PART (A) : PHYSICS

ANSWER KEY

1.	(2)	2.	(4)	3.	(3)	4.	(4)	5.	(1)
6.	(1)	7.	(A, C)	8.	(B, D)	9.	(A, B)	10.	(A, B, D)
11.	(A, B, C)	12.	(A, C, D)	13.	(8.00)	14.	(4.00)	15.	(3.00)
16.	(60.00)	17.	(10.00)	18.	(21.00)				

SOLUTION

1. (2)

Let they meet after time t sec

$V_1 \cos 45^\circ t = V_2 \cos 60^\circ t = a$

So $v_1 t = \sqrt{2}\,a,\ v_2 t = 2a$

For vertical $\left(V_2 \sin 60^\circ - V_1 \sin 45^\circ\right)t = h \Rightarrow\ h = 2a \times \frac{\sqrt{3}}{7} - \sqrt{2}a \times \frac{1}{\sqrt{2}} = \left(\sqrt{3}-1\right)a$

$\Rightarrow\ h = \left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right) = 2m$

2. (4)

$ma = 2T - mg\sin\theta$ and $ma = mg - 2T$

$\Rightarrow a = \frac{(M - \sin\theta)}{M+m} g = \frac{\left(15 - 10 \times \frac{1}{2}\right)}{15+10} \times 10 = 4$

3. (3)

$T_1 \cos 37^\circ - T_2 \cos 37^\circ = mg$

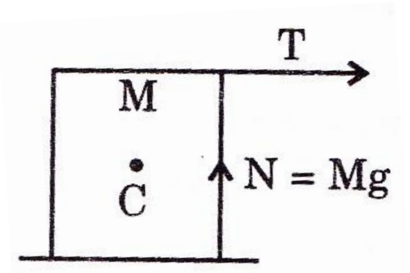
$T_1 \sin 37^\circ + T_2 \sin 37^\circ = m\omega^2 r$

4. (4)

$h_1 = e^2 h, h_2 = e^4 h, h_3 = e^6 h$ …..

$h_3 = e^6 h = \left(\frac{1}{2}\right)(64m) = 32m$

5. (1)



For toppling N will reach to one end

$mg - T = ma$ ……(i) and $T = Ma$

So, $a = \dfrac{mg}{M+m}$ $\quad \Rightarrow T = Ma = \dfrac{Mmg}{M+m}$

Considering torque about C

$T\dfrac{h}{2} = Mg\dfrac{h}{4}$ ......(iii) $\Rightarrow \dfrac{Mmg}{M+m}\dfrac{h}{2} = Mg\dfrac{h}{4}$

$2 = 1 + \dfrac{M}{m}$ $\quad \Rightarrow \dfrac{M}{m} = 1$

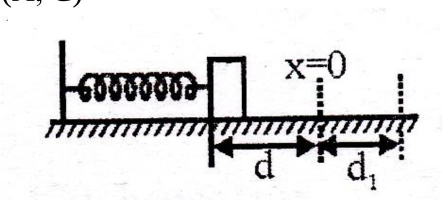
6. (1)

$\because \left(\dfrac{\Delta Q}{\Delta t}\right)_A = \left(\dfrac{\Delta Q}{\Delta t}\right)_B$

$\therefore \dfrac{K_A A(100-70)}{30} = \dfrac{K_B A(70-35)}{70}$

$\Rightarrow K_A = \dfrac{K_B}{2} \Rightarrow \dfrac{K_A}{K_B} = \dfrac{1}{2}$

7. (A, C)



The system will lose its entire mechanical energy if the block finally comes to rest at the mean position. Now suppose the block comes to rest for the first time after the start at a distance d_1 from the mean position on the other side, then

$\dfrac{1}{2}k\left(d^2 - d_1^{\,2}\right) - \mu mg\left(d + d_1\right) = 0$

(From work energy theorem, $\Delta K = 0$)

$\Rightarrow d - d_1 = \dfrac{2\mu mg}{k} \Rightarrow d_1 = d - \dfrac{2\mu mg}{k}$

