PERMUTATIONS & COMBINATIONS
Exercise 1(A)

Q.1 (C, D)

$\Rightarrow \dfrac{5!}{(5-r)!} = 120$

$\Rightarrow (5 - r)! = 1$

$\Rightarrow r = 5$ or 4

Q.2 (A)

$\Rightarrow n \cdot \dfrac{(n-1)!}{(n-r)!} = \dfrac{n!}{(n-r)!} = {}^nP_r$

Q.3 (D)

$\Rightarrow \dfrac{(k+5)!}{4!} = \dfrac{11}{2}(k-1)\dfrac{(k+3)!}{3!}$

$\Rightarrow \dfrac{(k+5)(k+4)}{4} = \dfrac{11}{2}(k-1)$

$\Rightarrow k = 6$ or 7.

Q.4 (D)

$\Rightarrow$ Number of words = 6! = 720

Q.5 (D)

$\Rightarrow$ Required number of numbers = ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$

Q.6 (A)

$\Rightarrow \underline{5}$ _ _ Required number of numbers = $5 \times 4 = 20$

Q.7 (A)

$\Rightarrow$ Total possible ways = $4 \times 3 = 12$

Q.8 (D)

$\Rightarrow$ Number of ways = $({}^2C_1)^{10} = 1024$.

Q.9 (B)

$\Rightarrow$ Required number of ways = ${}^5P_3 = 60$.

Q.10 (C)

$\Rightarrow$ Number of ways = 6! = 720.

Q.11 (B)

$\Rightarrow$ Number of permutations = $\frac{(4+3+2)!}{2!4!3!} = \frac{9!}{2!4!3!}$

Q.12 (B)

$\Rightarrow$ Required number of numbers = $\frac{5!}{2!} = 60$

Q.13 (A)

$\Rightarrow$ Number of different arrangements = $\frac{6!}{2!2!}$

Q.14 (C)

$\Rightarrow$ Number of words = $\frac{11!}{2!2!2!}$

Q.15 (A)

$\Rightarrow$ Required number of words = $\frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$

Q.16 (C)

$\Rightarrow$ Number of ways $= {}^5C_1 \times {}^5C_1 \times {}^5C_1 = 5^3$

Q.17 (A)

$\Rightarrow$ The number of T digit numbers = $\frac{7!}{2!3!} = 420$

Q.18 (B)

$\Rightarrow$ _ _ _ unit's digit can be filled in 3 different ways and Ten's and Hundred's place each has six options.

$\Rightarrow$ Hence, Number of 3 digit odd numbers = $3 \times 6 \times 6 = 108$

Q.19 (C)

$$\Rightarrow \begin{array}{ccccccccc} \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ & E & & E & & E & & E & \end{array}$$

$\Rightarrow$ Odd digits can occupy 2^{nd}. 4^{th}, 6^{th} & 8^{th} places.

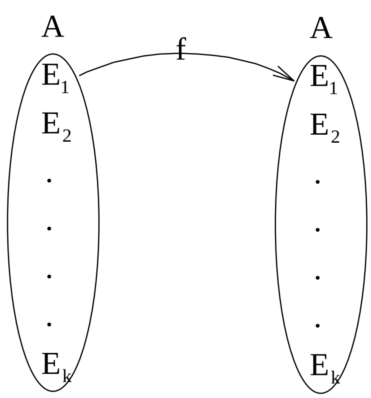
$\Rightarrow$ The number of different nine-digit numbers = $\dfrac{5!}{2!3!} \times \dfrac{4!}{2!2!} = 60$

Q.20 (C)

$\Rightarrow$ _ _ _ _ _ unit's place can be filled in 3 different ways.

$\Rightarrow$ Required number of even numbers = ${}^{3}C_{1} \times (4!) = 72$

Q.21 (B)



$\Rightarrow$ 1^{st} element E_1 from domain set 'A' has 'k' possibilities

$\Rightarrow$ Similarly, each element from domain set 'A' has 'k' possibilities

$\Rightarrow$ Hence, $n(S) = k \times k \times k \times ....k$ times

$\Rightarrow k^k$

Q.22 (A)

$\Rightarrow$Number of ways $= 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

Q.23 (B)

$\Rightarrow$Hence, required number = 3^4.

Q.24 (A)

$\Rightarrow$Number of ways = $4^3 - 1 = 63$

Q.25 (C)

$\Rightarrow$Number of ways = 5^4, as each parcel has 5 options.

Q.26 (A)

$\Rightarrow$ $\underline{C}$ _ _ _ _ _ _ $\underline{Y}$

$\Rightarrow$Number of words = 6!.

Q.27 (B)

$\Rightarrow$ _ _ $\underline{L}$ _ _

$\Rightarrow$Required number of ways = 4!

Q.28 (B)

$\Rightarrow$Number of arrangement of boys = (7!)

$\Rightarrow$Number of gaps created = 8.

$\Rightarrow$Hence, Required number of different ways = $7! \times {}^8P_3$.

Q.29 (C)

$\Rightarrow$Required number of ways = $5! \times {}^6C_5 \times 5! = 5! \times 6!$

Q.30 (A)

$$\Rightarrow \begin{array}{ccccccccc} \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ B & & B & & B & & B & & B \end{array}$$

$\Rightarrow$Number of arrangement of boys = 5!

$\Rightarrow$Number of possible permutations of girls = ${}^4P_3$.

$\Rightarrow$ Hence, Required number of arrangements = $5! \times {}^4P_3 = 2880$

Q.31 (C)

$\Rightarrow$ Required number of ways is the number of permutations of given 4 grades, taken 3 at a time. i.e., ${}^4P_3$.

Q.32 (B)

$\Rightarrow$ The number of ways in which husbands can be selected = ${}^7C_2$.

$\Rightarrow$ The number of ways in which wives can be selected after selecting husbands = ${}^5C_2$.

$\Rightarrow$ Also, a set of two men and two women can play 2 games.

$\Rightarrow$ Hence, Required number of ways = ${}^7C_2 \times {}^5C_2 \times 2$ = 420.

Q.33 (A)

$\Rightarrow$ Required number is number of circular permutations such that clockwise and anti-clockwise arrangements = $\dfrac{(5-1)!}{2} = \dfrac{4!}{2}$

Q.34 (D)

$\Rightarrow$ Required number = $(10-1)! \times 3! = 9! \times 3!$

Q.35 (B)

$\Rightarrow$ Number of arrangement of men = (7 – 1)! = 6!

$\Rightarrow$ Number of permutations of woman on the gaps created among man = 7!

$\Rightarrow$ Hence, required number = $6! \times 7!$

Q.36 (A)

Q.37 (C)

$\Rightarrow {}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 = {}^{52}C_4$ $\left(\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right)$

Q.38 (D)

$\Rightarrow n^2 - n = 2 + 10$

$\Rightarrow n = 4$ or -3

Q.39 (D)

$\Rightarrow {}^{20}C_{n+2} = {}^{n}C_{16} \Rightarrow 20 \geq n+2 \ \& \ n \geq 16$

$\Rightarrow n \leq 18 \ \& \ n \geq 16$

$\Rightarrow$Hence, n = {16, 17, 18}, but ${}^{20}C_{n+2} \neq {}^{n}C_{16}$ for $n \in \{16,17,18\}$

$\Rightarrow$Hence, No value.

Q.40 (A)

$\Rightarrow {}^{n}C_{3} + {}^{n}C_{4} > {}^{n+1}C_{3} \Rightarrow {}^{n+1}C_{4} > {}^{n+1}C_{3} \Rightarrow \left|\frac{n+1}{2} - 4\right| < \left|\frac{n+1}{2} - 3\right|$

$\Rightarrow |n-7| < |n-5|$

$\Rightarrow n > 6$

Q.41 (D)

$\Rightarrow$Required number of ways = ${}^{11}C_3$ = 165.

Q.42 (D)

$\Rightarrow$Number of ways of appointing clerks = ${}^{20}C_{16}$ = 4845.

Q.43 (B)

$\Rightarrow$Number of ways ${}^{11}C_4$ = 330.

Q.44 (D)

$\Rightarrow$Required number of words = $\left({}^{5}C_{3} \times {}^{4}C_{2}\right) \times 5!$

Q.45 (C)

$\Rightarrow$Number of ways of selection = ${}^{15}C_{1} \times {}^{10}C_{1} = 150$

Q.46 (C)

$\Rightarrow {}^{n}C_2$ = 66

$\Rightarrow$n = 13.

Q.47 (A)

$\Rightarrow$ Number of ways = ${}^8P_5 = 6720$.

Q.48 (B)

$\Rightarrow$ Number of Greeting card exchanged = ${}^{20}C_2 \times 2$

Q.49 (C)

$\Rightarrow$ The number of times he will go to the garden = ${}^8C_3 = 56$.

Q.50 (B)

$\Rightarrow$ The number of ways person can make selection of fruits = $(4 + 1)(5 + 1)(6 + 1) - 1 = 209$.

Q.51 (C)

$\Rightarrow$ Number of ways of selection = $(10 + 1)(9 + 1)(7 + 1) - 1 = 879$.

Q.52 (B)

$\Rightarrow$ Required number of ways = $2^6 - 1 = 63$.

Q.53 (D)

$\Rightarrow$ Number of ways in which a student can fail to get all answers correct = $4^3 - 1 = 63$.

Q.54 (C)

$\Rightarrow$ Total number of different combinations from the letters of word "MISSISSIPPI"
$(M \to 1, S \to 4, 1 \to 4, P \to 2)$ is $(1 + 1)(4 + 1)(4 + 1)(2 + 1) - 1 = 149$.

Q.55 (A)

$\Rightarrow$ Total number of ways of selecting six coins = coefficient of x^6 in $(x^o + x^1 + \ldots\ldots\ldots\ldots x^{20})(x^o + x^1 + \ldots\ldots + x^{10})(x^o + x^1 + \ldots\ldots x^7)$

$\Rightarrow$ coefficient of x^6 in $\dfrac{(x^{21}-1)(x^{11}-1)(x^8-1)}{(x-1)(x-1)(x-1)}$

$\Rightarrow$ coefficient of x^6 in $(x^{21} - 1)(x^{11} - 1)(x^8 - 1)(1 - x)^{-3}$

$\Rightarrow {}^{6+3-1}C_{3-1} = {}^8C_2 = \dfrac{8 \times 7}{2} = 28$

PERMUTATIONS & COMBINATIONS

Exercise 1(B)

Q.1 (C)

Arrangements of | L |AU|G|H | will be 4! i.e. 24
Arrangements of |A|U| will be 2! i.e. 2
Total arrangements = 48.

Q.2 (B)

To form natural numbers from 1000 to 9999 having all 4 distinct digits

9 choices	9 choices	8 choices	7 choices

Number of such numbers = $9\times9\times8\times7=4536$
Number of all the numbers from 1000 to 9999 = 9000.
Hence number of natural numbers from 1000 to 9999 not having all 4 distinct digits
= 9000 – 4536
= 4464.

Q.3 (A)

Two places to put 2s can be chosen in ${}^{7}C_{2}$ ways.
In each of the remaining 5 places we can put 1 or 3 hence these places can be filled in 2^5 ways.
Total number of numbers which can formed = ${}^{7}C_{2}\times2^{5}$ i.e. 672.

Q.4 (C)

Four consonants may be chosen in ${}^{7}C_{4}$ ways.
Two vowels may be chosen in ${}^{4}C_{2}$ ways.
The selected 6 letters may be arranged in 6! ways.
Total number of words = ${}^{7}C_{4}\times{}^{4}C_{2}\times6!$ i.e. 151200.

