PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(C)	3.	(D)	4.	(C)	5.	(A)
6.	(A)	7.	(B)	8.	(AC)	9.	(ACD)	10.	(AD)
11.	(AD)	12.	(AC)	13.	(10.00)	14.	(2.80)	15.	(4.24)
16.	(32.00)	17.	(1.00)	18.	(2.5)				

SOLUTIONS

1. (B)

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$\Rightarrow -ac - ab + ac + c^2 = 0$

$\Rightarrow c^2 = ab$

Hence, c is the geometric mean of a and b.

2. (C)

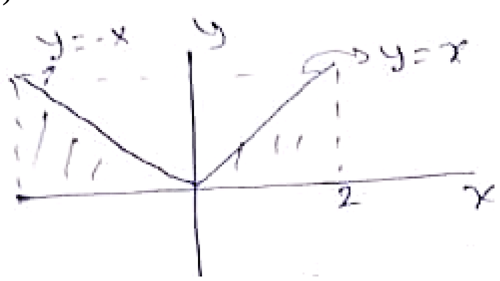
3. (D)

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_2y_3}{m_1 + m_2 + m_3}$$

Using this 0.9 cm right and 2.0 cm above 1 kg mass

4. (C)



Area under graph is 4 unit.

5. (A)

6. (A)

$AC = \frac{1}{2} g \sin\left(\theta + 55^{o}\right) t^2$

$AB = \frac{1}{2} g \sin\left(74^{o} - \theta\right) t^2$

$\frac{AC}{AB} = \frac{\sin 74^{o}}{\sin 55^{o}} = \frac{\sin\left(\theta + 55^{o}\right)}{\sin\left(74^{o} - \theta\right)}$

So, $\theta = 74^{o} - 55^{o} = 19^{o}$

7. (B)

Basic concept of magnification

8. (AC)

$V_{net} = \sqrt{4^2 + 4^2 - 2 \times 4 \times 4 \times \cos 60^{o}}$

$= 4$

& T = constant

9. (ACD)

10. (AD)

$U = 15 + (x-3)^2$

$U(5) = 19 \ \& \ KE(5) = 50$

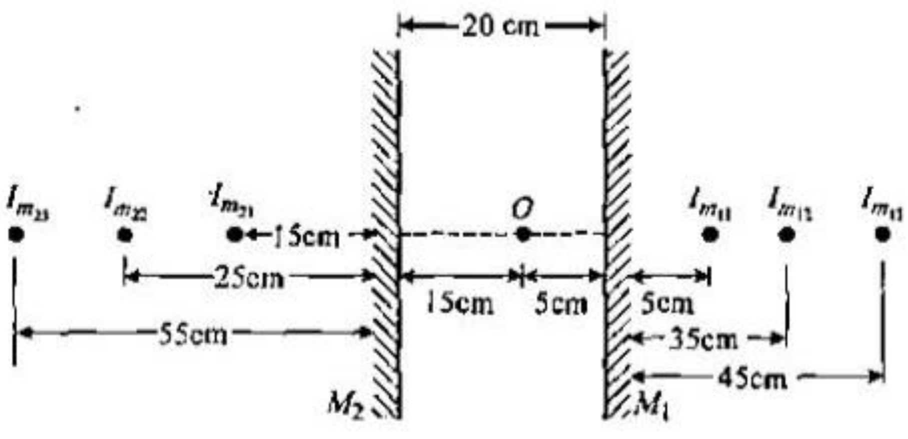
Total mechanical energy = 50 + 19 = 69 J

$U \equiv U(x-3) = 15 \text{J}$

$KE_{max.} = 69 - U_{min.} = 69 - 15 = 54 \text{ J}$

11. (AD)

First three images on M_1 are formed at distances 5 cm, 35 cm and 45 cm. First three images on M_2 are formatted at distances 15 cm, 25 cm and 55 cm.



