PART (A) : PHYSICS

ANSWER KEY

1.	(C)	2.	(A)	3.	(A)	4.	(B)	5.	(D)
6.	(B)	7.	(B)	8.	(A)	9.	(D)	10.	(A)
11.	(D)	12.	(A)	13.	(B)	14.	(A)	15.	(C)
16.	(B)	17.	(A)	18.	(B)	19.	(C)	20.	(D)
21.	(4)	22.	(2)	23.	(2)	24.	(158)	25.	(3)
26.	(19)	27.	(20)	28.	(101)	29.	(5)	30.	(3)

Solution

1. (C)

Work done by a variable force on the particle,

$$W=\int \mathbf{F}\cdot d\,\mathbf{r}$$
$$=\int \mathbf{F}\cdot\left(dx\,\hat{\mathbf{i}}+dy\,\hat{\mathbf{j}}\right)$$

$\therefore$ In two dimension. $d\,\mathbf{r}=dx\,\hat{\mathbf{i}}+dy\,\hat{\mathbf{j}}$ and it is given $\mathbf{F}=-x\,\hat{\mathbf{i}}++y\,\hat{\mathbf{j}}$

$$\therefore\quad W=\int\left(-x\,\hat{\mathbf{i}}+y\,\hat{\mathbf{j}}\right)\cdot\left(dx\,\hat{\mathbf{i}}+dy\,\hat{\mathbf{j}}\right)$$
$$=\int -x\,dx+y\,dy$$
$$=\int -x\,dx+\int y\,dy$$

As particle is displaced from A (1, 0) to B (0, 1), so x varies from 1 to 0 and y varies from 0 to 1.
So, with limits, work will be

$$W=\int_1^0 -x\,dx+\int_0^1 y\,dy$$
$$=-\left[\frac{x^2}{2}\right]_1^0+\left[\frac{y^2}{2}\right]_0^1$$
$$=-\frac{1}{2}\left[(0-(-1)^2\right]+\frac{1}{2}\left[(1)^2-0\right]$$
$$=1\,\text{J}$$

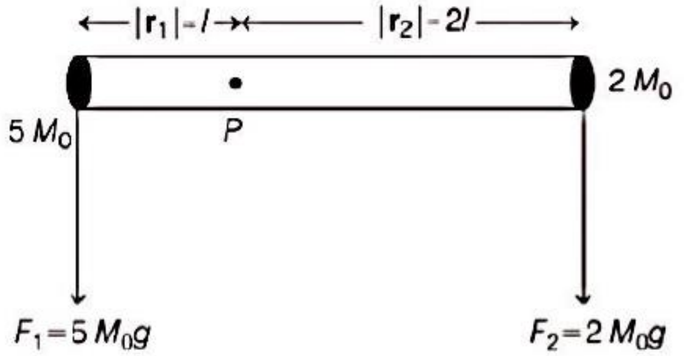
2. (A)

Given, $m_1=m_2=m, u_1=\upsilon$ and $u_2=0$

$$\upsilon_1=\frac{\upsilon}{2}(1-e)$$
$$\upsilon_2=\frac{\upsilon}{1}(1+e)$$
$$\therefore\quad \frac{\upsilon_1}{\upsilon_2}=\left(\frac{1-e}{1+e}\right)$$

3. (A)

The given condition can be drawn in the figure below



Torque (τ) about $P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$ … (i)

$\Rightarrow \quad \tau = l \times 5M_0\, g$ (outwards) – $2l \times 2M_0 g$ (inwards)

$\Rightarrow \quad \tau = 5M_0 gl - 4M_0 gl$ (outwards)

$\Rightarrow \quad \tau = M_0 gl$ (outwards) … (ii)

Now we know that, torque is also given by

$\tau = I\alpha$ … (iii)

Here, I = moment of inertia (w.r.t. point P) of rod and $\alpha =$ angular acceleration.

For point P, $I = (5M_0) \times l^2 + (2M_0)(2l)^2 \quad \left[\because I = MR^2\right]$

$\Rightarrow \quad I = 13M_0 l^2$ … (iv)

Putting value of I from Eq. (iv) in Eq. (iii), we get

$\tau = (13M_0 l^2)\alpha$ … (v)

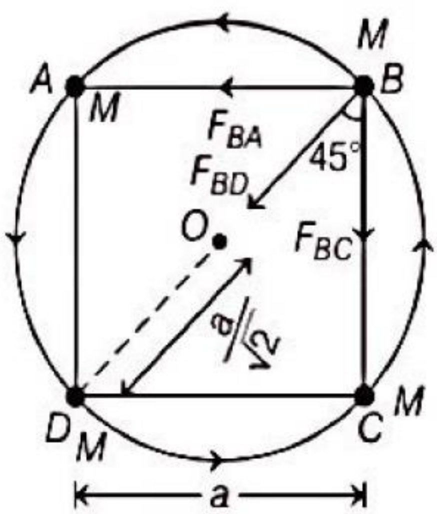
From Eqs. (ii) and (v), we get

$M_0 gl = 13M_0 l^2 \alpha$

$\Rightarrow \quad \alpha = \dfrac{g}{13l}$

4. (B)

In given configuration of masses, net gravitational force provides the necessary centripetal force for rotation.



Net force on mass M at position B towards centre of circle is

$$F_{BO(\text{net})} = F_{BD} + F_{BA}\sin 45^\circ + F_{BC}\cos 45^\circ$$
$$= \frac{GM^2}{(\sqrt{2}a)^2} + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right) \quad \text{[where, diagonal length } BD \text{ is } \sqrt{2}a]$$
$$= \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\left(\frac{2}{\sqrt{2}}\right) = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

Here, $F_{\text{centripetal}} = \frac{M\upsilon^2}{r} = \frac{M\upsilon^2}{\frac{a}{\sqrt{2}}} = \frac{\sqrt{2}M\upsilon^2}{a} \qquad \left(\because\ r = \frac{a}{\sqrt{2}}\right)$

So, for rotation about the centre, $F_{\text{centripetal}} = F_{BO(\text{net})}$

$$\Rightarrow \quad \sqrt{2}\frac{M\upsilon^2}{a} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$
$$\Rightarrow \quad \upsilon^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right) = \frac{GM}{a}(1.35)$$
$$\Rightarrow \quad \upsilon = 1.16\sqrt{\frac{GM}{a}}$$

5. (D)

Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

Where, $M =$ mass of earth,
$m =$ mass of the body
and $r =$ radius of earth.

