PART (A) : PHYSICS

ANSWER KEY

1.	(2)	2.	(5)	3.	(6)	4.	(4)	5.	(5)
6.	(2)	7.	(AC)	8.	(ABC)	9.	(ACD)	10.	(ABCD)
11.	(AC)	12.	(ABCD)	13.	(0.23)	14.	(1.50)	15.	(700.00)
16.	(0.39)	17.	(0.75)	18.	(5)				

SOLUTIONS

1. (2)

$$\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$$

For N_2 to be maximum,

$$\frac{dN_2}{dt} = 0 \Rightarrow \lambda N_1 = 2\lambda N_2 \text{ or } \frac{N_1}{N_2} = 2$$

2. (5)

$$\mu \sin\theta = 1 \times \sin 90^0$$

$$\sin\theta = \frac{1}{\mu}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{\mu^2 - 1}} = \frac{1}{\sqrt{1 + e^{x/d} - 1}} = e^{-x/2d}$$

$$\frac{dy}{dx} = e^{-x/2d}$$

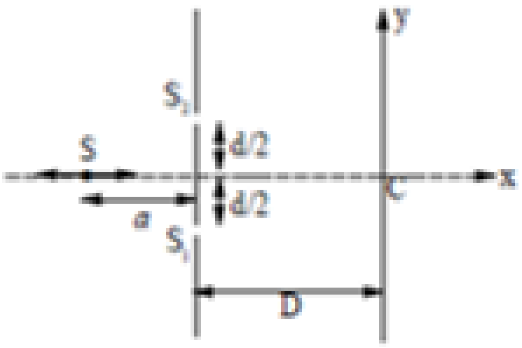
$$\Rightarrow y = 2d(1 - e^{-x/2d})$$

At B $y = d$

$$\Rightarrow x = 2d\ell n2 = \frac{50}{7} \times 0.7 = 5m$$

3. (6)

$$I = \frac{P}{4\pi r^2}$$



$$I = \frac{P}{4\pi\left(a^2 + \left(\frac{d}{2}\right)^2\right)} \Rightarrow I_{max} = 4I$$

Differentiate to find answer

4. (4)

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK_{max}}}{qB}$$

$$\Rightarrow K_{max} = \frac{q^2B^2r^2}{2m}$$

Purring the value of q, B, r and m

$$K_{max} = 2eV$$

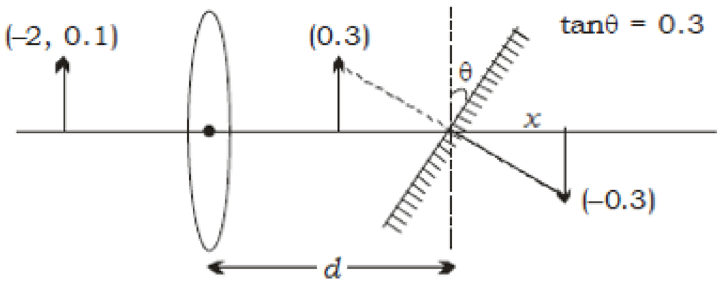
Energy of emitted photon

$$h\nu = 13.6 \times 3^2\left(\frac{1}{9} - \frac{1}{16}\right)eV = 5.95eV$$

$$h\nu = \phi + k_{max}$$

$$\Rightarrow \phi = 3.95eV \approx 4eV$$

5. (5)



$$\frac{1}{u} - \frac{1}{\upsilon} = \frac{1}{f} \Rightarrow \frac{1}{u} + \frac{1}{2} = \frac{\cancel{10}}{\cancel{15}}\frac{2}{3}$$

$$\Rightarrow \frac{1}{u} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \Rightarrow u = 6m$$

$$m = \frac{u}{\upsilon} = -3$$

$$\tan\theta = \frac{0.3}{x} \Rightarrow 0.3 = \frac{0.3}{x} \Rightarrow x = 1$$

$$x = 6 - d \Rightarrow 6 - d = 1$$

Ans.: d = 5m

6. (2)

For refraction at first surface of the lens

$$\frac{1.5}{-60} - \frac{4/3}{u} = \frac{1.5 - \frac{4}{3}}{60}$$

$\Rightarrow u = -48cm$

If the object is placed h cm above the water surface, after refraction at the water surface its image will be formed at distance $\frac{4h}{3}$ from the surface

$\Rightarrow \frac{4h}{3} + 30 = 48 \Rightarrow h \ \frac{27}{2} cm$

7. (AC)

If the friction offered by air is neglected, the speed of the bullet on returning to be staring point will be equal to its initial speed $v = 200 \ ms^{-1}$.

