LIMITS

EXERCISE 1

1. $\text{LHL} = \lim_{x\to 1^-} x^2 = \lim_{h\to 0^+}(1-h)^2 = \lim_{h\to 0^+}\left(1+h^2-2h\right) = 1$

$\text{RHL} = \lim_{x\to 1^+} x = \lim_{h\to 0^+}(1+h) = 1$

$\therefore \text{LHL} = \text{RHL} =$ a finite quantity.

Hence , $\lim_{x\to 1} f(x) = 1$

2. $\text{LHL} = \lim_{h\to 0^+}\frac{|2-h-2|}{(2-h)-2} = \frac{(-h)}{-h} = \frac{h}{-h} = -1$

$\text{RHL} = \lim_{h\to 0^+}\frac{|2+h-2|}{(2+h-2)} = \frac{|h|}{h} = \frac{h}{h} = 1$

$\therefore \lim_{x\to 2}\frac{|x-2|}{x-2} = 1$

3. $\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\left(\frac{2}{5-x}\right) = \lim_{h\to 0^+}\frac{2}{5-(3-h)} = \lim_{h\to 0^+}\frac{2}{2+h} = 1$

$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0^+}\left[5-(3+h)\right] = 2$

4. $\text{LHL} = \lim_{h\to 0^+} 3(1-h) = 3,\qquad \text{RHL} = \lim_{h\to 0^+} 5-3(1+h) = 2$

5. $\text{LHL} = \lim_{h\to 0^+}\frac{|h|}{h} = -1 \qquad \text{RHL} = \lim_{h\to 0^+}\frac{|h|}{h} = 1$

6. $\lim_{x\to 1}\left(3x^2+4x+5\right) = 3+4+5 = 12$

7. $\lim_{x\to 2}\frac{\left(3^{x/2}-3\right)}{\left(3^{x/2}-3\right)\left(3^{x/2}+3\right)} = \lim_{x\to 2}\frac{1}{\left(3^{x/2}+3\right)} = \frac{1}{6}$

8. $\lim_{x\to a}\frac{x^5-4^5}{x-a} = 5a^4$

9. $\lim_{h\to 0}\frac{1}{h}\left[\frac{x-(x+h)}{(x+h)x}\right] = \lim_{h\to 0}\frac{-1}{x(x+4)} = \frac{-1}{x^2}$

10. $\lim_{x\to 0}\frac{\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)}{x^2\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)} = \lim_{x\to 0}\frac{\left(1-x^2\right)-\left(1+x^2\right)}{x^2\left(\sqrt{1-x^2}+\sqrt{1+x^2}\right)}$

$= \lim_{x\to 0}\frac{-2x^2}{2x^2} = -1$

11. $$\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}=\lim_{x\to 3}\frac{1}{2}(2)=1$$

12. $$\lim_{x\to\infty}\frac{x^2}{x^2}\frac{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}{\left(d+\frac{e}{x}+\frac{f}{x^2}\right)}=\frac{a}{d}$$

13. $$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\frac{\left(\sqrt{x+\sqrt{x+x}}+\sqrt{x}\right)}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}=\lim_{x\to\infty}\frac{\left(x+\sqrt{x+\sqrt{x}}-x\right)}{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}$$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x}\left(1+\sqrt{\frac{1}{x}}\right)^{1/2}}{\sqrt{x}\left[\left(1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}\right)^{1/2}+1\right]}\right)=\frac{(1)^{1/2}}{(1)^{1/2}+1}=\frac{1}{2}$$

14. $$\lim_{x\to 1}(1+x)^{\frac{1}{x}}=2$$

$$\lim_{x\to\frac{\pi}{6}}\frac{\left(3\sin x-\sqrt{3}\cos x\right)}{6\left(x-\frac{\pi}{6}\right)}$$

$$=\frac{3}{6}\lim_{x\to\frac{\pi}{6}}\frac{\left(\sin x-\frac{1}{\sqrt{3}}\cos x\right)}{\left(x-\frac{\pi}{6}\right)}=\frac{1}{2}\lim$$

$$=\frac{2\sqrt{3}}{6}\lim_{x\to\frac{\pi}{6}}\frac{\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)}{\left(x-\frac{\pi}{6}\right)}$$

$$\lim_{x\to\frac{\pi}{6}}\left(\frac{1}{\sqrt{3}}\right)\frac{\sin\left(x-\frac{\pi}{6}\right)}{\left(x-\frac{\pi}{6}\right)}=\frac{1}{\sqrt{3}}$$

17. $$\lim_{x\to 3}\frac{\left(x^3-27\right)-\left(x^2-9\right)}{(x-3)}=\lim_{x\to 3}\frac{(x-3)}{(x-3)}\left[\left(x^2+3x+9\right)-(x+3)\right]=\lim_{x\to 3}\left(x^2+2x+6\right)=21$$

18. $$\lim_{x\to 2}\frac{(x-2)\left(x^2+2x+4\right)}{(x-2)(x+2)}=2$$

19. $$\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+2}+\sqrt{1-x}\right)}=\lim_{x\to 0}\frac{x\sqrt{1+x}-\sqrt{1-x}}{2x}=1$$

$$\lim_{x\to a}\frac{\left(\sqrt{a+2z}-\sqrt{3x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\sqrt{3a+x}-2\sqrt{x}\left(\sqrt{3a+x}+2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3}x\right)}=\frac{(a-x)(a+2x)+\sqrt{3x}}{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}$$

20. $$\lim_{x\to a}\frac{(a-x)}{3(a-x)}\frac{\left(\sqrt{2a+x}+2\sqrt{x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)}=\frac{1}{3}\frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}=\frac{2}{3\sqrt{3}}$$

21. $$\lim_{n\to\infty}\frac{\left(n^{49}+n^{98}+.......1^{99}\right)(n-1)}{n^{180}(n-1)}=\left(n^{100}-1\right)$$

22. $$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{\left(x^2-49\right)}\times\frac{\left(2+\sqrt{x-3}\right)}{\left(2+\sqrt{x-3}\right)}=\lim_{x\to 7}=\frac{(7-x)}{(x-7)(x+7)\left(x+\sqrt{x+3}\right)}=\frac{-1}{6}$$

23. $$\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=\lim_{n\to\infty}\frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6n^3}=\frac{1}{6}$$

24. $$\lim_{x\to 0}\left[\left(\frac{4^x-1}{x}\right)-\left(\frac{9^x-1}{x}\right)\right]\frac{1}{\left(4^x+9^x\right)}=\frac{1}{2}(\log 4-\log 9)=\log\left(\frac{2}{3}\right)$$

25. $$\lim_{x\to 3}\frac{2}{x-3}+\frac{x-3}{x+4}-\frac{2(2x+1)}{x^2+x-12}=\lim_{x\to 3}\frac{(2x+8-4x-2)}{\left(x^2+x-12\right)}=\lim_{x\to 3}\frac{-2(x-3)}{\left(x^2+x+2\right)}=\frac{-2}{7}$$

26. $$\lim_{x\to 0}\frac{\sin x+\log(1-x)}{x^2}=\lim_{x\to 0}\frac{x-\frac{x^3}{3!}-x-\frac{x^2}{2!}-\frac{x^3}{3!}}{x^2}=\frac{1}{2}$$

27. $$\lim_{x\to\infty}\left[\frac{x^3(1-a)+x^2(-b)+x(-a)+(a-b)}{x^2+1}\right]=2$$

$$\therefore 1-a=0\Rightarrow a=1$$

$$\Rightarrow\lim_{x\to\infty}\frac{-b+\left(\frac{-a}{x}\right)+\left(\frac{1-b}{x^2}\right)}{1+\frac{a}{x^2}}=2\Rightarrow -b=2$$

28. $$\lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\left(1+\frac{3}{x}\right)^{10}+........\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\left(\frac{10}{x}\right)^{10}\right]}=100$$

