0$, $x + \dfrac{2}{x} - 3 < 0$

$\dfrac{x^2 + 3x + 2}{x} > 0$, $\dfrac{x^2 - 3x + 2}{x} < 0$

$\dfrac{(x+1)(x+2)}{x} > 0$, $\dfrac{(x-1)(x-2)}{x < 0}$

$x \in (-2, -1) \cup (0, \infty)$ $x \in (-\infty, 0) \cup (1, 2)$

$\therefore x \in (-2, -1) \cup (1, 2)$

55.   (D)

$2^x + 2^{|x|} \geq 2\sqrt{2}$

Case I $x \geq 0$

$2^x + 2^x \geq 2\sqrt{2}$

$2^x \geq \sqrt{2}$

$\Rightarrow x \geq \dfrac{1}{2}$
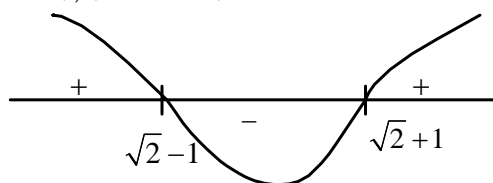
Case II $x < 0$

$2^x + \dfrac{1}{2^x} \geq 2\sqrt{2}$

Let $2^x = k \qquad k + \dfrac{1}{k} \geq 2\sqrt{2}$

$k^2 - 2\sqrt{2}k + 1 \geq 1$

$k = \dfrac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \dfrac{2\sqrt{2} \pm 2}{2}$

$K = \sqrt{2} \pm 1$

$\left(k - \left(\sqrt{2}+1\right)\right)\left(k - \sqrt{2}-1\right) \geq 0$

$-\infty < 2^x \leq \sqrt{2}-1 \ \& \ \sqrt{2}+1 \leq 2^x < \infty$

$\Rightarrow 0 < 2^x \leq \sqrt{2}-1 \ \& \ \sqrt{2}+1 \leq 2^x < \infty$

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \ \& \ \left[\log_2\left(\sqrt{2}+1\right), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup \left[\dfrac{1}{2}, \infty\right)$

56. (A)

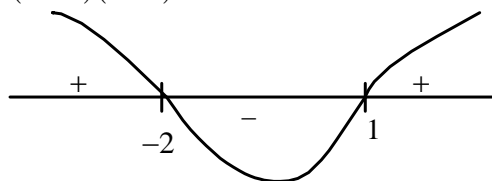$\log_{1/3}\left(x^2 + x + 1\right) + 1 < 0$

$\log_{1/3}\left(x^2 + x + 1\right) < -1$

$x^2 + x + 1 > \left(\dfrac{1}{3}\right)^{-1}$

$x^2 + x + 1 > 3$

$x^2 + x - 2 > 0$

$(x+2)(x-1) > 0$

$x \in (-\infty, -2) \cup (1, \infty)$

57. (B)

$x^2 = |x+2| + x > 0$

Case I $x \geq -2$

$x^2 - (x+2) + x > 0$

$x^2 - 2 > 0$

$x \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

Case II $x \le -2$

$x^2 + (x+2) + x > 0$

$x^2 + 2x + 2 > 0$

$(x+1)^2 + 1 > 0$

$x \in R$ i.e. $x < -2$

From (1) & (2)

$x \in \left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

58. (C)

$\log_{1/2} x \ge \log_{1/3} x$

$-\log_2 x \ge -\log_3 x$

$\log_2 x \le \log_3 x$

$\dfrac{\log x}{\log 2} \le \dfrac{\log x}{\log 3}$

$\log^x \left[\dfrac{1}{\log 2} - \dfrac{1}{\log 3} \right] \le 0$

$\log x \le 0 \left\{ \because \dfrac{1}{\log 2} - \dfrac{1}{\log 3} > 0 \right\}$

$\Rightarrow x \in (0, 1]$

59. (B)

$x^{\log_3 x^2} + (\log 3x)^2 - 10 = \dfrac{1}{x^2}$

Clearly one solution is $x = 1$

OR

$\log_3 x^2 + (\log_3 x)^2 - 10 = -2$

$2\log_3 x + (\log_3 x)^2 = 8$

Let $\log_3 x = k$

$\therefore 2k + k^2 - 8 = 0$

$k^2 + 2k - 8 = 0$

$(k+4)(k-2) = 0$

$\Rightarrow \log_3 x = 2$ or $\log_3 x = -4$

$\Rightarrow x = 0$ or $x = \dfrac{1}{81}$

$x \in \left\{ 1, 9, \dfrac{1}{81} \right\}$

60. (B)

$x = \log_a nc = \dfrac{\log bc}{\log a}$

$x + 1 = \dfrac{\log bc}{\log a} + 1$

$\Rightarrow x + 1 = \dfrac{\log bc + \log x}{\log a}$

$\Rightarrow x + 1 = \dfrac{\log abc}{\log a}$

Similarly

$y + 1 = \dfrac{\log abc}{\log b}$ & $z + 1 = \dfrac{\log abc}{\log c}$

$\therefore \dfrac{1}{x + 1} + \dfrac{1}{y + 1} + \dfrac{1}{z + 1}$

$\Rightarrow \dfrac{\log a}{\log abc} + \dfrac{\log b}{\log abc} + \dfrac{\log c}{\log abc}$

$= \dfrac{\log abc}{\log abc} = 1$

INEQUATIONS & EQUATION
EXERCISE 1 (B)

1. **(D)**

$$\Rightarrow \left|3^x - 1\right| > \left|3^x - 9\right|$$

Take square both side

$$\Rightarrow \left(3^x - 1\right)^2 - \left(3^x - 9\right)^2 > 0$$

$$\Rightarrow \left[\left(3^x - 1\right) + \left(3^x - 9\right)\right]\left[3^x - 1 - \left(3^x - 9\right)\right] > 0$$

$$\Rightarrow \left[2.3^x - 10\right][8] > 0$$

$$\Rightarrow 3^x - 5 > 0 \Rightarrow 3^x > 5$$

$$x > \log_3 5$$

2. **(B)**

$$\Rightarrow \sqrt{25 - 5^x} = \sqrt{4^x - 16} \qquad \qquad ………(1)$$

Here $25 - 5^x \geq 0$ & $4^x - 16 \geq 0$

$$\Rightarrow 5^x \leq 25 \qquad \& \qquad 4^x \geq 4^2$$

$$\Rightarrow 5^x \leq 5^2 \qquad \& \qquad x \geq 2$$

$$\Rightarrow x \in \left(-\infty, 2\right] \qquad \& \qquad x \in \left[2, \infty\right)$$

So only feasible region for given equation is $x = 2$

For $x = 2$, gives equation is satisfied

So no of solutions $= 1$

3. **(D)**

$$\sqrt{4x + 1} + \sqrt{7 - x} = 0$$

As square root is always positive so given equation is feasible only if

$$\Rightarrow 4x + 1 = 0 \qquad \& \qquad 7 - x = 0$$

$$\Rightarrow x = -\frac{1}{4} \qquad \& \qquad x = 7$$

So no common solution.

4. **(C)**

$$\Rightarrow 4^{x^2 + 2} - 9.2^{x^2 + 2} + 8 = 0$$

$$\Rightarrow 2^{2x^2 + 4} - 9.2^2 2^{x^2 + 8} = 0$$

$$\Rightarrow 16.2^{2x^2} - 36.2^{x^2} + 8 = 0$$

$$\Rightarrow 4.2^{2x^2} - 9.2x^2 + 2 = 0$$

Put $2^{x^2} = a$

so, $4a^2 - 9a + 2 = 0$

$$\Rightarrow (4a - 1)(a - 2) = 0$$

$$\Rightarrow a = \frac{1}{4}, 2$$

$$\Rightarrow 2^{x^2} = \frac{1}{4}, 2$$

$\Rightarrow x^2 = -2, 1$

$\Rightarrow x^2 = -2$ is not possible; $x^2 = 1$

$\Rightarrow x = \pm 1$

5. (A)

$\Rightarrow \sqrt{(x-1)^2} + \sqrt[4]{(2x+1)^4} - \sqrt[3]{\left(x - \dfrac{1}{2}\right)^3}$; $x \in (0,1)$

$\Rightarrow |x-1| + (2x+1) - \left(x - \dfrac{1}{2}\right)$ \qquad (as $1 + 2x > 0$ for $x \in (0,1)$)

$\Rightarrow -(x-1) + (2x+1) - \left(x - \dfrac{1}{2}\right)$

$\Rightarrow \dfrac{5}{2}$

6. (A)

$\Rightarrow \dfrac{2^{x-1}}{2^{x+1} + 1} < 2$

We can cross multiply $\left(2^{x+1} + 1\right)$ as $2^{x+1} + 1 > 0$ for $x \in R$

$\Rightarrow 2^{x-1} - 1 < 2\left(2^{x+1} + 1\right)$

$\Rightarrow \dfrac{2^x}{2} - 1 < 4.2^x + 4$

$\Rightarrow \dfrac{7}{2} 2^x > -5$

$\Rightarrow 2^x > \dfrac{-10}{7}$

This is true for $x \in R$

7. (C)

$\Rightarrow \sqrt{2^{2x} - 7} < 2^x - 1$ \qquad ....(1)

Here $2^{2x} - 7 \geq 0$

$\Rightarrow 2^{2x} \geq 7$

$\Rightarrow 2x \geq \log_2 7$

$\Rightarrow x \geq \dfrac{1}{2} \log_2 7$ \qquad ....(2) (feasible region)

From feasible region it is clear that $2^x - 1 > 0$

So by taking square of (1)

$\Rightarrow 2^{2x} - 7 < \left(2^x - 1\right)^2$

$\Rightarrow 2^{2x} - 7 < 2^{2x} + 1 - 2.2^x$

$\Rightarrow 2.2^x < 8$

$\Rightarrow 2^x < 4$

$\Rightarrow x < 2$ ....(2)

By taking intersection of (1) & (2)

We get $x \in \left[\dfrac{1}{2}\log_2 7, 2 \right)$

$\Rightarrow x \in \left[\log_4 7, 2 \right)$

8. (A)

$\Rightarrow \left| x^3 - 1 \right| \geq 1 - x$

Case I: $x^3 - 1 \geq 0$

$\Rightarrow x \geq 1$

So $\left(x^3 - 1 \right) \geq 1 - x$

$\Rightarrow x^3 + x - 2 \geq 0$

$\Rightarrow (x - 1)\left(x^2 + x + 2 \right) \geq 0$
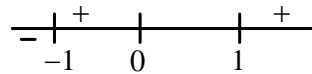
$\Rightarrow x \in [1, \infty)$ ....(1)

Case 2: $x^3 - 1 < 0$

$\Rightarrow x^3 < 1 \Rightarrow x < 1$

So, $-\left(x^3 - 1 \right) \geq 1 - x$

$\Rightarrow -x^3 + x \geq 0$

$\Rightarrow x^3 - x \geq 0$

$\Rightarrow x(x - 1)(x + 1) \leq 0$

$\Rightarrow x \in (-\infty, 1] \cup [0, 1]$ ....(2)

So take union of (1) & (2)

$\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$

9. (B)

$\Rightarrow \dfrac{|x + 2| - x}{x} < 2$

Case I: $x + 2 \geq 0$

$\Rightarrow x > -2$ ....(i)

$\Rightarrow \dfrac{(x + 2) - x}{x} < 2$

$\Rightarrow \dfrac{2}{x} - 2 < 0$

$\Rightarrow \dfrac{1 - x}{x} < 0$

$\Rightarrow \dfrac{x - 1}{x} < 0$

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

Taking intersection of (i) & (ii)

$\Rightarrow x \in [-2, 0) \cup (1, \infty)$ ....(iii)

Case 2: $\Rightarrow x + 2 < 0 \Rightarrow x \in (-\infty, -2)$ ....(A)

$\Rightarrow \dfrac{-(x+2) - x}{x} < 2$

$\Rightarrow \dfrac{-2x - 2 - 2x}{x} < 0$

$\Rightarrow \dfrac{2x - 1}{x} > 0$

$\Rightarrow x \in \left(-\infty, -\dfrac{1}{2}\right) \cup (0, \infty)$ ....(B)

Taking intersection of A & B

$\Rightarrow x \in (-\infty, -2)$ ....(C)

So by taking union of (iii) & (c)

$\Rightarrow x \in (-\infty, 0) \cup (1, \infty)$

10. (A)

$\Rightarrow \dfrac{x^2 - 5x + 4}{x^2 - 4} \le 1$

$\Rightarrow -1 \le \dfrac{x^2 - 5x + 4}{x^2 - 4} \le 1$

Case 1: $\dfrac{x^2 - 5x + 4}{x^2 - 4} \ge -1$

$\Rightarrow \dfrac{x^2 - 5x + 4 + x^2 - 4}{x^2 - 4}$

$\Rightarrow \dfrac{x(2x - 5)}{(x - 2)(x + 2)} \ge 0$

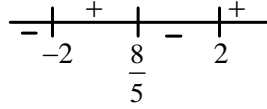$\Rightarrow x \in (-\infty, -2) \cup [0, 2) \cup \left[\dfrac{5}{2}, \infty\right)$ ....(1)

Case 2: $\dfrac{x^2 - 5x + 4}{x^2 - 4} \le 1$

$\Rightarrow \dfrac{x^2 - 5x + 4 - x^{2+4}}{x^2 - 4} \le 0$

$$\Rightarrow \frac{8-5x}{(x-2)(x+2)} \le 0$$

$$\Rightarrow \frac{(5x-8)}{(x-2)(x+2)} \ge 0$$

$$\Rightarrow x \in \left(-2, \frac{8}{5}\right] \cup (2, \infty)$$

By taking intersection of (1) & (2)

$$\Rightarrow x \in \left[0, \frac{8}{5}\right] \cup \left[\frac{5}{2}, \infty\right)$$

11. (C)

$$\Rightarrow \frac{x^2 - |x| - 12}{x - 2} \ge 2x$$

Case 1: $x \ge 0$

$$\Rightarrow \frac{x^2 - x - 12}{x - 3} \ge 2x$$

$$\Rightarrow \frac{x^2 - x - 12}{x - 3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2 - x - 12}{x - 3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2 - x - 12 - 2x^2 + 6x}{x - 3} \ge 0$$

$$\Rightarrow \frac{-x^2 + 5x - 12}{x - 3} \ge 0$$

$$\Rightarrow \frac{x^2 - 5x + 12}{x - 3} \le 0$$

as $x^2 - 5x + 12$ is always greater than zero for $x \in R$

so, $\dfrac{1}{x - 3} \le .0$

$$\Rightarrow x \in (-\infty, 3)$$

Case 2: $x < 0$

$$\Rightarrow \frac{x^2 + x - 12}{x - 3} - 2x \ge 0$$

$$\Rightarrow \frac{x^2 + x - 12 - 2x^2 + 6x}{x - 3} \ge 0$$

$$\Rightarrow \frac{-x^2 + 7x - 12}{x - 3} \ge 0$$

$$\Rightarrow \frac{x^2 - 7x + 12}{x - 3} \le 0$$

$\Rightarrow \dfrac{(x-4)(x-3)}{(x-3)} \le 0$

$\Rightarrow x - 4 \le 0$

$\Rightarrow x \in (-\infty, 4]$

But for this case $x < 0$

So we get

$\Rightarrow x \in (-\infty, 0)$

Take union of $(1) \& (2)$

$\Rightarrow x \in (-\infty, 3)$

12. (C)