The kinetic energy of the bullet is

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\left(50 \times 10^{-3}\right) \times \left(200\right)^2 = 1000 \text{ J}$$

Heat lost by bullet for its temperature to fall from 20°C to

$0^o \text{ C} = \left(50 \times 10^{-3}\right) \times 800 \times 20 = 800 \text{ J}$.

If x kg is the mass of ice melted, then

$\Rightarrow \quad x = 5.3 \times 10^{-3} \text{ kg} = 5.3 \text{ g}$.

Hence, the correct choices are (A) and (C).

8. (ABC)

$W_{A \to B}$ = Area of *ABED*

$$= \frac{1}{2} BC \times AC + CD \times DE = \frac{1}{2}\left(6 \times 10^{-3} \text{ m}^3\right) \times 4 \text{ Nm}^{-2} \times 6 \times 10^{-3} \text{ m}^3$$

$$= 0.0012 + 0.024 = 0.036 \text{ J}$$

$W_{B \to C}$ = Area of *BCDE* = –0.024 J.

The negative sign shows that the work is done on the gas.

$W_{C \to A} = P\Delta V = 0$ because $\Delta V = 0$.

9. (ACD)

10. (ABCD)

For the first interference maximum

$$2d \sin\theta = \frac{\lambda}{2}$$

$\Rightarrow \sin\theta = \frac{\lambda}{4d} = \frac{10^{-3}}{4}$

$\frac{y}{D} = \frac{10^{-3}}{4}$

$y = \frac{1}{4}\text{mm}$

Fringe width, $\omega = \frac{\lambda D}{2d} = 0.5\text{mm}$

$2d\sin\theta = \frac{\lambda}{4}$

$\Rightarrow \sin\theta = \frac{\lambda}{8d}$

$\therefore y = \frac{\lambda}{8d} = \frac{1}{8}\text{mm}$

11. (AC)

$-\frac{1}{F} = P = 2P_{l_1} + 2P_{l_2} + P_m$ .....(1)

$P_{l_1} = \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{l_1} = [(1.5 - 1)]\left[-\frac{1}{10} - \frac{1}{15}\right] = -\frac{1}{12}$ ......(2)

$P_{l_2} = \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$P_{l_2} = \left(\frac{4}{3} - 1\right)\left[\frac{2}{15}\right] = \frac{2}{45}$ ......(3)

$P_m = -\frac{1}{f} = +\frac{2}{15}$ ....(4)

$-\frac{1}{F} = P = 2\left[-\frac{1}{12} + \frac{2}{45}\right] + \frac{2}{15} = -\frac{1}{6} + \frac{4}{45} + \frac{2}{15} = \frac{1}{18}$

F = –18 cm. Focus is negative means system will behave as concave mirror.

12. (ABCD)

For maxima path difference = $n\lambda$

If d = path difference between waves reaching point O = 7λ

O will be maxima

For $d = \lambda$ only one maxima at O is possible, the screen being finite

13. (0.23)

$\mu = \frac{\text{velocity in air}}{\text{velocity in medium}}$

14. (1.50)

$$v = v_0 \frac{z^2}{n^3} \Rightarrow \frac{z^2}{n^3} = 2$$

$$E = E_0 \frac{z^2}{n^2} \Rightarrow \frac{z^2}{n^2} = 2$$

$$L = mvr = \frac{nh}{2\pi}, \Delta L = \tau \Delta t = \Delta n \frac{n}{2\pi}$$

$$\tau = \frac{\Delta n}{\Delta t} \frac{h}{2\pi} = 1.5 \times 10^{-26} \text{Nm}$$

15. (700.00)

Let t be the time required to raise to potential by 2V. Then number of β -particles emitted in t second is 5×10^{10} t. Now the number of β -particles escaping from sphere is 40% i.e., 2×10^{10} t. So, charge developed.

$$Q = \left(2\times10^{10}t\right)\left(1.6\times10^{-19}\right)\text{Coulomb} = \left(3.2\times10^{-9}t\right) \text{ Coulomb}$$

But $Q = \left(4\pi\varepsilon_0 R\right)V = \dfrac{10^{-2}\times2}{9\times10^9}$ Coulomb

$$\therefore \quad \frac{10^{-2}\times2}{9\times10^9} = 3.2\times10^9 t \text{ or } t = 700\times10^{-6} \text{sec}.$$

$$t = 700 \text{sec}.$$

16. (0.39)

