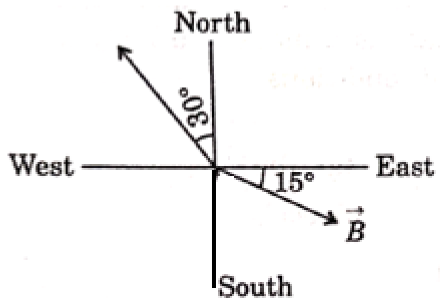
# PART (A) : PHYSICS

## SOLUTIONS

1. (D)




The direction of resultant must lie in between smaller angle between $\vec{A}$ and $\vec{B}$. Therefore resultant in south direction is to possible.

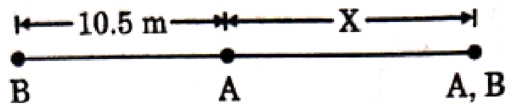
2. (C)
In option (C)
$c^2 = a^2 + b^2 - 2ab$ … (i)
$\vec{c} = \vec{a} + \vec{b}$ Squaring gives
$c^2 = a^2 + b^2 + 2ab\cos\theta$ … (ii)
Comparing eqns. (i) and (ii)
$\cos\theta = -1$
$\theta = 180°$

3. (B)
$\vec{a} + \vec{b} - \vec{c} = 4\hat{i} + 3\hat{j}, \qquad 2\vec{a} - \vec{b} = 4\hat{i} + 3\hat{j}$

4. (B)
$y = \ln\left(\cos^2 x\right)$
$\frac{dy}{dx} = \frac{1}{\cos^2 x}\left(-2\cos x \sin x\right) = -2\tan x$

5. (B)
$$\omega = \int_{r_0}^{\infty} \frac{K}{r^2}\,dr$$
$$= \left.\frac{kr^{-1}}{-1}\right|_{r_0}^{\infty} = \left.\frac{-K}{r}\right|_{r_0}^{\infty}$$
$$= 0 + \frac{K}{r_0}$$

6. (A)




$X = 10t$

$10.5 + X = \frac{1}{2} \times 1 \times t^2$

$\Rightarrow t^2 - 20t - 21 = 0$

$\Rightarrow t = -1, 21 \text{ sec}$

7. (A)

$a = \frac{u_1^2}{2s_1}, \; a = \frac{u_2^2}{2s_2}$

$\Rightarrow \frac{s_1}{s_2} = \frac{u_2^2}{\mu_1^2}$

$\Rightarrow s_2 = 6 \times \left(\frac{100}{50}\right) = 24 \text{ m}$

8. (B)

$T = \frac{2u \sin\theta}{g} \Rightarrow \frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$

For $\theta_1 = 0$ and $\theta_2 = 90 - \theta$

$\frac{T_1}{T_2} = \tan\theta : 1$

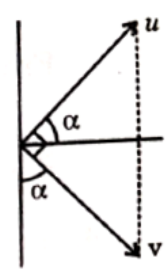
9. (B)

Naturally horizontal velocities remain same, thus

$\upsilon \sin\alpha = u \cos\alpha$

or $\upsilon = u \cot\alpha$

$\therefore$ *B* is correct.




10. (A)

From figure : (given in problem)

For time interval $t = 0$ to $t = 1$ sec

Slope of *x*-*t* graph is negative and increasing, so velocity increase in negative direction.

For $t = 1$ to 2 sec

The slope is +ve and decreasing, so velocity is decreasing in +ve direction and becomes zero at $t = 2$

So, (A) is correct.

11. (BCD)

(A) $\vec{A}\cdot\vec{B}=2-3=-1 \Rightarrow$ angle between $\vec{A}$ and $\vec{B}>90°$

(B) $\left(\vec{A}+\vec{B}\right)\cdot\vec{A}=\left(3\hat{i}-2\hat{j}\right)\cdot\left(\hat{i}+\hat{j}\right)=3-2=1$

$\Rightarrow$ Option (B) is correct.

(C) $\hat{k}$ is $\perp$ to all vectors in *x-y* plane

(D) $\left(\hat{i}+\hat{j}\right)\cdot\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right)=0$.

Therefore it is perpendicular to $\vec{A}$.

12. (ABCD)

Vector divided by magnitude of vector is unit vector. Hence (A) and (C) are correct. If two vector each of magnitude one are at 120° then their resultant has magnitude one. Hence (B) and (D) are correct.

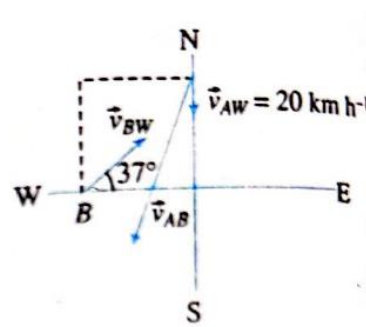
13. (AB)

$\vec{v}_{AW}=-20\,\hat{j}$

$\vec{v}_{BW}=32\hat{i}+24\hat{j}$

$\vec{v}_{AB}=\vec{v}_{AW}-\vec{v}_{BW}$

$=-32\hat{i}-44\hat{j}$




$\dfrac{d\vec{r}_{AB}}{dt}=\vec{v}_{AB}$

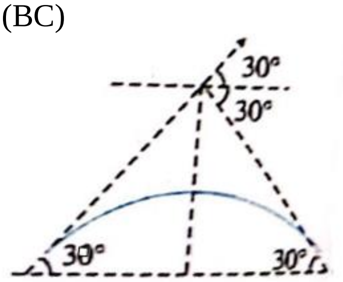
$\displaystyle\int_{3\hat{i}+4\hat{j}}^{\vec{r}_{AB}} d\left(\vec{r}_{AB}\right)=-\int_a^t 32dt\,\hat{i}-\int_0^t 44\,dt\,\hat{j}$