Now, similarly if $d_2, d_3, ...etc$ are the successive distances of the block from the mean position where it come to rest for the second, third times etc. then $d_2 - d_3 = d_1$

$-d_2 = d - d_1 = \dfrac{2\mu mg}{k}$ $\quad \Rightarrow d_2 = d_1 - \dfrac{2\mu mg}{k} = d - \dfrac{4\mu mg}{k}$

$d_3 = d_2 - \dfrac{2\mu mg}{k} = d - \dfrac{6\mu mg}{k}$

$d_n = d - \dfrac{2n\mu mg}{k}$

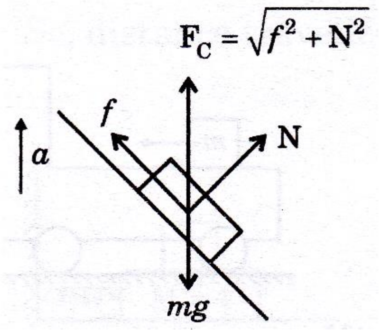
We want $d_n = 0$

$\Rightarrow d = n\left(\dfrac{2\mu mg}{k}\right) = n \times \dfrac{2 \times 0.4 \times 10}{200} = n \times 0.4\,m = n \times 4\,cm$

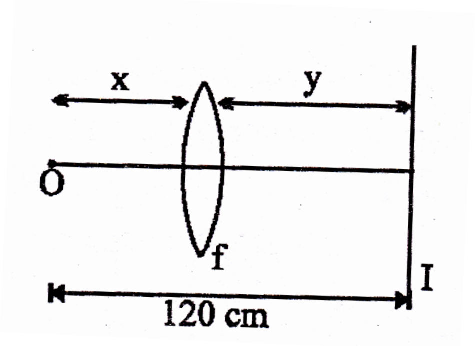
i.e. d should be an integral multiple of $\dfrac{2\mu mg}{k}$

8. (B, D)

Net force should be upwards, because mg is downwards so resultant contact, force (resultant of friction and normal) must be upwards.



9. (A,B)



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{22.5} = \frac{1}{y} - \frac{1}{-x} = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$$

$$x + y = 120 \quad xy = 120 \times 22.5$$

$$x(120 - x) = 120 \times 22.5$$

$$x^2 - 120x + 120 \times 22.5 = 0$$

$$x = \frac{120 \pm \sqrt{120^2 - 4 \times 120 \times 22.5}}{2}$$

$$= \frac{120 \pm \sqrt{120^{\circ} - 120 \times 90}}{2}$$

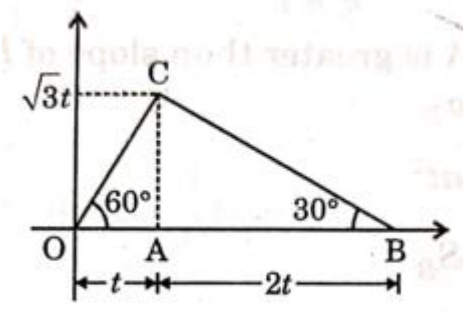
$$= \frac{120 \pm 10\sqrt{36}}{2} = 60 \pm 30 \qquad ; \qquad x = 30 \text{ or } 90$$

10. (A,B,D)
Plank may accelerate down due to component of gravitational force along the plane. It may accelerate up due to friction force acting by the man on the plank also it may be at rest. Ram can apply static friction on plank in any direction.

