PART (A) : PHYSICS

ANSWER KEY

1.	(ABCD)	2.	(ACD)	3.	(BD)	4.	(AC)	5.	(AD)
6.	(AD)	7.	(490)	8.	(100)	9.	(10)	10.	(7.5)
11.	(2)	12.	(17)	13.	(C)	14.	(D)	15.	(B)
16.	(C)	17.	(60)	18.	(2)	19.	(2)		

SOLUTION

1. (A, B, C, D)

(A) Only equilibrium position changes. Time period remains same $T = 2\pi\sqrt{\frac{m}{k}}$

(B) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$ $\therefore v$ at mean position is same amplitude will be same

(C) In case-4 Equilibrium position $x_0 = \frac{3mg}{k}$

2. (A, C, D)

$T_A = T_B = \frac{(300+400)}{2} = 350\,^{\circ}C$

$\therefore T_A = T_B = T_C$

3. (B, D)

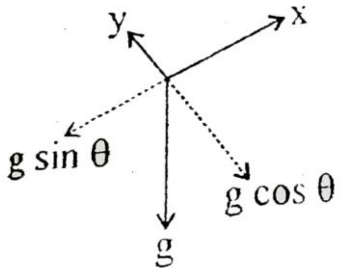
Suppose the dipole axis makes an angle θ with the electric field at an instant the magnitude of torque on it is

$|\tau| = |\vec{P} \times \vec{E}| = qlE\sin\theta$

$\tau \propto -\theta$

Hence, motion of dipole is SHM, as the motion is SHM it must be oscillatory.

4. (A, C)



$V_3^2 = V_1^2 - 2g\sin\theta x$

So $V_3 < V_1$

$V_4^2 = V_2^2 - 2g\cos\theta(\Delta y) = V_2^2 - 2g\cos\theta(0) = V_2^2$

$V_4 = V_2$

5. (A, D)

$$F_h = V_w \frac{dm}{dt}\cos\theta = V_w r\cos\theta$$

$$a_h = \frac{V_w r\cos\theta}{M - rt}$$

$$\frac{dv}{dt} = \frac{V_w r\cos\theta}{M - rt}$$

$$\int_0^v dv = V_w r\cos\theta \int_0^t \frac{dt}{M - rt}$$

$$v(t) = V_w r\cos\theta \ln\left(\frac{m - rt}{-r}\right)\Bigg|_0^t \Rightarrow v(t) = V_w\cos\theta\ln\left(\frac{M}{M - rt}\right)$$

6. (A, D)

(i) $n_1 C_{V_1} T_1 + n_2 C_{V_2} T_2 = (n_1 + n_2) C_V T$

$$2\left(\frac{5R}{2}\right)T_0 + .(3R)2T_0 = 5C_{V\,\text{mix}}T$$

$$\frac{23}{5}RT_0 = C_V T$$

(ii) $C_{V\,\text{mix}} = \dfrac{n_1 C_{V_1} + n_2 C_V}{n_1 + n_2} = \dfrac{2\times\frac{5R}{2} + 3\times 3R}{2+3} = \dfrac{14}{5}R$

$$C_{P\,\text{mix}} = \frac{19}{5}R$$

$\gamma_{\text{mix}} = \dfrac{19}{14}$; Put in (i) $C_V = \dfrac{14}{5}R$

$$T = \frac{23}{14}T_0$$

7. (490)

Let T be the final temperature of the gases when equilibrium is achieved.

Heat lost by monatomic gas at constant volume = Heat gained by diatomic gas at constant pressure

$\therefore \quad nC_{V_1}(700 - T) = nC_{p_2}(T - 400)$

$$\frac{3}{2}R(700 - T) = \frac{7}{2}R(T - 400)$$

$\Rightarrow \quad 2100 - 3T = 7T - 280 \Rightarrow 10T = 4900$

$\therefore \quad T = 490\text{ K}$

8. (100)