$\Rightarrow \left| \dfrac{x^2 - 3x - 1}{x^2 + x + 1} \right| < 3$

$\Rightarrow -3 \dfrac{x^3 - 3x - 1}{x^2 + x + 1} < 3$

Case 1: $\dfrac{x^2 - 3x - 1}{x^2 + x + 1} > -3$

As $(x^2 + x + 1)$ is always greater than zero

So $x^2 - 3x - 1 > -3(x^2 + x + 1)$

$\Rightarrow 4x^2 + 2 > 0$

$\Rightarrow x \in R$ ....(1)

Case 2: $\dfrac{x^2 - 3x + 1}{x^2 + x + 1} < 3$

$\Rightarrow x^2 - 3x - 1 < 3x^2 + 3x + 3$

$\Rightarrow 2x^2 + 6x + 4 > 0$

$\Rightarrow x^2 + 3x + 2 => 0$

$\Rightarrow (x+1)(x+2) > 0$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ ....(2)

Take intersection of $(1) \& (2)$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$
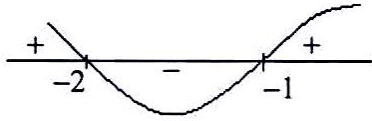
13. (D)

$\Rightarrow \dfrac{|x+3| + x}{x+2} > 1$

$\Rightarrow \dfrac{|x+3| + x}{x+2} - 1 > 0$

$\Rightarrow \dfrac{|x+3| + x - x - 2}{x+1 > 0} > 0$

$\Rightarrow \dfrac{|x+3| - 2}{x+2} > 0$

Case 1: $x + 3 \ge x \ge -3$

$$\Rightarrow \frac{x+3-2}{x+2}>0$$

$$\Rightarrow \frac{x+1}{x+2}>0$$

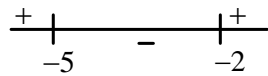$$\Rightarrow x \in (-\infty,-2)\cup(-1,-\infty)$$

But for this case $x \geq -3$

So we get $x \in [-3,-2)\cup(-1,\infty)$

Case 2: $x+3<0$

$$\Rightarrow x<-3$$

So, $\dfrac{-(x+3)}{x+2}>0$

$$\Rightarrow \frac{x+5}{x+2}<0$$

$$\Rightarrow x \in (-5,-2)$$

As $x<-3$ so $x \in (-5,-3)$ ......(2)

Take union of $(1)\&(2)$

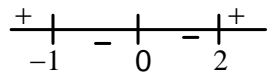$$\Rightarrow x \in (-5,-2)\cup(-1,\infty)$$

So least integral value of $x=-4$

14. (A)

$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)}<\frac{2}{3}$$

So, $1-\left(\dfrac{x+2}{x^2}\right)\geq 0$

$$\Rightarrow \frac{x^2-x-2}{x^2}\geq 0$$

$$\Rightarrow \frac{(x-2)(x+1)}{x^2}\geq 0$$

So $x \in (-\infty,-1]\cup[2,\infty)$ ......(1)

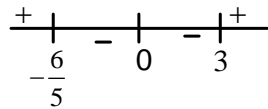$$\Rightarrow \sqrt{1-\left(\frac{x+2}{x^2}\right)}<\frac{2}{3}$$

$$\Rightarrow 1-\left(\frac{x+2}{x^2}\right)<\frac{4}{9}$$

$$\Rightarrow \frac{5}{9} - \frac{x+2}{x^2} < 0$$

$$\Rightarrow \frac{5x^2 - 9x - 18}{9x^2} < 0$$

$$\Rightarrow \frac{(5x+6)(x-3)}{x^2} < 0$$

So, $x \in \left(-\frac{6}{5}, 0\right) \cup (0, 3)$ ....(2)

By taking $(1) \cap (2)$

$$\Rightarrow x \in \left(-\frac{6}{5}, -1\right] \cup [2, 3)$$

15. $\Rightarrow \sqrt{4 - \sqrt{1-x}} - \sqrt{2-x} > 0$

Here $1 - x \geq 0$

$\Rightarrow 2 - x > 0$

$\Rightarrow x \leq 1$ ....(i)

$\Rightarrow 4 - \sqrt{1-x} \geq 0$

$\Rightarrow 4 \geq \sqrt{1-x}$

$\Rightarrow 16 \geq 1 - x$

$\Rightarrow x > -15$ ....(ii)

So, $(i) \cap (ii)$

$\Rightarrow x \in [-15, 1]$ .... (iii) feasible region

Take intersection of $(iv) \& (iii)$

$\Rightarrow x \in [-2, 1]$

Now

$\Rightarrow \sqrt{4 - \sqrt{1-x}} > \sqrt{2-x}$

Take square both side

$\Rightarrow 4 - \sqrt{1-x} > 2 - x$

$\Rightarrow 2 + x > \sqrt{1-x}$

Here $2 + x \geq 0 \Rightarrow x \geq -2$ ....(iv)

So, $2 + x > \sqrt{1-x}$

Take square $4 + x^2 + 4x > 1 - x$

$\Rightarrow x^2 + 5x + 3 > 0$

$$\Rightarrow \left(x - \left(\frac{-5 + \sqrt{13}}{2}\right)\right)\left(x - \left(\frac{-5 - \sqrt{13}}{2}\right)\right)$$

$$\Rightarrow x \in \left(-\infty, \frac{-5 - \sqrt{13}}{2}\right) \cup \left(\frac{\sqrt{13} - 5}{2}, \infty\right)$$ ....(2)

$(1) \cap (2)$

$$\Rightarrow x \in \left(\frac{\sqrt{13}-5}{2}, 1 \right)$$

16.   (A)

$$\Rightarrow \sqrt{4-x^2} + \frac{|x|}{x} \geq 0$$

Here $4 - x^2 \geq 0$

$\Rightarrow x^2 \leq 4$

$\Rightarrow x \in [-2, 2]$ ....(i)

Case 1: $x > 0$

$\Rightarrow \sqrt{4-x^2} + \frac{x}{x} \geq 0$ $(x \neq 0)$

$\Rightarrow \sqrt{4-x^2} + 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq -1$ true for $x \in R$

Case 2: $x < 0$

$$\Rightarrow \sqrt{4-x^2} + \frac{(-x)}{x} \geq 0$$

$\Rightarrow \sqrt{4-x^2} - 1 \geq 0$

$\Rightarrow \sqrt{4-x^2} \geq 1$

Take square $4 - x^2 \geq 1 \Rightarrow x^2 \leq 3$

$\Rightarrow x \in \left[-\sqrt{3}, \sqrt{3} \right]$

But $x < 0$

So $x \in \left[-\sqrt{3}, 0 \right)$

So case (1) $\cup$ case (2)

$\Rightarrow x \in R$ ....(iii)

But our feasible region is $x \in [-2, 2]$

So greatest integral $x = 2$

17.   (C)

$$\Rightarrow \frac{|x+2| - |x|}{\sqrt{4-x^3}} \geq 0 \text{ here } 4 - x^3 > 0$$

$\Rightarrow x^3 < 4 \Rightarrow x \in \left(-\infty, \sqrt[3]{4} \right)$ ....(1)

As $\left(4 - x^3 \right)$ is greater than zero

$\Rightarrow |x+2| - |x| \geq 0$

$\Rightarrow |x+2| \geq |x|$

$\Rightarrow (x+2)^2 \geq x^2$

$\Rightarrow (x+2-x)(x+2+x) \geq 0$

$\Rightarrow 2(2x+2) \geq 0$

$\Rightarrow x \geq -1$ ....(2)

So $(1) \cap (2)$

$\Rightarrow x \in \left[-1, \sqrt[3]{4}\right)$

18. (B)

$\Rightarrow \log_4 \left(\dfrac{x+1}{x+2}\right) > 0$

$\Rightarrow \dfrac{x+1}{x+2} > 0$

$\Rightarrow \dfrac{x+1-x-2}{x+1} > 0$

$\Rightarrow \dfrac{-1}{x+2} > 0$

$\Rightarrow \dfrac{1}{x+2} < 0$

$\Rightarrow x \in (-\infty, -2)$

19. (A)

$\Rightarrow x^3 + ax + b = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = a$

$\Rightarrow \alpha\beta\gamma = -b$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) + \gamma^3$

$= (-\gamma)\left((\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta\right) + \gamma^3$

$= (-\gamma)(\gamma^2 - 3\alpha\beta) + \gamma^3$

$= +3\alpha\beta\gamma = -3b$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow \dfrac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \dfrac{-3b}{-2a} = \dfrac{3b}{2a}$

20. (A)

$\Rightarrow f(x) = x^3 + 2x^2 + k$

If $(x-1)$ is divisor of $f(x)$

Then $f(1) = 0$

$\Rightarrow f(1) = 1 + 2 + k = 0$

$\Rightarrow k = -3$

21. (D)

$\Rightarrow 4x^3 + 20x^2 - 23x + 6 = 0$

Let's take roots are α, α, β

$$\Rightarrow 2\alpha + \beta = \frac{-20}{4} = -5 \qquad\qquad\qquad\qquad ....(1)$$

$$\Rightarrow \alpha^2 + \alpha\beta + \alpha\beta = \frac{-23}{4}$$

$$\Rightarrow \alpha^2\beta = \frac{-6}{4}$$

$$\Rightarrow \alpha^2(-5 - 2\alpha) = -\frac{6}{4}$$

$$\Rightarrow 5\alpha^2 + 2\alpha^3 - \frac{6}{4} = 0$$

$$\Rightarrow \alpha^3 + 20\alpha^3 - 6 = 0$$

Which gives $\alpha = \dfrac{1}{2}$

So $2\left(\dfrac{1}{2}\right) + \beta = -5 \Rightarrow \beta = -6$

So roots are $-6, \dfrac{1}{2}, \dfrac{1}{2}$

22. (D)

$$\Rightarrow 2x^3 - 5x^2 + 3x - 1 = 0$$

$$\Rightarrow \alpha\beta\gamma = \frac{1}{2}$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{5}{2}$$

$$\Rightarrow \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\beta\alpha} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{\left(\dfrac{5}{2}\right)}{\left(\dfrac{1}{2}\right)} = 5$$

23. (A)

$$\Rightarrow x^3 - 4x^2 + x + 6 = 0$$

Roots are $2\alpha, 3\alpha, \beta$

So, $2\alpha, 3\alpha, \beta = 4$

$$\Rightarrow 5\alpha + \beta = 4 \qquad\qquad\qquad\qquad ....(1)$$

$$\Rightarrow (2\alpha)(3\alpha)(\beta) = -6$$

$$\Rightarrow \alpha^2\beta = -1 \qquad\qquad\qquad\qquad ....(2)$$

By (1) & (2)

$$\Rightarrow \alpha^2(4 - 5\alpha) = -1$$

$$\Rightarrow 5\alpha^3 - 4\alpha^2 - 1 = 0$$

By this we get $\alpha = 1$

So roots are $2, 3, -1$

24. (B)

$$\Rightarrow 4x^3 - 12x^2 + 11x + k = 0$$

Sum of roots $(a-d+a+a+d) = \dfrac{12}{4} = 3$

$\Rightarrow a = 1$

As a is root of equation so it will satisfy if.

$\Rightarrow 4(a)^3 - 12a^2 + 11a + k = 0$

$\Rightarrow 4 - 12 + 11 + k = 0$

$\Rightarrow k - 3$

25. (B)

$\Rightarrow x^3 - 11x^2 + 37x - 35 = 0$

If root is $3 + \sqrt{2}$, second root has to be $3 - \sqrt{2}$

Let's take third root α

So, $\left(3 + \sqrt{2}\right) + \left(3 - \sqrt{2}\right) + \alpha = 11$

$\Rightarrow \alpha = 5$

26. (B)

As $x + 1$ is factor so $x = -1$ will satisfy the expression

$\Rightarrow (-1)^4 + (p-3)(-1)^3 - (3p-5)(-1)^2 + (2p+q)(-1) + 12 = 0$

$\Rightarrow p = 2$

27. (D)

$\Rightarrow x^3 - ix + 1 - i = 0$

Roots will be $1+i, 1-i, \alpha$

$\Rightarrow 1 + i + 1 - i + \alpha = 0$

$\Rightarrow \alpha = -2$

So equation will roots $(1-i) \& -2$ is

$\Rightarrow x^2 - (1-i-2)x + (1-i)(-2) = 0$

$\Rightarrow x^2 + (1+i)x - 2(1-i) = 0$

28. (D)

$\Rightarrow 2x^4 + 9x^3 + 8x^2 + 9x + 2 = 0$

29. (A)

$\Rightarrow x^3 - 1 = 0$

Roots will be α, β, γ

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \sum \alpha\beta = 0$

$\Rightarrow \alpha\beta\gamma = 1$

$\Rightarrow \sum (\alpha + 1) = (\alpha + \beta + \gamma) + 3 = 3$

$= \sum (\alpha + 1)(\beta + 1) = (\alpha + 1)(\beta + 1) + (\alpha + 1)(\gamma + 1)(\beta + 1)(\gamma + 1)$

$= \sum \alpha\beta + (\alpha + \beta + 1) + (\alpha + \gamma + 1) + (\beta + \gamma + 1)$

$= 3$

$\Rightarrow (\alpha+1)(\beta+1)(\gamma+1) = (\alpha\beta+\alpha+\beta+1)(\gamma+1)$

$= \alpha\beta\gamma+\alpha\beta+\alpha\gamma+\alpha+\beta\gamma+\beta+\gamma+1 = 2$

So equation with $(\alpha+1),(\beta+1),(\gamma+1)$ as roots

$= x^3 - 3x^2 + 3x - 2 = 0$

30.　(B)

$\Rightarrow x^3 + 3x + 2 = 0$

$\Rightarrow a + b + c = 0$

$\Rightarrow ab + bc + ca = 3$

$\Rightarrow abc = -2$

$\Rightarrow a^3 + b^3 + c^3 - 3abx = (a+b+c)(a^2+b^2+c^2+ab+bc+ca) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3(-2) = -6$

31.　(A)

$\Rightarrow x^3 + 1 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 0$

$\Rightarrow \alpha\beta\gamma = 1$

$\Rightarrow (\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow \alpha^2+\beta^2+\gamma^2 = 0$

$\Rightarrow (\alpha^2+\beta^2+\gamma^2)^2 = \alpha^4+\beta^4+\gamma^4+2(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)$

$\Rightarrow 0 = \alpha^4+\beta^4+\gamma^4+2\left[(\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2(\alpha\beta.\beta\gamma+\beta\gamma.\gamma\alpha+\gamma\alpha.\alpha\beta)\right]$

$= \alpha^4+\beta^4+\gamma^4+2\left[0 - 2(\beta+\gamma+\alpha)\right]$ 　　　　　　$(\because \alpha\beta\gamma = 1)$

$\Rightarrow 0 = \alpha^4+\beta^4+\gamma^4+2\left[0 - 2(0)\right]$

$\Rightarrow \alpha^4+\beta^4+\gamma^4 = 0$

32.　(A)

