PART (A) : PHYSICS

ANSWER KEY

1.	(A)	2.	(B)	3.	(B)	4.	(B)	5.	(B)
6.	(A)	7.	(BD)	8.	(BD)	9.	(ABCD)	10.	(AB)
11.	(ABD)	12.	(ABCD)	13.	(16.00)	14.	(5.55)	15.	(12.00)
16.	(97.50)	17.	(6.00)	18.	(0.15)				

SOLUTION

1. (A)

The wavelength λ_{min} emitted by the X-ray tube operating at a voltage V is given by $eV = \frac{hc}{\lambda_{min}}$.

Kinetic energy of the electron $= \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{m^2v^2}{m}\right) = \frac{h^2}{2m\lambda_1^2}$

Now the wavelength λ of an electron moving with the velocity v is given by

$\lambda = h/P \Rightarrow \lambda = \frac{h}{\sqrt{2m\,K.E}}$

$\therefore K.E. = 600eV$

2. (B)

Path difference at angular position θ is d sin θ.

3. (B)

Let K_1 & K_2 and P_1 & P_2 are the K.E and momentum of the α-particle and remaining nucleus, then

$K_1 + K_2 = 5.5\text{ MeV}$ ...(i)

From conservation of linear momentum $P_1 = P_2$

$\Rightarrow \sqrt{2K_1 \times 4m} = \sqrt{2K_2 \times 216m} \Rightarrow k_1 = 54K_2$ ...(ii)

From (i) and (ii)

$K_1 = \frac{5.5 \times 54}{55} = 5.4\text{ MeV}$

4. (B)

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-20} = \frac{1}{-40} \Rightarrow v = +40mm$

$m = \frac{-v}{u} = -\left(\frac{+40}{-20}\right) = +2$

$\Rightarrow$ Separation between the two images formed $= \sqrt{(20)^2 + (1)^2} \approx 20.025mm$

5. (B)

$$\frac{1}{-200} - \frac{1}{\infty} = \frac{1}{f}$$ Lens formula

6. (A)

$$\mu = \tan\theta$$

7. (BD)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} = -8\,cm/s$$

$$m_T = -\frac{v}{u} = \frac{f}{f-u}$$

$$h_i = \frac{f}{f-u}h_0$$

$$\frac{dh_i}{dt} = \frac{fh_0}{(f-u)^2}\frac{du}{dt} = \frac{-20\times 0.2\times 2}{100} = \frac{-2}{25}\,cm/s$$

8. (BD)

Use the concepts of refraction and TIR.

9. (ABCD)

$Q = \Delta K$

And use basic concept

10. (AB)

$$F = -\frac{dU}{dr} = \frac{Ke^2}{r^4} \text{ --- (i)}$$

$$\frac{Ke^2}{r^4} = \frac{mV^2}{r} \text{ --- (ii)}$$

And $mVr = \frac{nh}{2\pi}$ ---- (iii)

By solving (i), (ii) and (iii)

Total energy $\propto n^6$

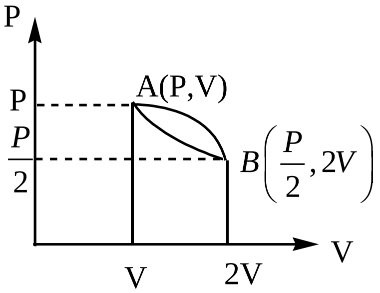
Total energy $\propto m^{-3}$

11. (ABD)

It is obbious that the gas does more work along the straight line as compared to that of the isothermal path.

Slope = –p/2V putting this in the ideal gas equation, V^2 = (constant) T which is the equation of parabola.

Similarly eliminating by V from ideal gas equation we get P^2 = (constant) T
which is equation of a parabola



12. (A, B, C D)

$y = 2a\cos(\omega t + \phi)\cos(Kx)$ comparison with given equation gives

$K = \frac{2\pi}{\lambda} = 10\pi \Rightarrow \lambda = \frac{1}{5}m$

$\omega = 2\pi f = 50\pi \Rightarrow f = 25Hz$

$\therefore v = 5ms^{-1}$

At x = 0.15m

$\cos(10\pi \times 0.15) = \cos(1.5\pi) = 0$

At x = 0.3

$\cos(10\pi \times 0.3) = \cos 3\pi = -1$ for all t

13. (16.00)

