PART (A) : PHYSICS

Solutions

1. (B)

$\vec{A} = \hat{i} + \hat{j} \Rightarrow |A| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos\alpha = \frac{A_x}{|A|} = \frac{1}{\sqrt{2}} = \cos 45°$

$\therefore \quad \alpha = 45°$

2. (B)

$(\hat{i} - 2\hat{j} + 2\hat{k}) + (2\hat{i} + \hat{j} - \hat{k}) + \vec{R} = \hat{i}$

$\therefore$ Required vector $\vec{R} = -2\hat{i} + \hat{j} - \hat{k}$

3. (D)

$A = 3\,\text{N}$, $B = 2\text{N}$ then $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R = \sqrt{9 + 4 + 12\cos\theta}$ ... (i)

Now, $A = 6\text{N}$, $B = 2\text{N}$ then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ ... (ii)

From (i) and (ii) we get

$\cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = 120°$

4. (B)

$\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

$|\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$

$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$

5. (D)

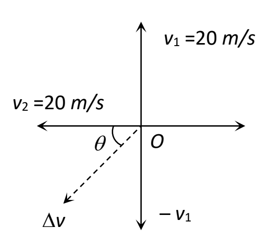
From figure

$\vec{v}_1 = 20\hat{j}$ and $\vec{v}_2 = -20\hat{i}$

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = -20(\hat{i} + \hat{j})$

$|\Delta\vec{v}| = 20\sqrt{2}$ and direction

$\theta = \tan^{-1}(1) = 45°$ *i.e.* S–W



6. (A)

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$; $\vec{B} = \hat{i} + \hat{j}$

$\therefore\ A=\sqrt{1^2+1^2+1^2}=\sqrt{3},\ B=\sqrt{1^2+1^2}=\sqrt{2}$

$\vec{A}.\vec{B}=2$

$\cos\theta=\frac{\vec{A}.\vec{B}}{AB}=\frac{2}{\sqrt{3}\sqrt{2}}=\left(\sqrt{\frac{2}{3}}\right)$

$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{2}{3}}=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$

$\therefore\quad \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

7. (D)

From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $\left(\vec{A}+\vec{B}\right)$ vector also, as it must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $\left(\vec{A}\times\vec{B}\right)$ gives a vector $\vec{C}$ which can not be perpendicular to itself. Thus the last statement is wrong.

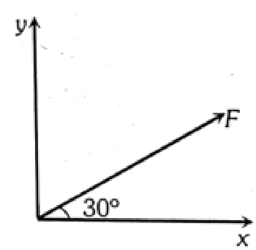
8. (C)

The X component of force F is

$F_x=F\cos 30°=F\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}F$

The Y component of force F is

$F_y=F\sin 30°=F\times\frac{1}{2}=\frac{1}{2}F$.



9. (C)

$\int \tan x\,dx-\int \sec^2 x\,dx-\int \text{cosec}^2 x\,dx$

$=\int\frac{\sin x}{\cos x}dx-\int \sec^2 x\,dx-\int \text{cosec}^2 x\,dx$

$=-\ln\cos x-\tan x+\cot x+C$

$=\ln|\sec x|-\tan x+\cot x+C$

10. (A)

$y=\ln(x)^{3/4}=\frac{3}{4}\ln x$

$\Rightarrow\ \frac{dy}{dx}=\left(\frac{3}{4}\right)\left(\frac{1}{x}\right)$

$\left.\frac{dy}{dx}\right|_{x=\frac{4}{3}}=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$

11. (B)

$$\frac{1}{2}g(3)^2=\frac{g}{2}(2n-1) \Rightarrow n=5\,\text{s}$$

12. (A)

Time taken by first stone to reach the water surface from the bridge be *t*, then

$$h=ut+\frac{1}{2}gt^2 \Rightarrow 44.1=0\times t+\frac{1}{2}\times 9.8t^2$$

$$t=\sqrt{\frac{2\times 44.1}{9.8}}=3\,\text{sec}$$

Second stone is thrown 1 sec later and both strikes simultaneously. This means that the time left for second stone

$=3-1=2\,\text{sec}$

Hence, $44.1=u\times 2+\frac{1}{2}9.8(2)^2$

$$\Rightarrow \quad 44.1-19.6=2u \Rightarrow u=12.25\,\text{m/s}$$

13. (C)