29. $\lim_{x\to 0}\frac{(xe^x - x)}{x^2}+\left(\frac{x-\log(1+x)}{x^2}\right)=L_1+L_2$

$L_1=\lim_{x\to 0}\frac{e^x-1}{x}=1,\ L_2=\lim_{x\to 0}\frac{x-\log(1+x)}{x^2}$, let $\log(1+x)=t$

$\therefore L_2=\lim_{t\to 0}\frac{e^t-1-t}{(e^t-1)^2}=\lim_{t\to 0}\frac{e^t-t-1}{t^2}=\frac{1}{2}$

30. $\lim_{x\to 0}\left(\frac{\sin^{-1}x-x}{x^3}\right)+\lim_{x\to 0}\left(\frac{x-\tan^{-1}x}{x^3}\right)=L_1+L_2$

Let $\sin^{-1}x=t_1\Rightarrow L_1=\lim_{t\to 0}\frac{t_1-\sin t}{\sin^3 t}=\lim_{t_1\to 0}\frac{t_1-\sin t}{t_1^{\ 3}}=\frac{1}{6}$

Let $\tan^{-1}x=t_2\Rightarrow L_2=\lim_{t_2\to 0}\frac{\tan t_2-t_2}{\tan^3 t_2}=\lim_{t_2\to 0}\frac{\tan t_2-t_2}{t_2^2}=\frac{1}{3}$

31. $RHL=\lim_{h\to 0^+}\frac{\sqrt{1-\sin\left(\frac{\pi}{2}+h\right)}}{2\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)}=\lim_{h\to 0^+}\frac{\sqrt{1-\cos h}}{2h}=\frac{\sqrt{2}\left|\ell n\frac{h}{2}\right|}{4\left(\frac{h}{2}\right)}=\frac{1}{2\sqrt{2}}$

$LHL\ =\lim_{h\to 0^+}\frac{\sqrt{2}\left|\sin\frac{h}{2}\right|}{(-2h)}=\frac{-1}{2\sqrt{2}}$

32. $\lim_{x\to 0}\left[\frac{1-\cos 2x}{(2x)^2}\right]\left(\frac{\sin 5x}{5x}\right)\left(\frac{3x}{\sin 3x}\right)\times\left(\frac{4\times 5}{3}\right)=\frac{10}{3}$

33. $\lim_{x\to 0}\frac{(x^2)}{\sin(x^2)}\times x=1\times 0=0$

34. $\lim_{x\to 0}3\left(\frac{\sin 3x}{3x}\right)+\lim_{x\to 0}\frac{\sin x}{x}=4$

35. $\lim_{x\to 0}\frac{(1+x)^8-(1+8x)}{x^2}=\lim_{x\to 0}\frac{\left(1+8x+\frac{8.7}{2}x^2+\frac{8.7.6}{3!}x^3+...\right)-(1+3x)}{x^2}\ =28$

36. $LHL=\lim_{H\to 0^+}\frac{\sin(-h)}{(-h)}=1$ $\qquad RHL=0$

37. $\lim_{x\to 0}8.\left(\frac{\sin 2x}{2x}\right)\left[\frac{1-\cos 2x}{(2x)^2}\right]\left(\frac{1}{\cos 2x}\right)=4$

38. $\lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{\sin(e^{x-2}-1)}{(e^{x-2}-1)}\times\frac{(e^{x-2}-1)}{(x-2)}\frac{(x-2)}{\log[1+(x-2)]}=1$

39. $\lim_{x\to\frac{\pi}{2}} \frac{a^{\cos x}\left(a^{\cot x-\cos x}-1\right)}{\cot x-\cos x}=\ell n(a)$

40. $\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\begin{vmatrix}\sin x & \cos^2 x & \tan x\\ x & 1 & 1/x\\ 2x & 1 & 1\end{vmatrix}=\lim_{x\to 0}\left(\frac{\begin{vmatrix}\sin x & \cos x & \tan x\\ x^2 & x & 1\\ 2x & 1 & 1\end{vmatrix}}{x}\right)$

41. $=\begin{vmatrix}\cos x & \cot x & \tan x\\ 2x & x & 1\\ 2x & 1 & 1\end{vmatrix}+\begin{vmatrix}\sin x & -\sin x & \tan x\\ x^2 & 1 & 1\\ 2x & 0 & 1\end{vmatrix}+\begin{vmatrix}\sin x & \cos x\\ x^2 & x\\ 2x & 1\end{vmatrix}$

42. $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)\times\frac{1}{(1+\tan x)}\times\left(\frac{1-\cos x}{x^2}\right)=1\times 1\times\frac{1}{2}=\frac{1}{2}$

43. $\lim_{x\to 0}\left[2(a-2)-\frac{\tan x}{x}\right]\frac{\sin 2x}{x}=0\Rightarrow 2[2a-4-1]=0\Rightarrow a=\frac{5}{2}$

44. $\lim_{h\to 0}\frac{\log_e\frac{(1+4h)}{(1+2h)^2}}{h^2}=\lim_{h\to 0}\frac{\ell n\left(1+\left(\frac{1+4h}{(1+2h)^2}-1\right)\right)}{\left(\frac{1+4h}{(1+2h)^2}-1\right)}\frac{\left(\frac{1+4h}{(1+2h)^2}-1\right)}{h^2}$

$\lim_{h\to 0}\frac{-4h^2}{h^2}=-4$

45. $\lim_{x\to a}\frac{\ell n(1+(x-a))}{(x-a)}=1$

46. $\lim_{h\to 0}\frac{\ell n(1+h)}{h}\times\log_{10}e=\log_{10}e$

47. $k=\lim_{x\to 0}\frac{\log\left(1+\frac{3+2x}{3-2x}-1\right)}{\left(\frac{4x}{3-2x}\right)}\times\frac{4x}{(3-2)x}=\frac{4}{3}$

48. $\lim_{x\to 0}\frac{(e^x-1)^2}{(x)^2}+\left(\frac{e^x-x-1}{x^2}\right)=1+\frac{1}{2}=\frac{3}{2}$

49. $\lim_{x\to 0}\frac{e^x-x-1}{x^2}=\lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.......\right)-(x+1)}{x^2}=\frac{1}{2}$

50. $$\lim_{x\to 0}\frac{\left(2^x-1\right)\left(\sqrt{1+x}+1\right)}{\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}=\lim_{x\to 0}2\frac{\left(2^x-1\right)}{x}=\ell n4$$

51. $$e\lim_{x\to 0}\left(x+3\right)\left(\frac{x+4}{x+1}-1\right)=e^{\lim\limits_{x\to\infty}3}=e^3$$

52. $$e\lim_{x\to 0}\left(c+dx\right)\left(\frac{1}{a+bx}\right)=e^{\frac{d}{b}}$$

53. $$\lim_{x\to 0}\left(2x\right)^{3x}=1$$

54. $$\lim_{x\to 0}\left(1+1+\frac{\frac{1}{x}\left(\frac{1}{x}-1\right)}{2}x^2+......\right)+\left(\frac{x}{2}-1\right)\left(1+1+\frac{1}{2!}+\frac{1}{3!}+.....\right)$$

55. $$e^{\lim\limits_{m\to\infty}}m\left(\cos\frac{x}{m}-1\right)=e^{\lim\limits_{m\to\infty}}\frac{\left(\cos\frac{x}{m}-1\right)}{\left(\frac{x^2}{m^2}\right)}\times\left(\frac{x^2}{m^2}\right)\times m=1$$

56. $$e^{\lim\limits_{n\to\infty}\ n(n-1)}\frac{(2)}{\left(n^2-n-1\right)}=e^2$$

57. $$\lim_{x\to 0}\left(\frac{1-\cos mx}{m^2x^2}\right)\times\left(\frac{n^2c^2}{1-\cos nx}\right)\times\frac{m^2}{n^2}$$
$$=\frac{m^2}{b^2}$$