11. (A, B, C)

12. (A, C, D)

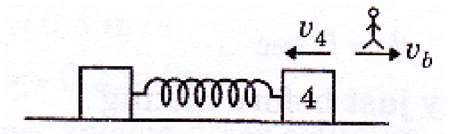
$$(V_{av})_{OA} = \frac{\text{Displacement}}{\text{Time}} = \frac{\text{Area}}{\text{Time}} = \frac{1}{2} \times \frac{t\sqrt{3}t}{t}$$



$$(V_{av})_{AB} = \frac{1}{2} \times \frac{3t \times \sqrt{3}t}{3t}$$

13. (8.00)

$\upsilon_b + \upsilon_4 = -7$ ....(i)



COM $0 = 10\upsilon_b - 4\upsilon_4$ .....(ii)

$$\upsilon_4 = \frac{70}{14} = 5$$

At υ_{max}, acceleration $= 0$

Spring is at normal length

$\Rightarrow 4 \times 5 = 1\upsilon_1 + 4\upsilon_4$ .....(i)

$$\Rightarrow \upsilon_4 = \frac{20 - \upsilon_1}{4}$$

$\frac{1}{2} \times 4 \times (5)^2 = \frac{1}{2} \times 1 \times \upsilon_1^2 + \frac{1}{2} \times 4 \times \upsilon_4^2$ .....(ii)

$$100 = \upsilon_1^2 + \frac{1}{4}\left(400 + \upsilon_1^2 - 4\upsilon_1\right)$$

$$\Rightarrow \frac{5\upsilon_1^2}{4} = \upsilon_1$$

$$\Rightarrow \upsilon_1 = 8\,\text{m/s}$$

14. (4.00)

At maximum compression of spring, velocities of blocks B and C are same $(\text{say } \upsilon_0)$

Then by conservation of linear momentum

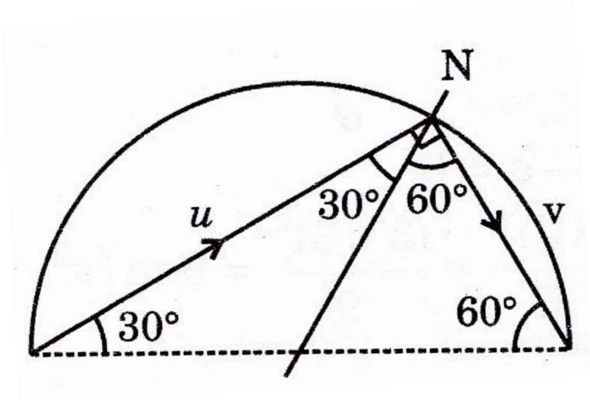
$$3(2) = (3+6)\upsilon_0$$

$$\upsilon_0 = \frac{2}{3}\,\text{m/s}$$

At this instant energy stored in spring

$$= \frac{1}{2}(3)(2)^2 - \frac{1}{2}(3+6)\left(\frac{2}{3}\right)^2 = 6 - 2 = 4\text{J}$$

15. (3.00)



For collision in normal direction: $eu\cos 30^{\circ} = \upsilon\cos 60^{\circ}$ and in tangential direction $u\sin 30^{\circ} = \upsilon\sin 60^{\circ}$. We get $e = \frac{1}{3}$

16. (60.00)

Beam is parallel to base $\Rightarrow$ min deviation

$$\mu = \frac{\sin\left(\frac{\delta+\gamma}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \Rightarrow \sqrt{3} = \frac{\sin\left(\frac{60+\gamma}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$\sin\left(\frac{60+\gamma}{2}\right) = \frac{\sqrt{3}}{2} \Rightarrow \frac{60+\gamma}{2} = 60$$

$$\gamma = 60^{\circ}$$

17. (10.00)

Refraction plane surface $h' = h\frac{\mu_r}{\mu_i} = \frac{20\times 3/2}{1} = 30\text{cm}$

Mirror

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{-45}=\frac{1}{-10}$$

$v = -\frac{90}{7}$ from pole of mirror distance of object from plane surface

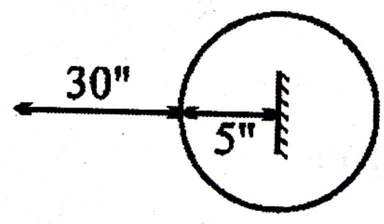
$$l = 15-\frac{90}{7} = \frac{105-90}{7} = \frac{15}{7}$$