$\vec{r}_{AB}=\left(3-32t\right)\hat{i}+\left(4-44t\right)\hat{j}$

At $t=1/11h$, $\hat{j}$ component of $\vec{r}_{AB}$ is zero. At this time, its $\hat{i}$ component is $3-32t=3-\dfrac{32}{11}=\dfrac{1}{11}$km

It means at $t=1/11h$, *A* will be east of *B*. So at no time, *A* will be west of *B*.

14. (BC)




$$T=\frac{2u\sin\theta}{g}=\frac{2\times10\times\frac{1}{2}}{10}=1\,\text{s}$$

15. (ABD)

$a = \frac{dv}{dt}$, if velocity changes, definitely there will be acceleration.

If speed changes, then velocity also changes, so definitely there will be acceleration.

Acceleration may be due to change in the direction of velocity only and not magnitude.

If body has acceleration, it speed may change if acceleration is due to change in the magnitude of velocity.

16. (C)

$V_{Rx} = 10\text{ m/s}$

$V_{Ry} = 10\text{ m/s}$

$V_{my} = 0$

Drops appear vertical to man

$\Rightarrow \quad V_{Rx} = V_{mx}$

$\Rightarrow \quad V_{mx} = 10\text{ m/s}$

17. (C)

18. (A)

To reach the port *B*, the *x*-component of the total velocity must be zero: $v\sin\theta - u = 0$.

So, $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

Take positive *x*-axis along east and positive *y*-axis along north.

19. (D)

To cross the river fastest, we need to maximize the *y*-component of the total velocity is $v_B \cos\theta$.

So $\theta = 0$. The boat should head straight to the north.

## PART (B) : CHEMISTRY

### SOLUTION

20. (C)
N has positive electron gain enthalpy due to half filled configuration.
N < O < S < Cl

21. (D)
$IE_3 > IE_2 > IE_1$
So (D).

22. (B)
$m_\ell = -\ell$ to $+L$
So, if $l = 2$
$m_e = -2, -1, 0, 1, 2$
So, (B)

23. (A)
$d_{x^2-y^2}$ is along $x$ and $y$ axis $f$ orbital has 3 angular nodes.
Neutron has higher mass than proton so lesser wave nature.

24. (A)
$m_{NH_3} = 1 \times 0.93 \times \frac{18.6}{100}$
$= 0.17$ g

25. (A)
$n_{Ca^{+2}} = \frac{1}{2} n_{resin}$
$= \frac{1}{2} \times \frac{1}{206} = 0.0024$

26. (B)
$2a + 3b = \frac{31.8}{0.3} = 106$
$a + 2b = \frac{18.6}{0.3} = 62$
$\Rightarrow a = 26, \; b = 18$

27. (A)
$K = \frac{2}{3} \times \frac{1240}{40} = 20.7$ eV
$\Rightarrow V_{stopping} = 20.7$ V

28. (C)

*Zn*

| | $1s^2$ | $2s^2$ | $2p^6$ | $3s^2$ | $3p^6$ | $4s^2$ | $3d^{10}$ |
|---|---|---|---|---|---|---|---|
| | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| | 0 | 0 | $2 \to 2e^-$ | 0 | $2 \to 2e^-$ | 0 | $4 \to 2e^-$ |
| $\ell + m$ | | | $1 \to 2e^-$ | | $1 \to 2e^-$ | | $3 \to 2e^-$ |
| | | | $0 \to 2e^-$ | | $0 \to 2e^-$ | | $1 \to 2e^-$ |
| | | | | | | | $0 \to 2e^-$ |

29. (C)

IE decreases down the group.

EA of F < EA of Cl

So, $F^- < Cl^- < Cl < F$

30. (AC)

| | $2Mg$ | + | $O_2$ | $\to$ | $2MgO$ |
|---|---|---|---|---|---|
| $i$ | $\frac{1}{24}$ | | $\frac{2}{32}$ | | 0 |
| $r$ | $\frac{1}{24}$ | | $\frac{1}{48}$ | | $\frac{1}{24}$ |
| $f$ | 0 | | $\frac{2}{32}-\frac{1}{48}$ | | $\frac{1}{24}$ |
| | | | $\Downarrow$ | | $\Downarrow$ |
| | | | $W = \frac{4}{3}g$ | | $W = \frac{5}{3}g$ |

31. (CD)

$$n_m = \frac{22.44}{374} = 0.06$$

$$AW = \frac{8}{0.06} = 133.33$$

$IE_2$ is higher than $IE_1$.

32. (ABCD)

Size increases down the group

Noble gases are abnormally bigger due to Vander Waal radius.

33. (ABD)

Radial node = $n - \ell - 1$

Angular node = $\ell$

For H energy depends only $n$.