At $r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $r = 3R,\ E_2 = -\frac{GMm}{(3R)}$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GMm}{R}\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{GMm}{6R}$$

6. (B)

As, $Y_S = \frac{FL_S}{A_S\Delta L_S}$ and $Y_C = \frac{FL_C}{A_C\Delta L_C}$

$$\therefore \quad \frac{L_C}{L_S} = \frac{\frac{Y_C A_C \Delta L_C}{F}}{\frac{Y_S A_S \Delta L_S}{F}} = \left(\frac{Y_C}{Y_S}\right)\left(\frac{A_C}{A_S}\right)\left(\frac{\Delta L_C}{\Delta L_S}\right) \qquad \ldots \text{(i)}$$

Here, $\frac{A_C}{A_S} = 2, \frac{\Delta L_C}{\Delta L_S} = 1, \frac{Y_C}{Y_S} = \frac{1.1}{2}$

Putting the value of ratios in Eq. (i), we get

$$\therefore \quad \frac{L_C}{L_S} = \frac{1.1}{2} \times 2 \times 1 = 1.1$$

Hence, $L_C : L_S = 1.1 : 1$

7. (B)

Figure (i) is incorrect. From equation of continuity, the speed of liquid is larger at smaller area. According to Bernoulli's theorem, due to larger speed, the pressure will be lower at smaller area (large velocity) and therefore height of liquid column will also be at lesser height, while in figure (i) height of liquid column at narrow area in higher.

8. (A)

If h is the initial height of liquid in drum above the small opening, then velocity of efflux, $\upsilon = \sqrt{2gh}$. As the water drains out, h decreases, hence υ decrease. This reduces the rate of drainage of water. Due to which, as the drainage continues, a longer time is required to drain out the same volume of water. So, clearly $t_1 < t_2 < t_3$.

9. (D)

Let R and R' be the radius of bubble of volume V and $2V$ respectively, then

$$\frac{4}{3}\pi R^3 = V \text{ and } \frac{4}{3}\pi R'^3 = 2V$$

So, $\frac{R'^3}{R^3} = 2$ or $R' = (2)^{1/3} R$

As $W = S \times (4\pi R^2)$ and $W' = S \times (4\pi R'^2)$

$$\frac{W'}{W} = \frac{R'^2}{R^2} = 2^{2/3} = (4)^{1/3}$$

or $W' = (4)^{1/3} W$

10. (A)

Terminal speed of a sphere falling in a viscous fluid is

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_f)g$$

where, $\eta =$ coefficient of viscosity of fluid,

$\rho_0 =$ density of falling sphere

and $\rho_f =$ density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $\upsilon_T \propto r^2$ … (i)

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then

Volume of sphere of radius R = 27 × Volume of sphere of radius r

$$\Rightarrow \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow R = 3r$$

$\Rightarrow \quad r = \frac{R}{3}$

So, from Eq. (i), we have

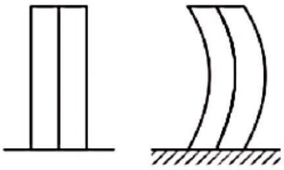
$$\frac{\upsilon_1}{\upsilon_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

11. (D)

Length of both strips will decrease

$\because \quad \alpha_A > \alpha_B$

$\therefore \quad \Delta L_A > \Delta L_B$



Therefore, it bend towards the left.

12. (A)

Let the temperature of junction be θ.

$$\left(\frac{\Delta Q}{\Delta t}\right)_{\text{copper}} = \left(\frac{\Delta Q}{\Delta t}\right)_{\text{steel}}$$

$$\Rightarrow \quad K_1 A \frac{(100-\theta)}{18} = \frac{K_2 A(\theta - 0)}{6}$$

$$\Rightarrow \quad 9K_2 \frac{(100-\theta)}{3} = K_2\theta$$

or $\quad 3\theta = 900 - 9\theta$

or $\quad 12\theta = 900$

or $\quad \theta = 75°\text{C}$

13. (B)

Here, $T_1 = 6000\,\text{K},\ \lambda_1 = 4800\,\text{Å}$,

$T_2 = 3000\,\text{K}, \lambda_2 = ?$

According to Wien's law, $\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2}$

$$\Rightarrow \quad \lambda_2 = \frac{T_1}{T_2} \times \lambda_1 = \frac{6000}{3000} \times 4800 = 9600\,\text{Å}$$

14. (A)

When n_1 moles of an ideal gas having specific heats C_{P_1} and C_{V_1} is mixed with n_2 moles of another ideal gas with specific heats C_{P_2} and C_{V_2}, specific heats of mixture are given by

$$C_{P(\text{mix})} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2} \text{ and } C_{V(\text{mix})} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

So, ratio of specific heats of mixture will be

$$\frac{C_{P(\text{mix})}}{C_{V(\text{mix})}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} \qquad \ldots \text{(i)}$$

Now, using $C_P = \left(\frac{\gamma}{\gamma - 1}\right) R$ and $C_V = \frac{R}{\gamma - 1}$

Note As, $\frac{C_P}{C_V} = \gamma$ and $C_P - C_V = R$

So, $C_P - \frac{C_P}{\gamma} = R$ or $C_P = \frac{\gamma}{\gamma - 1}$ and $C_V = \frac{R}{\gamma - 1}$

We can write Eq. (i) as

$$\frac{C_{P(\text{mix})}}{C_{V(\text{mix})}} = \frac{n_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right)R + n_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)R}{n_1\left(\frac{R}{\gamma_1 - 1}\right) + n_2\left(\frac{R}{\gamma_2 - 1}\right)}$$

$$= \frac{n_1\gamma_1(\gamma_2 - 1) + n_2\gamma_2(\gamma_1 - 1)}{n_1(\gamma_2 - 1) + n_2(\gamma_1 - 1)} \qquad \ldots \text{(ii)}$$

Here, we are given

$$n_1 = 2,\ \gamma_1 = \frac{5}{3},\ n_2 = 3 \text{ and } \gamma_2 = \frac{4}{3}$$

Substituting the above values in Eq. (ii), we have

$$\frac{C_{P(\text{mix})}}{C_{V(\text{mix})}} = \frac{2 \times \frac{5}{3}\left(\frac{4}{3} - 1\right) + 3 \times \frac{4}{3}\left(\frac{5}{3} - 1\right)}{2 \times \left(\frac{4}{3} - 1\right) + 3 \times \left(\frac{5}{3} - 1\right)}$$

$$= \frac{\frac{10}{9} + \frac{24}{9}}{\frac{2}{3} + \frac{6}{3}}$$

$$= \frac{34 \times 3}{9 \times 8} = \frac{17}{12} = 1.42$$

15. (C)

According to the first law of thermodynamics,

Heat supplied (ΔQ) = Work done (W) + Change in internal energy of the system (ΔU)