Q.5 (D)

Case I – The required numbers will be of form 57N, where N must be an even number.
Total number of such numbers = 5.
Case II – If 5 is not considered, then
for first digit from left we have 8 choices (not taking 0 & 5),
for second digit we have 9 choices (not taking 5),
for last digit we have 5 choices (only even numbers).
Total number of such numbers = $8\times9\times5$ i.e. 360.
Hence the number of such numbers = 365.

Q.6 (D)

First digit from left may be selected in 3 ways
last digit may be selected in 2 ways,
other 4 digits may be selected in $5 \times 4 \times 3 \times 2$ ways.
Total number of numbers = $5 \times 4 \times 3 \times 2 \times 3 \times 2$ i.e. 720.

Q.7 (A)

Flag can be designed by having following choices for six strips.

4 choices	3 choices	3 choices	3 choices	3 choices	3 choices

Hence number of designs = 12×81.

Q.8 (D)

In {0, 1, 2, 3, 4, 5} sum of all the digits is 15 which is divisible by 3.
Now we have to choose 5 numbers out of these 6 given numbers such as their sum is also divisible by 3.
Clearly either 0 or three must be discarded.
In the case when 0 is discarded, using {1, 2, 3, 4, 5} we can form 5! numbers.
In the case when 3 is discarded, using {0, 1, 2, 4, 5} we can form $4 \times 4!$ numbers.
Total possible numbers are 216.

Q.9 (C)

For each digit we have 5 choices
thus total possible numbers which can be formed are 5^9.

Q.10 (D)

All possible arrangements = 4!
Arrangements in which 'ab' kept together = $3! \times 2!$
Arrangements in which 'cd' kept together = $3! \times 2!$
Arrangements in which 'ab' as well 'cd' kept together = $2! \times 2! \times 2!$
Arrangements in which neither 'ab' nor 'cd' are together
= $4! - 3! \times 2! - 3! \times 2! + 2! \times 2! \times 2!$
= 8.

Q.11 (D)

To distribute books in given manner we need to select 5 persons and given one book each to these 5 persons.
Number of ways to do so will be ${}^{10}C_5 \times 5!$.

Q.12 (B)

To form a rectangle(including squares) we may choose any two vertical lines and any two horizontal lines.
Number of all possible rectangles, r = ${}^9C_2 \times {}^9C_2 = 1296$.
Number of squares of side length 1 = 8×8
Number of squares of side length 2 = 7×7
Number of squares of side length 3 = 6×6
Going by this pattern total number of squares, s = 1 + 4 + 9 + … + 64 = 204.
Hence $\frac{s}{r} = \frac{17}{108}$.

Q.13 (A)

Number of ways to select any 3, 4 or all 5 out of first 5 questions and then to select 7, 6 or 5, respectively, out of the remaining 8 questions = ${}^5C_3 \times {}^8C_7 + {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$ i.e. 276.

Q.14 (B)

Two consecutive digits we can have in two ways i.e. first two or last two.
Now if first two digits are identical, then we can chose these in 9 ways (excluding 0) and then third digit may be selected in 9 ways (excluding the number already placed in first 2 places).
If last two digits are identical, then we can choose first digit in 9 ways (excluding 0) and last two digits in 9 ways (excluding the number already placed in first place).
Hence number of all such numbers is $9 \times 9 + 9 \times 9$ i.e. 162.

Q.15 (A)

1, 3 & 5 we can put at 2nd, 4rd& 6th place in 3! ways.
Now 1st place can be filled by 2 or 4 and 3rd& 5th places can be filled in 2! Ways.
Hence number of all such numbers is $3! \times 2 \times 2!$ i.e. 24.

Q.16 (B)

Keeping those three men as one entity we have 4! arrangements.
These three men can be arranged in 3! ways within themselves.
Hence all possible arrangements = $4! \times 3!$ i.e. 144.

Q.17 (B)

We have to keep all the consonants together.
All the consonants as one entity and the 4 vowels may be arranged in $\frac{5!}{2!2!}$
Consonants with each other can be arranged in $\frac{5!}{2!}$ ways.

Hence all possible arrangements are $\frac{5!}{2!2!}\times\frac{5!}{2!}$ i.e. 1800.

Q.18 (A)

For the number to be divisible by four the last two digits must be 12, 16, 32, 36, 52 or 56.
First two digits can be selected from remaining three numbers in 3 ways.
Hence number of such numbers = $6\times3\times2$ i.e. 36.

Q.19 (B)

As the number has 9 distinct digits hence the middle digit must be 5, digits in last four places must be 6,7,8,9 and first four digits must be 1,2,3,4.
Now for first four place we have $4!$ ways.
For the last 4 places we have $4!$ ways.
Hence total number of such numbers is $(4!)^2$.

Q.20 (B)

6 digits can be selected in ${}^9C_6$.
Now as the digits are to be kept in decreasing order so no arrangement required.
Hence total number of such numbers is 84.

Q.21 (C)

Each of the 9 digits can be selected in 9 ways.
Hence total number of such numbers is 9^5.

Q.22 (A)

Let the sum of any five digits be N.
Now N can be any number from 1 to 45.
In each case the remaining digit can be chosen in two particular ways only so as sum of digits is divisible by 5.
{e.g. If N = 42, then remaining digit can be 3 or 8; if N = 23, then remaining digit is 2 or 7;…etc.}
Hence total possible number of such numbers is $9\times10^4\times2$ i.e. 180000.

Q.23 (B)

Each match can result in 3 ways.
forecast for 5 matches can be made in 3^5 ways in which one forecast is completely correct.
Hence there must be at least 243 people.

Q.24 (A)

Digits must be selected from {1, 3, 5, 7, 9}.

As the number must contain each of these so we need to permute 6 objects containing two identical objects and rest all distinct which can be done in $\frac{6!}{2}$ ways.

Also two identical digits can be selected in 5 ways.

Hence total number of such numbers is $5\times\frac{6!}{2}$.

Q.25 (D)

If one or more couple is there in the committee, then we can form the committee in the following manner

(i) committee having at least one couple : can be formed by choosing one couple in 4 ways and then choosing 2 more people from remaining 6 in ${}^6C_2$ ways i.e. $4\times{}^6C_2$ or 60 ways.

(ii) committee having two couples : can be formed by choosing two couples in ${}^4C_2$ or 6 ways.

Hence a committee having one or more couples can be formed in 60 – 6 = 54 ways.

Number of all possible committee = ${}^8C_4$ or 70 ways.

Thus number of ways to form a committee having no couples = 70 – 54 = 16.

Q.26 (D)

Let the numbers to fill be a, b, c, d as shown

a	b
c	d

Now a + b + c + d = 10 & a + d = b + c = k, then k = 5.
Now a + d = 5 = b + c gives choices for (a, d) as (1, 4) & (2, 3).
As a & d are interchangeable hence 4 ways to choose (a, d).
Similarly there are 4 ways to choose (b, c).
But as numbers are all distinct therefore for a given selection of (a, d) we can choose (b, c) in 2 ways only.
altogether a, b, c, d can be chosen in $4\times 2 = 8$ ways.

Q.27 (C)

Sum of internal angles of an n sided polygon = $(n-2)\times 180^o$.
Hence $150^o\times n = (n-2)\times 180^o$ gives n = 12.
Now a diagonal will be formed by joining two of these points but not adjacent two vertices.
Number of diagonals will be ${}^{12}C_2 - 12$ i.e. 54.

Q.28 (A)

Number of k digits numbers using just 1 & 2 = 2^k.

Now a number less than 100000000 can be formed in $\sum_{k=1}^{8} 2^k$ i.e. 510 ways.

A number between 10000000 & 20000000 can be formed in 2^8 or 256 ways.
Total possible numbers are 510 + 256 = 766.

Q. 29 (C)

Numbers formed using just 1 or 2 are only 2.
All possible numbers using 1 & 2 only = 2^n.
Hence numbers using at least one 1 & one 2 = $2^n - 2$.
Now $2^n - 2 = 510$ gives n = 9.

Q.30 (D)

To form a triangle we need to chose three non – collinear point.
We can chose 3 points in ${}^{18}C_3$ ways.
As each side of the triangle contains 6 points so we can choose all three points on any one side in $3\times{}^{6}C_3$ ways.
Total numbers of triangles = ${}^{18}C_3 - 3\times{}^{6}C_3$ i.e. 711.

Q.31 (A)

A student can answer 5 questions in two ways,
(i) one question each from two sections & 3 questions from one section. In this case
number of ways to chose two sections = 3,
number of ways to chose one question out of four from any section = 4,
number of ways to chose three questions out of four from any section = 4.
Hence number of ways to answer five questions = $3\times 4^2 \times 4 = 192$.
(ii) two questions each from two sections and one question from one section. In this case
number of ways to chose two sections = 3,
number of ways to chose two question out of four from any section = 6,
number of ways to chose one questions out of four from any section = 4.
Hence number of ways to answer five questions = $3\times 6^2 \times 4 = 432$.
Total number of ways = 192 + 432 = 624.

Q.32 (B)

Any 5 digits from {0, 1, 2,…,9} can be chosen in ${}^{10}C_5$ ways and put in descending order in just one way, hence m = ${}^{10}C_5$.
Any 5 digits from {1, 2, 3,…,9} can be chosen in ${}^{9}C_5$ ways and put in ascending order in just one way, hence n = ${}^{9}C_5$.
Thus m – n = ${}^{10}C_5 - {}^{9}C_5$ or ${}^{9}C_4$.

Q.33 (A)

(i) When A is excluded, to form a triangle we have to choose two points on one line & one point on one line number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1$

(ii) When A is included, other than the triangles formed in case (i) we can form more triangles by taking A as one vertex and choosing other two vertices one each on AB & AC.

Hence number of triangles = ${}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1$

Ratio of number of triangles in (i) & (ii) = $\dfrac{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1}{{}^{m}C_2 \times {}^{n}C_1 + {}^{n}C_2 \times {}^{m}C_1 + {}^{n}C_1 \times {}^{m}C_1} = \dfrac{m+n-2}{m+n}$.

Q.34 (A)

Number of ways to choose 4 prizes out of 9 = ${}^{9}C_4$.

Rest of the 5 prizes can be distributed to remaining 4 students in 4^5 ways.

Total number of ways to distribute 9 prizes among 5 students = ${}^{9}C_4 \times 4^5$.

Q.35 (C)

Number of ways in which m distinct objects can be permuted at n distinct places = ${}^{m}P_n$.

Q.36 (A)

The required number is maximum possible number of points in which the diagonals of an nonagon intersect inside the shape.

Now Each selection of four of the points gives three pair of lines out which exactly one pair intersects in the region enclosed within the four points.

Hence number of required points is ${}^{9}C_4$ i.e. 126.

Q.37 (B)

For sum of digits to be even : we can choose first four digits in general in 9×10^3 ways

Now if sum of first four digits is even, then last digit must be even

and if sum of first four digits is odd, then last digit must be odd

Hence in any possibility last digit can be chosen in 5 ways.

Number of all such numbers = $9 \times 10^3 \times 5$ or 45000.

Q.38 (D)

Possible way to distribute apples are such that

(i) one boy gets one apple, one boy gets three and remaining one gets four.

Hence number of ways to distribute = ${}^{8}C_1 \times {}^{7}C_3 \times 3!$.

(ii) Two boy gets two apples each, remaining one gets four.

Hence number of ways to distribute = $\dfrac{{}^{8}C_2 \times {}^{6}C_2 \times 3!}{2!}$.

(iii) two boys get three apples each and remaining one gets two.

Hence number of ways to distribute = $\frac{{}^{8}C_3 \times {}^{5}C_3 \times 3!}{2!}$.