34. (ABD)

$\lambda = 2\pi n a_0$

$= n \times 3.32\,\text{Å}$

35. (D)

Element is $S$

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^4$

$n_S = \frac{6.4}{32}$

No. of protons $= \frac{6.4}{32} \times 16 \times N_A$

$= 19.2 \times 10^{23}$

36. (B)

$\left.\begin{matrix} 2p^6 \\ 2p^4 \end{matrix}\right.$ $\Rightarrow$ So maximum 6 $e^-$, minimum 4 $e^-$.

37. (D)

$m + 19n = \frac{10.8}{0.1} = 108$ where $m$ is molar mass of $M$

$2m + (n-2)19 = \frac{1.02}{0.01} = 102$

$m + 19n = 108$

$2m + 19n = 140$

$\Rightarrow m = 32$

$\Rightarrow n = 4$

38. (C)

$M F_4 \rightarrow M F_2$

Remove 2 mole F.

## PART (C) : MATHEMATICS

### SSOLUTION

39. (C)

$$\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}=\cos\left(2\times 15^\circ\right)=\cos 30^\circ=\frac{\sqrt{3}}{2}$$

40. (D)

$\alpha\beta=36 \Rightarrow (1,36),(2,18),(3,12),(4,9),(6,6)$ including + & – signs

$\Rightarrow$ Total 10 values of $K$

41. (D)

$\tan^2\theta=2\tan^2\phi+1 \qquad \ldots$ (i)

$$\cos 2\theta+\sin^2\phi=\frac{1-\tan^2\theta}{1+\tan^2\theta}+\sin^2\phi=\frac{1-2\tan^2\phi-1}{1+2\tan^2\phi+1}+\sin^2\phi=\frac{-2\tan^2\phi}{2\left(1+\tan^2\theta\right)}+\sin^2\phi$$

$=-\sin^2\phi+\sin^2\phi=0$

Which is independent of $\phi$.

42. (B)

Given, $P(4)=Q(4)$

$\therefore \quad 64k+48-3=128-20+k$

$\quad 63k=63$

$\therefore \quad k=1$

43. (C)

$\alpha+\beta+y=8$

$\alpha+\beta=y$

$\therefore \quad y=4$

$\therefore \quad 4^3-8\times 4^2+19\times 4+k=0$

$\Rightarrow \quad k=-12$

44. (B)

$$\frac{1}{\cos\left(270^\circ+20^\circ\right)}+\frac{1}{\sqrt{3}\sin\left(270^\circ-20^\circ\right)}=\frac{1}{\sin 20^\circ}-\frac{1}{\sqrt{3}\cos 20^\circ}$$

$$=\frac{2\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)}{2\frac{\sqrt{3}}{2}\sin 20^\circ\cos 20^\circ}=\frac{4\sin\left(60^\circ-20^\circ\right)}{\sqrt{3}\sin 40^\circ}=\frac{4}{\sqrt{3}}\frac{\sin 40^\circ}{\sin 40^\circ}=\frac{4\sqrt{3}}{3}.$$

45. (B)

$\tan 60° \times \tan 60° = 3$

46. (B)

$P(x) = Q(x)\cdot(x-1)(x+1)(x+2) + ax^2 + bx + c$

$P(1) = a + b + c = 5$

$P(-1) = a - b + c = 3$

$P(-2) = 4a - 2b + c = 2$

$a = 0,\ b = 1,\ c = 4$

$\therefore$ remainder is $x + 4$

47. (B)

$D_1 = b^2 - 4ac$

$D_2 = b^2 + 4ac$

$\therefore$ $D_1 + D_2 = 2b^2 \geq 0$

$\therefore$ Atleast one $D$ is positive

$\therefore$ Atleast two real roots

48. (A)

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ $\qquad \frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \frac{\sin\theta(1 + 2\cos\theta)}{2\cos^2\theta + \cos\theta}$

$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)} = \tan\theta \in R$

49. (BCD)

**(A)** $S = \alpha^2 + \beta^2 = a^2 - 2b$ ; $\qquad P = \alpha^2\beta^2 = b^2$

$\therefore$ Equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

**(B)** $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $\qquad P = \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{b}$

$\therefore$ $x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow$ $bx^2 + ax + 1 = 0$

**(C)** $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{a^2 - 2b}{b}$; $\qquad P = \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2 - 2b}{b}x + 1 = 0 \quad \Rightarrow \quad bx^2 - (a^2 - 2b)x + b = 0$

**(D)** $S = \alpha + \beta - 2 = -a - 2$ ; $\qquad P = (\alpha - 1)(\beta - 1)$

$= \alpha\beta - (\alpha + \beta) + 1 = b + a + 1$

$\therefore$ equation is $x^2+(a+2)x+(a+b+1)=0$.

50. (BC)

$\cos(A-B)=\frac{3}{5}$ & $\tan A\tan B=2$

$\cos A\cos B+\sin A\sin B=\frac{3}{5} \Rightarrow (1+\tan A\tan B)\cos A\cos B=\frac{3}{5}$

$\therefore \sin A\sin B=\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$

$\cos(A+B)=\cos A\cos B-\sin A\sin B=\frac{1}{5}-\frac{2}{5}=-\frac{1}{5}$.

