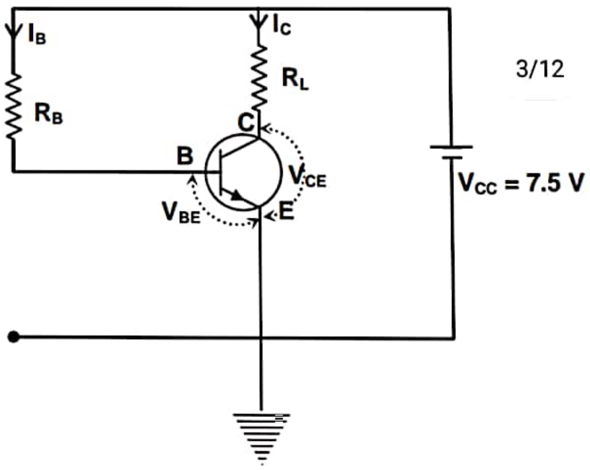
PART (A) : PHYSICS

ANSWER KEY

1.	(D)	2.	(A)	3.	(C)	4.	(A)	5.	(B)
6.	(D)	7.	(C)	8.	(A)	9.	(C)	10.	(C)
11.	(C)	12.	(B)	13.	(A)	14.	(D)	15.	(D)
16.	(A)	17.	(A)	18.	(D)	19.	(C)	20.	(D)
21.	(5)	22.	(7)	23.	(8)	24.	(20)	25.	(6)
26.	(4)	27.	(8)	28.	(18)	29.	(2)	30.	(14)

SOLUTIONS

1. (D)



$$V_{CC} = I_B R_B + V_{BE}$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{7.5-1}{130\times10^3} = 50\times10^{-6} = 50\mu A$$

$$I_C = \beta I_B = 100\times5\times10^{-5} = 5\times10^{-3} A = 5mA$$

Now, $I_C R_L + V_{CE} = V_{CC}$

$$R_L = \frac{V_{CC} - V_{CE}}{I_C} = \frac{7.5-3.5}{5\times10^{-3}} = \frac{4000}{5} = 800\Omega$$

2. (A)

$I = neAv_d$

$I = kV^2$

3. (C)

$$1 = \frac{mg-(0.5)(2g)-(0.3)(3g)}{3+2+m}$$

4. (A)

$I = mv$ ……(i)

Using WET: From initial point to top point of vertical circle.

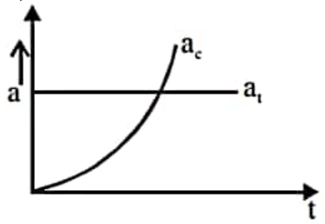
$$mg\ell = \frac{1}{2}m\left(v^2 - 0\right)$$
$$v = \sqrt{2g\ell}$$
$$I = m\sqrt{2g\ell}$$

5. (B)

When extension is x

$$\frac{1}{2}k\left(x_0^2 - x^2\right) = \frac{1}{2}mv^2$$
$$P = \vec{F}.\vec{V} = kxV$$
$$\frac{dp}{dx} = 0 \quad \Rightarrow x = \frac{x_0}{\sqrt{2}}$$

6. (D)



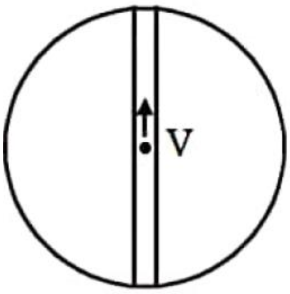
$$v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow x = \frac{\alpha^2 t^2}{4}$$
$$a_t = v\frac{dv}{dx} = \alpha\sqrt{x}.\frac{\alpha}{2\sqrt{x}} = \frac{\alpha^2}{2} = \text{constant}$$
$$a_C = \frac{v^2}{r} = \frac{\alpha^2 x}{r} = \frac{\alpha^2}{r} \times \frac{\alpha^2 t^2}{4}$$
$$a_C \propto t^2$$

7. (C)

$$\vec{E} = \frac{\delta v}{\delta r}\,\hat{r}$$

magnitude of electric field will be slope of $v - r$ graph, at point 3 slope is zero.