Total number of ways to distribute apples = ${}^{8}C_1 \times {}^{7}C_3 \times 3! + \frac{{}^{8}C_2 \times {}^{6}C_2 \times 3!}{2!} + \frac{{}^{8}C_3 \times {}^{5}C_3 \times 3!}{2!}$

i.e. 4620.

Now ${}^{7}P_3 = 210$ gives k = 22.

Q.39 (D)

If one or more pair is there in the selection, then we can select shoes in the following manner

(i) Selection having at least one pair : can be formed by choosing one pair in 5 ways and then choosing 2 more shoes from remaining 8 in ${}^{8}C_2$ ways i.e. $5 \times {}^{8}C_2$ or 140 ways.

(ii) Selection having two pairs : can be formed by choosing two pairs in ${}^{5}C_2$ or 10 ways.

Hence a selection having one or more pairs can be formed in 140 – 10 = 130 ways.

Number of all possible selections = ${}^{10}C_4$ or 210 ways.

Thus number of ways to make a selection having no pairs = 210 – 130 = 80.

Q.40 (C)

9 lines will intersect in ${}^{9}C_2$ points.

9 circles will intersect in $2 \times {}^{9}C_2$ points.

9 lines will intersect with 9 circles in $2 \times {}^{9}C_1 \times {}^{9}C_1$ points.

Total number of points = 270.

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Exercise 1(C)

Q.1 (D)

a_4	a_3	a_2	a_1

b_4	b_3	b_2	b_1

$a_1 + b_1 \le 9,\ a_2 + b_2 \le 9,\ a_3 + b_3 \le 9\ \&\, a_4 + b_1 \le 9$, where $a_1, a_2, a_3, b_1, b_2, b_3 \in W$ & $a_4, b_4 \in N$.
Now number of solutions of each of the first three equations will be ${}^{9+3-1}C_{3-1}$ or 55 and those of the fourth equation will be ${}^{7+3-1}C_{3-1}$ or 36.
Hence total number of ways = 36×55^3.

Q.2 (C)

Hand shakes by men with each other = ${}^{10}C_2$
Hand shakes by women with each other = ${}^{10}C_2$
Hand shakes by men with women = 9×5.
Total number of hand shakes = 135.

Q.3 (D)

Number of ways to choose superintendent = 3
Number of ways to choose teachers for English school = ${}^{6}C_2 \times 4!$
Number of ways to choose teachers for Vernacular school = $4!$
Total number of ways = $3 \times {}^{6}C_2 \times 4! \times 4!$ or 25920.

Q.4 (A)

Let A & B be the persons who want to serve together and C & D be those who don't want to serve together.
Number of committees including A, B & only one of C or D = $2 \times {}^{5}C_2$
Number of committees including A & B but none of C & D = ${}^{5}C_3$
Number of committees not including A & B and including C or D = ${}^{5}C_3$
Number of committees not including any of A, B, C & D = 1.
Total number of committees = 41.

Q.5 (B)

Only possible way to distribute in given manner is in groups of 1, 4 & 2.

Hence number of ways to distribute $= \dfrac{7!}{1!\,2!\,4!} \times 3!$ or 630.

Q.6 (B)

Trials needed for first four digits = $\dfrac{4!}{2!}$.

Trials needed for fifth digit = 2.
Trials needed for seventh & eighth digit = 10.

Total number of trials needed = $\dfrac{4!}{2!} \times 2 \times 10$ = 240.

Q.7 (C)

There is only one way to distribute the toys i.e. 3 to youngest one and 2 each to others.

Number of ways to distribute = $\dfrac{9!}{(2!)^3\,3!}$.

Q.8 (A)

Number of ways to distribute m + n + p objects in three groups containing m, n & p objects is given by $\dfrac{(m+n+p)!}{m!n!p!}$.

Required number of ways = $\dfrac{10!}{2!3!5!}$.

Q.9 (B)

We can put 10 red balls at a gap of one each in 10! Ways, then green balls can be put in 11 gaps so formed in ${}^{11}C_9$ ways.

Hence number of arrangements are ${}^{11}C_9 \times 10!$.

Q.10 (C)

Keeping volumes of the same together we have 7 objects to arrange.
Also keeping volumes of same book in order we can have two arrangements for each seat – ascending & descending.
Number of ways to arrange = $7! \times 2 \times 2 \times 2$.

Q.11 (C)

Number of words starting with E = $\dfrac{4!}{2!}$.

Number of words starting with QE = $\frac{3!}{2!}$.

Number of words starting with QUEE = 1.

Rank of QUEUE = 17.

Q.12 (A)

Marbles are to be distributed in 1 : 2 ratio therefore one child will get 4 & other will bet 8

Required number of ways = $\frac{12!}{4!8!} \times 2!$ i.e. 990.

Q.13 (B)

Numbers with 1 at first place = ${}^{8}C_{4}$ or 70.

Numbers with 23 at first two places = ${}^{6}C_{3}$ or 20.

Numbers with 245 at first three places = ${}^{4}C_{2}$ or 6.

As 70 + 20 + 6 = 96, hence 97th number is 24678.

Q.14 (A)

Dashes	5	4	3	2	1	0
Dots	2	3	4	5	6	7
Arrangements	${}^{7}C_{2}$	${}^{7}C_{3}$	${}^{7}C_{4}$	${}^{7}C_{5}$	${}^{7}C_{6}$	${}^{7}C_{7}$

Total Number of arrangements = $2^{7} - {}^{7}C_{0} - {}^{7}C_{1}$ or 120.

Q.15 (B)

Number of ways to arrange n identical red & r identical green balls = $\frac{(n+r)!}{n!\,r!}$ or ${}^{n+r}C_{n}$.

Now range of r is 0 to m.

Hence number of arrangements = ${}^{n}C_{n} + {}^{n+1}C_{n} + {}^{n+2}C_{n} + \ldots + {}^{n+m}C_{n} = {}^{n+m+1}C_{n+1}$.

x = n + m + 1 & y = n + 1 or m.

Q.16 (A)

D, P, M, L can be arranged in 4! Ways.

Two pairs of vowels can be chosen in 2 ways i.e. {EE, AA}, {EA, AE}

Now these pairs can be put in 5 gaps between D, P, M, L in $2 \times {}^{5}C_{2} \times 2!$ ways.

Total number of arrangements = ${}^{5}C_{2} \times 2 \times 2! \times 4!$ i.e. 960.

Q.17 (C)

Number of arrangements of 8 boys around a circular table = 7!.
Number of ways to put the two particular boys in 8 gaps = $^8C_2 \times 2!$.
Hence all possible arrangements = $7! \times {^8C_2} \times 2!$ i.e. 7 (8!).

Q.18 (A)

For India to win 5 matches before Pakistan does, there must be a minimum 5 matches and a maximum 9 matches.
In 5 matches – all must be won by India, hence 1 way.
In 6 matches – last match and in first 5 matches 4 must be won by India, hence $^5C_1$ ways.
In 7 matches – last match and in first 6 matches 4 must be won by India, hence $^6C_2$ ways.
In 8 matches – last match and in first 7 matches 4 must be won by India, hence $^7C_3$ ways.
In 9 matches – last match and in first 8 matches 4 must be won by India, hence $^8C_4$ ways.
Total number of ways = 1 + $^5C_1$ + $^6C_2$ + $^7C_3$ + $^8C_4$ i.e. 126.

Q.19 (D)

Put one of the distinct objects at 1 place in the circle. Now for remaining objects we can use linear permutations.
Hence 'r' identical & (n – r – 1) distinct objects can be arranged in $\frac{(n-1)!}{r!}$ ways.

Q.20 (A)

Number of ways to chose m objects out of (m + n) objects = $^{m+n}C_m$
Number of ways to arrange these m and other n on the two tables = $\frac{(m-1)!}{2} \times \frac{(n-1)!}{2}$.
Number of ways of arrangements = $^{m+n}C_m \times \frac{(m-1)!}{2} \times \frac{(n-1)!}{2}$ or $\frac{(m+n)!}{4mn}$.

Q.21 (D)

Number of ways in which delegates of A & B are together = $8! \times 2!$
Number of ways in which delegates of A & B as well of C & D are together = $7! \times 2! \times 2!$
Hence number of ways in which delegates of A & B are together but those of C & D are not together = $8! \times 2! - 7! \times 2! \times 2!$ i.e. $12 \times 7!$.

Q.22 (A)

Seats for the two specific persons can be chosen in 5 ways on any of the 2 sides between the master & mistress i.e. 10 ways.
Rest of the 10 places can be filled in $10! \times 2$ ways (differentiating between left and right of

master and mistress).
Hence total number of ways = $20 \times 10!$.

Q.23 (D)

Number of ways to select 3 people from n sitting in a row, $P_n = {}^{n-3+1}C_3$

Number of ways to select 3 people from n sitting in a circle, $Q_n = {}^{n-3+1}C_3 - {}^{n-3-1}C_1$.

Hence ${}^{n-3-1}C_1 = 6$ or n = 10.

Alternately

Number of ways to select 3 people out of n sitting in a circle = ${}^{n}C_3$

Number of ways to select two adjacent and one separated = $n \times {}^{n-4}C_1$

Number of ways to select all three adjacent = n.

Hence number of ways to select 3 people out of n sitting in a circle such that no two are adjacent $Q_n = {}^{n}C_3 - n \times {}^{n-4}C_1 - n$.

$P_n - Q_n = {}^{n-2}C_3 - {}^{n}C_3 + n \times {}^{n-4}C_1 + n$

$\Rightarrow n - 4 = 6$ i.e. $n = 10$.

Q.24 (C)

$$L = \frac{(p+q)!}{p!q!}, M = \frac{(p+q)!}{p!q!} \times 2! \ \& \ N = \frac{(p+q)!}{p!q!}.$$

Q.25 (C)

For 'm' choose 4 persons out of 10 and permute 4 books with 4 persons in ${}^{10}C_4 \times 4!$ ways.

For 'n' choose 4 persons out of 10 (no need to permute) in ${}^{10}C_4$ ways.

Q.26 (B)

Put ladies in (n – 1)! Ways.
Now put gentlemen in (n – 1) gaps between ladies in n! ways.
Total number of ways = $(n-1)! \times n!$.

Q.27 (A)

Number of ways to distribute 'n' identical objects in 'r' distinct groups such that no group is empty = ${}^{n-1}C_{r-1}$.

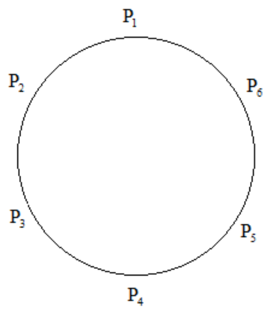
Hence required number of ways = ${}^{10-1}C_{6-1}$ i.e. 126.

Q.28 (C)

For each question we can have 3 choices namely (i) not selecting, (ii) selecting alternative 1 & (ii) selecting alternative 2.

Hence choices for 10 questions = 3^{10}.
But all the question cant be rejected hence required number of ways = $3^{10} - 1$.

Q.29 (D)



Let us put A at P_1.
If B is on right of A i.e. at P_2, then place on right of B i.e. at P_3 can be filled by C or D in 2 ways and remaining 3 places can be filled in 3! Ways. Hence this arrangements can be made in 12 ways.
If C is on right of A i.e. P_2, then we can make B & D sit $\{P_3, P_4\}$, {P4, P_5} or $\{P_5, P_6\}$ and the rest of the two places can be filled in 2 ways. Hence this arrangements can be made in 6 ways.
Total number of arrangements = 18.

