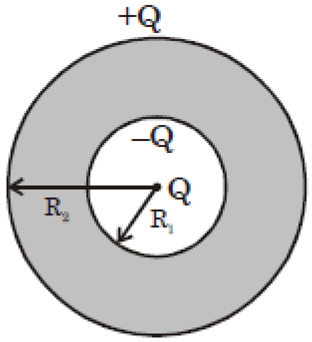
PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(A)	3.	(D)	4.	(B)	5.	(1.73)
6.	(3)	7.	(30)	8.	(1.5)	9.	(8)	10.	(2)
11.	(ABCD)	12.	(ABCD)	13.	(AC)	14.	(AC)	15.	(BD)
16.	(C)	17.	(10)	18.	(110)	19.	(320)		

SOLUTION

1. (B)



(A) $V_D = \dfrac{kQ}{R_2}$

(B) $V_B = \dfrac{kQ}{r_1} + \dfrac{kQ}{R_2} - \dfrac{kQ}{R_1}$

(C) As potential at D doesnot depends on positon of charge within cavity

(D) Shifting of charge within the cavity will change electric field distribution inside cavity. So potential at B will change.

2. (A)

For all positions of real object in case of diverging lens. Image will be virtual & diminished on the same side of lens.

3. (D)

4. (B)

Percentage error in $h = \dfrac{dh}{h} \times 100$

$= \dfrac{0.3}{46.2} \times 100 = 0.6\%$

Percentage error in $t = \dfrac{0.1}{1.6} \times 100 = 6.25\%$

Percentage error in g = (% error in h) + 2 × (% error in t)

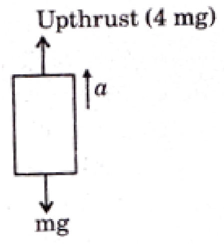
$\dfrac{dg}{g} = 13.1\%$

$g = 36 \pm 5 \text{ ft/s}^2$

5. (1.73)

6. (3)

7. (30)



8. (1.5)
The density of liquid is four times that of cylinder, hence in equilibrium position one-fourth of the cylinder is submerged. So as the cylinder is released from initial position, it moves by $\frac{3l}{4}$ to reach its equilibrium position. The upward motion in this time is SHM. Therefore required velocity is $\upsilon_{max} = \omega A$. $\omega = \sqrt{\frac{4g}{l}}$ and $A = \frac{3l}{4}$. Therefore $\upsilon_{max} = \frac{3}{2}\sqrt{gl}$

9. (8)

$$W_{OAC} = W_{OA} + W_{AC} = 0 + \int_0^4 x^2 y\, dc$$

$$= 0 + \int_0^4 y\, dy = 8\text{ J}$$

10. (2)

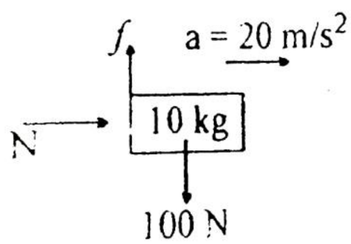
$$W_{OBC} = W_{OB} + W_{BC}$$

$$= 0 + \int_0^1 xy\, dx$$

$$= 0 + 4\int_0^1 x\, dx$$

$$= 2\text{ J}$$

11. (A, B, C, D)



$N = 10a = 200$ N

$f_{max} = 0.6 \times 200 = 120N$

$mg = 100\text{ N} < f_{max}$

$\therefore$ The friction is static and $f = 100$ N

The contact force $= \sqrt{N^2 + f^2} = 100\sqrt{5}$ N

12. (A, B, C, D)

Orbital speed $(v_0)=\sqrt{\dfrac{GM}{R}}$

Escapes speed $(v_e)=\sqrt{\dfrac{2GM}{R}}$

13. (AC)

Detector will detect maximum light when the image of source is formed at the detector. Let this happens after a time t.

At this time $u=-(90-10t);\upsilon=10t$

From lens formula $\dfrac{1}{\upsilon}-\dfrac{1}{u}=\dfrac{1}{f};\dfrac{1}{10t}-\dfrac{1}{-(90-20t)}=\dfrac{1}{20}$

This gives $t=3\text{sec}$ and $t=6\text{sec}$

14. (A, C)

$\theta>0_s$ for slipping

$\Rightarrow \tan\theta>\tan\theta_s$

$\Rightarrow \tan 37^\circ>\mu_s$

$\Rightarrow 0.75>\mu_s$

(B) $\therefore$ Bodies will not slip.

