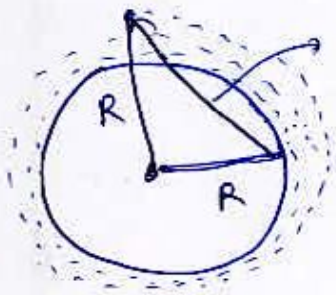
Kinematics I (Mains)

Q 1.

Ans. distance $\geq$ displacement

Q 2.

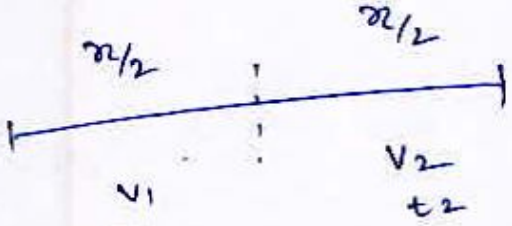
Ans.



displacement $= \sqrt{2}R$

Q 3.

Ans.



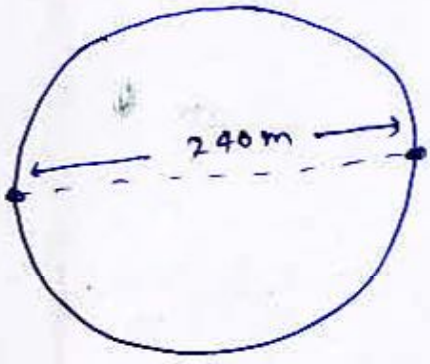
$$V_{avg} = \frac{x}{t_1 + t_2}$$

$$t_1 = \frac{x}{2v_1} \qquad t_2 = \frac{x}{2v_2}$$

$$V_{avg} = \frac{x}{\frac{x}{2v_1} + \frac{x}{2v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

Q 4.

Ans.



$$V_{avg} = \frac{\text{displacement}}{\text{time}}$$

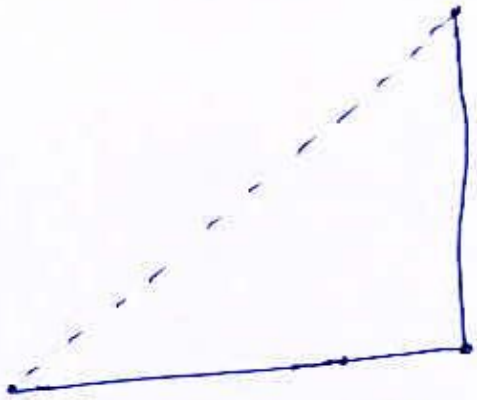
$$= \frac{240}{60}$$

$$= 4\ m/s$$

(2)

Q 5.

Ans.



$54 \times \frac{2}{3} = 36$ km

$54 \times \frac{1}{2} = 27$ km

$$V_{avg} = \frac{\sqrt{(27)^2 + (36)^2}}{\frac{1}{2} + \frac{2}{3}} = \frac{45}{7/6} = 38.6 \text{ km/h}$$

Q 6.

Ans.

$v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$a = \frac{u^2}{2s}$

} Question Correction with same acceleration.

$v = u - at$

$t = \frac{u}{u^2/2s} = \frac{2s}{u}$

$0 = nu - a' \frac{2s}{u}$

$a' = \frac{nu^2}{2s}$

$$v^2 = (nu)^2 - 2ax$$

$$0 = nu^2 - 2a\frac{u^2}{2s}x$$

$$x = n^2 s$$

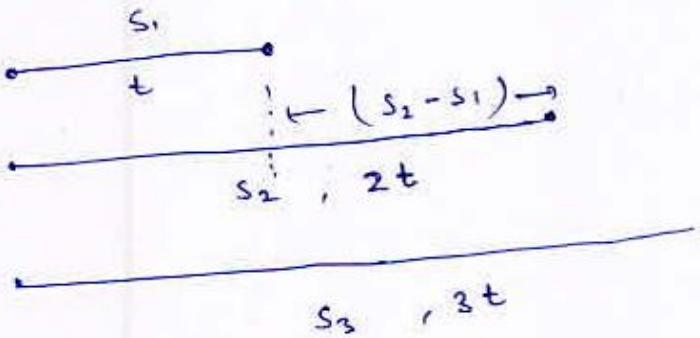
Q 7.

Ans.

$$a = \frac{\vec{v_f} - \vec{v_i}}{t} = \frac{-v-(v)}{t} = -\frac{2v}{t}$$

Q 8.

Ans.



$$s_1 = 0 + \frac{1}{2}at^2$$

$$s_2 = \frac{1}{2}a(2t)^2$$

$$s_3 = \frac{1}{2}a(3t)^2$$

$$s_1 : s_2 - s_1 : s_3 - s_2$$

$$\frac{1}{2}at^2 : \frac{1}{2}at^2(3^2-1) : \frac{1}{2}at^2(3^2-2^2)$$

$$1 : 3 : 5$$

Q 9.