$\Rightarrow x^3 + 3x^2 + 2x + 4 = 0$

$\Rightarrow \alpha\beta\gamma = -3$

$\Rightarrow \sum \alpha\beta = 2$

$\Rightarrow \alpha\beta\gamma = -4$

$\Rightarrow x^3 - (2\alpha+2\beta+2\gamma)x^2 + \left(\sum \alpha 2\beta\right)x - (2\alpha.2\beta.2\gamma) = 0$

$\Rightarrow x^3 + 6x^2 + 8x + 32 = 0$

33.　(A)

As α, β, γ satisfy cubic equation $x^3 - 5x^2 + 5x - 3 = 0$
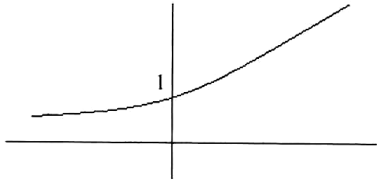
So, α, β, γ are roots of this equation

$\Rightarrow \alpha + \beta + \gamma = 5$

34. (C)

$$\Rightarrow 5^{x+2} > \left(\frac{1}{25}\right)^{\frac{1}{x}}$$

$$\Rightarrow 5^{x+2} > \frac{1}{5^{\frac{2}{x}}}$$

$$\Rightarrow 5^{x+2} \cdot 5^{\frac{2}{x}} > 1$$

$$\Rightarrow 5^{x+\frac{2}{x}+2} > 5^\circ$$

So $x + \dfrac{2}{x} + 2 > 0$

$$\Rightarrow \frac{x^2 + 2x + 2}{x} > 0$$

Numerator is always > 0

So $\dfrac{1}{x} > 0$

$$\Rightarrow x \in (0, \infty)$$

35. (B)

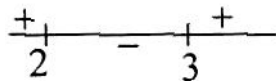$$\Rightarrow \frac{x^2 - 5x + 6}{|x| + 7} < 0$$

$$\Rightarrow \frac{(x-3)(x-2)}{|x| + 7} < 0$$

As $7 + |x| \geq 7$

So we can cross multiply $7 + |x|$

$$\Rightarrow (x-3)(x-2) < 0$$

$$\Rightarrow x \in (2, 3)$$

36. (D)

$$\Rightarrow \left|\frac{2x-1}{x-1}\right| > 2$$

$$\Rightarrow \frac{2x-1}{x-1} < -2 \qquad\qquad \text{or } \frac{2x-1}{x-1} > 2$$

$$\Rightarrow \frac{2x-1+2x-2}{x-1} < 0 \qquad \text{or} \quad \frac{2x-1-2x+2}{x-1} > 0$$

$$\Rightarrow \frac{4x-3}{x-1} \qquad\qquad \text{or} \quad \frac{1}{x-1} > 0$$

$$\Rightarrow x \in \left(\frac{3}{4}, 1\right) \qquad\qquad \text{Or} \quad x > 1$$

So, $x \in \left(\frac{3}{4}, \infty\right) - \{1\}$

37. (A)

$$\Rightarrow \frac{1}{|x|-3} < \frac{1}{2}$$

Case 1: $x \geq 0$

$$\Rightarrow \frac{1}{x-3} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{5-x}{2(x-3)} < 0$$

$$\Rightarrow \frac{x-5}{(x-3)} > 0$$

$$\Rightarrow x \in (-\infty, 3) \cup (5, \infty)$$

But $x \geq 0$

So, $x \in [0.3) \cup (5, \infty)$ \qquad\qquad ……..(1)

Case 2: $x < 0$

$$\Rightarrow \frac{1}{-x-3} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{1}{x+3} + \frac{1}{2} > 0$$

$$\Rightarrow \frac{5+x}{x+3} > 0$$

$$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$$

But $x < 0$

So, $x \in (-\infty, -5) \cap (-3, 0)$ \qquad\qquad ……..(2)

So (1) $\cup$ (2)

$$\Rightarrow x \in (-\infty, -5) \cup (-3, \infty)$$

So least positive integer value $= 1$

38. (C)

$$\Rightarrow (x-1)\sqrt{x^2 - x - 2} \geq 0$$

$$\Rightarrow (x^2 - x - 2) \geq 0 \qquad\qquad \& \ (x-1) \geq 0$$

$$\Rightarrow (x-2)(x+1) \geq 0 \qquad\qquad \& \ x \geq 1$$

$$\Rightarrow x \in (-\infty, -1] \cup [2, \infty) \ \& \qquad x \in [1, \infty)$$

So, $x \in [2, \infty)$

39. (B)

$\Rightarrow (x^2 - 1)\sqrt{x^2 - x - 2} \geq 0$

$\Rightarrow (x^2 - 1) \geq 0$ & $x^2 - x - 2 \geq 0$ xh2

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty) \ldots\ldots(1)$ & $(x - 2)(x + 1) \geq 0$

                                   $x \in (-\infty, -1] \cup [2, \infty)$ $\ldots\ldots(2)$

Take $(1) \cap (2)$

So, $x \in (-\infty, -1] \cup [2, \infty)$

Least positive integer $= 2$

40. (C)

$\Rightarrow 49^x + 7^{x+1} - 98 < 0$

$\Rightarrow 7^{2x} + 7 \cdot 7^x - 98 < 0$

$\Rightarrow 7^x = a; a^2 + 7a - 78 < 0$

$\Rightarrow (a + 14)(a - 7) < 0$ $\left(\because a + 14 = 7^x + 14 > 0 \text{ for } x \in R \right)$

$\Rightarrow 7^x < 7$

So, $x < 1$

41. (B)

$\alpha, \beta, \gamma \rightarrow x^3 + 2x^2 - 3x - 1 = 0$

The equation whose roots are $\dfrac{1}{\alpha^2}, \dfrac{1}{\beta^2}, \dfrac{1}{\gamma^2}$

$x = \dfrac{1}{\alpha^2}$

$\Rightarrow \alpha = \dfrac{1}{\sqrt{x}}$

$\left(\dfrac{1}{x} \right)^{\frac{3}{2}} + \dfrac{2}{x} - \dfrac{2}{x^{\frac{1}{2}}} - 1 = 0$

$\Rightarrow 1 + 2\sqrt{x} - 3x - x^{\frac{3}{2}} = 0$

$\Rightarrow 1 - 3x = x^{\frac{3}{2}} - 2x^{\frac{1}{2}}$

$\Rightarrow 1 + 9x^2 - 6x = x^3 + 4x - 4x^2 \text{ (squaring)}$

$\Rightarrow x^3 - 13x^2 + 10x - 1 = 0$

Hence, $\dfrac{1}{\alpha^4} + \dfrac{1}{\beta^4} + \dfrac{1}{\gamma^4} = S_1^2 - 2S_2$

               $= 169 - 20 = 149$

42. (C)

Let roots one diminished by k

So, $\alpha - k + \beta - k + \gamma - k = 0$

$\Rightarrow (\alpha + \beta + \gamma) - 3k = 0$

$\Rightarrow k = \dfrac{9/2}{3} = \dfrac{3}{2}$

43. (A)

$\alpha + \beta, \beta + \gamma, \gamma + \alpha$

$\alpha + \beta + \gamma = p$

Hence roots of the equations are $p - \alpha, p - \beta, p - \gamma$

$\qquad x = p - \alpha \Rightarrow \alpha = p - x$

Equation is

$(p - x)^3 - p(p - x)^2 + q(p - x) - r = 0$

$\Rightarrow x^3 - 3px^2 + 3p^2x - p^3 + p^3 - 2xp^2 + px^2 - qp + qx + r = 0$

$\Rightarrow x^3 - 2px^2 + 2(p^2 + q)x + r - pq = 0$

44. (B)

$x_1 = \beta\gamma + \dfrac{1}{\alpha} \quad = \dfrac{\alpha\beta\gamma + 1}{\alpha} = \dfrac{3 + 1}{\alpha} = \dfrac{4}{\alpha}$

Hence, $x_1 = \dfrac{4}{\alpha} \Rightarrow \alpha = \dfrac{4}{x_1}$

Equation

$\dfrac{64}{x_1^3} - \dfrac{2.16}{\alpha_1^2} + \dfrac{5.4}{\alpha_1} - 3 = 0$

$\Rightarrow 3x_1^3 - 20x_1^2 + 32x_1 - 64 = 0$

45. (B)

$x = \sqrt{3} - \sqrt{2}$

$x + \sqrt{2} = \sqrt{3}$

$\Rightarrow x^2 + 2\sqrt{2}x + 2 = 3 \qquad\qquad \left(\text{roots are} - \sqrt{2} \pm \sqrt{3} \right)$

$\Rightarrow x^2 - 1 = -2\sqrt{2}\, x$

$\Rightarrow x^2 - 2x^2 + 1 = 8x^2$

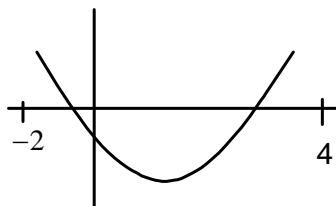$\Rightarrow x^4 - 10x^2 + 1 = 0 \qquad\qquad$ (roots are $\pm\sqrt{2} \pm \sqrt{3}$)

46. (B)

$(x - m)^2 = 1$

$= m - 1, m + 1$
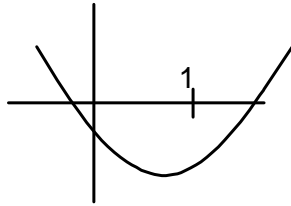
$m - 1 > -2 \qquad\qquad$ & $m + 1 < 4$

$$m > -1 \qquad\qquad m < 3$$
$$(-1, 3)$$

47. (A)

1 lies in between the roots

So, $\text{a.f}(1) < 0$

$$\Rightarrow (2k+1).[2k+1-k+k-2] < 0$$
$$(2k+1)(2k-1) < 0$$

$$k \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$

48. (C)

$$(r_1 + 2)(r_2 + 2)(r_3 + 2)$$

Equation whose roots are $(r_1 + 2), (r_2 + 2) \,\&\, (r_3 + 2)$ is

$$(x-2)^3 - 2(x-2)^2 + 4(x-2) + 5074 = 0$$
$$\Rightarrow x^3 - 6x^2 + 12x - 8 - 2x^2 + 8x - 8 + 4x - 8 + 5074 = 0$$
$$\Rightarrow 1x^3 - 8x^2 + 24x + 5050 = 0$$

Product $= -5050$

49. (A)

$$P(x) = (x-2)(4x-1)(x) + Ax + B$$
$$2A + B = 1$$
$$\frac{A}{4} + B = 2$$

_____

$$\frac{7A}{4} = -1 \qquad \Rightarrow A = -\frac{4}{7}, B = 1 + \frac{8}{7} = \frac{15}{7}$$

Hence, $-\dfrac{4x+15}{7}$

50. (D)

$$\frac{1}{\log_3\left(2^{2x}-1\right)} > \frac{1}{\log_3\left(2^x+1\right)}$$

Domain $x > 0, x \neq \dfrac{1}{2}$

$\log_3 2^x + 1$ is always positive

Hence, $\log_3\left(2^{2x} - 1\right) > 0$

$$x > \frac{1}{2}$$

And $\log_3\left(2^{2x} - 1\right) < \log_3\left(2^x + 1\right)$

$\Rightarrow 2^{2x} - 1 < 2^x + 1$

$\Rightarrow t^2 - t - 2 < 0$ $\qquad\qquad (t = 2x)$

$\Rightarrow t \in (-1, 2)$

$\Rightarrow x \in (-\infty, 1)$

Hence solution is $\left(\dfrac{1}{2}, 1\right)$

51. **(B)**

$A = 12^{300}$

$\log_{10} A = 300\left[\log_{10} 12\right]$

$= 300\left[0.6010 + 0.4771\right]$

$= 300 \times 1.0781 = 323.43$

$\Rightarrow A = 10^{323.43}$

Hence 324 digits

52. **(A)**

$\log_{10} 2 = 0.3010$

$\log_5 64 = \dfrac{\log_{10} 64}{\log_{10} 5} = \dfrac{6\log_{10} 2}{\log_{10} 10 - \log_{10} 2}$

$= \dfrac{6 \times 0.3010}{1 - 0.3010}$

$= \dfrac{1.8060}{0.6990}$

$= \dfrac{602}{233}$

53. **(B)**

$2^{\log_{10} 3\sqrt{3}} = 3^{k\log_{10} 2}$

$\Rightarrow 2^{\log_{10} 3\sqrt{3}} = 2^k \log_{10} 3$

$\Rightarrow \log_{10} 3\sqrt{3} = \log_{10} 3^k$

$\Rightarrow k = 3^{3/2}$

$\Rightarrow k = 3/2$

54. **(A)**

$x = \log_5(1000) = \log_5 125 + \log_5 8 > 4$

$y = \log_{57}(2056) = \log_7 343 + \log_7 6 < 4$

Hence $x > 4$

55. (B)

For compare $\dfrac{b}{a}$. Sum of the roots ≥ 3

Hence $\dfrac{-b}{a} \geq 3$ and $-\dfrac{b}{2a} \geq 2$

$\Rightarrow \left(\dfrac{b}{a}\right) \geq 3$ $\Rightarrow |b| \geq 4|a|$

$\Rightarrow |b| \geq 3|a|$ Hence $|b| \geq 4a$

56. (A)

$x = \dfrac{1+\alpha}{1-\alpha}$

$\Rightarrow \alpha = \dfrac{x-1}{x+1}$

$\left(\dfrac{x-1}{x+1}\right)^3 - \left(\dfrac{x-1}{x+1}\right) - 1 = 0$

$\Rightarrow (x-1)^3 - (x-1)(x+1)^2 - (x+1)^3 = 0$

$\Rightarrow -6x^2 - 2 - x^3 - x^2 + x + 1 = 0$

$\Rightarrow x^3 + 7x^2 - x + 1 = 0$

$\Rightarrow x^3 + 7x^2 - x + 1 = 0$

57. (D)

$x = \alpha(\beta + r) = \alpha\beta + \alpha v$

$= 3 - \beta r$

$= 3 - \dfrac{\beta r \alpha}{\alpha}$

$= 3 - \dfrac{(-2)}{\alpha} = 3 + \dfrac{2}{\alpha}$

$\Rightarrow x = 3 + \dfrac{2}{\alpha}$

$\Rightarrow \alpha = \dfrac{2}{x-3}$

58. (A)

$ax + by = 1$

$px^2 + qy^2 = 1$

$\Rightarrow px^2 + q\left(\dfrac{ax-1}{b}\right)^2 = 1$

$\Rightarrow b^2 px^2 + qa^2 x^2 - 2aqx + q - b^2 = 0$

Has one root

$$\Rightarrow 4a^2q^2 - 4(b^2q + a^2q)(q - b^2) = 0$$

$$\Rightarrow 4(a^2/q^2 - b^2pq - a^2/q^2 + b^4p + b^2a^2q) = 0$$
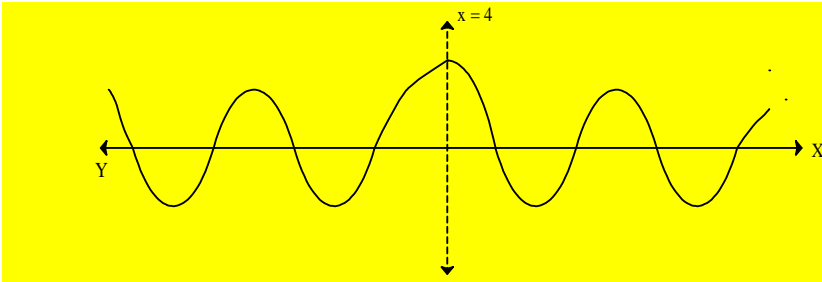
$$\Rightarrow b^2(-pq + b^2p + a^2q) = 0$$