Refraction at plane surface $x = 10\, l' = l\frac{\mu_r}{\mu_i}$

$$x = l = \frac{15}{7}\times\frac{1}{3/2} = \frac{10}{7}$$

$\Rightarrow 7x = 10$ (location of final image from plane surface)

18. (21.00)



$$\frac{4}{3v}-\frac{1}{-30}=\frac{\frac{4}{3}-1}{5}$$

$$\frac{4}{3v}=\frac{1}{15}-\frac{1}{30}=\frac{1}{30}$$

$$v=40" \qquad \Rightarrow u=35"$$

$$\frac{1}{v}-\frac{4/3}{+30}=\frac{1-4/3}{-5}$$

$$\frac{1}{v}-\frac{6}{90}+\frac{4}{90}$$

$$v=9"$$

Distance from observer $=21"$

PART (B) : CHEMISTRY

ANSWER KEY

19.	(3)	20.	(4)	21.	(3)	22.	(4)	23.	(5)
24.	(9)	25.	(A,B,C,D)	26.	(AB)	27.	(A,B,C,D)	28.	(A,B,D)
29.	(A,B)	30.	(ABC)	31.	(6.00)	32.	(60.00)	33.	(5.00)
34.	(90.00)	35.	(6.00)	36.	(4.00)				

SOLUTIONS

19. (3)
(B), (C) & (G) are correct statements.
After mixing of solutions & ppt. formation
$[Na^+]_{eqm} = 00.1M,\ [Cl^-]_{eqm} = 0.1M$
$[Ba^{2+}]_{eqm} = [SO_4^{2-}]_{eqm}$
$= \dfrac{0.20005 - 0.1 - 0.1}{2} = 0.000025\,M$
Hence, $K_{sp} = 6.25 \times 10^{-10}$

20. (4)
$$K_C = \frac{\frac{4}{4}}{\left(\frac{2}{4}\right)\left(\frac{1}{4}\right)^{1/2}} = 4$$

21. (3)
$$1.8 \times 10^{-5} = \frac{10^{-3} \times 10^{-3}}{\left[\frac{10/60}{V}\right]} \Rightarrow V = 3L$$

22. (4)

23. (5)

24. (9)

25. (A,B,C,D)

26. (AB)

27. (A,B,C,D)

28. (A,B,D)

29. (A,B)

30. (ABC)

31. (6.00)

$n \to 2; Z = 3$

$\Delta E = 10.2 + 17 = 27.2\,eV$

$27.2 = 13.6 \times 3^2 \left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

$\Rightarrow n = 6$

32. (60.00)

$V \times 0.25 = 50 \times 0.15 \times 2$

33. (5.00)

$\left[OH^-\right]_{Fe(OH)_2} = \sqrt{\frac{8 \times 10^{-16}}{0.08}} = 10^{-7}$

$\left[OH^-\right]_{Fe(OH)_3} = 10^{-9}$

$pH = 5$

34. (90.00)

$NH_4I(s) \rightleftharpoons NH_3(g) + HI(g)$

150 mm 150 mm $K_P = 150 \times 150$

$HI \rightleftharpoons \frac{1}{2}H_2 + \frac{1}{2}I_2$

$150 - P \quad \frac{P}{2} \quad \frac{P}{2}$

$\Rightarrow \frac{\frac{P}{2}}{150 - P} = 2 = K_P$

$P = 600 - 4P$

$5P = 600$

$P = 120$

$P_{HI} = 150 - 120 = 30$

$150 \times 150 = P_{NH_3} \times 30 \Rightarrow P_{NH_3} = 750mm$

Final total $P = P_{NH_3} + P_{HI} + P_{H_2} + P_{I_2}$

$= 750 + 30 + 60 + 60 = 900mm$

35. (6.00)

36. (4.00)

PART (C) : MATHEMATICS

ANSWER KEY

37.	(6)	38.	(3)	39.	(2)	40.	(7)	41.	(5)
42.	(6)	43.	(A,B)	44.	(A,B,C,D)	45.	(A,B,D)	46.	(A,B,C,D)
47.	(B,D)	48.	(B,C,D)	49.	(6.4)	50.	(3.14)	51.	(10.56)
52.	(16)	53.	(1.25 or 1.26)	54.	(4.00)				

SOLUTIONS

37 (6)

Here $\frac{\lfloor 10}{\lfloor p \lfloor q \lfloor r} 1^p . 3^{q/3} . 7^{r/7}$ where p + q + r = 10

case are possible which

p	q	r
10	0	0
7	3	0
4	6	0
1	9	0
3	0	7
0	3	7

Total cases are 6

38. (3)

3, P, K are in A.P.