58. $$\lim_{x\to\frac{\pi}{8}}\frac{\sin\left(2x-\frac{\pi}{4}\right)}{2\left(x-\frac{\pi}{8}\right)}\left(\sqrt{2}\right)=2\sqrt{2}$$

59. $$\lim_{x\to a}\frac{x^3-a^3}{x-a}=\lim_{x\to a}x^2+xa+a^2=3a^2$$

60. $$\lim_{h\to 0}\frac{2\left\{\left[1+\frac{\sin h}{4}\right]^{1/2}-1\right\}}{h}=li2\frac{\left[\frac{1}{8}\sin h+........\right]}{h}=$$

61. $$\lim_{\alpha\to\beta}\frac{\sin\left(\alpha+\beta\right)}{\left(\alpha+\beta\right)}\frac{\sin\left(\alpha-\beta\right)}{\left(\alpha-\beta\right)}=\frac{\sin\left(2\beta\right)}{\left(2\beta\right)}$$

62. $\lim_{x\to 0}\left(\frac{\tan 4x}{x}-2\right)\Big/\left(6-\frac{\sin 3x}{x}\right)=(4-2)/(6-3)=2/3$

63. $\lim_{x\to 1}\frac{\left(5-\sqrt{26-x^2}\right)\left(5+\sqrt{26-x^2}\right)}{\left(5+\sqrt{26-x^2}\right)(x-1)}=\lim_{x\to 1}\frac{\left(x^2-1\right)}{(x-1)}\frac{1}{10}=\frac{1}{5}$

64. $\lim_{x\to -\infty}\frac{|x|\left(1-\frac{1}{x^2}+\sqrt{1-\frac{2}{x^2}}\right)}{x\left(1+\frac{1}{x}\right)}=(-1)\times 2=-2$

65. $\lim_{x\to -\infty}\frac{\sqrt{x^2-1}+\sqrt{x^2-2}}{x+1}=-\lim_{y\to 0^-}\frac{\sqrt{1-y^2}+\sqrt{1-2y^2}}{1+y}=2$

LIMITS
EXERCISE 2

1 If $f(x) = \begin{cases} x^2 + 2, x \geq 1 \\ 2x + 1, x < 1 \end{cases}$, then $\lim\limits_{x \to 1} f(x)$ equals -

(A) 1 (B) 2
(C) 3 (D) Does not exist

Sol. $\lim\limits_{x \to 1-0} f(x) \lim\limits_{h \to 0} = [2(1-h)+1] = 3$

$\lim\limits_{x \to 1+0} f(x) = \lim\limits_{h \to 0} [(1+h)^2 + 2] = 3$

$\therefore$ LHL = RHL, so $\lim\limits_{x \to 1} f(x) = 3$. **Ans.[C]**

2 $\lim\limits_{x \to 0} \dfrac{1 + e^{-1/x}}{1 - e^{-1/x}}$ is equal to -

(A) 1 (B) –1
(C) 0 (D) Does not exist

Sol. LHL = $\lim\limits_{h \to 0} \dfrac{1 + e^{1/h}}{1 - e^{1/h}}$

$= \lim\limits_{h \to 0} = \dfrac{e^{-1/h} + 1}{e^{-1/h} - 1} -\!- 1$

RHL = $\lim\limits_{h \to 0} = \dfrac{1 + e^{-1/h}}{1 - e^{-1/h}} = \dfrac{1+0}{1-0} = 1$ **Ans.[D]**

LHL ≠ RHL, so given limit does not exist.

3 If $f(x) = \begin{cases} x - 1, x < 0 \\ 1/4, x = 0 \\ x^2, x > 0 \end{cases}$ then $\lim\limits_{x \to 0} f(x)$ equals -

(A) 0 (B) 1
(C) –1 (D) Does not exist

Sol. Here $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x^2 = 0$

and $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} (x-1) = -1$

$\because \lim\limits_{x \to 0^+} f(x) \neq \lim\limits_{x \to 0^-} f(x)$

$\therefore \lim\limits_{x \to 0} f(x)$ does not exist. **Ans.[D]**

4 $\lim\limits_{x \to 3} \dfrac{x - 3}{|x - 3|}$, is equal to -

(A) 1 (B) –1
(C) 0 (D) Does not exist

Sol. LHL = $\lim\limits_{h \to 0} \dfrac{(3-h) - 3}{|(3-h) - 3|}$

$= \lim\limits_{h \to 0} \dfrac{-h}{|-h|} = -\!-1$

$$\text{RHL} = \lim_{h\to 0} \frac{(3+h)-3}{|(3+h)-3|}$$

$$= \lim_{h\to 0} \frac{h}{|h|} = 1$$

LHL $\neq$ RHL, so limit does not exist. **Ans.[D]**

5 $\lim_{x\to\infty} \frac{2x^2+3x}{3x^2+4}$ equals -

(A) 1/2 (B) 2/3

(C) 3/4 (D) 0

Sol. $= \lim_{x\to\infty} \frac{2+(3/x)}{3+(4/x^2)} = \frac{2}{3}$ **Ans.[B]**

6 $\lim_{x\to\infty} \left(\sqrt{x^2+1}-x\right)$ equals -

(A) –1 (B) 0

(C) 1 (D) None of these

Sol. Limit $= \lim_{x\to\infty} x\left[\left(1+\frac{1}{x^2}\right)^{1/2}-1\right]$

$$= \lim_{x\to\infty} x\left[1+\frac{1}{2x^2}-\frac{1}{8x^4}+\ldots-1\right]$$

$$= \lim_{x\to\infty} \left[\frac{1}{2x}-\frac{1}{8x^3}+\ldots\right] = 0.$$ **Ans.[B]**

7 If $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty} f(x)$ equals -

(A) 0 (B) ∞

(C) 1 (D) None of these

Sol. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{\frac{\{1-(\sin x/x)\}}{\{1+(\cos^2 x/x)\}}}$

$$= \sqrt{\frac{1-0}{1+0}} = 1.$$ **Ans.[C]**

8 $\lim_{x\to a} \left[\frac{x^2-(a+1)x+a}{x^3-a^3}\right]$ is equal to -

(A) $\frac{a-1}{3a^2}$ (B) a – 1

(C) a (D) 0

Sol. $\lim_{x\to a} \left[\frac{x^2-(a+1)x+a}{x^3-a^3}\right]$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x\to a} = \frac{2x-a-1}{3x^2} = \frac{a-1}{3a^2}$$

(D.L.Hospital rule) **Ans.[A]**

9 $\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$ is equal to -

(A) 1/2 (B) 2

(C) 1 (D) 0

Sol. Limit = $\lim_{x\to 0} \frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)}$

$= \lim_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1.$ **Ans.[C]**

10 $\lim_{x\to 0} \frac{\tan 2x - x}{3x - \sin x}$ equals-

(A) 2/3 (B) 1/3

(C) 1/2 (D) 0

Sol. The given limit is in the form , therefore applying L 'Hospital's rule, we get

Limit $= \lim_{x\to 0} \frac{2\sec^2 2x - 1}{3-\cos x} = \frac{2-1}{3-1} = \frac{1}{2}$ **Ans.[C]**

11 $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to -

(A) e^3 (B) $e^{1/3}$ (C) 1 1 (D) e

Sol. Limit = $\lim_{x\to 0} \left(\frac{x + x^3/3 + ...}{x}\right)^{1/x^2}$

$= \lim_{x\to 0} \left(1+\frac{x^3}{3}\right)^{1/x^2}$

[$\because$ x $\to$ 0, so neglecting higher powers of x]

$= \lim_{x\to 0} \left[\left(1+\frac{x^2}{3}\right)^{3/x^2}\right]^{1/3} = e^{1/3}$ **Ans.[B]**