Total distance $=\frac{1}{2}gt^2=\frac{25}{2}g$

Distance moved in 3 sec $=\frac{9}{2}g$

Remaining distance $=\frac{16}{2}g$

It *t* is the time taken by the stone to reach the ground for the remaining distance then

$$\Rightarrow \quad \frac{16}{2}g=\frac{1}{2}gt^2 \Rightarrow t=4\,\text{sec}$$

14. (A)

Height travelled by ball (with balloon) in 2 sec

$$h_1=\frac{1}{2}at^2=\frac{1}{2}\times 4.9\times 2^2=9.8\,\text{m}$$

Velocity of the balloon after 2 sec

$v=at=4.9\times 2=9.8\,m/s$

Now if the ball is released from the balloon it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$$v^2=u^2-2gh_2 \Rightarrow 0=(9.8)^2-2\times(9.8)\times h_2$$

$\therefore \quad h_2=4.9\,m$

Greatest height above the ground reached by the ball

$=h_1+h_2=9.8+4.9=14.7\,m$

15. (B)

Let h distance is covered in n sec

$$\Rightarrow \quad h = \frac{1}{2} g n^2 \qquad \ldots \text{(i)}$$

Distance covered in n^{th} sec $= \frac{1}{2} g(2n-1)$

$$\Rightarrow \quad \frac{9h}{25} = \frac{g}{2}(2n-1) \qquad \ldots \text{(ii)}$$

From (i) and (ii),

$h = 122.5$ m

16. (D)

In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

17. (B)

$$\frac{(S)_{(\text{last } 2s)}}{(S)_{7s}} = \frac{\frac{1}{2} \times 2 \times 10}{\frac{1}{2} \times 2 \times 10 + 2 \times 10 + \frac{1}{2} \times 2 \times 10} = \frac{1}{4}$$

18. (B)

$$H = \frac{u^2 \sin^2 \theta}{2g} \text{ and } T = \frac{2u \sin \theta}{g} \Rightarrow T^2 = \frac{4u^2 \sin^2 \theta}{g^2}$$

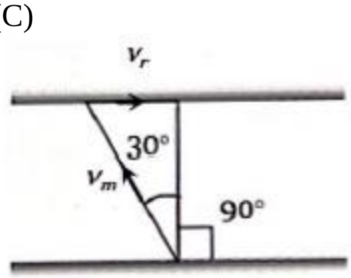
$$\therefore \quad \frac{T^2}{H} = \frac{8}{g} \Rightarrow T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$$

19. (B)

$$R_{\max} = \frac{u^2}{g} = 400\, m \qquad (\text{for } \theta = 45^\circ)$$

$$H_{\max} = \frac{u^2}{2g} = \frac{400}{2} = 200\, m \qquad (\text{for } \theta = 90^\circ)$$

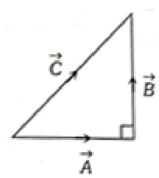
20. (C)



$\sin 30° = \frac{v_r}{v_m} = \frac{1}{2} \Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25 \text{ m/s}$

21. (1)
$\vec{A} + \vec{B} = 5\hat{i} - \hat{j} + \hat{k}$
The unit vector of $\vec{A} + \vec{B} = \frac{5\hat{i} - \hat{j} + \hat{k}}{\sqrt{25+1+1}} = \frac{5\hat{i} - \hat{j} + \hat{k}}{\sqrt{27}}$
Hence, $a = 1$

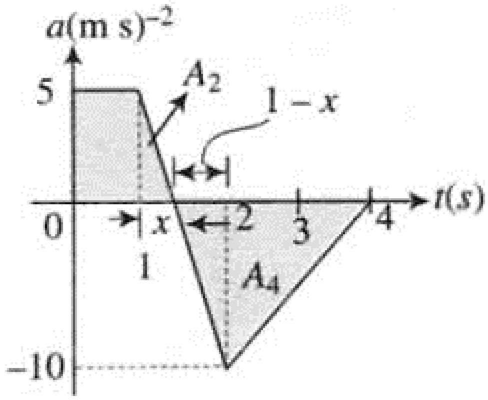
22. (5)
$C = \sqrt{A^2 + B^2}$
$= \sqrt{3^2 + 4^2} = 5$
$\therefore$ Angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{2}$.