Ans.

$$\frac{1}{2}a_1(5^2-4^2) = \frac{1}{2}a_2(3^2-2^2)$$

$$\frac{a_1}{a_2} = 5/9$$

Q 10. (4)

Ans.

$t_1 + t_2 = 3.12$ s.

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3 \text{ sec.}$$

$t_2 = .12$ sec.

$$V_{sound} = \frac{45}{0.12} = \frac{1500}{4} = 375 \text{ m/s}$$

Q 11.

Ans.

$$s_1 = \frac{1}{2} a 10^2$$

$$s_2 = \frac{1}{2} a (20^2 - 10^2)$$

$$s_2 = 3 s_1$$

Q 12.

Ans.

$$s = \frac{1}{2} a 4^2$$

$$s/4 = \frac{1}{2} a t^2$$

$$t^2 = \frac{s}{2a} = \frac{s}{2 \times s/8} = 4$$

$$t = 2 \text{ sec.}$$

Q 13.

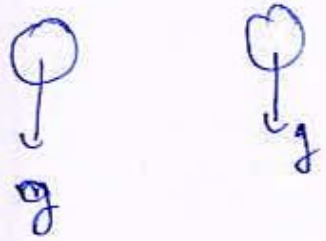
Ans.

$$u^2 = 2 a s_1 \quad \Rightarrow \quad x_1 = 4 s_1$$

$$(2u)^2 = 2 a x$$

Q 14.

Ans.



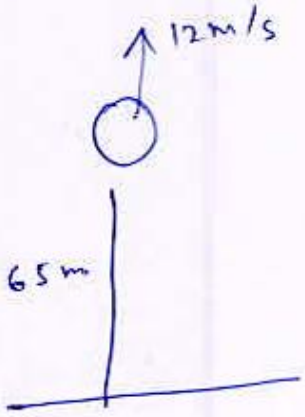
$$s = \frac{1}{2} g t_1^2$$

$$s = \frac{1}{2} g t_2^2$$

$$t_1 = t_2$$

Q 15.

Ans.



$$-65 = 12t - \frac{1}{2} \times 10^{5} \times t^2$$

$$5t^2 - 12t - 65 = 0$$

$$t = 5 \text{ sec.}$$

Q 16.

Ans.

$$s_1 = \frac{1}{2} g (1^2 - 0^2)$$

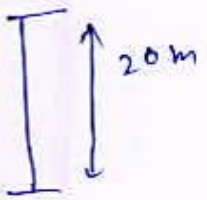
$$s_2 = \frac{1}{2} g (2^2 - 1^2)$$

$$\vdots$$

$$s_n = \frac{1}{2} g (n^2 - (n-1)^2) = \frac{1}{2} g (2n-1)$$

Q 17.

Ans.

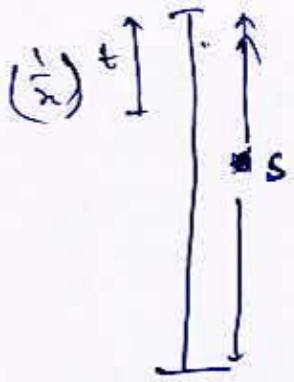


$$v^2 = u^2 + 2gh \quad (u = 0)$$

$$v = \sqrt{400} = 20 \text{ m/s}$$

Q 18. (6)

Ans.



$$\left(\frac{S}{n}\right) = \frac{1}{2} g t^2$$

$$S = \frac{1}{2} g (t')^2$$

$$(t')^2 = \left(\frac{S}{S/n}\right) t^2$$

$$t' = t\sqrt{n}$$

Q 19.

Ans.

$$h_A = \frac{1}{2} g_A t^2$$

$$h_B = \frac{1}{2} g_B t^2$$

$$\frac{h_A}{A_B} = \frac{g_A}{g_B}$$

Q 20.

Ans.

$$1 = \frac{1}{2} g t_1^2$$

$$2 = \frac{1}{2} g (t_2)^2$$

$$t_1 : (t_2 - t_1) = t_1 : (\sqrt{2} t_1 - t_1)$$

$$= 1 : (\sqrt{2} - 1)$$

Q 21.

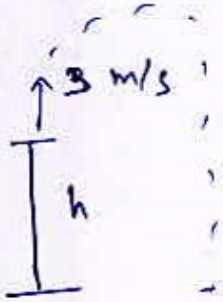
Ans.

$$v^2 = u^2 - 2g \left(\frac{u^2}{2g \times 2}\right)$$

$$v^2 = \frac{u^2}{2} \Rightarrow v = \frac{u}{\sqrt{2}}$$

Q 22.

Ans.