Applying Snell's law between the points O and P, we have

$$2 \times \sin 60^0 = (\sin 90^0) \times \frac{2}{(1+H^2)}, \ 2 \times \frac{\sqrt{3}}{2} = 1 \times \frac{2}{(1+H^2)}$$

$$(1+H^2) = \frac{2}{\sqrt{3}}, \ H = \sqrt{\left(\frac{2}{3} - 1\right)}$$

17. (0.75)

$$hv = 13.6\ (3)^2 \left[\frac{1}{4^2} - \frac{1}{5^2}\right] = 2.75 \text{ eV}$$

for $n = 4$ to $n = 3$

$$hv = (13.6) \times (3)^2 \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 5.95 \text{ eV}$$

for shorter wavelength

$3.95 = 5.95 - \phi \qquad \Rightarrow \phi = 2\text{eV}$

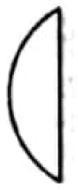
for longer wavelength $eV_S = 2.75 - 2 = 0.75$ eV

18. (5)

$$\frac{1}{f}=\frac{2\mu}{\infty}-\frac{2(\mu-1)}{+R}$$

$$=-\frac{2(1.5-1)}{10}$$

$$\therefore\ f=-10\,\text{cm}$$



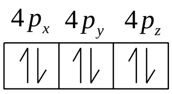
PART (B) : CHEMISTRY

ANSWER KEY

19.	(5)	20.	(6)	21.	(5)	22.	(3)	23.	(2)
24.	(4)	25.	(CD)	26.	(BD)	27.	(ABC)	28.	(ABD)
29.	(AB)	30.	(AB)	31.	(8.00)	32.	(8.00)	33.	(6.00)
34.	(6.00)	35.	(7.00)	36.	(3.00)				

SOLUTIONS

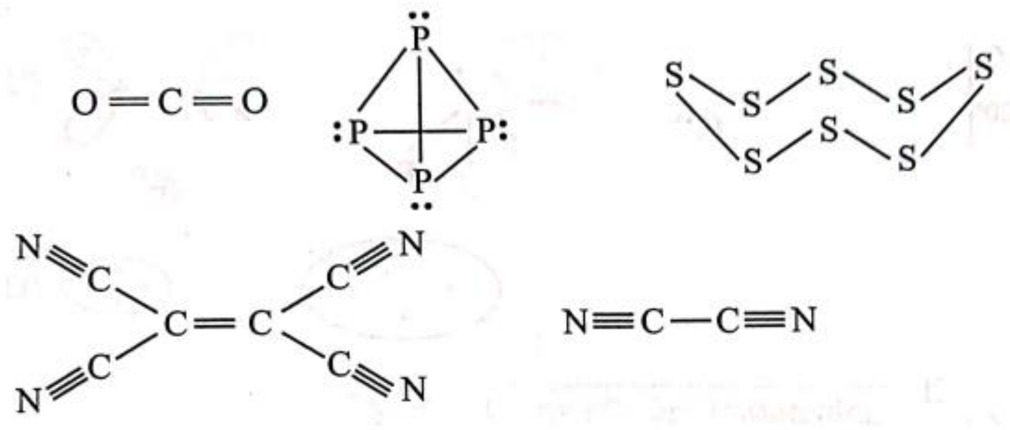
19. (5)



20. (6)

Total six planes contains HCH atoms in a plane.

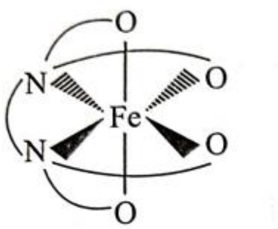
21. (5)



22. (3)

[*Mabcd*] type square planar complex has three G.I..

23. (2)



(Optically active complex)

24. (4)

$2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

$N_2O_4 + H_2O \rightarrow HNO_3 + HNO_2$

$N_2O_5 + H_2O \rightarrow 2HNO_3$

$N_2O_3 + H_2O \rightarrow 2HNO_3$ (Produce HNO_3 on disproportionation)

25. (CD)

(C) Cr_2O_3, BeO, SnO, SnO_2 all are amphoteric oxides

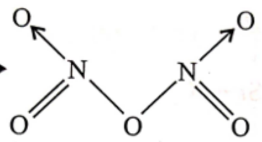
(D) ZnO, Al_2O_3, PbO, PbO_2 all are amphoteric oxides

26. (BD)

$$P_4O_{10} + 4HNO_3 \xrightarrow{\text{Dehydration of } HNO_3} 4(HPO_3) + \underset{\text{(required product)}}{2N_2O_5}$$

(A) $Pl4 + 20HNO_3 \longrightarrow 4H_3PO_4 + 20NO_2 \uparrow + 4H_2O$

(B) N_2O_5 is diamagnetic in nature

(C) $N_2O_5 \longrightarrow$ 

27. (ABC)