Hence $\dfrac{b^2}{q} + \dfrac{a^2}{p} = 1$

60. **(D)**

$x^2 + 2xy + 3y^2 - 6x + 6y \geq k$ (where k is min unvalue)

$$\Rightarrow x^2 + (2y - 6)x + 3y^2 + 6y - k \geq 0$$

Should have all real x

Hence $4(y - 3)^2 - 4(3y^2 + 6y - k) \leq 0$

$$\Rightarrow 4[y^2 + 9 - 6y - 3y^2 - 6y + k] \geq 0$$

$$\Rightarrow [2y^2 + 12y - (9 + k)] \leq (\forall y \in R)$$

Hence, $144 + 8(9 + k) \leq 0$

$$\Rightarrow k + 9 \leq -18$$

$$\Rightarrow k \leq -27$$

Hence $k = -27$

EXERCISE – 2 (A)

1.   (AC)

$$\Rightarrow 18^{4x-3} = \left(54\sqrt{2}\right)^{3x-4}$$

$$\Rightarrow 18^{4x-3} = \left(18\right)^{3x-4}\left(3\sqrt{2}\right)^{3x-4}$$

$$\Rightarrow 18^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$$

$$\Rightarrow \left[\left(3\sqrt{2}\right)^{2}\right]^{x+1} = \left(3\sqrt{2}\right)^{3x-4}$$

$$\Rightarrow \left(3\sqrt{2}\right)^{2x+2} \times \left(3\sqrt{2}\right)^{-3x+4} = 1$$

$$\Rightarrow \left(3\sqrt{2}\right)^{-x+6} = 1$$
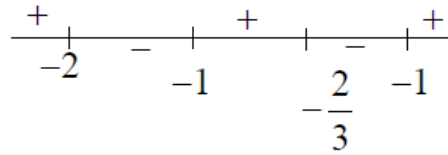
So $-x = 6 \Rightarrow x = 6$

2.   (BC)

$$\Rightarrow \frac{2x}{2x^{2}+5x+2} > \frac{1}{x+1} \Rightarrow \frac{2x}{2x^{2}+5x+2} - \frac{1}{x+1} > 0$$

$$\Rightarrow \frac{2x^{2}+2x-2x^{2}-5x+2}{\left(2x^{2}+5x+2\right)\left(x+1\right)} > 0$$

$$\Rightarrow \frac{3x+2}{\left(2x+1\right)\left(x+2\right)\left(x+1\right)} < 0$$

So, $x \in \left(-2,-1\right) \cup \left(-\dfrac{2}{3},-1\right)$

3.   (ABC)

$\Rightarrow$ 2576a456b

$\Rightarrow$ b has to 0 or 5

Sum of digits has to be multiple of 3, to divide it by 3.

$\Rightarrow 2 + 5 + 7 + 6 + a + 4 + 5 + 6 + b = 35 + a + b$

If a take value of 4, then b will also take 5.

If a take value of 4, then b has to be zero.

If take 6, then 35 + a + b can't become multiple of 3, for b = 0.5

4.   (ABD)

$$\Rightarrow x^{2} - 6x - 5\left|x-3\right| - 5 = 0$$

Case – I : $x \ge 3$

$$\Rightarrow^{2} - 6x - 5\left|x-3\right| - 5 = 0$$

$$\Rightarrow x^{2} - 11x + 10 = 0$$

$\Rightarrow x = 1, 10$

$\Rightarrow x = 1$ not a solution for $x \ge 3$

$\Rightarrow x = 10$

Case – 2: $x < 3$

$$\Rightarrow x^{2} - 6x + 5x - 15 - 5 = 0$$

$$\Rightarrow x^{2} - x - 20 = 0$$

$\Rightarrow (x - 5)(x + 4) = 0$

For $x < 3$, $x = 1 - 4$

So, $l = 1$, m = 1

5. (BCD)
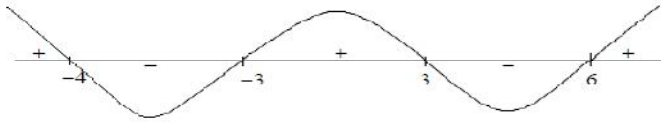
$\Rightarrow \dfrac{2x+3}{x^2+x-12} \le \dfrac{1}{2}$

$\Rightarrow \dfrac{2(2x+3)-(x^2+x-12)}{2(x^2+x-12)} \le 0$

$\Rightarrow \dfrac{-x^2+3x+18}{(x+4)(x-3)} \le 0$

$\Rightarrow \dfrac{(x-6)(x+3)}{(x+4)(x-3)} \ge 0$

$\Rightarrow \in (-\infty,-4) \cup [-3,3) \cup [6,\infty)$

6. (AD)

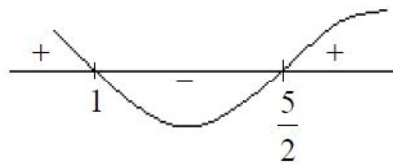$\Rightarrow |x-1|+|5-2x| = |3x-6|$

As $(x-1)-(5-2x) = 3x-6$

So, $(x-1)(5-2x) \le 0$

$\Rightarrow (x-1)(2x-5) \ge 0$

$\Rightarrow x \in (-\infty,1] \cup \left[\dfrac{5}{2},\infty\right)$

7. (AC)

$\Rightarrow \log_2 3 > 1$

$\Rightarrow \log_{12} 5 < 1$

So, $\log_2 3 > \log_{12} 5$

Similarly, $\log_6 5 < 1$

$\Rightarrow \log_7 11 > 1$

$\Rightarrow \log 82 > \log_3 3^4$

$\Rightarrow \log_3 81 > 4$

$\Rightarrow \log_2 15 < \log_2 16$

$\Rightarrow \log_2 15 < \log_2 2^4$

$\Rightarrow \log_2 15 < 4$

$\Rightarrow \log_{16} 15 < 1$

$\Rightarrow \log_{10} 11 > 1$

$\Rightarrow \log_7 6 < 1$

8. (BC)

$\Rightarrow 2x^2 + 6\sqrt{2}x + 1 = 0$

$\Rightarrow x = \dfrac{-6\sqrt{2} \pm \sqrt{72-8}}{4}$

$\Rightarrow x = \dfrac{-3\sqrt{2}}{2}+2, \dfrac{-3\sqrt{2}}{2}-2$

9. (BD)

$\Rightarrow \log_{x+1}(x - 0.5) = \log_{x-0.5}(x + 1)$

$\Rightarrow x - 5 > 0;\ x - 0.5 > 0;\ x - 0.5 \neq 1$

$\Rightarrow x > 0.5;\ x > 0.5;\ x \neq 1.5$

$\Rightarrow x + 1 > 0;\ x + 1 > 0$

$\Rightarrow x > -1;\ x > -1$

So, $x \in (0.5, 1.5) \cup (1.5, \infty)$ is feasible reason

$\Rightarrow \dfrac{\log(x - 0.5)}{\log(x + 1)} = \dfrac{\log(x + 1)}{\log(x - 0.5)}$

$\Rightarrow \log^2(x - 0.5) - \log^2(x + 1) = 0$

$\Rightarrow \log\left(\dfrac{x - 0.5}{x + 1}\right) \log\{(x - 0.5)(x + 1)\} = 0$

So, $\dfrac{x - 0.5}{x + 1} = 1$ or $(x - 0.5)(x + 1) = 1$

$\Rightarrow x - 0.5 = x + 1$ (no solution)

Or $x^2 + 0.5x - 1.5 = 0$

$\Rightarrow 2x^2 + x - 3 = 0$

$\Rightarrow 2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow x = 1$

As $x \neq \dfrac{-3}{2}$

So $x = 1$ is the only solution.

10. (CD)

$\Rightarrow \log\left[(1)^{\frac{1}{5}} + (32)^{\frac{1}{5}} + (243)^{\frac{1}{5}}\right]$

$\Rightarrow \log\{1 + 2 + 3\}$

$\Rightarrow \log 6$

$\Rightarrow \dfrac{1}{5}\{\log 1 + \log 32 + \log 243\} = \dfrac{1}{5}\log\{1 \times 32 \times 243\}$

$= \log\left\{(1)^{\frac{1}{5}}(32)^{\frac{1}{5}}(243)^{\frac{1}{5}}\right\}$

$= \log 6$

11. (BCD)

$\Rightarrow \alpha, \beta, \gamma$ are roots of $x^3 - 2x + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

$\Rightarrow \sum \alpha\beta = -2$

$\Rightarrow \alpha\beta\gamma = -3$

Equation with roots $-\alpha, -\beta, -\gamma$ is $x^3 - (0)x^2 + (-2)x - (3) = 0$

$\Rightarrow x^3 - 2x - 3 = 0$

$\Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} = \dfrac{\sum \alpha\beta}{\alpha\beta\gamma} = \dfrac{-2}{-3} = \dfrac{2}{3}$

$\Rightarrow \sum \dfrac{1}{\alpha} \cdot \dfrac{1}{\beta} = \dfrac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = 0$

$$\Rightarrow \frac{1}{\alpha} \cdot \frac{1}{\beta} \cdot \frac{1}{\gamma} = -\frac{1}{3}$$

So, equation $x^3 - \left(\frac{2}{3}\right)x^2 + (0)x - \left(-\frac{1}{3}\right) = 0$

$\Rightarrow 3x^3 - 2x^2 + 1 = 0$

$\Rightarrow \sum(\alpha + \beta) = 0$

$\Rightarrow \sum(\alpha + \beta)(\beta + \gamma) = (\alpha\beta + \beta\gamma + \gamma\alpha + \beta^2) + (\alpha\beta + \beta\gamma + \gamma\alpha + \alpha^2) + (\alpha\beta + \beta\gamma + \gamma\alpha + \gamma^2)$

$= 3(-2) + (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha s)$

$= 3(-2) + 0 - 2(-2)$

$\Rightarrow \sum(\alpha + \beta)(\beta + \alpha) = -2$

$\Rightarrow (\alpha\beta + \beta\gamma + \gamma\alpha) = (\alpha\beta + \beta\gamma + \gamma\alpha + \beta^2)(\gamma + \alpha)$

$= (-2 + \beta^2)(-\beta)$

$= 2\beta - \beta^3$

$= 3 \qquad\qquad\qquad (\because \beta^3 - 2\beta + 3 = 0)$

So equation $x^3 - 0.x^2 + (-2)x - (3) = 0$

$\Rightarrow x^3 - 2x - 3 = 0$

$\Rightarrow \sum \frac{\alpha + \beta}{\gamma + 1} = \frac{-\gamma}{\gamma + 1} - \frac{\alpha}{\alpha + 1} - \frac{\beta}{\beta + 1} = -3 + \left[\frac{1}{\gamma + 1} + \frac{1}{\alpha + 1} + \frac{1}{\beta + 1}\right]$

$= -3 + \left[\frac{\sum(\alpha + 1)(\beta + 1)}{(\alpha + 1)(\beta + 1)(\gamma + 1)}\right]$

$= -3 - \frac{1}{4} = \frac{-13}{4}$

12. **(B)**

$\Rightarrow S_1 : x^2 + |x| + 1 = 0$

$\Rightarrow x^2 + 1 = -|x|$
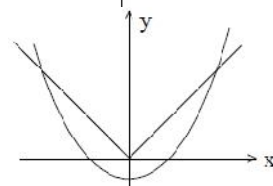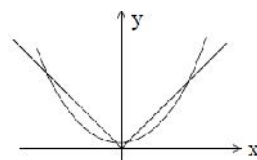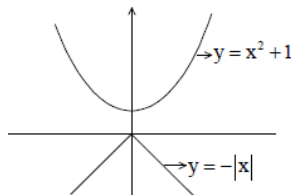
No solution

$\Rightarrow S_2 : x^2 - 5|x| + 6 = 0$

$\Rightarrow x^2 + 6 = 5|x|$

$\Rightarrow 4$ solution.

$\Rightarrow S_3 : x^2 - |x| - 2 = 0$

$\Rightarrow x^2 - 2 = |x|$

$\Rightarrow 2$ solution

13. **(BC)**

$\Rightarrow x^{1 - \log_5 x} = 0.04$

$\Rightarrow x > 0$

$\Rightarrow x.x^{\log \frac{1}{x}} = 0.04$

Only $x = 25, \frac{1}{5}$ satisfy the equation

14. **(AD)**
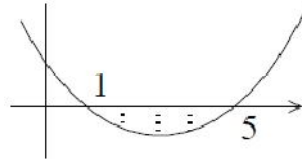 $\Rightarrow f(x) = x^2 - 6x + 5$

    Least value at $x = \dfrac{6}{2} = 3$

    Least value $= (3)^2 - 6(3) + 5 = -4$
    $\Rightarrow f(2) = (2)^2 - 6(2) + 5 = -3$
    $\Rightarrow f(4) = (4)^2 - 6(4) + 5 = -3$

15. **(AB)**
    $\Rightarrow 10^{\frac{2}{x}} + 25^{\frac{1}{x}} = \dfrac{17}{4}(50)^{\frac{1}{x}}$

    $\Rightarrow 100^{\frac{1}{x}} + 25^{\frac{1}{x}} = \dfrac{17}{4}(50)^{\frac{1}{x}}$

    $\Rightarrow 4^{\frac{1}{x}} + 1 = \dfrac{17}{4}2^{\frac{1}{x}}$

    $\Rightarrow 2^{\frac{2}{x}} - \dfrac{17}{4}2^{\frac{1}{x}} + 1 = 0$

    $\Rightarrow \left(2^{\frac{1}{x}} - 4\right)\left(2^{\frac{1}{x}} - \dfrac{1}{x}\right) = 0$

    $\Rightarrow 2^{\frac{1}{x}} = 4, \dfrac{1}{4}$

    $\Rightarrow x = \dfrac{-1}{2}, \dfrac{1}{2}$

16. **(BC)**
    $\Rightarrow |x^2 + 4x + 3| + 2x + 5 = 0$

    $\Rightarrow |(x+1)(x+3)| + 2x + 5 = 0$

    Case 1: $x \in (-\infty, -3] \cup [-1, \infty)$
    $\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$
    $\Rightarrow x^2 + 6x + 8 = 0$
    $\Rightarrow x = -4, -2$
    So $x = -4$

    Case 2: $x \in (-3, -1)$
    $\Rightarrow -(x^2 + 4x + 3) + 2x + 5 = 0$
    $\Rightarrow x^2 + 2x - 2 = 0$
    $\Rightarrow x = \dfrac{-2 \pm \sqrt{12}}{2}$
    $\Rightarrow x = -1 + \sqrt{3}, -1 - \sqrt{3}$
    So, $x = -1 - \sqrt{3}$

17. **(BC)**
    $\Rightarrow 4x^2 + 4x + 12\lambda^2 \geq 47 \qquad \forall x \in R$
    $\Rightarrow 4(x^2 + x + 3\lambda^2) \geq 47$

$\Rightarrow x^2 + x + 3\lambda^2 - \dfrac{47}{4} \geq 0$

So, $8 \leq 0$

$\Rightarrow 1 - 4\left(3\lambda^2 - \dfrac{47}{4}\right) \leq 0$

$\Rightarrow 4 - \lambda^2 \leq 0$

$\Rightarrow \lambda^2 \geq 4$

$\Rightarrow \lambda \in (-\infty, -2] \cup [2, \infty)$

18.　(ABD)

$\Rightarrow \dfrac{1}{2} \leq \log_{0.1} x \leq 2$; $x > 0$

$\Rightarrow \dfrac{1}{2} \leq -\log_{10} x \leq 2$

$\Rightarrow -\dfrac{1}{2} \geq \log_{10} x \geq -2$

$\Rightarrow \log_{10} x \leq -\dfrac{1}{2}$ 　　&　　 $\log_2 x \geq -2$

$\Rightarrow x \leq (10)^{-\frac{1}{2}}$ 　&　 $x \geq 10^{-2}$

$\Rightarrow x \leq \dfrac{1}{\sqrt{10}}$ 　…(1) 　$x \geq \dfrac{1}{100}$ 　　…(2)

$(1) \cap (2)$

$\Rightarrow x \in \left[\dfrac{1}{100}, \dfrac{1}{\sqrt{10}}\right]$

19.　(A)

$\Rightarrow \log_2\left(3^{2x+2} + 7\right) = 2 + \log_2\left(3^{x-1} + 1\right)$

$\Rightarrow \log_2\left(\dfrac{9.3^{2x} + 7}{\dfrac{3^x}{3} + 1}\right) = 2$

$\Rightarrow \dfrac{3\left(9.3^{2x} + 7\right)}{3^x + 3} = 4$

Put $3^x = a$

$\Rightarrow 27a^2 + 21 = 4a + 12$

$\Rightarrow 21a^2 - 4a + a = 0$

Which is not possible for values of a.