For refraction only

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ……..(i)

v = - 10cm

Differentiate equation (i) w.r.t. time

$-\frac{\mu_2}{v^2}\frac{dv}{dt} + \frac{\mu_1}{u^2}\frac{du}{dt} = 0$

$\frac{dv}{dt} = \frac{\mu_1}{\mu^2}\frac{v^2}{u^2}\frac{du}{dt} = \frac{4}{1\times 3} \times 1 \times 6 = 8m/s$ (towards left)

The velocity of image formed after refraction is 8m/s (towards left)
The velocity of image formed after reflection and then refraction = 8m/s (towards right)
The relative velocity between two images formed = 16m/s

14. (5.55)

$\beta = 10\log\frac{I}{I_0}$

$3 = \log\frac{I}{10^{-12}}$

$\Rightarrow I = 10^{-9} W/m^2$

Now, $I = \frac{(\Delta P_0)v}{2B} = \frac{(BAK)^2 v}{2B} = \frac{B\omega^2 A^2}{2v} = \frac{BA^2 4\pi^2 t^2}{2v}$

$A = \sqrt{\frac{Iv}{B2\pi^2 f^2}} = \sqrt{\frac{I}{\rho v^2 \pi^2 f^2}} = 5.55\overset{0}{A}$

15. (12.00)

$\lambda_A = \lambda_0 + v_s T = vT + \frac{v}{3}T = \frac{4}{3}\frac{v}{f}$

$\lambda_B = \frac{\frac{4}{3}\frac{v}{f}}{\frac{1}{4}} = \frac{16}{3}\frac{v}{f}$

16. (97.50)

$\sin i = \frac{4}{3}\sin r$

$\therefore r = 37^0$

Now $2\mu t \sec r = \lambda / 2$

$\Rightarrow t = \frac{\lambda}{4\mu \sec r} = 97.50\,\text{nm}$

17. (6.00)

1.5 hr = 3 $T_{Halflife}$

At beginning of 1.5 hr count rate = $(2)^3 \times 5 = 40\sec^{-1}$

$40\sec^{-1} = \frac{1}{9}\times 360 s^{-1} = \frac{1}{9}\times \text{initial rate}$

Intensity of radiation = $\frac{1}{9}$ $\times$ initial intensity at 2m

But intensity $\propto \frac{1}{d^2}$

So, new distance = 3 $\times$ initial distance = 3 $\times$ 2 = 6m

18. (0.15)

$\frac{1}{2}mv^2 = \left(\frac{hc}{\lambda} - \phi\right) \Rightarrow \upsilon = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi\right)}$

$r = \frac{m\upsilon}{qB} \Rightarrow r = \frac{m\sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi\right)}}{qB}$

Ans.: r = 0.15m

PART (B) : CHEMISTRY

ANSWER KEY

19.	(C)	20.	(D)	21.	(D)	22.	(D)	23.	(D)
24.	(D)	25.	(AD)	26.	(ABCD)	27.	(BC)	28.	(AB)
29.	(BCD)	30.	(BD)	31.	(4.00)	32.	(8.00)	33.	(0.00)
34.	(3.00)	35.	(5.00)	36.	(2.00)				

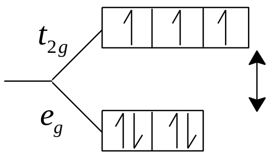
SOLUTIONS

19. (C)

20. (D)
As metallic character increases, basic strength of oxide increases
As non-metallic character increases, acidic strength of oxide increases

21. (D)
$ICl_4^- = sp^3d^2$
($s + p_x + p_y + p_z + d_{x^2-y^2} + d_{z^2}$ orbitals of iodine are involved in hybridisation)

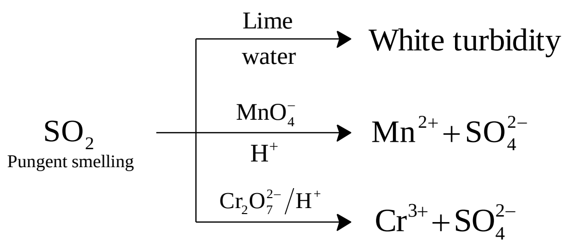
22. (D)

$Co^{II} = 3d^7 \ \Delta t$ 

because $[\overset{II}{Co}(SCN)_4]^2$ is a tetrahedral complex.

