PART (A) : PHYSICS

ANSWER KEY

1.	(B)	2.	(A)	3.	(A)	4.	(D)	5.	(C)
6.	(C)	7.	(C)	8.	(D)	9.	(A)	10.	(B)
11.	(B)	12.	(A)	13.	(A)	14.	(D)	15.	(A)
16.	(B)	17.	(B)	18.	(D)	19.	(D)	20.	(C)
21.	(2)	22.	(3)	23.	(5)	24.	(9)	25.	(500)
26.	(50)	27.	(70)	28.	(0)	29.	(50)	30.	(3)

SOLUTIONS

1. (B)

$\omega = 0+1\times 10 = 10\,rad/sec^2$

$\therefore v = r\omega = 1\times 10 = 10\,m/s$

$$\bar{B} = \frac{\mu_0}{4\pi}\frac{q\left(\vec{V}\times\vec{r}\right)}{r^3} \Rightarrow \left|\bar{B}\right| = \frac{\mu_0 qv}{4\pi r^2}$$

$$B = \frac{10^{-7}\times 0.1\times 10}{(1)^2} = 10^{-7}\,T$$

2. (A)

$$\phi = \frac{q}{\varepsilon_0}\times\frac{2\pi(1-\cos\theta)}{4\pi}$$

$$\phi = \frac{q}{2\varepsilon_0}(1-\cos\theta)$$

And $F = qE = q.\frac{\sigma}{2\varepsilon_0}(1-\cos\theta)$

3. (A)

If $n_2 \to n_1$ in $H(z=1)$ gives λ then $zn_2 \to z\,n_1$ gives λ in H-like ion for He^+ ion, $z=2$

4. (D)

No. of field lines $\propto \phi = \frac{q_{in}}{\varepsilon_0}$

5. (C)

$$(\chi) = \frac{\text{Intensity of magnetisation}(I)}{\text{Magnetizing field}(H)}$$

Or, $I = \chi H$

6. (C)

Pressure difference between lungs of student and atmosphere

$=(760-750)\,mm$ of $Hg = 1cm$ of $Hg = 13.6cm$ of water

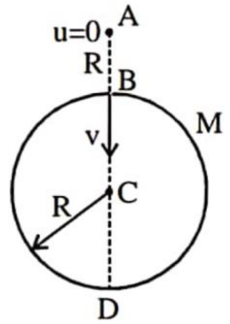
Hence $h = 13.6\text{cm}$

7. (C)

Let v be the velocity of the particle at point B. Applying conservation of mechanical energy at point A and B, we have

$$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv_B^{\ 2}$$

$$\Rightarrow v_B = \sqrt{\frac{GM}{R}}$$



Thereafter $V = \sqrt{\frac{GM}{R}} = \text{constant}$

$\left(\because \text{inside shell}, \vec{F}_g = m\vec{E}_g = 0\right)$

$\therefore t_{BD} = \frac{2R}{v}$

8. (D)

$W_{net} = \left(\sum \Delta Q\right)_{cycle} = 10 + 15 - 10 = 15J$

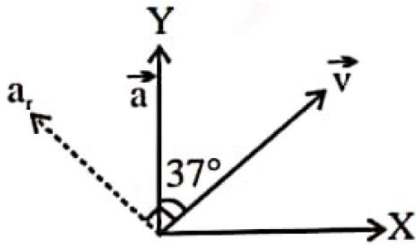
$\Delta Q_{in} = 10 + 15 = 25J$

$\Rightarrow \eta = \frac{15}{25} \times 100 = 60\%$

9. (A)

$a_r = a\sin 37^\circ = 3\,m/s^2$

Also, $r = \frac{v^2}{a_r} = \frac{25}{3}m$



10. (B)

$f \propto \sqrt{g}$

In water $f_w = 0.8 f_{air}$

$\therefore \frac{g'}{g} = (0.8)^2 = 0.64$

Or $\frac{\rho_w}{\rho_m} = 0.36$ ……(i)

In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$

Or $\frac{\rho_L}{\rho_m} = 0.64$

From equations (i) and (ii) $\frac{\rho_L}{\rho_w} = \frac{0.64}{0.36}$

$S_L = \rho_L / \rho_w = 1.77$

11. (B)

Let elongation of spring be x_0 in equilibrium.

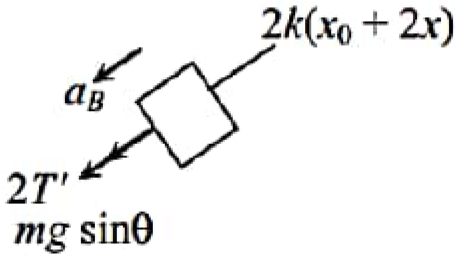
Then,

$2T + mg\sin\theta = 2kx_0$ …..(i)

And $T = mg$ ……(ii)

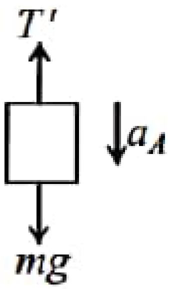
Let Block B is displaced by x down the inclination

F.B.D. of B



$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta$ …..(iii)

F.B.D of A



$mg - T = ma_A$

Also, $a_A = 2a_B$

$T' = mg - 2ma_B$

$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$

$-ma_B = 4kx + 4ma_B$

$a_B = -\frac{4k}{5m}x$

$\therefore T = 2\pi\sqrt{\frac{5m}{4k}}$

$T = 6.28s$

12. (A)
From perpendicular axis theorem
$I_z = I_x + I_y = 2I$

13. (A)
As the block moves out of the liquid, tension increases.

