PART (A) : PHYSICS

Answer Key

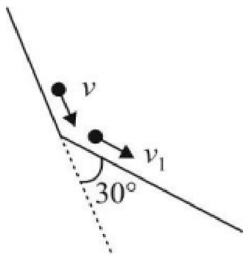
1.	(C)	2.	(B)	3.	(C)	4.	(A)	5.	(B)
6.	(B)	7.	(A)	8.	(D)	9.	(A)	10.	(B)
11.	(C)	12.	(C)	13.	(B)	14.	(C)	15.	(C)
16.	(D)	17.	(A)	18.	(A)	19.	(A)	20.	(A)
21.	(60)	22.	(3)	23.	(6)	24.	(30)	25.	(20)
26.	(2)	27.	(40)	28.	(3)	29.	(0)	30.	(15)

Solutions

1. (C)

Speed just before reaching B is given by energy conservation

$$mg(5) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2(10)(5)} = 10 \text{ m/s}$$



After collision at *B*, velocity perpendicular to the incline becomes zero while velocity along the incline remains unchanged

$\therefore \quad v_1 = v\cos 30^o = 5\sqrt{3}$ m/s

Velocity upon reaching *C* can be found by applying energy conservation again.

$$mg(7.5) = \frac{1}{2}mv_2^2 - \frac{1}{2},mv_1^2$$

$$\Rightarrow \quad v_2^2 = v_1^2 + 2g(7.5) = 75 + 150 = 225$$

$$\therefore \quad v_2 = 15 \text{m/s}$$

2. (B)

3. (C)

$$\frac{mgl}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)w^2$$

$$\Rightarrow w^2 = \frac{3g}{l} \qquad .....(1)$$

$$N - mg = mw^2\left(\frac{l}{2}\right)$$

$$\Rightarrow \quad N = mg + \frac{m.3g}{2}$$

$$= \frac{5}{2}mg$$

4. (A)

5. (B)

The moment of inertia about of A, B and C are $\frac{MR^2}{2}, \frac{MR^6}{6}$ and $\frac{2}{5}MR^2$ respectively.

The least value is for the square lamina.

6. (B)

If we join another similar triangle of mass M to complete the square we see that its moment of inertia about the given rotational axis would be that its moment of inertia about the given rotational axis would be $\frac{1}{12}(2M)(2a^2) = \frac{Ma^2}{3}$. The moment of inertia of half of this square plate would be $\frac{Ma^2}{6}$

7. (A)

8. (D)

$$R = \sigma T^4$$

$$\lambda = \frac{b}{T} \frac{b(\sigma)^{1/4}}{(R)^{1/4}}$$

9. (A)

We know,

The lengths of each rod increases by the same amount.

$$\therefore \ \Delta\ell_a = \Delta\ell_s$$

But, $\ell_1\alpha_a t = \ell_2\alpha_s t$

$$\therefore \ \frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a} \qquad \therefore \ \frac{\ell_2}{\ell_1} = \frac{\alpha_a}{\alpha_s}$$

$$\therefore \ \frac{\ell_2}{\ell_1} + 1 = \frac{\alpha_a}{\alpha_s} + 1$$

$$\therefore \ \frac{\ell_2}{\ell_1} + 1 = \frac{\alpha_a}{1\alpha_s} + 1$$

$$\therefore \ \frac{\ell_2 + \ell_1}{\ell_1} = \frac{\alpha_a + \alpha_s}{\alpha_s} \ \Rightarrow \ \frac{\ell_1}{\ell_1 + \ell_1} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

10. (B)

11. (C)

Momentum is conserved in collision. So after collision ball can not go in y direction

12. (C)

ma/K has the dimension equal to L.

13. (B)

$$\left|\hat{a} - \hat{b}\right| = \sqrt{a^2 + b^2 - 2ab\cos 60^o}$$

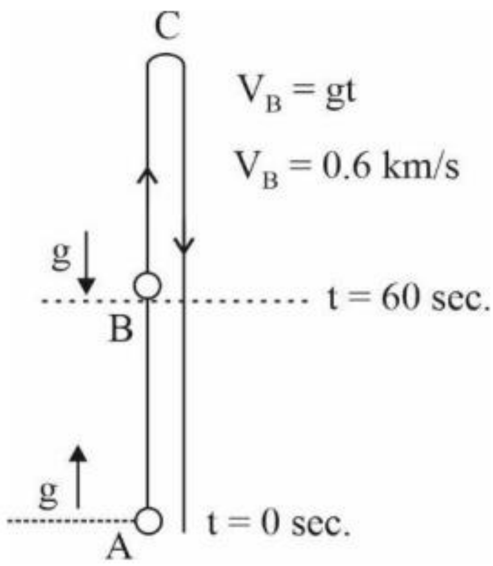
$$= 1$$

$$\Rightarrow \ \left|\vec{a} - \vec{b}\right| = \sqrt{1} = 1$$

14. (C)
Initial slope of x-t graph is negative.
Velocity at t = 0 should be negative.