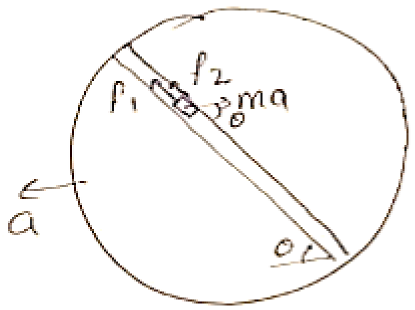
12. (AC)

Using energy conservation and circular motion

$$mg\left(\frac{R}{4}+R(1-\cos\theta)\right)=mgR\cos\theta$$

$$\Rightarrow \cos\theta=\frac{5}{6}$$

13. (10.00)



With respect to disc along the groove

$$ma_{net}=ma\cos\theta-f_1-f_2$$

$$\Rightarrow a_{net}=10.00\ m/s$$

14. (2.80)

Friction will not reach upto its maximum limit. So blocks will not accelerates.

$$2mg\sin 45^o-T-f_1=2ma \qquad ...(1)$$

$$T-mg\sin 45^o-f_2=ma \qquad ...(2)$$

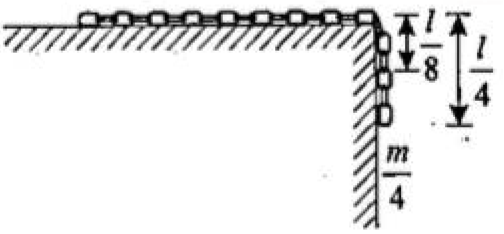
Solving it carefully

$$T=\frac{4mg}{3\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}\ N$$

Close to 2.80 N.

15. (4.24)

16. (32.00)



$$w=\frac{mg}{4}\left(\frac{l}{8}\right)=\frac{1}{32}mgl$$

17. (1.00)

18. (2.5)

PART (B) : CHEMISTRY

ANSWER KEY

19.	(B)	20.	(C)	21.	(C)	22.	(A)	23.	(B)
24.	(D)	25.	(AB)	26.	(ACD)	27.	(ABC)	28.	(ABD)
29.	(ABC)	30.	(BCD)	31.	(7.00)	32.	(2.00)	33.	(1.00)
34.	(5.00)	35.	(8.00)	36.	(4)				

SOLUTIONS

19. (B)

Gas cools on adiabatic expansion

20. (C)

$\underset{x\text{ moles}}{NH_3}$ $\quad\quad$ $\underset{y\text{ moles}}{N_2H_4}$ $\quad\quad$ $0.75\times V=(x+y)R300$

At T = 1200 K

$2NH_3 \longrightarrow N_2 + 3H_2$ $\quad\quad$ moles of $N_2 = \frac{x}{2}+y$

$N_2H_4 \longrightarrow N_2 + 2H_2$ $\quad\quad$ moles of $H_2 = \frac{3x}{2}+2y$

$$8.25\times V=(2x+3y)R\times 1200$$

$$0.75\times V=(x+y)R\times 300$$

$$\Rightarrow \frac{825}{75}=4\times\frac{2x+3y}{x+y}$$

$$\Rightarrow \frac{11}{4}=\frac{2x+3y}{x+y} \Rightarrow 11x+11y=8x+12y$$

$$\Rightarrow y=3x$$

$$\% \ N_2H_4 = \frac{y}{y+x}\times 100=\frac{3x}{4x}\times 100$$

$$=75\%$$

21. (C)

22. (A)

$$PM = dZRT \Rightarrow Z=\frac{PM}{dRT}$$

$$Z=\frac{1\times 30}{0.4\times 0.821\times 773}>1$$

$\therefore$ repulsive forces dominant

23. (B)
Back bonding in :$N(SiH_3)_3$

24. (D)

25. (AB)

26. (ACD)
B $\rightarrow$ non bonding molecular orbital.

27. (ABC)
Conceptual.

28. (ABD)

$$W_{AB} = \frac{1}{2}\times\frac{P}{2}\times V + \frac{P}{2}\times V = \frac{3pV}{4} = 00.75\,PV$$

Along the isotherm $W = PV\ln 2 = 0.693\,PV$

$$\frac{P-P_0}{V-V_0} = \frac{P_0}{2\times(V_0)} = -\frac{P_0}{2V_0}$$

$$\Rightarrow 2PV_0 - P_0 2V_0 = -P_0V + P_0V_0$$

$$\Rightarrow 2V_0\frac{RT}{V} = -P_0V + 3P_0C_0$$

$$\Rightarrow RTV_0 = -P_0V^2 + 3P_0V_0V$$

$$2RV_0\left(\frac{dT}{dV}\right) = 3P_0V_0 - P_0 2V = 0$$

$$\Rightarrow 3P_0V_0 = 2P_0V$$

$$\Rightarrow \text{ at } V = \frac{3V_0}{2} \text{ (T is maximum)}$$

$\frac{d^2T}{dV^2}$ is –ve.