14. (D)
In branch having R_2 and L

$i = \frac{E}{R_2}\left(1 - e^{-\frac{R_2 t}{L}}\right)$

$\frac{di}{dt} = \frac{E}{L} \cdot e^{-\frac{R_2 t}{L}}$

$V_L = L\frac{di}{dt} = 12e^{-5t}V$

15. (A)
$\frac{1}{2}mv_0^2 = \frac{(ze)(q)}{4\pi \in_0 r} \Rightarrow r \propto \frac{q}{m}$

16. (B)
$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega c} - \omega L\right)^2}}$

$I_0 \uparrow \omega \uparrow$ only possible if $\frac{1}{\omega C} > \omega L$.

17. (B)
$\rho(r) = A(r)^2$
Charge enclosed for sphere of radius $R/2$

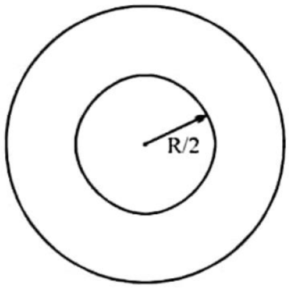
$Q = \int (4\pi r^2) dr\, \rho(r) = 4\pi A \int_0^{R/2} r^r dr$

$= 4\pi A \left[\frac{v^5}{5}\right]_0^{R/2}$

$= \frac{4\pi A}{5 \times 32}(R^5) = \frac{\pi A}{40}R^5$

Applying Gauss's law for this sphere

$4\pi(R/2)^2 E = Q/\varepsilon_0 = \frac{\pi A}{40}R^5$



$\Rightarrow E = \frac{AR^3}{40\varepsilon_0}$

18. (D)

$v = \frac{1}{5}S + 3$ $\quad 0 \le S \le 60$ $\quad \frac{dS}{dt} = \frac{1}{5}S + 3$

$t_1 =$ time taken to covered 60 m $\quad \int_0^{60} \frac{dS}{S+15} = \int_0^{t_1} \frac{dt}{5}$

$t_1 = 5\ln 5$ and $v = 15$ $\quad 60 \le S \le 120$

$t_2 =$ time taken to covered 60 m to 120 m $\quad t_2 = \frac{60}{15} = 4s$

Total time $= 5\ln 5 + 4$

19. (D)

Shortest wavelength comes from $n_1 = \infty$ to $n_2 = 1$ and longest wavelength comes from $n_1 = 6$ to $n_2 = 5$ in the given case.

Hence $\frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$

$\frac{1}{\lambda_{max}} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right) = R\left(\frac{36-25}{25\times 36}\right) = \frac{11}{900}R$

$\therefore \frac{\lambda_{max}}{\lambda_{min}} = \frac{900}{11}$

20. (C)

By definition, Bulk's modulus $K = -\frac{dP}{\left(\frac{dV}{V}\right)} = -\frac{mg}{dV/V}$

$V = \frac{4}{3}\pi R^3 \Rightarrow \frac{dV}{V} = 3\frac{dR}{R}, K = \frac{mg}{3A\left(\frac{dR}{R}\right)} \Rightarrow \frac{dR}{R} = -\frac{mg}{3AK}$

$\therefore (C)$

21. (2)

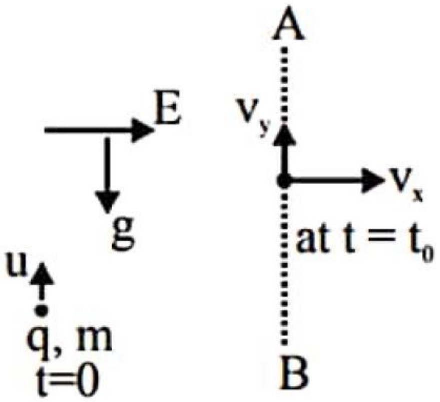
At $t = 0, \frac{A_{0_A}}{A_{0_B}} = \frac{25}{75} = \frac{1}{3}$ ......(1)

$$\text{at } t = t, \frac{A_{t_A}}{A_{t_B}} = \frac{A_{0_A} e^{-\lambda t}}{A_{0_B} e^{-2\lambda t}} = \frac{75}{25} = 3 \qquad \text{.......(2)}$$

$\therefore$ from (1) and (2), $e^{\lambda t} = 9$

$\Rightarrow \lambda t = 2\ln 3 \Rightarrow t = 2$

22. (3)



$t \le t_0 : V_x = \frac{qE}{m} t_0 = g t_0$

$V_y = u - g t_0$

Just after AB, $\vec{v} = \text{constant} \Rightarrow \vec{F}_{net} = 0$

$\Rightarrow q\vec{E} + q(\vec{v} \times \vec{B}) + m\vec{g} = 0$

$\Rightarrow qE\vec{i} + qv_x B\vec{j} - q v_y B\vec{i} - mg\vec{j} = 0$

$\Rightarrow E = B(u - g t_0)$ and $qB t_0 = m$

$\Rightarrow u = 2g t_0 = 3 m/s$

23. (5)

$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2} L i^2\right) = L i \frac{di}{dt}$

By KVL, $L\frac{di}{dt} + i(10) = 12$

$\Rightarrow L\frac{di}{dt} = 8$ when $i = 0.4A$

$\Rightarrow L i \frac{di}{dt} = 3.2 = \frac{16}{x} \Rightarrow x = 5$

24. (9)

