PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(A)	3.	(A)	4.	(A)	5.	(A)
6.	(C)	7.	(B,C)	8.	(A,D)	9.	(A,D)	10.	(A,B,C,D)
11.	(A, B, C)	12.	(A, B)	13.	(7.00)	14.	(15.00)	15.	(40.00)
16.	(4.00)	17.	(34.00)	18.	(40.00)				

SOLUTIONS

1. (B)

$[a] = T^2$

$[x] = L$

$[P] = ML^{-1}T^{-2} = \dfrac{T^2}{[b]L}$

$[b] = \dfrac{T^2}{ML^{-1}T^{-2}L} = M^{-1}T^4$

$\therefore \dfrac{[a]}{[b]} = \dfrac{T^2}{M^{-1}T^4} = MT^{-2}$

2. (A)

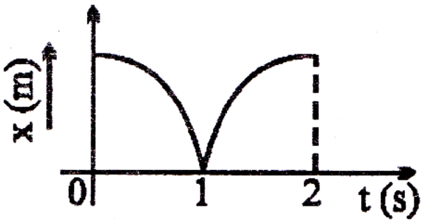
Zero error $= -(10-7)\times 0.1 = -0.3\,\text{mm}$

Diameter $= 77.0 + 8\times 0.1 - (0.3)$

$= 78.1\,\text{mm}$

3. (A)

From figure



For time interval $t = 0$ to $t = 1$ sec

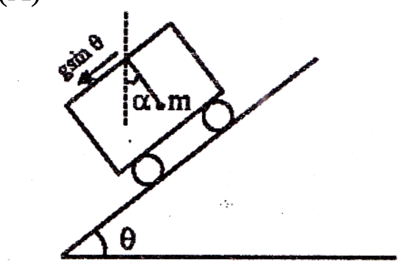
Slope of x-t graph is negative and increasing , so velocity increases in negative direction

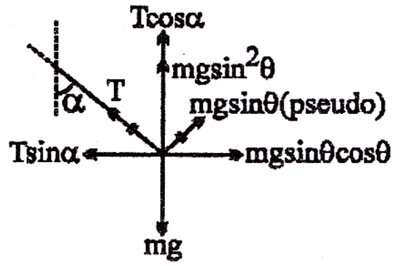
For $t = 1$ to 2 sec

The slope is positive and decreasing, so velocity is decreasing in positive direction and become zero at $t = 2$

So, (A) is correct

4. (A)





$T\sin\alpha = mg\sin\theta\cos\theta$ ......(1)

$T\cos\alpha = mg\cos^2\theta$ ....(2)

$\tan\alpha = \tan\theta$

$\Rightarrow \alpha = 0$

5. (A)

$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

So when PE is minimum then KE is maximum,

Or $\frac{d}{dx}\left(V(x)\right) = 0 \Rightarrow x^3 - x = 0$

$\Rightarrow x\left(x^2 - 1\right) = 0$

$\Rightarrow x = 0, \pm 1$

$V(0) = 0$ (maximum)

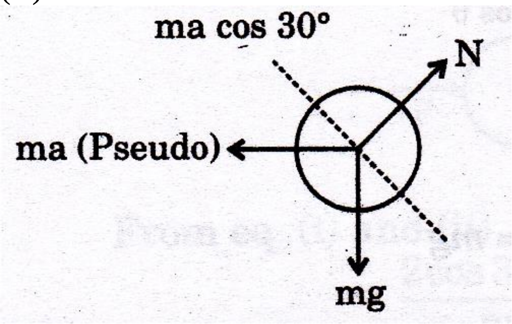
$V(\pm 1) = -\frac{1}{4}$ (minimum)

$\Rightarrow KE_{max} = TE - PE_{min}$

$\frac{1}{2}mV_{max}^2 = 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$

$V_{max} = \sqrt{\frac{2(9/4)}{1Kg}} = \frac{3}{\sqrt{2}}\, m/s$

6. (C)



Ball will leave frame when $ma\cos 30^\circ \geq mg\sin 30^\circ$ .So, $a \geq g\tan 30^\circ$.