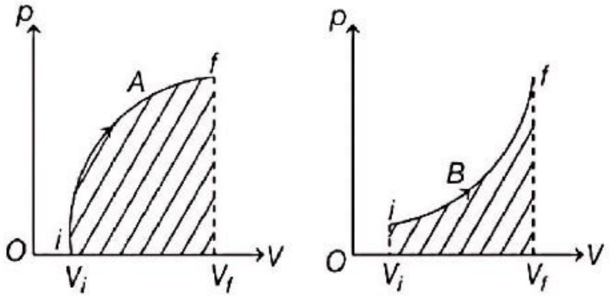
$$\Delta Q_A = \Delta U_A + W_A$$

Similarly, for process B,

$$\Delta Q_B = \Delta U_B + W_B$$

Now, we know that,

Work done for a process = Area under its p-V curve



Thus, it is clear from the above graphs,

$W_A > W_B$ ... (i)

Also, since the initial and final state are same in both process, so

$\Delta U_A = \Delta U_B$ ... (ii)

So, from Eqs. (i) and (ii), we can conclude that

$\Delta Q_A > \Delta Q_B$

16. (B)

Given, $x = 3\sin\omega t + 4\sin\left(\omega t + \frac{\pi}{3}\right)$

Comparing it with the equation

$x = r_1 \sin\omega t + r_2 \sin(\omega t + \phi)$

We have, $r_1 = 3\,\text{cm}, r_2 = 4\text{ cm}$ and $\phi = \frac{\pi}{3}$

The amplitude of combination is

$$r = \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\phi}$$

$$= \sqrt{3^2 + 4^2 + 2\times3\times4\times\cos\frac{\pi}{3}}$$

$$= \sqrt{37} \approx 6\,\text{cm}$$

17. (A)

Force on proton at point A,

$F_A = qE_A = 1.6\times10^{-19}\times40 = 6.4\times10^{-18}\text{ N}$

18. (B)

According to Gauss's theorem,

$$\oint \mathbf{E}.d\mathbf{S} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

$$\Rightarrow \quad E\cdot4\pi x^2 = \frac{Q}{\varepsilon_0}$$

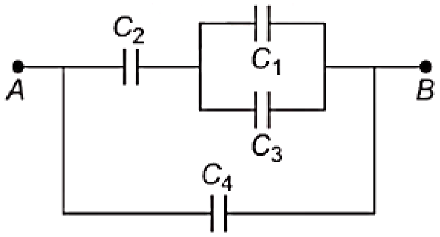
or $\quad E = \dfrac{Q}{4\pi\varepsilon_0 x^2}$

19. (C)

As, $E = \frac{dV}{dr} = -\frac{(1-3)}{0.3-0.1}\text{Vm}^{-1} = 10\,\text{Vm}^{-1}$

20. (D)

The arrangement can be redrawn as shown in the adjoining figure



$\therefore \quad C_{13} = C_1 + C_3 = 9 + 9 = 18\mu F$

and $C_{2-13} = \dfrac{C_2 \times C_{13}}{C_2 + C_{13}} = \dfrac{9\mu F \times 18\mu F}{(9+18)\mu F} = 6\mu F$

$\therefore \quad C = C_{2-13} + C_4 = 6\mu F + 9\mu F = 15\mu F$

21. (4)

Initial energy of charged capacitor,

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (220)^2$$

$$= 121 \times 10^{-3} \text{ J}$$

Common potential after redistribution of charges,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$\simeq \frac{C_1V_1}{C_1 + C_2}$$

$$\simeq \frac{5 \times 10^{-6} \times 220}{5 \times 10^{-6} + 2.5 \times 10^{-6}}$$

$$= 220 \times \frac{2}{3} \text{ V}$$

Final stored energy,

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2} \times 7.5 \times 10^{-6} \times \left(220 \times \frac{2}{3}\right)^2$$

$$\approx 80 \times 10^{-3} \text{ J}$$

Loss of energy

$$\Delta U = (121 - 80) \times 10^{-3} \text{ J}$$

$$= 41 \times 10^{-3} \text{ J}$$

$$= \frac{4.1}{100} \text{ J} \approx \frac{4}{100} \text{ J}$$

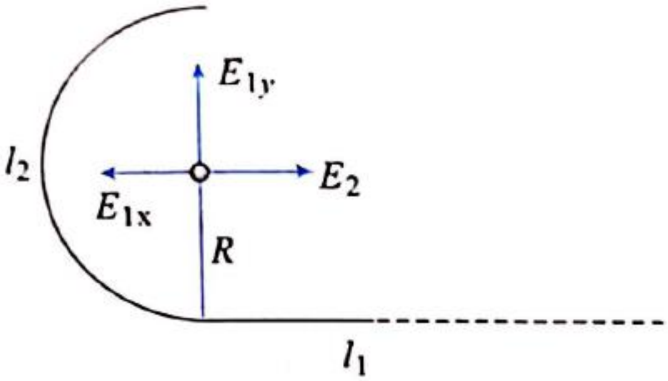
Given, $\Delta U = \dfrac{X}{100}$ J

$\therefore \quad X = 4$

22. (2)

Electric field due to semi-circular part:

$$E_2 = \frac{\lambda_2}{2\pi\varepsilon_0 R}$$



Electric field due to straight part:

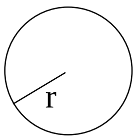
$$E_{1x} = E_{1y} = \frac{\lambda_1}{4\pi\varepsilon_0 R}$$

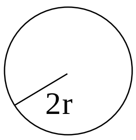
For net field to be along *y*-axis:

$$E_2 = E_{1x} \Rightarrow \frac{\lambda_2}{2\pi\varepsilon_0 R} = \frac{\lambda_1}{4\pi\varepsilon_0 R} \Rightarrow \frac{\lambda_1}{\lambda_2} = 2$$

23. (2)

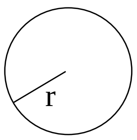
$-\sigma$ | $+2\sigma$

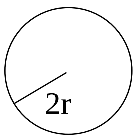




$-\sigma 4\pi r^2$
$= -\sigma 4\pi R^2$

$-2\sigma 4\pi\left(2r^2\right)^2$
$= 32\sigma\pi R^2$





Q | $32\sigma\pi R^2 - Q$

$$\frac{kQ}{r} = \frac{k\left(32\sigma\pi r^2 - Q\right)}{2r}$$

$$2Q = 32\sigma\pi r^2 - Q$$

$$3Q = 32\sigma\pi r^2$$

$$Q = \frac{32}{3}\sigma\pi r^2$$

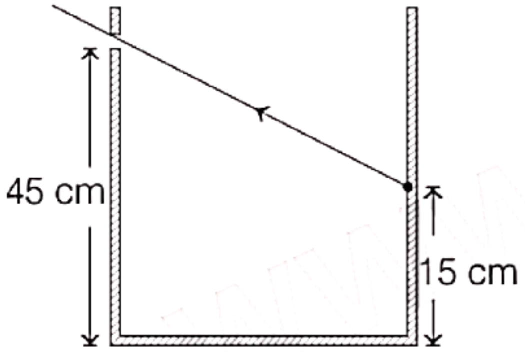
$$\frac{32\sigma\pi r^2}{3\times 4\pi r^2} = k\times\frac{\frac{64}{3}\sigma\pi r^2}{4\pi(2r)^2}$$