8. (A)



By conservation of energy

$$-\frac{3GMm}{2R} + \frac{1}{2}mv^2 = -\frac{GMm}{R} + 0$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{3GMm}{2R} - \frac{GMm}{R}$$
$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$
$$\Rightarrow v = \sqrt{\frac{GM}{R}}$$

9. (C)

Since the particle exists the magnetic field region with a velocity $\vec{v} = \left(3\hat{i} + 3\sqrt{3}\hat{j}\right) m/s$

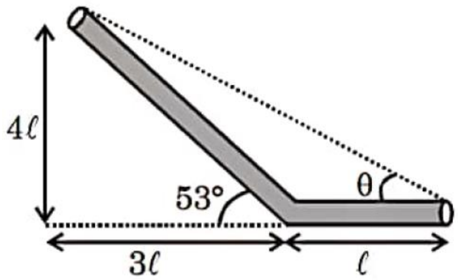
$$\tan\theta = \frac{3\sqrt{3}}{3}$$
$$\therefore \theta = \frac{\pi}{3}$$

The angular displacement of the particle in the magnetic field region is $\theta = \frac{\pi}{3}$

Angular velocity , $\omega = \frac{qB}{m}$

$$\therefore t = \frac{\theta}{\omega} = \frac{\pi m}{3qB}$$

10. (C)



$$\tan\theta = \frac{4\ell}{4\ell} = \frac{a}{g}$$
$$\Rightarrow a = g$$

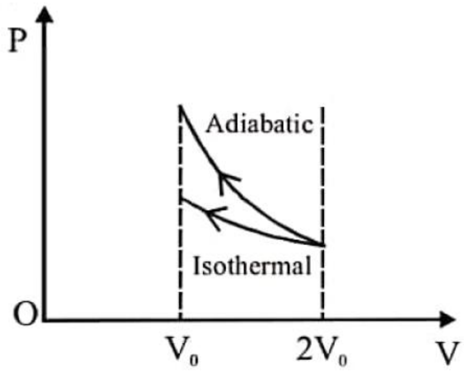
11. (C)

$C = \frac{\epsilon_0 A}{x}$, where x is separation between plates.

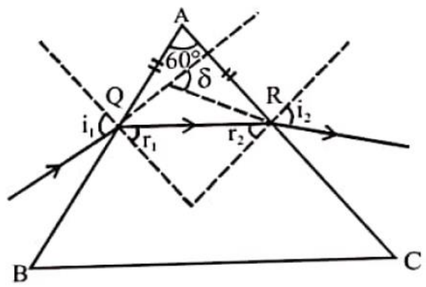
$$\frac{1}{C}\frac{dC}{dT} = \frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}$$
$$\text{For } \frac{dC}{dT} = 0, \frac{1}{x}\frac{dx}{dT} = \frac{1}{A}\frac{dA}{dT}$$
$$\Rightarrow \alpha_S = 2\alpha$$

12. (B)

W_{ext} = negative of area with volume-axis W(adiabatic) > W(isothermal)



13. (A)



Given $AQ = AR$ and $\angle A = 60^\circ$

$\therefore \angle AQR = \angle ARQ = 60^\circ$

$\therefore r_1 = r_2 = 30^\circ$

Applying Snell's law on face AB.

$1.\sin i_1 = \mu \sin r_1$

$\Rightarrow \sin i_1 = \sqrt{3}\sin 30^\circ = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$\therefore i_1 = 60^\circ$

In a prism deviation

$\delta = i_1 + i_2 - A = 60^\circ + 60^\circ - 60^\circ = 60^\circ$

14. (D)

For the first minima,

$\theta = \frac{\eta\lambda}{a} \Rightarrow \sin 30^\circ = \frac{\lambda}{a} = \frac{1}{2}$

First secondary maxima will be at,

$\sin\theta = \frac{3\lambda}{2a} = \frac{3}{2}\left(\frac{1}{2}\right) \Rightarrow \theta = \sin^{-1}\left(\frac{3}{4}\right)$

15. (D)

$3L(1 + \alpha_{eff}\Delta\theta) = L(1 + 2\alpha\Delta\theta) + 2L(1 + \alpha\Delta\theta)$,

$3L\alpha_{eff}\Delta\theta = 4\alpha L\Delta\theta$

$\therefore \alpha_{eff} = \frac{4}{3}\alpha$

16. (A)

$$nh\nu = \frac{100\times 60}{100} = 60J$$

$$n = \frac{60\lambda}{hc} = \frac{60\times 3.9\times 10^{-7}}{6.63\times 10^{-34}\times 3\times 10^{8}} = 1.18\times 10^{20}$$

17. (A)

Using conservation of momentum

$$mu = mv_1 + Mv_2$$

$$\Rightarrow 2v_2 + v_1 = u \qquad \ldots\ldots(i)$$

$$v_2 - v_1 = u \qquad \ldots\ldots..(ii)$$

From (i) and (ii)

$$v_2 = 2u/3$$

$$v_1 = \frac{u}{3}$$

$\therefore$ Fraction of kinetic energy lost be neutron

$$= 1 - \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mu^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

18. (D)

$$A = A_0 e^{-\frac{bt}{2m}}$$

$$A = A_0 e^{-\frac{\ell n 2}{10\times 2}\times\frac{20}{2}} = A_0 e^{-\ell n\sqrt{2}} = \frac{A_0}{\sqrt{2}}$$

19. (C)

According to Newton's law of cooling, the temperature goes on decreasing with time non-linearly.

