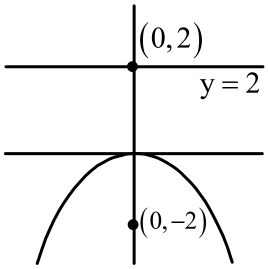
PARABOLA BOOKLET SOLUTION

1. (B)



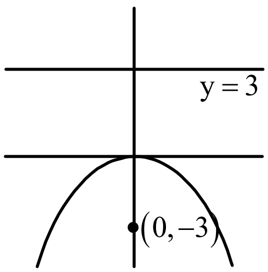
$x^2 = -8y$

$x^2 = -4Ay$

$\Rightarrow A = 2$

Directrix is $y = 2$

2. (A)



$x^2 = -12y$

3. (C)

Vertex $(0,0)$

Directrix $3x - 4y + 2 = 0$

$D = \frac{|2|}{\sqrt{3^2+4^2}} = \frac{2}{5} = a$

$L.R = 4a = 4.2/5 = 8/5$

4. (B)

Focus of $y^2 = 8x$ is $(-2,0)$

Focal chord $2x + y + d = 0$

$\Rightarrow 2(-2) + 0 + \lambda = 0$

$\Rightarrow \lambda = 4$

5. (A)

Focal distance is $x + a$

$\therefore x, +3$

6. (B)

Vertex is (a,b)

$L.R = \ell = 4A \Rightarrow A = e/4$

$(x-a)^2 = \ell(y-b)$

7. (A)

$x^2 = -y, \qquad x^2 = -4AV$

$\Rightarrow 4A = 1 = \ell$

8. (C)

$D = 2a$ between focus & Directrix

$x^2 = -8y$

$\Rightarrow 2a = 4$

9. (B)

$F(3,0) \qquad \ell = 4A = 8 \Rightarrow A = 2$

$\Rightarrow$ vertex $(1,0)$

10. (B)

$F(2,1)$

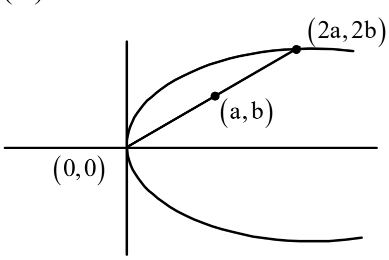
Let the L. R. be $2x - 3y + c = 0$

Now, $2(2) - 3(1) + c = 0$

$\Rightarrow c = -1$

$LR \equiv 2x - 3y - 1 = 0$

11. (D)



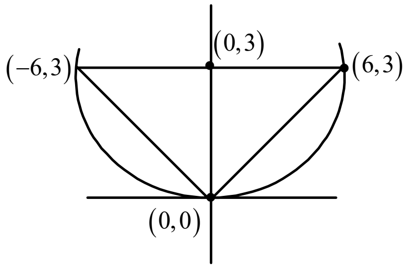
$y^2 = 4x$

$(2b)^2 = 4.2a$

$4b^2 = 8a$

$\Rightarrow b^2 = 2a$

12. (C)



$A = \frac{1}{2} \times 12 \times 3$

$A = 18\,\text{sq. units}$

13. (A)

$9x^2 - 6x + 36y + 9 = 0$

$(3x)^2 - 2.3x.1 + 1 + 36y + 8 = 0$

$(3x-1)^2 = -36y - 8$

$9(x - 1/3)^2 = -36\left(y + \frac{2}{9}\right)$

Vertex $(1/3, -2/9)$

14. (C)

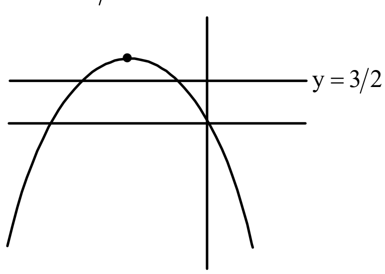
$x^2 + 4x + 2y = 0$

$(x+2)^2 = -2y + 4$

$(x+2)^2 = -2(y-2)$

Now, $4a = 2$

$\Rightarrow a = 1/2$



$LR \equiv y = 3/2$

15. (D)

$y^2 = x - 2y + 2 = 0$

$(y-1)^2 = x - 1$

Vertex $(1,1), a = 1/4$

Focus $\Rightarrow (5/4, 1)$

16. (D)

$y^2 + 2y + x = 0$

$(y+1)^2 = -(x-1)$

Vertex $(1,-1)$

17. (C)

$x^2 - 4x - 3y + 10 = 0$

$(x-2)^2 = 3(y-2)$

Axis is $x-2=0$

18. (D)

$x^2+4x+2y-7=0$

$(x+2)^2=-2(y-11/2)$

Vertex $(-2,11/2)$

19. (B)

$y^2=4y-2x-8=0$

$(y-2)^2=2(x+6)$

$LR=2$

20. (A)

$y^2+4x+2y-8=0$

$(y+1)^2=-4(x-9/4)$

It is focus $(5/4,-1)$

21. (C)

$x=ay^2+by+c$

$x=a\left[(y+b/2a)^2-\frac{b^2}{4a^2}+c/a\right]$

$\frac{1}{a}\left(x+\frac{b^2-4ac}{4a}\right)=(y+b/2a)^2$

$L.L.R=1/a$

22. (C)

For parabola $4^2=ab$

Option (C) $(4)\neq 2\times 1$

$2x^2+y^2-4xy=8$ is not a parabola

23. (B)

Option (B)

$\frac{x}{y}+\frac{2y}{x}=0$

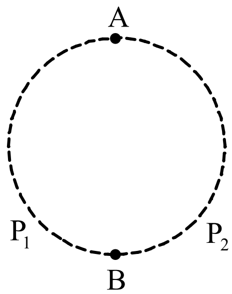
$x^2=-24y$

24. (C)

Theory based

25. (C)

Given ends $A\,\&\,B$ two parabolas are possible.



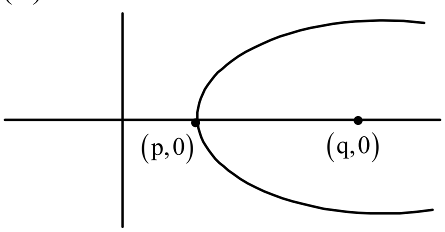
26. $v(0,a)$

$f(0,0)$

$(x-0)^2=-4a(y-a)$

$x^2=4a(a-y)$

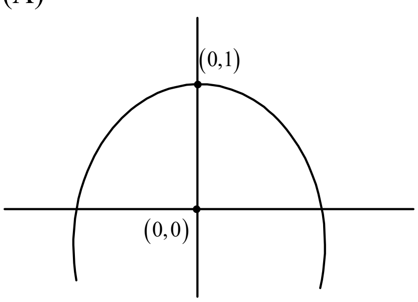
27. (C)



$y^2=4(q-p)(x-p)$

$\Rightarrow y^2=-4(p-q)(x-p)$

28. (A)



$(x-0)^2=-4\times1(y-1)$

$x^2+4y-4=0$

29. (B)

$v(2,0)$ $f(5,0)$

$y^2=4\times3(x-2)$

$y^2=12x-24$

30. (D)