As the pressure of gases in both compartments in the same

$\therefore \quad nC_{p_1}(700 - T) = nC_{p_2}(T - 400)$

$$\frac{5}{2}R(700 - T) = \frac{7}{2}R(T - 400)$$

$$3500 - 5T = 7T - 2800 \Rightarrow 12T = 6300$$

$\therefore \quad T = 525\text{ K}$

Applying first law of thermodynamics

$\Delta W_1 + \Delta U_1 = \Delta Q_1$ and $\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

or, $-(\Delta W_1 + \Delta W_2) = \Delta U_1 + \Delta U_2$

$= nC_{v_1}(525-700) + n_2 C_{v_2}(525-400)$

$= -2\times\frac{3R}{2}\times 175 + +2\times\frac{5R}{2}\times 125$

$= -525R + 625R = -100R$

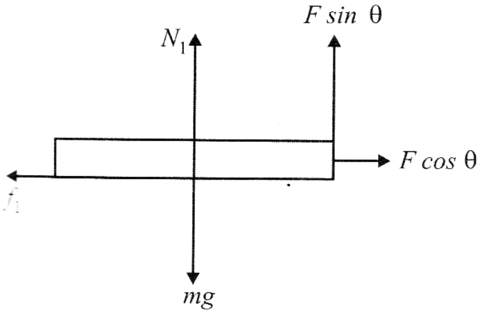
Therefore, total work done = –100 R

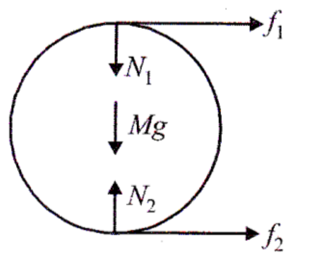
9. (10)

10. (7.5)

Solution for Que. 9 & 10

Drawing the F.B.D of the plank and the cylinder





Equations of motion are

$F\cos\theta - f_1 = ma$ ..... (1)

$F\sin\theta + N_1 = mg$ ..... (2)

$f_1 + f_2 = MA$ ..... (3)

$f_1 R - f_2 R = I\alpha$ ..... (4)

$A = R\alpha$ ..... (5)

$$a = \frac{4F\cos\theta}{3M+8m} = \frac{4\times 55\times\frac{1}{2}}{[(3\times 1)+(8\times 1)]} = 10\,\text{m/s}^2$$

$$f_1 = \frac{3MF\cos\theta}{3M+8M} = \frac{3\times 1\times 55\times\frac{1}{2}}{2\times 1+8\times 1} = 7.5\text{ N}$$

And $f_2 = \frac{3MF\cos\theta}{3M+8m} = \frac{1\times 55\times\frac{1}{2}}{3\times 1+8\times 1} = 25\text{ N}$

Solution for Q. 11 & 12

11. (2)

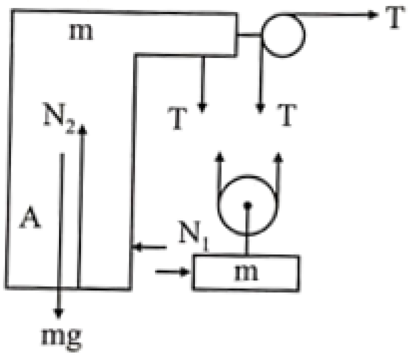
12. (17)

If acceleration of B is a, the acceleration of A will be $2a$.

$$Ta_A - 2T \times a_B = 0$$

or $a_A = 2a_B$

Motion of A :



$T - N_1 = m(2a)$ …(i)

and $2T + 2mg = N_2$ …(ii)

For B :

$mg - 2T = ma$ …(iii)

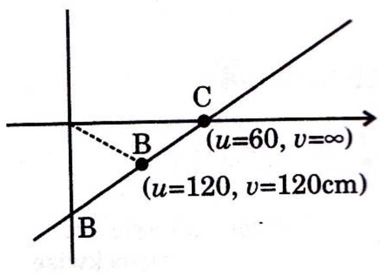
and $N_1 = m_2 a$ …(iv)

On solving above equations, we get

$$a = \frac{g}{9}, \text{ so } a_A = \frac{2}{9}g, \ N_1 = \frac{2mg}{9} \text{ and } N_2 = \frac{17}{9}mg.$$

13. (C)

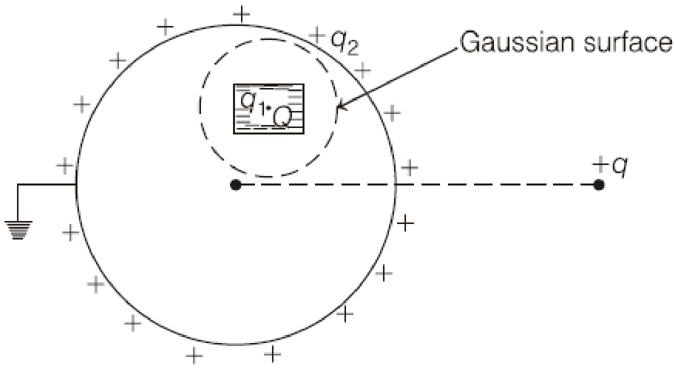
14. (D)



$$m = \frac{\upsilon}{u}.m = \frac{x}{y}$$

For region $BC\, x > y$ therefore $m > 1$ image formed will be magnified.