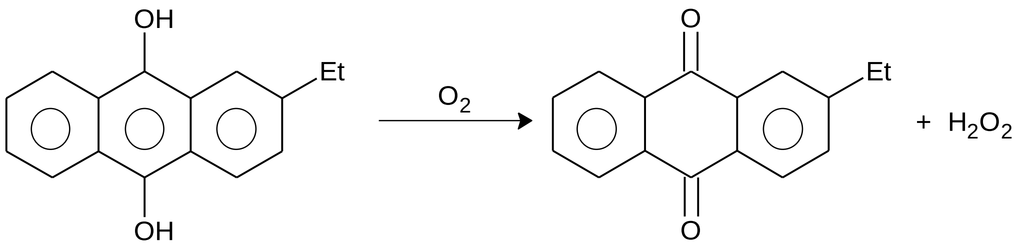
If ammonia considered as a liquid then reaction will be

(A) $M + (x+y)NH_3 \longrightarrow [M(NH_3)_x]^+ + \underset{\substack{\text{Ammoniated } e^- \text{ (Paramagnetic)} \\ \text{S.R.A}}}{[e(NH_3)y]^-}$

(B) $K + \underset{\text{(excess)}}{O_2} \xrightarrow{KO_2} (K^+, O_2^-)$ $[O_2^- = \text{Paramagnetic}]$

(C) $3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$ [NO = Paramagnetic]

[All diamagnetic]

(D) 

28. (ABD)

$$\overset{II}{[Co}(H_2O)]Cl_2 \xrightarrow[\text{Air}/O_2]{\text{Excess } NH_4OH/NH_4Cl} \underset{(Y)}{\overset{III}{[Co}(NH_3)_6]Cl_3}$$

Pink (X)

$$\underset{(X)}{[Co(H_2O)_6]^{2+}} + \underset{\text{(excess)}}{4Cl^-} \longrightarrow \underset{(X)\text{ blue colour}}{[CoCl_4]^{2-}}$$

(A) Hybridisation of (Y) is d^2sp^3 as NH_3 is strong field ligand

(B) $[CoCl_4]^{2-}$ have sp^3 hybridisation as Cl^- is weak field ligand

(C) $\overset{III}{[Co(NH_3)_6]Cl_3} + 3AgNO_3(aq.) \longrightarrow 3AgCl$
(Y)

(D) $[CoCl_4]^{2-} + 6H_2O \rightleftharpoons [Co(H_2O)_6]^{2+} + 4Cl^-$ $\Delta H = (-)ve$ (exothermic)
(Blue) (Pink)

When ice is added to the solution the equilibrium shifts right hence pink colour will remains predominant.
So, correct answer is (A, B, D).

29. (AB)
$$ZnS \xrightarrow{O_2} ZnO \xrightarrow{C} Zn \xrightarrow[\text{Refining}]{\text{Electrolytic}} Zn$$
$$PbS \xrightarrow{O_2} PbO \xrightarrow{C} Pb \xrightarrow[\text{Refining}]{\text{Electrolytic}} \underset{\text{(Pure)}}{Pb}$$

30. (AB)
Cr^{3+}, d^3 configuration shows same number of paired e^- and EAN but strength of all ligands are different so do not show same colour and splitting.

31. (8.00)
Ti^{2+}, V^{2+}, Sc^{2+}, Cr^{2+}, Mn^{2+}, Co^{2+}, Ni^{2+}, Cu^{2+}
Answer = 8

32. (8.00)
All $H_t - B - H_b$ bond angles are identical (t = terminal, b = bridging)

33. (6.00)

34. (6.00)

35. (7.00)

36. (3.00)
O.N. of Cr in X ($FeO \cdot Cr_2O_3$) = +3
O.N. of Cr in Z ($Na_2Cr_2O_7$) = +6

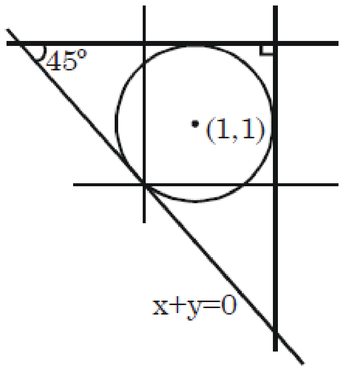
PART (C) : MATHEMATICS

ANSWER KEY

37.	(0)	38.	(5)	39.	(8)	40.	(5)	41.	(6)
42.	(1)	43.	(CD)	44.	(AD)	45.	(ABD)	46.	(ABC)
47.	(AD)	48.	(ABCD)	49.	(4.00)	50.	(9.00)		
51.	(16.66 or 16.67)			52.	(8.00)	53.	(5.00)	54.	(3)