P ↑ (Inserted number), K ↑ (Unknown number)

$\Rightarrow 2P = 3 + K$ …(1)

$\Rightarrow K = 2P - 3$

and 3, P – 6, K are in G.P.

$\Rightarrow (P-6)^2 = 3K$ …(2)

from (1) & (2), we get

$(P-6)^2 = 3(2P-3)$

$\Rightarrow P^2 - 18P + 45 = 0$

$\Rightarrow P = 3, 15$

$\therefore$ Unknown number K = 2P – 3 when P = 3

$K = 2 \times 3 - 3$

$K = 6 - 3$

K = 3 Ans.

When P = 15

$K = 2 \times 15 - 3$

$= 27$

$\therefore$ 3, 3, 3 A.P. 3, 15, 27 A.P.

3, –3, 3 G.P. 3, 9, 27 G.P.

$\therefore$ K = 3 and 27 Ans.

39. (2)

$$f(x)=\begin{cases}x^2+1, & x\ge 1\\ 3x-1, & x<1\end{cases}$$

L.H.L. = $\lim_{x\to 1^-}(3x-1)=2$

R.H.L. = $\lim_{x\to 1^+}(x^2+1)=2$

L.H.L. = R.H.L. = 2

40. (7)

A = letter has not reached B

B_1 = Son did not post the letter ; $P(B_1) = 1/4$

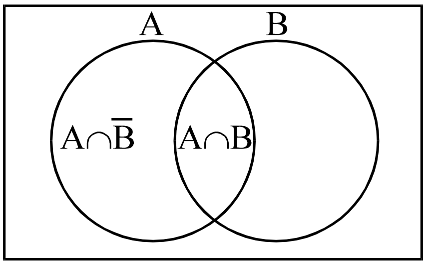
B_2 = son posted the letter ; $P(B_2) = 3/4$

$$P(A/B_1)=1;\ P(\bar{A}/B_2)=\frac{8}{9}\ \Rightarrow\ P(A/B_2)=\frac{1}{9}$$

$$P(B_1/A)=\frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1)+P(B_2).P(A/B_2)}$$

$$=\frac{\frac{1}{4}.1}{\frac{1}{4}.1+\frac{3}{4}.\frac{1}{9}}=\frac{1}{1+\frac{1}{3}}=\frac{3}{4}\qquad \therefore\ p+q=7.$$

41. (5)



$P(A^c) = 0.3 \Rightarrow P(A) = 0.7$

$P(B) = 0.4$; $P(A\cap B^c) = 0.5$

or $P(A) - P(A\cap B) = 0.5$

$\therefore$ $P(A\cap B) = 0.7 - 0.5 = 0.2$

Now $P(B / A\cup B^c) = \dfrac{P\left(B\cap(A\cup B^c)\right)}{P(A\cup B^c)}$

$$=\frac{P(B\cap A)+P(B\cap B^c)}{P(A\cup B^c)}$$

$$=\frac{P(A\cap B)}{P(A)+P(B^c)-P(A\cap B^c)}$$

$$=\frac{P(A\cap B)}{P(A)+1-P(B)-\left(P(A)-P(A\cap B)\right)}=\frac{P(A\cap B)}{1-P(B)+P(A\cap B)}=\frac{0.2}{1-0.4+0.2}$$

$$=\frac{0.2}{0.8}=\frac{1}{4}=\frac{p}{q}\Rightarrow\ (p+q)=1+4=5\ \text{Ans.}$$

42. (6)