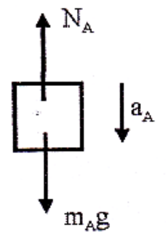
7. (B,C)

The frequency at which maximum intensity of emission occurs is proportional to $\frac{1}{T}$ while the rate of the emitted radiation is proportional to T^4 .

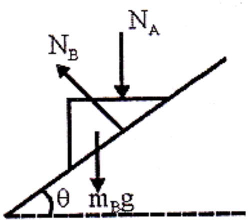
8. (A, D)

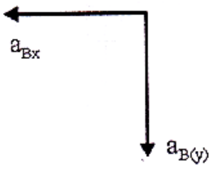
9. (A, D)
For rate of melting of ice to be doubled, the rate of flow must be doubled.
$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{L}$$

10. (A, B, C, D)
FBD of A



FBD of B

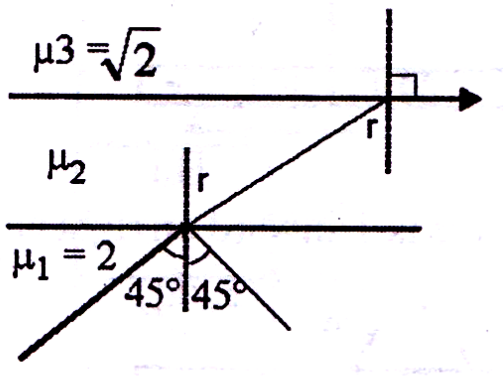




In frame B: B look A moves in positive x direction.

11. (A, B, C)
If tension is same in all parts of rope, the monkey will have to be at rest.

12. (A, B)



For $\mu_2 > \sqrt{2}$ (TIR will not take place)

$2\sin 45^\circ = \mu_2 \sin r$

$\mu_2 \sin r = \sqrt{2}\sin e$

$e = 90^\circ$

Hence deviation is 45°

$\mu_2 < \sqrt{2}$ TIR will take placed deviation is 90°

13. (7.00)

$v = u + at$

$v_1 = 10 + 4\times 4 = 26 m/s$

$S_1 = 10\times 4 + \frac{1}{2}\times 4\times 16 = 40 + 32 = 72m$

Now $a = 8 - t$

$\frac{dv}{dt} = 8 - t$

$\int_{26}^{v} dv = \int_{4}^{t} (8-t) = dt$

$v - 26 = \left[8t - \frac{t^2}{2}\right]_4^t$

$v = 26 + \left(8t - \frac{t^2}{2}\right) - \left(32 - \frac{16}{2}\right)$

$v = 26 + 8t - \frac{t^2}{2} 24$

$v = 2 + 8t - \frac{t^2}{2}$

$\therefore \frac{dx}{dt} = 2 + 8t - \frac{t^2}{2}$

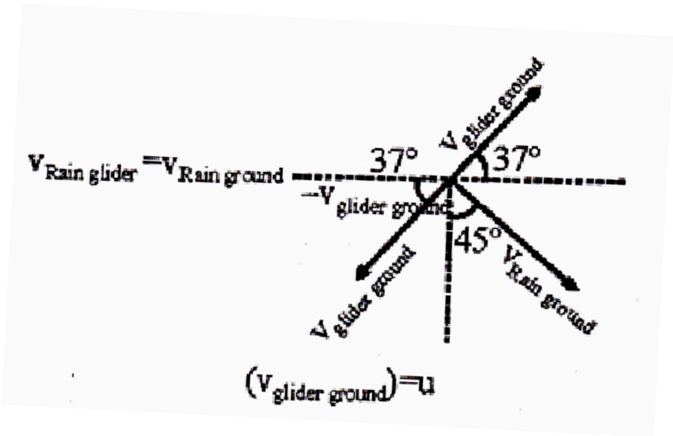
$x = \left[2t + 4t^2 - \frac{t^3}{6}\right]_4^8 = \left(16 + 64\times 4 - \frac{8^3}{6}\right) - \left(8 + 64 - \frac{64}{6}\right) = 8 + 64\times 3 - 74.6$