$$\frac{32\sigma}{12} = \frac{k64\sigma}{3\times 4\times 4}$$

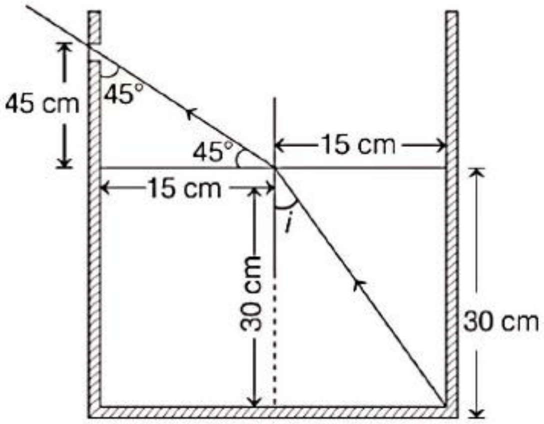
$$k = \frac{32 \times 4}{64} = 2$$

24. (158)

Initially, when the jar is empty.



Finally, when the jar is filled with liquid.



If i be the angle of incidence, then

$$\tan i = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \quad \sin i = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{5}}$$

According to law of refraction,

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow \quad \mu\left(\frac{1}{\sqrt{5}}\right) = 1 \times \sin 45°$$

$$\Rightarrow \quad \mu = \frac{\sqrt{5}}{\sqrt{2}} = 1.581$$

Also, given that,

$$\frac{N}{100} = \mu$$

$$\Rightarrow \quad N = 100\mu$$

$$= 100 \times 1.581 = 158.1$$

So, nearest integer value of N is 158.

25. (3)

Here reduced mass of the system, $\mu = \dfrac{m_1 m_2}{m_1+m_2} = \dfrac{3\times 1}{3+1}$

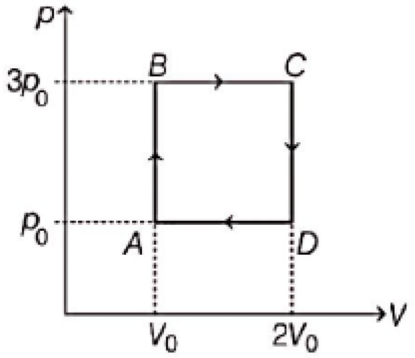
$= 0.75$ kg

$\therefore$ Vibrational frequency,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{300}{0.75}} = \frac{20}{2\pi} = \frac{10}{\pi} \cong 3\text{Hz}$$

26. (19)

From given p-V diagram,



Work done by gas in close loop, W = Area of loop

$$= (3p_0 - p_0)\times(2V_0 - V_0)$$

$$W = 2p_0V_0$$

During the cycle $ABCDA$, processes AB and BC absorb the heat.

Heat absorption during process AB,

$Q_{AB} = \mu C_V \Delta T$ (as AB = isochoric process)

$Q_{AB} = \mu\left(\frac{3}{2}R\right)(T_B - T_A)$ ($\because$ C_V for monatomic gas is $\frac{3}{2}R$)

$= \frac{3}{2}(\mu RT_B - \mu RT_A)$

$= \frac{3}{2}(p_BV_B - p_AV_A)$ ($\because pV = \mu RT$)

$= \frac{3}{2}(3p_0V_0 - p_0V_0)$

$= 3p_0V_0$

Heat absorption during process BC,

$Q_{BC} = \mu C_p \Delta T$ (as BC = isobaric process)

$= \mu\left(\frac{5}{2}R\right)(T_C - T_B)$ ($\because C_P = C_V + R$)

$= \frac{5}{2}(\mu RT_C - \mu RT_B)$

$= \frac{5}{2}(p_CV_C - p_BV_B)$

$= \frac{5}{2}(6p_0V_0 - 3p_0V_0)$

$$=\frac{15}{2}p_0V_0$$

Total heat absorption, $Q=Q_{AB}+Q_{BC}$

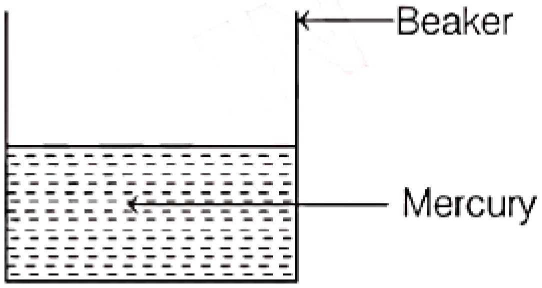
$$=3p_0V_0+\frac{15}{2}p_0V_0$$

$$=\frac{21}{2}p_0V_0$$

∴ Efficiency of engine,

$$\eta\%=\frac{W}{Q}\times100=\frac{2p_0V_0}{\frac{21}{2}p_0V_0}\times100=19\%$$

27. (20)



Volume of empty part = Volume of beaker – Volume of mercury

Also it is given that, there is no change in unfilled volume of the beaker with the varying temperature.

So, change in volume of beaker = change in volume of mercury

$$\Rightarrow V_b\gamma_b\Delta T=V_m\gamma_m\Delta T$$

$$\Rightarrow V_b\gamma_b=V_m\gamma_m$$

$$\Rightarrow V_m=\frac{V_b\gamma_b}{\gamma_m}=\frac{500\times6\times10^{-6}}{1.5\times10^{-4}}$$

$$=20\,\text{cm}^3=20\,\text{cc}$$

28. (101)

Height of liquid in capillary tube,

$$h=\frac{2S\cos\theta}{\rho g r}$$

$$\Rightarrow S=\frac{\rho g r h}{2\cos\theta}$$

Here, $\rho=900\,\text{kg m}^{-3}$,

$g=10\,\text{ms}^{-2}$

$r=0.015\,\text{cm}=15\times10^{-5}\,\text{m}$,

$h=15\,\text{cm}=15\times10^{-2}\,\text{m}$ and $\theta\approx0°$

$\Rightarrow \cos\theta=1$

Hence, $S=\dfrac{900\times10\times15\times10^{-5}\times15\times10^{-2}}{2\times1}$

$$=101.25\times10^{-3}\,\text{N/m}\approx101\,\text{mN/m}$$

29. (5)

Here, side of the square plate, l = 10 cm

Area of the plate = 100cm^2, $dv = 10\,\text{cms}^{-1}$, $F = 200\,\text{dyne}$, $\eta = 0.01$ poise, $dx = ?$