$\therefore$ No compression or expansion

(C) Block B slips, block A doesn't

15. (B, D)

$y=\sin\omega t+\cos\omega t=\left\{\sin\omega t.\dfrac{1}{\sqrt{2}}+\cos\omega t.\dfrac{1}{\sqrt{2}}\right\}.\sqrt{2}$

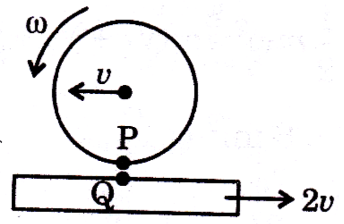
$=\sqrt{2}\left\{\sin\omega t\cos 45^\circ+\cos\omega t\sin 45^\circ\right\}$

$y=\sqrt{2}\sin\left(\omega t+\dfrac{\pi}{4}\right)$ …….(i)

Amplitude is $\sqrt{2}$ cm and when $t=0$,

$y=1\,\text{cm}$ [From eq. (i)]

16. (C)



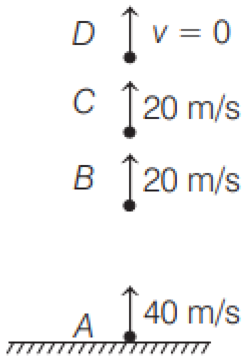
$-\vec{\upsilon}\hat{i}+\omega R\hat{i}=2\upsilon\hat{i}$

$\omega=\dfrac{3\upsilon}{R}$

$\vec{\omega}=\dfrac{3\upsilon}{R}\hat{k}$ (For Pure rolling direction must be $\hat{k}$)

17. (10)

18. (110)



From A to B,

$$v^2 = u^2 + 2as$$

$$20^2 = 40^2 + 2(-10)(AB)$$

$$AB = 60\,\text{m}$$

For next 2s,

$$s = ut + \frac{1}{2}at^2$$

$$BC = 20 \times 2 + \frac{1}{2}(0)(2)^2$$

$$BC = 40\,\text{m}$$

From C to D,

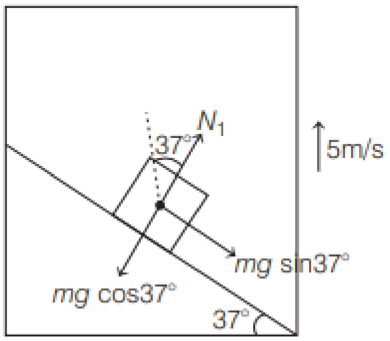
$$v^2 = u^2 + 2as$$

$$0^2 = 20^2 + 2(-20)(CD)$$

$$CD = 10\,\text{m}$$

$$AB + BC + CD = 60 + 40 + 10 = 110\,\text{m}$$

19. (320)



$$\sum F_y = 0$$

$$\Rightarrow \quad N_1 - mg\cos 37° = 0$$

$$\Rightarrow \quad N_1 = mg\cos 37°$$

$$W_{N_1} = Fs\cos\theta$$

$$= (mg\cos 37°)(vt)\cos 37° = 320\text{ J}$$

PART (B) : CHEMISTRY

ANSWER KEY

20.	(A)	21.	(B)	22.	(D)	23.	(C)	24.	(12.65)
25.	(120)	26.	(0.62)	27.	(0.53)	28.	(5)	29.	(3)
30.	(ABCD)	31.	(AC)	32.	(AC)	33.	(ABCD)	34.	(ABD)
35.	(CD)	36.	(8)	37.	(10)	38.	(5)		

SOLUTIONS

20. (A)

$\Delta_f G^\circ\ (NH_4Cl, s)$ at 310 K.

$$= S^\circ_{NH_4Cl(s)} - \left[\frac{1}{2}S^\circ_{N_2} + 2S^\circ_{H_2} + \frac{1}{2}S^\circ_{Cl_2}\right]$$

$$= -374\ JK^{-1}\ mol^{-1}$$

$\because \quad \Delta_r C_p = 0$

$\therefore \quad \Delta_f S^\circ_{310} = \Delta_f S^\circ_{300}$

$$= -374\ JK^{-1}\ mol^{-1}$$

$$\Delta_f H^\circ_{310} = \Delta_f H^\circ_{300} = -314.5$$

$$\Delta_f G^\circ_{310} = \Delta_f H^\circ - 310\Delta S^\circ$$

$$= -314.5 - \frac{310(-374)}{1000}$$

$$= -198.56\ kJ/mol$$

21. (B)