$E_{eq} = 8\varepsilon = 8 \times 1.5 = 12V$

$r_{eq} = 8r = 8 \times 0.5 = 4\Omega$

$\therefore$ For $P_{max}, R_{ext} = r_{eq} = 4\Omega$

$\Rightarrow P_{max} = \frac{\varepsilon_{eq}^2}{4 r_{eq}} = 9W$

25. (500)
Infront of upper slit
On screen $= \Delta x = d\left(\frac{d/2}{D}\right) - (\mu - 1)t = 0$

$$\Delta x = d\frac{(d/2)}{D} - (\mu - 1)t = 0$$

At centre on the screen

$$\Delta x = (\mu - 1)t = \frac{d^2}{2D}$$

26. (50)

$$P = \frac{nhc}{\lambda t}$$

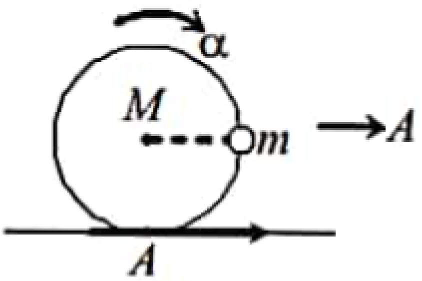
$$i = \left(\frac{n}{t}\right)ex\% = \frac{p\lambda e}{hc}x\%$$

$$= \frac{1.55\times10^{-3}\times4\times10^{-7}}{6.63\times10^{-34}\times3\times10^{8}}\times1.6\times10^{-19}\times\frac{10}{100}$$

$$= 50\mu A$$

27. (70)

$$(A_{CM})_x = \frac{mA + MA}{m + M} = A$$



$$(A_{CM})_y = \frac{M\times0 + mR\alpha}{m + M} = \frac{mR\alpha}{m + M}$$

$f = (M + m)A$ ……(i)

$(M + m)g - N = (M + m)(A_{CM})_y$ ……….(ii)

$mgR = I_A\alpha$ ………(iii)

$A = R\alpha$ ………(iv)

$\therefore N = 70N$

28. (0)
Doppler's effect depends upon velocity of approach and separation of source and observer, hence no change in frequency received by the observer.
$\therefore$ no beat is heard

29. (50)
Peak value of output voltage from the transformer is

$V_0 = \frac{200}{20} = 10V$

The junction diode conducts during the half cycle of the input when it is forward biased. During this half cycle, the capacitor is charged to the peak value of the supply voltage(which is 10V). Hence the final charge on capacitor plates is

$Q = CV_0 = \left(5 \times 10^{-6}\right) \times 10 = 50\mu C$

30. (3)

Diode D_1 is forward biased. Therefore, capacitors C and 3C are in series. If V_1 and V_2 are potential drops across C and 3C respectively, then

$\frac{V_1}{V_2} = 3$ or $V_1 = 3V_2$

Given $V_1 + V_2 = 12$ Thus, $3V_2 + V_2 = 12$ or $V_2 = 3V$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(D)	32.	(D)	33.	(C)	34.	(A)	35.	(A)
36.	(A)	37.	(B)	38.	(A)	39.	(D)	40.	(D)
41.	(C)	42.	(A)	43.	(D)	44.	(C)	45.	(C)
46.	(B)	47.	(C)	48.	(B)	49.	(B)	50.	(A)
51.	(13)	52.	(1)	53.	(0)	54.	(8)	55.	(4)
56.	(30)	57.	(10)	58.	(5)	59.	(3)	60.	(4)

SOLUTIONS

31. (D)

$$Na_2O_2 + \text{Dil. } H_2SO_4 \longrightarrow Na_2SO_4 + H_2O_4$$

32. (D)

33. (C)

34. (A)

35. (A)

N_2 bond order = 3

N_2^+ bond order = 2.5

O_2 bond order = 2

O_2^+ bond order = 2.5

Bond order $\propto$ Bond energy.

36. (A)

	A(g) $\rightarrow$	B(g) +	C(g) +	D(s)
t = 0	a	–	–	–
t	(a–x)	x	x	–

PV = nRT

$V \propto n$ (at constant P & T)

$V_0 \propto a$

$V_t \propto (a + x)$

$\Rightarrow x \propto (v_t - v_0)$

$$k = \frac{1}{t}\ln\frac{a}{a-x}$$

$$K = \frac{1}{t}\ln\frac{V_0}{2V_0 - V_t}$$

$$= \frac{1}{13.86} \ln \frac{100}{200-150}$$
$$= \frac{1}{20} \min^{-1}.$$

37. (B)

Bond order of O_2^{2-} is 1 & O_2^- is 1.5.

38. (A)

$$\wedge^0(\text{NaOH}) = \wedge^0(\text{NaCl}) + \wedge^0(\text{H}_2\text{O}) - \wedge^0(\text{HCl})$$

= 200 + 140 – 180 = 160 S cm^2 mol^{-1}

39. (D)

Van Arkel method

$$Ti(s) + 2I_2(g) \xrightarrow{523K} TiI_4(g)$$
$$TiI_4(g) \xrightarrow{1700K} Ti(s) + 2I_2(g)$$

40. (D)

$$N_2 + O_2 \rightleftharpoons 2NO \qquad K_C = 4\times10^{-4}$$
$$NO \rightleftharpoons \frac{1}{2}N_2 + \frac{1}{2}O_2 \qquad K_c^1 = \sqrt{\frac{1}{K_C}}$$
$$K_c^1 = \frac{1}{\sqrt{4\times10^{-4}}} = 50$$