Q.30 (C)

Let Miss C & Mr. B be included, then Mr. A cant be selected hence 1 more woman to be selected from 4 remaining and 2 more men to be selected from remaining 4.
Number of combinations = ${}^4C_1 \times {}^4C_2$ i.e. 24.
If Mr. A is included then Mr. B cant be selected hence 2 women to be selected from 5 choices and 2 more men to be selected from remaining 4.
Number of combinations = ${}^5C_2 \times {}^4C_2$ i.e. 60.
In none of A & B are included, then 2 women to be selected from 5 choices and 3 men to be selected from remaining 4.
Number of combinations = ${}^5C_2 \times {}^4C_3$ i.e. 40.
Total possible combinations = 124.

Q.31 (A)

Number of ways to arrange balls keeping balls of same color together $3! \times 4!$ i.e. $6 \times 4!$.

Number of all possible arrangements = $\frac{9!}{2!3!}$ i.e. $6 \times 7!$.

Number of arrangements in which at least one ball of same color is separated from others of same color = $6(7! - 4!)$.

Q.32 (B)

Trials needed for first three digits = 2.
If Last digit is 9, then rest three digits can be tried in 9^3 ways.
If last digit is one of 1, 3, 5 or 7, then one of the remaining three is 9(chosen in 3 ways) and other two digits can be chosen in 9^2 ways, hence all possible ways are $4 \times 3 \times 9^2$ ways.
Total number of ways to try the number = $2 \times (9^3 + 4 \times 3 \times 9^2)$ or 3402.

Q.33 (B)

-	AB	-	E	-	F	-

We can place AB, E & F in $3!\times 2!$ i.e. 12 ways.

Now C & D can be placed in gaps in ${}^{4}C_{2}\times 2!$ i.e. 12 ways.

Total number of arrangements = 144.

Q.34 (C)

Let ‘r’ boxes be filled by red balls and rest ‘5 – r’ with blue balls.
Blue balls must be put in r + 1 gaps between red balls.
Hence number of ways to put green balls = ${}^{r+1}C_{5-r}$.

Also $r+1\geq 5-r \Rightarrow 2\leq r\leq 5$.

Number of all possible arrangements = $\sum_{r=2}^{5}{}^{r+1}C_{5-r}={}^{3}C_{3}+{}^{4}C_{2}+{}^{5}C_{1}+{}^{6}C_{0}$ i.e. 13.

Q.35 (C)

Standard results.

Q.36 (B)

Using 6 points which are not cyclic we can draw ${}^{6}C_{3}$ circles.

Using 2 of the 5 cyclic points and 1 of the 6 noncyclic points we can draw ${}^{5}C_{2}\times{}^{6}C_{1}$ circles.

Using 1 of the 5 cyclic points and 2 of the 6 noncyclic points we can draw ${}^{5}C_{1}\times{}^{6}C_{2}$ circles.

Using the 5 cyclic points only 1 circle can be drawn.
Total number of circles = ${}^{6}C_{3}+{}^{5}C_{2}\times{}^{6}C_{1}+{}^{5}C_{1}\times{}^{6}C_{2}+1$ i.e. 156.
Alternately
Using 11 points we can draw ${}^{11}C_{3}$ circles, but 5 out of these are cyclic so ${}^{5}C_{3}$ circles will coincide into 1.
Total number of circles = ${}^{11}C_{3}-{}^{5}C_{3}+1$.

Q.37 (C)

Number of ways to divide ‘nr’ distinct objects into r groups containing equal number of objects $=\dfrac{(nr)!}{r!\times(n!)^{r}}$.

Hence 52 cards can be distributed into 4 groups of 13 cards each = $\dfrac{(52)!}{4!\times(13!)^{4}}$.

Q.38 (A)

Number of non negative integral solutions of $3x + y + z = 24$ is equal to number of no negative integral solutions of $y + z = 24 - 3r$, where $r = 0, 1,\dots,8$.

i.e. $\sum_{r=0}^{8} {}^{(24-3r)+2-1}C_{2-1}$ or $\sum_{r=0}^{8}(25-3r)$.

Hence required number of solutions are $225 - 3(1 + 2 +\dots+ 8)$ i.e. 117.

Q.39 (C)

A & B can be seated in 1 way on two identical tables.
Rest of the 6 persons can be seated in 6! Ways.
Hence total ways to seat 8 persons = 720.

Q.40 (D)

As $3^3+4^3+5^3=6^3$, hence for every $n \in N$, $(3n)^3+(4n)^3+(5n)^3=(6n)^3$.
There are infinitely many such numbers.

Q.41 (D)

Number of zeros at the end of 50! = exponent of 5 in 50!
i.e. $\left[\frac{50}{5}\right]+\left[\frac{50}{25}\right]$ or 12 , where [x] denotes greatest integer less than or equal to x.

Q.42 (B)

Let the subset A contain r elements, then subset B must contain remaining $n - r$ elements so as $A \cap B = \Phi$ & $A \cup B = S$.

Hence number of ordered pairs (A, B) = $\sum_{r=0}^{10} {}^{10}C_r$ i.e. 2^{10}

Total number of unordered pairs of subsets of S = $\frac{2^{10}}{2}$ i.e. 2^9.

Q.43 (A)

To form a number of r digits , where $1 \le r \le 9$, we have to chose r digits from $\{1, 2,\dots,9\}$ in ${}^9C_r$ number of ways and arrange them in ascending order which can be done in just 1 way.
Number of numbers of r digits = ${}^9C_r$.

Hence number of all such numbers = $\sum_{r=1}^{9} {}^9C_r = 2^9 - 1$.

Q.44 (C)

Odd digits – 3,3,5,5.
Even places 2^{nd}, 4^{th}, 6^{th} & 8^{th}.

Number of arrangements of odd digits at even places = $\frac{4!}{2!\times 2!}$.

Number of arrangements of remaining digits at rest of the places = $\frac{5!}{2!\times 3!}$.

Number of all such numbers = $\frac{4!}{2!\times 2!}\times\frac{5!}{2!\times 3!}$ i.e. 60.

Q.45 (D)

Place for A can be selected in 2×3 ways (leaving corner seats on both the sides) and then place for B can be selected in 6 ways(leaving places adjacent to A and opposite to A).
Place for A can be selected in 4 ways(seats at corners) and then for place for B in 7 ways (leaving place adjacent to A and opposite to A).
All the other people can sit in 8! Ways.
Hence number of seating arrangements = $(2\times 3\times 6 + 4\times 7)\times 8!$ i.e. $64\times 8!$.

Q.46 (A)

mn = 25! Gives mn = $2^{20}\times 3^{10}\times 5^{6}\times 7^{3}\times 11^{2}\times 13\times 17\times 19\times 23$.
Hence mn as 9 prime factors.
For gcd(m, n) = 1, we need to chose m and n from these only.

Number of ways to chose m & n = $\sum_{r=0}^{9} {}^{9}C_{r}$ or 2^9.

Now number of selection such that $\frac{m}{n}>1$ = Now number of selection such that $\frac{m}{n}<1$.

Hence number of ways to select m, n such that $\frac{m}{n}<1 = \frac{2^9}{2}$ i.e. 2^8.

Q.47 (D)

Given $x+1$ divides $P(x)$ hence $P(-1)=0$ i.e. $a+c=2b$.

Now if we chose any two odd numbers or any two even numbers from the set of given numbers as ‘a’ & ‘b’, then c gets selected automatically.
Required number of selections = $\left({}^{7}C_{2}+{}^{7}C_{2}\right)\times 2$.

Q.48 (B)

Number of positive integers of r–digits not having any digit as 1 = $8\times 9^{r-1}$.
Number of all the positive integers of r–digits = $9\times 10^{r-1}$.
Number of numbers having at least one digit as 1 = $9\times 10^{r-1}-8\times 9^{r-1}$.

Hence number of all the numbers up to 9-digit numbers = $\sum_{r=1}^{9}\left(9\times 10^{r-1}-8\times 9^{r-1}\right)$ i.e. 10^9-9^9.

Alternately

If we form a 9 digit number allowing consecutive 0s at places form beginning also, then r consecutive zeros from beginning will account for a number of (9 – r) digits.
Hence all numbers from 1 digit till 9 digit = 10^9.
Hence all numbers from 1 digit till 9 digit not including 1 = 9^9.
Number of numbers having at least one digit as 1 =$10^9 - 9^9$.

Q.49 (B)

A white square can be selected in 32 ways.
Leaving the 8 black squares in the row and column containing the previously selected white square, we are left with 24 black squares.
Number of ways to selected a white and a black square = 32×24.

Q.50 (B)

Red cards can be arranged with each other in 26! Ways.
Black cards can be arranged with each other in 26! Ways.
Ordering of red & black cards can be done in 2 ways.
Number of arrangements = $(26!)^2\times$ 2.

Q.51 (A)

$^{13}C_3+{}^{13}C_4={}^{14}C_4$ & $^{13}C_4+{}^{13}C_5={}^{14}C_5$
$\Rightarrow {}^{13}C_3+2\,{}^{13}C_4+{}^{13}C_5={}^{14}C_4+{}^{14}C_5$
$\Rightarrow {}^{n}C_r={}^{15}C_5$
Hence $r=5$ or 10.

Q.52 (C)

Number of outcomes on each die = 6.
Hence all the four dice will show same number I 6 ways.

Q.53 (B)

A & B along with any other two persons can sit on the straight table in $^6C_2\times 4!$ ways.
Remaining four persons can sit around the circular table in 3! Ways.
Total number of arrangements = $^6C_2\times 4!\times 3!$ i.e. 2160.

Q.54 (C)

Each suit can be arranged in 13! Ways and the suits can be arranged with each other in 4! Ways. Hence total number of arrangements = $(13!)^4 \times 4!$.

Q.55 (D)

To form a mixed doubles team we need to select 2 males and two females which can be done in $^8C_2 \times {}^8C_2$ ways and then the selected 4 players can be arranged in 2 ways.

Hence total number of ways = $2 \times {}^8C_2 \times {}^8C_2$ i.e. 1568.

Q.56 (D)

Number of pairings = number of ways to distribute 8 distinct objects in 4 groups each containing 2 objects i.e. $\dfrac{8!}{2!2!2!2!4!} = \dfrac{8.7.6.5}{16} = 105$.

Q.57 (C)

Let n parabola divide a plane in T_n regions.
1 parabola divides the plane in 2 regions.
If a second parabola is introduced, then 3 parts will increase.
If a third parabola is introduced, then 5 parts will increase.
If a fourth parabola is introduced, then 7 parts will increase.
Now if (n – 1) parabolas are already there and n^{th} parabola is introduced (2n – 1) new regions will be created, hence
$T_n = T_{n-1} + 2n - 1$
Also $T_1 = 2$.
Therefore $T_{10} = T_1 + 3 + 5 + ... + 19 = 101$.

Q.58 (A)

There are m number of ways to distribute each object and k objects are there so required number of ways = m^k.

Q.59 (D)

The word EQUATIONS contains {A, E, I, O, U} which are to be kept in the same order and {Q, T, N, S} which can be kept in any order.
Total number of arrangements are 9! But as vowels can't be arranged with each other hence required number of arrangements = $\dfrac{9!}{5!}$.

Q.60 (A)

As targets in same column can be shot only in a certain order hence number of ways to shoot will be $\frac{8!}{3!2!3!}$ i.e. 560.

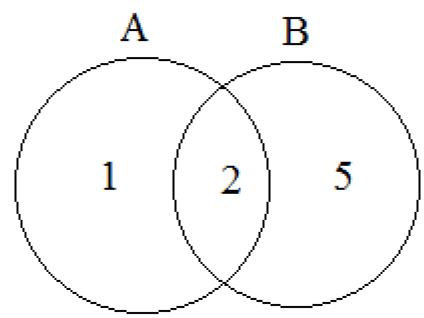
Q.61 (D)

We have to find umber of positive integral solutions of 3x + 5y = 283.