$= 200 - 74.6 = 125.33$

$\therefore$ Total distance $= 72 + 125.33 = 197.33 \simeq 197$]

14. (15.00)

$\vec{V}_{\text{Rain, glider}} = \vec{V}_{\text{Rain, ground}} - \vec{V}_{\text{glider, ground}}$

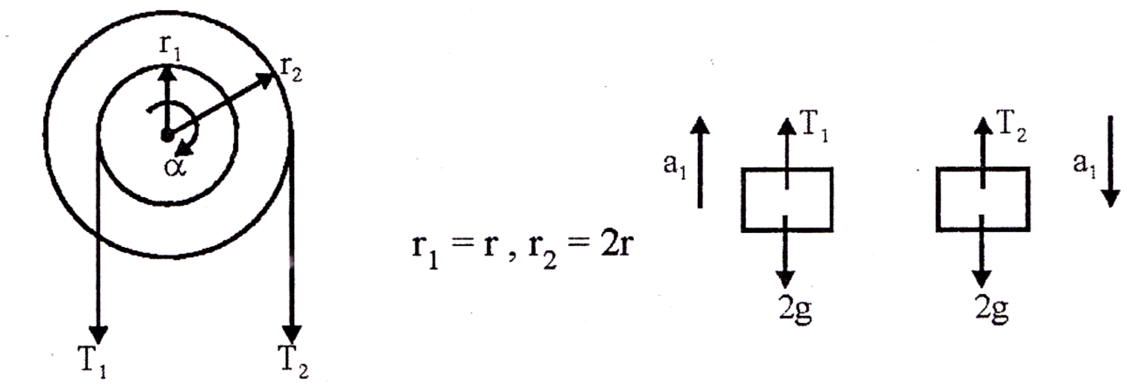


$u\cos 37^\circ = 12\sqrt{2}\sin 45^\circ \quad u = 15 m/s$

15. (40.00)

Both block moves downward with acceleation $g\left(=10 m/s^2\right)$ by using string constraint pully p will move upward with acceleration $40 m/s^2$.

16. (4.00)



$a_1 = r\alpha, a_2 = 2r\alpha$ ......(i)

$T_2 2r - T r = I\alpha$ ....(ii)

$\therefore \left(I = 10r^2\right)$

$T_1 - 2g = 2a_1$ ....(iii)

$2g - T_2 = 2a_2$ ......(iv)

On solving above equation

$a_2 = 4 m/s^2$

17. (34.00)

The rate of the heat transfer is approximately proportiona to the temperature differnce, between radiatior and room as well as between the room and the outside. The corresponding proportionality constant can be denoted as C ("radiator-room") and D ("room-outside")

Then, initially

$C(T-300)=D(300-260)$

For the second set of temperatures:

$C(T-290)=D(290-240)$

Solving the equation yields $T=340K$

18. (40.00)

The quantity of heat conducted per second through the bottom rod making the 'Y' gets divided equally at the junction of the three rods. If 'θ' is the temperature of the junction, we have $KA(\theta - 0)/L = 2KA(600 - \theta)/L$ where K is the thermal conductivity, A is the area of cross section and L is the length of the identical rods.

[Note that the L.H.S. is the quantity of heat conducted through the lower single rod making the 'Y' and the R.H.S. is the sum of the quantities of heat conducted through the upper two rods].