20. (D)

The electron ejected with maximum speed v_{max} are stopped by electric field $E = 4N/C$ after travelling a distance $d = 1m$

$$\frac{1}{2}mv_{max}^2 = eEd = 4eV$$

The energy of incident photon $= \frac{1240}{200} = 6.2eV$

From equation of photo electric effect

$$\frac{1}{2}mv_{max}^2 = h\nu - \phi_0$$

$$\therefore \phi_0 = 6.2 - 4 = 2.2eV$$

21. (5)

$$H = B/\mu_0$$

$$ni = 2\times 10^4$$

$$40\times 100i = 2\times 10^4$$

$i = \frac{20}{4} A = 5A$

22. (7)

$m_1g - T_1 = m_1a$ ....(i)

$T_2 - m_2g = m_2a$ ......(ii)

$T_1 - T_2 \pm \frac{m_1g}{2} = m_1a$

$(m_1 - m_2)g \pm \frac{m_1g}{2} = (2m_1 + m_2)a$

$a = \frac{2m_2g \pm \frac{3gm_2}{2}}{7m_2}$

$a_{max} = \frac{g}{2}$

$a_{min} = \frac{g}{14}$

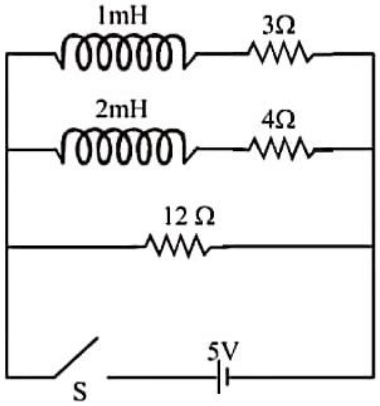
$\frac{a_{max}}{a_{min}} = 7$

23. (8)

At $t = 0$, current will flow only in 12Ω resistance

$\therefore I_{min} = \frac{5}{12}$

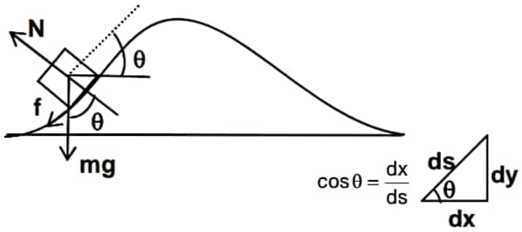
At At $t \to \infty$ both L_1 and L_2 behave as conducting wires



$\therefore R_{eff} = \frac{3}{2}, I_{max} = \frac{10}{3}$

$\frac{I_{max}}{I_{min}} = 8$

24. (20)



Work done by friction $= \int \vec{F}.\vec{ds} = \int_0^x \mu mg\cos\theta \frac{dx}{\cos\theta}$

$= \mu mgx = 20\,J$

25. (6)

For adiabatic process, $TV^{\gamma-1}$ =constant

$T\left(\frac{m}{\rho}\right)^{\gamma-1} = \text{constant}$

$\frac{T}{\rho^{\gamma-1}} = \text{constant}$

$\rho \propto T^{1/(\gamma-1)} \Rightarrow \frac{1}{\gamma-1} = 3 \Rightarrow \gamma = 4/3$

$f = \frac{2}{\gamma-1} = \frac{2}{\left(\frac{4}{3}-1\right)} = 6$

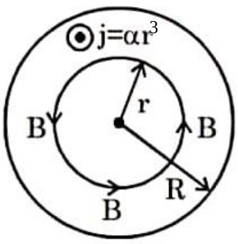
26. (4)

$Z = \sqrt{R^2 + X^2} = \sqrt{9 + X^2}$

But $\cos\phi = \frac{R}{Z} = \frac{3}{5}$

$X = 4\Omega$

27. (8)