For $y = \frac{283-3x}{5}$, (283 – 3x) must be a multiple of 5, hence last digit of 3x must be 3 or 8.

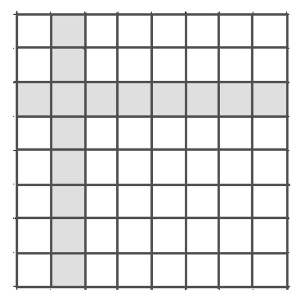
Now multiples of 3 having last digit 3 and less than 283 are {3, 33, 63, 93,…,273} i.e. 10
and multiples of 3 having last digit 8 and less than 283 are {18, 48, 78, 93,…,258} i.e. 9
Hence total 19 such points are there.

Q.62 (A)



There are two common elements in A & B.
If A has n elements and B has m elements, then $m \times n = 21$.
Hence {m, n} = {3, 7} or {7, 3} .
A possible arrangement is as shown in adjoining figure.
Hence $(A \cap B') \cup (A' \cap B)$ = 6.

Q.63 (B)



To place two rooks in attacking position they must be placed in the same row or column as shown in the adjoining figure.
As there are eight rows/columns and eight rooks hence each row/column must have exactly one rook.
So eight rooks can be placed in 8! Ways.

Q.64 (C)

n circles will intersect in $2 \times {}^nC_2$ ways.

n lines will intersect in ${}^nC_2$ ways.

n lines will intersect n circles in $2 \times {}^nC_1 \times {}^nC_1$ ways.

Hence $2 \times {}^nC_2 + {}^nC_2 + 2 \times {}^nC_1 \times {}^nC_1 = 80$ which gives n = 5.

Q.65 (B)

Let there be x number of 2s, y number of 5s and z number of 7s are used in forming an n digit number, then the total number of numbers formed will be

$\sum_{x=0}^{n}\sum_{y=0}^{n}\sum_{z=0}^{n}\frac{n!}{x!y!z!}\geq 900$, where $x+y+z=n$.

$\Rightarrow$ sum of coefficients in expansion of $(x+y+z)^n \geq 900$

$\Rightarrow 3^n \geq 900$.

Now $3^6 = 729$, hence least value of n = 7.

Q.66 (C)

12 dice can show any outcome in 12! Ways if all the outcomes could be distinct. Now as there are 2 each 1, 2, 3, 4, 5, 6 hence required number of ways = $\frac{12!}{(2!)^6}$.

Q.67 (C)

If there are n number of B used, then required number of numbers will be $\frac{(a+n)!}{a!n!}$.

Hence total possible number of numbers where $0 \leq n \leq b$ will be $\sum_{n=0}^{b}\frac{(a+n)!}{a!n!} = \sum_{n=0}^{b} {}^{a+n}C_n$

i.e. ${}^{a+b+1}C_b$.

Q.68 (D)

Given A = {1, 11, 21, …, 551}

There are 28 pairs whose sum is 552 such as (1, 551), (2, 550),…, (271, 281)

So if one number of a pair is taken in B then the other number can't be in B.

Hence B can contain maximum 28 elements.

Q.69 (A)

AAAAA, BBB, D, EE & F can be arranged in $\frac{12!}{5!\,3!\,2!}$ ways.

Now there will be 13 gaps where we can put 3 C in ${}^{13}C_3$ ways.

Hence total number of arrangements = ${}^{13}C_3 \times \frac{12!}{5!\,3!\,2!}$.

Q.70 (C)

EARTHQUAKE – {E,E}, {A,A}, {R, T, H, Q, U, K}

A 4 letter permutation can be of

(i) 2 alike & 2 alike = $\frac{4!}{2!\,2!}$ i.e. 6 ways

(ii) 2 alike & two distinct = $2 \times {}^7C_2 \times \frac{4!}{2!}$ i.e. 504 ways

(iii) all four distinct = ${}^{8}C_4 \times 4!$ i.e. 1680 ways

Hence total number of permutations = 6 + 504 + 1680 = 2190.

PERMUTATIONS & COMBINATIONS
Exercise 2(A)

Q.1 (A)

Case I:All 3 flowers of first are together = 1 way

Case II : 2 flowers of first kind are together. Keeping these as areference 3rd flower of second kind. No. of ways butanticlockwise arrangements are not different

$\therefore$ Number of ways = $\dfrac{11+1}{2}$

CaseIII : All 3 flowers of first kind are kept at 12 spaces generated by the flowers of 3rd kind

$\therefore$ Number of ways = 12

Ans. 1 + 6 + 12 = 19.

Q.2 (D)

$504 = 2^3 \cdot 7 \cdot 3^2$

For even divisor selecting at least one 2 & remaining factors inall possible ways, no. of possible divisors = $3 \cdot 2 \cdot 3$=18

Q.3 (D)

For$1 \le k \le p-1, n+k = p!+k+1$, is clearly divisible by k+1.

$\therefore$ There is no prime no. in the given list….......(Ans.:)

Q.4 (C)

Selecting places for A,B,C,D → ${}^6C_4$

Arranging other persons in 2! Ways : ${}^6C_4$.2! =30.

Q.5 (C)

A={2,3,5,7,11,13,17,19,23,29} $\therefore$ n(A) = 10

Selecting any 2 numbers& arranging them in 2 way = ${}^{10}C_2 \times 2 = 9$

(two numbers a & b give two rational numbers $\frac{a}{b}$ & $\frac{b}{a}$)

Also with repetition (i.e. $\frac{2}{2}, \frac{3}{3}, \frac{5}{5}, ...$ etc.) only the no.'1'can be formed

$\therefore$Total number of rational numbers, 90+1=91

Q.6 (B)

Each element in domain can be associated with any of the k elements in codomain

$\therefore$ Total number of mappings = k^k.

Q.7 (C)

Selecting any y form 0 to 9 & arranging them in one way (descending order only)

$= {}^{10}C_4 \times 1 = 210$.

Q.8 (C)

Number of ways to select one point each from three line = $p \times p \times p$ *i.e.* p^3

Number of ways to select 2 points from any one of three lines& one point from any one of the other two lines = ${}^3C_1 \times {}^pC_2 \times {}^2C_1 \times p$ *i.e.* $3p^2(p-1)$

$\therefore$ No. of triangles $= p^3 + 3p^2(p-1) = p^3(4p-3)$.

Q.9 (B)

First round $= {}^6C_2 + {}^6C_2 = 30\, matches$

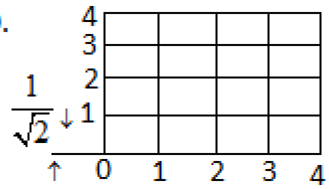
Second round $= {}^6C_2 = 15\, matches$

Third round $= {}^4C_2 = 6\, matches$

Min. no. of matches in best of the 3= 2

Total number of matches = 52.

Q.10.



A side of length $\sqrt{2}$ units vertically can be selected in 3 ways.
A side of length $\sqrt{2}$ units horizontally can be selected in 3 ways.
$\therefore$ no. of squares = 3×3 ....(Ans.: A)

Q.11 (A)

<u>Pattern I</u> : AAB $\rightarrow {}^3C_2 \times 2 = 6$ (two strips of one color and one of different color)

<u>Pattern II</u> : ABB $\rightarrow {}^3C_2 \times 2 = 6$ (two strips of one color and one of different color)

<u>PatternIII</u> : ABA $\rightarrow {}^3C_2 \times 2 = 6$ (two strips of one color and one of different color)

<u>PatternIV</u>: ABC $\rightarrow {}^3C_2 \times 2 = 6$ (One strip each of three colors)

$\therefore$ Total number of flags = 6+6+6+6=24

Q.12 (A)

For $\frac{p}{q}$, p & q both can be selected in $6 \times 6 = 36$ ways

The no. of ways for $\frac{p}{q} = 1$ is 6

The no. of ways for $\frac{p}{q} = \frac{1}{2}$ is 13 i.e. $\left(\frac{1}{2}, \frac{2}{4}, \frac{3}{6}\right)$

The no. of ways for $\frac{p}{q} = \frac{1}{2}$ is 3 i.e. $\left(\frac{2}{1}, \frac{4}{2}, \frac{6}{3}\right)$

The no. of ways for $\frac{p}{q} = \frac{1}{3}$ & 3 are 2 & 2 i.e. $\left(\frac{3}{1}, \frac{6}{2}; \frac{1}{3}, \frac{2}{6}\right)$

The no. of ways for $\frac{p}{q} = \frac{2}{3}$ & $\frac{3}{2}$ are 2 & 2 i.e. $\left(\frac{2}{3}, \frac{4}{6}; \frac{3}{2}, \frac{6}{4}\right)$

$\therefore$ Extra counting = 5+2+2+1+1+1+1=13

$\therefore$ No. of rational nos. = 36 – 13=23

Q.13 (D)

The no. of ways in which only one element be the element of $A \cap B$, is $= {}^{n}C_{1}$ *i.e.* n.

For other (n – 1) elements, each have possibilities of being a member of only P or of only Q or of none of P & Q i.e. 3 each

$\therefore$ required no. of ways = n.(3.3.3……….(n – 1)times)= $n \times 3^{n-1}$

Q.14 (B)

The no. of ways to select three numbers = ${}^{n}C_{3}$

No. of ways to arrange = 1

$\therefore$Total number of triplets = ${}^{n}C_{3} \times 1 = {}^{n}C_{3}$

Q.15 (D)

For each book number of ways to select (to select 0, 1, 2, …, l copies) is $l+1$

$\therefore$ for k books, number of ways to select = $(l+1)^{k}$

Out of these in one case no book will be selected.

$\therefore$ rejecting the case selection, total number of selections = $(l+1)^{k} - 1$

Q.16 (A)

The no. of ways to select any 2 stations = ${}^{20}C_{2}$

For tickets, (A $\longrightarrow$ B and B $\longrightarrow$ A) no. of ways =2

$\therefore$Total number of tickets required = ${}^{20}C_{2}$ .2=380.

Q.17 (D)

In MATHEMATICS we have M,A,T twice times each &H,E,I,C,S once each.

No. of words of the form (A,B,C,D i.e. all different)= ${}^{8}C_4 .4!$

No. of words of the form (A,A B,C i.e. two alike & two distinct)= ${}^{3}C_2 \times {}^{7}C_2 \times \frac{4!}{2!}$

No. of words of the form (A,A,B,B i.e. two alike & two alike)= ${}^{3}C_2 \times \frac{4!}{2!2!} = 18$

Total number of words = 2454.

Q.18 (A)

For y = 1, $x_1 \cdot x_2 \cdot x_3 = y$ has only 1 solution.

For y=2,3 & 5,number of solutions of $x_1 \cdot x_2 \cdot x_3 = y$ is $3 \times 3 = 9$

For y=2.3,2.5,3.5 i.e. for y=6,10 & 15 number of solutionsof $x_1 \cdot x_2 \cdot x_3 = y$ is

$3 \times \left(3! + {}^{3}C_2\right) = 27$

Explanation : $\begin{pmatrix} For\ \ x_1 \neq x_2 \neq x_3 \to 3! \\ For\ \ x_1 = x_2 \neq x_3 \to {}^{3}C_2 \end{pmatrix}$

For y=30 i.e.2.3.5, the no. of solutions of $x_1 \cdot x_2 \cdot x_3 = y$ is

$\left(3! + {}^{3}C_2 \times 3! + 3\right) = 27$

Explanation : $\begin{pmatrix} For\ \ x_1 \neq x_2 \neq x_3 \to 3! \\ For\ \ x_1 = 1 \neq x_2 \neq x_3 \to {}^{3}C_2 .3! \\ For\ \ x_1 = 1 = x_2 , x_3 \to 3 \end{pmatrix}$

Total number of solutions = 1 + 9 + 27 + 27 i.e. 64.