15. (B)



For the Gaussian surface,

$$E = 0$$

$$\Rightarrow \quad \phi = 0 \quad \Rightarrow \quad q_{enclosed} = 0$$

$$\Rightarrow \quad Q + q_1 = 0 \Rightarrow q_1 = -Q$$

Distribution of –Q on the inner surface of cavity will be non-uniform.

16. (C)

Since, the sphere is earthed, its potential will be zero.

At the centre,

$$V = \frac{kq}{l} + \frac{kq_2}{R} = 0$$

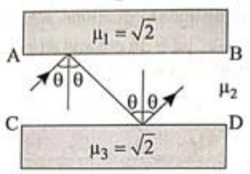
$$\Rightarrow \quad q_2 = \frac{-qR}{l}$$

17. (60)

Let θ is the angle of incidence and C_1 is the critical angle, then

$$\sin\theta > \sin C,$$

$$> \frac{\mu_1}{\mu_2}$$



$$> \frac{\sqrt{2}}{2}$$

$$\therefore \quad \theta > 45°.$$

For total internal reflection at the face CD

$$\sin\theta > \sin C_2$$

$$> \frac{\mu_3}{\mu_2}$$

$$> \frac{\sqrt{3}}{2}$$

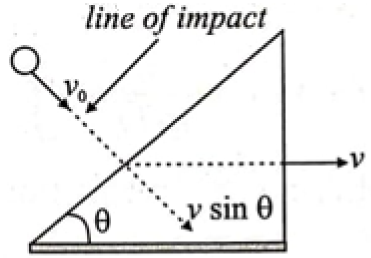
$$\therefore \quad \theta > 60°$$

For total internal reflection at both the surfaces

$$\theta_{min} = 60°.$$

18. (2)

If v is the horizontal velocity of the wedge, then its component along the line of impact will be $v\sin\theta$. Using conservation of momentum along horizontal direction.



$mv_0\sin\theta = mv_1\sin\theta + Mv$ ...(i)

Here, v_1 is the velocity of ball after collision.

Also $e = -\left[\frac{v_1 - v_2}{u_1 - u_2}\right] = -\left[\frac{v_1 - v\sin\theta}{v_0 - 0}\right]$

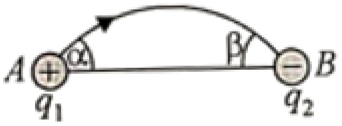
or $v_1 - v\sin\theta = -ev_0$ ...(ii)

After solving above equations, we get

$$v = \frac{m(1+e)v_0\sin\theta}{M + m\sin^2\theta}.$$

19. (2)

The number of lines of force emerges is proportional to the amount of changes. The line of force emanating from q_1 spread out equally in all directions. Hence lines of force per unit solid angle are $\frac{q_1}{4\pi}$ and the number of lines of force through cone of half angle α is $\frac{q_1}{4\pi}\cdot 2\pi(1-\cos\alpha)$. Similarly the number of lines of force terminating on $-q_2$ at angle β is $\frac{q_2}{4\pi}2\pi(1-\cos\beta)$



As the total lines of force emanating from q_1 is equal to the total lines of force terminating to q_2, so

$$\frac{q_1}{4\pi}2\pi(1-\cos\alpha) = \frac{q_2}{4\pi}2\pi(1-\cos\beta)$$

or $\frac{q_1}{2}\cdot 2\sin^2\frac{\alpha}{2} = \frac{q_2}{2}\cdot 2\sin^2\frac{\beta}{2}$

or $\sin\frac{\beta}{2} = \sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}$

$\therefore$ $\beta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\frac{\alpha}{2}\right]$

PART (B) : CHEMISTRY

ANSWER KEY

20.	(BCD)	21.	(CD)	22.	(BCD)	23.	(AC)	24.	(ABC)
25.	(ABCD)	26.	(10)	27.	(7)	28.	(190)	29.	(1.2)
30.	(30.146)	31.	(32.5)	32.	(B)	33.	(C)	34.	(D)
35.	(A)	36.	(5)	37.	(5)	38.	(4)		