$x-2=t^2, y=2t$

$x-2=(y/2)^2$

$\Rightarrow y^2 = 4(x-2)$

31. (B)

$x = \frac{t}{4}, y = \frac{t^2}{4}$

$\Rightarrow y = \frac{(4x)^2}{4} = 4x^2$ a parabola

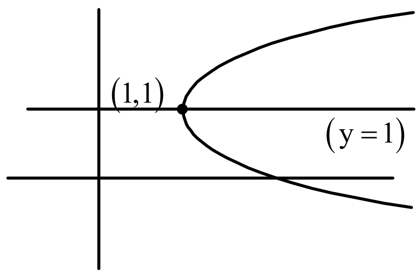
32. (A)

$x = t^2 + 1, y = 2t + 1$

$x = \left(\frac{y-1}{2}\right)^2 + 1$

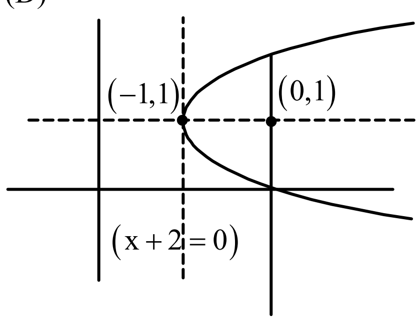
$4x = (y-1)^2 + 4$

$\Rightarrow (y-1)^2 = 4(x-1)$



Directrix is $x = 0$

33. (D)



Vertex $(-1,1)$

$f(0,1)$

$(y-1)^2 = 4 \times 1(x+1)$

Now, $x + 1 = 1 \times t^2 \Rightarrow x = t^2 - 1$

$y - 1 = 2 \times 1 \times t \Rightarrow y = 2t + 1$

Any point $(t^2 - 1, 2t + 1)$

34. (A)

$(a/m^2, 2/m)$ doesn't satisfy $y^2 = 4ax$

35. (A)

$y^2 - 12x - 2y - 11 = 0$

$(y-1)^2 = 12(x+1)$

$y-1 = 2\times3\times t \quad x+1 = 3.t^2$

$y = 1+6t \qquad x = 3t^2-1$

36. (B)

$y^2 = 6x$

Vertex $(0,0)$

Negative end of LR $(3/2,-3)$

$\frac{y-0}{x-0} = \frac{3-0}{3/2-0} \Rightarrow \frac{3}{2}y = -3x$

$\Rightarrow y+2x = 0$

37. (A)

$f(8,0)$

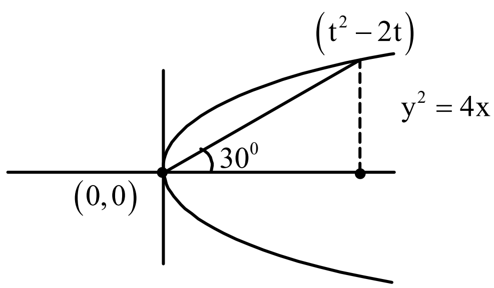
One end $(2,-8)$

$\frac{-2a}{t} = -8 = \frac{-2\times8}{t} \Rightarrow t = 2$

$(at^2, 2at) \equiv (8\times2^2, 2\times8\times2)$

$\equiv (32,32)$

38. (C)



$\tan 30^0 = \frac{2t}{t^2}$

$\frac{1}{\sqrt{3}} = \frac{2}{t} \Rightarrow$

$t = 2\sqrt{3}$

Length $= \sqrt{\left(\left(2\sqrt{3}\right)^2\right)^2 + \left(2\times2\sqrt{3}\right)^2}$

$= \sqrt{12^2 + 4^2\times3}$

$= 8\sqrt{3}$

39. (B)

$y^2 = 4x$

$(-x)^2 = 4x$

$x^2 = 4x \Rightarrow x = 0 \text{ or } x = 4$

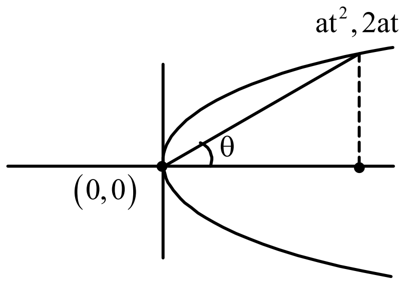
$y^2 = 0$ or $y^2 = 4\times 4$

$4 = 0$ or $y = \pm 4$

$A \equiv (4,4)$ $B \equiv (4,-4)$

$AB = 8$

40. (B)



$\tan\theta = \dfrac{2at}{at^2}$

$t = 2\cot\theta$

Length $= \sqrt{a^2t^4 + 4a^2t^2}$

$= at\sqrt{t^2+4}$

$= a\times 2\cot\theta\sqrt{4\cot^2\theta + 4}$

$= a\times 2\cot\theta \times 2\times \cos\sec\theta$

$= 4a\cos\theta.\cos ec^2\theta$

41. (C)

Put $y = 0$

$x^2 = 7x + 12 = 0$

$\Rightarrow x = 4 \text{ or } 3$

Intercept is 1 unit

42. (B)

Put $x = 0$

$2y^2 + 6y - 8 = 0$

$y^2 + 3y - 4 = 0$

$(y + 3/2)^2 - 9/4 - 4 = 0$

$(y + 3/2)^2 = \dfrac{25}{4}$

$y + 3/2 = \pm 5/2$

$y = 1, -4$

Intercept length $= 5$

43. (A)

$x^2 + 3x = 5 - x$

$x^2 + 4x + 4 = 9$

$(x+2)^2 = 9$

$x+2=\pm 3$
$x=1,-5$
$y=5-x$
$=5-1 \text{ or } 5-(-5)$
$y=4,10$
Points are $(1,4).\&(-5,10)$
Length $=6\sqrt{2}$

44. (C)
$2a=2\times 2=\dfrac{2\times sp\times sa}{sp+sq}$
$2=\dfrac{6\times s\theta}{6+s\theta}$
$12+2s\theta=65\theta$
$4s\theta=12$
$5\theta=3$

45. (B)
$2a=\dfrac{sp\times s\theta}{sp+s\theta}$ $\{A=2a \text{ in this case}\}$
$2a=\dfrac{2\times 4}{6}=4/3$
$a=2/3$

46. (B)
$y^2=K(1-y)$
$y^2+Ky0k=0$
$D=0\Rightarrow K^2=(4\times -K)=0$
$K^2+4K=0$
$K=0,-4$

47. (C)
$K-x=x-x^2$
$x^2-2x+K=0$
$D=0\Rightarrow 4-4K=0$
$\Rightarrow K=1$

48. (D)
$y^2=4a(x+a)$
$y=mx+c$
$y=m(x+a)+c-am$
Now, $c-am=\dfrac{a}{m}$

$\Rightarrow c = am + \frac{a}{m}$

49. (A)

$(1-2x)^2 = 4x$

$1 + 4x^2 - 4x = 4x$

$4x^2 - 8x + 1 = 0$

$\because D > 0,$ 2 real points

50. (C)