When $r \le R$

$B2\pi r = \mu_0 \int_0^r j\,2\pi r dr$

$Br = \frac{\mu_0 \alpha r^5}{5}$

$Br = \frac{\mu_0 \alpha r^4}{5}, r \le R$

When $r > R$

$$B2\pi r = \mu_0 \int_0^R j2\pi r\,dr$$

$$Br = \mu_0 \int_0^R \alpha r^3 rdr$$

$$Br = \frac{\mu_0 \alpha R^5}{5}$$

$$\therefore B = \frac{\mu_0 \alpha R^5}{5r}, r > R$$

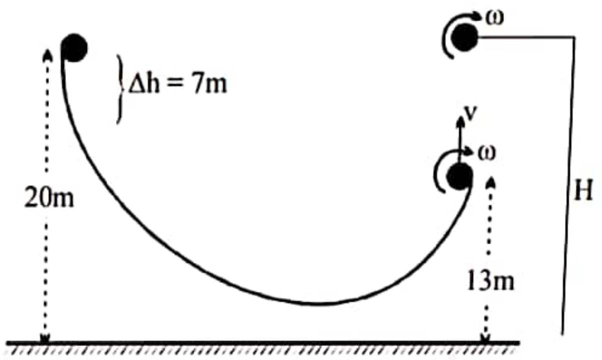
$$\text{At } r = \frac{R}{2}, B_1 = \frac{\mu_0 \alpha R^4}{80}$$

$$\text{At } r = 2R, B_2 = \frac{\mu_0 \alpha R^4}{10}$$

$$\therefore \frac{B_2}{B_1} = 8$$

28. (18)

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



$$= \frac{1}{2}m\left(2g\left(H-13\right)\right) + \frac{1}{2}\frac{2}{5}m\left(r^2\omega^2\right)$$

$$g\left(7\right) = g\left(H-13\right) + \frac{1}{5}\left(v^2\right)$$

$$= g\left(H-13\right) + \frac{1}{5}\left(2g\left(H-13\right)\right)$$

$$7 = \left(H-13\right) + \frac{2}{5}\left(H-13\right)$$

$$7 = \frac{7}{5}\left(H-13\right)$$

$$H = 18m$$

29. (2)

$$f = \frac{F}{1+\frac{mR^2}{I}}$$

For no slipping

$\mu mg \geq f$

30. (14)

$\lambda_1 = 50\text{cm}$ $\lambda_2 = 51\text{cm}$

$$v \propto \sqrt{T} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+20}{273}} \Rightarrow v_2 = 343.95\,\text{m/s}.$$

$$f_1 = \frac{v_2}{\lambda_1} = \frac{343.95}{0.50} = 688\,\text{Hz}\ .$$

$$f_2 = \frac{v_2}{\lambda_2} = \frac{343.95}{51\times10^{-2}} = 674\text{Hz}$$

No. of beats $= f_2 - f_1 = 14\text{Hz}$

PART (B) : CHEMISTRY

ANSWER KEY

31.	(C)	32.	(D)	33.	(B)	34.	(B)	35.	(A)
36.	(A)	37.	(D)	38.	(D)	39.	(D)	40.	(B)
41.	(C)	42.	(C)	43.	(D)	44.	(D)	45.	(B)
46.	(C)	47.	(B)	48.	(D)	49.	(A)	50.	(C)
51.	(316)	52.	(0)	53.	(2)	54.	(4)	55.	(4)
56.	(50)	57.	(8)	58.	(4)	59.	(5)	60.	(6)

SOLUTIONS

31. (C)

$$\Delta x.\Delta p = \frac{h}{4\pi}\ or\ \Delta x.m\Delta v = \frac{h}{4\pi};$$

$$\Delta v = \frac{0.011}{100}\times 3\times 10^{4} = 3.3 cms^{-1}$$

$$\Delta v = \frac{6.6\times 10^{-27}}{4\times 3.14\times 9.1\times 10^{-28}\times 3.3} = 0.175 cm$$

32. (D)

Zn > Cd < Hg

He > Ne > Ar

33. (B)

Species	N_2	O_2	N_2^-	O_2^-
Bond Order	3	2	2.5	1.5

The O-O bond is O_2^- decreases

34. (B)

Apply

$$\frac{P_1}{d_1T_1} = \frac{P_2}{d_2T_2};\frac{1}{2.86\times 273} = \frac{2}{1092\times d};d = 1.43 kg\ m^{-3}$$

35. (A)

$$\Delta S = \frac{\Delta H}{T} = \frac{900\times 18}{373} = 43.4\,\mathrm{J\,K^{-1}mol^{-1}}$$

36. (A)

$$I_2 \rightleftharpoons 2I$$

$$1-x \qquad 2x$$

$$K_c = \frac{(2x)^2}{(1-x)} = 10^{-6}$$

Soln, shows that $(1-x) > 2x \therefore \quad [I_2(g)] > [I^-(g)]$

37. (D)
The pH values indicate that NaD, NaC and NaB are salts of strong base and weak acid. pH of NaA = 7 (it is salt of strong acid and strong base).