29. (ABC)
Conceptual

30. (BCD)
Factual

31. (7.00)

32. (2.00)

$$Z = 1 + \frac{1}{RT}\left(b - \frac{a}{RT}\right)P + \frac{a}{(RT)^3}\left(2b - \frac{a}{RT}\right)P^2 + \text{(other terms)}$$

$$\frac{\partial Z}{\partial P}=\frac{1}{RT}\left(b-\frac{a}{Rt}\right)+\frac{a}{(RT)^3}\left(2b-\frac{a}{RT}\right)2P+\text{other terms of P}$$

$$\lim_{P\to 0}\frac{\partial Z}{\partial P}=\frac{1}{RT}\left(b-\frac{a}{RT}\right)$$

The expression $\frac{1}{RT}\left(b-\frac{a}{RT}\right)$ is maximum

$$y=\frac{1}{RT}\left(b-\frac{a}{RT}\right)$$

$$\frac{dy}{dT}=-\frac{b}{RT^2}-\frac{a}{R^2}\left(-\frac{2}{T^3}\right)=0$$

$$\frac{dy}{dT}=0 \text{ at } T=\frac{2a}{Rb}$$

33. (1.00)

34. (5.00)

$$\frac{3072/2}{20/32}=\left(\frac{240/2}{1600/32}\right)\left(\frac{32}{2}\right)^{\frac{n-1}{2}}$$

$\Rightarrow$ $n=6$, effusion steps = 5

35. (8.00)

$$\underset{\substack{x\,\text{moles}\\ \text{(initial)}}}{Cu}\longrightarrow Cu_2S$$

Let y moles get converted into Cu_2S

$$\therefore\quad \frac{y}{x}\times 100=31.75$$

Gain in wt. of copper = Final wt. – Initial wt.

$$=\frac{y}{2}\times M_{Cu_2S}+(x-y)M_{Cu}-xM_{Cu}$$

$$=y\left[\frac{159}{2}-63.5\right]=16y$$

$$\%\text{ gain in wt. }=\frac{16y}{x\times 63.5}\times 100=8$$

36. (4)

PART (C) : MATHEMATICS

ANSWER KEY

37.	(B)	38.	(D)	39.	(B)	40.	(A)	41.	(C)
42.	(B)	43.	(AC)	44.	(ABCD)	45.	(A)	46.	(ABC)
47.	(B)	48.	(BC)	49.	(240.00)	50.	(10.00)	51.	(4.00)
52.	(1.00)	53.	(5.00)	54.	(2.00)				

SOLUTIONS

37. (B)

We can write the given equation

$(x - 3a)^2 = 2a - 2$

Note the $a \geq 1$ and

$$x = 3a \pm \sqrt{2a-2}$$

Both the roots will exceed 3 if smaller of the two rots exceed 3, that is, if

$$3a - \sqrt{2a-2} > 3$$

$\Rightarrow \quad 3(a-1) > \sqrt{2}\sqrt{a-1}$

$\therefore \quad a > 1$ and $\sqrt{a-1} > \sqrt{2}/3$

$\Rightarrow \quad a > 1 + \frac{2}{9} = \frac{11}{9}$

38. (D)

$(2+x)^{40} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{40}x^{40}$

$A = a_0 + a_2 + a_4 + ...a_{40}$

$B = a_1 + a_3 + a_4 + ...a_{39}$

Put $x = 1$ and $x = -1$ in given expression to get

$3^{40} = a_0 + a_1 + a_2 + a_3.......a_{40}$

or $\quad 3^{40} = A + B \qquad ...(i)$

and $\quad 1 = a_0 - a_1 + a_2 - a_3.......$

or $\quad 1 = A - B \qquad ...(ii)$

From the equations (i) and (ii), we get

$$B = \left(\frac{3^{40}-1}{2}\right)$$

39. (B)