SOLUTIONS

37. (0)



$$\left(h^2+k^2-4\right)^2=\left(h^2+k^2-4h\right)\left(h^2+k^2-4k\right)+16$$

$\Rightarrow\ (h+k)\left(h^2+k^2-2h-2k\right)=0$

Circle $C_1 : x^2+y^2-2x-2y=0$ and $C_2 = x+y=0$

Circumcentre will be mid point of hypotenuse in right angle triangle (α, β) = (0, 0)

38. (5)

$\because\ P=(1,1)$ and $Q=(3,5)$

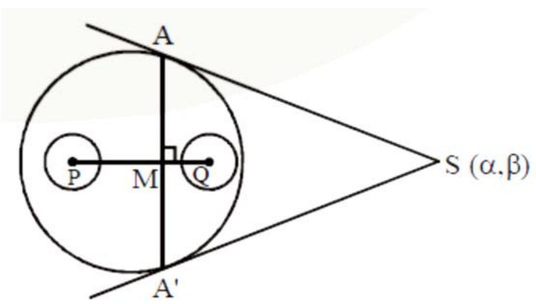
$\because$ Minimum value of $|AP-AQ|$ is zero

$\therefore$ A should be on perpendicular bisector of PQ

Min point of PQ is (2, 3) slope of PQ is 2

$\therefore$ Equation of perpendicular bisector of PQ (i.e. AA') is

$$y-3=-\frac{1}{2}(x-2)$$



$\Rightarrow\ 2y-6=-x+2$

$\Rightarrow \quad x+2y-8=0$ …… (1)

If S is (α, β) then equation of AA'

As C.O.C. will be $\alpha x+\beta y=100$ …… (2)

$\because$ (1) and (2) represent same line

$\therefore \quad \frac{\alpha}{1}=\frac{\beta}{2}=\frac{100}{8}$

$\therefore \quad \alpha=\frac{25}{2}$ and $\beta=25$

$\therefore \quad 2\alpha+\beta=50$

$\Rightarrow \quad \frac{2\alpha+\beta}{10}=5$

39. (8)

Let $\angle ABC=2\theta,\ \angle CBO=\theta,\ OC=2\sin\theta,\ C\equiv(2\sin\theta, 0)$.

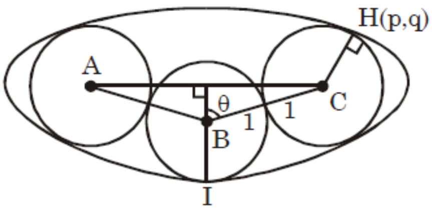
Let $H\equiv(p, q)$

$\therefore \quad \frac{p^2}{a^2}+\frac{q^2}{b^2}=1$ ...(i)

and $(2\sin\theta-p)^2+q^2=1$ ..(ii)

Since slope of CH must be same as slope of normal of the ellipse at H.

$\Rightarrow \quad \frac{q}{p-2\sin\theta}=\frac{a^2q}{b^2p}$ .....(iii)



From (i), (ii) and (iii) and using

$\sin^2\theta+\cos^2\theta=1$

$\Rightarrow \quad a=b\sqrt{\frac{2}{b-1}}$, area $=\pi ab$, $A=\pi.b^2\sqrt{\frac{2}{b-1}}$

$$\frac{dA}{db}=\sqrt{2}\pi\frac{\left(\sqrt{b-1}\right)(2b)-b^2\left(\frac{1}{2\sqrt{b-1}}\right)\cdot 1}{\left(\sqrt{b-1}\right)^2}$$

$\Rightarrow \frac{dA}{db}=0 \Rightarrow (b-1)2b=\frac{b^2}{2}$

$\Rightarrow b\neq 0$,

$\Rightarrow 2b-2=\frac{b}{2}$ & $a=\frac{4}{3}\sqrt{\frac{2}{\frac{4}{3}-1}}$

$\Rightarrow 4b-4=b$

$b=\frac{4}{3}$ $\quad\therefore\ a=\frac{4}{3}\sqrt{6}$

Now $b=2\cos\theta+1$

$\Rightarrow \cos\theta = \frac{1}{6}$

$\Rightarrow \tan\theta = \sqrt{35}$

$\Rightarrow \angle AOB = 2\theta = 2\tan^{-1}\sqrt{35}$

40. (5)

Common ratio of G.P. is $-\frac{2}{\pi}\tan^{-1}x$ whose modulus is less than 1.