As, $F = \eta A \dfrac{dv}{dx}$

$$dx = \frac{\eta A du}{F} = \frac{0.01 \times 100 \times 10}{200} = 0.05\,\text{cm}$$

$$= 5 \times 10^{-2}\,\text{m}$$

$$= x \times 10^{-2}\,\text{m (given)}$$

$\therefore \quad x = 5$

30. (3)

$$V_A = \begin{bmatrix}\text{Potential due to the} \\ \text{complete sphere at } A\end{bmatrix} - \begin{bmatrix}\text{Potential due to the} \\ \text{spherical cavity at } B\end{bmatrix}$$

$$= -\frac{3}{2}\frac{GM}{R} - \left(-\frac{GM'}{r}\right)$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$$M' = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{1}{8}\left(\frac{4}{3}\pi R^3 \rho\right),\ r = \frac{R}{2}$$

$$\therefore \quad V_A = \frac{G}{R}\left[\frac{\pi\rho R^3}{3} - 2\pi\rho R^3\right] = -\frac{5}{3}\pi G \rho R^2$$

Now, $V_B = \begin{bmatrix}\text{Potential due to the} \\ \text{complete sphere at } B\end{bmatrix} - \begin{bmatrix}\text{Potential due to the} \\ \text{spherical cavity at } B\end{bmatrix}$

$$= -\frac{GM}{2R^3}\left(3R^2 - r^2\right) - \left[-\frac{3}{2}\frac{GM'}{r}\right]$$

or $\quad V_B = -\dfrac{11}{8}\dfrac{GM}{R} + \dfrac{3GM'}{R}$

$$= \frac{G}{R}\left[\frac{\pi\rho R^3}{2} - \frac{11\pi\rho R^3}{6}\right]$$

$$= -\frac{4}{3}\pi G \rho R^2$$

Now, $V_B - V_A = \dfrac{1}{3}\pi G \rho R^2$

$$\frac{1}{2}m_0 \upsilon^2 = U_B - U_A = m_0\left(V_B - V_A\right)$$

$\Rightarrow \quad \upsilon = \sqrt{2\left(V_B - V_A\right)}$

or $\quad \upsilon \dfrac{\sqrt{2\pi G \rho R^2}}{\sqrt{3}}$

$\Rightarrow \quad \beta = 3$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(B)	33.	(D)	34.	(A)	35.	(C)
36.	(B)	37.	(C)	38.	(A)	39.	(C)	40.	(D)
41.	(B)	42.	(B)	43.	(D)	44.	(A)	45.	(D)
46.	(B)	47.	(B)	48.	(A)	49.	(C)	50.	(A)
51.	(9)	52.	(5)	53.	(9)	54.	(1)	55.	(3)
56.	(6)	57.	(2)	58.	(3)	59.	(8)	60.	(5)

SOLUTIONS

31 (D)

$$2\,Fe(s)+O_2(g)+4H^+(aq.)\rightarrow 2\,Fe^{2+}(aq.)+2H_2O(l)$$

$n=4$ (no. of moles of electron involved)

From Nernst's equation,

$$E_{cell}=E^\circ_{cell}-\frac{0.0591}{n}\log Q$$

$$=1.67-\frac{0.0591}{4}\log\frac{(10^{-3})^2}{0.1\times(10^{-3})^4}\left\{[H^+]=10^{-pH}\right\}$$

$$=1.67-0.103=1.57\text{ V}$$

32. (B)

33. (D)

Overall order of reaction can be decided by the data given $t_{75\%}=2t_{50\%}$.

$\therefore$ It is a first order reaction with respect to P.

From graph [Q] is linearly decreasing with time, *i.e.*, order of reaction with respect to Q is zero and the rate expression is $r=k[P]^1[Q]^0$.

Hence (D) is correct.

34. (A)

35. (C)

$$KE=\frac{3}{2}nRT$$

$$\therefore\ \frac{(KE)_{T_2}}{(KE)_{T_1}}=\left(\frac{313}{293}\right)$$

36. (B)

37. (C)

$$2MnO_4^-+10I^-+16H^+\longrightarrow 2Mn^{2+}+5I_2+8H_2O$$

38. (A)

39. (C)

$$\underset{+8}{Mn_3O_4} \longrightarrow \underset{+21}{MnO_4^-} + Mn^{2+}$$

$$2e^- + Mn_3O_4 \longrightarrow 3Mn^{2+}$$

$$n_{\text{eff}} = \frac{2\times 13}{2+13} = \frac{26}{15}$$

$$E = \frac{M}{26}\times 15$$

40. (D)

41. (B)

$$\frac{3.65}{1000}\times\frac{25}{36.5} = \frac{50\times N}{1000}$$

$$\Rightarrow n_{\text{NaOH}} = \frac{1}{20}$$

Now, $25\times\frac{1}{20} = 50\times N_{H_2SO_4}$

42. (B)

$$n\times 2 = [y\times 5 - 0.2\times x\times 5\times 5]$$

43. (D)

44. (A)

Since, $pK_a = pK_b$, hence pH = 7

Thus, $K_a = K_b = 1.8\times 10^{-5}$

At any state of hydrolysis

$[NH_4^+] + [NH_3] = 0.150$ M

If, $[NH_3] = x$, then $[NH_4^+] = (0.150 - x)\text{M} \approx 0.150$ M

$$NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+$$

$$\therefore \quad NH_4OH \rightleftharpoons NH_4^+ + OH^-$$

$$K_b = \frac{[NH_4^+][OH^-]}{[NH_4OH]} \text{ and } K_w = [H_3O^+][OH^-] = 1\times 10^{-14}$$

$$K_h = \frac{[NH_3][H_3O^+]}{[NH_4^+]} = \frac{K_w}{K_b}$$

$$\frac{x\times 10^{-7}}{0.150} = \frac{1\times 10^{-14}}{1.8\times 10^{-5}}$$

$$x = 8.3\times 10^{-4}\text{ M}$$

45. (D)

According to Arrhenius equation :

$$\ln\frac{k_2}{k_1}=-\frac{E_a}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)$$

$$2.303\log 2=-\frac{E_a}{8.314}\left(\frac{1}{310}-\frac{1}{300}\right)$$

$$\Rightarrow\quad E_a=53.6\ \text{kJ/mol}$$

46. (B)

47. (B)

For I order, $t_{1/2}=\dfrac{0.693}{K}$

For zero order, $t_{1/2}=\dfrac{[A]_0}{2K'}$

Initial rate for I order $r_1=K[A]_0$

Initial rate for zero order $r_0=K'$

$$\therefore\quad \frac{r_1}{r_0}=\frac{K[A]_0}{K'}$$

$$\because\quad \frac{0.693}{K}=\frac{[A]_0}{2K'}$$

$$\therefore\quad \frac{K[A]_0}{K'}=2\times 0.693\ \text{ or }\ \frac{r_1}{r_0}=2\times 0.693$$