$T_5 = {}^nC_4 a^{n-4}(-2b)^4$

and $T_6 = {}^nC_5\, a^{n-5}(-2b)^5$

As $\quad T_5 + T_6 = 0$, we get

$${}^nC_4 2^4 a^{n-4} b^4 = {}^nC_5 2^5 a^{n-5} b^5$$

$$\Rightarrow \quad \frac{a^{n-4}b^4}{a^{n-5}b^5} = \frac{n!2^5}{5!(n-5)!} \cdot \frac{4!(n-4)!}{n!2^4}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{2(n-4)}{5}$$

40. (A)

Let the equation of the line through (1, 2) be y – 2 = m(x – 1)

If p denotes the length of the perpendicular from (3, 1) on this line, then

$$p = \left|\frac{2m+1}{\sqrt{m^2+1}}\right|$$

$$\Rightarrow \quad p^2 = \frac{4m^2+4m+1}{m^2+1} = 4 + \frac{4m-3}{m^2+1} = s(\text{say}),$$ then p^2 is greatest if and only if s is greatest

$$\text{Now } \frac{ds}{dm} = \frac{(m^2+1)(4) - 2m(4m-3)}{(m^2+1)^2}$$

$$= \frac{-2(2m+1)(m-2)}{(m^2+1)^2}$$

$\frac{ds}{dm} = 0 \Rightarrow m = -1/2.$ Also $\frac{ds}{dm}$ < 0 if m < –1/2

< 0 if –1/2 < m < 2

< 0 if m > 2

So, s is greatest for m = 2

And thus the equation of the required line is y = 2x.

Alternative Solutions:

Distance maximum when line is perpendicular to line joining of point A(1, 2) & B(3, 1)

$$\Rightarrow m = \frac{-1}{m_{AB}}$$

So slope is $m = \frac{-1}{\frac{2-1}{1-3}} = 2$

Equation of line $y - 2 = 2(x-1)$

$\Rightarrow \ y = 2x$

41. (C)

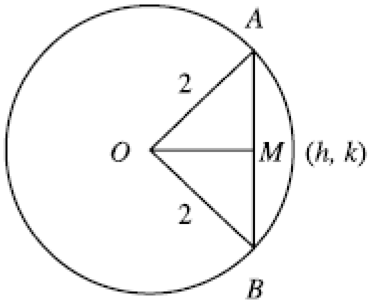
Let O be the centre of the circle $x^2 + y^2 = 4,$ and let AB be a chord of this circle, so that $\angle AOB = \pi/2.$ Let M(h, k) be the mid-point of AB. Then OM is perpendicular to AB.

$\therefore (AB)^2 = (OA)^2 + (OB)^2 = 4 + 4 = 8$

$\Rightarrow AM = (1/2)AB = \sqrt{2}$

$\Rightarrow (OM)^2 = (OA)^2 - (AM)^2 = 4 - 2 = 2 \Rightarrow h^2 + k^2 = 2$

Therefore, the locus of (h, k) is $x^2 + y^2 = 2.$



42. (B)

The given equation can be written as

$1-2\sin^2 x + a\sin x = 2a-7$

$\Rightarrow 2\sin^2 x - a\sin x + 2a - 8 = 0$

$\Rightarrow \sin x = \dfrac{a \pm \sqrt{a^2 - 8(2a-8)}}{4} = \dfrac{a \pm \sqrt{a^2 - 16a + 64}}{4}$

$= \dfrac{a \pm (a-8)}{4}$

Hence, $\sin x = (a-4)/2$ (the other value is not possible as $|\sin x| \le 1$). This value is possible only when

$-1 \le \dfrac{a-4}{2} \le 1$ or $-2 \le a-4 \le 2$

$\Rightarrow 2 \le a \le 6.$

43. (A), (C)