$t = 2 sec.$

$s = 3 \times 2 - \frac{1}{2} \times 10 \times 2^2$

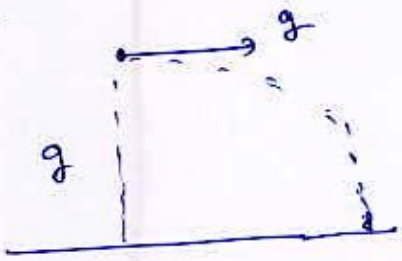
$= 6 - 20$

$= -14$

$h = 14 m.$

Q 23.

Ans.



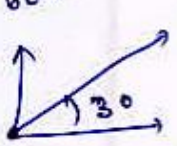
$v_y = \sqrt{2g \times g}$

$v_x = g$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3g}$

Q 24.

Ans. $60 \sin 30^\circ \hat{j}$



$60 \cos 30^\circ \hat{i}$

$= 30\sqrt{3}\,\hat{i} + 30\hat{j}$

Q 25.

Ans.

$60 = -10 \times \frac{1}{2} \times t + \frac{1}{2} \times 10 t^2$

$t = 4 s.$

Q 26.

Ans.

$\frac{2u^2 \sin\theta \cos\theta}{g} = \frac{u^2}{2g} \sin^2\theta$

$\tan\theta = 4$

(8)

$$H_{max} = \frac{u^2}{2g} \sin^2\theta$$

$$\sin\theta \times \sqrt{1-\sin^2\theta} = \frac{1}{4}$$

$$\sin^2\theta \times (1-\sin^2\theta) = \frac{1}{16}$$

$$\sin\theta = \frac{1}{2}$$

$$H_{max} = \frac{u^2}{2g} \times \frac{1}{4} \sin^2\theta$$

$$\sin\theta = \frac{4}{\sqrt{17}}$$

$$H_{max} = \frac{u^2}{2g} \times \frac{16}{17} = \frac{8u^2}{17g}$$

Q 27.

Ans.

$$S_y = 2m.$$

$$2 = 14 \sin\theta \, 1 - \frac{1}{2} g \times (1)^2$$

$$2 = 14\sin\theta - 5$$

$$\sin\theta = 1/2 \Rightarrow \theta = 30^\circ$$

Q 28.

Ans.

$$\frac{H_{max}}{T^2} = \frac{\frac{u^2}{2g}\sin^2\theta}{\left(\frac{2u\sin\theta}{g}\right)^2}$$

$$= \frac{g}{8} = \frac{5}{4}$$

Q 29.

Ans.

$$R_1 = \frac{2u^2}{g} \sin 35^\circ \cos 35^\circ$$

$$R_2 = \frac{2u^2}{g} \sin 55^\circ \cos 35^\circ$$

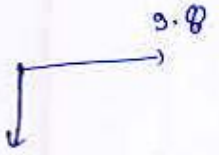
$R_1 \rightarrow \theta$

$R_2 \rightarrow 90 - \theta$

$\Big\} \rightarrow$ same.

Q 30.

Ans.



$$v_y = 0 + 9.8 \times 1$$

$$= 9.8$$

$$v = \sqrt{v_x^2 + v_y^2} = 9.8\sqrt{2} \text{ m/s}$$

Q 31.

Ans.

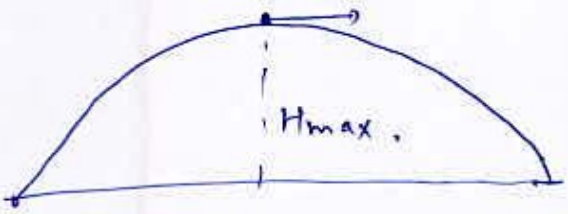
$$s = u \sin\theta \, t - \frac{1}{2} g t^2$$

$$\frac{2s}{g} + t^2 - \frac{2u \sin\theta}{g} t = 0 \quad \begin{matrix} \rightarrow t_1 \\ \rightarrow t_2 \end{matrix}$$

$$t_1 + t_2 = \frac{2u \sin\theta}{g} = T$$

Q 32.

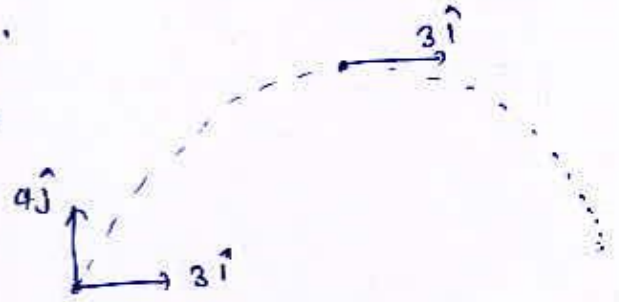
Ans.



at Hmax $\Rightarrow v_y = 0$

Q 33.

Ans.



$= 3 m/s$

Q 34.