38. (D)
Heavy water is D_2O (1 – C); temporary hard water contains the bi-carbonates of Mg and Ca (2 – A); soft water contains no foreign ions (3 – B); permanent hard water contains the sulphates and chlorides of Mg and Ca (4 – D) therefore the answer is D.

39. (D)
Ge (II) tends to acquire Ge (IV) state by loss of electrons. Hence it is reducing in nature. Pb (IV) tends to acquire Pb (II) O.S. by gain of electrons. Hence it is oxidising in nature. This is due to inert pair effect.

40. (B)
In AB AB AB …… packing, spheres occupy 74% & 26% is empty.

41. (C)
Mass of Cu deposited,

$m = \frac{31.75 \times 1 \times 965}{96500} = 0.3175\, g \Rightarrow \frac{0.3175}{63.5} = 0.005$ moles in one litre.

$\therefore$ Strength of $CuCl_2$ is 0.005 M.

42. (C)
It is electrophoresis (see definition of electrophoresis)

43. (D)
Most electropositive metals are obtained by electrolysis of their fused ionic salts.

44. (D)

45. (B)

46. (C)

47. (B)

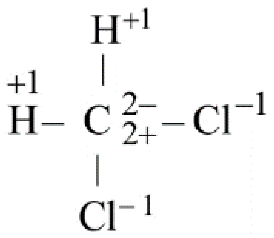
48. (D)

49. (A)

50. (C)

51. (316)
Eq. wt of $KMnO_4$ in acid medium is 31.6 g. hence this much amount must be dissolved in 1 litre to prepare normal solution.

52. (0)



Find O.N. by chemical bond method
O.N. of C is zero

53. (2)
It has two 3 Centre 2 electron bond (banana bond).

54. (4)

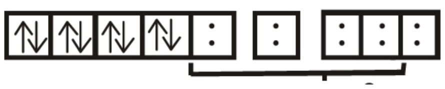
$$\frac{\Delta p}{p^o} = X_B;$$

$$\therefore \quad \Delta p = 0.25 \times 0.80 = 0.20$$

$$\therefore \quad 20x = 4$$

55. (4)

56. (50)
$[Ni(CN)_4]^{2-}$



Rearrangement dsp^2

57. (8)
(b) & (e) gives +ve test with $NaHCO_3$.

58. (4)

59. (5)

60. (6)

PART (C) : MATHEAMATICS

ANSWER KEY

61.	(C)	62.	(C)	63.	(B)	64.	(A)	65.	(D)
66.	(A)	67.	(C)	68.	(A)	69.	(B)	70.	(A)
71.	(D)	72.	(B)	73.	(C)	74.	(C)	75.	(B)
76.	(A)	77.	(C)	78.	(A)	79.	(D)	80.	(A)
81.	(6)	82.	(4)	83.	(0)	84.	(2)	85.	(3)
86.	(7)	87.	(8)	88.	(1)	89.	(9)	90.	(3)

SOLUTIONS

61. (C)

a, b, c are in A.P.

$\lambda a, \lambda b, \lambda c$ will be in A.P.

$\Rightarrow \sin A, \sin B, \sin C$ in A.P.

Also, $h_1 = \frac{2\Delta}{a}, h_2 = \frac{2\Delta}{b}; h_3 = \frac{2\Delta}{c}$ will be in H.P.

a, b, c in AP $\Rightarrow$ s – a, s – b, s – c in A.P.

$\Rightarrow \frac{\Delta}{s-a}, \frac{\Delta}{s-b}, \frac{\Delta}{s-c}$ will be in H.P.

$\Rightarrow r_1, r_2, r_3$ in H.P.

Also, $\frac{\Delta}{s(s-a)}, \frac{\Delta}{s(s-b)}, \frac{\Delta}{s(s-c)}$ will be in H.P.

62. (C)

$$\cot\left(\sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{3+\cos 2x}}\right)\right) = \cot\cot^{-1}\left(\sqrt{\frac{1+\cos 2x}{2}}\right)$$

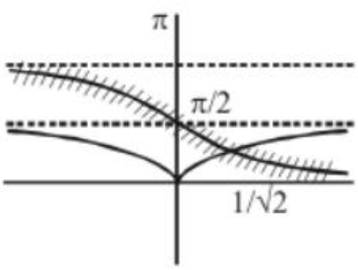
$$f(x) = |\cos x| \Rightarrow f'\left(\frac{2\pi}{3}\right) = \frac{\sin 2\pi}{2} = \frac{\sqrt{3}}{2}$$

63. (B)

Range : $\left[\frac{\pi}{4}, \pi\right)$

Non derivable at $x = \frac{1}{\sqrt{2}}$

I and II are wrong.