$HA \rightarrow H^+ + A^-\ ;\ \Delta_r H = 1.4\ kJ/mol$

$\Delta H_{neutralization} = \Delta H_{ionization} + \Delta_r H$

$(H^+ + OH^- \rightarrow H_2O)$

$-55.95 = \Delta H_{ionization} - 57.3$

$\Delta H_{ionization}$ for 1 M HA = 1.35 kJ/mol

% heat utilized by 1 M acid for ionization

$$= \frac{1.35}{1.4} \times 100 = 96.43\%$$

So, acid is 100 – 96.43 = 3.57% ionized

22. (D)

23. (C)

24. (12.65)

25. (120)

26. (0.62)
When X is connected to SHE, electron flow from X to SHE. This means that X is acting as anode and SHE as cathode and its oxidation potential is positive. Also, the reduction potential of Y is greater than the reduction potential of X (as electrons flow from X to Y).

$$\Rightarrow \quad Y^{2+} + 2e^- \rightarrow Y(s)$$

$$E^{\ominus}_{Y^{2+}|Y} = 0.34\ V;$$

$$X^{2+} + 2e^- \longrightarrow X(s)$$

$$E^{\ominus}_{X^{2+}|X} = -0.25\ V$$

Consider: $X(s)\ \left|X^{2+}(0.1\ M)\right|\left|Y^{2+}(1.0\ M)\right|Y(s)$

$$E^{\ominus}_{cell} = E^{\ominus}_{cell} - \frac{0.059}{2}\log_{10}\frac{[X^{2+}]}{[Y^{2+}]}$$

$$= [0.34 - (-0.25)] - \frac{0.059}{2}\log_{10}\frac{0.1}{1} = 0.62\ V$$

27. (0.53)
Also,

$$Y^{\oplus} + e^- \longrightarrow Y(s) \qquad E^{\ominus}_{req} = ?$$

$$\Rightarrow \Delta G^{\ominus}_{req} = -1 \times F \times E^{\ominus}_{req}$$

$$Y^{2+} + e^- \longrightarrow Y^{\oplus} \qquad E^{\ominus}_1 = 0.15\ V$$

$$\Rightarrow \Delta G^{\ominus}_1 = -1 \times F \times E^{\ominus}_1$$

$$Y^{2+} + 2e^- \longrightarrow Y(s) \qquad E^{\ominus}_2 = 0.34\ V$$

$$\Rightarrow \Delta G^{\ominus}_2 = -2 \times F \times E^{\ominus}_2$$

$$Y^{\oplus} + e^- \longrightarrow Y(s) \qquad -FE^{\ominus}_{req} = -2FE^{\ominus}_{req} - (-FE^{\ominus}_1)$$

$$\Rightarrow E^{\ominus}_{req} = -2E^{\ominus}_2 - E^{\ominus}_1$$

$$= 2(0.34) - 0.15$$

$$= 0.53\ V$$

28. (5)

29. (3)

30. (A, B, C, D)
At the point of maximum value of RDF

$$\frac{dP}{dr} = 0$$

$$\left(2r - \frac{2Z\,r^2}{a_0}\right) = 0; \ r = \frac{a_0}{Z}$$

Where $Z = 3$ for Li^{2+} and $Z = 2$ for the He^+; $Z = 1$ for hydrogen.

31. (A, C)

32. (A, C)

$$\frac{d}{dt}[NH_2CONH_2] = k_4[NH_3][HNCO] \text{ from (iii)}$$

Applying steady state approximation to HNCO or NH_3

$$\frac{d[HNCO]}{dt} = O = k_3[NH_4NCO] - k_4[NH_3][HNCO]$$

$$\therefore \quad \frac{k_3}{k_4} = \frac{[NH_3][HNCO]}{[NH_4CNO]}$$

$$\frac{d[\text{urea}]}{dt} = k_4 \times [NH_3][HNCO] = k_4 \times \frac{k_3}{k_4}[NH_4NCO]$$

Also, $[NH_4NCO] = \frac{k_1}{k_2} \times [NH_4CNO]$

$$\therefore \quad \frac{d[\text{urea}]}{dt} = k_3 \times \frac{k_1}{k_2} \times [NH_4CNO]$$

$$= k[NH_4CNO]$$

33. (ABCD)

34. (ABD)

35. (CD)

36. (8)
Statements (a), (b) and (c) are correct, so total score
= 4 + 1 + 3 = 8.