51. (BC)

$D<0 \therefore ax^2+bx+c<0 \; \forall \; x\in R$

$f(-1)<0 \therefore a-b+c<0$

$f(1)<0 \therefore a+b+c<0$

$\therefore f(-1)f(1)>0 \Rightarrow (a+c)^2-b^2>0$

Also $f(-2)<0 \Rightarrow 4a-2b+c<0$

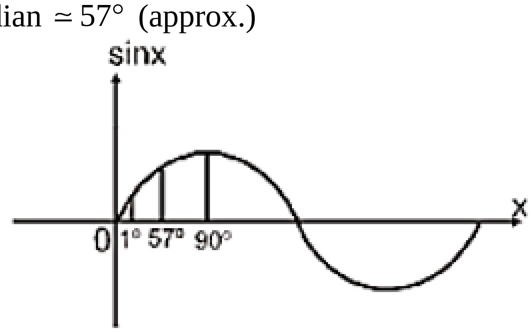
52. (AD)

$(2a-a)^2=(-22a+14a)(-14+11)$

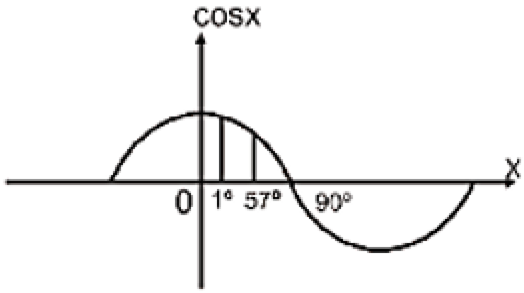
$a^2=24a \Rightarrow a=0, 24$

53. (BC)

1 radian $\simeq 57°$ (approx.)




$\therefore \sin 1>\sin 1°$




$\therefore$ $\cos 1° > \cos 1$

54. (A)

$f(5) = -3f(2)$

$25a + 5b + c = -3(4a + 2b + c)$

$37a + 11b + 4c = 0$

and $f(3) = 9a + 3b + c = 0$

$\therefore$ $36a + 12b + 4c = 0$

$\therefore$ $a - b = 0$

$\therefore$ $12a + c = 0$

$f(-4) = 16a - 4b + c = 12a + c = 0$

55. (D)

$a + b + c = a + a - 12a = -10a$

Cannot be uniquely determined

56. (A)

Let $f(x) = x^3 + 3x^2 + 4x + 12$

$f(-3) = -27 + 27 - 12 + 12 = 0$

$\therefore$ $x + 3$ is a factor of $f(x)$.

57. (B)

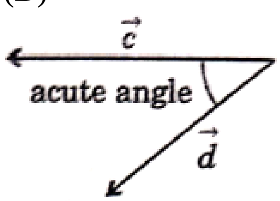
$P(1) = 1 - 3 + 2 + 1 = 1$

# PART (A) : PHYSICS

## SOLUTION

1. (C)
We know that –
$$\left||\vec{F}_1| - |\vec{F}_2|\right| \le |\vec{F}_1 + \vec{F}_2| \le |\vec{F}_1| + |\vec{F}_2|$$
So in this question, $4 \le |\vec{F}_1 + \vec{F}_2| \le 16$

2. (D)
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$
$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = -\vec{F}_4$$

3. (D)


Draw two vectors with their tail coinciding, then smaller angle between them is angle between vectors.

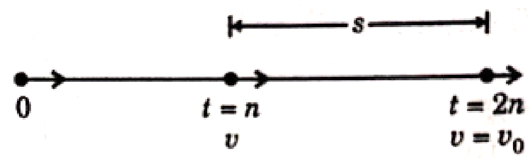
4. (A)
Projection of $=\left(3\hat{i} + 4\hat{k}\right) j$ on $y$-axis is 0 because angle between them is 90°.

5. (B)
$$y = \frac{\sin x}{x} + \frac{x}{\sin x}$$
$$\frac{dy}{dx} = \frac{(x\cos x - \sin x)}{x^2} + \frac{\sin x - x\cos x}{(\sin x)^2}$$
$$(x\cos x - \sin x)\left(\frac{1}{x^2} - \frac{1}{(\sin x)^2}\right)$$

6. (A)
Time taken by particle to collide $t = \sqrt{\frac{2H}{g}}$
Then $u\sqrt{\frac{2H}{g}} + V\sqrt{\frac{2H}{g}} = d$
$$\Rightarrow u + V = d\sqrt{\frac{g}{2H}} \text{ or } v = d\sqrt{\frac{g}{2H}} - u$$

7. (C)

Let $a$ be the acceleration




at $t = 2n$

$$\upsilon_0 = 0 + a \times 2n = 2na \quad \ldots \text{(i)}$$

at $t = n$

$$\upsilon = an \quad \ldots \text{(ii)}$$

Between $t = n$ and $t = 2n$

$$\upsilon_0^2 - \upsilon^2 = 2as$$

$$\Rightarrow \quad 4n^2a^2 - \alpha^2 n^2 = 2as$$

$$\Rightarrow \quad s = \frac{3n^2 a}{2}$$

From eq. (i) $a = \dfrac{\upsilon_0}{(2n)}$

So, $s = \dfrac{3n^2}{2}\left(\dfrac{\upsilon_0}{2n}\right) = \dfrac{3\upsilon_0 n}{4}$

8. (A)

$$< \vec{\upsilon} > = \frac{\vec{s}}{t} = \frac{\Delta \vec{x}}{t}$$

(A) +ve disp., +ve velocity
(B) zero disp.
(C) slope is +ve, so velocity is +ve.
(D) slope is –ve, so velocity is –ve.