41. (C)

$$[H^+] = \sqrt{K_a.C} \Rightarrow 10^{-3} = \sqrt{K_a \cdot 10^{-1}}$$

$\Rightarrow$ $K_a = 10^{-5}$

42. (A)

In RNA, the sugar is $\beta - D -$ Ribose, where as in DNA the sugar is $\beta - D - 2 -$ deoxy Ribose

43. (D)

$$\Delta T_f = K_f \times m = K_f \times \frac{w_2 \times 1000}{w_1 \times m_2}$$

w_1 & w_2 = wt. of solvent & solute respectively, m_2 = mw of solute

$$\Delta T_f = 0^o - (-6^o) = 6 = 1.86 \times \frac{w_2 \times 1000}{4000 \times 62}$$

Therefore w_2 = 800 g

44. (C)
In IF_7 Undergoes sp^3d^3 hybridisation

45. (C)

46. (B)

47. (C)

For a zero order reaction $k=\frac{x}{t}$ …(1)

Where x = amount decomposed
k = zero order rate constant
for a zero order reaction

$$k=\frac{[A]_0}{2t_{\frac{1}{2}}} \quad \ldots(2)$$

Since $[A_0]=2M, t_{1/2}=1hr; k=1$

$\therefore$ from equation (1)

$$t=\frac{0.25}{1}=0.25hr$$

48. (B)

$$IE_{He^+}=13.6\,Z^2_{He^+}\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right]=13.6Z^2_{He^+} \text{ where } (Z_{He^+}=2)$$

Hence $=13.6\times Z^2_{He^+}=19.6\times10^{-18}\,\text{J atom}^{-1}$.

$$(E_1)_{Li^{+2}}=-13.6\,Z^2_{Li^{+2}}\times\frac{1}{1^2}=-13.6\,Z^2_{He^+}\times\left[\frac{Z^2_{Li^{+2}}}{Z^2_{He^+}}\right]=-19.6\times10^{-18}\times\frac{9}{4}=-4.41\times10^{-17}\,\text{J atom}$$

49. (B)

$$Ag^++Br^-\rightleftharpoons AgBr$$

Precipitation starts when ionic product just exceeds solubility product

$$K_{sp}=\left[Ag^+\right]\left[Br^-\right]$$

$$\left[Br^-\right]=\frac{K_{sp}}{\left[Ag^+\right]}=\frac{5\times10^{-13}}{0.05}=10^{-11}$$

i.e., precipitation just starts when 10^{-11} moles of KBr is added to 1L of $AgNO_3$ solution.

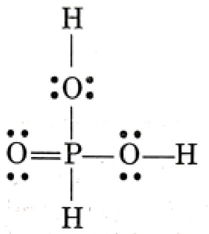
No. of moles of KBr to be added = 10^{-11}

$\therefore$ Weight of KBr to be added $=10^{-11}\times120$

$$=1.2\times10^{-9}\,g$$

50. (A)

51. (13)



$x = 6, y = 6, z = 1$

Total = 6 + 6 + 1 = 13

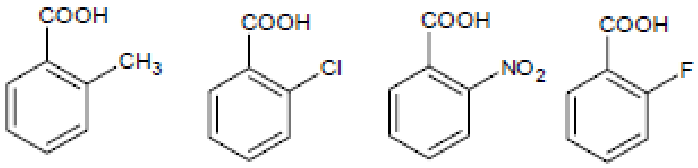
52. (1)

1 double bond equivalent.

53. (0)

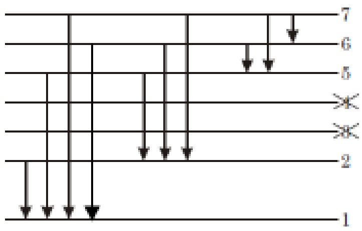
54. (8)

55. (4)



56. (30)

57. (10)



58. (5)

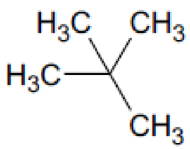
Three double bonds and one triple bond.

59. (3)

IUPAC Name of given compound is 4-ethoxycarbonylpent-3-enoic acid

60. (4)

Neopentane



PART (C) : MATHEAMATICS

ANSWER KEY

61.	(A)	62.	(C)	63.	(A)	64.	(B)	65.	(C)
66.	(C)	67.	(A)	68.	(C)	69.	(C)	70.	(B)
71.	(C)	72.	(A)	73.	(A)	74.	(B)	75.	(B)
76.	(B)	77.	(C)	78.	(B)	79.	(B)	80.	(D)
81.	(54)	82.	(4)	83.	(5)	84.	(6)	85.	(2)
86.	(2)	87.	(8)	88.	(11)	89.	(77)	90.	(34)

SOLUTIONS

61. (A)

Since $(1,2)\in S$ but $(2,1)\neq S$

Thus S is not symmetric

Hence, S is not an equivalence relation.

Given, $T=\{(x,y):(x-y)\in I\}$

Now, $x-x=0\in I$, it is reflexive relation.

Let $x-y=I_1$

And $y-z=I_2$

Then, $x-z=(x-y)+(y-z)$

$=I_1+I_2\in I$

So, T is also transitive. Hence , T is an equivalence relation.