$y^2 = 4ax$

$\ell x + my + n = 0$

$y = -\frac{\ell x}{m} - \frac{n}{m}$

$c = \frac{a}{M} \Rightarrow \frac{-n}{m} = \frac{a}{-\ell/m}$

$\Rightarrow \frac{\ell n}{m^2} = a$

51. (B)

$x^2 + 4\left(\frac{x-k}{2}\right) = 0$

$x^2 + 2x - 2k = 0$

$D = 0 \Rightarrow 4 + 8K = 0$

$\Rightarrow K = -1/2$

52. (D)

$\left(my + \frac{a}{m}\right)^2 = 4ay$

$m^2y^2 + \frac{a^2}{m^2} + 2ay = 4ay$

$(my - a/m)^2 = 0$

$y = \frac{a}{m^2}$

$\Rightarrow x = \frac{2a}{m}$

$\left(\frac{2a}{m}, \frac{a}{m^2}\right)$

53. (A)

$y^2 = \frac{1}{4}x$

$y = \sqrt{3}x + C$

Hence $c = a/m \Rightarrow c = \frac{1/16}{\sqrt{3}} = \frac{1}{16\sqrt{3}}$

$(x.y.) \equiv \left(\frac{1}{48}, \frac{1}{8\sqrt{3}}\right)$

54. (A)

$y.2 = 4.\frac{(x+1)}{2}$

$y = x + 1$

55. (B)

$1 - x = x - x^2$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1 \Rightarrow y = 0$

56. (A)

$4y^2 + 6x = 8y + 7$

$(2y)^2 - 2\times(2y)\times 2 + 4 = -6x + 11$

$(2y-2)^2 = -6\left(x - \frac{11}{6}\right)$

$(y-1)^2 = \frac{-3}{2}\left(x - \frac{11}{6}\right)$

Tangent at vertex $x = \frac{11}{6}$

57. (A)

$4x^2 = 4x = -y + 2$

$(2x)^2 - 2.2x \times 1 + 1 = -y + 3$

$(2x-1)^2 = -(y-3)$

$({}^{x-11}2)^2 == \frac{1}{4}(y-3)$

58. (D)

$y^2 = x$

$y = x + c \qquad (\tan 45^0 = 1)$

$c = \frac{1/4}{1} = \frac{1}{4}$

$(x + 1/4)^2 = x$

Point $(1/4, 1/2)$

59. (C)

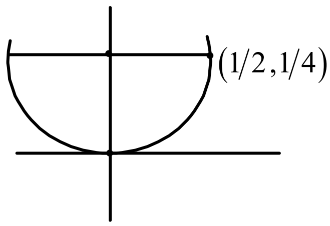
$(2x+2)^2 = 16x$

$(x+1)^2 = 4x$

$(x-1)^2 = 0$

$x = 1 \Rightarrow y = 4$

60. (D)



$x^2 = y$

$x^2 = 4 \times \left(\frac{1}{4}y\right)$

Tangent at $(1/2, 1/4)$ is

$x \times 1/2 = \frac{(y+1/4)}{2}$

$4x - 4y = 1$

61. (D)

Tangent of $y^2 = 4x$ is

$y = mx + 1/m$

Also for $x^2 = 32y$

$x^2 = 32(mx + 1/m)$

$x^2 - 32mx - \frac{32}{m} = 0$

For tangency $D = 0$

$\Rightarrow (-32m)^2 + \frac{32}{m} \times 4 = 0$

$\Rightarrow m = -1/2$

$\therefore y = \frac{-1}{2}x - 2$

$\Rightarrow x + 2y + 4 = 0$

62. ()

$$\Delta = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ a\left(t_2^2 t_1^2\right) & 2a\left(t_2 - t_1\right) & 0 \\ a\left(t_3^2 - t_1^2\right) & 2a\left(t_3 - t_1\right) & 0 \end{vmatrix}$$

$$= \frac{1}{2} \times a^2\left(t_2 - t_1\right)\left(t_3 - t_1\right)$$

$$\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ t_1 + t_2 & 2 & 0 \\ t_1 + t_3 & 2 & 0 \end{vmatrix}$$

$$\Delta = \frac{1}{2}a^2\left(t_2 - t_1\right)\left(t_3 - t_1\right) \times 2\left(t_2 - t_3\right)$$

Required area is $1/2\,\Delta$

$\therefore$ Area $= \frac{a^2}{2}\left(t_1 - t_2\right)\left(t_2 - t_3\right)\left(t_3 - t_1\right)$

63. ()

$t, y = x + at_1^2$ & $t_2 y = x + at_2^2$

Solving $x = at_1 t_2$

$\therefore$ abscissa is G. M. between ponts.

EXERCISE 1(B)

1. (A)

$at_1^2 = p$

$2ah = q$

At conditions of θ be $\left(\frac{a}{h^2}, \frac{-2a}{h}\right)$

Or $\left(\frac{a^2}{p}, \frac{-4a^2}{q}\right)$

2. (A)

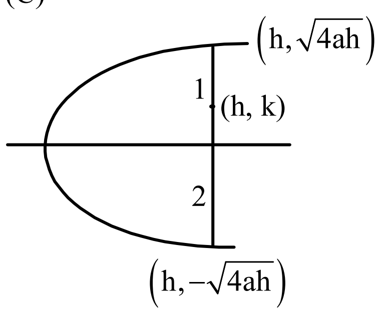
$a + at^2 = 4$

Here a = 2 $\Rightarrow t^2 = 1$

$\Rightarrow t = \pm 1$

Coordinate $(2, \pm 4)$

3. (C)



By section formula

$\frac{2\sqrt{4ah} - \sqrt{4ah}}{3} = k$

$\frac{4ax}{J} = y^2$

4. (B)

Check option (B)

$x^2 - 2 = -2\left(2\cos^2\frac{t}{2} - 1\right)$

$x^2 - 2 = -4\cos^2\frac{t}{2} + 2$

$x^2 - 4 = -y$

5. (C)

$x_1 = at_1^2$ $\qquad t_1t_2 = -1$

$x_2 = at_2^2$

$x_1x_2 = a^2$

Square be G.M. of $x_1x_2 = a^2$

6. (A)

y = mx +c is tangent to

$y^2 = 4ax \Rightarrow c = \frac{a}{m}$

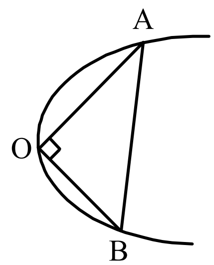
& of $x^2 = 4by \Rightarrow c = -bm^2$

$$\Rightarrow m^3 = -\frac{a}{b}$$

$$\text{Or } m = \frac{-a^{1/3}}{b^{1/3}}$$

Equation of tangent is $a^{1/3}x + b^{1/3}y + (ab)^{2/3} = 0$

7. (A)



Homogenize line with $y^2 = 4ax$

$y^2 - 4ax\left(\frac{y+mx}{c}\right) = 0$

Since $\angle AOB = 90^0$

$\Rightarrow$ coefficient x^2 + coefficient $y^2 = 0$

c + 4ax = 0

8. (A)

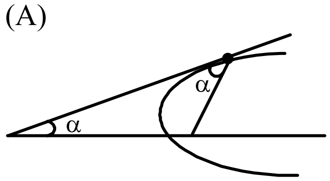
We write equation as

$\left(\frac{2u^2\cos^2\alpha}{g}\right)y = \left(\frac{u^2\sin 2\alpha}{g}\right)x - x^2$

When we form a perfect square

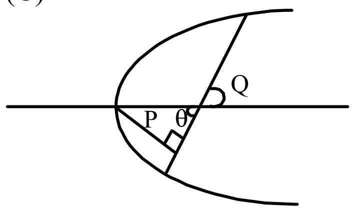
We get $x = \frac{2u^2\cos^2\alpha}{g}$

9. (A)



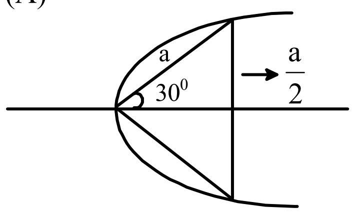
Properly these two angles are same.