37. (10)

38. (5)

PART (C) : MATHEMATICS

ANSWER KEY

39.	(A)	40.	(B)	41.	(C)	42.	(A)	43.	(2.8)
44.	(22.4)	45.	(4)	46.	(2)	47.	(48)	48.	(24)
49.	(AC)	50.	(ABD)	51.	(BC)	52.	(ABCD)	53.	(AB)
54.	(AD)	55.	(5)	56.	(21)	57.	(6)		

SOLUTIONS

39. (A)

$$\left|2x-\sqrt{(2x-1)^2}\right|=1 \Rightarrow \left|2x-|2x-1|\right|=1$$

Case – I: $x<\frac{1}{2} \Rightarrow |2x+2x-1|=1 \Rightarrow |4x-1|$

Further : $x \in \left(\frac{1}{4},\frac{1}{2}\right) \Rightarrow 4x-1=1 \Rightarrow x=\frac{1}{2}$

And $x \in \left[-4,\frac{1}{4}\right] \Rightarrow -4x+1=1 \Rightarrow x=0$

Case – II: $x \geq \frac{1}{2} \Rightarrow |2x-2x+1|=1$

Possible values of x are {0, 1, 2, ….., 100}

Sum of all the value of x $=1+2+.....+100=5050$

40. (B)

S S S C C U E

P = $\dfrac{\text{‘S’ separate and ‘C’ anywhere – ‘S’ separate and ‘C’ together}}{\text{Total}}$

$$P=\frac{\frac{4!}{2!}.{}^5C_3-3!\,{}^4C_3}{\frac{7!}{3!2!}}$$

41. (C)

Area of $\Delta BPC=$ Area of ΔBGC

$$=\frac{1}{2}\cdot a\cdot h=\frac{1}{3}(\text{Area of }\Delta ABC)=\frac{1}{2}\{\text{Area of }(\Delta APB)+\text{Area of }(\Delta APC)\}=\frac{1}{2}\left\{\frac{2}{3}(\text{Area of }\Delta ABC)\right\}$$

$$=\frac{1}{2}a\cdot h=\frac{1}{2}\left\{\frac{1}{2}h_1\cdot 0+\frac{1}{2}\cdot h_2\cdot a\right\} \Rightarrow h=\frac{h_1+h_2}{2}$$

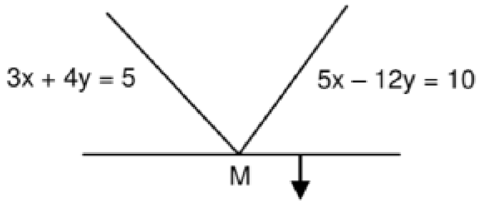
42. (A)

$\frac{dy}{dx}$ for a homogeneous equation is always $\frac{y}{x}$

Solution for Question Stem Que. No. 43 & 44

43. (2.8)

44. (22.4)



Equation of Angle bisector

$$\frac{3x+4y-5}{5}=\pm\left(\frac{5x-12y-10}{13}\right)$$

Taking (+ve) sign

$$14x+112y-15=0 \Rightarrow 14x+112y=15$$

$$\frac{14}{15}x+\frac{112}{15}y=1 \Rightarrow a=\frac{14}{5} b=\frac{112}{5}$$

Taking (–ve) sign

$$(39+25)\times+(52-60)y-65-50=0$$

$$64x-8y=115 \Rightarrow a=\frac{64}{115}\ b=\frac{-8}{115}\text{(rejected)}$$

Solution for Question Stem Q. 45 & 46

45. (4)

Here $\lambda=1, b=2$

$\therefore$ 4 solutions

46. (2)

Here, $\lambda=4, b=3$

$\therefore$ 2 solutions

Solution for Question Stem Q. 47 & 48

47. (48)