62. (C)

Let $|z|=|\omega|=r$ and let $\arg\omega=\theta$

Then, $\omega=r(\cos\theta+i\sin\theta)=re^{i\theta}$

And $\arg z=\pi-\theta$

Hence, $z=r(\cos(\pi-\theta)+i\sin(\pi-\theta))$

$=r(-\cos\theta+i\sin\theta)$

$=-r(\cos\theta-i\sin\theta)$

$z=-\bar{\omega}$

63. (A)

Point of intersection is $A(-2,0)$. The required line will be one which passes through $(-2,0)$ and is perpendicular to the line joining $(-2,0)$ and $(2,3)$

64. (B)

Equation of the family of circles passing through $A(3,7)$ and $B(6,5)$ is

$(x-3)(x-6)+(y-7)(y-5)+\lambda(2x+3y-27)=0$

$\Rightarrow$Equation of common chord is: $S_1-S_2=0$

$\Rightarrow (2\lambda-5)x+(3\lambda-6)y+(56-27\lambda)=0$

$\Rightarrow \lambda(2x+3y-27)-(5x+6y-56)=0$

$\Rightarrow$ This represents family of lines passing through the point of intersection of $2x+3y-27=0 \& 5x+6y-56=0$

$\Rightarrow$ fixed point $=\left(2,\dfrac{23}{3}\right)$

65. (C)

Equation of tangent to $y^2=4x$

$y=mx+\dfrac{1}{m}$

$\Rightarrow m^2x-my+1=0$

$\tan(45^\circ)=\tan(Q_1+Q_2)=1$

$=\dfrac{\tan Q_1+\tan Q_2}{1-\tan Q_1.\tan Q_2}$

$\Rightarrow 1-\dfrac{1}{x}=\dfrac{y}{x}\Rightarrow y=(x-1)$

66. (C)

$\lim\limits_{x\to1^+} f(x)=\lim\limits_{x\to1^+}\left(\left[x^2\right]+\sqrt{(x)^2}\right)$

$\Rightarrow \lim\limits_{x\to1^+}(1+0)=1$

$\lim\limits_{x\to1^+} f(x)$ $\Rightarrow \lim\limits_{x\to1^+}(0+1)=1$

And $f(1)=1$ $\Rightarrow \lim\limits_{x\to1} f(x)=f(1)$

$\therefore$ continuous at $x=1$

Similarly we check for another integers

67. (A)

$h'(x)=2f(x)f'(x)+2g(x).g'(x)$

$=2f'(x)(f(x)+g'(x))$

$=2f'(x)(f(x)+f''(x))$

$=0$

$\Rightarrow h(x)$ is a constant function

$\Rightarrow h(10)=h(0)=1$

68. (C)

$V=\pi r^2h$

Let k be the thickness of sides then that of the top will be $\dfrac{5}{4}k$.

$\therefore S=k\left(\dfrac{2v}{r}+\dfrac{5}{4}\pi r^2\right)$

$$\frac{dS}{dr} = k\left(-\frac{2v}{r^2} + \frac{5}{2}\pi r\right) \text{ and } \frac{d^2s}{dr^2} = k.\left(\frac{4v}{r^3} + \frac{5}{2}\pi\right) > 0 \text{ for positive r}$$

$$\frac{dS}{dr} = 0 \text{ will give } \frac{r}{h} = \frac{4}{5}$$

69. (C)

$$\int \sqrt{1+\sec x}\, dx = \int \frac{\sec x.\tan x}{\sec x\sqrt{\sec x - 1}} dx = \int \frac{\tan x\, dx}{\sec x - 1}$$

$$\sec x = t \Rightarrow \sec x \tan x\, dx = dt$$

$$= \int \frac{dt}{t\sqrt{t-1}} = 2\int \frac{y\,dy}{(y^2+1)y}, \text{ where } y^2 = t - 1$$

$$= 2\tan^{-1} y + c = 2\tan^{-1}\sqrt{\sec x - 1} + c$$

70. (B)

$$y^2 = \frac{x^3}{2a - x}, x = 2a \text{ is the vertical asymptote of the curve}$$

Domain $y = \sqrt{\dfrac{x^3}{2a - x}}$

$0 \le x < 2a$

x	0	$2a^-$	
Y	0	∞^+	

Plot some point on the curve

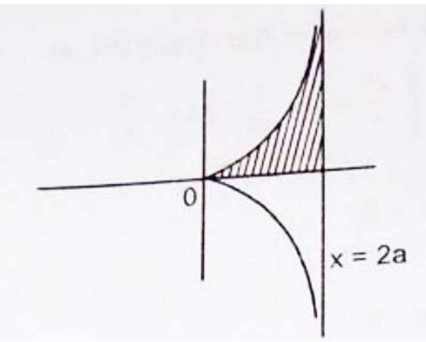
$$A = \int_0^{2a} \sqrt{\frac{x^3}{2a - x}} dx \text{ [area above x-axis]}$$

Put $x = 2a\sin^2\theta$

$$\frac{dx}{d\theta} = 4a\sin\theta\cos\theta$$

$$A = \int_0^{\pi/2} \sqrt{\frac{8a^3\sin^6\theta}{2a\cos^2\theta}}\, 4a\sin\theta\cos\theta\, d\theta$$

$$A = 8a^2 \int_0^{\pi/2} \sin^4\theta d\theta$$



$$A = 8a^2.\frac{3.1}{4.2}\frac{\pi}{2} \quad \Rightarrow \frac{3\pi a^2}{2}$$

71. (C)

$$x = e^{\left(xy\cdot\frac{dy}{dx}\right)}$$

$$\log x = xy\left(\frac{dy}{dx}\right)$$

$$y\,dy = \left(\frac{\log x}{x}\right)dx \Rightarrow \frac{y^2}{2} = \frac{(\log x)^2}{2} + c$$

$$\Rightarrow y^2 = (\log_e x)^2 + 2c$$

$$y = \pm\sqrt{(\log_e x)^2 + 2c}$$

72. (A)

$$\left(\frac{ac+ab-bc}{abc}\right)\left(\frac{ab+bc-ac}{abc}\right) \Rightarrow \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$$