9. (D)

$$X = R + \frac{1}{2}\left(\frac{g}{2}\right)\left[\frac{2u\sin\theta}{g}\right]^2$$

$$= R + \frac{1}{4}\left[\frac{4u^2\sin^2\theta}{g}\right]$$

$$= R + \frac{u^2\sin^2\theta}{g}$$

$$= R + 2H$$

10. (D)

$u_x = 0,\ u_y = u$

$$a_x = g\sin 30° = \frac{g}{2}$$

and $a_y = -g\cos 30° = \frac{-(g\sqrt{3})}{2}$

$$0 = u_y T + \frac{1}{2}a_y T^2$$

$$T = \frac{4u}{g\sqrt{3}}$$

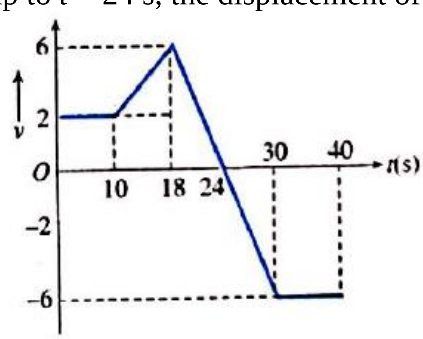
$\therefore$ $X = u_x T + \frac{1}{2}a_x T^2$

$$X = R = \frac{1}{2}g\sin\theta\left(\frac{4u}{g\sqrt{3}}\right)^2 = \frac{40}{3}\text{ m}$$

11. (AB)
The forces whose magnitude can from a triangle can be in equilibrium.
$\therefore$ Sum of two should be such that it should be equal or greater than the magnitude of biggest force.

12. (ABCD)
$C^2 = A^2 + B^2 + 2AB\cos\theta$
If $\theta = 90°$, then $C^2 = A^2 + B^2$
If $\theta > 90°$, then $C^2 = A^2 + B^2 + 2AB\cos\theta < A^2 + B^2$
$\therefore$ $\cos\theta$ will be negative
If $\theta < 90°$, then $C^2 = A^2 + B^2 + 2AB\cos\theta > A^2 + B^2$
$\therefore$ If $C = A - B \Rightarrow \theta = 180°$

13. (ACD)
Maximum value of position coordinate = Initial coordinate + Area under the graph up to $t$ = 24 s (as up to $t$ = 24 s, the displacement of the particle will be positive (figure).




Maximum value of position coordinate

$$= -16 + \left[(2\times10) + \left(\frac{2+6}{2}\right)\times(18-10) + \frac{1\times6}{2}\times(24-18)\right]$$

$$= -16 + [20 + 32 + 18] = 54\text{ m}$$

At time $t = 18\,\text{s}$

Position $= -16 + \text{Area of graph up to } t = 18\,\text{s}$

$$= -16 + [20 + 32] = 36\,\text{m}$$

At time $t = 30\,\text{s}$

Positive $= -16 + \text{Area of graph up to } t = 30\,\text{s}$

$$= -16 + \left[70 - \frac{1}{2} \times 6 \times 6\right] = 36\text{ m}$$

14. (AD)

Since the graph is a straight line, its slope is constant. It means acceleration of the particle is constant. Velocity of the particle changes from positive to negative at $t = 10\,\text{s}$, so particle change direction at this time.

The particle has zero displacement up to 20 s, but not for the entire motion.

The average speed in the interval of 0 to 10 s is the same as the average speed in the interval of 10 s to 20 s because distance covered in both time intervals is same.

15. (8)

$$t_1 = t_2 - t,\ v_1 - v_2 = v,\ S = \frac{1}{2} a_1 t_1^2,\ S = \frac{1}{2} a_2 t_2^2$$

$$v_1 = a_1 t_1,\ v_2 = a_2 t_2 \qquad \Rightarrow\ v_2 + v = a_1 t_1$$

$$\Rightarrow\ a_2 t_2 + v = a_1 t_1 = a_1 (t_2 - t) \Rightarrow t_2 = \frac{v + a_1 t}{a_1 - a_2}$$

$$\sqrt{\frac{a_2}{a_1}} = \frac{t_1}{t_2} = 1 - \frac{t}{t_2} \Rightarrow \sqrt{\frac{a_2}{a_1}} = 1 - \frac{t(a_1 - a_2)}{(v + a_1 t)}$$

$$\Rightarrow\ \frac{\sqrt{a_2}}{\sqrt{a_1}} = \frac{v + a_2 t}{v + a_1 t} \Rightarrow \sqrt{a_2}\,v + a_1\sqrt{a_2}\,t = v\sqrt{a_1} + a_2\sqrt{a_1}\,t$$

$$\Rightarrow\ v = \left(\sqrt{a_1 a_2}\right) t = 8\,\text{ms}^{-1}$$

16. (5)

For rat $S_R = \frac{1}{2}\beta t^2$ … (i)

For cat $S_C = ut + \frac{1}{2}\alpha t^2 = d + S_R$ … (ii)

$$\frac{\beta t^2}{2} + 5 = 5t + \frac{2.5}{2} t^2$$

$$t^2 (2.5 - \beta) + 10t - 10 = 0$$

$$t = \frac{-10 \pm \sqrt{100 + 40(2.5 - \beta)}}{2(2.5 - \beta)}$$

For real value of $t$

$100+40(2.5-\beta)\geq 0$

$200\geq 40\beta \Rightarrow \beta=5$

17. (5)

$s=u+\frac{a}{2}(2n-1)$

$u=100\text{ ms}^{-1}, a=-10\text{ ms}^{-2}$ and $s=5\text{ m}$

$5=100-5(2n-1)$ gives $n$ = 10 s

Body when thrown up with velocity 200 ms$^{-1}$ will take 20 s to reach the highest point.
Distance travelled in 20$^{th}$ second is

$200-5(20\times 2-1)=5\text{m}$

In the last second of upward journey, the bodies will travel same distance.

