

PART (A) : PHYSICS

ANSWER KEY

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (C) | 2. (A) | 3. (C) | 4. (B) | 5. (A) |
| 6. (B) | 7. (C) | 8. (D) | 9. (B) | 10. (A) |
| 11. (C) | 12. (D) | 13. (D) | 14. (B) | 15. (B) |
| 16. (C) | 17. (B) | 18. (D) | 19. (B) | 20. (C) |
| 21. (5) | 22. (3) | 23. (4) | 24. (4) | 25. (1) |
| 26. (5) | 27. (4) | 28. (1) | 29. (7) | 30. (2) |

SOLUTIONS

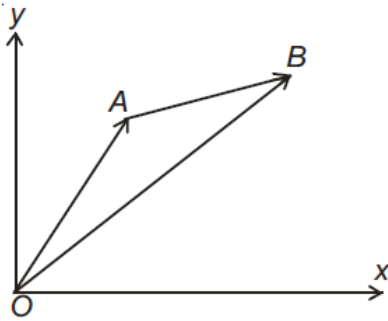
1. (C)

$$\text{Acceleration} = \frac{v^2}{r} \text{ towards centre}$$

$$\therefore |\vec{a}| = \frac{4^2}{2} = \frac{16}{2} = 8 \text{ towards centre}$$

$$\text{i.e., } \vec{a} = 8(-\hat{j})$$

2. (A)



$$m_A = 2\text{kg}$$

$$m_B = 5\text{kg}$$

$$\vec{OA} = 2\hat{i} + 3\hat{j}, \vec{AB} = 5\hat{i} + 8\hat{j}$$

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= 7\hat{i} + 11\hat{j}$$

$$\vec{r}_{cm} = \frac{m_A \vec{r}_{OA} + m_B \vec{r}_{OB}}{m_A + m_B} = \frac{39\hat{i} + 61\hat{j}}{7}$$

$$\therefore \text{co-ordinate of centre of mass} = \left(\frac{39}{7}, \frac{61}{7} \right)$$

3. (C)

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m \cdot (0) + m \left(\frac{\ell}{2}\right)}{2m} = \frac{\ell}{4}$$

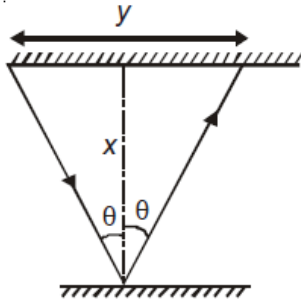
$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \cdot \left(\frac{\ell}{2}\right) + m(\ell)}{2m} = \frac{3\ell}{4}$$

4. (B)

$$V_{\text{cm}} = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} = \frac{2 \times 25 - 2 \times 5}{2 + 2} = 10 \text{ m/s}$$

5. (A)

Let the angle of incident is θ then from figure we can see that



$$\tan \theta = \frac{y}{2x}$$

So as x decreases then y will also decrease.

6. (B)

$$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

$$\frac{n^2 a}{V^2} = P$$

$$\Rightarrow a = \frac{PV^2}{n^2}$$

$$\therefore \text{Dimension of } a = [ML^5 T^{-2} \text{mol}^{-2}]$$

7. (C)

$$x = \frac{V_0}{\alpha} (1 - e^{-\alpha t})$$

$$[\alpha] = T^{-1}$$

$$\frac{V_0}{\alpha} = [L]$$

$$\Rightarrow V_0 = [LT^{-1}]$$

8. (D)

$$v \rightarrow [T^{-1}]$$

$$\text{Also, } \sqrt{\frac{F}{ML}} \rightarrow [T^{-1}]$$

9. (B)