23. (D)
At 710°C
$CO_2 + C \rightleftharpoons 2CO; \quad \Delta G^o = 0$

24. (D)



25. (AD)
Test tube-1 contains CO_3^{2-} and HCO_3^-
Test tube-2 contains CO_3^{2-} or HCO_3^- or both.
Test tube-3 contains CO_3^{2-} or HCO_3^- or both.

26. (ABCD)
Plaster of Parisf $= CaSO_4 \cdot \frac{1}{2} H_2O$ or $2CaSO_4 \cdot Ca(OH)_2$

27. (BC)
$$Cl_2 \xrightarrow[\text{disproportionation}]{\text{Hot and Conc. NaOH}} Cl^- + ClO_3^-$$

28. (AB)
Cu, Ag and Au are the coinage metal.
Cu and Ag dissolve in HNO_3.

29. (BCD)
MnO_4^- have highest oxidation number of Mn = +7.

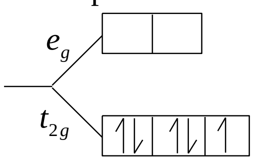
30. (BD)

	NO	$\overset{+}{NO}$
B.O. =	2.5	3.0
	paramagnetic	diamagnetic

31. (4.00)
Isoelectronic species are (i), (ii), (iv) and (v).

32. (8.00)
O=C=C=C=O
$\sigma = 4, \ \pi = 4$

33. (0.00)
In the given complex Cr is in +1 oxidation state

$Cr^+ = d^5$ 

(because $\mu = \sqrt{3}$ B.M.)

Zero electron in e_g orbital.

34. (3.00)
Ca, CaH_2 and Ba produce H_2 gas with cold water.

35. (5.00)

36. (2.00)
Cations belong to first group of basic radical produce insoluble chlorides.

PART (C) : MATHEMATICS

ANSWER KEY

37.	(B)	38.	(B)	39.	(B)	40.	(A)	41.	(D)
42.	(D)	43.	(ABC)	44.	(AB)	45.	(ABC)	46.	(BCD)
47.	(ABC)	48.	(AB)	49.	(2.83)	50.	(0.50)	51.	(9.00)
52.	(4.00)	53.	(5.00)	54.	(4.00)				

SOLUTIONS

37. (B)

$$\cos\theta = \frac{OA^2 + OB^2 - AB^2}{2OA.OB}$$

$$= \frac{OA^2 + OB^2 - \left(\frac{OA+OB}{2}\right)^2}{2OA.OB} = \frac{3}{8}\frac{\left(OA^2+OB^2\right)}{OA.OB} - \frac{1}{4}$$

For maximum $\cos\theta$,

$$\frac{3}{8}\left(\frac{OA^2+OB^2}{OA.OB}\right) - \frac{1}{2} \geq \frac{3}{8}\times\frac{2.OA.OB}{OA.OB} - \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

38. (B)

39. (B)

40. (A)

Let the circle be $x^2 + (y-\alpha)^2 = a^2$. Let the point of intersection of tangents at P and Q be (h, k)

Then equation of PQ, is $hx + (k-\alpha)(y-\alpha) - a^2 = 0$.

As it passes through (a, 0), so,

$ha - \alpha(k-\alpha) - a^2 = 0$

$\Rightarrow a^2 - k\alpha + a(h-a) = 0.D \geq 0 \Rightarrow k^2 - 4a(h-a) \geq 0$

i.e. $y^2 \geq 4a(x-a)$

41. (D)

The given expression can be rearranged as

$$\left[x - (1+\cos\theta)\right]^2 + \left[\frac{4}{x} - (1+\sin\theta)\right]^2$$

which is square of distance between point $\left(x, \frac{4}{x}\right)$ on hyperbola $xy = 4$ and point $(1+\cos\theta, 1+\sin\theta)$ on a circle centre (1, 1) and radius 1. Clearly the minimum distance occurs at (2, 2) on the hyperbola and $\left(1+\frac{1}{\sqrt{2}}, 1+\frac{1}{\sqrt{2}}\right)$ on the circle.