10. (C)



$P = a\sin\theta$

11. (A)



The length of double ordinate will be a

$\text{Area} = \frac{1}{2} \times a\cos 90^0 \times a$

$= \frac{\sqrt{3}}{4}a^2$

Here $P^2\left(\frac{\sqrt{3}a}{2}, a/2\right)$ lies on parabola

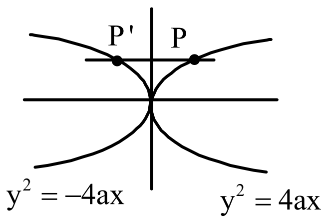
$\frac{a^2}{4} = \frac{4\sqrt{3}\,a}{2}$ $\quad \because A = 48\sqrt{3}$

$a = 8\sqrt{3}$

12.	(B)	13.	(D)	14.	(A)	15.	(C)		
16.	(B)	17.	(A)	18.	(C)	19.	(C)	20.	(D)
21.	(A)	22.	(A)	23.	(C)	24.	(D)	25.	(C)
26.	(D)	27.	(A)	28.	(C)	29.	(C)	30.	(C)
31.	(A)	32.	(D)	33.	(D)	34.	(C)	35.	(C)
36.	(C)	37.	(D)	38.	(B)	39.	(B)	40.	(C)
41.	(C)	42.	(B)	43.	(C)	44.	(A)	45.	(B)
46.	(A)	47.	(D)	48.	(B)				

(2-A) PARABOLA BOOKLET SOLUTION

1. (A)



$P \equiv (at^2, 2at)$. $P' \equiv (-at^2, 2at)$

Let unit point be $\theta(h, k)$

$h = \frac{at^2 - bt^2}{2} \Rightarrow t^2 = \left(\frac{2h}{a-b}\right)$ .... (I)

$K = \frac{2at - 2bt}{2} \Rightarrow t = \left(\frac{K}{a-b}\right)$ .... (II)

Eliminating t from (I) and (II)

$\left(\frac{K}{a-b}\right)^2 = \frac{2h}{(a-b)}$

$\Rightarrow K^2 = 2h(a-b)$

$\therefore$ Locus is $y^2 = 2(a-b)x$. Which is a parabola

2. ()

$y = m(x-a)$ .... (I)

$y^2 = 4ax$ .... (II)

Solving (I) and (II)

$[m(x-a)]^2 = 4ax$

$\Rightarrow m^2(x-a)^2 = 4ax$

$\Rightarrow m^2x^2 - 2am^2x + m^2a^2 = 4ax$

$\Rightarrow m^2x^2 - 2ax(m^2+2) + m^2a^2 = 0$

For 2 real solutions $D > 0$ & $m^2 \neq 0$

$\Rightarrow 4a^2(m^2+2)^2 - 4m^2(m^2a^2) > 0$

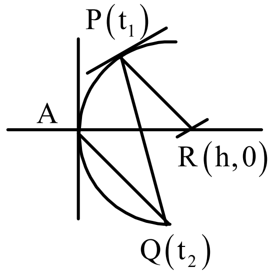
$\Rightarrow 4a^2((m^2+2)^2 - m^4) > 0$

$\Rightarrow 4a^2((2m^2+2)(2)) > 0$

$\Rightarrow m \in R$

$\therefore m \in R \sim \{0\}$

3. (C)



$\because$ Pθ is a normal chord

$\Rightarrow t_2 = -t_1 \frac{-2}{t_1}$ ………. (I)

Slope of $AQ = \frac{2}{t_2}$ = slope of PR.

Equation of PR.

$$y - 2ati = \frac{2}{t_2}\left(x - at_1^2\right)$$

For R.

$$0 - 2at_1 = \frac{2}{t_2}\left(h - at_1{}^2\right)$$

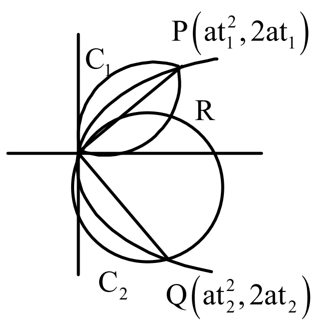
$\Rightarrow$ $h = at1^2 - at_1t_2$

$$= at_1^2 - at_1\left(-t_1 - \frac{2}{t_1}\right)$$

$$= at_1{}^2 + at_1{}^2 + 2a \qquad = 2a\left(1 + t^2\right)$$

$= 2 \times$ focal distance of P.

4. ()



$\tan\theta_1 = \frac{2}{t_1}$

$\tan\theta_2 = \frac{2}{t_2}$

$C_1 : x^2 + y^2 - at_1{}^2x - 2at_1y = 0$

$C_2 : x^2 + y^2 - at_2{}^2x - 2at_2y = 0$

Equation of OR

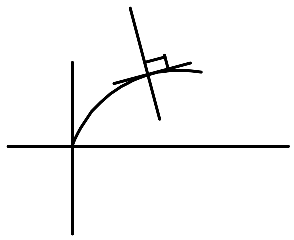
$C_1 - C_2 = 0 \Rightarrow a\left(t_2^2 - t_1^2\right)x + 2a\left(t_2 - t_1\right)y = 0$

$\Rightarrow y = -\frac{1}{2}\left(t_1 + t_2\right)x$

$\therefore \tan\phi = -\frac{1}{2}(t_1 + t_2) = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2)$

$\Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

5. ()



$y = \sqrt{x} \Rightarrow y^2 = x$ ……. (I)

Equation of normal to (I) in parametic form.

$$y + tx = \frac{t}{2} + \frac{t^3}{4}$$

$\because$ This passes through $(3,6)$

$\Rightarrow 6 + 3t = \frac{t}{2} + \frac{t^3}{4} \Rightarrow t^3 - 10t - 24 = 0$

$\Rightarrow (t-4)(t^2 + 4t + 6) = 0$

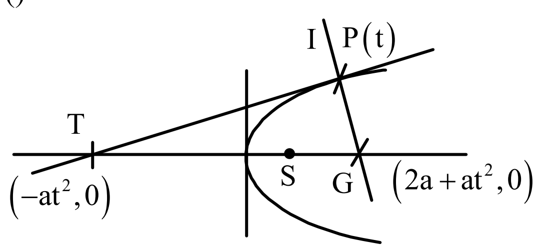
$\Rightarrow t = 4$

$\therefore$ Normal is

$y + 4x = 2 + 16$

$\Rightarrow 4x + y - 18 = 0$

6. ()