$\therefore f(x) = \dfrac{\tan^{-1}x}{1+\frac{2}{\pi}\tan^{-1}x}$

Now, domain of $(f(x))^2 + (\sin^{-1}x)^2 = a$ is $x \in [-1, 1]$

Now, $f(x) = \frac{\pi}{2}\left(1 - \dfrac{1}{1+\frac{2}{\pi}\tan^{-1}x}\right)$

$\therefore f(x)\big|_{min} = -\frac{\pi}{2}$ at $x = -1$ and $f(x)\big|_{max} = \frac{\pi}{6}$ at $x = 1$

$\therefore f^2(x) \in \left[0, \frac{\pi^2}{4}\right]$ and $(\sin^{-1}x)^2 \in \left[0, \frac{\pi^2}{4}\right]$

$\therefore$ minimum $f^2(x) + (\sin^{-1}x)^2$ is 0 and maximum is $\frac{\pi^2}{2}$

$\therefore a \in \left[0, \frac{\pi^2}{2}\right]$

$\therefore$ integral values are 0, 1, 2, 3, 4, i.e. 5.

41. (6)

If $xy < 1$ then equation has no solution

If $xy = 1$, then equation has no solution

If $xy > 1 \Rightarrow x^2y^2 > 1$

$\Rightarrow \pi + \tan^{-1}\left(\dfrac{x^2+y^2}{1-x^2y^2}\right) = [xy]\frac{\pi}{4}$

$= \tan^{-1}\left(\dfrac{x^2+y^2}{(xy)^2-1}\right) = \pi\left(1 - \frac{[xy]}{4}\right)$ is possible, when $3 \le xy < 4$

$\Rightarrow \dfrac{x^2+y^2}{x^2y^2-1} = 1 \Rightarrow x^2+y^2 = x^2y^2 - 1$

$\Rightarrow 9 \le x^2+y^2+1 < 16$

$\therefore x^2+y^2 \in [8, 15)$

42. (1)

Let $\alpha+\beta=2\gamma$

Where $\tan\alpha=\frac{2x+1}{x+1}$, $\tan\beta=\frac{2x-1}{x-1}$ and $\tan\gamma=x+1$

Taking tan in Eq. (i), we get

$$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{2\tan\gamma}{1-\tan^2\gamma}$$

$$\Rightarrow \frac{\frac{2x+1}{x+1}+\frac{2x-1}{x-1}}{1-\frac{4x^2-1}{x^2-1}}=\frac{2(x+1)}{1-(x+1)^2}$$

$$\Rightarrow \frac{2x^2-x-12x^2+x-1}{x^2-1-4x^2+1}=\frac{2(x+1)}{-x^2-2x}$$

$$\Rightarrow \frac{2(2x^2-1)}{-3x^2}=\frac{2(x+1)}{-(x^2+2x)}$$

$x=0$

or $2x^3+4x^2-x-2=3x^2+3x$

$\Rightarrow 2x^3+x^2-4x-2=0$

$\Rightarrow x^2(2x+1)-2(2x+1)=0$

$\Rightarrow (x^2-2)(2x+1)=0$

$\Rightarrow x=\sqrt{2},-\sqrt{2}$ or $x=-\frac{1}{2}$

$x=-\sqrt{2}$ is rejected

$\because$ It does not satisfy (i)

$\Rightarrow x_1=-\frac{1}{2}, x_2=0$ and $x_3=\sqrt{2}$

$\Rightarrow 2x_1+x_2+x_3^2=1$

43. (CD)

$$\frac{(\sin 36^\circ+\cos 36^\circ)^2-2\sin^2 27^\circ}{2\sin 54^\circ}$$

$$=\frac{\cos 54^\circ+\cos 18^\circ}{2\sin 54^\circ}=\frac{2\cos 36^\circ\cos 18^\circ}{2\cos 36^\circ}$$

$=\cos 18^\circ<\cos 9^\circ,\ \cos 15^\circ$

44. (AD)

$$\cos\frac{\pi}{9}\cdot\cos\frac{4\pi}{9}\cdot\cos\frac{7\pi}{9}$$

$$= \cos\frac{\pi}{9}\left(-\cos\frac{8\pi}{9}\right)\cdot\left(\cos\frac{4\pi}{9}\right)\left(-\cos\frac{2\pi}{9}\right)$$

$$= \cos\frac{\pi}{9}\cdot\cos\frac{2\pi}{9}\cdot\cos\frac{4\pi}{9}\cdot\cos\frac{8\pi}{9}$$

$$= \frac{\sin\frac{16\pi}{9}}{16\sin\frac{\pi}{9}} = \frac{-\sin\frac{7\pi}{9}}{16\sin\frac{\pi}{9}}$$

$$= \frac{-\sin\left(\frac{\pi}{9}+\frac{2\pi}{3}\right)}{16\sin\frac{\pi}{9}}$$

$$= \frac{-\left(\sin\frac{\pi}{9}\cdot\left(-\frac{1}{2}\right)+\cos\frac{\pi}{9}\cdot\frac{\sqrt{3}}{2}\right)}{16\sin\frac{\pi}{9}}$$

$$= \frac{1}{32}\left(1-\sqrt{3}\cot\frac{\pi}{9}\right)$$

$b=1, a=3$

45. (ABD)