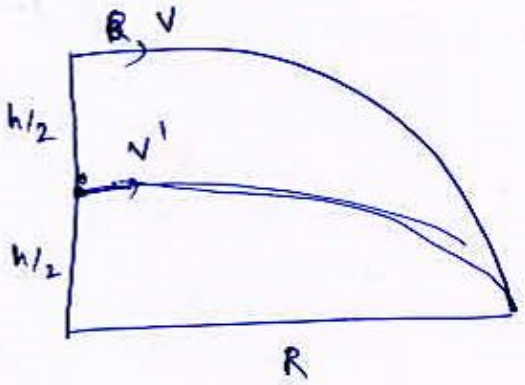
Ans.

$$\frac{H_{max}}{R} = \frac{\frac{u^2}{2g} \times \sin^2\theta}{\frac{2u^2}{g} \sin\theta\cos\theta} = \frac{\tan\theta}{4} = \frac{1}{4}$$

$\tan\theta = 1 \Rightarrow \theta = 45^\circ$

Q 35

Ans.



$$R = V \times \sqrt{\frac{2h}{g}}$$

$$R = V' \times \sqrt{\frac{2h/2}{g}}$$

$V' = \sqrt{2}\, V$

Q 36.

Ans.

$v_x = v_y \quad (\theta = 45^\circ)$

$v_x = V = gt = 9.8 \times 4$

$V = 39.2 m/s$

Q 37.

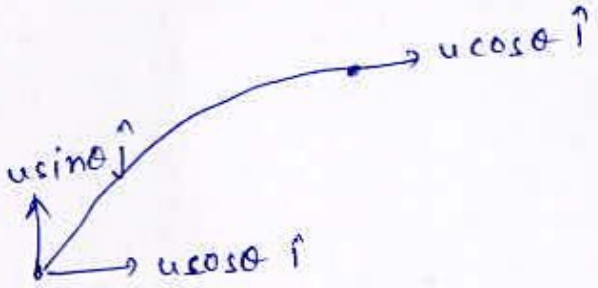
Ans.

$$t_1 = \frac{2u\sin\theta}{g} \qquad t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_1 t_2 = \frac{2u^2 2\sin\theta\cos\theta}{g} = 2R$$

Q38.

Ans.



$\Delta \vec{v} = -u \sin\theta \, \hat{j}$

Q39.

Ans.

~~$T = \frac{2 \times 5}{g} = 1 \text{ sec}$~~

$30 = -5 \times t + \frac{1}{2} g t^2$

$t^2 - 10t - 60 = 0$

$t = 3 \text{ sec.}$

Q40.

Ans.

$V_g = \frac{x}{t_1}$ $\quad V_{es} = \frac{x}{t_2}$

$t = \frac{x}{\frac{x}{t_1} + \frac{x}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

Q41.

Ans.

$d = 100 \text{ m.}$

$V_{rel} = 25 \text{ m/s}$

$t = \frac{100}{25} = 4 \text{ sec.}$

42.

Ans.

$$v^2 = u^2 + 2as$$

$$0 = (50)^2 - 2 \times 5 \, s$$

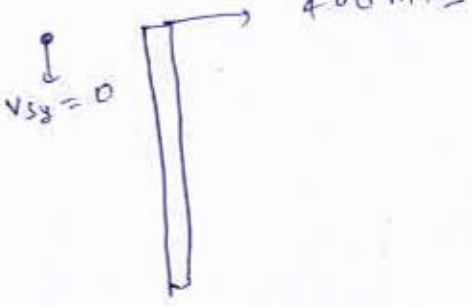
$$s = 100 \text{ m.}$$

(43)

Ans.

relative $a = 0$

(44)



$v_{sy} = v_{By} = 0$

time will be same.

Kinematics II (Mains)

Q 1.

Ans. $s = 6t^2 - t^3$

$$\frac{ds}{dt} = v = 12t - 3t^2$$

$$= 3t(4 - t^4) = 0$$

$t = 4$ sec.

Q 2.

Ans. $t = (r^2 - 1)^{1/2}$

$$t^2 = r^2 - 1 \quad \Rightarrow \quad t^2 + 1 = r^2$$

$$\boxed{r^2 - t^2 = 1}$$

$$2t = 2r\frac{dr}{dt}$$

$$\frac{dr}{dt} = v = \frac{t}{r}$$

$$t = r\frac{dr}{dt}$$

$$1 = \left(\frac{dr}{dt}\right)^2 + r\left(\frac{d^2r}{dt^2}\right)$$

$$a = \frac{1 - v^2}{r} = \frac{1 - \frac{t^2}{r^2}}{r} = \frac{r^2 - t^2}{r^3}$$

$$a = \frac{1}{r^3}$$

Q 3.

$$x = at^2 - bt^3$$

$$\frac{dx}{dt} = 2at - 3bt^2$$

$$a = 2a - 6bt = 0 \quad \Rightarrow t = \frac{a}{3b}$$

Q4. (14)

Ans. $x = u(t-2) + a(t-2)^2$

$$\frac{dx}{dt} = u + 2a(t-2)$$

$$\frac{d^2x}{dt^2} = 2a$$

Q5.