Now $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$$\left[\frac{1}{2a}+\frac{1}{2c}+\frac{1}{c}-\frac{1}{a}\right]\left[\frac{1}{c}+\frac{1}{a}-\frac{1}{2a}-\frac{1}{2c}\right] \Rightarrow \left[\frac{3}{2c}-\frac{1}{2a}\right]\left[\frac{1}{2a}+\frac{1}{2c}\right] = \frac{(3a-c)(a+c)}{4a^2c^2}$$

73. (A)

$$\left(x^8+1\right)^{60}\left\{\left(x^4+\frac{1}{x^4}\right)^3\right\}^{-10}$$

$$\left(x^8+1\right)^{60}\left(x^4+\frac{1}{x^4}\right)^{-30}$$

$$\frac{\left(x^8+1\right)^{60}\left(x^8+1\right)^{-30}}{x^{-120}}$$

$$x^{120}\left(1+x^8\right)^{30}$$

Now coefficient of $x^{160} = {}^{30}C_5 = \frac{30\times29\times28\times27\times26}{120} = 142506$

74. (B)

$P(E_1) = \frac{3}{6} = \frac{1}{2}$ $\qquad$ $P(E_2) = \frac{3}{6} = \frac{1}{2}$

P_1 (ball drawn from first box is white) $= \frac{4}{9}$

P_2 (ball drawn fro 2^{nd} box is white) $= \frac{5}{9}$

$\therefore$ By Bayer's theorem probability of ball draw from first box $= \dfrac{\frac{1}{2}\cdot\frac{4}{9}}{\frac{1}{2}\cdot\frac{4}{9}+\frac{1}{2}\cdot\frac{5}{9}} = \frac{4}{9}$

75. (B)

$\because 0 \le [x] < 2 \Rightarrow [x] = 0, 1$

$-1 \le [y] < 1 \quad \Rightarrow [y] = -1, 0$

And $1 \le [z] < 3 \Rightarrow [z] = 1, 2$

Now $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\begin{vmatrix} [x]+1 & [y] & [z] \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

$= ([x]+1) - [y](-1) + [z](1)$

$= [x] + [y] + [z] + 1$

For maximum value of determinant $[x] = 1, [y] = 0, [z] = 2$

maximum value $= 1 + 0 + 2 + 1 = 4$

76. (B)

$x^7 - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)......(x-\alpha_6)$

Put $x = 3$

$3^7 - 1 = 2(3-\alpha_1)(3-\alpha_2)......(3-\alpha_6)$

But $|3-\alpha_1| = |3-\alpha_6|$

$|3-\alpha_2| = |3-\alpha_5|$

$|3-\alpha_3| = |3-\alpha_4|$

So $|(3-\alpha_1)(3-\alpha_3)(3-\alpha_5)| = \sqrt{\dfrac{3^7-1}{2}} = \sqrt{1093}$

77. (C)

Required unit vector $= \pm \dfrac{\vec{a} \times (\vec{a} \times \vec{b})}{|\vec{a} \times (\vec{a} \times \vec{b})|}$

Now $\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}$

$\vec{a} \times \vec{b} = -2\hat{j} - 2\hat{k}$

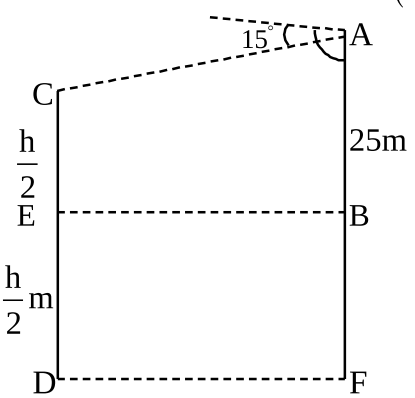
$\vec{a} \times (\vec{a} \times \vec{b}) = -4\hat{i} + 2\hat{j} - 2\hat{k}$

$|\vec{a} \times (\vec{a} \times \vec{b})| = 2\sqrt{6}$

Unit vector $= \pm \dfrac{\vec{a} \times (\vec{a} \times \vec{b})}{|\vec{a} \times (\vec{a} \times \vec{b})|} = \pm \dfrac{(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{6}}$

78. (B)
Since PQ is perpendicular to the plane and direction ratio of PQ are $\alpha-\alpha', \beta-\beta', \gamma-\gamma'$ hence equation of plane can be written as
$$(\alpha-\alpha')x+(\beta-\beta')y+(\gamma-\gamma')z=0$$
Also plane passes through $\left(\frac{\alpha+\alpha'}{2}, \frac{\beta+\beta'}{2}, \frac{\gamma+\gamma'}{2}\right)$
Hence $(\alpha-\alpha')\left(\frac{\alpha+\alpha'}{2}\right)+(\beta-\beta')\left(\frac{\beta+\beta'}{2}\right)+(\gamma-\gamma')\left(\frac{\gamma+\gamma'}{2}\right)=0$
$$\Rightarrow \alpha^2+\beta^2+\gamma^2=(\alpha')^2+(\beta')^2+(\gamma')^2$$