The above equation yields $\theta = 400°C$

PART (B) : CHEMISTRY

ANSWER KEY

19.	(D)	20.	(C)	21.	(C)	22.	(A)	23.	(B)
24.	(D)	25.	(A,B,C)	26.	(A,B,C,D)	27.	(B,C)	28.	(A,B,D)
29.	(B)	30.	(A,C,D)	31.	(75.00)	32.	(5.00)	33.	(50.00)
34.	(85.10)	35.	(5.00)	36.	(5.00)				

SOLUTIONS

19. (D)

20. (C)

$PM = dRT$

$\Rightarrow \frac{P_1M_1}{d_1} = \frac{P_2M_2}{d_2}$

$\therefore \frac{P_1}{P_2} = \left[\frac{d_1}{d_2}\right] \times \left[\frac{M_2}{M_1}\right] = \frac{4}{1}$

21. (C)

$H_2O(l) \longrightarrow H_2O(g)(100^\circ C) \longrightarrow H_2O(g)(300^\circ C)$

1 bar 2 bar

$q_1 = 2 \times 10.8 kcal$

$= \Delta H_1 = 21.6 kcal$

$\Delta H_2 = nC_P\Delta T = 2 \times \frac{4 \times 2 \times 200}{1000} = 3.2 kcal$

$\Delta H_2 = \Delta H_1 + \Delta H_2 = 24.8 kcal$

22. (A)

23. (B)

24. (D)

(A) As $T\uparrow, k_{eq}\uparrow \Rightarrow 1\Delta H > 0$

(B) $\Delta G^\circ_{300} > 0; \Delta G^\circ_{400K} < 0$

(C) $\text{In}\left(\frac{1.2}{0.98}\right) = \frac{\Delta H}{R}\left[\frac{1}{300} - \frac{1}{400}\right]$

$\Rightarrow \Delta H^\circ_R = 1200R \times \ln\left(\frac{1.2}{0.98}\right)$

(D) $\Delta S^\circ_R = \frac{\Delta H^\circ - \Delta G^\circ}{T}$

$= \left[\frac{1200R}{300}\ln\left(\frac{1.2}{0.98}\right) + R\ln(0.98)\right]$

25. (A,B,C)

26. (A,B,C,D)

$AgCl(s) \longrightarrow Ag^{+} + Cl^{-}(aq)$ $\qquad K_{sp} = sx$

$\underset{x}{Ag^{+}} + \underset{0.2}{2NH_3} \rightleftharpoons \underset{s}{Ag(NH_3)_2^{+}}$ $\qquad 10^{8} = \dfrac{s}{(0.2)^2 \times x}$

$10^{-2} = \dfrac{s^2}{(0.2)^3}$

$\left[Cl^{-}\right] = s = 2\times10^{-2}M = \left[Ag(NH_3)_2^{+}\right]$

$x = \left[Ag^{+}\right] = \dfrac{10^{-10}}{2\times10^{-2}} = 5\times10^{-9}M$

$Ag^{+} + NH_3 \rightleftharpoons Ag(NH_3)^{+}$ $\qquad K = 10^{3} = \dfrac{\left[Ag(NH_3)^{+}\right]}{5\times10^{-9}\times0.2}$

$\left[Ag(NH_3)^{+}\right] = 10^{-6}M$

27. (B,C)

28. (A,B,D)

29. (B)

30. (A,C,D)

31. (75.00)

$300\times(0.5)^{\gamma-1} = T_2\times(4)^{\gamma-1}$

$\Rightarrow T_2 = \dfrac{300}{(8)^{1/3}} = 150K$

$\omega = \dfrac{0.88}{44}\times3\times\dfrac{25}{3}\times150 = 75\,J$

32. (5.00)

$2A(g)$	$+\ H_2O(g)$	$\rightleftharpoons$	$C(g)$	$+\ 3D(g)$
2	0.05		2	2
~ 0	0.05		3	5

$3\times10^{22} = \dfrac{3\times5^3}{5\times10^{-2}\times P_A^2} \Rightarrow P_A = 5\times10^{-10}$