12 $\lim_{x\to 0} \frac{\sin x^o}{x}$ is equal to -

(A) 1 (B) π (C) x (D) $\pi/180$

Sol. Limit = $\lim_{x\to 0} \frac{\sin(\pi/180)x}{x}$

$= \lim_{x\to 0} \frac{(\pi/180)\cos(\pi/180)x}{1}$

$= \frac{\pi}{180}$ **Ans.[D]**

13 $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$ equals -

(A) 0 (B) 1 (C) ∞ (D) -1

Sol. Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$

$= \lim_{x\to\infty} (\cot^{-1}x)^{1/x}$

$\therefore \log y = \lim_{x\to\infty} \frac{\log\cot^{-1} x}{x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} -\frac{1}{(1+x^2)\cos^{-1} x}$ (0x ∞ form)

$= -\lim_{x\to\infty} \frac{(1+x^2)^{-1}}{\cot^{-1} x}$ $\left(\frac{0}{0}\text{form}\right)$

$= -\lim_{x\to\infty} \frac{\frac{-2x}{(1+x^2)^2}}{\frac{-1}{1+x^2}}$ $= -2\lim_{x\to\infty} \frac{x}{1+x^2}$

$= -2\lim_{x\to\infty} \frac{1}{2x} = 0$ $\therefore y = e^0 = 1.$ **Ans.[B]**

14 If $G(x) = -\sqrt{25 - x^2}$,

then $\lim_{x\to 1} \frac{G(x) - G(1)}{x - 1}$ equals -

(A) 1/24 (B) 1/5

(C) $-\sqrt{24}$ (D) None of these

Sol. Here $G(1) = -\sqrt{25 - x^2} = -\sqrt{24}$

$\therefore$ Given limit

$= \lim_{x\to 1} \frac{-\sqrt{25-x^2} + \sqrt{24}}{x - 1}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to 1} \frac{x}{\sqrt{25 - x^2}}$ (By L Hospital ruel)

$= \frac{1}{\sqrt{24}}$ **Ans.[D]**

15 If f(9) = 9 and f'(9) = 4, then $\lim_{x\to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}$ is equal to -

(A) 1 (B) 3 (C) 4 (D) 9

Sol. Given limit is in 0/0 form, so using Hospital rule, we get

$$\text{Limit} = \lim_{x\to 9} \frac{\frac{1}{2\sqrt{f(x)}}.f(x)}{\frac{1}{2\sqrt{x}}}$$

$= \frac{f(9).\sqrt{9}}{\sqrt{f(9)}} = \frac{4.3}{3} = 4$ **Ans.[C]**

16 By L'hospital's rule

$$\lim_{x\to 1} \frac{f(1)\,g(x) - f(1) - g(1)\,f(x) + g(1)}{g(x) - f(x)} = \lim_{x\to 1} \frac{f(1)\,g'(x) - g(1)\,f'(x)}{g'(x) - f'(x)} = k = 4$$ **Ans.[A]**

17 By L'hospital's rule

$$\lim_{x\to 0}\left(\frac{\int_0^{2x^2}\sec^2 2t\,dt}{x\sin x}\right)=\lim_{x\to 0}\left(\frac{4x\sec^2 4x^2}{x\cos x+\sin x}\right)=\lim_{x\to 0}\left(\frac{4\sec^2 4x^2}{\cos x+\frac{\sin x}{x}}\right)=2$$

Ans.[B]

18 $$\lim_{x\to 0}\frac{\sin^{-1}\left(3x-4x^3\right)}{\ln(1+2x)}=\lim_{x\to 0}\frac{3\sin^{-1}x}{\ln(1+2x)}=\lim_{x\to 0}\frac{3\frac{\sin^{-1}x}{x}}{2\frac{\ln(1+2x)}{2x}}=\frac{3}{2}.$$

Ans.[C]

19 $$\lim_{x\to 2}\frac{\left(x^2+5\right)^{\frac{1}{2}}-\left(x^3+1\right)^{\frac{1}{2}}}{x^2-4}=\lim_{x\to 2}\frac{\left(x^2+5\right)-\left(x^3+1\right)}{\left(x^2-4\right)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}$$

$$=\lim_{x\to 2}\frac{-(x-2)\left(x^2+x+2\right)}{(x-2)(x+2)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}=\lim_{x\to 2}\frac{-\left(x^2+x+2\right)}{(x+2)\left(\left(x^2+5\right)^{\frac{1}{2}}+\left(x^3+1\right)^{\frac{1}{2}}\right)}=-\frac{1}{3}$$

Ans.[C]

20 $$\lim_{x\to 0}\frac{\tan 2x-\sin x}{x}=2\lim_{x\to 0}\frac{\tan 2x}{2x}-\lim_{x\to 0}\frac{\sin x}{x}=1$$

Ans.[A]

21 $$\lim_{x\to 0}\frac{\sin 4x-\sin 2x+\sin x}{x\cot x+1}=\lim_{x\to 0}\frac{\sin 4x-\sin 2x+\sin x}{\left(\frac{x}{\tan x}+1\right)}=\frac{0}{2}=0$$

22 $$\lim_{x\to 0}\frac{\ln(1+\sin x)}{e^{\tan 2x}-1}=\lim_{x\to 0}\frac{\ln(1+\sin x)}{\sin 2x}\times\frac{\tan 2x}{e^{\tan 2x}-1}\times\frac{\frac{\sin 2x}{2x}}{\frac{\tan 2x}{2x}}=1$$

Ans.[A]

23 $$\lim_{x\to\frac{\pi}{4}}(1+\cos 2x)^{\frac{1}{1-\tan x}}=e^{\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{1-\tan x}}$$

$$=e^{\lim_{x\to\frac{\pi}{4}}\frac{1-\tan^2 x}{1+\tan^2 x}\times\frac{1}{1-\tan x}}=e^{\lim_{x\to\frac{\pi}{4}}\frac{1+\tan x}{1+\tan^2 x}}=e.$$

Ans.[A]

24 $$\lim_{x\to 0}\frac{a\sin x+b\cos x+ce^x}{x^2}=\lim_{x\to 0}\frac{a\left(x-\frac{x^3}{6}\right)+b\left(1-\frac{x^2}{2}\right)+c\left(1+x+\frac{x^2}{2}\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{(b+c)+(a+c)x+(c-b)\frac{x^2}{2}-a\frac{x^3}{6}}{x^2}$$

$$\Rightarrow b+c=0, a+c=0\ \&\ c-b=4$$

$$\Rightarrow a=b=-2, c=2.$$

25 $$\lim_{x\to\infty}\left(\frac{ax+1}{ax+2}\right)^{2x}=e^{\lim_{x\to\infty}2x\left(\frac{ax+1}{ax+2}-1\right)}$$

$= e^{\lim_{x\to\infty}\left(\frac{-2x}{ax+2}\right)} = e^{-\frac{2}{a}} = e^{\frac{1}{2}} \Rightarrow a = -4$ **Ans.[C]**

26 In RHL $[x] = 0$ hence limit is not defined. **Ans.[D]**

27 $\lim_{x\to 0^-}\left(1+[x]\right)^{\frac{1}{x}}$ not defined as $1+[x]=0$ & $\frac{1}{x}\to -\infty$ **Ans.[D]**

$\lim_{x\to 0^+}\left(1+[x]\right)^{\frac{1}{x}} = 1$ as $[x]=0$

28 $\lim_{x\to\infty}\left(\sqrt{x^2+8x+3}-\sqrt{x^2+4x+3}\right) = \lim_{x\to\infty}\left(\frac{4x}{\sqrt{x^2+8x+3}+\sqrt{x^2+4x+3}}\right)$

$= \lim_{x\to\infty}\left(\frac{4}{\sqrt{1+\frac{8}{x}+\frac{3}{x^2}}+\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}}\right) = 2$ **Ans.[C]**