Q.19 (B)

$7! = 2^4 \cdot 3^2 \cdot 5^1 \cdot 7^1$

Factors of the form 3t+1 & odd are 1 & 7.

$\therefore$ sum = 8

Q.20 (B)

No. of ways to select 6 digits form 0,1,2………..,9=${}^{10}C_6$

No. of ways to choose a_4 (the smallest)=1

For other 3 the no. of ways to choose = ${}^5C_3$

$\therefore$ reqd. no. of ways ${}^{10}C_6 \times {}^5C_3 = 2100$.

Q.21 (A)

60 = $2^2 \cdot 3 \cdot 5$

Let the powers of 2 in x_1, x_2 & x_3 bei , j , k, then i+j+k=2 where i, j, k are whole numbers.

$\therefore$ no. of non-negative integral solution = ${}^{2+3-1}C_{3-1} = {}^4C_2 = 6$

Also 3 & 5 can be factors of a, b & c in 3 ways each

$\therefore$ no. of non-negative integral solution of $x_1x_2x_3 = 60$ is ${}^4C_2 \times 3 \times 3 = 54$.

Q.22 (A)

For $2^n \rightarrow$ n=2,3,4,5 & 6 $\therefore$ no. of ways =5

For $3^n \rightarrow$ n=2,3, 4 $\therefore$ no. of ways =3

For $5^n \rightarrow$ n=2 $\therefore$ no. of ways =1

For $7^n \rightarrow$ n=2 $\therefore$ no. of ways =1

For $6^n \rightarrow$ n=2 $\therefore$ no. of ways =1

Also For $10^n \rightarrow$ n=2 $\therefore$ no. of ways =1

Total number of ways =12

Q.23 (C)

First digit can be chosen in 9 ways (let first digit be a, a$\neq$0)

Second digit can be chosen in 9 ways (let second digit be b$\neq$a)

Third digit can be chosen in 9 ways (let third digit be c$\neq$b)

Hence each digit can be chosen in 9 ways.

$\therefore$ number of n digit nos. = 9^n

Q.24 (B)

$3630 = 2 \cdot 3 \cdot 5 \cdot 11^2$

For no. of factors of the form (4n+1) i.e. $3^a \cdot 5^b \cdot 11^c$ the values of a , b & c required are

a= 1, b=0 or 1 c= 2 or a=0, b= 0 or 1& c=2

$\therefore$No. of ways = $1 \times 2 \times 2 + 1 \times 2 \times 1 = 6$.

Q.25 (D)

(3m+1)+(3n+2) $\rightarrow$ is divided by 3 $\therefore$ No. of ways = $4 \times 4 = 16$

3m+3n $\rightarrow$ is div. by 3 $\therefore$ No. of ways = ${}^4C_2 = 6$

$\therefore$ all possible number of ways = 16+6=22.

Q.26 (A)

$30 = 2 \cdot 3 \cdot 5 = x_1 \cdot x_2 \cdot x_3$

Each factor 2, 3&5 has 3 choices for being a factor of x_1, x_2 *or* x_3

$\therefore$ No. of solution $= 3 \times 3 \times 3$= 27.

PERMUTATIONS & COMBINATIONS

Exercise 2(B)

Q.1 (A)(B)(C)(D)

$^{2n}P_n = n!\left(^{2n}C_n\right)$

$\Rightarrow {}^{2n}P_n = \dfrac{2n(2n-1)(2n-2)...3.2.1}{n(n-1)(n-2)...3.2.1}$

$\Rightarrow {}^{2n}P_n = 2n(2n-1)(2n-2)...(n+1)$

Hence $^{2n}P_n = \dfrac{2^n\left(n(n-1)(n-2)...1\right)\left((2n-1)(2n-3)...3.1\right)}{n(n-1)(n-2)...1}$

or $^{2n}P_n = 2^n\left[1.3.5....(2n-1)\right]$ i.e. $2.6.10....(4n-2)$

Q.2 (A)(C)(D)

Exponent of any prime P in 100! = $\left[\dfrac{100}{P}\right]+\left[\dfrac{100}{P^2}\right]+\left[\dfrac{100}{P^3}\right]+...$, where [x] denotes greatest integer less than or equal to x.

Now exponent of 2, α = 50 + 25 + 12 + 6 + 3 + 1 = 97.
Exponent of 3, β = 33 + 11 + 3 + 1 = 48.
Exponent of 5, γ = 20 + 4 = 24.
Exponent of 7, δ = 14 + 2 = 16.

Q.3 (A)(C)(D)

Number of ways to select any 3, 4 or all 5 out of first 5 questions and then to select 7, 6 or 5, respectively, out of the remaining 8 questions = $^5C_3 \times {}^8C_7 + {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$ i.e. 276.

Q.4 (A)(B)

$$\max\left(^nC_r\right) = \begin{cases} ^nC_{n/2}, & \text{if } n \text{ is even} \\ ^nC_{(n-1)/2}, & \text{if } n \text{ is odd} \end{cases}$$

Also $^nC_r = {}^nC_{n-r} \Rightarrow {}^nC_{(n-1)/2} = {}^nC_{(n-1)/2}$.

Q.5 (B)(D)

$x_1 + x_2 + x_3 + x_4 \le n \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = n$.
Now number of non negative integral solutions = $^{n+5-1}C_{5-1}$ or $^{n+4}C_4$.

Also $^nC_r = {}^nC_{n-r} \Rightarrow {}^{n+4}C_4 = {}^{n+4}C_n$.

Q.6 (A)(B)(C)

Arrangements of 8 balls keeping two particular colors together = $7!\times 2!$
Unrestricted arrangements = 8!
Arrangements of 8 balls keeping two particular colors separated = $8! - 7!\times 2!$.

Q.7 (A)(B)(C)(D)

$$S = 1.1! + 2.2! + 3.3! + ... + n.n!$$
$$\Rightarrow S = (2-1).1! + (3-1).2! + (4-1).3! + ... + (n+1-1).n!$$
$$\Rightarrow S = 2! - 1! + 3! - 2! + 4! - 3! + ... + (n+1)! - n!$$
$$\Rightarrow S = (n+1)! - 1$$

Q.8 (A)(B)(C)(D)

When two vowels are together : $\frac{5!\times 2!}{2!}$.

When vowels occur in alphabetical order : $\frac{6!}{2!\times 2!}$.

When vowels and consonant occupy their respective places : $\frac{4!\times 2!}{2!}$.

When vowels don't occur together : $\frac{6!}{2!} - \frac{5!\times 2!}{2!}$.

Q.9 (A)(B)

Arrangements of 10 objects such that two objects are not be arranged with each other
= $\frac{10!}{2!}$.

Q.10 (B)(C)(D)

Number of ways to select r distinct objects out of n distinct such that no two or more of the selected objects were adjacent = ${}^{n-r+1}C_r$.

Hence required number of arrangements is ${}^{7}C_4 \times 4!$.

Q.11 (B)(C)

Let the number of students be n, then everyone will have to send (n – 1) cards.
Number of greeting cards which were sent will be n(n – 1).
Now n(n – 1) = 1640 gives n = 41.

Q.12 (A)(B)(C)

For sum of digits to be even : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be even
and if sum of first four digits is odd, then last digit must be even
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers, x = $9\times10^3\times5$ or 45000.
For sum of digits to be odd : we can choose first four digits in general in 9×10^3 ways
Now if sum of first four digits is even, then last digit must be odd
and if sum of first four digits is odd, then last digit must be even
Hence in any possibility last digit can be chosen in 5 ways.
Number of all such numbers, y = $9\times10^3\times5$ or 45000.
Hence x = y = 45000.

Q.13 (A)(D)

To deal 5 consecutive cards irrespective of any preference for suite we can deal each value card in 4 ways and there are 10 ways to chose a sequence of 5 consecutive values.
Hence total number of ways = 10×4^5.
5 consecutive cards of the same suite can be dealt in 10×4 ways.
Hence number of ways to deal a straight = $10\times\left(4^5-4\right)$ i.e. 10200.

Q.14 (B)

There are two ways to form a triangle –
(i) Choosing one point each on AB, BC & CA.
Number of triangles = ${}^3C_1\times{}^4C_1\times{}^5C_1$ or 60.
(ii) Choosing one point on one side & two points on one side.
Number of triangles = ${}^3C_2\times\left({}^4C_1+{}^5C_1\right)+{}^4C_2\times\left({}^3C_1+{}^5C_1\right)+{}^5C_2\times\left({}^4C_1+{}^3C_1\right)$ or 172.
Total number of triangles = 205.

Q.15 (B)(D)

Number of ways to chose two ice-creams of same flavor & one different = 8×7.
Number of ways to chose all three ice-creams of same flavor = 8.
Hence total number of ways = 64.
Also ${}^{10}C_3-{}^8C_3$ = 64.

Q.16 (A)

1 + 2 + 3 + …+ 9 = 45.
Now to chose seven out of these numbers such that their sum is divisible by 3, sum of the two rejected numbers must be divisible by three.
Possible pairs are {1, 2}, {1, 5}, {1, 8}, {2, 4}, {2, 7}, {3, 6}, {3, 9}, {4, 5}, {4, 8}, {5, 7}, {6, 9} & {7, 8}.
Hence 12 selections are possible leaving these pairs out of 1, 2, 3, …,9.
Total number of such possible numbers are $12\times 7!$.

Q.17 (B)(C)

Number of ways to divide 10 students in 3 teams such that one contains 4 students and others 3 each will be $\frac{{}^{10}C_4\times{}^{6}C_3\times{}^{3}C_3}{2!}$ or 2100.

Q.18 (B)(C)

${}^{1000}C_{500}=\frac{1000!}{500!\times 500!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 7 in 1000! = $\left[\frac{1000}{7}\right]+\left[\frac{1000}{49}\right]+\left[\frac{1000}{243}\right]$ or 164.

Exponent of 7 in 500! = $\left[\frac{500}{7}\right]+\left[\frac{500}{49}\right]+\left[\frac{500}{243}\right]$ or 82.

Hence exponent of 7 in $\frac{1000!}{500!\times 500!}$ = 0.

Exponent of 13 in 1000! = $\left[\frac{1000}{13}\right]+\left[\frac{1000}{169}\right]$ or 81

Exponent of 13 in 500! = $\left[\frac{500}{13}\right]+\left[\frac{500}{169}\right]$ or 40.

Hence exponent of 13 in $\frac{1000!}{500!\times 500!}$ = 1.

Exponent of 191 in 1000! = $\left[\frac{1000}{191}\right]$ or 5

Exponent of 191 in 500! = $\left[\frac{500}{191}\right]$ or 2.

Hence exponent of 13 in $\frac{1000!}{500!\times 500!}$ = 1.

Exponent of 201 in 1000! = $\left[\frac{1000}{201}\right]$ or 4

Exponent of 201 in 500! = $\left[\frac{500}{201}\right]$ or 2.

Hence exponent of 201 in $\frac{1000!}{500!\times 500!}$ = 0.

Hence ${}^{1000}C_{500}$ is divisible by 13 & 191 but not by 7 &201.

Q.19 (B)(C)

(A) Number of zeros in the end of 125! = exponent of 5 i.e. $\left[\frac{125}{5}\right]+\left[\frac{125}{25}\right]+\left[\frac{125}{125}\right]$ or 31.