48. (24)

$$2^{m-2n}e^{i\frac{m\pi}{6}}=e^{i\frac{n\pi}{4}} \Rightarrow 2^{m-2n}=e^{i\left(\frac{n\pi}{4}-\frac{m\pi}{6}\right)}$$

$$\Rightarrow \cos\left(\frac{n\pi}{4}-\frac{m\pi}{6}\right)=2^{m-2n} \text{ and } \sin\left(\frac{n\pi}{4}-\frac{m\pi}{6}\right)=0 \Rightarrow \cos\left(\frac{n\pi}{4}-\frac{m\pi}{6}\right)=2^{m-2n}=1$$

$$\Rightarrow m=2n \text{ and } \frac{m\pi}{6}=\left(\frac{n\pi}{4}\right)+2k\pi \Rightarrow n=24k$$

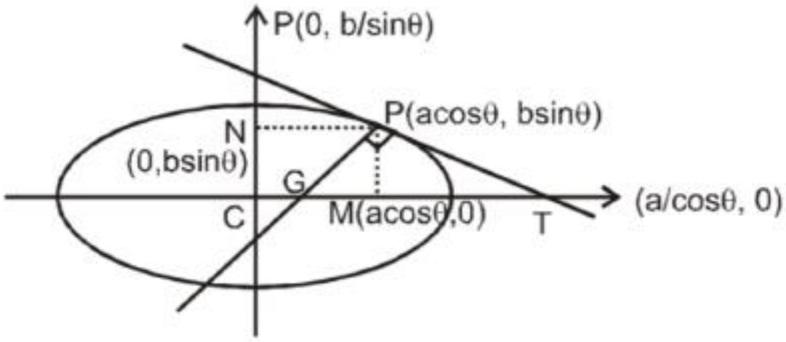
$\therefore$ For least positive integer

$n = 24$

$m = 48$

49. (A, C)

Equation of tangent is $\frac{x\cos x}{a}+\frac{y\sin\theta}{b}=1$



$CM.CT = a\cos\theta\dfrac{a}{\cos\theta} = a^2; CN.Ct = b^2$

$a^2 = 16; b^2 = 9$

$LRs = \dfrac{2b^2}{a} = \dfrac{2\times 9}{4} = \dfrac{9}{2}$

Equation of normal $\dfrac{x}{\frac{7\cos\theta}{4}} - \dfrac{y}{\frac{7\sin\theta}{3}} = 1$

$MG = \left|\dfrac{7\cos\theta}{4} - 4\cos\theta\right| = \dfrac{9\cos\theta}{4}$

$CM = a\cos\theta = 4\cos\theta$

$\dfrac{MG}{MC} = \dfrac{9}{16}$

50. (ABD)

Let roots are z_1, z_2, z_3 such that $|z_1| = |z_2| = |z_3| = 1$

Now we know $z_1 + z_2 + z_3 = -a$

$|a| = |z_1 + z_2 + z_3| \le |z_1| + |z_2| + |z_3|$

$|a| \le 3$

Similarly, $|b| = |z_1z_2 + z_2z_3 + z_3z_1| \le |z_1z_2| + |z_3z_1| + |z_1z_3|$

$|b| \le 3$

$|c| = |z_1||z_2||z_3| = 1$

51. (B, C)

$f(x) = e^{2x} - (a+1)e^x + 2x$

$f'(x) = 2e^{2x} - (a+1)e^x + 2$

Now, $2e^{2x} - (a+1)e^x + 2 \ge 0$ for all $x \in R$

i.e. $2\left(e^x + \dfrac{1}{e^x}\right) - (a+1) \ge 0$ for all $x \in R$

i.e. $4 - (a+1) \ge 0$

i.e. $a \le 3$

52. (A, B, C, D)

(A) We have, $y^2 = 4ax \Rightarrow m_1 = y' = \frac{2a}{y}$ and $y = e^{-x/2a} \Rightarrow m_2 = y' = \frac{1}{2a}e^{-x/2a} = -\frac{1}{2a}y$

$\therefore\ m_1 \cdot m_2 = -1$

Hence, orthogonal.

(B) We have, $y = 4ax \Rightarrow y' = \frac{4a}{2y} = \frac{2a}{y}(m_1)$ [not defined at (0, 0)

$\therefore$ The two curves are orthogonal at (0, 0).