Decrease in potential energy of 1 kg = increase in kinetic energy of both

$$\therefore 1 \times 10 \times 1 = \frac{1}{2}(4+1)v^2$$

$$v = 2 \text{ m/s}$$

10. (A)

$$t = px^2 + qx$$

$$\frac{1}{v} = \frac{dt}{dx} = 2px + q$$

$$\Rightarrow v = \frac{1}{2px + q}$$

$$\text{Also acceleration } a = \frac{dv}{dt} = \frac{-1 \times 2p}{(2px + q)^2} \times \frac{dx}{dt}$$

$$a = -2pv^2 \times v$$

$$\therefore a \propto v^3$$

11. (C)

$$\text{Using } y = ut + \frac{1}{2}gt^2,$$

$$t_1 t_2 = \frac{2h}{g} \Rightarrow h = \frac{t_1 t_2 g}{2} = \frac{6 \times 2 \times 10}{2} = 60\text{m}$$

$$\vec{s} = \vec{ut} + \frac{1}{2}\vec{at}^2 \Rightarrow -60 = u \times 6 - \frac{1}{2} \times 10 \times 6^2 \Rightarrow u = 20\text{m/s}$$

12. (D)

$$\text{Let } |\vec{A}| = (P+Q)$$

$$|\vec{B}| = (P-Q)$$

$$\text{Given } |\vec{A} + \vec{B}| = \sqrt{P^2 + Q^2}$$

$$\Rightarrow \sqrt{P^2 + Q^2} = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

$$\Rightarrow \cos \theta = \frac{P^2 + Q^2}{2(Q^2 - P^2)}$$

13. (D)

If \vec{A} and \vec{B} are two vectors, then

$$\left| |\vec{A}| - |\vec{B}| \right| \leq |\vec{A} + \vec{B}| \leq |\vec{A}| + |\vec{B}|$$

14. (B)

$$v_{y_1} = v_{y_2}$$

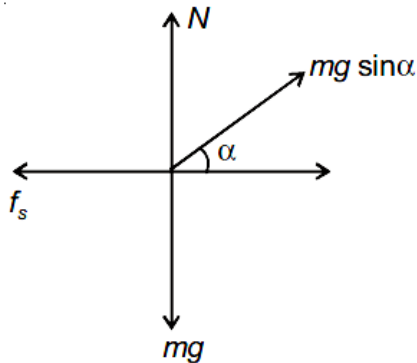
$$\Rightarrow v_1 \sin 30 = v_2$$

$$(v_1) \frac{1}{2} = v_2$$

$$\frac{v_1}{v_2} = 2$$

15. (B)

FBD of m (w.r.t. $2m$)



For no sliding,

$$(mg \sin \alpha) \cos \alpha \leq f_s^{\max}$$

$$(mg \sin \alpha) \cos \alpha \leq (\mu)(mg - mg \sin^2 \alpha)$$

$$mg \sin \alpha \cos \alpha \leq \mu mg \cos^2 \alpha$$

$$\sin \alpha \leq \mu \cos \alpha$$

$$\tan \alpha \leq \mu$$

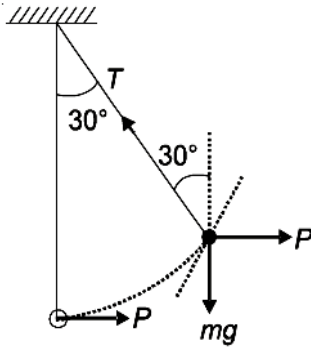
$$\alpha \leq \tan^{-1}(\mu)$$

16. (C)

$$F = T + mg \sin \theta$$

$$F = 2mg + mg \sin \theta$$

17. (B)



Tension = T

$$P = T \sin \theta \quad \dots\dots(i)$$

$$mg = T \cos \theta \quad \dots\dots(ii)$$

Divide (i) by (ii), $\frac{P}{mg} = \tan \theta \quad P = mg \tan \theta$

Suppose the bob swings through arc length x and it becomes x + dx when the angular displacement increases from θ to $\theta + d\theta$, $x = l\theta$, $dx = l d\theta$

Work done by force P is $W = \int \vec{P} \cdot d\vec{x}$

$$= \int P \cos \theta dx$$

$$= \int_0^{30^\circ} P \cos \theta l d\theta$$

$$= |P \sin \theta l|_0^{30} = \frac{Pl}{2}$$

18. (D)

$$\frac{1}{2} mv^2 = 12 \times \frac{3}{4} = 9$$

$$\Rightarrow v = 3 \text{ m/s}$$

19. (B)