23. (2)
Here, $\vec{F}_1 = 3\hat{k}$
$\vec{F}_2 = 5\cos 37° \hat{j} + 5\sin 37° \hat{i} = 4\hat{j} + 3\hat{i}$
and $\vec{F}_3 = -4\sqrt{2}\cos 45° \hat{i} - 4\sqrt{2}\sin 45° \hat{j}$
$= -4\hat{i} - 4\hat{j}$
For equilibrium of the particle
$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$
or $3\hat{k} + 4\hat{j} + 3\hat{i} - 4\hat{i} - 4\hat{j} + \vec{F}_4 = 0 \Rightarrow \vec{F}_r = \hat{i} - 3\hat{k}$
$\therefore$ $\vec{F}_4 = \sqrt{1+9} = \sqrt{10}$
$\therefore$ $n = 2$

24. (1)
If a and b are perpendicular, $a \cdot b = 0$
or $2x - 3 + 1 = 0$
$\therefore$ $x = 1$

25. (3)
The velocity of the particle at $t = 4$ s can be given as
$v_4 = v_0 + \Delta v$ … (i)



Where $\Delta v \equiv A$ (= area under *a*-*t* graph during first 4 s)

Referring to *a*-*t* graph (shown in figure), we have

$$A = A_1 + A_2 - A_3 - A_4 \qquad \ldots \text{(ii)}$$

Where $A_1 = 5\times1 = 5$, $A_2 = \frac{1}{2}\times x\times 5$

$$A_3 = \frac{1}{2}\times(1-x)\times 10 \text{ and } A_4 = \frac{1}{2}\times 2\times 10 = 10$$

We can find he value of *x* as follows:

Using properties of similar triangles, we have $\frac{x}{5} = \frac{1-x}{10}$.

This yields $x = \frac{1}{3}$.

Substituting $x = \frac{1}{3}$ in $A_2 = \frac{1}{2}\times x\times 5$ and $A_3 = \frac{1}{2}(1-x)\times 10$,

We have $A_2 = \frac{5}{6}$ and $A_3 = \frac{10}{3}$.

Then substituting A_1, A_2, A_3 are A_4 in Eq. (ii), we have

$$A = -7.5 \Rightarrow \Delta v = -7.5 \text{ and } \vec{v}_0 = 10.5\ \text{ms}^{-1}.$$

Hence, we have $v_4 = v_0 + \Delta v = 10.5 - 7.5 = 3\ \text{ms}^{-1}$

26. (2)

Taking upward direction as positive, let us work in the frame of lift. Acceleration of ball relative to lift = (*g* + *a*), initial velocity: $u_{rel} = v$, final velocity: $v_{rel} = -v$ as the ball will reach the man with same speed w.r.t. lift

Apply $v_{rel} = u_{rel} + a_{rel}t \Rightarrow -v = v + (-g-a)t \Rightarrow t = 2\text{s}$

27. (1)

$V_p = 90\ \text{km h}^{-1} = 25\ \text{m s}^{-1}$

$V_c = 72\ \text{km h}^{-1} = 20\ \text{m s}^{-1}$

In 10 s culprit reaches point *B* from *A*. Distance covered by culprit,

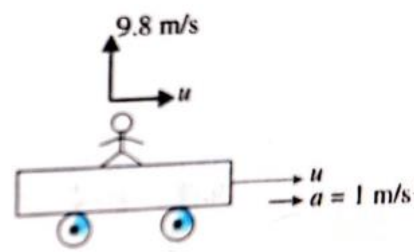
$S = vt = 20\times 10 = 200\ \text{m}$.

At time $t = 10\,\text{s}$, the police jeep is 200 m behind the culprit.
Relative velocity between jeep and culprit is 25 – 20 = 5 ms^{-1}.

Time $= \frac{S}{V} = \frac{200}{5} = 40\,\text{s}$ (Relative velocity is considered)

In 40 s, the police jeep will move from A to a distance S. Where
$S = vt = 25 \times 40 = 1000\,\text{m} = 1.0\,\text{km}$ away.
The jeep will catch up with the bike 1 km far from the turning.

28. (2)
Let the velocity of car be u when the ball is thrown.
Initial velocity of car = Horizontal velocity of ball.
Distance travelled by ball B,
$S_b = ut$ (in horizontal direction)
Car has travelled extra distance



$$= S_c - S_b = \frac{1}{2}at^2$$

Ball can be considered a projectile having $\theta = 90°$

$$t = \frac{2u\sin\theta}{g} = \frac{2 \times 9.8}{9.8} = 2\,\text{s}$$

$$S_c - S_b = \frac{1}{2}at^2 = \frac{1}{2}(1)2^2 = 2\,\text{m}$$

Hence, the ball will drop 2 m behind the boy.