SOLUTION

20. (BCD)

(A) $S=\sqrt{K_{sp}}=10^{-5}\text{ mol lit}^{-1}$

(B) $s=10^{-5}\text{ mol lit}^{-1}$

(C) $AgCl(s)\rightleftharpoons Ag^{+}+Cl^{-}$

$\quad\quad\quad\quad S+2\quad S$

$$10^{-10}=(S+2)S\ \Rightarrow\ S=\frac{10^{-10}}{2}=5\times10^{-11}$$

(D) $AgCl(s)\rightleftharpoons[Ag(NH_3)_2]^{+}+Cl^{-}$

$$Ag^{+}+2NH_3\rightleftharpoons[Ag(NH_3)_2]^{+}+Cl^{-}$$

$$AgCl(s)+2NH_3\rightleftharpoons[Ag(NH_3)_2]^{+}+Cl^{-}$$

$$K=\frac{[Ag(NH_3)_2]^{+}[Cl^{-}]}{[NH_3]^2}=K_{sp}\times K_f$$

$$\frac{S^2}{(2-S)^2}=10^{+8}\times10^{-10}=10^{-2}$$

$$\frac{S^2}{(2-S)^2}=10^{-1}$$

$$10S=2-S$$

$$S=\frac{2}{11}=0.182\ M$$

21. (CD)

Le Chatelier's principle is not quantitative.

If both stress would cause the same direction of shift, the shift is determinable. If the two stresses would cause shifts in opposite directions, no deduction is possible.

22. (BCD)

23. (AC)

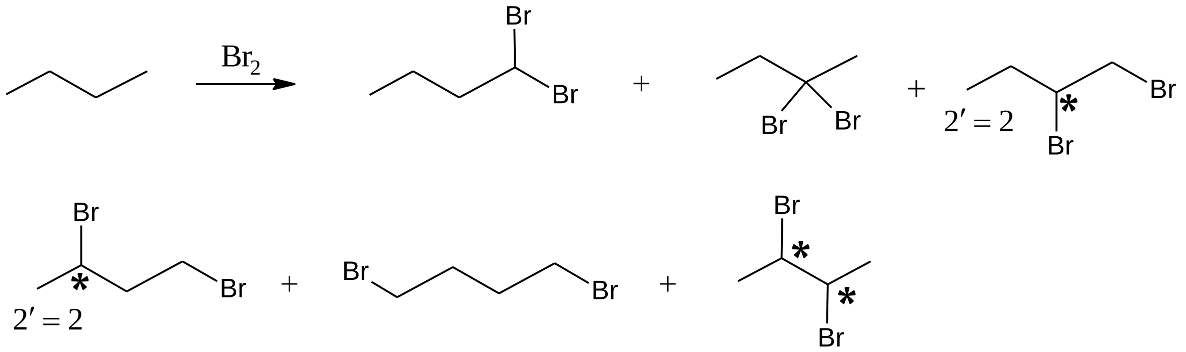
24. (ABC)

25. (ABCD)

Solution for Stem Question Nos. 26 and 27

26. (10)

27. (7)



Three stereomers

$X = 10$
$Y = 7$

28. (190)

$$h_L = \frac{76 \times 13.6}{5.44} = 190\,\text{cm}$$

29. (1.2)

$$P_{\text{gas}} = P_{\text{atm}} + P_L = 1 + \frac{38}{190} = 1.2$$

30. (30.146)

At 2ºC (275 K), the reaction is three times slower than at 27ºC (300 K).
This implies that for souring of milk.

$$\frac{k_{300}}{k_{275}} = 3,\ T_1 = 275\text{ K},\ T_2 = 300\text{ K}$$

Applying Arrhenius equation

$$\log \frac{k_{300}}{k_{275}} = \frac{E_a}{2.303R}\left(\frac{T_2 - T_1}{T_2 T_1}\right)$$

$$\log 3 = \frac{E_a}{2.303 \times 8.314}\left(\frac{300 - 275}{275 \times 300}\right)$$

or $\quad E_a = 30.146\text{ kJ mol}^{-1}$

31. (32.5)