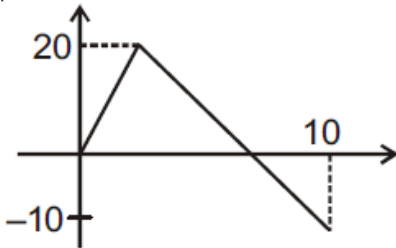
$$W = \frac{1}{2} mv_f^2 - \frac{1}{2} mv^2 \Rightarrow v_f = \sqrt{\frac{2W}{m} + v^2}$$

20. (C)

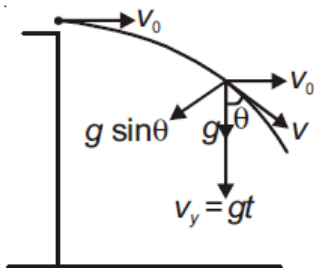
$$\frac{1}{2} kx^2 = \frac{1}{2} m \left(v^2 - \frac{v^2}{4} \right)$$

$$\therefore k = \frac{3mv^2}{4x^2} = \frac{6mv^2}{8x^2}$$

21. (5)
 Total distance travelled is $20 + 20 + 10 = 50$
 Time = 10s
 Average speed = $\frac{50}{10} = 5 \text{ m/s}$



22. (3)



$$R = \frac{v^2}{g \sin \theta}$$

$$\sin \theta = \frac{v_0}{v}$$

$$\sin \theta \propto \frac{1}{v}$$

$$R \propto v^3$$

23. (4)
 $16t = 8$
 $t = \frac{1}{2}$
 At $t = \frac{1}{2}$ $P = 2$
 So, $F_{av} = \frac{2}{1/2} = 4$

24. (4)
 For maximum,
 $m_2g = Mg \sin 37^\circ + \mu Mg \cos 37^\circ$
 For minimum,

$$m_1 g = Mg \sin 37^\circ - \mu Mg \cos 37^\circ$$

$$\text{So, } (m_2 - m_1)g = 2\mu Mg \cos 37^\circ$$

$$(m_2 - m_1) = 2 \times \frac{1}{2} \times 5 \times \frac{4}{5} = 4 \text{ kg}$$

25. (1)

$$\frac{1}{2} kd^2 = \mu mgd$$

$$d = \frac{2\mu mg}{k} = \frac{2 \times 0.5 \times 4 \times 10}{50} = 0.8 \text{ m}$$

So, answer is 1m.

26. (5)

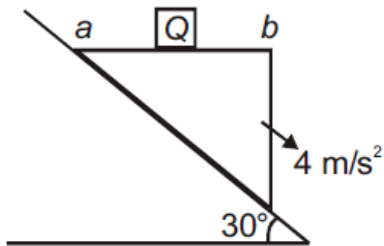
$$a_{\text{cm}} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 g$$

$$a_{\text{cm}} = \left(\frac{3-1}{3+1} \right)^2 g$$

$$a_{\text{cm}} = \frac{g}{4} = 2.5 \text{ m/s}^2$$

$$\therefore x = 5$$

27. (4)



$$a_Q = a_p \sin 30^\circ = 4 \times \frac{1}{2} = 2 \text{ m/s}^2$$

$$Mg - N = ma_Q$$

$$\Rightarrow N = mg - ma_Q$$

$$= 0.5 \times 10 - 0.5 \times 2 = 5 - 1 = 4 \text{ N}$$

28. (1)

$$v = \frac{p^b d^c}{l^a}$$

Equating dimension of both side

$$[M^0 L^0 T^{-1}] = [ML^{-1} T^{-2}]^b [ML^{-3}]^c [L^{-a}]$$

$$0 = b + c$$

$$0 = -b - 3c - a$$

$$-1 = -2b$$

Solving these,

$$c = -\frac{1}{2}$$

$$a = -\frac{1}{2} + \frac{3}{2} = 1$$

$$b = \frac{1}{2}$$

$$a + b + c = 1$$

29. (7)