Centre of circle passing through P, T, G is $S(a,0)$

$\therefore$ Slope of tangent to circle at

$P = \frac{-1}{\text{slope of PS.}}$

$= \frac{-1}{\left(\frac{2at}{at^2 - a}\right)} = \frac{1-t^2}{2t} = m_1$

Slope of tangent to parabola at P $= \frac{1}{t} = m_2$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{t} - \frac{1}{2t} + \frac{t}{2}}{1 + \frac{1-t^2}{2t}}\right| = t$

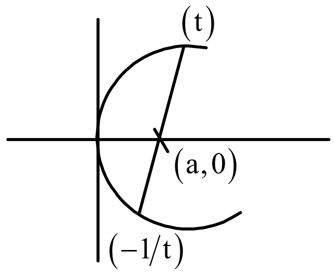
$\therefore \theta = \tan^{-1}(t)$

7. ()

Clearly the point of intersection of the tangents will always lie in the line. Hence locus will be the given line its effect.

8. ()

Length of focal chord $= a\left(t+\frac{1}{t}\right)^2$



Also $\tan\alpha = \frac{2}{t-\frac{1}{t}}$

$\Rightarrow t\frac{-1}{t} = 2\cot\alpha$

$\Rightarrow \left(t+\frac{1}{t}\right)^2 = \left(t\frac{1}{t}\right)^2 + 4$

$= 4\cot^2\alpha + 4$

$= 4\cos ec^2\alpha$

$\therefore$ Length of focal chord $= 4\cos ec^2\alpha$

$\because \alpha \in (0, \pi/4)$

$\therefore$ Minimum length $= 4\cos\sec^2\frac{\pi}{4} = 8a$

9. $y = k\sqrt{x} \Rightarrow y^2 = k^2x$

Let $k^2 = 4a \Rightarrow y^2 = 4ax$

For ABCD

$at_1^2 = 2at_1 \Rightarrow t_1 = 2$

For EFGC

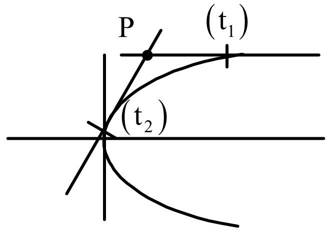
$at_2^2 - ya = 2at_2$

$\Rightarrow at_2^2 - 2at_2 - 4a = 0$

$\Rightarrow t_2 = 1+\sqrt{5}$

$\therefore \frac{FG}{BC} = \frac{2at_2}{2at_1} = \frac{2a(1+\sqrt{5})}{2a(2)} = \frac{\sqrt{5}+1}{2}$

10. ()



Let $P \equiv (h, k)$ and pair of tangents drawn touches parabola at $A(t_1)$ and $B(t_2)$

$\tan\theta_1 = \frac{1}{t_1} \qquad \tan\theta_2 = \frac{1}{t_2}$

Also, $h = t_1 t_2 \qquad k = (t_1 + t_2)$

Given, $\theta_1 + \theta_2 = \frac{\pi}{4}$

$$\Rightarrow \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = 1$$

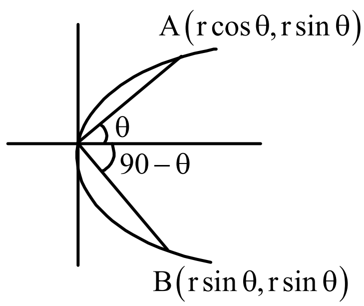
$$\Rightarrow \frac{\frac{1}{t_1} + \frac{1}{t_2}}{1 - \frac{1}{t_1 t_2}} = 1$$

$$\Rightarrow \frac{t_1 + t_2}{t_1 t_2 - 1} = 1$$

$$\Rightarrow \frac{k}{h-1} = 1 \Rightarrow h - k = 1$$

$\therefore$ Locus is $x - y = 1$

11. ()



$\because$ A and B lies on the parabola

For A

$$r^2 \sin^2\theta = r\cos\theta$$

$$\Rightarrow r = \frac{\cos\theta}{\sin^2\theta} = |VA|$$

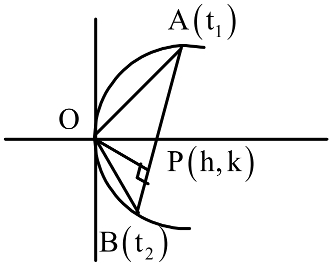
For B

$$r^2 \cos^2\theta = r\sin\theta$$

$$\Rightarrow r = \frac{\sin\theta}{\cos^2\theta} = |VB|$$

$$\therefore \frac{|VA|}{|VB|} = \frac{\cos^3\theta}{\sin^3\theta} = \cot^3\theta$$

12. ()



$\because$ AB subtends 90^0 at vertex

$\Rightarrow t_1t_2 = -4$ ------ (I)

Equation of AB

$2x - y(t_1 + t_2) + 2at_1t_2 = 0$

$\Rightarrow 2x - y(t_1 + t_2) - 8a = 0$ ……….. (II)

Equation of OP

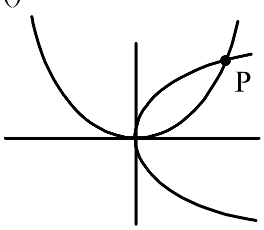
$y = -\frac{(t_1 + t_2)}{2}x$ ……… (III)

To find locus of P eliminate t_1 and t_2 from (II) And (III)

Required locus is

$x^2 + y^2 - 4ax = 0$

13. ()



Let $P \equiv (at^2, 2at)$

$\because$ P lies on $x^2 = 4by$ as well.

$\Rightarrow a^2t^4 = 4b.2at$

$\Rightarrow t = 2\left(\frac{b}{a}\right)^{1/3}$

Slope of tangent at P to on $y^2 = 4ax$

$$= \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t}.(m_1)$$

Slope of tangent at P on $x^2 = 4by$

$$= \frac{x}{2b} = \frac{at^2}{2b} = \frac{a}{2b}4.\frac{b^{2/3}}{a^{2/3}}$$

$$= 2\left(\frac{a}{b}\right)^{1/3} = \frac{4}{t}(m_2)$$

$\therefore$ Angle of enter section of parabola

=angle b/w their tangents

$$\therefore \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{4}{t} - \frac{1}{t}}{1 + \frac{4}{t^2}}\right|$$

$$= \left|\frac{3}{t + \frac{4}{t}}\right| = \left|\frac{3/2}{\left(\frac{a}{b}\right)^{1/3} + \left(\frac{b}{a}\right)^{1/3}}\right| = \sqrt{3}. \Rightarrow \theta = \pi/3$$

14. ()

Let point be $P(h,k)$

Equation of polar $xh + yk = r^2$

$$\Rightarrow y = \left(\frac{-h}{k}\right)x + \frac{r^2}{k}$$

For this to be tangent to $y^2 = 4ax$

$$\frac{r^2}{k} = \frac{a}{\left(\frac{-b}{k}\right)} \Rightarrow k^2 = -\frac{r^2}{a}h$$