(C) We have, $xy = a^2 \Rightarrow m_1 = y' = \frac{-a^2}{x^2}$ and $x^2 - y^2 = b^2 \Rightarrow m_2 = y' = \frac{2x}{2y} = \frac{x}{y}$

$\therefore\ m_1 \cdot m_2 = \frac{-a^2}{xy} = \frac{-a^2}{a^2} = -1 \quad \left[\because\ xy = a^2\right]$

Hence, orthogonal.

(D) We have, $y = ax \Rightarrow y' = a(m_1)$ and $x^2 + y^2 = c^2$

$\Rightarrow\ m_2 = y' = \frac{-x_1}{y_1}$

$\therefore\ m_1 \cdot m_2 = \frac{-ay_1}{y_1} = \frac{-x_1}{y_1} = -1$

Hence, orthogonal.

53. (A, B)

Equation of tangent to circle $y = mx + 4\sqrt{1+m^2}$

Equation of tangent to ellipse $y = mx + \sqrt{25m^2 + 4}$

Both represent the same line so

$4\sqrt{1+m^2} = \sqrt{25m^2+4} \Rightarrow m = \pm\frac{2}{\sqrt{3}}$

The tangent is at the point in first quadrant so $m < 0 \Rightarrow m = -\frac{2}{\sqrt{3}}$

So common tangent is $y = -\frac{2}{\sqrt{3}}x + 4\sqrt{\frac{7}{3}}$

It meets the co-ordinate axis at $A\left(2\sqrt{7}, 0\right)$ and B $\left(0, 4\sqrt{\frac{7}{3}}\right)$

So, $AB = \frac{14}{\sqrt{3}}$

54. (A, D)

Given, $x = kt,\ y = \frac{k}{t},\ k > 0$

$\Rightarrow\ \frac{dx}{dt} = k,\ \frac{dy}{dt} = \frac{-k}{t^2}$ and $\frac{dy}{dx} = \frac{-k/t^2}{k} = \frac{-1}{t^2}$

$\therefore$ Slope of tangent = Slope of line $\left(\frac{x}{a} + \frac{y}{b} = 1\right)$

$$\Rightarrow -\frac{1}{t^2}=\frac{-1/a}{1/b}$$

$$\Rightarrow \frac{1}{t^2}=\frac{b}{a}$$

$$\Rightarrow t^2=\frac{a}{b}$$

$$\Rightarrow t=\pm\sqrt{\frac{a}{b}}$$

So, a and b are of same sign

i.e. $a>0, b>0$ or $a<0, b<0$

55. (5)

Common roots of the equation $z^{15}-1=0$ *and* $z^{25}-1=0$ will be roots of the equation $z^5-1=0$

56. (21)

$$f(x)=e^{g(x)}$$

$$\Rightarrow f(x+1)=e^x.f(x)\Rightarrow e^{g(x+1)}=e^x.e^{g(x)}\Rightarrow g(x+1)=x+g(x)$$

$$\Rightarrow g(x+1)-g(x)=x\Rightarrow g'(x+1)-g'(x)=1$$

57 (6)

$$\lim_{n\to\infty}\left(1-\frac{d^2}{a_1^2}\right)\left(1-\frac{d^2}{a_2^2}\right)\left(1-\frac{d^2}{a_3^2}\right).....\left(1-\frac{d^2}{a_{n-1}^2}\right)\left(1-\frac{d^2}{a_n^2}\right)=\frac{1}{4}$$

$$\lim_{n\to\infty}\frac{(a_1-d)(a_1+d)}{a_1^2}\times\frac{(a_2-d)(a_2+d)}{a_2^2}\times\frac{(a_3-d)(a_3+d)}{a_3^2}\times\frac{(a_n-d)(a_n+d)}{a_n^2}=\frac{1}{4}$$

$$\Rightarrow \lim_{n\to\infty}\frac{a_1-d}{a_1}\times\frac{a_n+d}{a_n}=\frac{1}{4}\Rightarrow \lim_{n\to\infty}\frac{8-d}{8}\times\frac{8(n-1)d+d}{8+(n-1)d}=\frac{1}{4}$$

$$\Rightarrow \lim_{n\to\infty}(8-d)\times\left(\frac{\frac{8}{n}+d}{\frac{8}{n}+\left(1-\frac{1}{n}\right)d}\right)=2\ \Rightarrow 8-d=2\Rightarrow d=6$$