33. (50.00)

$$2CH_3COOH(l) \longrightarrow 2CH_4(g) + 2CO_2(g)$$

$$\Delta H = 2\left[\{50+50+350+350+450\}-\{400+800+25\}\right]$$

$$= 50\text{ kJ}$$

34. (85.10)

Solubility of $Ca(OH)_2$ in saturated solution

$$= \sqrt[3]{\frac{3.2\times10^{-5}}{4}} = 0.02M$$

$$x(0.2+2x)^2 = 3.2\times10^{-5}$$

$$\Rightarrow x = 0.8\times10^{-3}M$$

$$Ca(OH)_2 \text{ pptd.} = \left[[0.02\times0.25]-[0.8\times10^{-3}\times0.5]\right]\times74\times1000\text{ mg}$$

$$= 85.10\text{ mg}$$

35. (5.00)

36. (5.00)

PART (C) : MATHEMATICS

ANSWER KEY

37.	(C)	38.	(D)	39.	(A)	40.	(C)	41.	(A)
42.	(B)	43.	(A,B,C)	44.	(A,C)	45.	(B,C)	46.	(A, B)
47.	(B,C)	48.	(A,C)	49.	(12.50)	50.	(0.66 or 0.67)	51.	(46.00)
52.	(20.00)	53.	(81.00)	54.	(16.00)				

SOLUTION

37. (C)

Given line is 3x + y = 8

and given parabola is

$(y - 2)^2 = 4(x - 1)$

its vertex is = (1, 2)

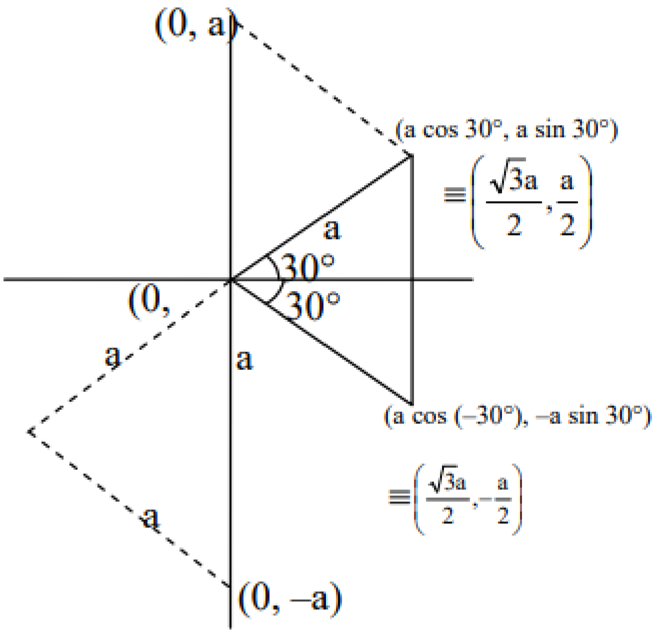
and its focus is = (1 + 1, 2) = (2, 2)

Since given line 3x + y = 8 passes through focus. therefore given line is the focal chord of parabola Therefore A and B are end points of focal chord. the tangents drawn at A and B meets its directrix at right angle.

Therefore points of intersection lie on its directrix is

$x + 1 - 1 = 0 \Rightarrow x = 0$

38. (D)



Hence, co-ordinates of the third vertex will be given by (0, a), (0, –a) and $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$

39. (A)

4^{th} term from the end

$= (7 - 4 + 2)^{th}$ term from beginning

$\Rightarrow 5^{th}$ term from beginning

$$T_5 = {}^7C_4 \left(\frac{x^3}{2}\right)^3 \left(-\frac{2}{x^2}\right)^4 = \frac{35}{8} . 16x = 70x$$

40. (C)

$$A = \frac{2+3}{2} = \frac{5}{2}$$

$$H = \frac{2.2.3}{2+3} = \frac{12}{5}$$

$$\Rightarrow A + \frac{6}{H} = \frac{5}{2} + \frac{5}{2} = 5$$

41. (A)