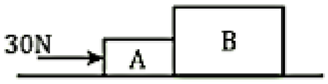
15. (C)
For lens 1 the object is at infinity thus $u=\infty$, hence its image will form at focal length, and as per the figure provided image distance v = 5 cm and is real, inverted and diminished, thus the focal length of lens 1 $f_1 = -5$ cm.
Now for lens 2 image so formed by lens 1 will be the object for lens 2 which is at a distance of 5 cm away from the lens 2.
Assuming that the image will be formed at infinity we can conclude that the object for lens 2 is at focal length of lens 2, thus focal length of lens 2 $f_2 = +5$ cm.

16. (D)
$$\overline{AB} = \frac{1}{2}gt^2 = 5\times 60\times 60m = 18\,km$$
$$\overline{BC} = \frac{(600)(600)}{20} = 18\,km,\ \overline{AC} = 36\,km$$
$$v^2 = \begin{cases} +0.02h;\ 0 \le h \le 18\,km \\ -0.02(h-36);\ 18\,km \le h \le 36\,km \\ +0.02(36-h);\ 36km \ge h \ge 0\,km \end{cases}$$



17. (A)
When the mass just begins to move up, then
$\Rightarrow\ Mg\sin\theta = P\cos\theta + P$
$\Rightarrow\ Mg\sin\theta = P(\cos\theta + 1)$
$\Rightarrow\ P = \dfrac{Mg\sin\theta}{(\cos\theta+1)}$
We know that, $\sin\theta = 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$ and $1+\cos = 2\cos^2\left(\frac{\theta}{2}\right)$.
So, $P = \dfrac{Mg\sin\theta}{\cos\theta+1} = \dfrac{2Mg\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}$
$\Rightarrow\ P = Mg\tan\left(\frac{\theta}{2}\right)$

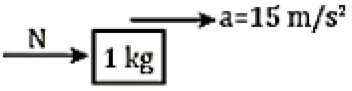
18. (A)
The correct option is (A)



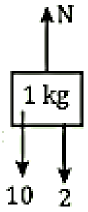
Case A
Net acceleration of masses

$a = \frac{30}{2} = 15\ \text{m/s}$

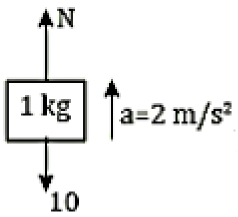
FBD of block B:



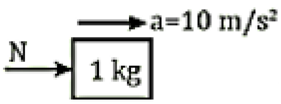
Normal force N = 1 × 15 = 15 N
Case B-
FBD of block A:



N = 10 + 2 = 12 N
Case C-
FBD of block A:



N – 10 = 1 × 2
N = 12 N
Case D-
FBD of Block A-



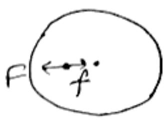
$N = 1 \times 10 = 10\ \text{N}$

19. (A)
Given $\mu = 0.5$
Angular velocity $\omega = 5\,\text{rad/s}$
If F = centrifugal force & f = frictional force then, in the limiting case

$f > F$

$m(\mu g) \geq mr\omega^2$

$\frac{\mu g}{\omega^2} \geq r$



or $\quad r \le \dfrac{0.5 \times 10}{(5)^2}$ $\quad\quad$ (Assuming g = 10 m/s^2)

$r \le \dfrac{5}{25}$

$\Rightarrow r \le \dfrac{1}{5}$

$\Rightarrow r \le 0.2\,m$

20. (A)

CM if a solid hemisphere of radius $R = 3R/8$ from the centre.

21. (60)

Let θ^{o} C be temperature at B. Let Q be the heat flowing per second from A to B on account of temperature difference by conductivity

$\therefore \quad Q = \dfrac{KA(90 - \theta)}{1}$ $\quad\quad$ ...(i)

Where k = thermal conductivity of the rod, A = Area of cross section of the rod, l = length of the rod.
By symmetry, the same will be the case for heat flow from C to B.

$\therefore$ The heat flowing per second from B to D will be

$2Q = \dfrac{KA(\theta - 0)}{1}$ $\quad\quad$ ...(ii)

Dividing Eq. (ii) by Eq. (;i)

$2 = \dfrac{\theta}{90 - \theta} \Rightarrow \theta = 60^{o}\,C$.

22. (3)

When the liquid is not there-
$u = -0.5, v = -0.5$
Hence, f = –0.25 m focal length of mirror.
Lateral shift by the liquid is 0.2 m
The ray travel 0.4 m in the liquid

$0.2 = 0.4 \times (\mu - 1)$

$\mu = 0.5 + 1$

$= 1.5$

23. (6)

The equilibrium position of the given system will be given by

$\left[m \times g \sin(37) = k \times x\right]$

$\left[\dfrac{(m \times g \times \sin 37)}{k} = x\right]$

Now the block is at is extreme position in the beginning, hence maximum elongation will be given by, $\left[\dfrac{(2 \times m \times g \times \sin 37)}{k} = x\right]$.