29. (5)

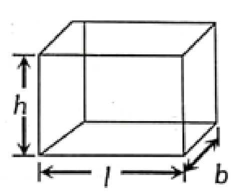
$$v_x = \frac{dx}{dt} = 3 \text{ and } v_y = \frac{dy}{dt} = 4 - 10t = 4 - 10(0) = 4$$

$$v = \left[v_x^2 + v_y^2\right]^{1/2} = \left[3^2 + 4^2\right]^{1/2} = 5\,\text{ms}^{-1}$$

30. (7)

Diagonal of the hall $= \sqrt{l^2 + b^2 + h^2}$

$$= \sqrt{10^2 + 12^2 + 14^2}$$
$$= \sqrt{100 + 144 + 196}$$
$$= \sqrt{440} = 20.9 \approx 21m.$$



PART (B) : CHEMISTRY

Solutions

31. (B)
Let % abundance of $B^{10} = x$
So % abundance of $B^{11} = (100 - x)$
$$10.10 = \frac{10 \times x + 11 \times (100 - x)}{100}$$
$$1010 = 10x + 1100 - 11x$$
$$x = 90$$

32. (A)
$$0.2 = \frac{n_{\text{glucose}}}{n_{\text{glucose}} + n_{H_2O}}$$
Let $n_{\text{glucose}} = x$
$$\Rightarrow \quad n_{H_2O} = 4x$$
$$\text{Molality} = \frac{n_{\text{glucose}}}{W_{H_2O}(kg)} = \frac{x}{4x \times \frac{18}{1000}} = \frac{250}{18} = 13.8$$

33. (D)
For minimum MW, number of oxygen atoms per molecule = 1
$$\text{So } 3.2 = \frac{16 \times 1}{MW_{\min}} \times 100$$
$$\Rightarrow MW_{\min} = \frac{1600}{3.2} = 500$$

34. (D)
(A) Hund's rule violated
(B) Aufbau's principle violated
(C) Pauli's rule violated
(D) Aufbau's principle & Hund's rule violated

35. (A)
$$v = \frac{Z}{n} = v_{1,\,H}$$
$$v = \frac{2}{3} \times v_{1,\,H}$$
$$\Rightarrow V_{1,\,H} = \frac{3}{2} v$$

$$V_{2,\,Li^{2+}} = \frac{3}{2} v_{1,\,H}$$
$$= \frac{3}{2} \times \frac{3}{2} v$$
$$= \frac{9}{4} v$$

36. (B)

$x + 1 + 2 \times 2 = y \times 1.5$

$\Rightarrow \; x + 4 = 1.5y$

Since final molarity is average of both molarities, the volume of both solution has to be equal

$\Rightarrow \; x = 2$

$\Rightarrow \; y = 4$

37. (D)

(A) $Mg^{2+} \rightarrow 1s^2\, 2s^2\, 2p^6$

Unpaired electron = 0

(B) $Ti^{3+} \rightarrow 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^1$

Unpaired electron = 1

(C) $V^{3+} \rightarrow 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^2$

Unpaired electron = 2

(D) $Fe^{2+} \rightarrow 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^6$

Unpaired electron = 4

38. (A)

IE increase along a period left to right Mg has unusually high IE

Na < Mg > Al < S.

39. (D)

For $n = 3$

$\Rightarrow \; \ell = 0, 1, 2$

$\Rightarrow \; 0, \pm 1, \pm 2$

So, (D) nor possible.

40. (B)

$n_{Cl^-} = 2 \times 1 \times 2 + 2 \times 1 \times 1 + 1 \times 1 \times 3$

$= 4 + 2 + 3 = 9$

$[Cl^-] = \frac{9}{5} = 1.8\text{ M}$

41. (A)

EA decrease down the group but F has abnormally low value

F < Cl > Br > I

42. (B)

$r_2 = x \Rightarrow r_1 = \frac{x}{4}$

$\Rightarrow x_4 = 4x$

$2\pi \times 4x = 4\lambda \Rightarrow \lambda = 2\pi x$

43. (D)

$Fe^{2+} \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6$

No. of electrons = 6

$p \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$

No. of *s* electrons = 6

44. (B)

$IE_5 >> IE_4$

$\Rightarrow$ group 14

$\Rightarrow$ Si

45. (C)