18. (5)

$v_x=u_x+a_xt$

$\Rightarrow \quad 0=u\cos 30°-g\sin 30°t$

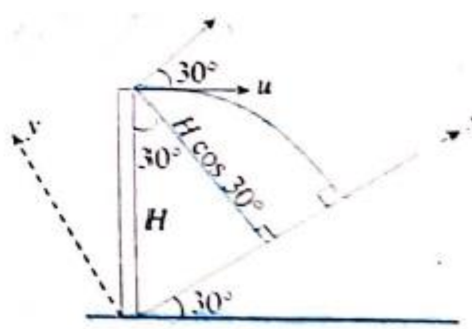
$\Rightarrow \quad t=\frac{u\sqrt{3}}{g}$

$S_y=u_yt+\frac{1}{2}a_yt^2$

$\Rightarrow \quad -H\cos 30°=-u\sin 30°t-\frac{1}{2}g\cos 30°t^2$

$\Rightarrow \quad -H\frac{\sqrt{3}}{2}=\frac{-u}{2}\frac{u\sqrt{3}}{g}-\frac{1}{2}g\frac{\sqrt{3}}{2}\frac{u^2 3}{g^2}$

$\Rightarrow \quad u=\sqrt{\frac{2gH}{5}}=\sqrt{2\times 10\times\frac{6.25}{5}}=5\text{ ms}^{-1}$




19. (2)

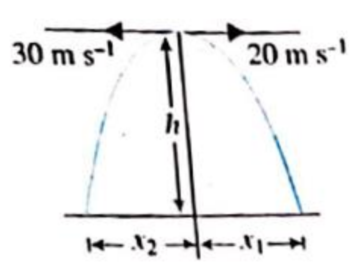
$80=\frac{1}{2}\times 10$

$t^2=\frac{80}{5} \Rightarrow t=4$

$x_1=20\times 4=80$

$x_2=30\times 4=120$

Separation $x_1+x_2=80+120=200\text{ m}$



## PART (B) : CHEMISTRY

### SOLUTION

20. (D)

$268\,g \longrightarrow 126\,g$ water

$\Rightarrow$ 142 g $Na_3SO_4 \longrightarrow 126$ g water

$\Rightarrow$ 1 mole $Na_2SO_4 \rightarrow \frac{126}{18}$ mole water

$\Rightarrow n = 7$

21. (B)

$P_i = -2y$

$P_f = -\frac{y}{2}$

$P_f - P_i = \frac{3y}{2}$

22. (B)

$$\frac{\frac{E_0}{4} - \frac{E_0}{9}}{\frac{E_0}{9} - \frac{E_0}{16}} = \frac{20}{7}$$

23. (C)

$m_2 + n_1 = 4 \quad \Rightarrow \quad n_2 = 3$

$n_2 - n_1 = 2 \quad \Rightarrow \quad n_1 = 1$

$$\frac{1}{\lambda} = 4 \times R \times \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\lambda = \frac{9}{32R}$$

24. (C)

Sample has $6 \times 10^{23}$ molecule

So can be divided in 6 students, $10^{23}$ each

25. (B)

$n_i = 0.01 \times 0.2 = 0.002$

(A) $m_f = \frac{0.002}{\frac{150}{1000}} = \frac{2}{150}$

(B) $n_{\text{added}} = \frac{0.18}{180} = 0.001$

$n_f = 0.003$

$m_f = \frac{0.003}{0.20} = 0.015$

26. (B)

27. (A)
Acidic nature increase as oxidation state increase.

28. (C)
V $\rightarrow$ 3 unpaired $e^-$
Cr $\rightarrow$ 6 unpaired $e^-$
Mn $\rightarrow$ 5 unpaired $e^-$
So, $x < z < y$

29. (C)
$N_2O$ is neutral

30. (AC)
$\ell \rightarrow 0$ to $n-1$
So, $3f$ & $5h$
Not possible.

31. (ABC)
(A) IE of N > IE of O > IE of O
(B) IE of $N^+$ < IE of $O^+$
(C) $IE_1$ of N < $IE_2$ of N
(D) IE of $O^+$ > IE of O

32. (ABCD)

| | $1s^2$ | $2s^2$ | $2p^6$ | $3s^2$ | $3p^6$ | $4s^1$ | $3d^5$ |
|---|---|---|---|---|---|---|---|
| $n$ | 1 | 2 | 2 | 3 | 3 | 4 | 3 |
| $\ell$ | 0 | 0 | 1 | 0 | 1 | 0 | 2 |
| $m_\ell = 0$ | $2e$ | $2e^-$ | $2e^-$ | $2e^-$ | $2e^-$ | $1e^-$ | $1e^-$ |
| $m_s = \frac{1}{2}$ | 1 | 1 | 3 | 1 | 3 | 0 or 1 | 0 or 5 |
| $= -\frac{1}{2}$ | 1 | 2 | 3 | 1 | 3 | 0 or 1 | Or 5 |