Let $\cot^{-1}(3) = \theta \Rightarrow \cot\theta = 3 \Rightarrow \cos\theta = \frac{3}{\sqrt{10}}$

Now, $\tan^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\frac{3}{\sqrt{10}}}{1+\frac{3}{\sqrt{10}}} = \frac{\sqrt{10}-3}{\sqrt{10}+3} = \frac{1}{\left(\sqrt{10}+3\right)^2}$

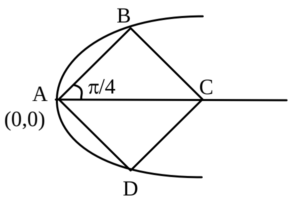
$\tan\left(\frac{\theta}{2}\right) = \left(\sqrt{10}+3\right)^{-1}$ as $\tan\left(\frac{\theta}{2}\right) > 0$

42. (B)

$$\lim_{x\to\infty} \frac{x^3+x^2+1}{x^3-x^2+1}$$

$$\lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{1}{x^3}}{1-\frac{1}{x}+\frac{1}{x^3}} = 1$$

43. (A, B, C)



AC is one diagonal along x-axis then the other diagonal is BD where both B and D lie on parabola.

Also slope of AB is tan $\frac{\pi}{4} = 1$

If B is $(at^2, 2at)$ then the slope of AB

$= \frac{2at}{at^2} = \frac{2}{t} = 1$

$\therefore$ t = 2

$\therefore$ B is (4a, 4a) and hance D is (4a, – 4a)

Clearly C is (8a, 0)

44. (A, C)

$x^2 + 9x - 4x - 6 = 0$

$\Rightarrow x^2 + 5x - 6 = 0$

$\Rightarrow x^2 + 6x - x - 6 = 0$

$\Rightarrow x(x + 6) - 1(x + 6) = 0$

$\Rightarrow (x + 6)(x - 1) = 0$

$\Rightarrow x = 1, -6$

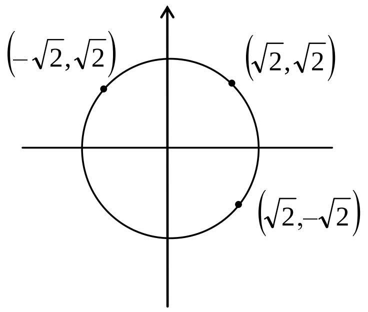
when x = 1, y = ±3 x = – 6x

A(1, 3) & B(1, –3) are common points

Length of common chord AB

$AB = \sqrt{(6)^2} = 6$

45. (B, C)

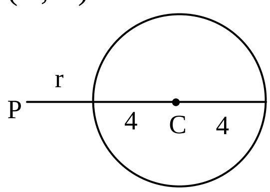


Clearly from above diagram

Required co-ordinates of Q are given by

$(-\sqrt{2}, \sqrt{2})$ and $(\sqrt{2}, -\sqrt{2})$

46. (A, B)



Slope of PC $= \frac{-3-2}{2-7} = \frac{-5}{-5} = 1$

If tanθ = 1, $\because$ θ = 45°

Equation of PA is

$\frac{x-7}{1/\sqrt{2}} = \frac{y-2}{1/\sqrt{2}} = r$

$\therefore \left(7 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$ lie on circle then

$\left(7+\frac{r}{\sqrt{2}}\right)^2 + \left(2+\frac{r}{\sqrt{2}}\right)^2 - 4\left(7+\frac{r}{\sqrt{2}}\right) + 6\left(2+\frac{r}{\sqrt{2}}\right) - 3 = 0$