48. (A)

49. (C)

$$E^\circ_{cell}=E^\circ_{OP_{Sn/Sn^{2+}}}+E^\circ_{RP_{Fe^{2+}/Fe^{2+}}}$$

$$=0.14+0.77=0.91\ \text{V}$$

50. (A)

51. (9)

$$k=\frac{2.303}{t}\log\frac{a_0}{a},\quad a=\frac{a_0}{8}\ \text{at}\ t_{1/8}$$

$$t_{1/8}=\frac{2.303}{k}\log\frac{a_0}{a_0/8}=\frac{2.303}{k}\log 8 \qquad \ldots\text{(i)}$$

When $t=t_{1/10},\ a=\dfrac{a_0}{10}$

$$t_{1/10}=\frac{2.303}{k}\log\frac{a_0}{a_0/10}=\frac{2.303}{k}\log 10 \qquad \ldots\text{(ii)}$$

From eq. (i) and (ii),

$$\frac{[t_{1/8}]}{[t_{1/10}]}\times 10=\frac{2.303}{k}\log 8\times\frac{k}{2.303\times\log 10}\times 10$$
$$=\frac{\log 8}{\log 10}\times 10=\frac{\log 2^3}{\log 10}\times 10=\frac{3\log 2}{\log 10}\times 10$$
$$=\frac{3\times 0.3\times 10}{1}=\mathbf{9}$$

52. (5)

53. (9)

At constant T

$$\Delta S=2.303nR\log\frac{V_2}{V_1}$$
$$=2.303\times 2\times 2\times\log\frac{20}{2}\text{ » }\mathbf{9.2\ cal\ K^{-1}\ mol^{-1}}$$

54. (1)

$$Sb_2S_3(s)\ +\ 3H_2(g)\ \rightleftharpoons\ 2Sb(s)\ +\ 3H_2S(g)$$
$$0.01-x\qquad 0.01-3x\qquad 2x\qquad 3x$$

Where $3x=0.005$

$$H_2S+Pb^{2+}\longrightarrow PbS+2H^+$$

Number of moles of PbS formed = $\frac{1.19}{238}=0.005$ mole

At equilibrium, $[H_2]=\left(\frac{0.005}{250}\right)$

$$K_c=\left(\frac{0.005}{0.005}\right)=\mathbf{1}$$

55. (3)

56. (6)

57. (2)

$$M\longrightarrow M_{n_A}^{n^+}+ne^-\qquad\text{(at anode)}$$
$$M_{n_C}^{n^+}+ne^-\longrightarrow M\qquad\text{(at cathode)}$$

Net reaction: $M_{n_C}^{n^+}\longrightarrow M_{n_A}^{n^+}$

$$\therefore\quad 0.0295=\frac{0.059}{n}\log\frac{\left[M_{n_C}^{n^+}\right]}{\left[M_{n_A}^{n^+}\right]}$$
$$n=\frac{0.059}{0.0295}\log\frac{0.1}{0.01}\quad\Rightarrow\quad n=2$$

58. (3)

59. (8)

60. (5)

PART (C) : MATHEMATICS

ANSWER KEY

61.	(D)	62.	(B)	63.	(A)	64.	(B)	65.	(B)
66.	(A)	67.	(A)	68.	(C)	69.	(C)	70.	(D)
71.	(D)	72.	(D)	73.	(C)	74.	(A)	75.	(A)
76.	(D)	77.	(C)	78.	(A)	79.	(B)	80.	(B)
81.	(9)	82.	(5)	83.	(5)	84.	(3)	85.	(5)
86.	(3)	87.	(18)	88.	(6)	89.	(4)	90.	(727)

SOLUTIONS

61. (D)

$$\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)=x$$

Let $\operatorname{cosec}\alpha=\frac{5}{3}$ and $\tan\beta=\frac{2}{3}$

$$\therefore\quad x=\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$$

$$\cot\alpha=\frac{4}{3},\ \cot\beta=\frac{3}{2}$$

$$\therefore\quad x=\frac{\frac{4}{3}\times\frac{3}{2}-1}{\frac{4}{3}+\frac{3}{2}}=\frac{6}{17}$$

62. (B)

MOD: $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+.....\infty}}}$

$$y^2=\cos x+\sqrt{\cos x+\sqrt{\cos x+......}}$$

$$y^2=\cos x+y$$

Diff. on both side

$$2y\frac{dy}{dx}=-\sin x+\frac{dy}{dx}$$

$$\therefore\frac{dy}{dx}=\frac{\sin x}{1-2y}$$

63. (A)

$$x^m y^n=(x-y)^{m+n},$$

Divide by x^{m+n}

$$\frac{y^n}{x^n}=\left(\frac{x-y}{x}\right)^{m+n}$$

$\left(\frac{y}{x}\right)^n = \left(1-\frac{y}{x}\right)^{m+n}$

Let $y = xt \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$diff \swarrow \therefore t^n = (1-t)^{m+n}$

$w.r.t \nearrow nt^{n-t}\frac{dt}{dx} = -(m+n)(1-t)^{m+n-1}$

$\left(nt^{n-1} + (m+n)(1-t)^{m+n-1}\right)\frac{dt}{dx} = 0$

$\therefore dt/dx = 0$

$\therefore \frac{dy}{dx} = t = \frac{y}{x}$

64. (B)

$f(x) = \sin x - bx + c$

$f'(x) = \cos x - b$

For decreasing

$\cos x - b < 0$

$\cos x < b$

$\therefore b \in (1, \infty)$

65. (B)

If a year is selected at random, there are 2 possibilities year being leap or not

p(leap year) $= 1/4$

p(non-leap year) $= 3/4$

p(53 Sunday in leap year) = 2/7

p(53 Sunday in non-leap year) =1/7

$\therefore$ p(Selected year having 53 Sunday) $= \frac{1}{4}\times\frac{2}{7}+\frac{3}{4}\times\frac{1}{7} = \frac{5}{28}$

66. (A)

$2y^2 = x^3$ and $y^2 = 32x$

Solving we get $\frac{x^3}{2} = 32x$

$x(x^2 - 64) = 0$

$x = 0, x = 8, x = -8$

$y = 0, y = 16, n.p$

$\frac{dy}{dx}$ for $C_1 = \frac{3x^2}{2y}$ $\left.\frac{dy}{dx}\right|_{C_2} = \frac{32}{2y}$

For (0, 0)