So no solution.

20.　(AB)

$\Rightarrow \log_{a_1 a_2} a_1 = 4$

$\Rightarrow \log a_1 = 4\left[\log_{a_1} + \log_{a_2}\right] 3\log_{a_1} + 4\log_{a_2} = 0$

$$\Rightarrow \log_{a_1 a_2} \frac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} \Rightarrow \log_{a_1 a_2} (a_1)^{\frac{1}{3}} - \log_{a_1 a_2} (a_2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{3}[4] - x$$

$$\Rightarrow x = \frac{1}{2}\left[\frac{\log(a_2)}{\log a_1 a_2}\right] = \frac{1}{2}\left[\frac{\log a_1 a_2}{\log a_1 a_2}\right]$$

$$= \frac{1}{2}\left[1 - \log_{a_1 a_2} a_1\right]$$

$$= \frac{1}{2}[1 - 4] = -\frac{3}{2}$$

So, $\log_{a_1 a_2} \dfrac{(a_1)^{\frac{1}{3}}}{(a_2)^{\frac{1}{2}}} = \dfrac{4}{3} + \dfrac{3}{2} = \dfrac{17}{6}$

21. **(AD)**
$f(x) = x^3 - 3x^2 + 4 = 0$
clearly $x = -1$ is a root of given equation.
$$\frac{x^3 - 3x^2 + 4}{x + 1} = (x - 2)^2$$
$x = 2$ is a repeated root & $x = -1$ is a root

22. **(AD)**
Given $a < \alpha, \beta$
(i) $f(a) > 0$
(ii) $D > 0$
(iii) $-\dfrac{B}{2A} > a$
(i) $f(a) = a^2 + a + a = a^2 + 2a > 0$

$a \in (-\infty, -2) \cup (0, \infty)$ … (1)
(ii) $D > 0$
$1 - 4a > 0$
$a < \dfrac{1}{4}$ …(2)
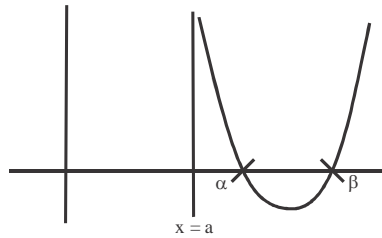
(iii) $-\dfrac{B}{2A} > a$
For $x^2 + x + a$, $A = 1$ & $B = 1$
$-\dfrac{1}{2} > a$ …(3)
From (1), (2), (3)
$a < -2$

23. **(CD)**
$\tan x \geq 3\cot x, \; x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \& x \neq 0$

$\{\because \cos x > 0$ for given integral$\}$
(i) for $x > 0$

$\tan x \geq \dfrac{3}{\tan x}$ {here $\tan x > 0$}

$\tan^2 x \geq 3$

$\Rightarrow \tan x \geq \sqrt{3}$

$x \in \left[\dfrac{\pi}{3}, \dfrac{\pi}{2}\right)$

(ii) for $x < 0$

$\tan x \geq \dfrac{3}{\tan x}$ {here $\tan x < 0$}

$\tan^2 x \leq 3$

$\tan x \leq -\sqrt{3}$

$\Rightarrow x \in \left(-\dfrac{\pi}{3}, 0\right]$

24. (AD)

$\dfrac{3}{\sqrt{2-x}} - \sqrt{2-x} < 2$

$3 - (2-x) < 2\sqrt{2-x}$

$1 + x < 2\sqrt{2-x}$

Here $2 - x > 0$

$\Rightarrow x < 2$

$(1 + x)^2 < 4(2 - x)$

$1 + x^2 + 2x < 8 - 4x$

$x^2 + 6x - 7 < 0$

$(x + 7)(x - 1) < 0$

$x \in (-7, 1)$

25. (ABC)

$xy + 3y^2 - x + 4y - 7 = 0$

& $2xy + y^2 - 2x - 2y + 1 = 0$

$xy + 3y^2 - x + 4y - 7 = 0$

$xy - x + 3y^2 - 3 + 4y - 4 = 0$

$x(y - 1) + 3(y - 1)(y + 1) + 4(y - 1) = 0$

$(y - 1)[x + 3(y + 1) + 4] = 0$

$\Rightarrow$ either $y = 1$ or $x + 3y + 7 = 0$ …(i)

$2xy + y^2 - 2x - 2y + 1 = 0$

$2xy - 2x + y^2 - 2y + 1 = 0$

$2x(y - 1) + (y - 1)^2 = 0$

$(y - 1)[2x + y - 1] = 0$

Either $y = 1$ or $2x + y - 1 = 0$

26. (ABC)

$kx^2 + kx - (2k + 1) > 0$

If true for at least one real x

$\Rightarrow D > 0$

$k^2 - 4k \times (-1)(2k + 1) > 0$

$k^2 + 8k^2 + 4k > 0$

$k(9k + 4) > 0$

$\Rightarrow k \in \left(-\infty, \dfrac{-4}{9}\right) \cup (0, \infty)$

27. (BC)

$\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2$

$\dfrac{\log 2}{\log x} \cdot \dfrac{\log 2}{\log 2x} = \dfrac{\log 2}{\log 4x}$

$\log 2 (\log 4x) = \log x \cdot \log 2x$

$\log 2 (2 \log 2 + \log x) = \log x (\log 2 + \log x)$

$2(\log 2)^2 + \log 2 \cdot \log x = \log 2 \cdot \log x + (\log x)^2$

$\log x = \pm\sqrt{2}\, \log 2$

$x = 2^{\pm\sqrt{2}}$

28. (a)

$\log_{(2x+3)}(6x^2 + 23x + 21)$

$= 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

$\Rightarrow \log_{(2x+3)}\left[(2x+3)(3x+7)\right]$

$= 4 - \log_{(3x+7)}\left[(2x+3)^2\right]$

$\Rightarrow 1 + \dfrac{\log(3x+7)}{\log(2x+3)} = 4 - \dfrac{2\log(2x+3)}{\log(3x+7)}$

Let $\dfrac{\log(3x+7)}{\log(2x+3)} = k$

$\Rightarrow 1 + k = 4 - \dfrac{2}{k}$

$k + \dfrac{2}{k} - 3 = 0$

$k^2 - 3k + 2 = 0$

$(k - 1)(k - 2) = 0$

$\dfrac{\log(3x+7)}{\log(2x+3)} = 1, 2$

$3x + 7 = 2x + 3$ or $3x + 7 = (2x + 3)^2$

$x = -4$ or $3x + 7 = 4x^2 + 9 + 12x$

$4x^2 + 9x + 2 = 0 \qquad 4x^2 + 8x + x + 2 = 0$

$4x(x + 2) + (x + 2) = 0$

$x = -2$ or $x = \dfrac{-1}{4}$

So, $x = \dfrac{-1}{4}, -2, -4$

Also $2x + 3 > 0 \qquad \& \qquad 3x + 7 > 0$

$\qquad\qquad x > \dfrac{-3}{4} \qquad \& \qquad x > \dfrac{-7}{4}$

$\Rightarrow$ only value of $x = -\dfrac{1}{4}$

29. **(BD)**

$2^{\log_{\sqrt{2}}(x-1)} > x + 5$

$2^{\log_2(x-1)^2} > x + 5$

$(x - 1)^2 > x + 5$

$x^2 + 1 - 2x > x + 5$

$x^2 - 3x - 4 > 0$

$(x - 4)(x + 1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

30. **(BCD)**

Given $\dfrac{1}{3} < \dfrac{x^2 - 2x + 4}{x^2 + 2x + 4} < 3$

$\Rightarrow \dfrac{1}{3} < \dfrac{x^2 + 2x + 4}{x^2 - 2x + 4} < 3$

$\dfrac{9.3^{2x} + 6.3^x + 4}{9.3^{2x} - 6.3x + 4}$

$\dfrac{3^{2x+2} + 2.3.3^x + 4}{3^{2x+2} - 2.3.3x + 4}$

$\dfrac{\left(3^{x+1}\right)^2 + 2.\left(3^{x+1}\right) + 4}{\left(3^{x+1}\right)^2 - 2.\left(3^{x+1}\right) + 4}$

Let $3^{x+1} = y$

$\Rightarrow \dfrac{y^2 + 2y + 4}{y^2 - 2y + 4}$

It will also lie between (1/3, 3)

31. **(A)**

$\log_2 x + \log_x 2 = \dfrac{10}{3} = \log_2 y + \log_y 2$

Also $x \neq y$

$\dfrac{\log x}{\log 2} + \dfrac{\log 2}{\log x} = \dfrac{10}{3}$

Solving we get

$\dfrac{\log x}{\log 2} = 3, \dfrac{1}{3}$

$x = 8$ or $x = 2^{\frac{1}{3}}$

If $x = 8 \Rightarrow y = 2^{\frac{1}{3}}$

& if $x = 2^{\frac{1}{3}} \Rightarrow y = 8$

$\therefore \dfrac{x}{y} = \dfrac{2^3}{2^{\frac{1}{3}}}$ or $\dfrac{2^{\frac{1}{3}}}{2^3}$

$\dfrac{x}{y} = 2^{\frac{8}{3}}$ or $2^{-\frac{8}{3}}$

32.　(ACD)

$2x^2 - 2(2a+1)x + a(a+1) = 0$

(i) $f(a) < 0$

(ii) $D > 0$

(i) $f(a) = 2a^2 - 2(2a+1)a + a(a+1)$

$\qquad = 2a^2 - 4a^2 - 2a + a^2 + a$

$\qquad = -a^2 - a < 0$

$\qquad = a(a+1) > 0$

$a \in (-\infty, -1) \cup (0, \infty)$

(ii) $D > 0$

$4(2a+1)^2 - 4 \cdot 2 \cdot a(a+1) > 0$

$4a^2 + 1 + 4a - 2a^2 - 2a > 0$

$2a^2 + 2a + 1 > 0$

$\Rightarrow a \in R$

$\therefore a \in (-\infty, -1) \cup (0, \infty)$

33.　(D)

$2^x + 2^{|x|} \geq 2\sqrt{2}$

Case – 1: $x \geq 0$

$2^x + 2^x \geq 2\sqrt{2}$

$2^x \geq \sqrt{2}$

$\Rightarrow x \geq \dfrac{1}{2}$

Case – II: $x < 0$

$2^x + 2^{-x} \geq 2\sqrt{2}$

Let $2^x = k$

$k + \dfrac{1}{k} \geq 2\sqrt{2}$

$k^2 - 2\sqrt{2}k + 1 \geq 0$

$\left(k - \left(\sqrt{2}+1\right)\right)\left(k - \left(\sqrt{2}-1\right)\right) \geq 0$

$-\infty < 2^x \leq \sqrt{2} - 1 \ \& \ \sqrt{2} + 1 \leq 2^x < \infty$

$\Rightarrow 0 < 2^x < \sqrt{2} - 1 \ \& \ \sqrt{2} + 1 \leq 2^x < \infty$

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right) \& \left(\log_2\left(\sqrt{2}+1\right), \infty\right)$

From case I & II

$x \in \left(-\infty, \log_2\left(\sqrt{2}-1\right)\right] \cup \left[\dfrac{1}{2}, \infty\right)$

34.　(AC)

$(3-x)^4 + (2-x)^4 = (5-2x)^4$

$(3-x)^4 + (2-x)^4 = (3-x+2-x)^4$

Let $3 - x = a \ \& \ 2 - x = b$

$a^4 + b^4 = (a+b)^4$

clearly either $a = 0$ or $b = 0$

$\Rightarrow$ either $x = 3$ or $x = 2$

Now

$a^4 + b^4 = (a^2 + b^2 + 2ab)^2$

$a^4 + b^4 = a^4 + b^4 + 4a^2b^2 + 2a^2b^2 + 4ab^3 + 4a^3b$

$\Rightarrow 4ab^3 + 6a^2b^2 + 4a^3b = 0$

$2ab\,[2b^2 + 3ab + 2a^2] = 0$

Either $a = 0$ or $b = 0$

Or $2a^2 + 3ab + 2b^2 = 0$

$2\left(\dfrac{a}{b}\right)^2 + 3\dfrac{a}{b} + 2 = 0$

Here $D < 0$

$\Rightarrow$ two real roots & two non real roots

35. (AB)

$4^x - (a - 3)2^x + a - 4 = 0$

Roots are non positive

$x \le 0$

$\Rightarrow 0 < 2^x \le 1$

$\left(2^x\right)^2 - \left(a - 3\right)2^x + a - 4 = 0$

$k^2 - \left(a - 3\right)k + \left(a - 4\right) = 0$

(i) $f(0) > 0$

(ii) $f(1) \ge 0$

(iii) $D > 0$

(i) $f(0) = a - 4 > 0$

$\qquad a > 4$

(ii) $f(1) \ge 0$

$1 - \left(a - 3\right) + a - 4 \ge 0$

(iii) $D > 0$

$(a - 3)^2 - 4(a - 4) > 0$

$a^2 + 9 - 6a - 4a + 16 > 0$

$a^2 - 10\,a + 25 > 0$

$(a - 5)^2 > 0$

$a \in R$

(iv) $0 < -\dfrac{B}{2A} \le 1$

$0 < -\dfrac{\left(-(a - 3)\right)}{2 \times 1} \le 1$

$0 < \dfrac{a - 3}{2} \le 1$

$3 < a \le 5$

From (i), (ii), (iii), (iv)

$a \in \left(4, 5\right]$

36. (BCD)

$27^x + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$3^{3x} + 2\cos 3y + 8 + 6\cos y = 4.3^{x+1}.\cos y$