$\Rightarrow r^2 + 10\sqrt{2}\,r + 34 = 0$

$\therefore r = -5\sqrt{2} \pm 4$

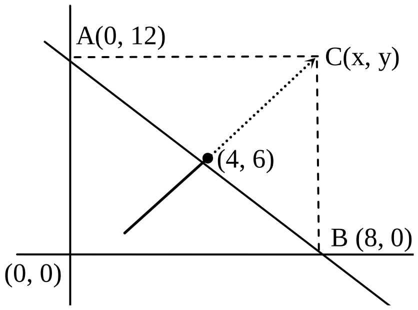
$\therefore$ points are $\left(7+\frac{-5\sqrt{2}\pm 4}{\sqrt{2}}, 2+\frac{-5\sqrt{2}\pm 4}{\sqrt{2}}\right)$

$\Rightarrow (2 \pm 2\sqrt{2}, -3 + 2\sqrt{2})$

$\therefore$ B $(2 + 2\sqrt{2}, -3 + 3\sqrt{2})$

A $(2 - 2\sqrt{2}, -3 - 2\sqrt{2})$

47. (B, C)



Equation of perpendicular bisector of AB is

$2x - 3y + \lambda = 0$

It passes through (4, 6)

$2(4) - 3(6) + \lambda = 0$

$\Rightarrow 8 - 18 + \lambda = 0$

$\Rightarrow \lambda = 10$

So equation is $2x - 3y + 10 = 0$ ... (1)

Area of $\Delta ABC = 91$

$\frac{1}{2}\begin{vmatrix} 0 & 8 & x \\ 12 & 0 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm 91 \Rightarrow \begin{vmatrix} 0 & 8 & x \\ 12 & 0 & y \\ 1 & 1 & 1 \end{vmatrix} = \pm 182$

$\Rightarrow -8(12 - y) + x(12) = 182$

$\Rightarrow -96 + 8y + 12x = 182$

$\Rightarrow 12x + 8y - 278 = 0$

$\Rightarrow 6x + 4y - 139 = 0$ (+ve) ... (2)

By Eqn. (1) and (2)

$\Rightarrow 3(6x - 9y + 30 = 0)$

$6x + 4y - 139 = 0$

$-\quad -\quad +$

$-13y = -169$

$y = 13$

From (1), $2x - 39 + 10 = 0$

$\Rightarrow 2x = 29$ $\Rightarrow x = 29/2$

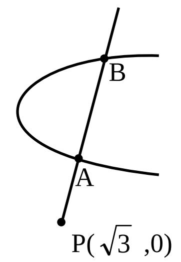
$\therefore$ C is (29/2, 13) Ans.

Also
$12x + 8y + 86 = 0$ (–ve)
$\Rightarrow 6x + 4y + 43 = 0$ ... (1)
$\Rightarrow 6x + 4y + 43 = 0$ ... (1)
$6x - 9y + 30 = 0$ ... (2)
$-\quad +\quad -$
$13y = -13$
$y = -1;\ 6x - 4 + 43 = 0;\quad \Rightarrow 6x = -39$
$\Rightarrow x = -\frac{39}{6} = -\frac{13}{2}$
$\therefore \left(-\frac{13}{2}, -1\right)$

48. (A, C)
$\because 7^9 + 9^7 = (8-1)^9 + (8+1)^7$
$= ({}^9C_0\, 8^9 - {}^9C_1\, 8^8 + \ldots.. {}^9C_8\, 8 - 1) + ({}^7C_0\, 8^7 + {}^7C_1\, 8^6 + \ldots\ldots + 1)$
$= 8^9 - 9.8^8 + \ldots.. + 9.8 + 8^7 + 7.8^6 + \ldots.. + 7.8$
$= 128 + (8^9 - 9.8^8 + \ldots\ldots + {}^9C_7\, 8^2) + (8^7 + 7.8^6 + \ldots\ldots + {}^7C_6\, 8^2)$
= It is divisible by 16 and 64

49. (12.50)
Length of focal chord
$= a\left(t + \frac{1}{t}\right)^2$
Here $4a = 8$
$a = 2$
and $2at = 8$
$\Rightarrow 2 \times 2 \times t = 8$
$\Rightarrow t = 2$
$\therefore$ length of focal chord
$= a\left(t + \frac{1}{t}\right)^2$
$= 2\left(2 + \frac{1}{2}\right)^2$
$= 2\left(\frac{5}{2}\right)^2$
$= \frac{25}{2}$