Now $T_1 = 300\text{ K},\ T_2 = 37°\text{C} = 310\text{K}$

$E_a = 30146\text{ J mol}^{-1}$

$$\log \frac{k_{310}}{k_{300}} = \frac{30146\text{ J mol}^{-1}}{2.303 \times 8.314} \times \frac{310 - 300}{300 \times 310} = 0.1693$$

$$\therefore\ \frac{k_{310}}{k_{300}} = \text{antilog } (0.1693) = 1.477$$

Higher the rate constant, faster is the reaction, i.e., lesser is the time taken, hence

$$\frac{t_{310}}{t_{300}} = \frac{k_{300}}{k_{310}} = \frac{1}{1.477}$$

$$\therefore\ t_{310} = t_{300} \times \frac{1}{1.477} = \frac{48}{1.477} = 32.5\ \text{hr}$$

32. (B)

$k_{310} > k_{300}$ and $k \propto \frac{1}{E_a}$ and also $E_a \propto \frac{1}{T}$

But E_a at 310K < E_a at 300K

$\therefore\ \frac{k_{310}}{k_{300}}$ will be maximum for the reaction having high E_a.

33. (C)

For first reaction:

$$\log k_1 = \log A - \frac{E_a}{RT_1}$$

$$\log k_2 = \log A - \frac{E_a}{RT_2}$$

$$\therefore\ \log\frac{k_2}{k_1} = \frac{E_a}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

Similarly, for second reaction,

$$\log\frac{k_2'}{k_1'} = \frac{E_a'}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

From Eqs. (i) and (ii),

$\frac{k_2}{k_1} \propto E_a$ and $\frac{k_2'}{k_1'} \propto E_a'$

Since $E_a > E_a'$

$$\therefore\ \frac{k_2}{k_1} > \frac{k_2'}{k_1'} \Rightarrow \frac{k_1'}{k_1} > \frac{k_2'}{k_1}$$

34. (D)

35. (A)

36. (5)

37. (5)

For second order reaction:

$[R]_{initial} = 00.08\ M;\ [R]_{final} = 0.01\ M$

$x = 0.08 - 0.01 = 0.07\ M$

$\therefore\ (a - x) = 0.08 - 0.07 = 0.01\ M$

$$k_2 = \frac{1}{t}\cdot\frac{x}{a(a-x)}$$

$$= \frac{1}{70\ \text{min}} \times \frac{0.07\ \text{M}}{0.08\ \text{M} \times 0.01\ \text{M}} \qquad \ldots \text{(i)}$$

Now, time required to become concentration = 0.04 M.

i.e., x = 0.04 M

$$k_2 = \frac{1}{\text{t}} \times \frac{0.04\ \text{M}}{0.08\,\text{M} \times (0.08 - 0.04)\text{M}} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii)

$$\frac{0.07}{70 \times 0.08 \times 0.01} = \frac{0.04}{t \times 0.08 \times 0.04}$$

$$t = 10\,\text{min} = 2x\,\text{min}$$

$$\therefore\ x = 5\,\text{min}$$

38. (4)

PART (C) : MATHEMATICS

ANSWER KEY

39.	(CD)	40.	(A)	41.	(BC)	42.	(ABCD)	43.	(AB)
44.	(AD)	45.	(1)	46.	(0.5)	47.	(2)	48.	(8)
49.	(4)	50.	(4)	51.	(D)	52.	(A)	53.	(B)
54.	(C)	55.	(2)	56.	(9)	57.	(2)		

SOLUTION

39. (C, D)

Clearly, $p_1 = \frac{1}{2}$

$$p_2 = p(HH) + p(T) = \frac{3}{4}$$

Now the score n can be obtained in two distinct ways:

(i) By throwing head when the score is $n-1$

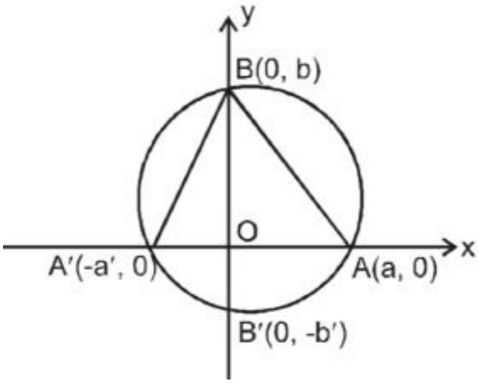
(ii) By throwing tail when the score is $n-2$

$$\Rightarrow p_n = (p_{n-1})\frac{1}{2} + (p_{n-2})\frac{1}{2} = \frac{1}{2}(p_{n-1} + p_{n-2})$$

Also, from this $p_4 = \frac{11}{16}$

40. (A)