If α is a common root of the two equations,

$\alpha^2 + b\alpha + c = 0,\ b\alpha^2 + c\alpha + 1 = 0$

Multiply the first by b and subtracting we get

$(b^2 - c)\alpha + bc - 1 = 0 \Rightarrow \alpha = \dfrac{1-bc}{b^2 - c}$

Thus,

$(1-bc)^2 + b(1-bc)(b^2-c) + c(b^2-c)^2 = 0$

$\Rightarrow \quad b^3 + c^3 + 1 - 3bc = 0$

$\Rightarrow \quad (b+c+1)(b^2+c^2+1-bc-b-c) = 0$

$\Rightarrow \quad \dfrac{1}{2}(b+c+1)\left[(b-c)^2 + (b-1)^2 + (c-1)^2\right] = 0$

44. (A), (B), (C), (D)

$\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

$\Rightarrow \quad \dfrac{a+b+c}{a}, \dfrac{a+b+c}{b}, \dfrac{a+b+c}{c}$ are in A.P.

$\Rightarrow \quad \dfrac{b+c}{a}, \dfrac{c+a}{b}, \dfrac{a+b}{c}$ are in A.P.

[subtracting 1 from each term]

$\Rightarrow \quad \frac{b+c}{a}-1, \frac{c+a}{b}-1, \frac{a+b}{c}-1$ are in A.P.

[subtracting 1 from each term]

$\Rightarrow \quad \frac{b+c-a}{a}-1, \frac{c+a-b}{b}-1, \frac{a+b-c}{c}$ are in A.P.

Also b = 2ac/(a + c), so

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{2b-(a+c)}{(b-a)(b-c)}=\frac{2b-(a+c)}{b^2-b(a+c)+ac}$$

$$\frac{2b-2ac/b}{b^2-b(2ac)/b+ac}=\frac{2}{b}\cdot\frac{b^2-ac}{b^2-ac}=\frac{2}{b}$$

Lastly. $\left(a-\frac{b}{2}\right)\left(c-\frac{b}{2}\right)=ac-\frac{b}{2}(a+c)+\frac{b^2}{4}$

$=ac-\frac{b}{2}\cdot\frac{2ac}{b}+\frac{b^2}{4}=\frac{b^2}{4}$

$\therefore \quad a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$ are in G.P.

45. (A)

We are given

$$^mC_0+{}^mC_1+{}^mC_2=46\Rightarrow 2m+m(m-1)=90$$

$\Rightarrow \quad m^2+m-90=0\Rightarrow m=9$ as $m>0$

Now, (r + 1)th term of $\left(x^2+\frac{1}{x}\right)^m$ is

$$^mC_r\left(x^2\right)^{m-r}\left(\frac{1}{x}\right)^r={}^mC_r x^{2m-3r}$$

For this to be independent of x is

$2m - 3r = 0 \Rightarrow r = 6$

$\therefore$ term independent of x is $^9C_6=84$

46. (A), (B), (C)

The given equation can be written as

$(2x + 3y - 5)\cos\theta + (3x - 5y + 2)\sin\theta = 0$

or $(2x + 3y - 5) + \tan\theta\ (3x - 5y + 2) = 0$

This passes through the point of intersection of the lines 2x + 3y – 5 = 0 and 3x – 5y + 2 = 0 for all values of θ. The coordinates of the point P of intersection are (1, 1). Let Q(h, k) be the reflection of P(1, 1) in the line

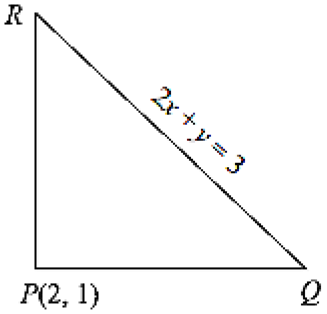
$x + y = \sqrt{2}$ (1)

Then PQ is perpendicular to (1) and the mid-point of PQ lies on (1).