$\therefore$ Locus is $y^2 = \frac{-r^2}{a}x$

15. ()

$x\cos\alpha + y\sin\alpha = P \Rightarrow y = (-\cot\alpha)x + P\operatorname{cosec}\alpha$ ………….. (I)

Tangent to $y^2 = 4a(x+a)$ will be

$$y = m_1(x+a) + \frac{a}{m}$$

$$\Rightarrow y = mx + am + \frac{a}{m} \quad \text{……….. (II)}$$

Comparing (i) and (ii)

$m = -\cot\alpha$ and

$$P\operatorname{cosec}\alpha = am + \frac{a}{m}$$

$$\Rightarrow P\operatorname{cosec}\alpha = -a\cot\alpha - \frac{a}{\cot\alpha}$$

$$\Rightarrow P\operatorname{cosec}\alpha = -\frac{a\operatorname{cosec}^2\alpha}{\cot\alpha}$$

$$\Rightarrow P\cos\alpha + a = 0$$

16. ()

Normal to $y^2 = 4c(x-d)$ can be

$$y = m(x-d) - 2cm - cm^3$$

$$\Rightarrow y = mx - md - 2cm - cm^3 \text{ …… (I)}$$

Normal to $y^2 = 4ax$ can be

$$y = mx - 2am - am^3 \quad .... (II)$$

For common normal comparing (I) and (II)

$$-md - 2cm - cm^3 = -2am - am^3$$

$$\Rightarrow d + 2c + cm^2 = 2a + am^2$$

$$\Rightarrow (a-c)m^2 = (2c + d - 2d)$$

$$\because a > c > 0 \Rightarrow 2c + d - 2a > 0$$

$$\Rightarrow 2a < 2c + d$$

17. ()

Let normal be $y + tx = 2at + at^3$

$\Rightarrow -t = \tan\phi \Rightarrow t = -\tan\phi$

Normal cells curve again at t_2

$$\Rightarrow t_2 = -t - \frac{2}{t} = \tan\phi + 2\cot\phi$$

Slope of tangent at $t_2 = \frac{1}{t_2}$

Angle of intersection of normal at t_2

$$\tan\theta = \left|\frac{\frac{1}{t_2} + t}{1 - \frac{t}{t_2}}\right| = \left|\frac{1 + tt_2}{t_2 - t_1}\right| = \left|\frac{-1 + t^2}{t_2 - t_1}\right|$$

$$= \left|\frac{1 + \tan^2\phi}{\tan\phi + 2\cot\phi + \tan\phi}\right| = \left|\frac{\tan\phi}{2}\right|$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{2}\tan\phi\right)$$

18. ()

Equation of focal chord

$$2x - y\left(t - \frac{1}{t}\right) - 2a = 0 \text{ and its length } = a\left(t + \frac{1}{t}\right)^2$$

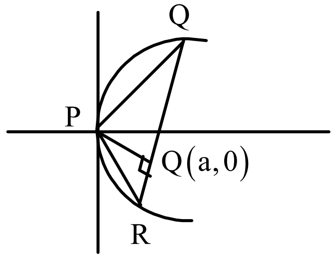
Distance of vertex focus this chord

$$P = \left|\frac{2a}{\sqrt{4 + (t - 1/t)^2}}\right| = \left|\frac{2a}{\sqrt{(t + 1/t)^2}}\right|$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^2 = \frac{4a^2}{P^2}$$

$$\therefore \text{length of chord } = a\left(t + \frac{1}{t}\right)^2 = \frac{4a^3}{P^2}$$

19. ()



Area of $\Delta = A = \frac{1}{2} \times a\left(t+\frac{1}{t}\right)^2 \times \frac{2a}{(t+1/t)}$

$$\left[\because QR = a\left(t+\frac{1}{t}\right)^2, PQ = \frac{2a}{\left(t+\frac{1}{t}\right)}\right]$$

$\Rightarrow A = a^2\left(t+\frac{1}{t}\right) \Rightarrow t+\frac{1}{t} = \frac{A}{a^2}$

Difference between ordinates $= 2at - \left(\frac{2a}{-t}\right) = 2a\left(t+\frac{1}{t}\right)$

$$= 2a\frac{A}{a^2} = \frac{24}{a}$$

20. ()

Same question as in Q. 3

21. ()

Slope of normal at $P(t_1)$ and $Q(t_2)$ is

$-t_1$ and $-t_2$ respectively

$\because$ Normal are perpendicular to each other

$\Rightarrow t_1t_2 = -1$

Equation of chord PQ

$2x - y(t_1+t_2) + 2at_1t_2 = 0$

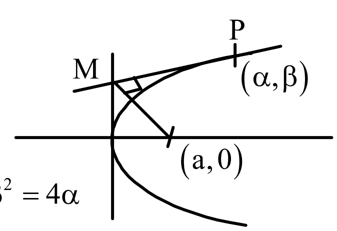
$\Rightarrow 2(x-a) - (t_1+t_2)(y) = 0$

Which passes through intersection of lines

$x - a = 0$ and $y = 0$

$\therefore$ Fixed Pt is $(a,0)$

22. ()



$\beta^2 = 4\alpha$

Equation of tangent at P

$y\beta = 2a(x-\alpha)$

Coordinates of $M \equiv \left(0, \frac{2a\alpha}{\beta}\right)$

Area of $\Delta = \frac{1}{2}\begin{vmatrix} \alpha & \beta & 1 \\ 0 & \frac{2\alpha}{\beta} & 1 \\ 1 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left(\frac{2\alpha^2}{\beta} + \left(\beta - \frac{2\alpha}{\beta}\right)\right) = \frac{1}{2}\left(\frac{\beta^3}{8} + \frac{\beta}{2}\right)$

Maximum area $= (\text{for } \beta = 2) = \frac{1}{2}\left(\frac{8}{8} + \frac{2}{2}\right) = 1$

23. (A)

$y^2 - 2y - 4x + 5 = 0$

$\Rightarrow (y-1)^2 = 4(x-1)$

Equation of normal

$y - 1 = m(x-1) - 2m - m^3$

Let point on axis of parabola be $(x, 1)$

$\Rightarrow 0 = m(x-1) - 2m - m^3 \Rightarrow 0 = x - 3 - m^2$

$\Rightarrow m^2 = x - 3 \geq 0 \Rightarrow x \geq 3$

24. ()

Slope of line $= \sqrt{3} = \tan\theta \Rightarrow \theta = 60^0$

Coordinate of a point at distance r from $(\sqrt{3}, 0)$. On the line $y - \sqrt{3x} + 3 = 0$

$\equiv \left(\sqrt{3} + r\cos 60^0, r\sin 60^0\right)$

$\equiv \left(\sqrt{3} + \frac{r}{2}, \frac{\sqrt{3}r}{2}\right)$

Putting this on parabola

$y^2 = x + 2$

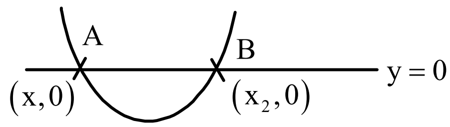
$\Rightarrow \left(\frac{\sqrt{3}r}{2}\right)^2 = \left(\sqrt{3} + \frac{r}{2}\right) + 2$