$x = \frac{6mg}{5k}$.

Since, $\sin 37 = \frac{3}{5}$

24. (30)

$90 - i + 2x + \theta = 180$

$x = \frac{90 + i - \theta}{2}$

$\alpha + 2i + x = 180$

$\theta + 90 - i + 2i + 90 - x = 180$

$\theta + i - x = 0$

$\theta + i = \frac{90 + i - \theta}{2}$

$2\theta + 2i = 90 + i - \theta$

$i = 90 - 3\theta = 90 - 3 \times 20$

$= 30$

25. (20)

We already know that there are frictional forces acing on the horizontal wall.
And also, we have weight acting downwards.
Frictional force in this case is opposite and equal to the horizontal component of weight.
Balancing the components:

$mg \cos\theta = \mu N$

For the body:

$F_{fr} = 3 \times 10 \times \cos 37^{o} = 20\,\text{N}$

26. (2)

From the conservation of energy,

$mgh = \frac{1}{2} mv^2$

$v = \sqrt{2gh}$

The condition for completing the loop is given as,

$\sqrt{2gh} = \sqrt{5gR}$

$R = \frac{2 \times 5}{5}$

$= 2\,\text{cm}$

Thus, the maximum value of R for body to successfully complete the loop is 2 cm.

27. (40)

$mgh \sin 30 = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$v = r\omega$

$v^2 = 40$

$\omega = 40\sqrt{10}$ rad/s

28. (3)

$\vec{\tau} - = \vec{r} \times \vec{F}$ or, $\tau = rF \sin\theta$

$3\sqrt{3} = 6 \times 1 \times \sin\theta$; $\sin\theta = \frac{\sqrt{3}}{2}$; $\theta = 60^{\circ}$ or $\frac{\pi}{3}$

Thus, $n = 3$

29. (0)

If F = 0

Then assuming no relative motion acceleration of

$A + B = \frac{300}{15} = 20\ \text{m/s}^2$

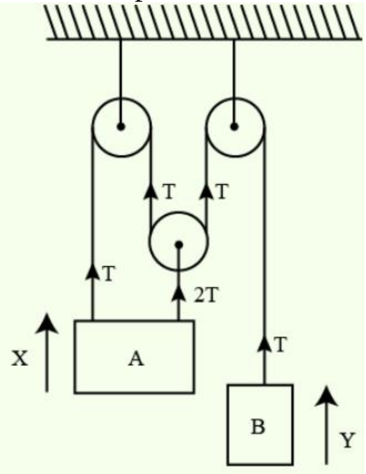
$\because\ 20\ \text{m/s}^2 > \mu g$

Where $\mu = 0.5$ and $g = 10\,\text{m/s}^2$

$\therefore$ Relative motion shall exist. Hence F = 0 N.

30. (15)

Let the displacement of A and B be X and Y respectively.



Using work done by tension method, (net work-done by tension is zero, since it is internal force)

$3TX + TY = 0$

$\Rightarrow Y = -3X$

Differentiating,

$V_B = -3V_A$

$= -3(+5)$

$= -15\ \text{ms}^{-1}$

$= 15\ \text{ms}^{-1}$ downwards

PART (B) : CHEMISTRY

Answer Key

31.	(B)	32.	(B)	33.	(D)	34.	(B)	35.	(B or C)
36.	(B)	37.	(B)	38.	(C)	39.	(D)	40	(C)
41.	(A)	42.	(B)	43.	(C)	44.	(D)	45.	(C)
46.	(B)	47.	(D)	48.	(B)	49.	(B)	50.	(C)
51.	(8)	52.	(10)	53.	(0)	54.	(5)	55.	(9)
55.	(3)	57.	(9)	58.	(5)	59.	(5)	60.	(3)

Solutions

31. (B)

$PT^{-3} = \text{const}, \quad PV = RTn$

$\Rightarrow \dfrac{P}{P^3V^3} = \text{const} \Rightarrow P^{-2}V^{-3} = \text{const}$

$\Rightarrow$

$\Rightarrow PV^{3/2} = \text{const}$

$C_{process} = \dfrac{5R}{2} + \dfrac{R}{1-\frac{3}{2}} = \dfrac{5R}{2} - 2R$

$= R/2$

32. (B)

$NH_3 + HCl \rightarrow NH_4Cl$

1V   1V   1V → only salt

end 0   0

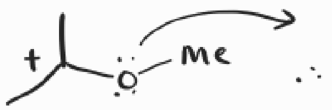
formed ∴ not a buffer

33. (D)

$\dfrac{5}{300} = \dfrac{5+\Delta V}{T}$

$\Rightarrow \dfrac{1.5}{240} = \dfrac{\Delta V}{T} \Rightarrow T = 480$

34. (B)