64. (A)

$\ln\left(\left(1+\sin^2 x\right)\left(5+x^2\right)\right)=1$

Least value of LHS = $\ln 5(>1)$

$\therefore$ No solution.

65. (D)

Homogeneous equation $\Rightarrow \frac{dy}{dx}=\frac{y}{x}$

$\therefore$ Slope = 4

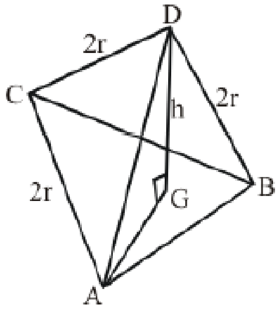
66. (A)

BC = 2r = AD

G is centroid of ΔABC

$AG=\frac{2}{3}(2r\sin 60)$

$AG=\frac{2r}{\sqrt{3}}$



From ΔAGD

$AG^2+h^2=(2r)^2$

$h=\sqrt{4r^2-\frac{4r^2}{3}}=2r\sqrt{\frac{2}{3}}$

$\because$ ABCD are centres of Balls so required height

= h + 2r

$=2r\left(\sqrt{\frac{2}{3}}+1\right)=4\left(\sqrt{\frac{2}{3}}+1\right)$

67. (C)

(1, 1), (1, 2) ….. (1, 99) $\Rightarrow 98\times 2+1$ pairs

(2, 2), …… (2, 49) $\Rightarrow$ $47\times 2+1$

(3, 3), …… (3, 33) $\Rightarrow$ $30\times 2+1$

(4, 4), …… (4, 24) $\Rightarrow 20\times 2+1$

(5, 5), …… (5, 19) $\Rightarrow 10\times 2+1$

(6, 6), …… (6, 16) $\Rightarrow 7\times 2+1$

(7, 7), …… (7, 14) $\Rightarrow 4\times 2+1$

(8, 8), …… (8, 12) $\Rightarrow 4\times 2+1$

(9, 9), …… (9, 11) $\Rightarrow 2\times 2+1$ pairs

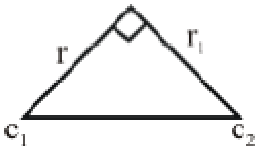
total = 946

68. (A)

$\pi r_1^2 = 2\pi r^2 \Rightarrow r_1 = \sqrt{2}\, r$

$\Rightarrow r_1^2 + r^2 = c_1c_2^2$

$\Rightarrow c_1c_2 = \sqrt{3}\, r$



69. (B)

$b^2 - 4a \geq 0$

$b = 1 \Rightarrow a \in \phi$

$b = 2 \Rightarrow a \in \{1\}$ total = 7

$b = 3 \Rightarrow a \in \{1, 2\}$

$b = 4 \Rightarrow a \in \{1, 2, 3, 4\}$

70. (A)

$s = \frac{1^2}{7^0} + \frac{2^2}{7^1} + \frac{3^2}{7^2} + \frac{4^2}{7^3} \ldots\ldots\infty$

$\frac{s}{7} = \frac{1^2}{7} + \frac{2^2}{7^2} + \frac{3^2}{7^3} + \ldots\ldots\infty$

$\frac{6s}{7} = 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{9}{7^3} + \ldots\ldots\infty$

$\frac{6s}{7^2} = \frac{1}{7} + \frac{3}{7^2} + \frac{5}{7^3} + \frac{9}{7^4} + \ldots\ldots\infty$

$\therefore \frac{36s}{7^2} = 1 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots\ldots\infty$

$\Rightarrow \frac{36s}{49} = \frac{4}{3} \Rightarrow s = \frac{49}{27}$

71. (D)

$f(x)$ is continuous for all non integers

for integers

$f(I^+) = I - 0 = I$

$f(I^-) = I - 1 + \sqrt{1} = I$

$\therefore f(x)$ is continuous for integers

$\therefore f(x)$ is continuous $\forall\, x \in R$

72. (B)

$\int_{-3\pi}^{3\pi} \sin^2\theta \sin^2 2\theta\, d\theta = 2\int_0^{3\pi} \sin^2\theta \sin^2 2\theta\, d\theta$

$= 8\int_0^{3\pi} \sin^4\theta \cos^2\theta\, d\theta = 24\int_0^{\pi} \sin^4\theta \cos^2\theta\, d\theta$

$= 48\int_0^{\pi/2} \sin^4\theta\cos^2\theta d\theta$

$= \frac{48.(3.1).(1)}{6.4.2}.\frac{\pi}{2} = \frac{3\pi}{2}$

73. (C)

$\vec{a} : 2\hat{i}+4\hat{j}+5\hat{k}$; $\vec{p} = 3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{b} : \hat{i}+2\hat{j}+3\hat{k}$; $\vec{q} = 2\hat{i}+3\hat{j}+4\hat{k}$

$SD = \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$= \left|\frac{-(\hat{i}+2\hat{j}+2\hat{k})\times(\hat{i}-2\hat{j}+\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}|}\right| = \frac{1}{\sqrt{6}}$

74. (C)

$\sec x = \frac{1}{1-\cos x} \Rightarrow \cos x = 1-\cos x$

$\Rightarrow \cos x = \frac{1}{2}$

2 solutions in $[0, 2\pi]$.