We have $T_r-3T_{r-1}=6^r$

Dividing both sides by 3^r, we get

$$\frac{T_r}{3^r}-\frac{T_{r-1}}{3^{r-1}}=2^r$$

$$\Rightarrow \sum_{r=2}^{n}\left(\frac{T_r}{3^r}-\frac{T_{r-1}}{3^{r-1}}\right)=\underbrace{\sum_{r=2}^{n}2^r}_{\text{G.P.}}$$

$$\Rightarrow \left.\begin{array}{l}\frac{T_2}{3^2}-\frac{T_1}{3}\\ +\frac{T_3}{3^3}-\frac{T_2}{3^2}\\ +\frac{T_4}{3^4}-\frac{T_3}{3^3}\\ \quad\vdots \qquad \vdots\\ +\frac{T_n}{3^n}-\frac{T_{n-1}}{3^{n-1}}\end{array}\right\}=\underbrace{\sum_{r=2}^{n}2^r}_{\text{G.P.}}$$

$$\frac{T_n}{3^n}-\frac{T_1}{3}=4\left(2^{n-1}-1\right)$$

$$\frac{T_n}{3^n}-\frac{6}{3}=4\left(2^{n-1}-1\right) \quad [\because T_1=6]$$

$$\Rightarrow T_n=2\cdot3^n+4\cdot3^n\cdot2^{n-1}-4\cdot3^n=2^{n+1}\cdot3^n-2\cdot3^n$$

$$\Rightarrow T_n=2\left(6^n-3^n\right)$$

$$\text{So, } S_n=2\left(\sum_{1}^{n}6^n-\sum_{1}^{n}3^n\right)$$

$$=2\left[\frac{6}{5}(6^n-1)-\frac{3}{2}(3^n-1)\right]$$

$$\Rightarrow S_n=\frac{3}{5}\left[4\cdot6^n-5\cdot3^n+1\right]$$

Now on comparing, we get

$$\frac{(n^2-12n+39)(4\cdot6^n-5\cdot3^n+1)}{5}=\frac{3}{5}\left[4\cdot6^n-5\cdot3^n+1\right]$$

$$\Rightarrow n^2-12n+39=3 \Rightarrow (n-6)^2=0$$

Hence n = 6.

43. (A, B)

Given that $P(A\cap B)=\frac{1}{6}$ and $P(\overline{A}\cap\overline{B})=\frac{1}{3}$

$\therefore$ P(A) P(B) = $\frac{1}{6}$ and $P(\overline{A})\ P(\overline{B})=\frac{1}{3}$

$\Rightarrow$ P(A) P(B) = $\frac{1}{6}$ and [1 – P(A)] [1 – P(B)] = $\frac{1}{3}$

$\Rightarrow 1 - [P(A) + P(B)] + \frac{1}{6} = \frac{1}{3}$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ and $P(A)\,P(B) = \frac{1}{6}$

Solving we get

$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}$

or $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$

44. (A, B, C, D)

Let A, B, C be the events that the student is successful in test I, II and III respectively then

P(The student is successful)

$= P(A)\,P(B)\,\{1 - P(C)\} + P(A)\,\{1 - P(B)\}\,P(C) + P(A)\,P(B)\,P(C)$

$= pq\left\{1 - \frac{1}{2}\right\} + p(1 - q)\,\frac{1}{2} + p.q.\,\frac{1}{2}$

$= \frac{pq}{2} + \frac{p}{2}$

$\therefore \frac{1}{2} = \frac{pq + p}{2} \Rightarrow pq + p = 1$

Hence $p(1 + q) = 1$

This satisfies by $p = 1, q = 0$

and $p = \frac{2}{3}, \; q = \frac{1}{2}$

and $p = \frac{3}{5}, \; q = \frac{2}{3}$

so there are many values of p and q

so option A,B,C,D are correct

45. (A, B, D)

$7 \cos x - 24 \sin x = \lambda \cos(x + \alpha), \; 0 < \alpha < \pi/2$

$25\left(\frac{7}{25}\cos x - \frac{24}{25}\sin x\right) = \lambda \cos(x + \alpha)$