Ans. $x = \alpha t^3$, $y = \beta t^3$

$$v_x = 3\alpha t^2 \qquad v_y = 3\beta t^2$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2}$$

$$= 3t^2 \sqrt{\alpha^2 + \beta^2}$$

Q6.

Ans. $t = ax^2 + bx$

$$1 = 2ax \frac{dx}{dt} + b \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = v = \frac{1}{2ax+b}}$$

$$0 = 2a\left(\frac{dx}{dt}\right)^2 + 2ax\frac{d^2x}{dt^2} + b\frac{d^2x}{dt^2}$$

$$0 = 2av^2 + (2ax+b)a$$

$$a = -\frac{2av^2}{(2ax+b)} = -2av^3$$

Q7.

Ans. $s = 2 - 3t + 4t^3$

$$\frac{ds}{dt} = -3 + 12t^2$$

$$v_{avg} = \frac{s_3 - s_2}{1} = \frac{(2 - 3\times3 + 4\times3^3) - (2 - 3\times2 + 4\times2^3)}{1}$$

$$= \frac{4 \times (3^3 - 2^3) - 3}{1}$$

$$= 19 \times 4 - 3 = 73 \text{ cm/s}$$

Q8.

Ans. $x = at^2$, $y = bt^2$, $z = 0$

$$v_x = 2at \qquad v_y = 2bt \qquad v_z = 0$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = 2t\sqrt{a^2 + b^2}$$

Q9.

Ans. $s = t^3 - 6t^2 + 3t + 4$

$$\frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = 6t - 12 = 0 \Rightarrow t = 2$$

v. at $t = 2^\circ \Rightarrow$ $v = 3 \times 4 - 12 \times 2 + 3$

$$= -9 \text{ m/s}$$

Q 10.

Ans. $y = a \sin \omega t$

$$\frac{dy}{dt} = a\omega \cos \omega t = V$$

Q 11.

Ans. $t = \sqrt{x} + 3$

$$1 = \frac{1}{2\sqrt{x}} \frac{dx}{dt}$$

$$v = 2\sqrt{x} = 0 \Rightarrow x = 0 \text{ Ans.}$$

Q 12.

Ans. $a = kt + c$

$$\frac{dv}{dt} = kt + c$$

$$v = \frac{kt^2}{2} + ct$$

$$\int_0^v dv = \int_0^t kt + c$$

$$v = \frac{kt^2}{2} + ct$$

Q 13.

Ans. $x = \sqrt{v+1} \Rightarrow v = x^2 - 1$

$$1 = \frac{1}{2\sqrt{v+1}} \times \frac{dv}{dx}$$

$$a = v \frac{dv}{dx} = (x^2 - 1)(2x) = 10 \times (25 - 1) = 240$$

Q 14.

Ans.

$$a = -kx$$

$$v\frac{dv}{dx} = -kx$$

$$\int_u^v v\,dv = -\int_0^x kx\,dx$$

$$\frac{(v^2-u^2)}{2} = -\frac{kx^2}{2}$$

$$\frac{1}{2}m(v^2-u^2) = -\frac{km}{2}x^2$$

$$\Delta KE \propto x^2$$

Q 15.

Ans.

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

$$\int \frac{dv}{\sqrt{v}} = -\int 2.5\,dt$$

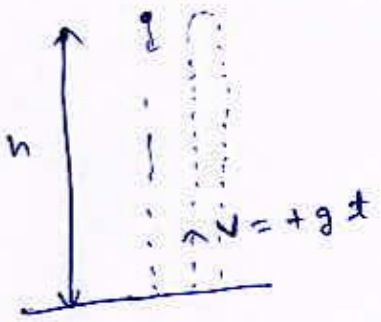
$$\left[2\sqrt{v}\right]_{6.25}^{0} = -2.5\,t$$

$$-2 \times 2.5 = -2.5 \times t$$

$$t = 2 \text{ sec.}$$

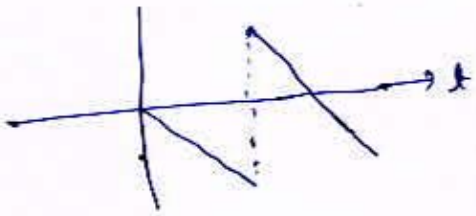
Q 16.

Ans.