$\therefore$ 100 solutions.

75. (B)

$r = \frac{2}{2}\cot\frac{\pi}{10}$ $\therefore \pi R^2 - \pi r^2 = \pi$ sq. units

$R = \frac{2}{2}\text{cosec}\frac{\pi}{10}$

Note that it is independent of 'n'.

76. (A)

$(\vec{a}-\vec{d})\times\vec{b} = 0 \Rightarrow \vec{a} = \vec{d}+\lambda\vec{b}$

Dot with $\vec{c}$

$2+3+1 = 8+\lambda(1-1+1)$

$\Rightarrow \lambda = -2$

$\therefore \vec{d} = \vec{a}+2\vec{b}$ -

$= 4\hat{i}+\hat{j}+3\hat{k}$

$\therefore \vec{a}.\vec{d} = 4-1+3 = 6$

77. (C)

$f'(x)=f(x);\ f(1)=0$

$\therefore\ \frac{f'(x)}{f(x)}=1 \Rightarrow \ln(f(x))=x+c$

$\Rightarrow f(x)=k.e^x \Rightarrow k=0$

$\therefore\ f(x)=0$

78. (A)

$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=12$

$\frac{1}{6}\left(\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{a}-\vec{c}+\vec{b}\right]\right)=\frac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]=2$

79. (D)

Let mid point be (h, k)

Chord of hyperbola : $hx-ky=h^2-k^2$

this is tangent to $x^2=4by$

form of tangent for parabola : $y=mx-bm^2$

comparing we get

$m=\frac{h}{k};-bm^2=-\left(\frac{h^2-k^2}{k}\right)$

$\therefore\ b\left(\frac{h^2}{k^2}\right)=-\left(\frac{h^2-k^2}{k}\right)$

Clearly, locus is dependent on b, but not a.

80. (A)

$2xy\,dx+x^2dy=\frac{ydx-xdy}{y^2}$

$\Rightarrow d(x^2y)=d\left(\frac{x}{y}\right)$

$\Rightarrow x^2y=\frac{x}{y}+c$

At $x=2;\ y=1 \Rightarrow 4=2+c \Rightarrow c=2$

At $x=-1;\ y=-\frac{1}{y}+2 \Rightarrow y=1$

81. (6)

$\left|z-\frac{6}{z}\right|\ge|z|-\frac{6}{|z|}\Rightarrow 5\ge|z|-\frac{6}{|z|}$

$\Rightarrow |z|^2-5|z|-6\le 0$

$\Rightarrow |z| \in [-1, 6]$

$\therefore$ maximum value = 6

82. (4)

$f'(4) = 0$ at $x = 0, \pm\sqrt{2}$

$f(x)_{max} = \frac{4}{e^2}; f(x)_{min} = 0$

$\therefore a = 4$

83. (0)

$f'(x) = \text{coeff. of 'x' in } f(x)$

$$= \text{coeff. of 'x' in} \begin{vmatrix} -x & 2(1+2x) & 0 \\ 1 & x & 1-x \\ 3x & -2x & x+1 \end{vmatrix} = 0$$

84. (2)

Equation of tangent to circle $x^2 + y^2 = 16$ is

$$y = mx \pm 4\sqrt{m^2+1}$$

It passes through P(0, h); h > 0 $\Rightarrow h = 4\sqrt{m^2+1}$

Hence, equation of tangent PA or PB will be $y = mx \pm h$

They intersect at x-axis, where

$$0 = mx \pm h \Rightarrow mx \Rightarrow \pm h \Rightarrow x = \pm\frac{h}{m} \Rightarrow AB = \frac{2h}{|m|}$$

Therefore,

$$\text{Area of } \Delta PAB = \frac{1}{2}\left(\frac{2h}{|m|}\right)\cdot h = \frac{h^2}{|m|}$$