33. (AD)
$r_{\text{cation}} < r_{\text{atom}} < r_{\text{anion}}$

34. (2)

$$1g \longrightarrow \frac{1}{12x+6} \times (x+6) = \frac{4}{26}$$

$$\Rightarrow 26x + 156 = 48x + 24$$

$$22x = 132$$

$$\Rightarrow x = 6$$

$$n = \frac{156}{78} = 2$$

35. (7)

Endothermic

| | |
|---|---|
| $N \to N^-$ | $S \to S^+$ |
| $N \to N^+$ | $Be \to Be^-$ |
| $Ne \to Ne^+$ | $Se \to Se^{2-}$ |
| $P \to P^+$ | |

36. (7)

$$\sqrt{n(n+2)} = 3.873$$

$$\Rightarrow n = 3 \Rightarrow y = 3$$

$$Mn^{4+} [Ar] 4s^0\ 3d^3$$

$$\Rightarrow n = 4$$

37. (5)

Paramagnetic

$Co^{2+}, Fe^{2+}, Ni^{2+}, Cr^{3+}, Co^{3+}$

38. (4)

| | $1s^2$ | $2s^2$ | $2p^6$ | $3s^2$ | $3p^6$ | $4s^2$ | $3d^6$ |
|---|---|---|---|---|---|---|---|
| $n$ | 1 | 2 | 2 | 3 | 3 | 4 | 3 |
| $\ell$ | 0 | 0 | 1 | 0 | 1 | 0 | 2 |
| $m$ | 0 | 0 | $0 \to 2e^-$<br>$-1 \to 2e^-$<br>$+1 \to e^-$ | 0 | 0<br>–1<br>+1 | 0 | –2<br>–1<br>0<br>1<br>2 |
| | × | $2e^-$ | $2e^-$ | × | × | × | × |

## PART (C) : MATHEMATICS

### SOLUTION

39. (A)

$$\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8}-3\cos\frac{\pi}{8}\right]+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8}-4\sin^3\frac{\pi}{8}\right]$$

$$=4\cos^6\frac{\pi}{8}-4\sin^6\frac{\pi}{8}-3\cos^4\frac{\pi}{8}+3\sin^4\frac{\pi}{8}$$

$$=4\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]\left[\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]-3\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]$$

$$=\cos\frac{\pi}{4}\left[4\left(1-\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)-3\right]=\frac{1}{\sqrt{2}}\left[1-\frac{1}{2}\right]=\frac{1}{2\sqrt{2}}$$

40. (A)

$\sin\theta=1$ & $\operatorname{cosec}\theta=1$

41. (C)

$$S=\frac{-(2a+3)}{a+1}=-1 \quad\Rightarrow\quad 2a+3=a+1 \quad\Rightarrow\quad a=-2;\ p=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$$

42. (D)

$$A+B+C=\frac{3\pi}{2}\ \Rightarrow\ \cos 2A+\cos 2B+\cos 2C=2\cos(A+B)\cdot\cos(A-B)+1-2\sin^2 C$$

$$=2\cos\left(\frac{3\pi}{2}-C\right)\cdot\cos(A-B)-2\sin^2 C+1 \qquad \left(\because\ A+B+C=\frac{3\pi}{2}\right)$$

$$=-2\sin C\{\cos(A-B)+\sin C\}+1=-2\sin C\left\{\cos(A-B)+\sin\left(\frac{3\pi}{2}-(A+B)\right)\right\}+1$$

$$=-2\sin C\{\cos(A-B)-\cos(A+B)\}+1=1-4\sin A\sin B\sin C.$$

43. (B)

$$(a-1)\left(x^2+x+1\right)^2-(a+1)\left(x^4+x^2+1\right)=0 \qquad ...(1)$$

$\because\quad x^4+x^2+1=\left(x^2+x+1\right)\left(x^2-x+1\right)$

$\therefore\quad$ (1) becomes

$\Rightarrow\quad \left(x^2+x+1\right)\left[\left(x^2+x+1\right)(a-1)-(a+1)\left(x^2-x+1\right)\right]=0$

$\Rightarrow\quad \left(x^2+x+1\right)\left(x^2-ax+1\right)=0$

Here two roots are imaginary and for other two roots to be real $D>0$

$\Rightarrow\quad a^2-4>0$

$\Rightarrow\quad a\in(-\infty,-2)\cup(2,\infty)$

44. (C)

$a_{10} - 6a_9 - 2a_8 = 0$

$\frac{a_{10} - 2a_8}{2a_9} = 3$

45. (B)

Clearly $\alpha = 30°$ and $\theta \in (60°, 90°)$.

Hence $\theta + \alpha$ lies in $(90°, 120°)$.