$\left.\frac{dy}{dx}\right|_{C_1} = \lim_{\substack{x\to 0\\ y\to 0}} \frac{3x^2}{4y} = 0, \left.\frac{dy}{dx}\right|_{C_2} = \infty$

$\therefore$ Angle between them $\frac{2\pi}{2}$

For [8, 16)

$$\left.\frac{dy}{dx}\right|_{C_1} = \frac{3}{4}\times\frac{64}{16} = 3$$

$$\left.\frac{dy}{dx}\right|_{C_1} = \frac{16}{16} = 1$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{3-1}{1+3}\right| = \frac{1}{2}$$

$$\therefore\ \theta = \tan^{-1}\frac{1}{2}$$

67. (A)

$\Rightarrow$ m is 5th root of unit

i.e. of $x^5 - 1 = 0$

$\therefore$ Other roots are $1, m^2, m^3, m^4$

We got $1 + m + m^2 + m^3 + m^4 = 0$ (Sum of roots)

$\therefore\ 1 + m + m^2 + m^3 = -m^4$

$\therefore\ m^5 = \frac{1}{m}$

$\therefore$ Required $= \log_2|-m_4 - m_4| = \log_2|2m^4|$

$= \log_2 2 = 1$

68. (C)

$S_n = 1 + (1+2) + (1+2+3) + \ldots\ldots\ n$ brackets

$S_{20} = ?$

$$T_n = 1 + 2 + \ldots. + n = \frac{n(n+1)}{2}$$

$$S_{20} = \sum_{n=1}^{20}\frac{n(n+1)}{2} = \sum_{n=1}^{20}\frac{n^2}{2} + \frac{n}{2}$$

$$= \left.\frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}\right|_{n=20}$$

$$= \frac{20\times21\times41}{12} + \frac{20\times21}{4}$$

$= 1540$

69. (C)

Putting $y = mx + 2$ in equation of circle $x^2 + m^2x^2 + 4mx + 4 = 4x + 8mx + 21$

$(1+m^2)x^2 - 4(m+1)x - 17 = 0$

For x to have 2 values D > 0

$\therefore\ \overset{}{\underset{4}{\cancel{16}}}(m+1)^2 + \underset{1}{\cancel{4}}\times17(1+m^2) > 0$

$4m^2+8m+4+17+17m^2>0$

$21m^2+8m+21>0$

Which is true for all m.

$\therefore m\in R$

70. (D)

$S: a_1{}^nC_1+a_2{}^nC_2+a_3{}^nC_3+a_1{}^nC_1+.....+a_n{}^nC_n=?$

$S: a_1[{}^nC_1+3^nC_2+3^{2\,n}C_3+.....+3^{n-1\,n}C_n]$

$S: \frac{a_1}{3}\left[3^nC_1+3^{2n}C_2+3^{3n}C_3+....3^{nn}C_n\right]$

$S: \frac{a_1}{3}[4^n-1]$

71. (D)

$f(x)=\log(x^2-8/x)$

$\therefore x^2-\frac{8}{x}>0$

$\frac{x^3-8}{x}>0\Rightarrow\frac{(x-2)\overset{D<0}{(x^2+2x+4)}}{x}>0$

$\therefore x\in(-\infty,0)\cup(2,\infty)$

72. (D)

A………..=5! =120

E………..= 5! = 120

L………..=5! = 120

PA………=4! = 24

PE………=4! = 24

PLAE…..=2! = 2

PLAR ….=2! = 2

PLAYER = 1

$\therefore$ Rank = 413

73. (C)

$13x^2-18xy+37y^2+2x+14y-2=0$ [Conic section]

$a=13, b=37, h=-9, g=1, f=7, c=-2$

$$\Delta=\begin{vmatrix}a&h&g\\h&b&f\\g&f&c\end{vmatrix}=\begin{vmatrix}13&-9&1\\-9&37&7\\1&7&-2\end{vmatrix}=-1600$$

$h^2-ab=81-481=-400$

$\because \Delta\neq0, h^2-ab<0$, Ellipse

74. (A)

$y^2=8x$ $\qquad x^2-\frac{y^2}{3}=1$

E.O.T for Parabola: $y = mx + \frac{2}{m}$

E.O.T. for H. B: $y = mx \pm \sqrt{m^2 - 3}$

Since tangent is same, $\frac{2}{m} = \sqrt{m^2 - 3}$

$\frac{4}{m^2} = m^2 - 3$

$\therefore t = m^2$

$t^2 - 3t - 4 = 0$

$m^2 = 4$ $\quad$ $m^2 = -1$

$m = \pm 2$ $\quad$ n. p

$\therefore E.O.T. \Rightarrow y = \pm 2x \pm 1$

75. (A)

$$z = \frac{\left(1+\sqrt{3}i\right)\left(\sqrt{3}-i\right)^2}{(1+i)\left(3-\sqrt{3}i\right)^3}$$

Arg (z) = $Arg(1+\sqrt{3}i) + 2Arg\left(\sqrt{3}-i\right) - Arg(1+i) - 3Arg(3-\sqrt{3}i)$

$= \frac{\pi}{3} + 2\left(-\frac{\pi}{6}\right) - \frac{\pi}{4} - 3\times\left(-\frac{\pi}{6}\right)$

$Arg\left(z\right) = \frac{\pi}{4}$

76. (D)

$x^2 + y^2 - 2x + 4y + 3 = 0 \Rightarrow (x-1)^2 + (y+2)^2 = 2$

Centre = (1, –2)

Radius = $\sqrt{2}$

Since sides are parallel to axis diagonal will be at angle of $\frac{\pi}{4}$.

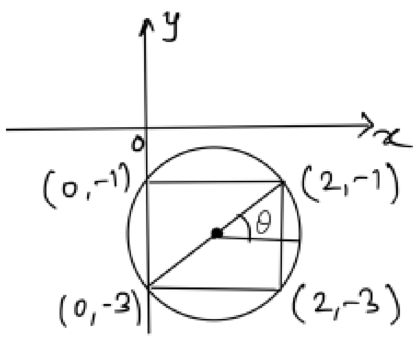
$x_1 = 1 + r\cos\theta$

$y_1 = -2 + r\cos\theta$

$\therefore$Coordinates of the vertices will be

(2, –1), (2, –3), (0, –3), (0, –1)

$\therefore$ None of these



77. (C)

Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ …….(1)

For circle $\Rightarrow r = ae \Rightarrow r = \sqrt{a^2 - b^2}$

$\therefore EOC ! x^2 + y^2 = a^2 - b^2$ …….(2)