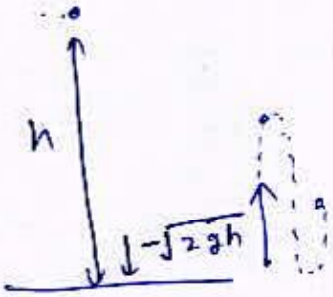
$v = 0 - gt$

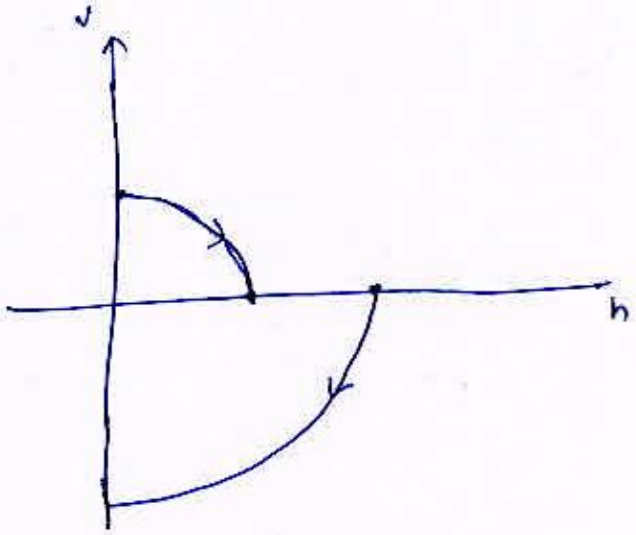
$v = -gt$



Q 17.

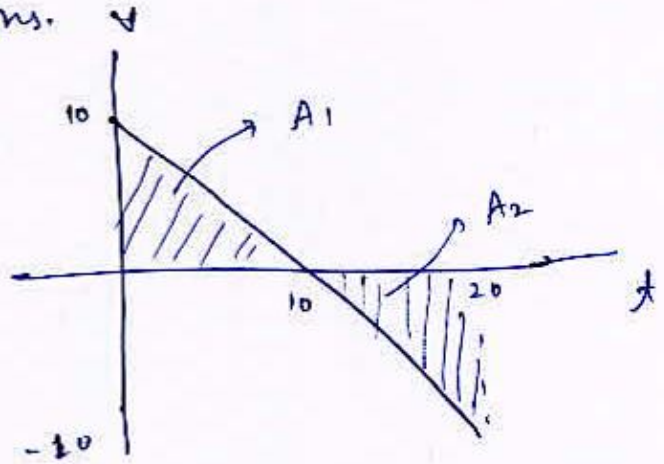
Ans.





Q 18.

Ans.



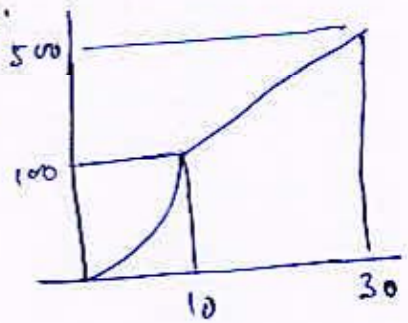
$\frac{dv}{dt} = -1$

$A_1 + A_2 = 0$

$s = 0$

Q 19.

Ans.



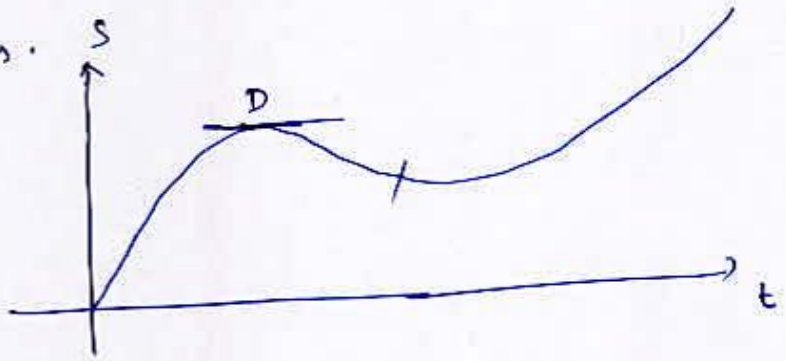
$\frac{ds}{dt} = \frac{500 - 100}{30 - 10} = 20$ for $t > 10$

$s = t^2$

$\frac{ds}{dt} = 2t$ for $t < 10$

Q 20.

Ans.



$\frac{ds}{dt} = 0$ at D

Q 21.

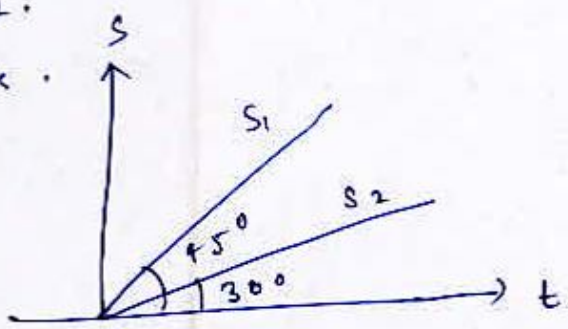
Ans. area of v-t graph

$\frac{1}{2} \times 3.6 \times 2 + 8 \times 3.6 + \frac{1}{2} \times 3.6 \times 2$

$3.6 \times (10) = 36$ m.