Back Bonding

∴ more stable

35. (B or C)

$$d = 15/5 = 3\,g/L = 3\,kg/m^3$$

$$V_{rms} = \sqrt{\frac{3P}{d}} = \sqrt{\frac{3\times10^4}{3}}$$

36. (B)

$$\Delta x = 0.529\times10^{-10}\times\frac{10}{100}$$

$$\Delta x\,\Delta V = \frac{h}{4\pi m}$$

$$\Delta V = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times.529\times10^{-11}}$$

$$\approx 10^7\ m/sec$$

37. (B)

$$2SO_2 + O_2 \rightleftharpoons 2SO_3$$

	SO_2	O_2	SO_3
	2	1	2
initial	2		
pressure at eq	$2-2x$	$1-x$	$2+2x$
	$\simeq 2y$	$\simeq y$	$\simeq 4$

since K_p is very large almost reactants converted into products

$$K_p = \frac{(4)^2}{(2y)^2(y)} = 4\times10^{24}$$

$$\Rightarrow y \simeq 10^{-8}$$

$$P_{SO_2}\text{ at eq.} = 2y = 2\times10^{-8}$$

38. (C)

39. (D)
Conceptual

40. (C)

$CaCO_3 \longrightarrow CaO + CO_2$

0.5 $\quad$ 11.2 L = 0.5 mole

$CaO + H_2O \rightarrow Ca(OH)_2$

0.5 mole

$Ca(OH)_2 + 2HCl \rightarrow CaCl_2 + 2H_2O$

↳ moles = 1 = 36.5 gm

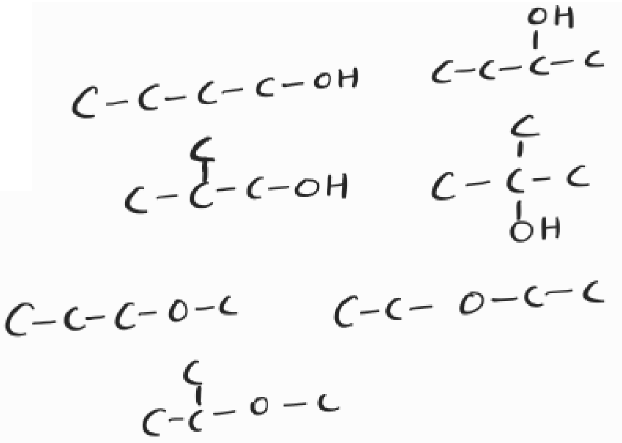
41. (A)

$\left(P + \frac{a}{V^2}\right)(V) = RT, \quad n = 1$

$\Rightarrow Z = 1 - \frac{a}{VRT}$

$Z = 1 - \frac{96}{20 \times .08 \times 300} = 0.8$

42. (B)



43. (C)

$CN^- + H^+ \longrightarrow HCN$

	CN^-	H^+	HCN
	12 mmole	8 mmole	
end	4	0	8

$pH = 10 - \lg 5 + \lg \frac{1}{2} = 9$

44. (D)

$A(g) \rightleftharpoons B(g) + C(g) + D(g)$

eq $\quad a-x \quad x \quad x \quad x$

$120 = \frac{(Mwt_A) \times a}{a+2x}$

$x/a = 1/5$

$\Rightarrow 120 \times \left(1+\frac{2}{5}\right) = \text{Mol. wt}_A$

45. (C)

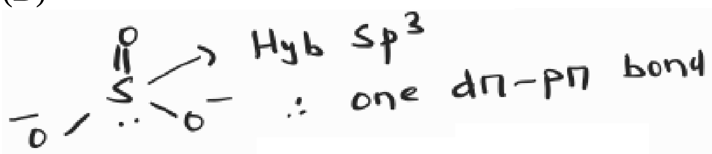
Greater the BO more is Bond energy. If BO same then greater no. of e^- in ABMO lesser is the Bond energy

46. (B)

ΔS –ve, ΔH –ve

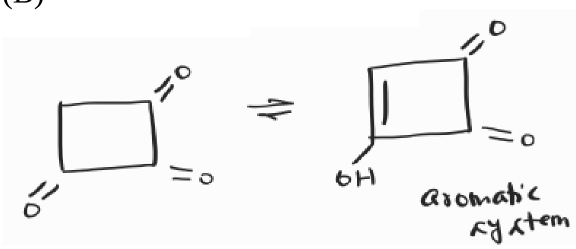
For spontaneous $\Delta G < 0$

47. (D)



48. (B)

49. (B)



50. (C)

51. (8)