29 $\lim_{x\to 0}\frac{2x\left(3^x-1\right)}{1-\cos 2x} = \lim_{x\to 0}\frac{2x\left(3^x-1\right)}{2\sin^2 x} = \lim_{x\to 0}\frac{\left(\frac{3^x-1}{x}\right)}{\frac{\sin^2 x}{x^2}} = \ln 3$ **Ans.[A]**

30 $\lim_{n\to\infty}\frac{1^5+2^5+3^5+........+n^5}{n^6} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^5$

$= \int_0^1 x^5 dx = \frac{1}{6}.$ **Ans.[D]**

31 $\lim_{n\to\infty}\left[\frac{3}{1+n^3}+\frac{12}{8+n^3}+\frac{27}{27+n^3}+....+n \text{ terms}\right] = 3\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{\left(\frac{r}{n}\right)^2}{\left(\frac{r}{n}\right)^3+1}$

$= 3\int_0^1\frac{x^2}{1+x^3}dx = \int_1^2\frac{1}{t}dt \quad \{\text{by substitution } 1+x^3=t\}$

= ln 2. **Ans.[B]**

32 $\lim_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{\sin^{-1}2x} = \lim_{x\to 0}\frac{2x}{\sin^{-1}2x}\times\frac{1}{\sqrt{4+x}+\sqrt{4-x}} = \frac{1}{4}.$ **Ans.[D]**

33 $\lim_{x\to\infty}\left(\frac{1^x+3^x+5^x+...+\left(2n-1\right)^x}{n}\right)^{\frac{1}{x}}$

$= \lim_{x\to\infty}\left(2n-1\right)\left(\frac{\left(\frac{1}{2n-1}\right)^x+\left(\frac{3}{2n-1}\right)^x+\left(\frac{5}{2n-1}\right)^x+...+\left(\frac{2n-3}{2n-1}\right)^x+1}{n}\right)^{\frac{1}{x}}$

$$= 2n - 1 \quad \left\{ \text{as all } \frac{1}{2n-1}, \frac{3}{2n-1}, ..., \frac{2n-3}{2n-1} < 1 \right\}$$ **Ans.[A]**

34 $$\lim_{x \to 0} \left(\frac{1^x + 2^x + 3^x + ...n^x}{n} \right)^{\frac{1}{x}} = e^{\lim_{x \to 0} \frac{1}{x} \left(\frac{1^x + 2^x + 3^x + ...n^x}{n} - 1 \right)}$$

$$= e^{\lim_{x \to 0} \frac{1}{n} \left(\frac{1^x - 1 + 2^x - 1 + 3^x - 1 + ...n^x - 1}{x} \right)} = e^{\frac{\ln 2 + \ln 3 + ... + \ln n}{n}} = (n!)^{\frac{1}{n}}$$ **Ans.[B]**

35 $$\lim_{x \to 0} \frac{8(2^x - 3^x)\tan x}{1 - \cos 4x} = \lim_{x \to 0} \frac{8\left(\frac{2^x - 1}{x} - \frac{3^x - 1}{x} \right) \frac{\tan x}{x}}{8 \frac{\sin^2 2x}{4x^2}}$$

$$= \ln 2 - \ln 3 = \ln \frac{2}{3}$$ **Ans.[D]**

LIMITS
EXERCISE 3

1. $\lim_{x\to 1}\frac{x^{\frac{1}{13}}-x^{\frac{1}{7}}}{x^{\frac{1}{5}}-x^{\frac{1}{3}}}$ $\left(\frac{0}{0}\text{form}\right)$

Apply L'Hospital rule

$$\lim_{x\to 1}\frac{\frac{1}{13}x^{-1+\frac{1}{13}}-\frac{1}{7}x^{-1+\frac{1}{7}}}{\frac{1}{5}x^{\frac{1}{5}-1}-\frac{1}{3}x^{\frac{1}{3}-1}}$$

$$\lim_{x\to 1}\frac{\frac{1}{13}-\frac{1}{7}}{\frac{1}{5}-\frac{1}{3}}=\frac{45}{91}$$ **Ans.**

2. $\lim_{x\to 1}\frac{\sum_{k=1}^{100}x^k-100}{x-1}$

$\Rightarrow \lim_{x\to 1}\frac{x+1}{x-1}+\lim_{x\to 1}\frac{x^2-1}{x-1}+\lim_{x\to 1}\frac{x^3-1}{x-1}.....\lim_{x\to 1}\frac{x^{100}-1}{x-1}$

Applying L'Hospital rule

$\Rightarrow \lim_{x\to 1}1+\lim_{x\to 1}2x+\lim_{x\to 1}3x^2+\lim_{x\to 1}4x^3+.........\lim_{x\to 1}100x^9$

= 1 + 2 + 3 + 4...........100

$= \frac{100(100+1)}{2} = 50 \times 101 \left(1+2+3...........n=\frac{n(n+1)}{2}\right)$

= 5050

3. $\lim_{x\to 1}\left(\frac{p}{1-x^p}-\frac{q}{1-x^2}\right)$ $(\infty-\infty\text{ form})$

$$\lim_{x\to 1}\left(\frac{p(1-x^q)-q(1-x^p)}{(1-x^p)\,(1-x^q)}\right)$$

$$\lim_{x\to 1}\left(\frac{p-px^2-q+qx^p}{1-x^q-x^p+x^{p+q}}\right)$$ $\left(\frac{0}{0}\text{form}\right)$

Apply L'Hopital rule

$$\lim_{x\to 1}\left(\frac{-pqx^{q-1}+qpx^{p-1}}{-qx^{q-1}-px^{p-1}+(p+q)x^{(p+q-1)}}\right) \qquad \left(\frac{0}{0}\text{form}\right)$$

$$\lim_{x\to 1}\left(\frac{-pq(q-1)x^{q-2}+qp(p-1)x^{p-2}}{-q(q-1)x^{q-2}-p(p-1)x^{p-2}+(p+q)(p+q-1)x^{(p+q-2)}}\right)$$

$$\Rightarrow \frac{-pq^2+pq+p^2q-pq}{-q^2+q-p^2+p+p^2+pq-p+qp+q^2-q}$$

$$\Rightarrow \frac{pq(p-q)}{2pq}=\frac{p-q}{2}$$

4. $$\lim_{x\to 3\pi/4}\frac{1+(\tan x)^{\frac{1}{3}}}{1-2\cos^2 x}$$

$$\lim_{x\to 3\pi/4}\frac{1+(\tan x)^{\frac{1}{3}}}{-\cos 2x} \qquad \left(\frac{0}{0}\text{form}\right)$$

Applying L'Hospital rule

$$\lim_{x\to \frac{3\pi}{4}}\frac{\frac{1}{3}(\tan x)^{\frac{-2}{3}}\times\sec^2 x}{+2\sin 2x}$$

$$\Rightarrow \frac{-1}{6}\left(-\sec\frac{\pi}{4}\right)^2$$

$$\frac{-1}{6}\times 2=-\frac{1}{3}$$

5. $$\lim_{x\to 0}\frac{\ell n(1+\sin^2 x)}{\tan[\ell n^2(1+x)]}$$

$$\Rightarrow \lim_{x\to 0}\ell n(1+\sin^2 x)\frac{1}{\tan[\ell n^2(1+x)]}$$

$$\Rightarrow \lim_{x\to 0}\frac{(\sin x/x)^2}{\frac{\tan(\ell n^2(1+x))}{\ell n^2(1+x)}}\frac{1}{\left(\frac{1}{x^2}\right)\ell n^2(1+x)}$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\frac{\ell n(1+x)}{x} \; \frac{\ell n(1+x)}{x}} = 1$$