Q 22.

Ans.



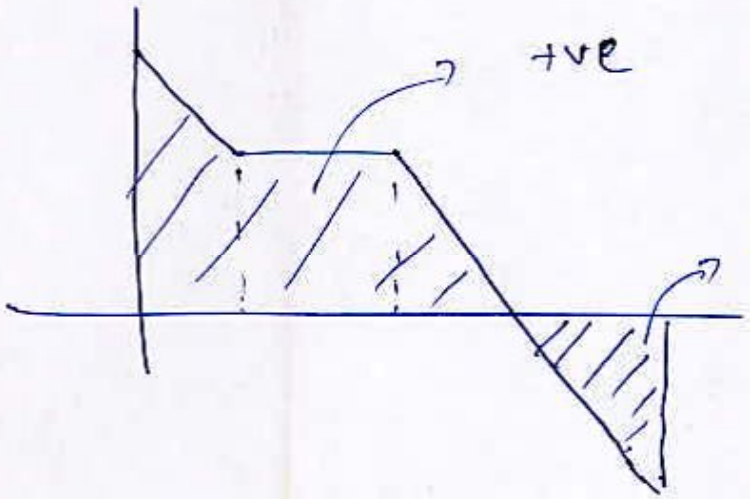
$v_1 = \frac{ds_1}{dt} = 1$

$v_2 = \frac{ds_2}{dt} = \frac{1}{\sqrt{3}}$

$v_1/v_2 = \sqrt{3}$

Q 23.

Ans. Area of v-t graph.



distance = $20 + 20 \times 2 + \frac{1}{2} \times 20 \times 2 + 2 \times \frac{1}{2} \times 20$

$= 100$ m

Q24.

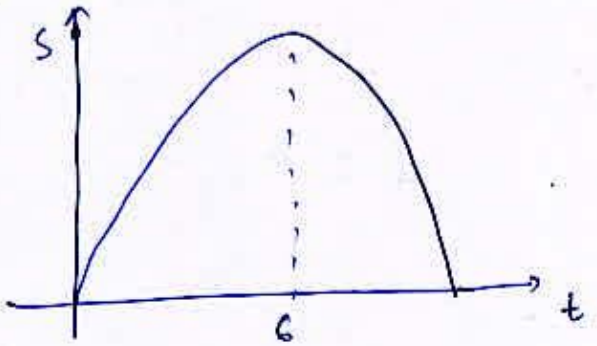
Ans. $v_1 = \frac{20}{2} = 10$

$v_2 = \frac{20}{4} = 5$

$v_1/v_2 = 2$

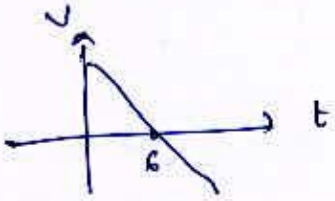
Q25.

Ans.



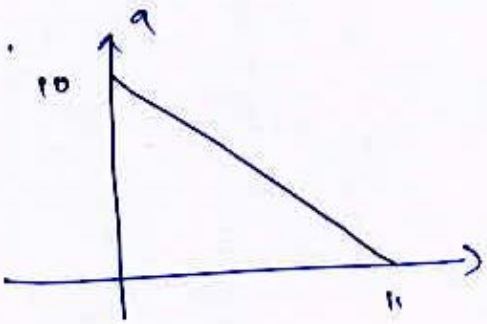
$\frac{ds}{dt} > 0$ for $t < 6$

$\frac{ds}{dt} < 0$ for $t > 6$



Q26.

Ans.



area of a-t graph. = Δv

$v_f - v_i = \frac{1}{2} \times 10 \times 11$

$v_f - 0 = 5 \times 11 = 55 m/s$

$v_f = 55 \ m/s$

Q27.

Ans. $t_1 = \frac{d}{v \sin\theta}$

$t_2 = \frac{d}{v}$

$t_1 - t_2 = \frac{d}{v}\left(\frac{13}{12} - 1\right)$

$= \frac{156}{13}\left(\frac{1}{12}\right) = 1 \text{ sec.}$

$v\cos\theta = u$

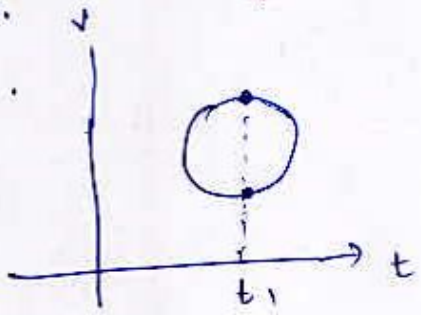
$13\cos\theta = 5$

$\cos\theta = 5/13$

$\sin\theta = 12/13$

Q 28.

Ans.