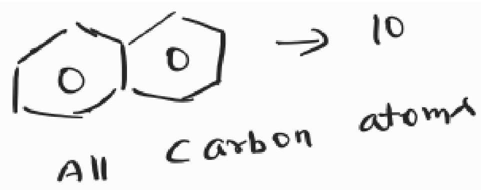
$$M + 3N \rightleftharpoons 4P$$

	M	3N	4P
	a	a	0
eq	a − x	a − 3x	4x

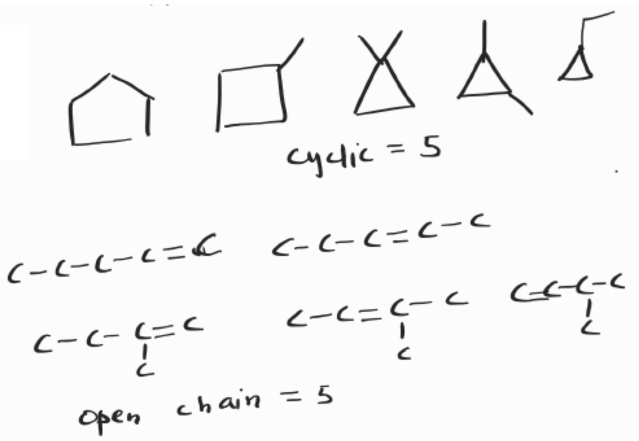
$$a - x = 4x \Rightarrow a = 5x$$

$$K_c = \frac{(4x)^4}{(2x)^3(4x)} = 8$$

52. (10)



53. (0)



54. (5)

55. (9)

$$NaOH + CH_3COOH \rightarrow CH_3COONa + H_2O$$

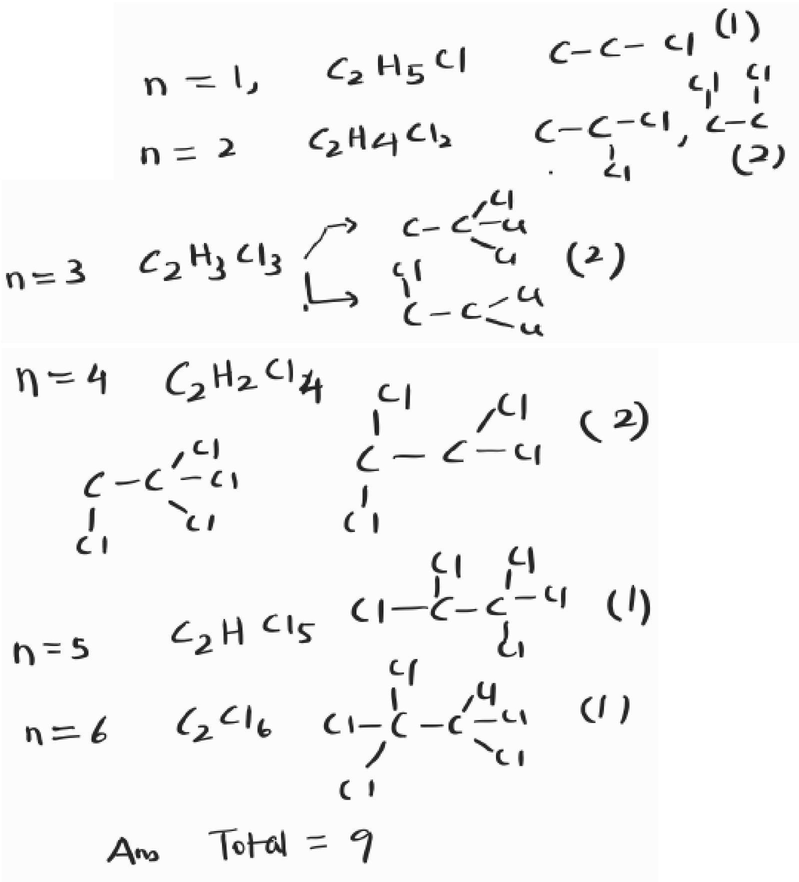
	NaOH	CH_3COOH	CH_3COONa
	20	20	
end	0	0	20

$$pH = 7 + \frac{1}{2}\left(5 + \lg\frac{20}{200}\right)$$

$$= 7 + 2.5 + \frac{1}{2}(-1) = 9$$

56. (3)
(i), (ii), (iv)

57. (9)



58. (5)
Conceptual

59. (5)

$$30 + 15x = 105$$
$$\Rightarrow 15x = 75 \Rightarrow x = 5$$

60. (3)

Those which will form a stable carbocation on removal of Br^- will react

PART (C) : MATHEMATICS

Answer Key

61.	(D)	62.	(A)	63.	(D)	64.	(B)	65.	(C)
66.	(A)	67.	(C)	68.	(B)	69.	(B)	70.	(B)
71.	(D)	72.	(C)	73.	(B)	74.	(A)	75.	(D)
76.	(D)	77.	(D)	78.	(B)	79.	(C)	80.	(A)
81.	(56)	82.	(53)	83.	(24)	84.	(495)	85.	(0.6)
86.	(3)	87.	(8)	88.	(16)	89.	(150)	90.	(2)

Solution

61. (D)

$n = 5! \times 6!$

Where 5! = arranging five girls and 6! = arranging five boys and five consecutive girls

Now consider for m

__ B __ B __ B __ B __ B __

$m = 5! \times {}^5C_4 \times 4! \times {}^6C_2 \times 2!$

where 5! = arranging of five boys,

${}^5C_4$ = selecting of four girls,

4! = arranging of four girls,

${}^6C_2$ = selecting two gaps in between five boys,

and 2! = arranging of four consecutive girls and one single girl

$\therefore m/n=5$

62. (A)

We know that |sin x| <= 1, then x <= 100.