6. $$\lim_{x\to\infty} \frac{2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 5x^{\frac{1}{5}}}{(3x-2)^{\frac{1}{2}} + (3x-3)^{\frac{1}{3}}}$$

divide numerator & denominator by $x^{1/2}$

$$\Rightarrow \lim_{x\to\infty} \frac{2 + \frac{3}{x^{\frac{1}{6}}} + \frac{5}{x^{\frac{3}{10}}}}{\left(3-\frac{2}{x}\right)^{\frac{1}{2}} + \frac{1}{x^{6}}\left(2-\frac{3}{x}\right)^{\frac{1}{3}}} \Rightarrow \frac{2+0+0}{\sqrt{3}+0} \Rightarrow \frac{2}{\sqrt{3}}$$

7. $$\lim_{x\to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$$

$$\lim_{x\to 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$$

$$\lim_{x\to 0} \frac{\cos 2x - \cos 4x}{\cos x - \cos 3x} \times \frac{\cos x . \cos 3x}{\cos 4x . \cos 2x}$$

$$\lim_{x\to 0} \frac{+2\sin 3x . \sin x}{2\sin 2x . \sin x}$$

$$\lim_{x\to 0} \frac{\sin 3x}{\sin 2x} \times \frac{2x}{3x} \times \frac{3x}{2x}$$

$$\lim_{x\to 0} \frac{3x}{2x} = \frac{3}{2}$$

8. Let first term of on infinite G.P. is a & common ratio of infinite G.P. is r

given $a = \lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x} \left(\frac{0}{0} \text{form}\right)$

Apply L'Hospital rule

$$\lim_{x\to 0} \frac{\sec^2 x - \cos x}{3\sin^2 x \cos x}$$

$$\lim_{x\to 0} \frac{1-\cos^3 x}{3\sin^2 x \cos 3x}$$

$$\lim_{x\to 0} \frac{1-\cos 3x}{3\sin^2 x \cos 2x} \times \lim_{x\to 0} \frac{1}{\cos x}$$

$$\lim_{x\to 0}\frac{4}{4}\times\frac{1-\cos^3 x}{3\sin^2 x\cos^2 x}\times 1$$

$$\lim_{x\to 0}\frac{4(1-\cos 3x)}{3(\sin 2x)^2}\qquad\left(\frac{0}{0}\text{form}\right)$$

Apply L'Hospital rule

$$\lim_{x\to 0}\frac{4}{3}\times\frac{(+3\cos 2x\sin x)}{2(\sin 2x)\times 2\cos 2x}$$

$$a=\lim_{x\to 0}\frac{\cos^2 x\sin x}{(2\sin x\cos x)\cos 2x}=\frac{1}{2}$$

$$r=\lim_{x\to 1}\frac{1-\sqrt{x}}{(\cos^{-1}x)^2}$$

$$\lim_{x\to 1}\frac{1\frac{1}{2\sqrt{x}}}{2(\cos^{-1}x)\times-\frac{1}{\sqrt{1-x^2}}}$$

$$\lim_{x\to 1}\frac{1\times\sqrt{1-x^2}}{4\sqrt{x}(\cos^{-1}x)}$$

$$\lim_{x\to 1}\frac{\sqrt{1-x^2}}{4}\ \cos^{-1}x\qquad\qquad \text{put } x=\cos t$$

$$r=\lim_{t\to 1}\frac{\sin t}{4t}=\frac{1}{4}$$

so sum of infinite G.P. is $\int\infty=\frac{a}{1-r}$

$$=\frac{\frac{1}{2}}{1-\frac{1}{4}}=\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$$

9. $\lim_{x\to\infty}(x-\ell n(\cos nx))$ $\qquad$ $\cosh x=\frac{e^x+e^{-x}}{2}$

$$\lim_{x\to\infty}\left(x-\ell n\left(\frac{e^x+e^{-x}}{2}\right)\right)$$

$$\lim_{x\to\infty}\left(x-\ell n\left(\frac{e^{2x}+1}{2e^x}\right)\right)$$

$$\lim_{x\to\infty}\left[x-[\ell n(e^{2x}+1)-\ell n 2e^{x}]\right]$$

$$\lim_{x\to\infty}[x-\ell n(e^{2x}+1)-\ell n2-\ell n e^{x}]$$

$$\lim_{x\to\infty}[x-\ell n(e^{2x}+1)-\ell n2-x]$$

$$\lim_{x\to\infty}[-\ell n^{2}-\ell n(e^{2x}+1)]$$

$$-\ell n2-\lim_{x\to\infty}\ell n\frac{(e^{2x}+1)}{e^{2x}}.e^{2x}$$

$$-\ell n2-\lim_{x\to\infty}.\ell n\left(\frac{1+\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}}\right)$$

10. $$\underset{x\to 0}{\mathrm{L\,im}}\frac{8}{x^{8}}\left[\left(1-\cos\frac{x^{2}}{2}\right)-\cos\frac{x^{2}}{4}\left(1-\cos\frac{x^{2}}{2}\right)\right]$$

$$\underset{x\to 0}{\mathrm{L\,im}}=\frac{8}{x^{8}}\left[\left(1-\cos\frac{x^{2}}{2}\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\underset{x\to 0}{\mathrm{L\,im}}=\frac{8}{x^{8}}\left[1-\left(2\cos^{2}\frac{x^{2}}{4}-1\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\underset{x\to 0}{\mathrm{L\,im}}=\frac{8}{x^{8}}\left[2-2\cos^{2}\frac{x^{2}}{4}\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\underset{x\to 0}{\mathrm{L\,im}}=\frac{8\times 2}{x^{8}}\left[\left(1-\cos^{2}\frac{x^{2}}{4}\right)\left(1-\cos\frac{x^{2}}{4}\right)\right]$$

$$\underset{x\to 0}{\mathrm{L\,im}}=\frac{16}{x^{8}}\left[\sin^{2}\frac{x^{2}}{4}\quad 2\sin^{2}\frac{x^{2}}{8}\right]$$

$$\lim_{x\to 0} = \frac{32}{x^8}\left[\sin^2\frac{x^2}{4}\sin^2\frac{x^2}{8}\right]$$

$$\lim_{x\to 0} = \frac{1}{32}\left(\frac{\sin\frac{x^2}{4}}{\left(\frac{x^2}{4}\right)}\right)\left(\frac{\sin\frac{x^2}{8}}{\left(\frac{x^2}{8}\right)}\right)^2 = \frac{1}{32}$$

11. $\lim_{\theta\to\frac{\pi}{4}} \frac{\sqrt{2}-\cos\theta-\sin\theta}{(4\theta-\pi)^2}$ $\left(\frac{0}{0}\text{form}\right)$

Apply L'Hospital rule

$\lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta-\cos\theta}{2(4\theta-\pi).4}$ $\left(\frac{0}{0}\text{form}\right)$

Again apply L'Hospital rule

$$\lim_{\theta\to\frac{\pi}{4}} \frac{\cos\theta+\sin\theta}{8.4} = \frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{32} = \frac{\sqrt{2}}{32} = \frac{1}{16\sqrt{2}}$$

12. $\lim_{\theta\to\frac{\pi}{2}} \frac{2^{-\cos x}-1}{x\left(x-\frac{\pi}{2}\right)}$

$\lim_{\theta\to\frac{\pi}{2}} \frac{1-2^{\cos x}}{2^{\cos x}\left(x^2-\frac{\pi}{2}x\right)}$ $\left(\frac{0}{0}\text{ form}\right)$

$\lim_{x\to\frac{\pi}{2}} \frac{1-2^{\cos x}}{x^2-\frac{\pi}{2x}}$ $\left(\frac{0}{0}\text{form}\right)$

again apply L'Hospital rule

$$\lim_{x\to\frac{\pi}{2}} \frac{-2^{\cos x}\log 2}{2x-\frac{\pi}{2}} \Rightarrow \frac{-2\log 2}{\pi}$$