So, minimum distance $= 2\sqrt{\left(2-\left(1+\frac{1}{2}\right)\right)^2} = \sqrt{2}-1$

42. (D)

$$\frac{\sin(2-1)}{\sin 1 \sin 2} = \cot 1 - \cot 2$$

$$\frac{\sin(4-2)}{\sin 2 \sin 4} = \cot 2 - \cot 4$$

$$\frac{\sin(128-64)}{\sin 64 \sin 128} = \cot 64 - \cot 128$$

$$\therefore \frac{\cos 1}{\sin 1} - \frac{\cos 128}{\sin 128} = \sin 127^{o} \operatorname{cosec} 1^{o} \operatorname{cosec} 128^{o}$$

43. (ABC)

44. (AB)

$x^2 - y^2 = a^2$

Equation of normal at $P(\theta)$ is

$$\frac{ax}{\sec\theta} + \frac{ay}{\tan\theta} = 2a$$

$$\Rightarrow \frac{x}{\sec\theta} = \left(2 - \frac{y}{\tan\theta}\right)$$

Squaring & substituting $x^2 = a^2 + y^2$

We get

$$\sec^2\theta\left(2-\frac{y}{\tan\theta}\right) - y^2 = a^2 \text{ or } \left(\frac{\sec^2\theta}{\tan^2\theta}-1\right)y^2\left(\frac{4a\sec\theta}{\tan\theta}\right)y + a^2\left(4\sec^2\theta - 1\right) = 0$$

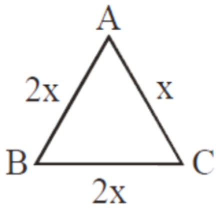
Hence $y_1y_2 = a^2\left(4\sec^2\theta - 1\right)\tan^2 2\tan\theta\tan\phi\, a^2$

$$\Rightarrow \tan\phi = \tan\theta\left(4\sec^2\theta - 1\right)$$

45. (ABC)

46. (BCD)

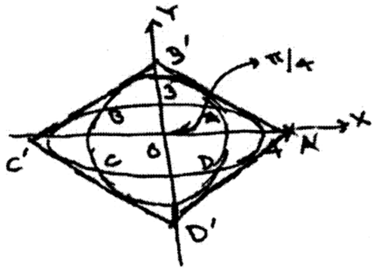
$(a-c)^2 + (a-2b)^2 = 0 \Rightarrow a = c = 2b$



$\therefore \quad \cos A = \dfrac{(2x)^2 + x^2 - (2x)^2}{2 \cdot 2x \cdot x} = \dfrac{1}{4}$

and $\cos B = \dfrac{4x^2 + 4x^2 - x^2}{2 \cdot 2x \cdot 2x} = \dfrac{7}{8}$

47. (ABC)



$OA = r \qquad A \equiv \left(r\cos\dfrac{\pi}{4}, r\sin\dfrac{\pi}{4} \right)$

A lies on $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1 \Rightarrow \dfrac{r^2}{16\times 2} + \dfrac{r^2}{9\times 2} = 1 \Rightarrow r = \dfrac{12}{5}\sqrt{2}$

Tangent : $y = mx \pm \sqrt{16m^2 + 9}$; where $m = \pm 1$

$y = \pm x \pm 5$ is also tangent to circle $\Rightarrow \dfrac{5}{\sqrt{2}} = r$

Circumcircle of $\Delta A'B'C'$ has centre O and radius $= OA' = 5$

Area (cir. of $\Delta A'B'C'$) $= \pi 5^2$

Area (circle C_1) $= \pi\left(\dfrac{5}{\sqrt{2}}\right)^2$

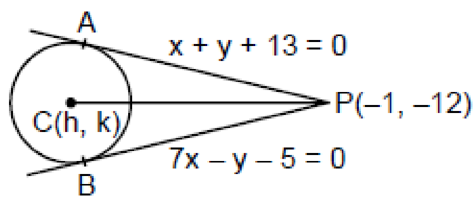
Ratio $= 1/2$

48. (AB)

Clearly, the point lies on $7x - y = 5$

Also, centre of the circle must lie on the bisector of the lines $x + y + +13 = 0$ and $7x - y - 5 = 0$ given by

$\dfrac{x+y+13}{\sqrt{2}} = \pm\dfrac{7x-y-5}{\sqrt{50}} \Rightarrow x - 3y = 35$ and $3x + y + 15 = 0$



Let (h, k) be the centre of the circle, then

$h - 3k = 35 \qquad .....(1)$

and $3h + k = 15 \qquad .....(2)$

Clearly CB is perpendicular $BP \Rightarrow \frac{k-2}{h-1}\times 7=-1 \Rightarrow h+7k-15=0$ .....(3)

On solving, we get centre as $C_1 \equiv (29-2)$ and $C_2 \equiv (-6,3)$

$\Rightarrow r_1^2 = 800$ and $r_1^2 = 50 \Rightarrow$ smaller circle has radius $=\sqrt{50}$