Let $\cos\alpha = \frac{7}{25}$ then $\sin\alpha = \frac{24}{25}$

$\Rightarrow 25 \cos(x + \alpha)$

So $\lambda = 25$

$\alpha = \cos^{-1}\frac{7}{25}$ and $\alpha = \sin^{-1}\frac{24}{25}$

46. (A, B, C, D)

$(x^2 + 1)^2 = x(3x^2 + 4x + 3)$

$\Rightarrow x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$

Let roots are $\alpha, \beta, \gamma, \delta$

$\Rightarrow \alpha + \beta + \gamma + \delta = 3$ and $\alpha\beta\gamma\delta = 1$

From option A, B, C, D,
Satisfy these conditions
$\Rightarrow$ option A, B, C, D are correct

47. (B, D)
$E = \pi - 3 + \pi - 4 + 5 - 2\pi = -2$

48. (B, C, D)
$\cos\left(-\frac{14\pi}{5}\right) = \cos\frac{14\pi}{5} = \cos\frac{4\pi}{5}$

Hence, $\cos\left(\frac{1}{2}\cos^{-1}\left(\cos\frac{4\pi}{5}\right)\right) = \cos\frac{4\pi}{10} = \cos\frac{2\pi}{5} \Rightarrow$ **BCD**

49. (6.4)
Probability of drawing a white ball is = $\frac{1}{2}$

Probability of not drawing a white ball is = $\frac{1}{2}$

we want to drawn a white ball 4^{th} time in 7^{th} draw
so a white ball drawn in 7^{th} draw and 3 white balls are drawn in first 6 draws
So required probability
$= {}^6C_3\, p^3 q^3 \,.\, p$

$= {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 . \frac{1}{2} = \frac{5}{32}$

$\therefore\ \frac{1}{p} = \frac{32}{5} = 6.4$

50. (3.14)
F.P. is $\pi = 3.14$ Ans.

51. (10.56)
$f(x) + f(-x) = 4\pi$

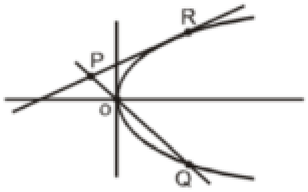
Put $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) + f\left(-\frac{1}{2}\right) = 4\pi$

So, $f\left(-\frac{1}{2}\right) = 4\pi - 2$

= 10.56 Ans.

52. (16)



$y = mx + \frac{a}{m}$ ..(i)

Equation of OP is

$y = -\frac{1}{m}x$ ...(ii)

$OP = \frac{a/m}{\sqrt{1+m^2}}$

Equation (ii) meets the parabola at Q

$\frac{1}{m^2}x^2 = 4ax \Rightarrow x = 4am^2, y = -4am$

$\therefore OQ = 4am\sqrt{1+m^2}, OP.OQ = 4a^2$

53. (1.25 or 1.26)

$\frac{\pi}{5} + \frac{\pi}{5} = \frac{2\pi}{5} = 1.25$

54. (4.00)

Add 1 to each fraction to get

$$\frac{r_4^2 + r_1^2 + r_2^2 + r_3^2}{r_2^2 + r_3^2 + r_4^2} + \frac{r_1^2 + r_2^2 + r_3^2 + r_4^2}{r_1^2 + r_3^2 + r_4^2} + \frac{r_1^2 + r_2^2 + r_3^2 + r_4^2}{r_1^2 + r_2^2 + r_4^2} + \frac{r_1^2 + r_2^2 + r_3^2 + r_4^2}{r_1^2 + r_2^2 + r_3^2}$$

This seems like a difficult problem until one realizes that

$$r_1^2 + r_2^2 + r_3^2 + r_4^2 = (r_1 + r_2 + r_3 + r_4)^2 - 2(r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4) = 4^2 - 2\cdot 8 = 0$$

Thus, out correct expression is 0.

Nothing that we added 4, the original value had to be – 4.