$\left(3^x\right)^3 + 8\cos^3 y - 6\cos y + 8 + 6\cos y = 3.2.3^x.2\cos y$

$\left(3^x\right)^3 + \left(2\cos y\right)^3 + 2^3 = 3.\left(3^x\right)\left(2\cos y\right)(2)$

$3^x + 2\cos y + 2 = 0$

Or $3^x = 2\cos y = 2$

$x = \log_3 2, \cos y = 1$

37. **(ABD)**

$x + y = \sqrt{3}$ ...(1)

$y + z = \sqrt{5}$ ...(2)

$z - x = \sqrt{7}$ ...(3)

(1) & (2) we get

$x - z = \sqrt{3} - \sqrt{5}$

Add in (3) we get

$0 = \sqrt{3} - \sqrt{5} + \sqrt{7}$

Not possible

No real values of x, y, z

38. **(ABCD)**

$y = \dfrac{x^2 + 2x - 7}{x^2 + 2x + 7}$

Range in given D

For $x = 1, \quad y = -\dfrac{2}{5}$

              $x = -1 \quad y = -\dfrac{4}{3}$

              $x = 0 \quad y = -1$

              $x = 2 \quad y = -1$

39. **(CD)**

$\log_2 x = \log_4 y + \log_4 (4 - x)$

$4 - x > 0 \Rightarrow x < 4$

& x, y > 0

$\log_2 x = \dfrac{1}{2} \log_2 \big(y.(4 - x) \big)$

$x^2 = (4 - x).y$ ...(1)

$\log_3 (x + y) = \log_3 x - \log_3 y$

$x + y > 0$ & $x > 0$ & $y > 0$

$x + y = \dfrac{x}{y}$

$y = \dfrac{x}{y} - x$

$y = \left(\dfrac{1}{y} - 1 \right)^x$ ... (2)

$x = \dfrac{4}{3}, y = \dfrac{2}{3}$

40. **(AC)**

$2\cos^2 x > \dfrac{3}{2}$

$$\left(\cos x - \frac{\sqrt{3}}{2}\right)\left(\cos x + \frac{\sqrt{3}}{2}\right) > 0$$

$$-1 \le \cos x < -\frac{\sqrt{3}}{2} < \cos x \le 1$$

41. (ABCD)
$f(x) = x^8 - x^5 + x^2 - x + 1$
$f(x) = x^5(x^3 - 1) + x(x - 1) + 1$
$f(x) = x(x - 1)[x^4(x^2 + x + 1) + 1] + 1$
$x^4(x^2 + x + 1) > 0$
Also $x(x - 1) > 0$ for $n \in (-\infty, 0) \cup (1, \infty)$

Now for $x \in [0, 1]$
$x^8 + x^2 + 1 - x^5 - x > 0$
$\therefore f(x) > 0 \Rightarrow x \in R$

42. (ABC)
$x^2 + y^2 - xy - x - y + 1 \ge k$
$x^2 - (y + 1)x + y^2 - y + 1 = k \ge 0$
$D \le 0 (y + 1)^2 - 4y^2 + 4y - 4 + 4k \le 0$
$3y2 - 6y + 3 - 4k \ge 0 (\forall y \in R)$
$(D \le 0) 36 - 36 + 48k \le 0$
Minimum value of expected = 0
$k \le 0$
Have minimum value is 0
In–equation has solution $x, y \in R$
$\therefore$ All equations with real values if x & y are true.

43. (ABCD)
$x^2 + \left(a \log(1 - a^2)\right)x - \left(a^2 - 1\right) = 0$
Roots are opposite in sign
$\frac{C}{A} <)$
$-(a^2 - 1) < 0$
$a \in (-\infty, -1) \cup (1, \infty)$ ...(1)
As we have tan log $(1 - a^2)$
$\therefore a^2 < 1$ ...(2)
From (1) & (2) no value of a for which roots are opposite in sign

44. (CD)
$x^4 - 4x^3 + ax^2 - bx + 1 = 0$
$\alpha + \beta + \gamma + \delta = 4$
$\alpha . \beta . \gamma . \delta = 1$
Now by $AM \ge am$
$AM = \dfrac{\alpha + \beta + \gamma + \delta}{4} = 1$

$$GM = \left(\alpha.\beta.\gamma.\delta\right)^{1/4} = 1$$
Now, $\because$ AM = GM
$$\Rightarrow \alpha = \beta = \gamma = \delta = 1$$
Now, $a = \alpha\beta + \beta\gamma + \gamma\delta + \delta\gamma + \alpha\gamma + \beta\delta$
$$\Rightarrow b = 4$$

45. (BC)
$$\log_{(a)} x \le \log_a x^2$$
For $0 < a < 1$
$$x \ge x^2$$
$$x \in (0,1] \qquad \{as\ x \ne 0\}$$
For $1 < a$
$$x \le x^2$$
$$x(x-1) \ge 0$$
$$x \in [1,\infty)$$

46. (AC)
$$\log_3 (x+2) > \log_{x+2} 81$$
$$\frac{\log(x+2)}{\log 3} > \frac{4\log 3}{\log(x+2)}$$
(i) for $x > -1$
$$\log(x + 2) > 0$$
$$\therefore\ [\log(x+2)]^2 > 4\,(\log 3)^2$$
$$(\log(x+2) - 2\log 3)(\log(x+2) + 2\log 3) > 0$$
$$\Rightarrow \log(x+2) < -2\log 2\ \&\ \log(x+2) > 2\log 3$$
$$\&\ \log(x+2) > 2\log 3$$
$$x < \frac{1}{9} - 2\ \&\ x > 7$$
$$x < \frac{-17}{9}$$
(ii) for $x < -1$
$$\log(x+2)^2 \le 4(\log 3)^2$$
$$-2\log 3 < \log(x+2) < 2\log 3$$
$$\Rightarrow \frac{-17}{9} < x < 7$$

47. (BC)
$$\frac{1}{\log_4 \left(\dfrac{x+1}{x+2}\right)} \le \frac{1}{\log_4 (x+3)}$$
$$\frac{x+1}{x+2} > 1$$
$$\frac{x+1-x-2}{x+2} > 0$$

$$-\frac{1}{x+2} > 0$$

$$\frac{1}{x+2} < 0$$

$$\Rightarrow x < -2 \qquad \qquad \dots (1)$$

$$\frac{x+1}{x+2} > 0$$

$$\Rightarrow x > -1 \ \& \ x < -2 \qquad \dots (2)$$

$$\Rightarrow 0 < \frac{x+1}{x+2} < 1$$

For $x > -1$

Case I take $x > -1$

$$\frac{1}{\log_4\left(\dfrac{x+1}{x+2}\right)} \le \frac{1}{\log_4(x+3)}$$

$$\log_4(x+3) \ge \log_4\left(\frac{x+1}{x+2}\right)$$

$$\left\{ \text{as } \log_4\left(\frac{x+1}{x+2}\right) < 0 \text{ for } x > -1 \right\}$$

$$x+3 \ge \frac{x+1}{x+2}$$

$$x+3 - \left(\frac{x+1}{x+2}\right) \ge 0$$

$$\Rightarrow x > -2$$

{we have taken $x > -1$}

i.e. $x > -1$

case II: $-3 < x < -2$

$$x+3 = \frac{x+1}{x+2} \qquad \frac{x^2+4x+5}{x+2} \ge 0$$

$$x > -2$$

no solution

48. (ABC)

$n = x(x+1)(x+2)(x+3)$

$= x^4 + 6x^3 + 11x^2 + 6x$

Now $(x^2 + 3x + 1)^2 = x^4 + 6x^3 + 11x^2 + 6x + 1$

$\Rightarrow n + 1$ is a perfect square

n is a never a perfect square.

Also in four consecutive integers there is a factor of 2, a factor of 4, a factor of 3

$\therefore$ n is always divisible by $2 \times 4 \times 3 = 24$

49. (BC)

Possible values 10 & 12 from options

50. (C)

$28x + 30y + 31z = 365$

One of solution is (1, 4, 7) for (x, y, z)

$z - 2x = 5$

1. (D)

    $\|x+1|-2| = 1$

    $\Rightarrow |x-1| = 3 \quad or \quad |x-1| = 1$

    $\Rightarrow (x-1)^2 = 9 \qquad x-1 = \pm 1$

    $\Rightarrow (x-4)(x+2) = 0 \quad x = 2, 0$

    $\Rightarrow x = -2, 4$

2. (B)

    $\|x-2|-3| = 4$

    $\Rightarrow |x-2| = 7$

    $\Rightarrow (x-2)^2 = 7^2$

    $\Rightarrow (x-9)(x-5) = 0$

    $\Rightarrow x = 5, 9$

3. (C)

    $|x-2| = 10, 0$

    $x = 2, 12, -8$

4. (A)

    $A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$

    $\Rightarrow A = 2^2 + 3^{\log_2 4} + 4^1 - 3^{\log_2 4} = 8$

    $D = (\log_5 49)(\log_7 125)$

    $\Rightarrow D = \dfrac{\log 49}{\log 5} \times \dfrac{\log 125}{\log 7} = \dfrac{2\log 7}{\log 5} \times \dfrac{3\log 5}{\log 7}$

    $\Rightarrow D = 6$

    $\Rightarrow a = \log_A D = \log_8 6$

    $\Rightarrow a = \dfrac{\log_2 6}{3}$

    Now $\log_6 12 = \log_6 6 + \log_6 2$

    $\Rightarrow \log_6 12 = 1 + \dfrac{1}{\log_2 6}$

    $\Rightarrow \log_6 12 = 1 + \dfrac{1}{3a} = \dfrac{1+3a}{3a}$

5. (A)

    $N = 7^{\log_{49} 900}$

    $\Rightarrow N = 7^{\log_{7^2} 30^2} = 7^{\log_7 30} = 30$

    $\log_6 12 = \dfrac{1+ma}{ma} \Rightarrow m = n = 3$

    $\log_N m = \log_{30} 3$

    $\log_m N = \log_3 30$

    $\log_n N = \log_3 30$

    Clearly $\log_{30} 3 < \log_3 30$

$\therefore \log_N m < \log_m N = \log_n N$

6. **(B)**

$\log_{\left(A - \frac{N}{10}\right)} |N + A + D + m + n| - \log_5 2 = \log_{\left(8 - \frac{30}{10}\right)} |30 + 8 + 6 + 3 + 3| - \log_5 2$

$= \log_5 50 - \log_5 2 = \log_5 25 + \log_5 2 - \log_5 2$

$= \log_5 5^2 = 2$

7. **(A)**

$y = 4 - |4x^2 - 9| \Rightarrow |4x^2 - 9| = 4 - y \geq 0$

$\Rightarrow y \leq 4$

8. **(A)**

$\Rightarrow (4x^2 - 9)^2 = 7^2$

$\Rightarrow (4x^2 - 16)(4x^2 - 2) = 0$

$\Rightarrow x = \pm 2, \pm \dfrac{1}{\sqrt{2}}$

9. **(B)**

$|Z| + 4 = 4 - |4x^2 - 9| \Rightarrow |Z| + |4x^2 - 9| = 0$

$\Rightarrow 4x^2 - 9 = 0, Z = 0$

$\Rightarrow x = \pm \dfrac{3}{2}, Z = 0$

10. **(D)**

$y = \log_x (4 - x) + \log_3 (2 + x)$

$x > 0, x \neq 1, 4 - x > 0 \, \& \, 2 + x > 0$

$x \in (0, 1) \cup (1, 4)$

11. **(A)**

$\Rightarrow \dfrac{x}{x+3} \geq 1 \Rightarrow \dfrac{3}{x+3} \leq 0$

$\Rightarrow x < -3$

12. **(C)**

$y = \log_{\frac{1}{2}} \left(\dfrac{x+4}{x+1}\right) + \sqrt{x-1}$

$\Rightarrow \dfrac{x+4}{x+1} > 0 \, \& \, x - 1 \geq 0$

$\Rightarrow x < -4 \text{ or } x > -1 \, \& \, x \geq 1$

$\Rightarrow (-\infty, -4) \cup [1, \infty)$

13. **(A)**

$f(x) = |x - 2| + |x - 1| + |x + 1| + |x + 2|$

$f(-x) = f(x)$ then given functions is symmetric about y – axis

14. (B)
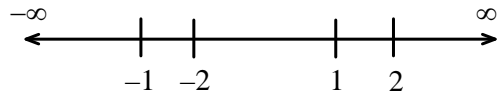
$$|x-1|+|x-2|+|x+1|+|x+2|=6$$

Case I: $x < -2$

$\therefore \not{1} - x + \not{2} - x - n - \not{1} - n - \not{2} = 6$

$-4x = 6$

$x = {}^{-3}\!/_2$

Case II: $-2 \le x < -1$

$\not{1} - n + \not{2} - n - \not{n} - \not{1} + \not{x} + 2 = 6$

$-2x = 2 \qquad \therefore x = -1$

Case III: $-1 \le x < 1$

$1 - \not{x} + 2 - \not{x} + \not{x} + 1 + \not{x} + 2 = 6$

$6 = 6$

Case IV: $1 \le x < 2$

$\not{x} + 1 + 2 - \not{x} + n + 1 + n + 2 = 6$

$2x = 0 \qquad \therefore n = 0$

Case V: $x \ge 2$

$4x = 6 \qquad \therefore x = {}^{3}\!/_2$

$\therefore x \in [-1, 1]$
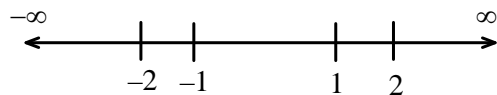
15. (C)

$$f(n) = |x-2| + |x-1| + |x+1| + |x+2|$$

$$f(x)_{min} = 6$$

$$\therefore k = 3$$

16. (C)

$$|x-1| + |x-2| + |x+1| + |x+2| \le 20$$

Case I: $x < -2$

$\not{1} - x + \not{2} - x - x - \not{1} - x - \not{2} \le 20$

$-4x \le 20, \qquad x \ge -5$

$\therefore x \in [-5, -2]$

Case II: $-2 \le x < -1$

$\not{1} - x + \not{2} - x - \not{x} - \not{1} + \not{x} + 2 \le 20$

$-2x \le 16 \qquad \therefore x \ge -8$

$x \in [-2, -1]$

Case III: $-1 \le x < 1$

$1 - \not{x} + 2 - \not{x} + \not{n} + 1 + \not{n} + 2 \le 20$

$6 \le 20$

Case IV: $1 \le x < 2$

$\not{x} - \not{1} + 2 - \not{x} + n + \not{1} + x + 2 \le 20$

$2x \le 16 \qquad x \le 8$

Case V: $x \geq 2$

$4x \leq 20 \qquad x \leq 5$

$x \in [-5, 5]$

17. (D)

If $x \in (-\infty, -1), y \in (0, 1)$
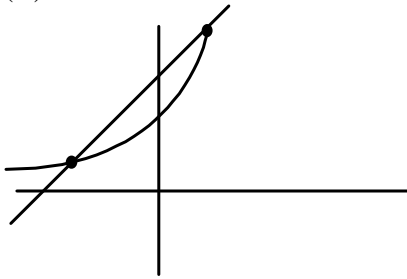
18. (B)

If $x \in (-1, 2)$ then $y \in (-\infty, 0)$

19. (C)

If $n \in (2, 4)$, then $y \in (-\infty, \infty)$

20. (A)