$\Rightarrow \frac{3}{4}r^2 - \frac{r}{2} - \left(\sqrt{3} + 2\right) = 0$

$PA.PB = |\text{Product of roots}| = \frac{\sqrt{3} + 2}{3/4}$

$= \frac{4\left(2 + \sqrt{3}\right)}{3}$

25. ()



Equation of circle passing through A and B can be given by

$(x-x_1)(x-x_2)+y^2+\lambda y=0$

Where x_1 and x_2 are roots of $ax^2+bx+c=0$

$\therefore$ Equation will be

$$x^2+y^2+\frac{b}{a}x+\lambda y+\frac{c}{a}=0$$

Length of tangent from origin $=\sqrt{\frac{c}{a}}$

26. $\because (ap^2,2ap),(aq^2,2aq),(ar^2,2ar)$ are

Co – normal point

$\therefore p+q+r=0$

$\Rightarrow x=1$ is a root of $px^2+qx+r=0$

Also $x=1$ satisfies

$a(b-c)x^2+b(c-a)x+c(a-b)=0$

$\therefore$ common root is $x=1$

27. ()

$$y=\frac{a^3}{3}\left(x^2+\frac{3}{2a}x\right)$$

$$\Rightarrow \frac{3y}{a^3}=\left(x+\frac{3}{4a}\right)^2-\frac{9}{16a^2}$$

$$\Rightarrow \left(\frac{3y}{a^3}+\frac{9}{16a^2}\right)=\left(x+\frac{3}{4a}\right)^2$$

Vertex $h=\frac{-3}{4a}$ $k=\frac{-3a}{16}$

$hk=\frac{9}{64}$ $\therefore$ locus is $xy=\frac{9}{64}$

28. ()

Equation of tangent at $(1,2)$

$$2y=2(x+1)\Rightarrow x-y+1=0$$

Image of a variable point $(t^2,2t)$ in the tangent

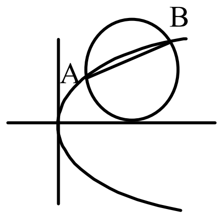
$$\frac{x-t^2}{1}=\frac{y-2t}{-1}=\frac{-\cancel{2}(t^2-2t+1)}{\cancel{2}}$$

$\Rightarrow x=2t-1$ & $y=t^2+1$

$\Rightarrow t=\left(\frac{x+1}{2}\right)$ & $t^2=(y-1)$

$$\left(\frac{x+1}{2}\right)^2 = (y-1) \Rightarrow (x+1)^2 = 4(y-1)$$ is the required image

29. ()



$\because$ circle touches $x-$axis

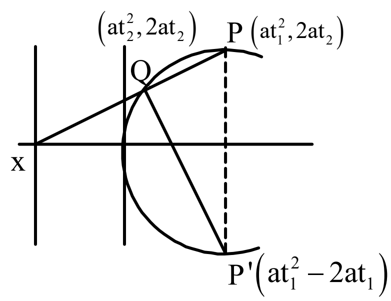
y coordinate of center = radius

$$\Rightarrow \frac{2at_1 + 2at_2}{2} = r$$

$$\Rightarrow t_1 + t_2 = \frac{r}{a}$$

Slope of $AB = \frac{2}{t_1 + t_2} = \frac{2a}{r}$

30. ()



Equation of PQ

$2x - y(t_1 + t_2) + 2at_1t_2 = 0$

$\because$ It passes through $(-a, 0)$

$\Rightarrow -2a + 0 + 2at_1t_2 = 0$

$\Rightarrow t_1t_2 = 1$ .... (I)

$\Rightarrow t_2 = \frac{1}{t_1}$

Equation of P'Q $\Rightarrow 2x - y(t_2 - t_1) - 2at_1t_2 = 0$

$$\Rightarrow 2x - y(t_2 - t_1) - 2a = 0$$

$$\Rightarrow 2(x-a) - (t_2 - t_1)y = 0$$

Which passes through $(a, 0)$ i.e. focus

31. Let mid point of PQ be (h, k)

Equation of chord PQ

$$xh + yk = h^2 + k^2$$

$$\Rightarrow y=\left(\frac{-h}{k}\right)x+\frac{h^2+k^2}{k}$$

For this to be tangent to parabola

$$c=a/m$$

$$\Rightarrow \frac{h^2+k^2}{k}=\frac{a}{(-h/k)} \Rightarrow h\left(h^2+k^2\right)+ak^2=0$$

$\therefore$ locus is

$$x\left(x^2+y^2\right)+ay^2=0$$

32. ()

Let mid point be $P(h,k)$

Equation of chord $\Rightarrow xh-2(y+k)=h^2-4k$

$$\Rightarrow x=\left(\frac{2}{h}\right)y+\frac{h^2-2k}{h} \qquad \ldots\ldots \text{(I)}$$

Equation of normal to $x^2=4y$

$$x=my-2m-m^3 \qquad \ldots\ldots\ldots \text{(II)}$$

Comparing (I) and (II)

$$\frac{h^2-2k}{h}=-2\left(\frac{2}{h}\right)-\left(\frac{2}{h}\right)^3$$

$$\Rightarrow h^2-2k=-4-\frac{8}{h^2} \Rightarrow 2k=h^2+\frac{8}{h^2}+4$$

$\therefore$ Locus is $2y=x^2+\frac{8}{x^2}+4$

33. ()

Equation of tangent to parabola $y^2=4ax$ is

$$yt=x+at^2 \qquad \ldots\ldots\ldots\ldots \text{(I)}$$

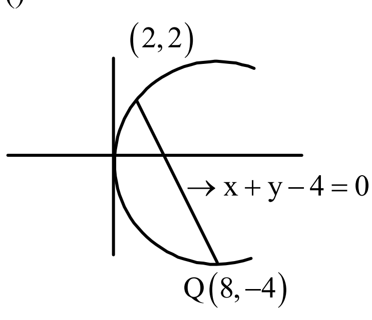
For (I) to be normal to circle $x^2+y^2-2ax-2by+c=0$ it should pass through its centre (a,b)

$$\therefore bt=a+at^2 \Rightarrow at^2-bt+a=0$$

For 2 distinct tangent $D>0$

$$\Rightarrow b^2-4a^2>0 \Rightarrow b^2>4a^2$$

34. ()



Point $\left(P^2, P-2\right)$ should lie inside the parabola as well as on the same side of the chord PQ as the origin

$(P-2)^2 - 2P^2 < 0 \Rightarrow -P^2 - 4P + 4 < 0$

$\Rightarrow P^2 + 4P - 4 > 0$

$\Rightarrow P \in \left(-\infty, -2-2\sqrt{2}\right) \cup \left(-2+2\sqrt{2}, \infty\right)$ ……….. (I)

Also, $P^2 + P - 2 - 4 < 0 \Rightarrow P^2 + P - 6 < 0$

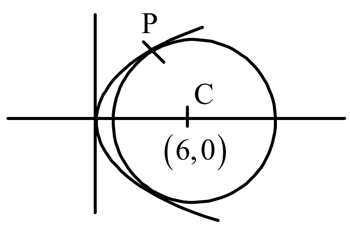
$\Rightarrow (P+3)(P-2) < 0$

$\Rightarrow -3 < P < 2$ …………… (II)

$(I) \cap (II)$

$\Rightarrow -2 + 2\sqrt{2} < P < 2$

35. ()



For having a common tangent circle should touch the parabola

$\therefore$ It centre well lie on normal to the parabola at $P\left(t^2 - 2t\right)$

Substituting $(6,0)$ in equation of normal at P, we get $t = 0, 2 \; \& -2$

Least distance $CP = \sqrt{20}$

$\therefore$ For no common tangent

$CP >$ radius

$\Rightarrow r < \sqrt{20}$

36. ()

Equation of tangent to $y^2 = 4ax$ with slope m

$y = mx + \dfrac{a}{m}$ ………. (I)

Equation of normal to $x^2 = 4by$ of slope m.