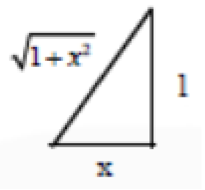
(A) $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$ $\cos\left[\cos^{-1}x+\cos^{-1}x+\sin^{-1}x\right]$

$$\cos\left[\cos^{-1}x+\frac{\pi}{2}\right] = -\sin\cos^{-1}x = -\sin\sin^{-1}\sqrt{1-x^2}$$

$-\sqrt{1-x^2} = -\frac{\sqrt{24}}{5}$, Option (A) is correct.

(B) $\cos\left(\tan^{-1}\left(\sin\left(\cos^{-1}\right)\right)\right)$

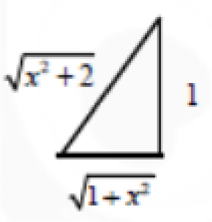
$\sin\left(\cos^{-1}\right)x$ $\quad \cos^{-1}x = A, \cot A = x$



$\sin\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$ $\quad \sin A = \frac{1}{\sqrt{1+x^2}}$

$\cos\left(\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$ $\quad \tan^{-1}\frac{1}{\sqrt{1+x^2}} = A$

$\tan A = \frac{1}{\sqrt{1+x^2}}$



$\cos\cos^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)$

$\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}$ $\therefore$ (B) is correct.

(C) Principle value of $\sin^{-1}\left(\sin\frac{2\pi}{2}\right)=\frac{2\pi}{3}$ is correct option

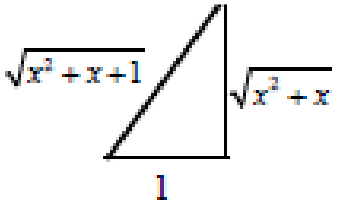
$\because$ $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

(D) $\tan^{-1}\sqrt{x^2+x+1}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

$\sin^{-1}\sqrt{x^2+x+1}+\cos^{-1}\frac{1}{\sqrt{x^2+x1}}=\frac{\pi}{2}$

$\tan^{-1}\sqrt{x^2+x}=A$, $\tan A=\sqrt{x^2+x}$ it is true.

If $\sqrt{x^2+x+1}=\frac{1}{\sqrt{x^2+x+1}}$



$x^2+x+1=1$, $x^2+x=0$, $x=0,-1$

(D) is correct option.

46. (ABC)

$f(x)=\frac{\cos 9x+\cos 6x}{2\cos 5x-1}=\frac{2\cos\frac{15}{2}x\cos\frac{3x}{2}}{1-4\sin^2\frac{5}{2}x}$

$=\frac{2\cos\frac{3x}{2}\cos\frac{5x}{2}\left(1-4\sin^2\frac{5}{2}x\right)}{\left(1-4\sin^2\frac{5}{2}x\right)}$

$=\cos 4x+\cos x=2\cos\frac{3x}{2}\cos\frac{5x}{2}$

$\Rightarrow$ $f(x)\le 2$

$f(x)=1$

$\Rightarrow \quad \cos x = 1 - \cos 4x = 2\sin^2 2x$

$\cos x = 8\sin^1 x \cos^2 x$

$\cos x = 0$ or $8\cos^3 x - 8\cos x + 1 = 0$

$\cos x \in [-1, 1]$.

47. (AD)

$AP = \sqrt{(1-\cos\alpha)^2 + \sin^2\alpha}$

$= 2\left|\sin\frac{\alpha}{2}\right| = 2\sin\frac{\alpha}{2}$

Similarly $AQ = 2\sin\frac{\beta}{2}$ and $AR = 2\sin\frac{\gamma}{2}$

Now as AP, AQ, AR are in GP.

$\therefore \sin\frac{\alpha}{2}, \sin\frac{\beta}{2}, \sin\frac{\gamma}{2}$ are in GP.