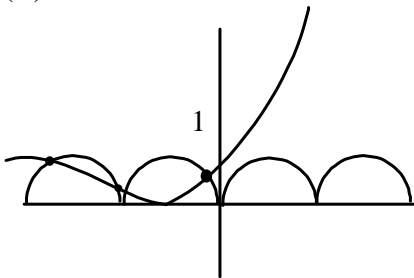$f(x) = k$ has root in $(4, \infty)$ then $k \in (1, \infty)$

21. (C)

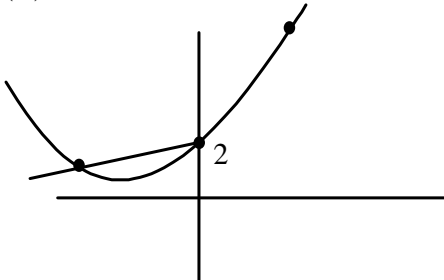

$2^x + 3^x - 1 = 2x + 3$

$2^x + 3^x = 2x + 4$

22. (B)

23. (C)

$2^x + 3^x - 1 = 2x^2 - 2$

$-2^x - 3$

$\Rightarrow 2^x + 3^x = 2(x^2 + x + 1)$

For 24 – 26

$$\tan\left(\theta+\frac{\pi}{4}\right)=3\tan 3\theta$$

$$\frac{1+t\theta}{1-t\theta}=\frac{3\left(3t\theta-t^3\theta\right)}{\left(1-3\tan^2\theta\right)}$$

Get $\tan\theta = t$ $(1 + t)(1 - 3t^2) = 3(1 - t)(3t - t^3)$

$1-3t^2 +t-3t^3 = 9t-3t^3 -9t^2 +3t^4$

$3t^4 - 6t^2 + 8t - 1 = 0$

24. **(D)**

$$\sum\tan\alpha = 0$$

25. **(A)**

$$\pi\tan\alpha = -\frac{1}{3}$$

26. **(B)**

$$\frac{1}{t\alpha}+\frac{1}{t\beta}+\frac{1}{t\gamma}+\frac{1}{t\delta}=\frac{\sum t\alpha\, t\beta\tan\gamma}{t\alpha\, t\beta\, t\gamma\, t\delta}$$

$$=\frac{-8/3}{-1/3}=8$$

27. **(A)**

Statement – 1 is correct as 13 doesn't $x^4 - 5x^3 + 56|x-3| = 0$

Statement – 2: $a_0 x^n + a_1 x^{n-1} +...+ a_{n-1}x + a_n = 0$ where $a_0,a_1,....,a_n \in I, a_0 \neq 0, a_n \neq 0$

$$\Rightarrow \left(a_0 x^{n-1} + a_1 x^{n-2} +...+ a_{n-1}\right) = -\frac{a_n}{x}$$

Clearly for an integral root an must be divisible by x.

Hence statement –2 is also correct & is a correct explanation of statement – 1

28. **(D)**

For common root of equations $x^2 - 4x + 1 = 0$ and $x^2 - ax + b = 0$, if $a = 4$ & $b = 1$, then both the roots will be common.

If quadratic equations $ax^2 + bx + c = 0$ and $a_1 x^2 + b_1 x + c_1 = 0$ with rational coefficient have both the roots common, then we must have $a = a_1 k$, $b = b_1 k$ and $c = c_1 k$, for some real number $k \neq 0$.

29. **(D)**

Let the polynomial be P (x) = (x – 1) (x – 2) Q (x) + ax + b

P (1) = –1 $\Rightarrow$ a + b = –1

P (2) = 1 $\Rightarrow$ 2a + b = 1

$\Rightarrow$ a = 2, b= –3

$\Rightarrow$ remainder is 2x – 3

Statement –1 is false.

Statement – 2 is correct.

30. (D)
As $x^2 + 1 \geq 1$ hence $\log_3(x^2 + 1) \geq 0$, also $\left|\log_4 y^2\right| \geq 0$

Hence $\log_3(x^2 + 1) + \left|\log_4 y^2\right| = 0 \Rightarrow \log_3(x^2 + 1) = 0$ & $\left|\log_4 y^2\right| = 0$

$\Rightarrow x^2 + 1 = 1$ & $y^2 = 1$

Statement –1 is false
Statement –2 is correct

31. (A)
$\log_{0.1}\left(\sqrt{14} - \sqrt{13}\right) = \log_{10}\left(\sqrt{14} + \sqrt{13}\right)$

Also $\sqrt{13} - \sqrt{12} < \sqrt{14} + \sqrt{13} \Rightarrow \log_{0.1}\left(\sqrt{14} - \sqrt{13}\right) > \log_{10}\left(\sqrt{13} - \sqrt{12}\right)$

Statement –1 is true
Statement –2 is also correct & correct explanation of statement –1

32. (C)
Statement –1 $\log_{\sin^2\theta} b < \log_{\sin^2\theta} C$

$\Rightarrow b > c$
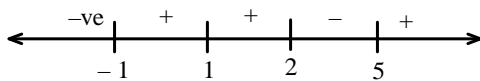
As $\log_x b < \log_x c$

If $x \in (0,1)$ $b > c$

& if $x > 1$ $b < c$

True

$\Rightarrow$ statement 2 is false. It's not always true.

33. (A)
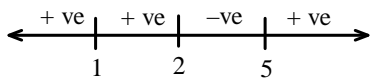
34. (A) $\dfrac{(x-1)^2(x+1)(x-2)}{(x-5)} \geq 0$

x = 1, 1, –1, 2 (roots)
x = 5 (pole)

$x \in [-1, 2] \cup (5, \infty)$

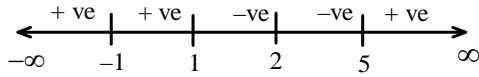(B) $\dfrac{|x|(x-5)}{(x-2)} \geq 0$

$x \in (-\infty, 2] \cup [5, \infty)$

(c) $\log_{10} x$
If x is two digit number then Q = [1, 2]
P = $\log_{10} x$, x is four digit number then $P = [3, 4]$

(d) $\dfrac{(x-1)(x+1)(x-2)^2}{(x-5)} \le 0$

$x \in [1,5]$

35. (A) – (r, s)

$\Rightarrow \log_{49} 7 = \dfrac{1}{2}$

$\Rightarrow \log_3 \left(5 + 8\log_{49}\left(5 + 4\log_{49} 7\right)\right)$

$= \log_3 \left(5 + 8\log_{49} 7\right)$

$= \log_3 \left(5 + \dfrac{8}{2}\right)$

$= \log_3 9 = 2$

$|k| = 2 \Rightarrow k = \pm 2$

(B) $\sqrt{\left(\log_{2^{-1}} . 2^2\right)^2} = \sqrt{(-2)^2} = |-2| = 2$

(C) $\log_x \left(x^2 - 1\right) = 0$

$\therefore x^2 - 1 = 1$

$x^2 = 2$

$x = \pm\sqrt{2}$

But $x = -\sqrt{2}$ (rejected)

$\therefore$ only one solution

(D) $\dfrac{1}{x} = \dfrac{\sqrt{9 - \sqrt{77}}}{\sqrt{81 - 77}}$ $\qquad \therefore \qquad \dfrac{2}{x} = \sqrt{9 - \sqrt{77}}$

$x + \dfrac{2}{x} = \sqrt{9 + \sqrt{77}} + \sqrt{9 - \sqrt{77}}$
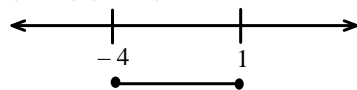
$\left(x + \dfrac{2}{x}\right)^2 = 18 + 2\times 2 = 22$

$\dfrac{1}{11}\left(x + \dfrac{2}{x}\right)^2 = 2$

36. (A) $x^2 + 4x - x - 4 \le 0$

$x(x+4) - (x+4) \le 0$

$(x-1)(x+4) \le 0$

$x \in [-4, 1]$

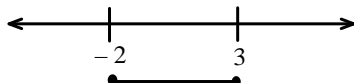(B) $x^2 - 2x + 2 > 0$

$\therefore x \in R$

(C) $x^2 - 3x + 2x - 6 < 0$

$x(x-3) + 2(x-3) < 0$

$(x+2)(x-3) < 0$
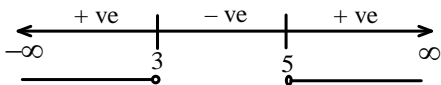

$x \in (-2, 3)$

(D) $x^2 - 3x - 5x + 15 > 0$

$x(x-3) - 5(x-3) > 0$

$(x-3)(x-5) > 0$

$x \in (-\infty, 3) \cup (5, \infty)$

37.   (A) – (QR)

$1 - x^2 > 0$ $\qquad$ $\therefore x^2 < 1$

$2 - x^2 > 2\sqrt{1-x^2}$ $\qquad$ $|x| < 1$

$4 + x^4 - \cancel{4x^2} > 4 - \cancel{4x^2}$

$x^4 > 0$ $\qquad$ $\therefore x \in (-1, 1)$

(B) – (PQRS)

$x^8 - x^5 + x^2 - x + 1 > 0 \ x \in R$

For $x < 0$ (time)

For $x \in (011)$

$\dfrac{x^8}{+ve} + \dfrac{(x^2 - x^5)}{+ve} + \dfrac{(x)}{+ve} > 0$

For $x > 1$

$\dfrac{(x^8 - x^5)}{+ve} + \dfrac{(x^2 - \alpha)}{+ve} + \dfrac{1}{+ve} > 0$

(C) – (PQRS)

$x^{12} - x^9 + x^4 - x + 3 > 0$ (same)

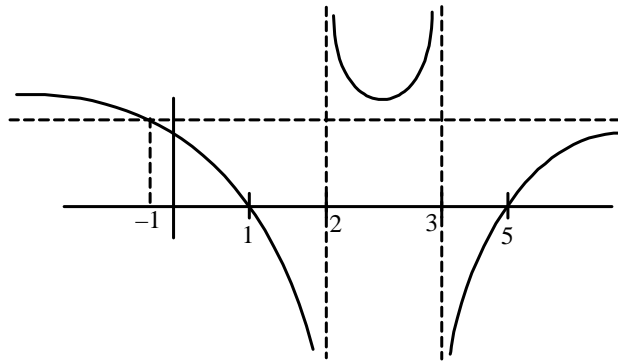(D) – (PQRS)

$2x^2 + 1 > \sqrt{4x^2 + 1}$

$4x^4 + 4x^2 + 1 > 4x^2 + 1$

$x^4 > 0 \ n \in (-\infty, \infty) - \{0\}$

38. $\dfrac{(x-1)(x-5)}{(x-2)(x-3)}$

(A) – (PRS)
$-1 < x < 1,$ $f(x) \in (0,1)$

(B) – (QS)
$1 < x < 2,$ $f(x) \in (-\infty, 0)$

(C) – (QS)
$3 < x < 5,$ $f(x)(-(-\infty,0))$

(D) – (PRS)
$x > 5,$ $f(x) \in (0,1)$

39. (A) – (R)
$|x-2| < 5$
$-5 < x - 2 < 5$
$-3 < x < 7$

(B) – (P)
$|x-1| + |x-2| < 6$
Case (i) $x \geq 2$
$2x - 3 < 6$
$x < 9.5$
Case (ii) $1 \leq x < 2$
$x - 1 - x + 2 < 6$
$1 < 6$
Case (iii) $x \leq 1$
$-x + 1 - x + 2 < 6$
$3 < 6 + 2x$
$x > -3$

(C) – (S)
$|4 - 3x| \leq \frac{1}{2}$
$-\frac{1}{2} \leq 4 - 3x \leq \frac{1}{2}$
$-\frac{9}{2} \leq -3x \leq -\frac{7}{2}$
$\frac{3}{2} \geq x \geq \frac{7}{6}$

(D) – (Q)
$|x^2 - 2| < 7$

$-7 < x^2 - 2 < 7$

$-5 < x^2 < 9$

$0 < x^2 < 9$

$-3 < x < 3$

40. (A) – (QS)

$x^{\log_{10} x} = 100x$

$\Rightarrow \left(\log_{10} \lambda\right)^2 = 2\left(\log_{10} \alpha\right)$

$\Rightarrow \log_{10} x = 2, -1$

$x = 100, \dfrac{1}{10}$

$x_1 . x_2 = 10$

(B) – (QS)

$\log_2 (9 - 2^x) = 3 - x$

$\Rightarrow 9 - 2^x = \dfrac{8}{2^x}$

$\Rightarrow \left(2^x\right)^2 - 9 \cdot \left(2^x\right) + 8 = 0$

$2^x = 1, \ 2^x = 8$

$x = 0, \ x = 3$

$\lambda_1^{\,2} + \alpha_2^{\,2} = 9$

(C) – (P)

$\log_{1/8} \log_{1/4} \log_{1/2} (x) = \dfrac{1}{3}$

$\Rightarrow \log_{1/4} \log_{1/2} x = \dfrac{1}{2}$

$\Rightarrow \log_{1/2} x = \left(\dfrac{1}{4}\right)^{1/2} = \dfrac{1}{2}$

$\Rightarrow x = \left(\dfrac{1}{2}\right)^{1/2} = \dfrac{1}{\sqrt{2}}$

(D) – (QR)

$\log_b a = 3 \qquad \log_b c = -4$

$a = b^3, \qquad c = \dfrac{1}{b^4}$

$b^{9x} = \dfrac{1}{b^{4x-4}}$
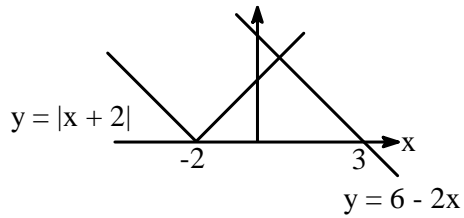
$\Rightarrow b^{9x} = b^{-4x+4}$

$9x = -4x + 4$

$\Rightarrow x = \dfrac{4}{13}$

p + q = 17

1. (1)

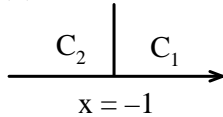$y = |x + 2|$

$y = 6 - 2x$

2. (2)
    Same as above.

3. (3)

$x = -1$

C_1: If $x \geq 1$ … (a)
Then, $2(x + 1) > x + 4$
$x > 2$ … (b)
(a) n (b)
$x \in (2, \infty)$ (C – 1)
C_2: If $x < -1$ … (a)
Then, $-2(x + 1) > x + 4$
$\Rightarrow x < -3$ … (b)
(a) n (b)
$x \in (-\infty, -3) \cup (2, \infty)$
$C - 1 \cup C - 2$
So required $x = 3$
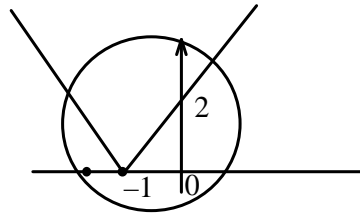
4. (2)
    Same as above take three cases.

5. (5)
    $(|x| - 2)(|x| - 3) < 0 \Rightarrow 2 < |x| < 3$
    $\Rightarrow x \in (-3, -2) \cup (2, 3)$
    So $(a + b) = 5$

6. (4)
    $-5 < x^2 - 4x < 5$
      (1)   (2)
    (1) $x^2 - 4x + 5 > 0 \Rightarrow x \in R (\because D < 0)$ …(1)
    (2) $x^2 - 4x - 5 < 0 \Rightarrow (x - 5)(x + 1) < 0 \Rightarrow -1 < x < 5$ …(2)
    (1) n (2)
    $x \in (-1, 5)$
    So, $m = 0$, $n = 4 \Rightarrow (n - m) = 4$

7. (4)
    Same as question 6.