$y = mx + 2b + \dfrac{b}{m^2}$ ……. (II)

Comparing (I) and (II)

$\dfrac{a}{m} = 2b + \dfrac{b}{m^2}$

$\Rightarrow 2bm^2 - am + b = 0$

For 2 tangents $D > 0$

$\Rightarrow a^2 - 8b^2 > 0 \Rightarrow b^2 < \dfrac{a^2}{8}$

$\Rightarrow |b| < \dfrac{|a|}{2\sqrt{2}}$

37. ()

Let mid point of chord be $\left(at^2, 2at\right)$

Equation of chord

$x.at^2 + y.2at = a^2t^4 + 4a^2t^2$

$\because$ It passes through (a, a)

$\Rightarrow a^2t^2 + 2a^2t = a^2t^4 + 4a^2t^2$

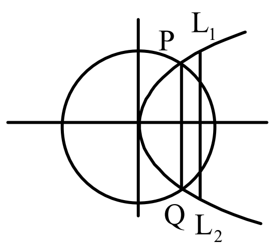
$\Rightarrow t\left(t^3 + 3t^2 - 2\right) = 0$

$t = 0$ and one real root from $t^3 + 3t^2 - 2 = 0$

$\therefore 2$ values of t.

$\therefore 2$ chords are possible

38. ()



$P \equiv \left(at^2, 2at\right)$

Equation of circle

$x^2 + y^2 = \frac{9a^2}{4}$

$\because$ P lies on circle also

$\Rightarrow \left(at^2\right)^2 + (2at)^2 = \frac{9}{4}a^2$

$\Rightarrow 4t^4 + 16t^2 - 9 = 0 \Rightarrow \left(2t^2 + a\right)\left(2t^2 - 1\right)$

$\Rightarrow t = \pm\frac{1}{\sqrt{2}}$

$PQ = 4at = 2\sqrt{2}a$

$L_1L_2 = 4a$

Distance between PQ and $L_1L_2 = \frac{a}{2}$

$\therefore \text{area} = \frac{1}{2} \times \frac{a}{2}\left(4a + 2\sqrt{2}a\right) = a^2\left(\frac{2+\sqrt{2}}{2}\right)$

39. ()

Equation of normal

$y + tx = 2at + at^3$

$A \equiv (\alpha, \beta)$ lies on it

$\Rightarrow \beta + t\alpha = 2at + at^3$

$\Rightarrow at^3 + (2a - \alpha)t - \beta = 0 \qquad \ldots\ldots\ldots \text{(I)}$

$at^3 + (2a - \alpha)t = \beta$

Squaring

$a^2t^6 + (2a-\alpha)^2 t^2 + 2a(2a-\alpha)t^4 = \beta^2$

Focal distance $= sp = r = a(1+t^2)$

$$\Rightarrow t^2 = \left(\frac{r-a}{a}\right)$$

$\therefore$ Equation becomes

$$a^2\left(\frac{r-a}{a}\right)^3 + (2a-\alpha)^2\left(\frac{r-a}{a}\right) + 2a(2a-\alpha)\left(\frac{r-a}{a}\right)^2 - \beta^2 = 0$$

SP. SQ. SR = (Product of roots)

$$= -\left(\frac{-a^2 - (2a-\alpha)^2 + 2a(2a-\alpha) - \beta^2}{\frac{1}{a}}\right)$$

$$= -a\left(-a^2 - 4\cancel{a^2} + 4a\alpha - \alpha^2 + 4\cancel{a^2} - 2a\alpha - \beta^2\right)$$

$$= -a\left(-a^2 + 2a\alpha - \alpha^2 - \beta^2\right)$$

$$= a\left(a^2 - 2a\alpha + \alpha^2 + \beta^2\right)$$

$$= a\left((\alpha - a)^2 + \beta^2\right)$$

$$= a(SA)^2$$

$$\therefore I = a$$

40. $P \equiv (h,k)$

Equation of chord of contact

$$yk = 2a(x+h)$$

$$\Rightarrow y = \left(\frac{2a}{k}\right)x + \left(\frac{2ah}{k}\right) \qquad \text{………..(I)}$$

Equation of tangent to $x^2 = 4by$

$$y = mx - bm^2 \quad \text{………..(II)}$$

From (I) and (II)

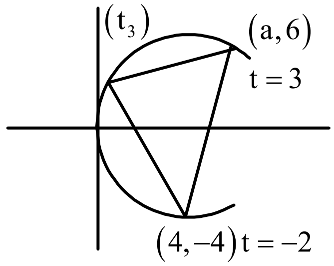
$$\frac{2ah}{k} = -\left(\frac{2a}{k}\right)^2$$

$\Rightarrow 2akh = -4a^2b$

$\Rightarrow$ locus is $xy = -2ab$

i.e. a hyperbola

41. ()



Substituting P and Q in the parabola we get $a=1, b=0$

Area of $\Delta = a^2\left|(t_1-t_2)(t_2-t_3)(t_3-t_1)\right|$

$= a^2\left|(t_3+2)(t_3-3)(5)\right|$

$= 5a^2\left|t^2-t-6\right|$

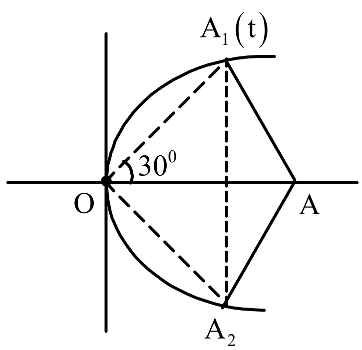
$= 5a^2\left|(t-1/2)^2-\dfrac{25}{4}\right|$

Area $= 5a^2\left(\dfrac{25}{4}-(t-1/2)^2\right)$

Area will maximum when $t=1/2$

$\therefore$ Pt is $\left(\dfrac{1}{4},\dfrac{1}{2}\right)$

42. ()



For OAA_1 to be equilateral

$$\frac{2}{t}=\tan 30^0=\frac{1}{\sqrt{3}} \Rightarrow t=2\sqrt{3}$$

Also, equation of normal $y+tx=2at+at^{-3}$

$\because$ A lies on it

$\Rightarrow ht=4t+2t^3$

$\Rightarrow h=4+2t^2$

$=4+2\times 12=28$