[x] denotes greatest integer less than or equal to x.

(B) Total possible combinations of positions of all the arms = 10^{10}.

Now when all the arms are at rest no signal is transmitted hence

Number of signals = $10^{10} - 1$.

(C) A number greater than 400000 will have first digit 4 or 5 and will be of 6 digits as each given choice can be used only once.

If first digit is 4, then rest of the digits can be placed in $\frac{5!}{2!}$.

If first digit is 5, then rest of the digits can be placed in $\frac{5!}{2!2!}$.

Total number of ways = 90.

(D) If there are n players, then number of games played will be ${}^{n}C_2$.

Now ${}^{n}C_2 = 5050 \Rightarrow n(n-1) = 10100$.

Hence n = 101.

Q.20 (A)(C)(D)

[x] denotes greatest integer less than or equal to x.

(A) Number of zeros at the end of 20! = exponent of 5 in 20! = $\left[\frac{20}{5}\right]$ or 4.

(B) Exponent of 2 in 20! = $\left[\frac{20}{2}\right]+\left[\frac{20}{4}\right]+\left[\frac{20}{8}\right]+\left[\frac{20}{16}\right]$ or 18.

Exponent of 3 in 20! = $\left[\frac{20}{3}\right]+\left[\frac{20}{9}\right]$ or 8.

Exponent of 5 in 20! = $\left[\frac{20}{5}\right]$ or 4.

Exponent of 7 in 20! = $\left[\frac{20}{7}\right]$ or 2.

Exponent of 11, 13, 17 & 19 in 20! = 1 each.

Now $\frac{20!}{10^4}=\frac{2^{18}\times3^8\times5^4\times7^2\times11\times13\times17\times19}{2^4\times5^4}$ i.e. $2^{14}\times3^8\times7^2\times11\times13\times17\times19$.

Last digit of 2^{14} = 4, last digit of 3^8 = 1, last digit of 7^2 = 9.

Hence last digit of $\frac{20!}{10^4}$ will be last digit of $4\times1\times9\times1\times3\times7\times9$ or 4.

(C) Exponent of 5 in 20! = 4 & exponent of 5 in 10! = 2.

(D) Exponent of 7 in 20! = 2 & exponent of 7 in 10! = 1.

Passage – 1

In CURRICULUM, there are
3 – U, 2 – C, R, 1 – I, L, M.

Q.21 (B)

Words which can be formed using only three letters can be formed using
(i) 2 identical letters, 2 identical letters & 1 different letter
(ii) 3 identical letters & 2 different letters
In first case, we can choose two pairs of identical letters in 3 ways & a distinct letter in 4 ways and the letters can be arranged in $\frac{5!}{2!2!}$ ways.

Hence total number of words = $3\times4\times\frac{5!}{2!2!}$ or 360.

In second case three identical letters must be 3 Us and two distinct letters can be chosen in ${}^5C_2$ ways and the letters can be arranged in $\frac{5!}{3!}$ ways.

Hence total number of words = ${}^5C_2\times\frac{5!}{3!}=200$.

Number of all such possible words are = 560.

Q.22 (A)

All possible arrangements = $\frac{10!}{3!2!2!}$ or 30.7!

To find arrangements in which vowels are separated –

Keeps consonants at a gap of one each in $\frac{6!}{2!2!}$ or 180 ways,

Now put vowels in 7 gaps in ${}^7C_4\times\frac{4!}{3!}$ or 140 ways.

Hence number of arrangements = $180\times140=25200=5(7!)$.

Q.23 (B)

Consonants can be arranged with each other in $\frac{6!}{2!2!}$ or 180 ways.

Vowels can be arranged with each in $\frac{4!}{3!}$ or 4 ways.

Hence all possible arrangements are $180 \times 4 = 720$ ways.

Passage – 2

Q.24 (D)

Increasing order : Any selection of 5 digits can be made in ${}^{10}C_5$ ways and arranged in 1 way.
Hence number of numbers = 252.
Decreasing order : Any selection of 5 digits can be made in ${}^{9}C_5$ ways (not including 0 in any selection) and arranged in 1 way.
Hence number of numbers = 126.
Number of all possible numbers = 378.

Q.25 (A)

Two alike & three alike can be chosen in 9×8 or 72 ways (not selecting 0) and arranged in $\frac{5!}{3!2!}$ or 10 ways.
Number of such numbers = 720.
If two '0's and three other identical digits are chosen (in 9 ways) then after placing one nonzero digit at the left most place rest of the digits can be arranged in $\frac{4!}{2!2!}$ or 6 ways.
Number of such numbers = 54.
If three '0's and two other identical digits are chosen (in 9 ways) then after placing one nonzero digit at the left most place rest of the digits can be arranged in $\frac{4!}{3!}$ or 4 ways.
Number of such numbers = 36.
Number of all possible numbers = 810.

Q.26 (A)

Sum of all the digits of S is 45 which is divisible by 3 hence every number made up of all 10 digits will be divisible by 3.
None of the numbers is prime.

Passage – 3

Q.27 (C)

Number of ways to de-arrange 5 objects = $\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)\times 5!$ i.e. 44.

Q.28 (B)

Number of all integers less than or equal to 100 = 100.
Number of integers divisible by 2 = 50
Number of integers divisible by 3 = 33
Number of integers divisible by 5 = 20
Number of integers divisible by 2 & 3 = 16
Number of integers divisible by 2 & 5 = 10
Number of integers divisible by 3 & 5 = 6
Number of integers divisible by all of 2, 3 & 5 = 3.
Number of integers not divisible by any of 2, 3 or 5
= 100 – (50 + 33 + 20) + (16 + 10 + 6) – 3 i.e. 26.

Q.29 (A)

Required number of solutions will be coefficient of x^{30} in $\left(1+x+x^2+...+x^9\right)^4$.

i.e. coefficient of x^{30}in $\left(1-x^{10}\right)^4\left(1-x\right)^{-4}$.

Now general term of $\left(1-x^{10}\right)^4\left(1-x\right)^{-4}$ will be $\left(-1\right)^p\ {}^4C_p\ {}^{q+3}C_3\ x^{10p+q}$.

coefficient of x^{30} = ${}^4C_0\ {}^{33}C_3-{}^4C_1\ {}^{23}C_3+{}^4C_2\ {}^{13}C_3-{}^4C_3\ {}^3C_3$ or 84.

Passage – 4

Q.30 (A)

${}^{1000}C_{500}=\frac{1000!}{500!\times 500!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 7 in 1000! = $\left[\frac{1000}{7}\right]+\left[\frac{1000}{49}\right]+\left[\frac{1000}{243}\right]$ or 164.

Exponent of 7 in 500! = $\left[\frac{500}{7}\right]+\left[\frac{500}{49}\right]+\left[\frac{500}{243}\right]$ or 82.

Hence exponent of 7 in $\frac{1000!}{500!\times 500!}$ = 0.

Q.31 (C)

Exponent of 5 in 50! = $\left[\frac{50}{5}\right]+\left[\frac{50}{25}\right]$ or 12 .

Hence number of zeros at the end of 50! = 12.

Q.32 (D)

${}^{200}C_{100}=\frac{200!}{100!\times 100!}$. Let [x] denote greatest integer less than or equal to x.

Exponent of 59 in 200! = $\left[\frac{200}{49}\right]$ or 4.

Exponent of 59 in 100! = $\left[\frac{100}{49}\right]$ or 2.

Hence exponent of 59 in ${}^{200}C_{100}$ = 0.

Exponent of 53 in 200! = $\left[\frac{200}{53}\right]$ or 3.

Exponent of 53 in 100! = $\left[\frac{100}{53}\right]$ or 1.

Hence exponent of 53 in ${}^{200}C_{100}$ = 1.

Exponent of 59 in 200! = $\left[\frac{200}{59}\right]$ or 3.

Exponent of 59 in 100! = $\left[\frac{100}{59}\right]$ or 1.

Hence exponent of 59 in ${}^{200}C_{100}$ = 1.

Thus 59 divides ${}^{200}C_{100}$. None of the numbers given options is largest divisor of ${}^{200}C_{100}$.

ASSERTION REASONING

Q.33 (A)

${}^{n}C_{r}=\frac{n(n-1)(n-2)...(n-r+1)}{r!}$.

As ${}^{n}C_{r}$ is an integer hence $n(n-1)(n-2)...(n-r+1)$ is divisible by r!.

Q.34 (A)

${}^{40}C_{r}{}^{60}C_{0}+{}^{40}C_{r-1}{}^{60}C_{1}+...+{}^{40}C_{0}{}^{60}C_{r}$ = coefficient of x^r in the expansion of $(1 + x)^{40}(1 + x)^{60}$.

i.e. coefficient of x^r in $(1 + x)^{100}$.
i.e. ${}^{100}C_r$.
Now ${}^{n}C_r$ is maximum for r = n/2 if n is even, hence r = 50.

Q.35 (D)

Statement – 2 is a standard result.
By the result given in statement – 2, number of solutions of $x_1 + x_2 + x_3 + ... + x_{20} = 100$ is ${}^{100+20-1}C_{20-1}$ or ${}^{119}C_{19}$ hence statement – 1 is false.

Q.36 (A)

Statement – 2 is a standard result (Using ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$).
For statement – 1

$$S = {}^{n}C_m + 2\,{}^{n-1}C_m + 3\,{}^{n-2}C_m + ... + (n-m+1)\,{}^{m}C_m$$

Using ${}^{n}C_r = {}^{n+1}C_{r+1} - {}^{n}C_{r+1}$

$$\Rightarrow S = {}^{n+1}C_{m+1} - {}^{n}C_{m+1} + 2\,{}^{n}C_{m+1} - 2\,{}^{n-1}C_{m+1} + 3\,{}^{n-1}C_{m+1} - 3\,{}^{n-2}C_{m+1} + ... + (n-m+1)\,{}^{m+1}C_{m+1} - (n-m+1)\,{}^{m}C_{m+1}$$

$$\Rightarrow S = {}^{n+1}C_{m+1} + {}^{n}C_{m+1} + {}^{n-1}C_{m+1} + {}^{n-2}C_{m+1} + ... + {}^{m+1}C_{m+1}$$

$$\Rightarrow S = {}^{n+2}C_{m+1}.$$

Q.37 (D)

Consider the following arrangement.

1		2		3		4		5		6	
	7				8						9

Here f(1) = 7, f(2) = f(3) = 8 & f(4) = f(5) = f(6) = 9
Clearly any such arrangement can be made by putting 1,2,3,4,5,6 at a gap each in on column(domain) and choosing 3 gaps to put 7, 8, 9 in second column(codomain) such that each element of codomain will be the value of function for all elements of domain on left till the previous element of codomain

1		2		3		4		5		6	

Also 9 must be put in last cell only as f(6) has to be the greatest.
Hence we have to chose 2 out of 5 gaps to put 7 & 8.
Total number of functions ${}^{5}C_2$ i.e. 10.
Statement 1 is false.
Statement 2 is clearly correct.

MATRIX MATCH TYPE

Q.38 $(\mathbf{A})\rightarrow(\mathbf{Q}),(\mathbf{B})\rightarrow(\mathbf{S}),(\mathbf{C})\rightarrow(\mathbf{S}),(\mathbf{D})\rightarrow(\mathbf{P})$

(A) Let the children be denoted as $C_1, C_2, C_3, C_4, C_5, C_6$ in increasing order of heights, where C_1 is the shortest child.
If C_6 is placed at first place, only arrangements possible is descending. Similarly for C_6 standing at last place only ascending order is possible. This can be done in 2 ways.
If C_6 is placed at second place, first place can be filled in 5 ways & rest in only descending order. Similarly for C_6 standing at second last place, last place can be filled in 5 ways & rest only in descending order. This can be done in 2×5 ways.
If C_6 is placed at second place, first & second place can be filled by any two children in 1 way & rest in only descending order. Similarly for C_6 standing at third from last place, last two places can be filled by any two of the remaining five in 1 ways & rest only in descending order. This can be done in $2\times {}^5C_2$ ways.
Hence all possible arrangements are 32.