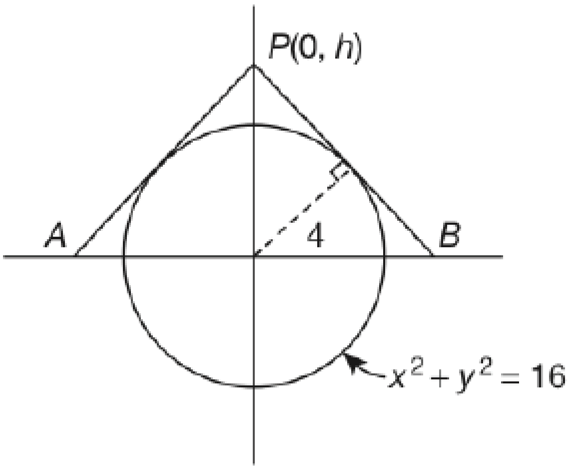
Also,

$$\frac{|h|}{\sqrt{m^2+1}} = 4$$

$$\Rightarrow \sqrt{m^2+1} = \frac{|h|}{4} \Rightarrow m^2+1 = \frac{h^2}{16}$$

$$\Rightarrow m^2 = \frac{h^2-16}{16} \Rightarrow |m| = \frac{\sqrt{h^2-16}}{4}$$

Therefore, Area of $\Delta PAB = \frac{4h^2}{\sqrt{h^2-16}} = f(h)$ (say)



$$\Rightarrow \quad f'(h) = \frac{4\left(h^3 - 32h\right)}{\left(h^2 - 16\right)^{3/2}} = 0 \Rightarrow h = 4\sqrt{2}$$

Hence, for minimum area, $h = 4\sqrt{2}$.

85. (3)

$$\tan\theta = \frac{3\tan\frac{\theta}{3} - \tan^3\frac{\theta}{3}}{1 - 3\tan^2\frac{\theta}{3}} = \lambda$$

$$\Rightarrow \quad \tan^3\frac{\theta}{3} - 3\lambda\tan^2\frac{\theta}{3} - 3\tan\frac{\theta}{3} + \lambda = 0$$

$$\Rightarrow \quad \sum \text{tab}\frac{\theta_1}{3}\frac{\tan\theta_2}{3} = -3.$$

86. (7)

The digits are 1, 1, 1, 1, 1, 2, 3 or 1, 1, 1, 1, 2, 2, 2

Hence, the number of seven-digit numbers formed is

$$\frac{7!}{5!} + \frac{7!}{4!3!} = 77$$

$$\therefore \quad \frac{N}{11} = \frac{77}{11} = 7$$

87. (8)

Given expression is $(1+x)\left(1+x^2\right)\left(1+x^3\right)....\left(1+x^{100}\right)$.

Coefficient of x^9 in $(1+x)\left(1+x^2\right)+\left(1+x^3\right).....\left(1+x^9\right)$, that is,

Terms containing x^9

$$= \left(1\cdot x^9 + x^1\cdot x^8 + x^2\cdot x^7 + x^3\cdot x^6 + x^4\cdot x^5 + x^1\cdot x^2\cdot x^6 + x^1\cdot x^3\cdot x^5 + x^1\cdot x^4\cdot x^5\right)$$

$\Rightarrow$ Term containing x^9 is $8x^9E$

Therefore, coefficient of $x^9 = 8$.

88. (1)

Any tangent to circle is given by

$$y = m(x-6) \pm \sqrt{2}\sqrt{1+m^2}$$

If it is a focal chord to parabola, then

$$0 = m(4-6) \pm \sqrt{2}\sqrt{1+m^2}$$

$\Rightarrow \quad m = \pm 1$

Hence $|m| = 1$

89. (9)

Slope of the first curve is

$$\left(\frac{dy}{dx}\right)_I = -\frac{4x}{py}$$

Slope of the second curve is

$$\left(\frac{dy}{dx}\right)_{II} = \frac{x}{4y}$$

For the orthogonal intersection, we have

$$\left(-\frac{4x}{py}\right)\left(\frac{x}{4y}\right) = -1 \Rightarrow x^2 = py^2$$

On solving the equations of the given curves, we get x = 3 and y = 1.

Therefore,

$$p(1) = (3)^2 = 9 \Rightarrow p = 9$$

90. (3)

$$f(x) = (x^2 + +5x + 4)(x^2 + 5x + 6) + 5$$

$$= \left[(x^2 + 5x + 5) - 1\right]\left[(x^2 + 5x + 5) + 1\right] + 5$$

$$= (x^2 + 5x + 5)^2 - 1 + 5 = (x^2 + 5x + 5)^2 + 4$$

Therefore, minimum value of $f(x) = 4$ and maximum value occurs at x = 6.

$$f(x)_{max} = (36 + 30 + 5)^2 + 4 = 5045$$

Now, $a = 4, b = 5045$.

Hence, $\frac{a+b}{1683} = 3$.