79. (B)
Let the tower be $AB=25\text{m}$ and the pole be $CD=h\,\text{m}$ whose middle point be E. So $CE=DE=\frac{h}{2}\text{m}$
$$\angle ADF=45^\circ, AF=\left(25+\frac{1}{2}\right)\text{m}$$
$$\therefore \sin 45^\circ=\frac{AF}{AD}=\frac{25+h/2}{AD} \quad \Rightarrow AD=\sqrt{2}\left(25+\frac{h}{2}\right)\text{m}$$
Now, in the ΔACD
$$\angle DAC=30^\circ, \angle ACD=15^\circ+90^\circ=105^\circ$$
$$\therefore \frac{AD}{\sin 105^\circ}=\frac{CD}{\sin 30^\circ} \quad \text{or} \quad \frac{\sqrt{2}\left(25+\frac{h}{2}\right)}{\sin(60^\circ+45^\circ)}=\frac{h}{1/2}$$



80. (D)
$$=\tan^{-1}1+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+\text{.......n terms}$$
$$=\tan^{-1}1+\tan^{-1}\frac{2-1}{1+1(2)}+\tan^{-1}\frac{3-2}{1+3(2)}+\tan^{-1}\frac{4-3}{1+4(3)}+\text{........n terms}$$
$$=\tan^{-1}1+\tan^{-1}2-\tan^{-1}1+\tan^{-1}3-\tan^{-1}2+\tan^{-1}4-\tan^{-1}3+\text{....n terms}=\tan^{-1}n$$

81. (54)

Corrected $\sum x = 40\times200-50+40=7990$

Corrected $\bar{x}=\frac{7990}{200}=39.95$

Incorrect $\sum x^2 = n\left(\sigma^2+\bar{x}^2\right)=200\left(5^2+40^2\right)=365000$

Correct $\sum x^2 = 365000-2500+1600=364100$

Corrected $\sigma=\sqrt{\frac{364100}{200}-(39.95)^2}=\sqrt{(1820.5-1596)}$

$=\sqrt{224.5}$
$=14.98$

82. (4)

Let $P(x_1,y_1)$ be a point on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x_1{}^2}{a^2}-\frac{y_1{}^2}{b^2}=1$

Chord of contact of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$ from (x_1,y_1) is

$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=2$ ....(1)

Equations of asymptotes are $\frac{x}{a}-\frac{y}{b}=0$ and $\frac{x}{a}+\frac{y}{b}=0$

Pts. Of intersection of (1) with two asymptotes are

$x_1=\frac{2a}{\frac{x_1}{a}-\frac{y_1}{b}}, y_1=\frac{2b}{\frac{x_1}{a}-\frac{y_1}{b}}$

$x_2=\frac{2a}{\frac{x_1}{a}+\frac{y_1}{b}}, y_2=\frac{-2b}{\frac{x_1}{a}+\frac{y_1}{b}}$

$\therefore \text{or } \Delta=\frac{1}{2}(x_1y_2-x_2y_1)=\frac{1}{2}\left(\frac{4ab\times2}{\frac{x_1{}^2}{a^2}-\frac{y_1{}^2}{b^2}}\right)=4ab$

83. (5)

Equation of normal to the ellipse is

$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

It meets the ellipse again at $Q(3\theta)$'

$\therefore \frac{a^2.\cos3\theta}{\cos\theta}-\frac{b^2\sin3\theta}{\sin\theta}=a^2-b^2\left(a^2=14\right)$

$\Rightarrow \frac{14\left(4\cos^3\theta-3\cos\theta\right)}{\cos\theta}-\frac{5\left(3\sin\theta-4\sin^3\theta\right)}{\sin\theta}$

$\Rightarrow 56\cos^2\theta - 42 - 15 + 20\sin^2\theta = 9$

$\Rightarrow 36\cos^2\theta = 46$

$\Rightarrow \cos^2\theta = \frac{23}{18}$

$\therefore \cos 2\theta = 2\cos^2\theta - 1$

$= \frac{23}{9} - 1 = \frac{14}{9} = \frac{a}{b}$

$\therefore a - b = 5$

84. (6)

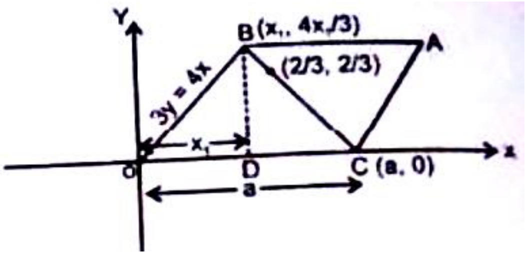
Let $OC = a$

$\therefore OC = CA = AB = BO = a$

Let $\left(x_1, \frac{4x_1}{3}\right)$

$\therefore A\left(a + x_1, \frac{4x_1}{3}\right)$

$\therefore x_1^2 + \frac{16x_1^{\ 2}}{9} = a^2$ ($\because$ ODB is a right angles triangle)



$a = \frac{5x_1}{3}$

Equation of BC is

$y - 0 = \frac{\frac{4x_1}{3} - 0}{x_1 - a}(x - a)$ $\quad\quad \because a = \frac{5x_1}{3}$

$y = -2x + \frac{10x_1}{3}$ $\quad\quad \therefore$ BC passes through $\left(\frac{2}{3}, \frac{2}{3}\right)$