$\Rightarrow \quad \frac{\sin\frac{\alpha}{2} + \sin\frac{\gamma}{2}}{2} \geq \sin\frac{\beta}{2}$

$\Rightarrow \quad \sin\frac{\alpha+\gamma}{2}\cos\frac{\alpha-\gamma}{2} \geq \sin\frac{\beta}{2}$

Also, $\sin\frac{\alpha}{2}\sin\frac{\gamma}{2} \leq \sin\frac{\beta}{2}$

48. (ABCD)

Point of intersection of P_1 and L is given by $mx^2 + \ell x + n = 0$

Line is tangent if $\ell^2 = 4mn \Rightarrow m, \frac{\ell}{2}, n$ are in G.P.

If point of intersection is rational (let $x = \frac{p}{q}$) where p and q are co-prime

Then $mp^2 + lpq + nq^2 = 0 \qquad \ldots(1)$

Now, if one of p and q is even and other is odd then (1) can not hold as sum of an even and an odd integer can't be zero.

If p, q are odd then (1) can not hold true as sum of three odd numbers can't be zero

Common tangent to P_1 and P_2 is $2x - y - 1 = 0$

Common chord of P_1 and P_2 is $2x + y = 0$

49. (4.00)

Perimeter of $\Delta DEF = a\cos A + b\cos B + c\cos C$

$= R[\sin 2A + \sin 2B + \sin 2C]$

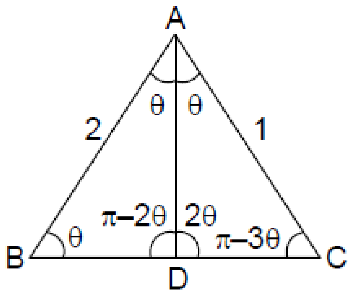
$= R[4\sin A + \sin B + \sin C]$

$= 4R\dfrac{abc}{8R^3} = \dfrac{abc}{2R^2} = \dfrac{2\Delta}{R} = 4\,\text{cm}$.

50. (9.00)

Since $\dfrac{AB}{AC} = \dfrac{BD}{DC} = \dfrac{2}{1}$

Using since law in ΔABC we get



$\dfrac{1}{\sin\theta} = \dfrac{2}{\sin(\pi-3\theta)} = \dfrac{2}{\sin 3\theta}$

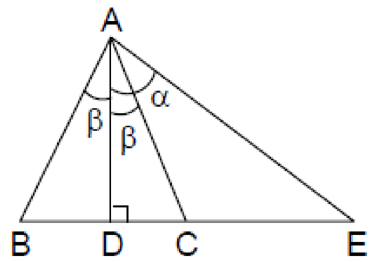
$3 - 4\sin^2\theta = 2$

$\Rightarrow \sin^2\theta = \dfrac{1}{4} \Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30^\circ \Rightarrow 2\theta = 60^\circ$

Area of $\Delta ABC = \dfrac{1}{2}\times 2\times 1\times \sin 60^\circ = \dfrac{\sqrt{3}}{2}$

Hence $12\Delta^2 = 12\times\dfrac{3}{4} = 9$

51. (16.66 or 16.67)



$\angle EAC = \alpha - \beta$

$\angle EAB = \alpha + \beta$

It is given that $\tan(\alpha-\beta)$, $\tan\alpha$ and $\tan(\alpha+\beta)$ form a G.P.

Thus $\tan^2\alpha = \tan(\alpha-\beta)\tan(\alpha+\beta) = \dfrac{\tan^2\alpha - \tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

$\tan\alpha = 1, \alpha = 45^\circ$, thus

$AD = DE = 54\sqrt{2}$, so that area $\Delta(ABC) = \dfrac{1}{2}BC\cdot AD = CD\cdot AD = 50\tan\beta$

Now, $\cot\alpha, \cot(\alpha-\beta), \cot\beta$ form an A.P.

$2\cot(45^\circ - \beta) = 1 + \cot\beta \Rightarrow \cot\beta = 3$

$\Rightarrow \Delta(ABC) = \dfrac{50}{3}$

52. (8.00)

$P(2,4) = (2t^2, 4t) \Rightarrow t = 1$

$t' = -t - \dfrac{2}{t} \Rightarrow t' = -3 \Rightarrow R$ is $(18,-12)$

Also $tt' = 2$ gives $t' = 2$ hence $Q(8,8)$

Perpendicular bisector of PR : $x - y = 14$ and of $PQ : 3x + 2y = 27$

Solving together gives $h = 11, k = -3$

$h + k = 8$

53. (5.00)

Note that the two given lines are mutually perpendicular, hence their point of intersection i.e. (1) will lie on director circle of the ellipse.

If centre of the ellipse is (h, k) the equation of direction circle will be $(x-h)^2+(y-k)^2=25$

As (1, 1) lies on this circle hence locus of center will be

$(h-1)^2+(k-1)^2=25$

$\therefore$ Radius of circle is 25.

54. (3)