$$F = \frac{mv^2}{r}$$

For maximum error,

$$\frac{\Delta F}{F} \times 100 = \left(\frac{\Delta m}{m} + \frac{2\Delta v}{v} + \frac{\Delta r}{r} \right) \times 100$$

$$= \left(\frac{0.1}{4} + 2 \times \frac{0.1}{10} + \frac{0.2}{8} \right) \times 100$$

$$= \frac{10}{4} + \frac{20}{10} + \frac{20}{8}$$

$$= 2.5 + 2 + 2.5 = 7\%$$

30. (2)

$$a = \frac{v dv}{dx} = (x^2 + x)(2x + 1) = 30$$

$$\Rightarrow x = 2\text{m}$$

PART (B) : CHEMISTRY

ANSWER KEY

| | | | | |
|---------|---------|---------|---------|---------|
| 31. (D) | 32. (C) | 33. (D) | 34. (D) | 35. (D) |
| 36. (D) | 37. (D) | 38. (D) | 39. (B) | 40. (C) |
| 41. (A) | 42. (A) | 43. (C) | 44. (C) | 45. (A) |
| 46. (A) | 47. (C) | 48. (C) | 49. (B) | 50. (B) |
| 51. (1) | 52. (4) | 53. (3) | 54. (3) | 55. (9) |
| 56. (4) | 57. (6) | 58. (6) | 59. (9) | 60. (7) |

SOLUTIONS

31. (D)

32. (C)

33. (D)

34. (D)

35. (D)

36. (D)

37. (D)

38. (D)

39. (B)

40. (C)

41. (A)

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{Mass} = \text{density} \times \text{volume}$$

$$\text{Density} = 0.93 \text{ g/cc}$$

$$\text{Mass of 1 cc of solution} = 0.93 \times 1 = 0.93 \text{ g}$$

The solution contains 18.6% ammonia by weight.

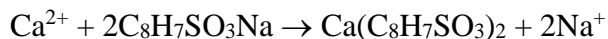
So, 1 cc of solution = 18.6% of 0.93 g ammonia

$$= \frac{18.6 \times 0.93}{100} = 0.173 \text{ g/cm}^3$$

Hence, the correct option is (A).

42. (A)

According to the reaction:



2 mole of the resin requires 1 mole of Ca^{2+} ions.

$(2 \times 206 \text{ g})$ of the resin requires 1 mole of Ca^{2+} ions.

So, 1 g of the resin requires

$$= \frac{1}{2 \times 206} \times 1 = 0.00243 \text{ mole of } \text{Ca}^{2+} \text{ ions.}$$

43. (C)

44. (C)

45. (A)

46. (A)

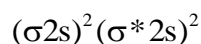
Maximum polarization is brought about by a cation of high charge.

According to Fajan's rule, polarisation of anion is influenced by charge and size of cation. More is the charge on cation, more is polarisation of anion.

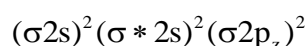
The correct option is (A).

47. (C)

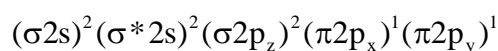
(A) Be_2 : Valence electrons = 4



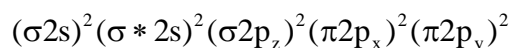
(B) B_2 : Valence electrons = 6



(C) C_2 : Valence electrons = 8



(D) N_2 : Valence electrons = 10



C_2 has two unpaired electrons, thus its paramagnetic.

48. (C)

Let x be that volume of the cylinder.

Initially at lower end -

$$P = 76 \text{ cm of Hg} + 76 \text{ cm of air} = 152 \text{ cm}$$

$$T = 300 \text{ K}$$

$$V = \frac{x}{2}$$

Finally at lower end-

$$P = 57 \text{ cm of hg} + 76 \text{ cm of air} = 133 \text{ cm}$$

$$T = ?$$

$$V = \frac{5x}{8}$$

Therefore, for an ideal gas $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\frac{152 \times \frac{x}{2}}{300} = \frac{133 \times \frac{5}{8} x}{T}$$

$$\Rightarrow T = \frac{133 \times 5 \times 2 \times 300}{152 \times 8}$$

$$\Rightarrow T = 328.12 \text{ K}$$

49. (B)

0.1 g of $\text{CO}_2 = \frac{0.1}{44}$ moles of CO_2 occupies 0.32 L.