Put (2) and (1) we get

$\frac{a^2 - b^2 - y^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2}=\frac{b^2+y^2}{a^2}$$

$$y=\frac{b^2}{\sqrt{a^2-b^2}}$$

Area $=\frac{1}{2}(y)F_1F_2$ (∵ Area of Δ)

$$30=\frac{1}{\not{2}}\times\not{2}\times\frac{b^2}{\sqrt{a^2-b^2}}\times ae$$

$$30=B^2\frac{\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}}$$

$$\therefore b^2=30$$

$$F_1F_2=2ae=2\sqrt{a^2-b^2}$$

$$=2\sqrt{\left(\frac{17}{2}\right)^2-30}$$

$$F_1F_2=13$$

78. (A)

$$\sum_{n=1}^{10}\sum_{m=1}^{10}\tan^{-1}\left(\frac{m}{n}\right)=\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+......+\tan^{-1}\frac{1}{10}$$

$$\tan^{-1}2+\tan^{-1}1+\tan^{-1}\frac{2}{3}+........+\tan^{-1}\frac{2}{10}$$

$$\tan^{-1}3+\tan^{-1}\frac{3}{2}+\tan^{-1}1+......+\tan^{-1}\frac{3}{10}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\tan^{-1}10+\tan^{-1}\frac{10}{2}+........+\tan^{-1}1$$

⇒ Here apply $\tan^{-1}(a)+\tan^{-1}\left(\frac{1}{a}\right)=\frac{\pi}{2}$ and $\tan^{-1}1=\frac{n}{4}$,

We get

$$S=45\times\frac{\pi}{2}+\frac{10\pi}{4}=25\pi$$

79. (B)

$$\lim_{x\to0}\frac{e^x-e^{-x}-2x}{x^3}=\frac{1+x+\frac{x^2}{2}+\frac{x^3}{6}-\left(1-x+\frac{x^2}{2}-\frac{x^3}{6}\right)-2x}{x^3}$$

$$=\lim_{x\to0}\frac{x^3}{3x^3}=\frac{1}{3}$$

80. (B)

LHL = $\lim\limits_{h\to 0^+}(-h)e^{-\left(\frac{1}{-h}+\frac{1}{h}\right)}=0$

$RHL=\lim\limits_{h\to 0^+} he^{-2/h}=0$

$f(0)=0$

$\therefore f(x)$ is cont. at $x=0$

For differentiability

LHD = $\lim\limits_{x\to 0}\dfrac{-f(h)}{h}=\dfrac{0-(-h)}{h}=1$

RHD = $\lim\limits_{h\to 0^+}\dfrac{f(h)-f(0)}{h}=\dfrac{he^{-2h}-0}{h}=0$

$\because LHD\neq RHD$

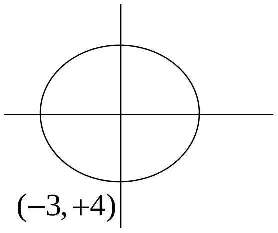
$\therefore$ cont. but not diff at $x=0$

81. (9)

Complex and locus

$\max(iz+3-4i)=$ distance from centre + radius [max. distance from point on centre]

= 5 + 4 = 9



82. (5)

If $f(x)$ is diff. $f(x)$ is cont. as well [LCD]

$\therefore f(1)=f(1)^-$

$A+B=3A-B+2$

$A-B+1=0$

Since diff LHD = RHD

$2B=3A$

Solving equation we get

$B=3,\ A=2$

$\therefore A+B=5$

83. (5)

$f(x)=x^{x+1}+x^2+1$

$f(x)=e^{(x+1)\ln x}+x^2+1$

$f'(x)=e^{(x+1)\ln x}\left[\ln x+\dfrac{x+1}{x}\right]+2x+1$

$f'(1)=e^0\left(0+\dfrac{2}{1}\right)+2+1$

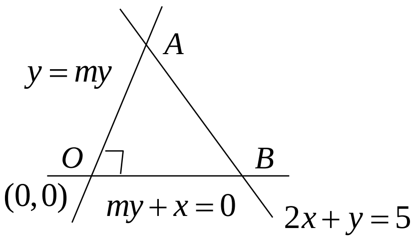
$=5$

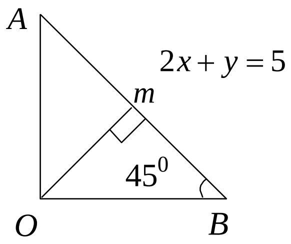
84. (3)

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{2t} = 3$$

Now for $x = 1;\ t = 0$ but $\frac{dy}{dx} = 3\ \forall\ x \in R$

85. (5)





Let $\perp er$ lines be $y = mx$ and $y = -\frac{1}{mx}$ is ($\perp$ er lines p.t. origin)

$$OM = \left|\frac{2\times 0 + 0 - 5}{\sqrt{5}}\right|$$

Parallel $AM = \sqrt{5}$

In ΔOBM

$$\tan 45° = \frac{OM}{MB}$$

$\therefore MB = \sqrt{5}$

11y AM $= \sqrt{5}$

$$(\Delta AOB) = \frac{1}{2} OM \times AB = \frac{1}{2} \times \sqrt{5} \times 2\sqrt{5}$$

$= 5$

86. (3)

$\tan x + \sec x = 2\cos x$

$1 + \sin x = 2\cos^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$\sin x = -1$ or $\sin x = \frac{1}{2}$

$x = \frac{3\pi}{2},\ x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore$ 3 values of x

87. (18)

$$2\operatorname{cosec}^2\theta + 8\sec^2\theta = 10 + \underbrace{2\cot^2\theta + 8\tan^2\theta}_{AM \geq GM}$$

$= 10 + 2\sqrt{2\times 8}$

$\min(2\operatorname{cosec}^2\theta + 8\sec^2\theta) = 18$

88. (6)

$$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+......+n \text{ terms}=\frac{kn}{n+1}$$

$$T_r=\frac{2r+1}{\sum r^2}=\frac{2r+1}{r(r+1)(2r+1)}=\frac{6}{r(r+1)}$$

$$\sum\frac{6}{r(r+1)}=\sum\frac{6}{r}-\frac{6}{r-1}=6-\frac{6}{2}+\frac{6}{2}-\frac{6}{3}+......+\frac{6}{n}-\frac{6}{n+1}$$

$$=6-\frac{6}{n+1}=\frac{6n}{n+1}$$

$\therefore K=6$

89. (4)

Let n be required no. of coins

Then P (at least one head) > 0.9

$1-P(\text{no head})>0.9$

$$1-\left(\frac{1}{2}\right)^n>0.9$$

$$1-\left(\frac{1}{2}\right)^n<0.1$$

$$\left(\frac{1}{2}\right)^n<0.1$$

$2^n>10$

$\therefore$ least tosses =4

90. (727)

The given $P(x)$ should be

$P(x)=(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)+x$

For all given condition to be true

$\therefore P(7)=6\times5\times4\times3\times2\times1+7$

$=727$