13. $$\lim_{x\to 1}\left[\ell n\left(\frac{1+x}{2}\right)\frac{1}{\sin(x-1)}\right].3.\left[\frac{4^{x-1}-x}{(7+x)^{1/3}-(1+3x)^{1/2}}\right]$$

here $(1)^{\infty}$

$$=\frac{x-1}{2\sin(x-1)}.3\left[\frac{\ell n4\times(4^{x-1})-1}{\frac{1}{3}(7+x)^{-2/3}-\frac{3}{2}(1+3x)^{-1/2}}\right]\Rightarrow\frac{1}{2}.3\left[\frac{\ell n4-1}{\frac{1}{3}.\frac{1}{4}-\frac{3}{2}.\frac{1}{2}}\right]$$

$$\Rightarrow\frac{1}{2}.3.\left[\frac{\ell n4-1}{\frac{1-9}{12}}\right]\Rightarrow\boxed{\frac{-9}{4}\ell n\frac{4}{e}}$$

14. If $\ell=\lim_{n\to\infty}\sum_{r=2}^{n}\left((r+1)\sin\frac{\pi}{r+1}-r\sin\frac{\pi}{r}\right)$

$$\ell=\lim_{n\to\infty}\left(3\sin\frac{\pi}{3}-2\sin\frac{\pi}{2}+4\sin\frac{\pi}{4}-3\sin\frac{\pi}{3}+5\frac{\sin\pi}{5}-4\frac{\sin\pi}{4}+n\sin\frac{\pi}{4}-(n-1)\frac{\sin\pi}{n-1}\right.$$
$$\left.+(n+1)\frac{\sin\pi}{n+1}-n\frac{\sin\pi}{n}\right)$$

$$\ell=\lim_{n\to\infty}\left((n+1)\frac{\sin\pi}{n+1}-2\sin\frac{\pi}{2}\right)$$

$$\ell=\lim_{n\to\infty}\left((n+1)\frac{\frac{\sin\pi}{n+1}}{\frac{\pi}{n+1}}\times\frac{\pi}{n+1}-2\right)$$

$$\ell=\lim_{n\to\infty}(\pi-2)=3.14-2$$
$$=1.14$$

so $\{\ell\}=\ell-[\ell]$

$=1.14-1=.14$ or $\pi-3$

15. $$\lim_{x\to-\infty}\frac{(3x^4+2x^2)\left(\sin\frac{1}{x}\right)\frac{1}{x}}{(|x|^3+|x|^2+|x|+1)\frac{1}{2}}+\frac{|x|^3+5}{|x|^3+|x|^2+|x|+1}$$

$$\lim_{x\to-\infty}\frac{3x^3+2x}{-(x)^3-(x)^2-(x)+1}+\frac{-(x)^3+5}{-(x)^3+(x)^2-(x)+1}$$

$$\lim_{x\to-\infty}\frac{3+\frac{2}{x^2}}{-1-\frac{1}{x}-\frac{1}{x^2}-\frac{1}{x^3}}+\frac{-1+\frac{5}{x^3}}{-1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$\lim_{x\to-\infty}\frac{-3}{1}+1=-2$$

16. $$\lim_{x\to 3}\frac{(x^3+27)\ell n(x-2)}{(x^2-9)}$$

$$\lim_{x\to 3}\frac{(x-3)\ (x^2+9-6x)\ \ell n(1+(x-2))}{(x+3)\ (x-3)}$$

$$\lim_{x\to 3}x^2+9-6x\ =18-9=9$$

17. $$\lim_{x\to 0}\frac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}\times\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$\lim_{x\to 0}\frac{2\sqrt{2}(27^x-9^x-3^x+1)}{2-1-\cos x}$$

$$\lim_{x\to 0}\frac{2\sqrt{2}(27^x-9^x-3^x+1)}{1-\cos}\times x^2$$

$$\lim_{x\to 0}\frac{4\sqrt{2}(27^x-9^x-3^x+1)}{x^2}$$

$$\lim_{x\to 0}4\sqrt{2}\left(\frac{9^x(3^x-1)-1(3^x-1)}{x^2}\right)$$

$$\lim_{x\to 0}4\sqrt{2}\left(\frac{(9^x-1)\ (3^x-1)}{x^2}\right)\quad\Rightarrow\ \lim_{x\to 0}4\sqrt{2}\left(\frac{9^x-1}{x}\right)\times\lim_{x\to 0}\frac{3^x-1}{x}\ \Rightarrow\ 8\sqrt{2}(\ell n3)^2$$

$$\Rightarrow\ 4\sqrt{2}(\ell n9)\ (\ell n3)$$

18. $$\lim_{x\to 0}\left[\frac{(1+x)^{1/x}}{e}\right]^{1/x}\quad(1)^\infty$$

$$L=\boxed{e^{\lim_{x\to 0}}\frac{(1+x)^{1/x}-e}{e}.\frac{1}{x}}$$

Act.: $\lim\limits_{x\to 0} \dfrac{e[e^{\frac{\ell n(1+x)}{x}-1}-1]}{\dfrac{e(\ell n(1+x)-x)}{x}} \cdot \left(\dfrac{\ell n(1+x)-x}{x^2}\right)$

$$\lim_{t\to 0} \frac{t-e^t+1/t^2}{(e^t-1)^2/t^2} = -\frac{1}{2}$$

(taking commone)

$= e^{-1/2}$

19. $e^{\lim\limits_{n\to\infty}\left[\frac{\sqrt{n^2+n}-1-n}{n}\right](2\sqrt{n^2+n}-1)}$

$e^{\lim\limits_{x\to\infty}\frac{[(n^2+n)-(1+n)^2](2\sqrt{n^2+n}-1)}{n\left\{\sqrt{n^2+n}+(1+n)\right\}}}$

$e^{\lim\limits_{n\to\infty}\frac{(-n-1)(2\sqrt{n^2+n}-1)}{n\left\{\sqrt{n^2+n}+(1+n)\right\}}}$ [taking n^2 as common]

$e^{\lim\limits_{n\to\infty}\frac{\left(-1-\frac{1}{n}\right)\left(2\sqrt{1+\frac{1}{n}}-\frac{1}{n^2}\right)n^2}{n^2\left\{\sqrt{1+\frac{1}{n}}+\frac{1}{n}+1\right\}}} = e^{-1}$

20. $\lim\limits_{x\to 1}\left[\tan\dfrac{\pi x}{4}\right]^{\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\left(\tan\frac{\pi x}{4}-1\right)\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\frac{\sin\left(\frac{\pi x}{4}-\frac{\pi}{4}\right)}{\cos\frac{\pi}{4}.\cos\frac{\pi x}{4}}.\tan\frac{\pi x}{2}}$

$e^{\lim\limits_{x\to 1}\frac{\sin\frac{\pi}{4}(x-1)}{\frac{\pi}{4}(x-1)\cos\frac{\pi}{4}\cos\frac{\pi}{4}x}.\tan\frac{\pi x}{2}.\frac{\pi}{4}(x-1)}$

$e^{\lim\limits_{x\to 1}\frac{\pi}{2}(x-1)\cot\left(\frac{\pi}{2}-\frac{\pi}{2x}\right)} = e^{-1} = (e^{-1})$

21. $\lim\limits_{x\to 0}\left[\tan\left(\dfrac{\pi}{4}+x\right)\right]^{1/x} = e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\frac{2\tan x}{1-\tan x}\right]} = e^{\lim\limits_{x\to 0}\left[\frac{\tan x}{x}\right]\frac{2}{1-\tan x}} = e^2$ •

22. $\lim_{x\to0}\left(\frac{3^x-1}{x}\right)=\ln 3$

23. $\lim_{x\to0}\frac{\cos(\sin x)-1}{x^2}=\lim_{x\to0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2}$

$$=\lim_{x\to0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{\frac{\sin^2 x}{4}}\times\frac{\sin^2 x}{4x^2}=\frac{1}{2}$$

24. $\lim_{x\to0^+}\frac{e^{1/x}}{e^{\frac{1}{x}}+1}=\lim_{x\to0^+}\frac{1}{e^{-\frac{1}{x}}+1}=1$ & $\lim_{x\to0^-}\frac{e^{1/x}}{e^{\frac{1}{x}}+1}=0$

LHL $\neq$ RHL

Limit does not exist.