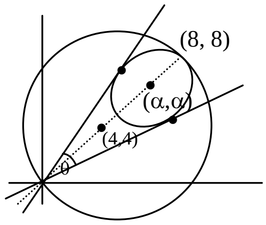
Therefore area of quadrilateral $ACBP = 2\left[\frac{1}{2}\times\sqrt{50}\times\sqrt{200}\right]$ sq. units

49. (2.83)

$\tan\theta = \frac{2\sqrt{81-49}}{14}$

$= \frac{4\sqrt{2}}{7}$

$\therefore \sin\frac{\theta}{2} = \frac{1}{3}$



radius = $\sqrt{2}(8-\alpha)$

$\sin\frac{\theta}{2} = \frac{\sqrt{2}(8-\alpha)}{\alpha\sqrt{2}}$ or, $\frac{1}{3}=\frac{8-\alpha}{\alpha}$

$\therefore \alpha = 6$

Radius = $2\sqrt{2}$

50. (0.50)

$(\sec(A-B)-1)(\sec(A-B)+1)=\sec^2(A-B)-1=\tan^2(A-B)=\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)^2$ ..(1)

$y=\cot B-\cot A=\frac{\tan A-\tan B}{\tan A\tan B}=\frac{x}{\tan A.\tan B}$

$\therefore \tan A\tan B=\frac{x}{y}$ .....(ii)

From (i) and (ii), we get

$(\sec(A-B)-1)(\sec(A-B)+1)=\left(\frac{x}{1+(x/y)}\right)^2=\frac{x^2y^2}{(x+y)^2}$

51. (9.00)

Let the centre be $(2\cos\theta, 3\sin\theta)$

$\frac{x_1+x_2+x_3+x_4}{4}=\frac{2\cos\theta}{2}$ and $\frac{y_1+y_2+y_3+y_4}{4}=\frac{3\sin\theta}{2}$

If (h, k) is the centroid of Δ then $x_4 = 4\cos\theta - 3h,\ _4 = 6\sin\theta - 3k$

Hence, $\left(h-\frac{4}{3}\cos\theta\right)(k-2\sin\theta)=\frac{2}{9}$

52. (4.00)

Director circle of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2+b^2$.

It intersects n$(x-1)^2-y^2+\frac{19}{2}=0$ at only two points.

Solving the two equations simultaneously $2x^2-2x+\frac{21}{2}-(a^2+b^2)=0$

$\Rightarrow\ 4-8\left(\frac{21}{2}-(a^2+b^2)\right)=0 \Rightarrow a^2+b^2=10 \Rightarrow a+b=4$

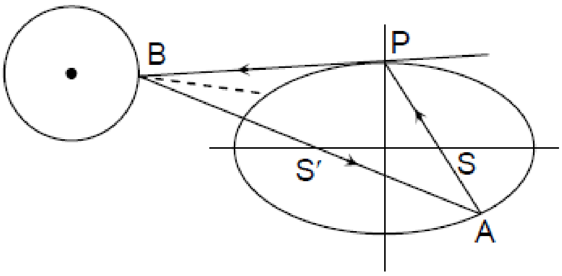
53. (5.00)

Using reflection property of ellipse equation of reflected ray SP is

$4x+3y=12$ ...(1)

Solving equation (1) with ellipse, we get

$A\equiv\left(\frac{75}{17},-\frac{32}{17}\right)$



Equation of reflected ray BS' is

$16x+63+48=0$ ...(2)

Equation of incident ray PB is

$y=4$ ...(3)

Equation of bisector of (2) and (3) with negative slope gives normal to circle at 3 which is $4x+32y-53=0$

Point $B\left(-\frac{75}{4},4\right)$, using parametric equation of line ordinate of centre is 5.

54. (4.00)

Let hyperbola be $xy=c^2$. Then points of intersection is $\left(ct,\frac{c}{t}\right)$

$\Rightarrow\ c^2t^4-2ct^3-20t^2-4ct+c^2=0$

If t_1,t_2,t_3 and t_4 are its roots

Then, $\sum t_1=\frac{2}{c};\sum t_1t_2=-\frac{20}{c^2};\sum t_1t_2t_3=\frac{4}{c}$ and $t_1t_2t_3t_4=1$

$\Rightarrow \ell=2, m=44, n=56$