$\frac{0.2}{M}$ moles of X occupies 0.44 L

$$PV = nRT$$

$V \propto n$ [At constant T and P]

$$\frac{V_{(\text{CO}_2)}}{V_X} = \frac{n_{\text{CO}_2}}{n_X} \Rightarrow \frac{0.32}{0.44} = \frac{0.1 \times M}{44 \times 0.2}$$

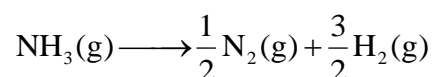
$$\Rightarrow M = 64$$

& Molar mass of $\text{SO}_2 = 32 + 32 = 64$

50. (B)

Let the volume of NH_3 be x ml, then the volume of H_2 would be (50 – x) ml.

On sparking,



Since 40 ml of O_2 is added and sparked, it must have reacted with H_2 to form liquid water.

Since 6 ml contraction in volume is there with alkaline pyrogallol as it is known to absorb O_2 , so 34 ml is the volume of O_2 used up.

\therefore Total volume of H_2 would be 68 ml. ($\because 2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$)

$$\therefore (50 - x) + \frac{3}{2}x = 68, 50 + \frac{x}{2} = 68, x = 36 \text{ ml}$$

\therefore % volume of NH_3 in the original mixture

$$= \frac{36}{50} \times 100 = 72\%$$

51. (1)

52. (4)

53. (3)

54. (3)

55. (9)

56. (4)

57. (6)

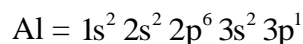
58. (6)

59. (9)

Given: $\frac{\ell \times m}{n} = 0$ (For Al)

To find : maximum no. of electrons

$\frac{\ell \times m}{n} = 0 \Rightarrow$ This means $\ell = 0$ or $m = 0$ electrons.



For $n = 1$, where $n =$ shell no.

ℓ m_ℓ $\ell =$ angular momentum quantum no.

0 0 $m_\ell =$ magnetic quantum no.

For $n = 2$

ℓ m_ℓ

0 0

1 -1, 0, +1

For $n = 3$

ℓ m_ℓ

0 0

1 -1, 0, +1

2 -2, -1, 0, +1, +2

Total no. of electrons with $\ell = 0$ or $m = 0$

$\Rightarrow 2 + 3 + 4 = 9$

Hence maximum no. of electrons = 9

60. (7)

PART (C) : MATHEMATICS

ANSWER KEY

| | | | | |
|----------|---------|---------|----------|------------|
| 61. (A) | 62. (B) | 63. (D) | 64. (A) | 65. (C) |
| 66. (C) | 67. (B) | 68. (A) | 69. (C) | 70. (D) |
| 71. (A) | 72. (A) | 73. (B) | 74. (B) | 75. (C) |
| 76. (B) | 77. (A) | 78. (B) | 79. (B) | 80. (A) |
| 81. (22) | 82. (2) | 83. (2) | 84. (6) | 85. (1002) |
| 86. (1) | 87. (2) | 88. (8) | 89. (21) | 90. (48) |

SOLUTIONS

61. (A)

Starting with M mean you have to arrange AAR which can be done in $\frac{(3)!}{(2)!} = 3$.

62. (B)

$|x - 1| + |x| + |x + 1| \geq 6$, gives four cases

Case I. When $x < -1$ (i)

$$-(x + 1) - x - (x - 1) \geq 6$$

$$\Rightarrow -x - 1 - x - x + 1 \geq 6$$

$$\Rightarrow -3x \geq 6 \text{ or } x < -1 \text{ ... (ii)}$$

From Eqs. (i) and (ii), $x \leq -2$

Case II. When $-1 \leq x \leq 0$ (iii)

$$\Rightarrow x + 1 - x - (x - 1) \geq 6$$

$$\Rightarrow 1 - x + 1 \geq 6 \Rightarrow x \leq -4 \text{ (iv)}$$

\therefore no value of x [From (iii) and (iv)]