Empirical formula $C_{\frac{9}{12}}\ H_{\frac{1}{1}}\ N_{\frac{3.5}{15}}$

$\Rightarrow C_2H_4N$

Empirical mass = 36 + 4 + 14 = 54

MW = 108

$\Rightarrow$ Molecular formula $C_6H_8N_2$

46. (D)

$n_{OH^-} = V \times \frac{1}{10} \times 2$

$n_{H^+} = 0.5 \times 2 \times 2$

$V \times \frac{1}{10} \times 2 = 0.5 \times 2 \times 2$

$V = 10\ L$

47. (C)

$m_\ell = +4$

$\Rightarrow \ell = 4$

$\Rightarrow n = 5$

$\Rightarrow$ No. of waves = 5

48. (B)

$\frac{1}{\lambda} = R \times 4 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda = \frac{1}{3R}$

49. (B)

$h \times 2x - h\nu_0 = y$

$h \times 3x - h\nu_0 = 4y$

$3hx - h\nu_0 = 8hx - 4h\nu_0$

$3h\nu_0 = 5hx$

$\nu_0 = \frac{5}{3}x$

50. (D)

O^- resist addition of electron due to electron-electron repulsion.

51. (5)

No. of radial nodes = $n - \ell - 1$

$= 6 - 0 - 1$

$= 5$

52. (4)

$28.9 = \frac{14 \times x}{194} \times 100$

$x = 4$

53. (3)

d_{xy}, d_{yz}, d_{xz} are between axes, not along axes.

54. (0)

Since mass of both samples is same, no. of (protons + neutrons) will be same.

55. (1)

$n_{Cl^-} = 0.5 \times 1 \times 1 + 0.25 \times 1 \times 2$

$= 1$

56. (1)

Let wt. of solution = x g

$\Rightarrow$ wt. of solute = $0.2x$ g

$V_{solution} = \frac{x}{1.25}$ ml

$= \frac{x}{1250}$ L

$n_{solute} = \frac{0.2x}{250}$

$$\text{Molarity} = \frac{\frac{0.2x}{250}}{\frac{x}{1250}} = \frac{0.2 \times 1250}{250} = \frac{250}{250} = 1$$

57. (2)

$$1s^2 \quad 2s^2 \quad 2p^6 \quad 3s^2 \quad 3p^6 \quad 4s^1 \quad 3d^5$$

$$m = 0 \quad m = 0 \quad n = 2 \quad m = 0 \quad 1e^- \quad m = 0 \quad 1e^-$$

58. (4)
He, Ne, N, Be have positive electron gain enthalpy.

59. (0)
IE of cation's always higher than atom.

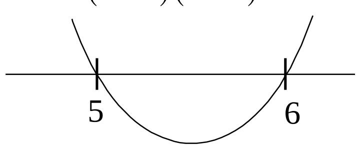
60. (6)
Atomic no. = 114
1 + 1 + 4 = 6

PART (C) : MATHEMATICS

Solutions

61. (B)
The true relation is $\sin 1 > \sin 1^\circ$
Since value of $\sin\theta$ is increasing $\left[0 \to \frac{\pi}{2}\right]$.

62. (C)
Since $(x-5)(x-6) \le 0$



$x \in [5, 6]$

63. (D)
We have,
$\sin\theta + \text{cosec}\,\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$
$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$
$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$
Required value of $\sin^{10}\theta + \text{cosec}^{10}\theta = (1)^{10} + \frac{1}{(1)^{10}} = 2$.

64. (C)
$\sin\theta = \frac{24}{25} \Rightarrow \cos\theta = \frac{-7}{25}, \tan\theta = \frac{-24}{7}$
$\therefore \sec\theta + \tan\theta = \frac{-25}{7} + \frac{-24}{7} = -7$

65. (C)
$\text{cosec}\,A + \cot A = \frac{11}{2} \Rightarrow \text{cosec}\,A - \cot A = \frac{2}{11}$
Therefore $2\cot A = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$.

66. (C)
$5\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{5}$
$\therefore \sin\theta = \frac{4}{\sqrt{41}}$ and $\cos\theta = \frac{5}{\sqrt{41}}$

$$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{5\times\frac{4}{\sqrt{41}}-3\times\frac{5}{\sqrt{41}}}{5\times\frac{4}{\sqrt{41}}+2\times\frac{5}{\sqrt{41}}}$$

$$\frac{20-15}{20+10}=\frac{5}{30}=\frac{1}{6}.$$

67. (A)

$\sin\theta+\cos\theta=1$

Squaring on both sides, we get

$\Rightarrow\ \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=1$

$\therefore\ \sin\theta\cos\theta=0$.