We partition the interval [0,100] into segments $(0, 2\pi], (2\pi, 4\pi], ..., (28\pi, 30\pi], (30\pi,100]$.

We can see that there are two solutions in each interval except for $(0, 2\pi]$ in which there is one solution, in total there are 31 positive solutions.

So there must be 31 negative solutions.

Since x = 0 is also a solution, there are 31 + 31 + 1 = 63 solutions.

63. (D)

$\sin^4\theta - 2\sin^2\theta - 1 = 0 \Rightarrow \sin^2\theta = \dfrac{2 \pm \sqrt{4+4}}{2}$

$\Rightarrow \sin^2\theta = 1 \pm \sqrt{2}$.

So, no value of θ can satisfy the given equation.

64. (B)

Let $y = m_1x$ and $y = m_2x$

$\therefore \quad m_1 + m_2 = \frac{-2h}{b}, \quad m_1 m_2 = \frac{a}{b}$

In new position $m_1 m_1' = -1 \Rightarrow m_1' = -\frac{1}{m_1}$

Similarly, $m_2' = -\frac{1}{m_2}$

New lines are $y = \left(-\frac{1}{m_1}\right)x$ and $y = \left(-\frac{1}{m_2}\right)x$

$\therefore \quad (m_1 y + x)(m_2 y + x) = 0 \quad \Rightarrow \quad bx^2 - 2hxy + ay^2 = 0$

65. (C)

Any line through (1, 2) can be written as $\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$

Where θ is the angle which this line makes with positive direction of x-axis.

Any point on this line is (r cosθ + 1, r sin θ + 2) when $|r| = \frac{1}{3}\sqrt{6}$,

this point lies on the line $x + y = 4$.

i.e. r cosθ + 1 + r sinθ + 2 = 4,

$\Rightarrow \quad r(\cos\theta + \sin\theta) = 1, |r| = \frac{1}{3}\sqrt{6}$

$\Rightarrow \quad r^2(1 + 2\sin\theta\cos\theta) = 1, r^2 = \frac{6}{9} \quad \Rightarrow \quad 1 + \sin 2\theta = \frac{1}{r^2} = \frac{9}{6} \quad \Rightarrow \quad \sin 2\theta = \frac{1}{2}$

$\Rightarrow \quad 2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6} \quad \Rightarrow \quad \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

66. (A)

Let $S \equiv x^2 + y^2 - 15x + 5y = 0$ …(i)

Any point circle through (1, 2) is given as

$S_1 \equiv (x-1)^2 + (y-2)^2 = 0$ …(ii)

Family of circles passing through $S_1 = 0$ and $S_2 = 0$ is $S_1 + \lambda S_2 = 0$ …(iii),

where, λ is a parameter (λ ≠ –1).

Now, circle (iii) passes through (0, 2) ⇒ λ = –14

Putting, λ = –14 in (iii), we get the required equation.

67. (C)

$x = a\left(y^2 + \frac{b}{a}y + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$ or $a\left(y + \frac{b}{2a}\right)^2 = x + \frac{b^2}{4a} - c$

or $\left(y + \frac{b}{2a}\right)^2 = \frac{1}{a}\left\{x + \frac{b^2 - 4ac}{4a}\right\}$ ∴ the length latus rectum = $\frac{1}{|a|}$

68. (B)

Let PQ be a double ordinate of length $8a$. Then $PR = RQ = 4a$.

Coordinates of P and Q are $(OR, 4a)$ and $(OR, -4a)$ respectively.

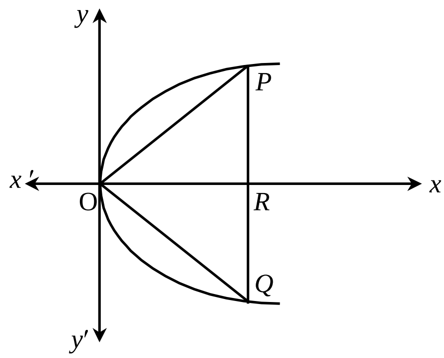
Since P lies on the parabola $y^2 = 4ax$, therefore

$(4a)^2 = 4a\,(OR) \Rightarrow OR = 4a$

Thus, the coordinates of P and Q are $(4a, 4a)$ and $(4a, -4a)$

Now, $m_1 = \text{Slope of } OP = \dfrac{4a-0}{4a-0} = 1$,

and $m_2 = \text{Slope of } OQ = \dfrac{-4a-0}{4a-0} = -1$



Clearly, $m_1 m_2 = -1$.

Thus, PQ makes a right angle at the vertex of the parabola.