Case III. When $0 \leq x \leq 1$ (v)

$$\Rightarrow x + 1 + x - (x - 1) \geq 6$$

$$\Rightarrow x \geq 4 \text{ ... (vi)}$$

No solution, using Eqs. (v) and (vi)

Case IV. When $x > 1$... (vii)

$$x + 1 + x + x - 1 \geq 6$$

$$\Rightarrow 3x \geq 6 \text{ or } x \geq 2 \text{(viii)}$$

From Eqs. (vii) and (viii), $x \geq 2$

Thus, from above four cases,

$$x \leq -2 \text{ or } x \geq 2$$

$$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$$

Option (B)

63. (D)

Given circle $x^2 + y^2 - 2gx + 6y - 19c = 0$

Passes through (6, 1)

$$12g + 19c = 43 \text{(1)}$$

Centre (g, -3) lies on given line

$$\text{So, } g + 6c = 8 \text{(2)}$$

Solve equation (1) & (2)

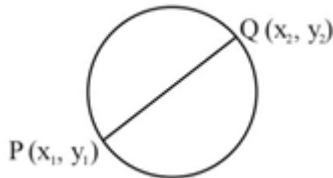
$$c = 1 \text{ \& } g = 2$$

$$\text{equation of circle } x^2 + y^2 - 4x + 6y - 19 = 0$$

Length of intercept on x-axis

$$= 2\sqrt{g^2 - c} = 2\sqrt{23}$$

64. (A)



Equation of circle diameter form

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(Where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the root of $y^2 + 2y - 7 = 0$)

$$x^2 + y^2 - 4x + 2y - 13 = 0$$

Now,

Compare in with the given equation, we get

$$a = -2, b = 1, c = -13$$

Now

$$a + b - c = 12$$

65. (C)

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\text{let } f(a) = (3 - a)^2 - 2(1 - 2a)$$

$$f(a) = a^2 - 2a + 7$$

$$f(a) = (a - 1)^2 + 6$$

$$f(a)_{\min} = 6$$

66. (C)

Let the common difference is 'd'

$$a_1 + a_7 + a_{16} = 40$$

$$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40$$

$$\Rightarrow 3a_1 + 21d = 40$$

$$\Rightarrow a_1 + 7d = \frac{40}{3}$$

$$S_{15} = \frac{15}{2} [2a_1 + 14d]$$

$$= 15(a_1 + 7d)$$

$$= 15\left(\frac{40}{3}\right)$$

$$= 200$$

67. (B)

Let the centroid of ΔPQR is (h, k) & P is (α, β) , then

$$\frac{\alpha+1+3}{3} = h \quad \text{and} \quad \frac{\beta+4-2}{3} = k$$

$$\alpha = (3h - 4) \quad \beta = (3k - 2)$$

Point $P(\alpha, \beta)$ lies on line $2x - 3y + 4 = 0$

$$\therefore 2(3h - 4) - 3(3k - 2) + 4 = 0$$

$$\Rightarrow \text{locus is } 6x - 9y + 2 = 0$$

68. (A)

$$2 \sin \frac{\pi}{2^{10}} \cos \frac{\pi}{2^{10}} \dots \cos \frac{\pi}{2^2}$$

$$2 \times \frac{\sin \left(2^9 \times \frac{\pi}{2^{10}} \right)}{2^9 \times \sin \left(\frac{\pi}{2^{10}} \right)} \times \sin \left(\frac{\pi}{2^{10}} \right) = \frac{1}{256}$$

69. (C)

$$\frac{a}{1-r} = 3 \quad \dots\dots(1)$$

$$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ as } |r| < 1$$

70. (D)

$$\frac{17-\beta}{-8} \times \frac{2}{3} = -1$$

$$\beta = 5$$

71. (A)

Given $\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$

$$\Rightarrow \cos(\alpha + \beta) = 1$$

$$\Rightarrow \sin(\alpha + \beta) = 0$$

$$\Rightarrow \sin\alpha \cos\beta + \cos\alpha \sin\beta = 0$$

$$\Rightarrow \cot\alpha \tan\beta = -1.$$

Hence (A) is the correct answer.