41. (B, C)



Equation of line perpendicular to $A'B$ and passing through A

$$y - 0 = -\frac{a'}{b}(x - a) \qquad \ldots(A)$$

Equation of the line BO is x = 0 …(B)

So by solving (A) and (B) orthocentre $\left(0, \frac{aa'}{b}\right)$

Again $OA.OA' = OB.OB'$

$$a(-a') = b(-b') \Rightarrow \frac{aa'}{b} = b'$$

42. (A, B, C, D)

$2^x + 2^y = 2^{x+y}$ .....(i)

Diff. both sides w.r.t. x

$$2^x \ell n2 + 2^y.\ell n2\frac{dy}{dx} = 2^{x+y}, \ell n2\left(1+\frac{dy}{dx}\right)$$

$$2^x - 2^{x+y} = (2^{x+y} - 2^y)\frac{dy}{dx}$$ .....(ii)

$$\frac{2^x(1-2^y)}{2^y(2^x-1)} = \frac{dy}{dx}$$

From (i) and (ii)

$$2^x - 2^x - 2^y = (2^x + 2^y - 2^y)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2^y}{2^x}$$

$$\frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y} - 2^y} = \frac{-2^y}{2^y(2^x-1)} = \frac{1}{1-2^x} = 1-2^y$$

43. (A, B)

$$2\sin 11x + 2\sin\left(3x+\frac{\pi}{6}\right) = 0$$

$$\sin 11x = -\sin\left(3x+\frac{\pi}{6}\right)$$

$$11x = n\pi + (-1)^n\left(-3x-\frac{\pi}{6}\right)$$

If n = even

$$11x = 2m\pi - 3x - \frac{\pi}{6}$$

$$\Rightarrow 14x = 2m\pi - \frac{\pi}{6} \Rightarrow x = \frac{m\pi}{7} - \frac{\pi}{84}$$

If x = odd

$$11x = (2m+1)\pi + 3x + \frac{\pi}{6} \Rightarrow 8x = 2m\pi + \frac{7\pi}{6} \Rightarrow x = \frac{m\pi}{4} + \frac{7\pi}{48}$$

44. (A, D)

$(x + a)(x + 1991) + 1 = 0$

$\Rightarrow (x+a)(x+1991) = -1$

$\Rightarrow (x+a) = 1$ and $x+1991 = -1$

$\Rightarrow a = 1993$

Or $x+a = -1$ and $x+1991 = 1$

$\Rightarrow a = 1989$

Solution for Que. 45 & 46

45. (1)
46. (0.5)

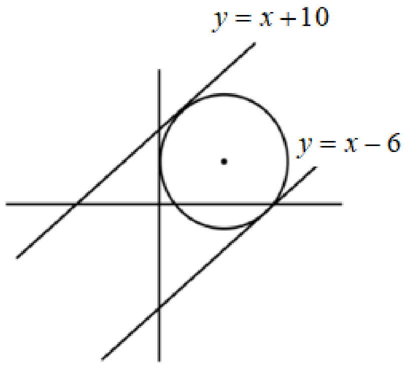
$$\left|\frac{2}{3+ie^{i\theta}}\right| = \frac{2}{|(3-\sin\theta)+i\cos\theta|} = \frac{2}{\sqrt{10-6\sin\theta}} \therefore M = \frac{2}{\sqrt{10-6}} = 1$$

$$m = \frac{2}{\sqrt{10+6}} = \frac{1}{2}$$

Solution for Que. 47 & 48

47. (2)

48. (8)



Centre is (h, k) and touch the line $x = 0 \Rightarrow$ radius h

Distance between both lines

$$2r = \frac{16}{\sqrt{2}} \Rightarrow r = 4\sqrt{2} = h \qquad \ldots\ldots(\text{i})$$

Length of perpendicular from center $c(h, k)$ = radius

$$\Rightarrow \left|\frac{h-k+10}{\sqrt{2}}\right| = 4\sqrt{2}$$

$$\Rightarrow 4\sqrt{2}-k+10 = 8 \Rightarrow 4\sqrt{2}+2 = k$$

$$h+k = 4\sqrt{2}+4\sqrt{2}+2 = 8\sqrt{2}+2 = a+b\sqrt{a} \Rightarrow a = 2 \text{ and } b = 8$$

Solution for Que. 49 & 50

49. (4)