50. (0.66 or 0.67)
Given parabola is $y^2 = x + 2$ and
given line is $y = \sqrt{3}x - 3$ and $P \equiv (\sqrt{3}, 0)$



AB makes an angle of 60° with the positive direction of x-axis. Co-ordinates of any point on this line may be taken as ($\sqrt{3}$ + r cos 60°, 0 + r sin 60°) i.e. $\left(\sqrt{3}+\frac{r}{2},\frac{r\sqrt{3}}{2}\right)$

If this point lies on $y^2 = x + 2$ then.

$\frac{3}{4}r^2 = \sqrt{3} + \frac{r}{2} + 2$ or $3r^2 = 4\sqrt{3} + 2r + 8$

or $3r^2 - 2r - 4(2+\sqrt{3}) = 0$ ....(1)

Let r_1 and r_2 be the roots of equation (1)

Then $r_1 + r_2 = \frac{2}{3}$ but $r_1 r_2 < 0$

$\therefore \ |PA - PB| = \frac{2}{3}$

51. (46.00)

Length of intercept = $2\sqrt{a^2 - p^2}$

where a is radius of circle and p is length of perpendicular

p = $\frac{2+3-2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$ and a = $\sqrt{4+9+3} = 4$

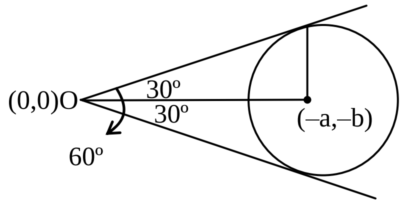
$\therefore$ length of intercept = $2\sqrt{16 - \frac{9}{2}}$

= $2\sqrt{\frac{32-9}{2}}$ = $2\sqrt{\frac{23}{2}}$

= $\sqrt{2}.\sqrt{23}$ = $\sqrt{46}$

$\Rightarrow \ \ell^2 = 46$

52. (20.00)



We know that tan $\frac{\theta}{2}$ = $\frac{\sqrt{g^2+f^2-c}}{\sqrt{s_1}}$

$\therefore$ tan 30º = $\frac{\sqrt{a^2+b^2-5}}{\sqrt{5}}$

$$\therefore \frac{1}{\sqrt{3}} = \frac{\sqrt{a^2+b^2-5}}{\sqrt{5}}$$

$\therefore$ on squaring

$$\frac{1}{3} = \frac{a^2+b^2-5}{5} \Rightarrow a^2 + b^2 - 5 = \frac{5}{3}$$

$$\Rightarrow a^2 + b^2 = 5 + \frac{5}{3} \Rightarrow a^2 + b^2 = \frac{20}{3}$$

$\therefore$ (a, b) lies on circle $x^2 + y^2 = r^2$

$$\therefore r^2 = \frac{20}{3}$$

$$\therefore r = \sqrt{\frac{20}{3}} = \sqrt{\frac{k}{3}}$$

$$\Rightarrow k = 20$$

53. (81.00)

Total case = $^{20}C_2 = \frac{20.19}{2} = 190$

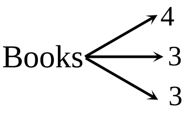
Prime no.s = 2, 3, 5, 7, 11, 13, 17, 19 = 8

Favourable case = $^{8}C_2 = 28$

Required probability = $\frac{28}{190} = \frac{14}{95}$

$$\Rightarrow q - p = 81$$

54. (16.00)



$$\left[\frac{10!}{4!3!3!2!}\right] \times 3! \times 2!$$

Total = $\frac{12\,!}{4!\,4!\,4!\,3!} \times 3!$

$$\therefore p = \frac{8}{11} \Rightarrow 22p = 16$$