68. (A)

$\alpha+\beta=4$

$\alpha\cdot\beta=10$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{10}=\frac{2}{5}$

Option (A).

69. (D)

$3\tan A+4=0\Rightarrow\tan A=-\frac{4}{3}$

$\Rightarrow\ \sin A=\pm\frac{\tan A}{\sqrt{1+\tan^2 A}}=\pm\frac{-4/3}{\sqrt{1+16/9}}=\frac{4}{5}$ $(\because A$ is in 2^{nd} quadrant)

and $\cos A=-\frac{3}{5}$. Thus, $2\cot A-5\cos A+\sin A$

$=2\left(-\frac{3}{4}\right)-5\left(-\frac{3}{5}\right)+\frac{4}{5}=\frac{23}{10}$.

70. (A)

Since $\cos(90°)=0$

So, product is zero.

71. (A)

$x=\sqrt{1+\sqrt{1+\sqrt{1+.....}}}$ to ∞

We have $x=\sqrt{1+x}$

$\Rightarrow\ x^2=1+x\ \Rightarrow x^2-x-1=0$

$\Rightarrow\ x=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt{5}}{2}$

As $x>0$, we get $x=\frac{1+\sqrt{5}}{2}$

72. (C)
Given equations are $2x^2+3x+5\lambda=0$
and $x^2+2x+3\lambda=0$ have a common root if
$\frac{x^2}{-\lambda}=\frac{x}{-\lambda}=\frac{1}{1} \Rightarrow x^2=-\lambda, x=-\lambda$ or $\lambda=-1, 0$.

73. (B)
We know that range of $a\sin\theta+b\cos\theta$ is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$
So, required range is [–5 + 6, 5 + 6] or [1, 11]
Option (B).

74. (A)
From $k=\frac{x^2-x+1}{x^2+x+1}$
We have $x^2(k-1)+x(k+1)+k-1=0$
As given, x is real $\Rightarrow$ $(k+1)^2-4(k-1)^2\geq 0$
$\Rightarrow$ $3k^2-10k+3\leq 0$
Which is possible only when the value of k lies between the roots of the equation $3k^2-10k+3=0$
That is, when $\frac{1}{3}\leq k\leq 3$ {Since roots are $\frac{1}{3}$ and 3}

75. (A)
Given equation can be rewritten as
$$3x^2-(a+c+2b+2d)x+(ac+2bd)=0$$
Its discriminant D
$$=(a+c+2b+2d)^2-4.3(ac+2bd)$$
$$=\{(a+2d)+(c+2b)\}^2-12(ac+2bd)$$
$$=\{(a+2d)-(c+2b)\}^2+4(a+2d)(c+2b)-12(ac+2bd)$$
$$=\{(a+2d)-(c+2b)\}^2-8ac+8ab+8dc-8bd$$
$$=\{(a+2d)-(c+2b)\}^2+8(c-b)(d-a)$$
which is +*ve*, since $a<b<c<d$.
Hence roots are real and distinct.

76. (A)
The given equations are
$qx^2+px+q=0$ .....(i)

and $x^2 - 4qx + p^2 = 0$ .....(ii)

Roots of (i) are complex, therefore $p^2 - 4q^2 < 0$

Now discriminant of (ii) is $16q^2 - 4p^2 = -4(p^2 - 4q^2) > 0$

Hence, roots are real and unequal.

77. (D)

We know that the expression $ax^2 + bx + c > 0$ for all x, if $a > 0$ and $b^2 < 4ac$

$\therefore$ $(a^2 - 1)x^2 + 2(a-1)x + 2$ is positive for all x if

$a^2 - 1 > 0$ and $4(a-1)^2 - 8(a^2 - 1) < 0$

$\Rightarrow$ $a^2 - 1 > 0$ and $-4(a-1)(a+3) < 0$

$\Rightarrow$ $a^2 - 1 > 0$ and $(a-1)(a+3) > 0$

$\Rightarrow$ $a^2 > 1$ and $a < -3$ or $a > 1$

$\Rightarrow$ $a < -3$ or $a > 1$

78. (A)

Given equation can be written as

$(m+1)x^2 - \{m(a+b) - (a-b)\}x + c(m-1) = 0$.