72. (A)

$$\text{We have } \frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$$

$$\Rightarrow \tan(3x - 2x) = 1 \quad \Rightarrow \tan x = 1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{4} \quad \Rightarrow x = n\pi + \frac{\pi}{4}$$

But for this value of x , $\tan 2x = \tan(2n\pi + \pi/2) = \infty$ which does not satisfy the given equation as it reduces to indeterminate form.

Hence (A) is the correct answer.

73. (B)

$$a + b + c = 18 \Rightarrow 2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 4 \cdot \frac{c}{4} = 18$$

$$\Rightarrow \left(\left(\frac{a}{2} \right)^2 \cdot \left(\frac{b}{3} \right)^3 \cdot \left(\frac{c}{4} \right)^4 \right)^{1/9} \leq \frac{1}{9} \left(2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 4 \cdot \frac{c}{4} \right)$$

$$\Rightarrow a^2 b^3 c^4 \leq 2^9 \cdot 2^2 \cdot 3^3 \cdot 4^4$$

Thus the maximum value of $a^2 b^3 c^4 = 2^{19} \cdot 3^3$.

Option (B)

74. (B)

Let the first five terms of the given G.P. be a_1, a_2, a_3, a_4, a_5 .

Hence $a_3 = 4$. Now $a_1 a_5 = a_2 a_4 = a_3^2$

$$\Rightarrow a_1 a_2 a_3 a_4 a_5 = 4^5$$

Hence (B) is the correct answer.

75. (C)

$$\left(x^2 + \frac{1}{x^3} \right)^n$$

$$\text{General term } T_{r+1} = {}^n C_r (x^2)^{n-r} \left(\frac{1}{x^3} \right)^r$$

$${}^n C_r \cdot x^{2n-5r}$$

For coefficient of x , $2n - 5r = 1$

$$\text{Given } {}^n C_r = {}^n C_{23}$$

$$r = 23 \quad \text{or} \quad n - r = 23$$

$$\Rightarrow n = 58 \quad \text{or} \quad n = 38$$

Minimum value is $n = 38$

76. (B)

The equation of the line L is $x/a + y/b = 1$ (1)

After the rotation of the axes, the line L has intercepts p and q on the new axes.

In this system equation of the line is $x/p + y/q = 1$.

Since the origin and the line, both are fixed, the distance between them remains the same.

$$\Rightarrow \left| \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = \left| \frac{-1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \right| \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$$

Hence (B) is the correct answer

77. (A)

Given $P \equiv (\alpha, \beta)$.

$$\text{Given line is } \frac{x}{a} + \frac{y}{b} = 1. \quad \dots \dots (1)$$

If line (1) cuts x and y axes at A and B respectively, then $A \equiv (a, 0)$ and $B \equiv (0, b)$. Also the area of $\Delta OAB = s$

i.e. $(1/2)ab = s \Rightarrow ab = 2s$.

Since line (1) passes through $P(\alpha, \beta)$,

$$\frac{\alpha}{a} + \frac{\beta}{b} = 1 \Rightarrow \frac{\alpha}{a} + \frac{a\beta}{2s} = 1 \Rightarrow a^2\beta - 2as + 2\alpha s = 0.$$

Since a is real, $4s^2 - 8\alpha\beta s \geq 0 \Rightarrow s \geq 2\alpha\beta$.

Hence the least value of $s = 2\alpha\beta$.

Hence (A) is the correct answer.

Alternative solution:

Since (α, β) lies on the given line, $\frac{\alpha}{a} + \frac{\beta}{b} = 1$

$$\Rightarrow ab = a\beta + b\alpha \geq 2\sqrt{ab\alpha\beta} \Rightarrow 2s = ab \geq 4\alpha\beta$$

\Rightarrow least value of $s = 2\alpha\beta$.

Hence (A) is the correct answer.