69. (B)

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$$

$$f^3(x) = f(f^2(x)) = f\left(-\frac{1}{x}\right) = \frac{x+1}{1-x}$$

$$\Rightarrow f^4(x) = f\left(\frac{x+1}{x-1}\right) = -\frac{1}{x}$$

$$\Rightarrow f^6(x) = -\frac{1}{x} \Rightarrow f^6(6) = -\frac{1}{6}$$

$$f^7(x) = \left(-\frac{1}{x}\right) = \frac{x+1}{10-x}$$

$$\Rightarrow f^7(7) = \frac{8}{-6} = -\frac{4}{3}$$

$$\therefore\ -\frac{1}{6} + -\frac{4}{3} = -\frac{3}{2}$$

70. (B)

$5|x| - x^2 - 6 \ge 0 \Rightarrow x^2 - 5|x| + 6 \le 0$

when $x < 0$, $x^2 + 5x + 6 \le 0$, $-3 \le x \le -2$

when $x > 0$, $x^2 - 5x + 6 \le 0$, $2 \le x \le 3$

$x = 0$ will not satisfy the condition.

Domain is $[-3, -2] \cup [2, 3]$.

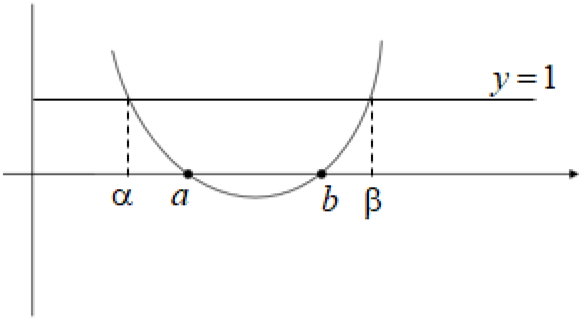
71. (D)

Consider $(x-a)\ (x-b)\ =0$

It has two roots a and b.

Now we need to find roots of $(x-a)\ (x-b)\ =1$

This will have one root less than a and other more than b, as shown in the figure.



From graph it is clear that one root of the equation lies in $(-\infty, a)$ and other in (b, ∞) .

72. (C)

A family has two children.

Let the girls be denoted by 'g'& boys be denoted by 'b'.

Therefore, sample space of the experiment is

S = { (g, b), (b, g), (b, b)}

Required probability is 1/3

73. (B)

Consider the value of function $[\log_{10}x]$in different intervals-

When x is a single digit number then $[\log_{10}x] = 0$

When x is a two digit number then $[\log_{10}x] = 1$

When x is a 3 digit number then $[\log_{10}x] = 2$ and so on.

Now consider the given equation $[\log_{10}1] + [\log_{10}2] + \ldots . [\log_{10}x] = x$

Since there are 9 single digit numbers and for those the values $[\log_{10}x] = 0$

That 9 numbers will be compensated by 9 three digit numbers hence. Hence x = 108.

74. (A)

75. (D)

Here $t_r = \frac{1}{\{(2r-1).(2r+1)\}} = \frac{1}{2}\left[\frac{1}{2r-1} - \frac{1}{2r+1}\right]$

$t_r = \frac{1}{2}\left[\frac{1}{2r-1} - \frac{1}{2r+1}\right]$

$t_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$

$t_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$

$t_3 = \frac{1}{2}\left[\frac{1}{5} - \frac{1}{7}\right]$

… … …

… … …

$t_{n-1} = \frac{1}{2}\left[\frac{1}{2n-3} - \frac{1}{2n-1}\right]$

$t_n = \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$

On adding these terms we will get

$S_n = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right] = \frac{n}{2n+1} = \frac{100}{201}$

76. (D)

Sum of 1st k positive odd integers $1 + 3 + 5 + \cdots + (2k-1) = k^2$

Thus, the given equation can be written as $\left(\frac{p+1}{2}\right)^2 + \left(\frac{q+1}{2}\right)^2 = \left(\frac{r+1}{2}\right)^2$

$\Rightarrow (p+1)^2 + (q+1)^2 = (r+1)^2$

Therefore, $(p+1, q+1, r+1)$ forms a Pythagorean triplet. As p > 6, p + 1 > 7.

The first Pythagorean triplet containing a number > 5 is (6, 8, 10).

∴ We may take p + 1 = 8, q + 1 = 6, r + 1 = 10 ⇒ p + q + r = 21.

77. (D)

$3^{2003} = 3^{2001} \cdot 3^2 = 9(27)^{667} = 9(28-1)^{667} = 9({}^{667}C_0\, 28^{667} - {}^{667}C_1\, (28)^{666} + \cdots + {}^{667}C_{667}\, (-1)^{667})$.

That means, if we divide 3^{2003} by 28, remainder is 19. Thus, $\left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$

78. (B)

${}^{2n}C_1,\ {}^{2n}C_2,\ {}^{2n}C_3$ are in AP. $\therefore 2\ {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow 2n^2 - 9n + 7 = 0$.