$\therefore \frac{k-1}{h-1}=1\Rightarrow k=h$

And $\frac{h+1}{2}+\frac{k+1}{2}=\sqrt{2}\Rightarrow h=k=\sqrt{2}-1$

47. (B)
Let the slopes of PQ and PR be m and – 1/m respectively.
Since PQR is an isosceles triangle
$\angle PQR = \angle PRQ$



$$\Rightarrow \left|\frac{m+2}{1-2m}\right| = \left|\frac{-\frac{1}{m}+2}{1+\frac{2}{m}}\right|$$

$[\because \text{slope of QR} = -2]$

$\Rightarrow m+2 = \pm(1-2m) \Rightarrow m = 3 \text{ or } -1/3$

So the equations of PQ and PR are
$(y – 1) = 3(x – 2)$ and $y – 1 = (–1/3)(x – 2)$
Thus, joint equation representing PQ and PR is
$[3(x – 2) – (y – 1)]\,[(x – 2) + 3(y – 1)] = 0$
$\Rightarrow 3(x – 2)^2 – 3(y – 1)^2 + 8(x – 2)(y – 1) = 0$
$\Rightarrow 3x^2 – 3y^2 + 8xy – 20x – 10y + 25 = 0$

48. (B), (C)
The given relation can be written as
$(m+2)\tan\theta + (2m-1) = (2m+1)\sec\theta$

$\Rightarrow (m+2)^2\tan^2\theta + 2(m+2)(2m-1)\tan\theta + (2m-1)^2 = (2m+1)^2(1+\tan^2\theta)$

$\Rightarrow \left[(m+2)^2 - (2m+1)^2\right]\tan^2\theta + 2(m+2)(2m-1)\tan\theta + (2m-1)^2 - (2m+1)^2 = 0$

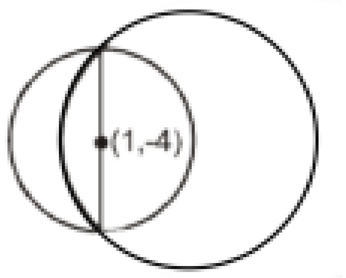
$\Rightarrow 3(1-m^2)\tan^2\theta + (4m^2+6m-4)\tan\theta - 8m = 0$

$\Rightarrow (3\tan\theta - 4)\left[(1-m^2)\tan\theta + 2m\right] = 0$

Which is true if $\tan\theta = 4/3 \text{ or } \tan\theta = 2m/(m^2-1)$.

49. (240.00)
M R T I $\boxed{\text{A U}}$
No. of ways $((5!))2 = (120)(2) = 240$

50. (10.00)
Common chord of given circle
$6x + 4y + (p+q) = 0$
This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



Centre (1, –4)

$6-16+(p+q)=0 \Rightarrow p+q=10$

51. (4.00)

x = 1 does not satisfy (1)

for $x \neq 1$, we get

$$\left(\frac{x^3-1}{x-1}\right)\left(\frac{x^{11}-1}{x-1}\right)=\left(\frac{x^7-1}{x-1}\right)^2$$

$\Rightarrow \quad x^{14}+x^3+x^{11}+1=x^{14}-2x^7+1$

$\Rightarrow \quad x^{11}-2x^7+x^3=0$

$\Rightarrow \quad x^8-2x^4+1=0 \qquad [\because x \neq 0]$

$\Rightarrow \quad \left(x^4-1\right)^2=0 \Rightarrow x^4=1$

$\Rightarrow \quad x=-1$

Thus, x + 5 = 4

52. (1.00)

Coefficient of x^6 in exp. of ${}^6C_0(1+x)^{12}-{}^6C_1(1+x)^{11}+{}^6C_2(1+x)^{10}-.....+{}^6C_6(1+x)^6$

= coefficient of x^6 in exp. of $(1+x)^6\left[{}^6C_0(1+x)^6-{}^6C_1(1+x)^5+{}^6C_2(1+x)^4-......+{}^6C_6(1+x)^0\right]$

= coefficient of x^6 in $x^6(1+x)^6$

= coefficient of x^0 in $(1+x)^6=1$

53. (5.00)

$$S=\sum_{r=1}^{25}\left[(3r-2)+(3r-1)(3r)\right]$$

$$=\sum_{r=1}^{25}\left(9r^2-2\right)$$

$$=\frac{9}{6}(25)(26)(51)-50$$

= 49675

$S/9935=5$

54. (2.00)

All different letters $\rightarrow {}^5C_2=10$

2 same 1 different $(1)\left({}^{4}C_{1}\right)=4$

No. of ways $14 \Rightarrow \frac{K}{7}=2$