46. (A)

$S_1$ : $x^2 - bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\because |\alpha - \beta| = 1 \Rightarrow (\alpha - \beta)^2 = 1 \Rightarrow b^2 - 4c = 1.$

$S_2$ : $\because \alpha + \beta = 1$ and $\alpha\beta = 3$

$\because \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = \left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2 = (1-6)^2 - 2(9) = 25 - 18 = 7$

$S_3$ : $\because \Sigma\alpha = 7 \Rightarrow \Sigma\alpha\beta = 16 \Rightarrow \alpha\beta\gamma = 12$

$\therefore \Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta) = 49 - 32$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = 17$

47. (A)

$3\cos x + 2\cos 3x = \cos y$ .....(i)

$3\sin x + 2\sin 3x = \sin y$ .....(ii)

$(i)^2 + (ii)^2$ gives

$9 + 4 + 12\cos x\cos 3x + 12\sin x\sin 3x = 1 \Rightarrow 13 + 12(\sin x\sin 3x + \cos x\cos 3x) = 1$

$\Rightarrow 13 + 12\cos 2x = 1 \Rightarrow 12\cos 2x = -12 \Rightarrow \cos 2x = -1$

48. (A)

$\alpha + \beta + \gamma = 0 \Rightarrow \frac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \frac{3\alpha\beta\gamma}{-2(\alpha\gamma + \beta\gamma + \gamma\alpha)} = \frac{3b}{2a}$

49. (BC)

$ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\Rightarrow \alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a} \Rightarrow Ax^2 + Bx + C = 0 \begin{cases} \alpha + \delta \\ \beta + \delta \end{cases}$

$(\alpha + \delta) + (\beta + \delta) = -\frac{B}{A},\ (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$

$$\therefore\ 2\delta=\left(\frac{b}{a}-\frac{B}{A}\right)$$

$$\therefore\ \left|(\alpha+\delta)-(\beta+\delta)\right|=\left|(\alpha-\beta)\right|$$

$$\Rightarrow \sqrt{\frac{B^2}{A^2}-\frac{4C}{A}}=\sqrt{\frac{b^2}{a^2}-\frac{4c}{a}} \Rightarrow \frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$$

50. (AC)

Let $y=2\sin t \ \Rightarrow\ y=\dfrac{1-2x+5x^2}{3x^2-2x-1}$

$$(3y-5)x^2-2x(y-1)-(y+1)=0$$

$$x\in R-\left\{1,-\frac{1}{3}\right\}$$

$$\therefore\ D\ge 0 \ \Rightarrow\ y^2-y-1\ge 0$$

$$\therefore\ y\ge\frac{1+\sqrt{5}}{2} \text{ or } y\le\frac{1-\sqrt{5}}{2} \Rightarrow \sin t\ge\frac{1+\sqrt{5}}{4} \text{ or } \sin t\le\frac{1-\sqrt{5}}{4}$$

$$\therefore \text{ range of } t \text{ is } \left[-\frac{\pi}{2},-\frac{\pi}{10}\right]\cup\left[\frac{3\pi}{10},\frac{\pi}{2}\right]$$

51. (ABCD)

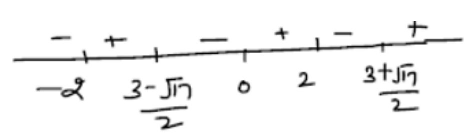
$$\frac{1+\tan 11^\circ}{1-\tan 11^\circ}=\tan(45^\circ+11^\circ)=\tan 56^\circ$$

52. (ACD)

$$\frac{x(x+2)-(x-2)(x+2)-2x(x-2)}{(x-2)(x)(x+2)}\le 0$$

$$\frac{-2x^2+6x+4}{(x-2)(x)(x+2)}\le 0 \Rightarrow \frac{x^2-3x-2}{(x-2)(x)(x+2)}\ge 0$$

$$\frac{\left(x-\frac{3-\sqrt{17}}{2}\right)\left(x+\frac{3-\sqrt{17}}{2}\right)}{(x-2)(x)(x+2)}\ge 0$$




$$\therefore\ x\in\left(-2,\frac{3-\sqrt{17}}{2}\right]\cup(0,2)\cup\left[\frac{3+\sqrt{17}}{2},\infty\right)$$

$$\therefore\ \alpha=2,\ \beta=\frac{3-\sqrt{17}}{2},\ \gamma=0,\ \delta=\frac{3+\sqrt{17}}{2}$$

53. (2)

$$\frac{\cos 20° + 8\sin 70° \sin 50° \sin 10°}{\sin^2 80°} = \frac{\cos 20° + 4\{(\cos 60° - \cos 80°)\sin 50°\}}{\sin^2 80°}$$

$$= \frac{\cos 20° + 2(1 - 2\cos 80°)\sin 50°}{\sin^2 80°} = \frac{\cos 20° + 2\sin 50° - 2(\sin 130° - \sin 30°)}{\sin^2 80°}$$

$$= \frac{\cos 20° + 2\sin 50° - 2\sin(180° - 50°) + 2\sin 30°}{\sin^2 80°} = \frac{1 + \cos 20°}{1 + \cos 20°} \times 2$$

$\because \quad \sin^2(80°) = \cos^2 10° = 2$

54. (4)

$12\sin\theta - 9\sin^2\theta = 4 - (3\sin\theta - 2)^2$ whose maximum value is 4

55. (2)

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca) = 4$

$\therefore \; ab + bc + ca = \frac{3}{2}$

$\therefore \; abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{3}{2}$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$

56. (5)

$x^2 + 1 < x^2 + 4x + 4$ and $x^2 + 4x + 4 < 2x^2 + 4x - 12$

$4x > -3$ and $x^2 > 16$

$x \in (-\infty, -4) \cup (4, \infty)$

$\therefore \; x \in (4, \infty)$

$\therefore$ least integral value of $x$ is 5.

57. (1)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6 = (a-1)^2 + 5$

This is minimum when $a = 1$