Roots are equal and of opposite sign. So sum of roots is equal to zero.

$\Rightarrow$ $0 = m(a+b) - (a-b)$

$\Rightarrow$ $m = \dfrac{a-b}{a+b}$.

79. (B)

Let the equation (in correctly written form) be $x^2 + 17x + q = 0$.

Roots are – 2, – 15. So 30 = q, so correct equation is $x^2 + 13x + 30 = 0$.

Hence roots are – 3, – 10.

80. (B)

Let the roots are α and $n\alpha$

Sum of roots, $\alpha + n\alpha = -\dfrac{b}{a} \Rightarrow \alpha = -\dfrac{b}{a(n+1)} \Rightarrow$ .....(i)

and product, $\alpha . n\alpha = \dfrac{c}{a} \Rightarrow \alpha^2 = \dfrac{c}{na}$ ....(ii)

From (i) and (ii), we get

$\Rightarrow$ $\left[-\dfrac{b}{a(n+1)}\right]^2 = \dfrac{c}{na} \Rightarrow \dfrac{b^2}{a^2(n+1)^2} = \dfrac{c}{na}$

$\Rightarrow$ $nb^2 = ac(n+1)^2$.

81. (1)

$\tan 1^\circ \tan 2^\circ .... \tan 89^\circ$

$=(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)......=1\times 1\times 1....=1.$

82. (8)

Since $(x-2)$ is a factor so $f(2)=0$

$4-12+K=0$
$K=12-4=8$

83. (2)

Max value of the expression is 2

84. (4)

3/4 = 15/r so r = 20

$\frac{r}{5}=4$

85. (4)

Let the roots are α, β of $x^2-bx+c=0$ and α', β' be roots of $x^2-cx+b=0$

Now $\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{b^2-4c}$ .....(i)

and $\alpha'-\beta'=\sqrt{(\alpha'+\beta')^2-4\alpha'\beta'}=\sqrt{c^2-4b}$ .....(ii)

But $\alpha-\beta=\alpha'-\beta'$

$\Rightarrow \sqrt{b^2-4c}=\sqrt{c^2-4b} \Rightarrow b^2-4c=c^2-4b$

$\Rightarrow b^2-c^2=4c-4b$

$\Rightarrow (b+c)(b-c)=4(c-b)$

$\Rightarrow b+c=-4$

86. (6)

We have $\alpha+\beta=-\frac{7}{2}$ and $\alpha\beta=\frac{c}{2}$

$\therefore \left|\alpha^2-\beta^2\right|=\frac{7}{4} \Rightarrow \alpha^2-\beta^2=\pm\frac{7}{4}$

$\Rightarrow (\alpha+\beta)(\alpha-\beta)=\pm\frac{7}{4} \Rightarrow -\frac{7}{2}\sqrt{\frac{49}{4}-2c}=\pm\frac{7}{4}$

$\Rightarrow \sqrt{49-8c}=\mp 1 \Rightarrow 49-8c=1 \Rightarrow c=6$

87. (4)

$a_n=19^n-12^n$

$$\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31\left(19^9-12^9\right)-\left(19^{10}-12^{10}\right)}{57\alpha_8}$$

$$= \frac{19^9(31-19)-12^9(31-12)}{57\alpha_8}$$

$$= \frac{19^9 \cdot 12 - 12^{19} \cdot 19}{57\alpha_8}$$

$$= \frac{12 \cdot 19\left(19^8 - 12^8\right)}{57\alpha_8} = 4$$

88. (4)

Consider $\sin 80° = \sin(90° - 10°) = \cos(10°)$

So, $\sin^2(10°) + \sin^2 80° = \sin^2(10°) + \cos^2(10°) = 1$

Similarly,

$\sin^2(20°) + \sin^2(70°) = 1$

$\sin^2(30°) + \sin^2(60°) = 1$

$\sin^2(40°) + \sin^2(50°) = 1$

So, required value is 4.

89. (2)

90. (1)

We will substitute the zero of $q(x)$ into the polynomial $p(x)$ to find the remainder r.

The zero of $q(x)$ is $x = 1$.

So by remainder theorem, $r = p(1)$

$= 3(1)^5 - (1)^4 + (1)^3 - 4(1)^2 + 2$

$= 3 - 1 + 1 - 4 + 2$

$= 1$