78. (B)

The lines $x - 2y = 0$ and $x - 2y + k = 0$ are parallel. The distance between these two lines =

$$\left| \frac{k}{\sqrt{1+(-2)^2}} \right| = 3 \Rightarrow k = \pm 3\sqrt{5}.$$

Hence (B) is the correct answer.

79. (B)

We can select 2 men from 5 men in 5C_2 ways, and 3 women from 7 women in 7C_3 ways. Hence total number of ways is ${}^5C_2 \times {}^7C_3$

Option (B)

80. (A)

$$\begin{aligned} 9(2!) &= 3.3.2! = 3.3! = (4-1)3! = 4! - 3! \\ 16(3!) &= 4.4.3! = 4.4! = (5-1)4! = 5! - 4! \\ 25(4!) &= 5.5.4! = 5.5! = (6-1)5! = 6! - 5! \\ 2601(50!) &= 51.51.50! = 51.51! = (52-51)51! = 52! - 51! \\ \text{Sum} &= 52! - 3! = 52! - 6 \end{aligned}$$

Option (A)

81. (22)

Here we have following cases-

Case (i) – when 4 students are selected then number of ways is ${}^6C_4 = 15$ ways.

Case (ii) – when 5 students are selected then number of ways is ${}^6C_5 = 6$ ways.

Case (iii) – when 6 students are selected then number of ways is ${}^6C_6 = 1$ way.

Then total number of ways is $15 + 6 + 1 = 22$ ways.

82. (2)

We know that $\cos \frac{5\pi}{8} = \cos \left(\frac{\pi}{2} + \frac{\pi}{8} \right) = -\sin \frac{\pi}{8}$

and $\cos \frac{7\pi}{8} = \cos \left(\frac{\pi}{2} + \frac{3\pi}{8} \right) = -\sin \frac{3\pi}{8}$

$$\Rightarrow \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} = 2$$

- 83. (2)**
 Given lines are $ax + y + 1 = 0$, ... (1)
 $x + by = 0$, ... (2)
 $ax + by = 1$ (3)
 Joint equation of (1) and (2) is $(ax + y + 1)(x + by) = 0$
 $\Rightarrow ax^2 + by^2 + (ab + 1)xy + x + by = 0$
 Making (4) homogeneous with the help of (1) we have
 $ax^2 + by^2 + (ab + 1)xy + x(ax + by) + by(ax + by) = 0$
 Since the angle between these two lines is 90° ,
 coefficient of $x^2 + \text{coeff. of } y^2 = 0$
 $\Rightarrow 2a + b + b^2 = 0$
- 84. (6)**
 The given function will attain a minimum value at $x = 6$
- 85. (1002)**
 ${}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_8 = ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_9 + {}^{10}C_{10}) - 22 = 1024 - 22 = 1002$
- 86. (1)**
 The given equation yields $1 + \log_b\{1 + \log_c(1 + \log_px)\} = 1$
 $\Rightarrow \log_b\{1 + \log_c(1 + \log_px)\} = 0$
 $\Rightarrow 1 + \log_c(1 + \log_px) = 1$
 $\Rightarrow \log_c(1 + \log_px) = 0 \Rightarrow 1 + \log_px = 1$
 $\Rightarrow \log_px = 0 \Rightarrow x = 1$.
- 87. (2)**
 Since $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$,
 $\tan A + \tan B + \tan A \tan B + 1 = 1 + 1$
 or $\tan A(1 + \tan B) + (1 + \tan B) = 2$
 or $(1 + \tan A)(1 + \tan B) = 2$.
- 88. (8)**
 Remainder when 5^{162} divided by 17 is same as when $25/17$
- 89. (21)**
 A ticket will be between two stations so 2 stations from n stations can be selected in nC_2 ways i.e. $n(n-1)/2$ which is given to us equal to 210 so $n(n-1) = 210 \times 2 = 420$ or $n = 21$
- 90. (48)**
 $(1 - x^4)(1 - x)^9 = (1 - x)^9 - x^4(1 - x)^9$
 Coefficient $x^7 = -{}^9C_7 + {}^9C_3 = 48$