8. (7)
   Domain $x \geq 3$
   So $x \in (3, 10]$
   So number of integer $n = 7$
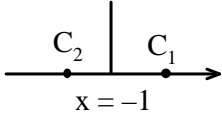
9. (2)
   $0 \leq x^2 + 2x - 3 < 1$
      (1)   (2)
   (1) $(x+3)(x-1) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [1, \infty)$ ... (1)
   (2) $x^2 + 2x - 4 < 0 \Rightarrow x = -1 \pm \sqrt{5} \; -1-\sqrt{5} < x < -1 + \sqrt{5}$ ...(2)
   (1) n (2)
   $x \in \left(-1-\sqrt{5}, -3\right] \cup \left[1, \sqrt{5}-1\right)$
   So integer $x = \{-3, 1\}$

10. (1)
    $0 \leq \dfrac{x^2 - x - 2}{x^2} < \dfrac{4}{3}$ & solve as above

11. (9)
    $\dfrac{\sqrt{2x^2 + 15x - 17}}{10 - x} \geq 0$
    $2x^2 + 15x - 17 \geq 0$
    $2x^2 + 17x - 2x - 17 \geq 0$
    $x(2x + 17) - (2x + 17) \geq 0$
    $(x - 1)(2x + 17) \geq 0$
    $x \in (-\infty, -8.5] \cup [1, \infty)$
    Also $x < 10$
    $\therefore$ No. of integers positive are
    $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
    No. of positive integers are 9.

12. (6)
    Domain: $x \in [1, 9]$ & on solving $x \in [1, 5]$
    So $(m + n) = 6$

13. (5)
    Domain (a) $-x^2 + 2x + 2y \geq 0 \Rightarrow x^2 - 2x - 2y \leq 0$
    $\Rightarrow (x - 6)(x - 4) \leq 0 \Rightarrow -4 \leq x \leq 6$ ... (a)
    (b) $8x - x^2 \geq 0 \Rightarrow x^2 8x \leq 0 \Rightarrow x \in [0, 8]$ ...(b)
    (a) n (b) $x \in [0, 6]$ ...(1)
    (b) $-x^2 + 2x + 2y \geq 8x - x^2 \Rightarrow x \leq y$ ... (2)
    (1) & (2) $x \in [0, 4]$
    So answer is (5)

14. (1)
    As function is increasing so equation change at most one solution

15. (1)
Same as Q. 13

16. (1)
Same as above

17. (2)
$\left|2^{x+1}-1\right|+\left|2^{x+1}+1\right|=2^{|x+1|}; x \in R$

$$\underset{x=-1}{\underset{\bullet}{\underline{C_2\Big|C_1}}}\longrightarrow$$

C_1 : if $x \geq -1$ …(a)
$2.2^x - 1 + 2.2^x + 1 \Rightarrow 2^x.2$
$\Rightarrow 2^x = 0 \Rightarrow x \in \phi$ … (b)
(a) & (b) $x \in \phi$ … C_1
C_2: If $x < -1$
$1 - 2.2^x + 2.2^x + 1 = \dfrac{1}{2}.2^{-x}$

$\Rightarrow 2^{-x} = 4 \Rightarrow x = -2$ … (b)
(a) & (b) $x = -2$ … C_2
$C-1 \cup C-2$
$x = -2 \Rightarrow |x| = 2$

18. (7)
$(x - 2) = 0$ or $1 \Rightarrow x = 2, 3$

or $\log_2 x^3 - 3\log_x{}^4 = 3 \Rightarrow 3\log_2{}^x - \dfrac{6}{\log_2 x} = 3$

$\Rightarrow \log_2^2 x - \log_2 x - 2 = 0 \Rightarrow \left(\log_2 x - 2\right)\left(\log_2 x + 1\right) = 0$

$\log_2 x = -1, 2 \Rightarrow x = \dfrac{1}{2}, 4$

So sum $= 9.5$

19. (8)
As function is increasing so only one solution
$x = (16)^2 = 2^8$

20. (2)
$\sqrt{\log_2^x} + \sqrt[3]{\log_2^x} = 2$
$x = 2$

21. (0)
$\left(|x| - 3\right)\left(|x| + 2\right) = 0 \Rightarrow |x| = -2, 3 \Rightarrow x = \pm 3$

22. (0)
$\log(x - 2)(1 - x) \geq -1$
$x \in \phi \Rightarrow$ No solution

23. (0)

$$(x+1)(x-1)-2\sqrt{x^2-1}=4x-1$$

$$\Rightarrow -2\sqrt{x^2-1}=(2x-1)$$

$$\Rightarrow 4(x^2-1)=4x^2+1-4x \Rightarrow x=\frac{5}{4}$$

But $x=\dfrac{5}{4}$. Doesn't satisfy so no solution

24. (2)

Same as Q. 18

25. (4)

$$D \geq 0 \Rightarrow 16-\log_2 n \geq 0 \Rightarrow \log_2 n \leq 2$$

$$(0 < n \leq 4)$$

26. (1)

$$(2x)^2 = x^2 + 75 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$$

Checking at $x=\pm 5$ then only $x = 5$ possible

27. (2)

Same as above

28. (7)

(1) Domain: a) $x>0, x \neq \dfrac{1}{2}$ …(a)

   b) $x^2-5x+6>0$

   $\Rightarrow (x-2)(x-3)>0 \Rightarrow x \in (-\infty,2)\cup(3,\infty)$ … (b)

(a) & (b) $x \in (0,2)\cup(3,\infty)-\left\{\dfrac{1}{2}\right\}$ … (1)

(2) C–1: If $x>\dfrac{1}{2}$ … (a)

$x^2-5x+6>2x$

$\Rightarrow x^2-7x+6>0 \Rightarrow (x-6)(x-1)>0$

$\Rightarrow x \in (-\infty,1)\cup(6,\infty)$ …(b)

(a) & (b) $x(6,\infty)$ … (C–1)

C–2 if $x<\dfrac{1}{2}$ … (a)

$x^2-5x+6<2x$

$x \leq (1,6)$ … (b)

n $x \in \phi$ … (C–2)

$(C-1)\cup(C-2)$

$x \in (6,\infty)$ …(2)

(1) n (2) $x \in (6,\infty)$

29. (2)

$y^{(6-y)^2+7(6-y)+12}=1$

$\Rightarrow y^{y^2-19y+90}=1 \Rightarrow y^{(y-10)(y-9)}=1$

So $y = 1, 9, 10, -1 \Rightarrow x = 5, -3, -4, 7$

So $(x, y) \equiv (5,1)$ or $(7,-1)$

30. (1)

$3+|x+2|=\sqrt{9-y^2}$

So $x = -2, y = 0$

So $(-2, 0)$

31. (2)

$\log(-x) = \sqrt{\log|x|}$

Domain: $x < 0$

$\log^2(-x) = \log(-x)$

$\log(-x) = 0, 1$

$-x = 1, 10 \Rightarrow x = -1, -10$

i.e. two solutions

32. (2)

Let $\log_a^x = t$

$\dfrac{2}{t} + \dfrac{1}{1+t} + \dfrac{3}{2+t} = 0$

$\Rightarrow 2(1 + t)(2 + t) + t(2 + t) + 3t(1 + t) = 0$

$\Rightarrow 6t^2 + 11t + 2 = 0$

$t = \dfrac{-11 \pm \sqrt{121-48}}{12} = \dfrac{-11 \pm \sqrt{73}}{12}$

i.e. two values

33. (3)

$N = \log_3(3^3.5).\log(5.3) - \log_3^5.\log_3(3^4.5)$

$= (3 + \log_3^5)(1 + \log_3^5) - (\log_3^5)(4 + \log_3^5)$

$= 3$

34. (0)

$\log(x-3)(x-1)$

$\left(\dfrac{1}{10}\right)(x-3) \geq 1$

$\log(x-3)(x+1)$

$\Rightarrow 10(x-3) \leq 1$

Domain: $x \in (3, \infty) - \{4\}$

$10^{1+\log_{(x-3)}^{(x-1)}} \leq 1$

$\Rightarrow \log_{(x-3)}^{(x-1)} \leq -1$

Not possible as $x > 3$ so no solution

35. (2)

Same as Q. 32

36. (2)

$$\sqrt{(x-1)+4-2.\sqrt{4}\sqrt{2-1}}+\sqrt{(x-1)+9-2\sqrt{9}\sqrt{-1}}=1$$

$$\Rightarrow \left|\sqrt{x-1}-2\right|+\left|\sqrt{x-1}-3\right|=1$$

C_1: If $x \geq 4$ ... (a)

$\left(\sqrt{x-1}-2\right)+\left(\sqrt{x-1}-3\right)=1 \Rightarrow \sqrt{x-1}=3$

$x = 10$ ... (b)

(a) n (b) x = 10   ... (C–1)

C_2: If $x \in (3,4)$ ... (a)

$\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1 \Rightarrow 1=1 \Rightarrow x \in R$ ...(b)

(a) n (b) $x \in (3,4)$ ...(C – 2)

C_3: If $x \leq 3$ ... (a)

$-\left(\sqrt{x-1}-2\right)-\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \left(\sqrt{x-1}-2\right)\left(\sqrt{x-1}-3\right)=1$

$\Rightarrow \sqrt{x-1}=2 \Rightarrow x=5$ ... (b)

(a) n (b) $x \in \phi$ ... (C – 3)

$(C-1) \cup (C-2) \cup (C-3)$

$x \in (3,4) \cup \{10\}$

So, x = 10

37. (0)

$$2x^2\left(2^x\right)+4.2^{|x-3|}=16x^2.2^{|x-3|}+\frac{2^x}{2}$$

$$\Rightarrow 4x^2.\left(2^x\right)+8.2^{|x-3|}=32x^2.2^{|x-3|}+2^x$$

$$\Rightarrow 4x^2\left(2^x-8.2^{|x-3|}\right)=\left(2^x-8.2^{|x-3|}\right)$$

So, $4x^2 = 1$ or $2^x = 2^{|x-3|+3}$

$x = \pm\dfrac{1}{2}$ or $x = |x-3|+3$

$|x-3|=(x-3)\ x \geq 3$

So negative integral $x = \phi$

38. (0)

Domain: $x \geq -1$

$$(x+1)+(x+4)+2\sqrt{(x+1)(x+4)}=(x+2)+(x+3)+2\sqrt{(x+2)(x+3)}$$

$\Rightarrow (x + 1)(x + 4) = (x + 2)(x + 3)$

$\Rightarrow y = 6 \Rightarrow x \in \phi$

39. (2)

$x^{x-y}=y^{x+y}, \left(\sqrt{x}\right)y=1$

$$x^{x-y}=\left[\left(\frac{1}{x}\right)^{1/2}\right]^{x+y}$$

$$x - y = -\frac{(x + y)}{2}$$

$$\Rightarrow 3x = y$$

$$\Rightarrow (3x).\sqrt{x} = 1$$

$$\Rightarrow x = \left(\frac{1}{3}\right)^{\frac{2}{3}}$$

Also x = y = 1 is a possible solution

So two pairs possible

40. (1)

$$2^{x+1} = y^2 + 4 \quad \& \quad 2^{x-1} \le y$$

$$\Rightarrow 2^x = \frac{y^2 + 4}{2} \quad \& \quad 2^x \le 2y$$

$$\Rightarrow \frac{y^2 + 4}{2} \le 2y$$

$$\Rightarrow (y - 2)^2 \le 0$$

$$\Rightarrow y = 2$$

So no. of values is 1

41. (6)

$$|x| + |y| = 1, k > 0$$

xy (x + y) = 0

$$\Rightarrow x = 0 \text{ or } y = 0 \text{ or } x + y = 0$$

(i) If x = 0 $\quad \Rightarrow |y| = k$

$$\Rightarrow y = \pm k$$

(ii) If y = 0 $\quad \Rightarrow |x| = k$

$$\Rightarrow x = \pm k$$

(iii) If x + y = 0 $\Rightarrow x = -y$

$$\Rightarrow |x| = |y|$$

$$\Rightarrow |x| + |x| = k$$

$$\Rightarrow x = \pm\frac{k}{2} = y$$

Hence total 6 solutions

42. (7)

$$x^2 - mx + 2m = 0$$

$$x_1 + x_2 = lm \qquad \ldots (1)$$

$$x_1 x_2 = 2m \qquad \ldots (2)$$

Now, $x_1^3 + x_2^3 = x_1^2 + x_2^2$

$$(x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) = (x_1 + x_2)^2 - 2x_1 x_2$$

$$(m)^3 - 3.2m. \, m = (m)^2 - 2.2m$$

$$m^3 - 6m^2 = m^2 - 4m$$

$$m^3 - 7m^2 + 4m = 0$$

$$\Rightarrow m_1 = 0 \, \& \, m_2 + m_3 = 7$$

$$\Rightarrow m_1 + m_2 + m_3 = 7$$

43. (2)

$\left|x^2 + 3x\right| + x^2 - 2 = 0$

Case – I

$x^2 + 3x \geq 0$

$x(x+3) \geq 0$

$x \in (-\infty, -3) \cup (0, \infty)$

Now, $x^2 + 3x + x^2 - 2 = 0$

$2x^2 + 3x - 2 = 0$

$x = \dfrac{1}{2} \ \& \ x = -2$

$\therefore x = \dfrac{1}{2}$ is the solution

Case – II

$x^2 + 3x < 0$

$x(x+3) < 0$

$x \in (-3, 0)$

Now, $-(x^2 + 3x) + x^2 - 2 = 0$

$-3x - 2 = 0$

$\Rightarrow x = \dfrac{-2}{3}$

$\therefore x = \dfrac{-2}{3}$ is the solution

$\Rightarrow$ Total 2 solutions

44. (1)

$4^{\frac{x}{y} + \frac{y}{x}} = 32$

$2^{2\left(\frac{x}{y} + \frac{y}{x}\right)} = 2^5$

$\Rightarrow 2\left(\dfrac{x}{y} + \dfrac{y}{x}\right) = 5$ ...(1)

$\log_3 (x - y) = 1 - \log_3(x + y)$

$\Rightarrow \log_3 \left(x^2 - y^2\right) = 1$

$\Rightarrow x^2 - y^2 = 3$

$\Rightarrow x = \pm\sqrt{3 + y^2}$ ... (2)

Solving (1) & (2) we get

$x = \pm 2 \ \& \ y = \pm 1$

Also $x - y > 0 \ \& \ x + y > 0$

Only solution which satisfy is

$x = 2 \ \& \ y = 1$

So one solution

45. (0)

$x_1 + x_2 - x_3 - x_4 = 1$ ... (1)

$x_1 + 2x_2 + 3x_3 - x_4 = 2$ ... (2)

$3x_1 + 5x_2 + 5x_3 - 3x_4 = 6$ ... (3)

Here (1) & (2) gives

$x_2 + 4x_3 = 1$ ... (4)

Also $2 \times (2) - (3)$ gives

$x_2 + 4x_3 = 0$ ... (5)

$\therefore$ From (4) & (5)

No solution