25. $\lim_{x\to0}\frac{a^{\sin x}-1}{b^{\sin x}-1}=\lim_{x\to0}\frac{\frac{a^{\sin x}-1}{\sin x}}{\frac{b^{\sin x}-1}{\sin x}}=\frac{\ln a}{\ln b}$

26. $\lim_{x\to0}\left(\frac{e^x-1}{x}\right)=1$

27. $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+1}=e^{\lim_{x\to\infty}\left(\frac{x+3}{x+1}-1\right)(x+1)}=e^{\lim_{x\to\infty}\left(\frac{2}{x+1}\right)(x+1)}=e^2$

28. $\lim_{x\to0}(1-ax)^{\frac{1}{x}}=e^{\lim_{x\to0}\frac{-ax}{x}}=e^{-a}$

29. By L'hospitals rule

$$\lim_{x\to0}\frac{x^n-\sin x^n}{x-\sin^n x}=\lim_{x\to0}\frac{nx^{n-1}\left(1-\cos x^n\right)}{1-n\sin^{n-1}x\cos x}$$

Now for any value of n greater than 1, denominator will be nonzero and numerator will be zero.

But for n = 1, limit becomes $\lim_{x\to0}\frac{1-\cos x}{1-\cos x}=1$.

30. $\lim_{x\to 0}\frac{2x+ax\cos x+b\sin x}{x^3}=\lim_{x\to 0}\frac{2x+ax\left(1-\frac{x^2}{2}\right)+b\left(x-\frac{x^3}{6}\right)}{x^3}$

$$=\lim_{x\to 0}\frac{(a+b+2)x-\left(\frac{a}{2}+\frac{b}{6}\right)x^3}{x^3}=2$$

$\Rightarrow a+b+2=0\ \ \&\ 3a+b=-12$

$\Rightarrow a=-5, b=3$.

31. $\lim_{h\to 0}\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}=\lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\frac{\sqrt{3}}{2}\cos h-\frac{1}{2}\sin h\right)}$

$$=\lim_{h\to 0}\frac{\cos\frac{\pi}{6}\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\cos\frac{\pi}{6}\cos h-\sin\frac{\pi}{6}\sin h\right)}$$

$$=\lim_{h\to 0}\frac{\sinh}{\sqrt{3}h\cos\left(\frac{\pi}{6}+h\right)}=\frac{2}{3}.$$

32. $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}=\lim_{x\to 0}\frac{x\left(1-\frac{x^2}{2}\right)-\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)}{x^2}$

$$=\lim_{x\to 0}\frac{\frac{x^2}{2}-\frac{2x^3}{3}}{x^2}=\frac{1}{2}.$$

33. $\lim_{x\to\frac{\pi}{4}}\left(\frac{1-\tan x}{1-\sqrt{2}\sin x}\right)=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x}\left(\frac{\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x}{\frac{1}{\sqrt{2}}-\sin x}\right)$

$$= \lim_{x\to\frac{\pi}{4}} \frac{1}{\cos x}\left(\frac{\sin\left(\frac{\pi}{4}-x\right)}{\sin\frac{\pi}{4}-\sin x}\right)$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{1}{\cos x}\left(\frac{2\sin\left(\frac{\pi}{8}-\frac{x}{2}\right)\cos\left(\frac{\pi}{8}-\frac{x}{2}\right)}{2\cos\left(\frac{\pi}{8}+\frac{x}{2}\right)\sin\left(\frac{\pi}{8}-\frac{x}{2}\right)}\right)$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{1}{\cos x}\left(\frac{\cos\left(\frac{\pi}{8}-\frac{x}{2}\right)}{\cos\left(\frac{\pi}{8}+\frac{x}{2}\right)}\right) = 2$$

34. $\lim_{x\to 0}\frac{3^x-1}{\sqrt{4+x}-2} = \lim_{x\to 0}\frac{3^x-1}{x}\left(\sqrt{4+x}+2\right) = 4\ln 3$

35. $\lim_{x\to 0}\left(\frac{\sin 2(1+x)+\sin 2(1-x)-2\sin 2}{x\sin x}\right) = \lim_{x\to 0}\left(\frac{2\sin 2\cos 2x-2\sin 2}{x\sin x}\right)$

$$= -2\sin 2\lim_{x\to 0}\left(\frac{2\sin^2 x}{x\sin x}\right) = -4\sin 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = -4\sin 2$$

36. By L'hospitals rule

$$\lim_{x\to\frac{\pi}{4}}\frac{\int_{\pi/4}^{x} t^2\,dt}{\cos 2x} = \lim_{x\to\frac{\pi}{4}}\frac{x^2}{-2\sin 2x} = -\frac{\pi^2}{32}$$

37. By L'hospitals rule

$$\lim_{h\to 0}\frac{(x+h)\sec(x+h)-x\sec x}{\sin h} = \lim_{h\to 0}\frac{(x+h)\sec(x+h)\tan(x+h)+\sec(x+h)}{\cos h}$$

$$= x\sec x\tan x+\sec x = \sec x\left(x\tan x+1\right)$$

Alternately

$$\lim_{h\to 0}\frac{(x+h)\sec(x+h)-x\sec x}{\sin h} = \lim_{h\to 0}\frac{(x+h)\sec(x+h)-(x+h)\sec x-x\sec x+(x+h)\sec x}{\sin h}$$

$$=\lim_{h\to 0}\frac{(x+h)\left(\sec(x+h)-\sec x\right)}{\sin h}-\lim_{h\to 0}\frac{\left(x-(x+h)\right)\sec x}{\sin h}$$

$$=\lim_{h\to 0}\frac{(x+h)\left(\cos x-\cos\left(x+h\right)\right)}{\sin h\cos x\cos\left(x+h\right)}+\lim_{h\to 0}\frac{h\sec x}{\sin h}$$

$$=\lim_{h\to 0}\frac{2(x+h)\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{2\sin\frac{h}{2}\cos\frac{h}{2}\cos x\cos\left(x+h\right)}+\lim_{h\to 0}\frac{h\sec x}{\sin h}$$

$$=\frac{x\sin x}{\cos^2 x}+\sec x=x\sec x\tan x+\sec x\,.$$

38. $$\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)}{2\cos 2x-1}=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)}{4\cos^2 x-3}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{3}-2x\right)\cos x}{\cos 3x}$$

$$=\lim_{y\to 0}\frac{\sin 2y\cos\left(\frac{\pi}{6}-y\right)}{\sin 3y}\quad\left\{x=\frac{\pi}{6}-y\right\}$$

$$=\lim_{y\to 0}\frac{2\frac{\sin 2y}{2y}\cos\left(\frac{\pi}{6}-y\right)}{3\frac{\sin 3y}{3y}}=\frac{2}{3}\times\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{3}}$$

39. $$\lim_{x\to 0}\frac{e^{x^4}-\cos x^2}{x^4}=\lim_{x\to 0}\frac{\left(1+x^4+\frac{x^8}{2}\right)-\left(1-\frac{x^4}{2}+\frac{x^8}{4!}\right)}{x^4}=\frac{3}{2}\,.$$

40. $$\lim_{x\to 0}\frac{x\cos^2 x-\sin^2 x}{x+\sin x}=\lim_{x\to 0}\frac{\cos^2 x-x\frac{\sin^2 x}{x^2}}{1+\frac{\sin x}{x}}=\frac{1}{2}$$