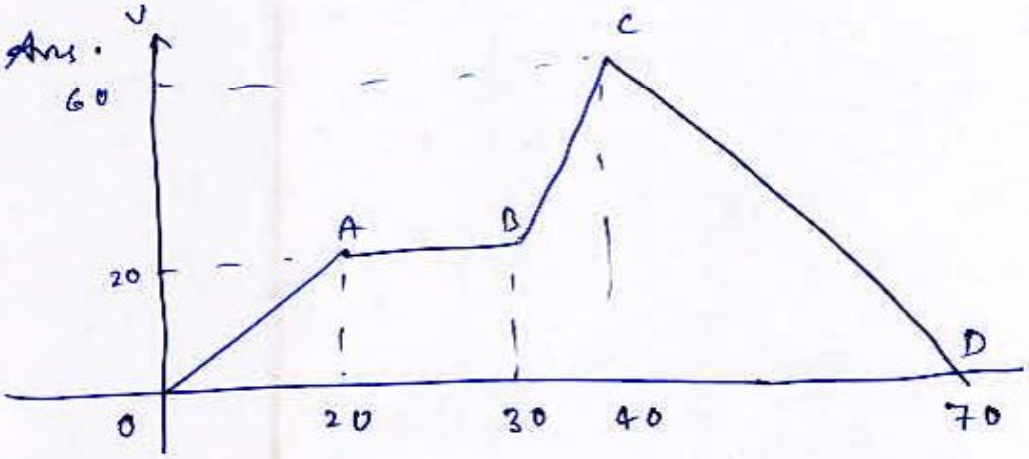
at time t_1 only one velocity is possible

Q 29.

Ans. uniform motion $a = 0$

Q 30.

Ans.

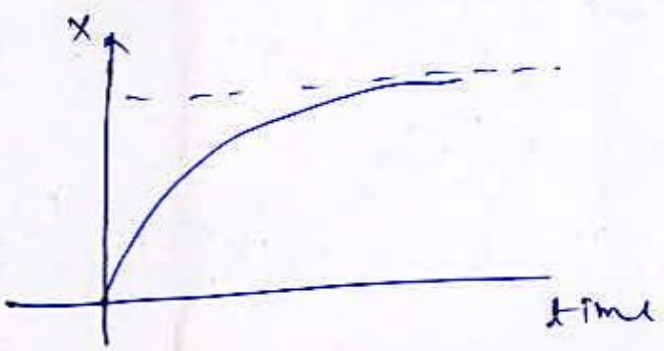


$a_{OA} = \frac{20}{20} = 1$ $\quad a_{AB} = 0$

$a_{BC} = \frac{40}{10} = 4$ $\quad a_{CD} = -\frac{60}{30} = -2$

$a_{max} = 4\ m/s^2$

Q 31.



$\frac{dx}{dt} \Rightarrow 0$ and decreasing

v & positive decreasing

$\frac{d^2x}{dt^2} < 0 \Rightarrow a$ -ve

Q32 (22)

Ans. $\frac{ds_A}{dt} = \frac{ds_B}{dt} \Rightarrow v_A = v_B$

Q33.

Ans.

↓ 10km/hr

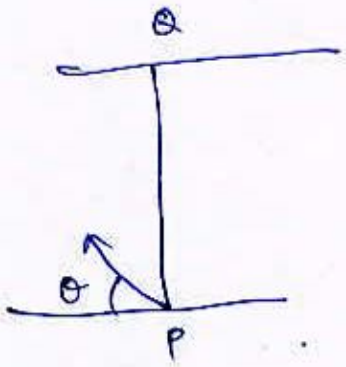
⟶ 3km/hr

$v_{rain} = \sqrt{10^2 + 3^2}$

$= \sqrt{109}$ km/h

Q34.

Ans.

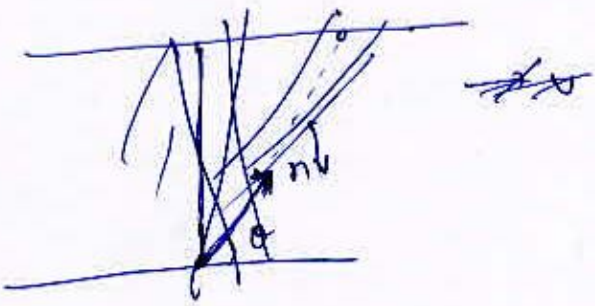


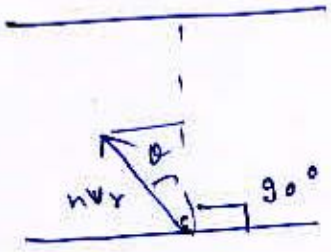
$10 \cos\theta = 4$

$\theta = \cos^{-1}(2/5)$

Q35.

Ans.





$nv_r \sin\theta = v_r$

$\sin\theta = 1/n$

$\theta = \sin^{-1}(1/n)$

angle $= \pi/2 + \sin^{-1}(1/n)$