$x_1 = 3/5$ $\quad\quad \therefore a = 1$

$A\left(1 + \frac{3}{5}, \frac{4}{3} \times \frac{3}{5}\right)$ $\quad\quad \therefore A\left(\frac{8}{5}, \frac{4}{5}\right)$ $\quad\quad \therefore \frac{5}{2}(\alpha + \beta) = 6$

85. (2)

$\lim_{x \to 0^+} \frac{-1 + \sqrt{1 + 4(\tan x - \sin x)}}{-1 + \sqrt{1 + 4x^3}}$ $\quad\quad \left(\frac{0}{0} \text{form}\right)$

Rationalizing numerator and denominator

$= \lim_{x \to 0^+} \frac{4(\tan x - \sin x)\left(1 + \sqrt{1 + 4x^3}\right)}{4x^3\left(1 + \sqrt{1 + 4(\tan x - \sin x)}\right)}$

$$= \lim_{x\to 0^+}\left(\frac{\sin x\left(1-\cos x\right)}{x^3\cos x}\right)\left(\frac{\left(1+\sqrt{1+4x^3}\right)}{\left(1+\sqrt{1+4\left(\tan x-\sin x\right)}\right)}\right)$$

$$= \lim_{x\to 0^+}\left(\frac{\sin x}{x}\right).\left(\frac{2\sin^2\frac{x}{2}}{4\times\frac{x^2}{4}}\right).\left(\frac{\left(1+\sqrt{1+4x^3}\right)}{\left(1+\sqrt{1+4\left(\tan x-\sin x\right)}\right)}\right)$$

$$=1.\frac{1}{2}.1\frac{(1+1)}{(1+1)}=\frac{1}{2}$$

86. (2)

$f(x)=(1+a)\sin x$,

$$a=\int_0^{\pi/2} f(t)\cos t\,dt=(1+a)\int_0^{\pi/2}\sin t\cos t\,dt$$

$$=\frac{1+a}{2} \qquad \Rightarrow a=1, f(x)=2\sin x$$

$$\int_0^{\pi/2} f(x)dx=2\int_0^{\pi/2}\sin x\,dx=2$$

87. (8)

$\sin 3\theta = 2\sin\theta\left(2\sin 2\theta\sin 4\theta\right)$

$\Rightarrow \sin 3\theta = 2\sin\theta\left(\cos 2\theta-\cos 6\theta\right)$

$\Rightarrow \sin 3\theta = 2\sin\theta\cos 2\theta - 2\sin\theta\cos 6\theta$

$\Rightarrow \sin 3\theta = \sin 3\theta - \sin\theta - 2\sin\theta\cos 6\theta$

$\Rightarrow \sin\theta\left(1+2\cos 6\theta\right)=0$

$\Rightarrow \sin\theta = 0, 1+2\cos 6\theta = 0$

$\Rightarrow \theta = n\pi, 6\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = \frac{n\pi}{3}\pm\frac{\pi}{9}$

$\Rightarrow \theta = 0, \pi, \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}\left(\text{as } 0\le\theta\le\pi\right)$

$\therefore$ total number of solutions $=8$

88. (11)

α, β, γ are root of $x^3-5x^2+x-2=0$

Now $y=\frac{\alpha+2}{\alpha-2} \qquad \Rightarrow \alpha=\frac{2(y+1)}{y-1}$

It is root of given equation so

$$\frac{8(y+1)^3}{(y-1)^3}-5.4\frac{(y+1)^2}{(y-1)^2}+\frac{2(y+1)}{(y-1)}-2=0$$

$$8(y+1)^3-20(y+1)^2(y-1)+2(y+1)(y-1)^2-2(y-1)^3=0$$

$$\Rightarrow 3y^3-2y^2-9y-8=0$$

Roots of this equation are $\frac{\alpha+2}{\alpha-2}, \frac{\beta+2}{\beta-2}, \frac{\gamma+2}{y-2}$

So product of roots

$\left(\frac{\alpha+2}{\alpha-2}\right)\left(\frac{\beta+2}{\beta-2}\right)\left(\frac{\gamma+2}{\gamma-2}\right)=\frac{8}{3}$

89. (77)

Taking 3 once, 2 once and 1 five times

Numbers $=\frac{7!}{5!}=42$

Taking 2 thrice and 1 four times

Numbers $=\frac{7!}{3!4!}=35$

Total $=77$

90. (34)

Let $a_1=a$ and $d=$ common difference

$\because a_1+a_5+a_9+......+a_{49}=416$

$\therefore a+(a+4d)+(a+8d)+......(a+48d)=41$

$\Rightarrow \frac{13}{2}(2a+48d)=416$

$\Rightarrow a+24d=32$ ....(i)

Also, we have $a_9+a_{43}=66$

$\therefore a+8d+a+42d=66$

$\Rightarrow 2a+50d=66$

$\Rightarrow a+25d=33$ ....(ii)

Solving Eqs. (i) and (ii), we get

$a=8$ and $d=1$

Now, $a_1^2+a_2^2+a_3^2+......+a_{17}^2=140m$

$8^2+9^2+10^2+......+24^2=140m$

$\Rightarrow (1^2+2^2+3^2+......+24^2)-(1^2+2^2+3^2+......+7^2)=140m$

$\Rightarrow \frac{24\times25\times49}{6}-\frac{7\times8\times15}{6}=140m$

$\Rightarrow \frac{3\times7\times8\times5}{6}(7\times5-1)=140m$

$\Rightarrow 7\times4\times5\times34=140m$

$\Rightarrow 140\times34=140m$

$\Rightarrow m=34$