79. (C)

When there is no restriction 10 students can be arranged in 10! ways out of these consider the arrangements of A, B and C, the possible arrangements are (A, B, C), (A, C, B), (B, C, A), (B, A, C), (C, A, B), (C, B, A) in total 3! Or 6 ways out of these 6 only two are (B, A, C) and (C, A, B) which satisfy the given condition hence we can say that every 6 only two case satisfy the given condition.

Hence total number of ways is $\frac{2\times 10!}{6} = \frac{10!}{3}$

80. (A)

$$\lim_{n\to\infty} \frac{(n+2)!+(n+3)!}{(n+1)!-(n+3)!} = \lim_{n\to\infty} \frac{(n+2)![1+n+3]}{(n+1)![1-(n+3)\,(n+2)]}$$

$$= \lim_{n\to\infty} \frac{(n+1)!(n+2)\,(n+4)}{(n+1)![1-(n^2+5n+6)]} = \lim_{n\to\infty} \frac{n^2+6n+8}{-n^2-5n-5} = \lim_{n\to\infty} \frac{1+\frac{6}{n}+\frac{8}{n^2}}{-1-\frac{5}{n}-\frac{5}{n^2}} = -1$$

81. (56)

Equation of radical axis (i.e. common chord) of the two circles is

$10x + 4y - a - b = 0 \quad \ldots(i)$

Centre of first circle is H(–4, –4).

Since second circle bisects the circumference of the first circle, therefore, Centre

H(–4,–4) of the first circle must lie on the common chord (i)
$\therefore$ $-40 - 16 - a - b = 0 \Rightarrow a + b = -56$

82. (53)
Here $f(x_i) \neq x_i \forall x_i \in \{1, 2, 3, ... 6\}$, so de-arrangement of 6 article can be done in 265 ways
Since f(1) =2 so required number of ways is 265/5 = 53

83. (24)
Here Identify the form of limit, here given limit is in the form of $\frac{0}{0}$
So $\underset{x \to 1}{Lt} \frac{2(x-1)}{7+x-8} \times (7+x)^{2/3} + 8^{2/3} + \{8(7+x)\}^{1/3}$
$= 2[8^{2/3} + 8^{2/3} + (64)^{1/3}] = 2[12] = 24$

84. (495)
We have to select 4 days.
Not selected days =11 days
Gaps = 12
$\therefore$ Required number of ways $= {}^{12}C_4 = \frac{12 \times 11 \times 10 \times 9}{24} = 495$

85. (0.6)
There are two cases when Abhay will say 'Yes'-
Case (i)
The number that came out is a prime and Abhay is speaking truth, probability for this case is
$P(P) \times P(T)$
Here P(P) = probability of getting a prime = 3/6 = 1/2 = 0.5
P(T) is probability that Abhay is speaking truth and P(T) = 0.6
So probability for this case is $0.5 \times 0.6 = 0.3$
Case (ii)
The number that came out is not a prime and Abhay is not speaking truth, probability for this case is
$P(P') \times P(T') = 0.5 \text{ x } 0.4 = 0.2$
So total probability for the given case is 0.3 + 0.2 = 0.5
New sample space is 0.5 and we have to find the probability of case (i) which is 0.3/0.5 = 0.6

86. (3)
$\because \alpha, \beta$ are the roots of $x^2 - 6x - 2 = 0$
$\therefore \alpha^2 - 6\alpha - 2 = 0$
$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$
$\Rightarrow \alpha^{10} - 2\alpha^8 = 6\alpha^9$ . . . (i)
Similarly $\beta^{10} - 2\beta^8 = 6\beta^9$ . . . (ii)
From equation (i) and (ii)
$\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9)$
$\Rightarrow a_{10} - 2a_8 = 6a_9$

$\Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3.$

87. (8)
The function will attain its minimum value at x =-2
Required minimum value is 8.

88. (16)
If a number is both perfect square and perfect cube then its factors must have all the powers of prime factors must be divisible by 6.
Perfect square factor must be $2^{(0 \text{ or } 6)}3^{(0 \text{ or } 6)}5^{(0 \text{ or } 6)}7^{(0 \text{ or } 6)}$

(2 ways) (2 ways) (2ways) (2ways)

Hence total number of perfect cube factors are $2 \times 2 \times 2 \times 2 = 16$

89. (150)
Here we have two cases-
Case (i) 2 + 2 + 1 = 5, total number of ways in this case is $({}^5C_2)({}^3C_2)({}^1C_1)(3) = 90$
Case (ii) 3 + 1 + 1 = 5, total number of ways in this case is $({}^5C_1)({}^4C_1)({}^3C_3)(3) = 60$
Hence total number of ways is 90 + 60 = 150

90. (2)
$\tan x \tan 4x = -1 \Rightarrow \cos 4x \cos x + \sin 4x \sin x = 0$

$\Rightarrow \cos 3x = 0 \Rightarrow 3x = (2n+1)\dfrac{\pi}{2} \Rightarrow x = \dfrac{(2n+1)\pi}{6},\ n \in I \Rightarrow x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$, but $x \neq \dfrac{\pi}{2}$