50. (4)

$$10^n \frac{\sum_{r=0}^{n} {}^nC_r 10^{n-r}.2^r}{\sum_{r=0}^{n} {}^nC_r (10)^{n-r}(-2)^r} = \frac{10^n(10+2)^n}{(10-2)^n}$$

$$= (15^n) = 5^n.3^n = (625).3^P$$

$$\Rightarrow n = 4 \text{ and } p = 4$$

51. (D)

For intersection of C_1 and C_2 subtract $C_1 - C_2$

$(2x)$ (4) =0

$x = 0$

$y = \pm 2\sqrt{15}$

$P\left(0, 2\sqrt{15}\right),\ Q\left(0, -2\sqrt{15}\right)$

52. (A)

Foot of focus on any tangent of ellipse lie on auxiliary circle.

Let image of focus be (h, k)

$e=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$ae = 2$

mid-point $m\left(\frac{h+2}{2},\frac{k}{2}\right)$

It is lie on auxiliary circle $x^2+y^2=16$

$\frac{(h+2)^2}{4}+\frac{k^2}{4}=16$

$(h+2)^2+k^2-64$

Similarly $(h-2)^2+k^2=64$ (From 2[nd] focus)

$C_1:(x+2)^2+y^2=64$

$C_2:(x+2)^2+y^2=64$

53. (B)

Area of quadrilateral $PRQS$ = $\frac{1}{2}PQ\times RS$

$=\frac{1}{2}\times\left(2\sqrt{3}\right)\left(\frac{15}{2}\right)$

$=\frac{15\sqrt{3}}{2}$

54. (C)

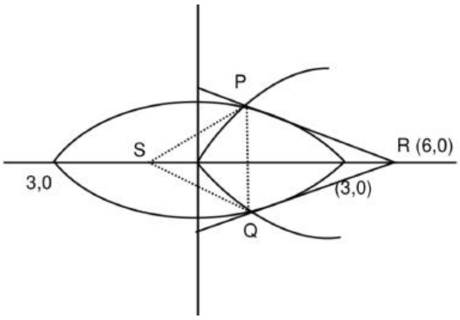
$\frac{x^2}{9}+\frac{y^2}{4}=1, y^2=2x$

$\frac{x^2}{9}+\frac{2x}{4}=1$

$\Rightarrow 2x^2+9x-18=0$

$\Rightarrow (2x-3)(x+6)=0$

$\Rightarrow x=3/2,-6$



$P\equiv\left(3/2,\sqrt{3}\right)$ $Q\equiv\left(3/2,-\sqrt{3}\right)$

$R\equiv(6,0)$ $S\equiv(-3/2,0)$

Area of $\Delta PQS = \frac{1}{2}\left(2\sqrt{3}\right)(3) = 3\sqrt{3}$

Area of $\Delta PQRs = \frac{1}{2}\left(2\sqrt{3}\right)\left(\frac{9}{2}\right) = \frac{9\sqrt{3}}{2} \Rightarrow \frac{Area\ of\ PQS}{Area\ of\ \Delta PQR} = \frac{2}{3}$

55. (2)

$x^3 + 2x^2 + 2x + 1 = 0$

$\Rightarrow (x+1)(x^2 + x + 1) = 0$

$\Rightarrow x = -1, \omega, \omega^2$

$x = \omega$ and ω^2 are also satisfying the equation $x^{2008} + x^{2009} + 1 = 0$

56. (9)

Since, $x + 4y = 14$ is normal to the curve $y^2 = \alpha x^3 - \beta$ at (2, 3).

$\therefore$ Slope of $(x + 4y = 14) = \left(-\frac{dx}{dy}\right)_{(2,3)}$

$$\Rightarrow \quad -\frac{1}{4} = \left[-\frac{1}{\left(3\alpha\frac{x^2}{2y}\right)}\right]_{(2,3)}$$

$$\Rightarrow \quad -\frac{1}{4} = -\frac{1}{2\alpha} \Rightarrow \alpha = 2$$

From Eq. (i), we have

$(3)^2 = 2(2)^3 - \beta$

$\Rightarrow \quad 9 = 16 - \beta$

$\Rightarrow \quad \beta = 7$

$\therefore \quad \alpha + \beta = 2 + 7 = 9$

57. (2)

$$y = \frac{\sin x}{1 + \frac{\cos x}{1+y}}$$

$\Rightarrow y(1 + y + \cos x) = \sin x \cdot (1 + y)$

$y'(0) = \frac{1}{2}$