(B) The children must be arranged such that C_1, C_2, C_3 are put in one order only(not to be arranged with each other.
Hence all possible arrangements are $\dfrac{6!}{3!} = 120$

(C) Let every child get 1 marble. Now remaining 4 marbles must be distributed such that nobody gets more that 3 marbles. Number of ways in which all 4 marbles can be given to one child in 6 ways and in general 4 marbles can be given to 6 boys in ${}^{4+6-1}C_{6-1}$ i.e. 126 ways.
Hence required number of ways = 126 – 6 or 120.

(D) C_6 must be put in first row of one column. Chose two more in ${}^5C_2$ ways to put in the column in which C_6 is in 1 way. The other three can be put in the other column in just 1 way.
Hence all possible arrangements are 20.

Q.39 $(\mathbf{A})\rightarrow(\mathbf{R}),(\mathbf{B})\rightarrow(\mathbf{R}),(\mathbf{C})\rightarrow(\mathbf{Q}),(\mathbf{D})\rightarrow(\mathbf{P})$

(A) Number of subsets of set $\{x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ = 2^7 or 128.

(B) Number of subsets of $\{x_1, x_2, x_3, ..., x_{10}\}$ which necessarily contain $\{x_1, x_2, x_3\}$ will also be same as that in (A) as after including $\{x_1, x_2, x_3\}$ rest of the elements have to be chosen from $\{x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ in 2^7 or 128 ways.

(C) Total number of subsets = 2^{10}.

Number of subsets not containing any of $\{x_1, x_2, x_3\}$ = 2^7.

Number of subsets containing at least one of $\{x_1, x_2, x_3\}$ = $2^{10} - 2^7$ or 896.

(D) Number of subset containing exactly one of $\{x_1, x_2, x_3\}$ = ${}^3C_1 \times 2^7$.

Number of subsets not containing any of $\{x_1, x_2, x_3\}$ = 2^7.

Number of subsets containing at the most one of $\{x_1, x_2, x_3\}$ = ${}^3C_1 \times 2^7 + 2^7$ or 512.

Q.40 $\mathbf{(A) \to (P), (B) \to (S), (C) \to (Q), (D) \to (Q)}$

(A)

ENDEA	N	O	E	L

Total ways to permute = 5!.

(B)

E								E

To fill remaining 7 places N-2, DAOEL-1 each.

Number of permutations = $\frac{7!}{2!}$ or $21 \times 5!$.

(C)

Arrange DLNN in first 4 places in $\frac{4!}{2!}$ ways and EEEAO in last 5 places in $\frac{5!}{3!}$ ways.

Number of permutations = $\frac{4! \times 5!}{2! \times 3!}$ or $2 \times 5!$.

(D)

AEEEO must be put in 1st, 3rd, 5th, 7th & 9th place only. Number of ways to do so $\frac{5!}{3!}$.

Now DNNL are to be put in rest of the places in $\frac{4!}{2!}$.

Number of permutations = $\frac{4! \times 5!}{2! \times 3!}$ or $2 \times 5!$.

PERMUTATIONS & COMBINATIONS

Exercise 2(C)

Q.1 [8]

The 2 specific persons can be allotted a seat in $2\times2\times2$ ways (2 ways to chose side, 2 to chose adjacent seats & 2 to arrange them).
Now remaining 4 seats can be allotted in ${}^{5}P_{4}$ ways.
Number of possible seating arrangements = $2\times2\times2\times{}^{5}P_{4}$ i.e. $8\times5!$.

Q.2 [8]

Number of ways in which 'r' people can be selected out of 'n' people sitting in a row, if no two of them are consecutive = ${}^{n-r+1}C_{r}$.
Hence $P_{n}={}^{n-2}C_{2}$.
Now ${}^{n-1}C_{3}-{}^{n-2}C_{3}=15\Rightarrow(n-2)(n-3)=45$ or $n=8$.

Q.3 [52]

Possible digits can be{ 3,3,2}, {3,2,2} or {2,3,x} where x can be any of {0,1,4,5,6,7,8,9}.
In first two cases number of numbers will be 3 each.
In third case if 0 is not taken, then number of numbers will be $7\times3!$.
If 0 is taken as one digit then number of numbers will be 2×2.
Total number of numbers = 6 + 42 + 4 = 52.

Q.4 [35]

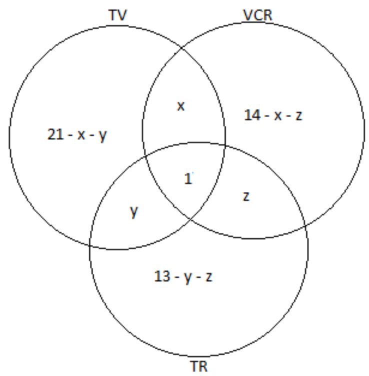
	+		+		+		+		+		+	

Put the 6 '+' signs at a gap each. Any 4 gaps out of these 7 gaps can be selected in ${}^{7}C_{4}$ ways to put the 4 '–' signs.
Hence number of arrangements = ${}^{7}C_{4}$ or 35.

Q.5 [8]

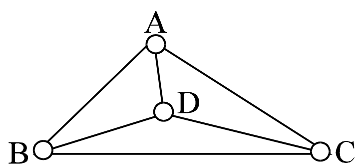
For the first letter we have 3 choices A, B & C and for rest of the places 2 choices each.
Hence an n – lettered word can be formed in $3\times2^{n-1}$ ways.
Now $3\times2^{n-1}$ = 384 gives n = 8.

Q.6 [10]



In adjoining Venn diagram
$x + y + z = 9$
Number of teachers who own none of three =
$50 - (21 - x - y) - (14 - x - z) - (13 - y - z) - x - y - z - 1.$
$= 50 - (49 - x - y - z) = 10.$

Q.7 [12]



To move ahead from A – 3 choices.
To move ahead of B/C/D – 2 choices.
To move ahead from next point – 2 choices.
Hence total number of ways $3\times2\times2$ or 12 ways.

Q.8 [21]

One Green ball (GRRRRR) – 6 ways
Two Green balls (GGRRRR) – 5 ways
Three Green balls (GGGRRR) – 4 ways
Four Green balls (GGGGRR) – 3 ways
Five Green balls (GGGGGR) – 2 ways
Six Green balls (GGGGGG) – 1 way.
All possible ways to fix 6 boxes = 1 + 2 + 3 + …+ 6 = 21.

Q.9 [82]

Exponent of 13 in 1000! = $\left[\frac{1000}{13}\right]+\left[\frac{1000}{13^2}\right]=76+5$ or 81.

Hence if $\frac{1000!}{13^n}$ is not an integer, then n must be at least 82.

Q.10 [36]

Rectangles of size 1×2 = 8,
Rectangles of size 1×3 = 7,
Rectangles of size 1×4 = 6,
$\vdots$
Rectangles of size 1×9 = 1.
Total number of non – congruent rectangles = 1 + 2 + 3 + … + 8 = 36.

Q.11 [23]

7056 = $2^4 \times 3^2 \times 7^2$, hence 7056 has (4 + 1)(2 + 1)(2 + 1) or 45 divisors.
Thus we can write 7056 as a product of two factors in 23 ways.

Q.12 [10]

Number of subsets containing 3 elements in which 3 is the least element = ${}^{n-3}C_2$

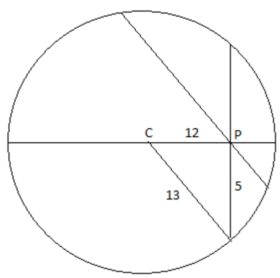
Number of subsets containing 3 elements in which 7 is the greatest element = ${}^{6}C_2$

Number of subsets containing 3 elements in which 3 is the least element as well as 7 is the greatest element = ${}^{3}C_2$

Hence number of subsets containing 3 elements in which 3 is the least element or is the greatest element = ${}^{n-3}C_2 + {}^{6}C_2 - 3$.

Now $\frac{(n-3)(n-4)}{2} + 15 - 3 = 33 \Rightarrow n^2 - 7n - 30 = 0$ or $n = 10$.

Q.13 [32]



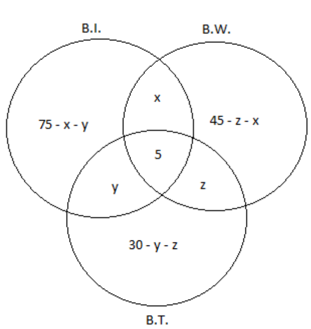
Longest possible chord will be the diameter through P, of length 26.
Shortest possible chord will be perpendicular to diameter through P, of length 10 units.
Possible lengths between 10 & 26 are 11, 12, 13, …, 25.
There will be two chords of each of these lengths and one chord each of length 10 & 26.Total number of chords = $2 \times 15 + 2$ or 32.

Q.14 [2]

${}^{20}C_0{}^{20}C_1 + {}^{20}C_1{}^{20}C_2 + \ldots + {}^{20}C_{19}{}^{20}C_{20}$ = coefficient of x^{19} in the expansion of $(1+x)^{20}(x+1)^{20}$.

= coefficient of x^{19} in $(1+x)^{40}$ = ${}^{40}C_{19}$ or ${}^{40}C_{21}$.As $r > 20$ hence $r = 21$.

Q.15 [50]



In adjoining Venn diagram
(75 – x – y) + (45 – z – x) + (30 – y – z) + x + y + z + 5 = 100.
Hence x + y + z + 5 = 50.

Q.16 [50]

Number of ways in which room A can be filled in $A_n = {}^nC_{25} \times 25!$ ways.

$$A_n - A_{n-1} = {}^{49}C_{25} \times 25!$$

$$\Rightarrow {}^nC_{25} - {}^{n-1}C_{25} = {}^{49}C_{25}$$

$$\Rightarrow \frac{n!}{25!(n-25)!} - \frac{(n-1)!}{25!(n-26)!} = \frac{49!}{25!24!}$$

$$\Rightarrow \frac{(n-1)!}{(n-25)!} = \frac{49!}{25!} \text{ or } n = 50$$

Q.17 [41]

108900 = $2^2 \times 3^2 \times 5^2 \times 11^2$.
Now number of divisors of 108900 = $(1 + 2)^4$ or 81.
Hence number of ways to write 108900 as a product of two factors = 41.

Q.18 [56]

Getting more than 25% marks means getting more than 3 marks out of maximum 12 marks.
(i) All 6 questions correct(12 marks) – 1 way (5 correct means all 6 correct).
(iii) 4 correct 2 wrong(6 marks) – ${}^6C_4$ ways.
(iv) 3 correct 3 wrong(3 marks) – ${}^6C_3 \times 2$ ways(3 wrong answers can be marked in 2 ways).
Total number of ways = 1 + 15 + 40 or 56.

Q.19 [9]

To form a 'k' letter palindrome we need to chose only 1st, 2nd& 3rd letter each in k ways.
4th& 5th digits must be same as 1st, 2nd digits.
Hence number of ways to form a palindrome = k^3.
$k^3 = 729 \Rightarrow k = 9$.

Q.20 [15]

Let there be n players.
2 players who withdraw after 3 games will play 6 matches.
Remaining (n – 2) players will play ${}^{n-2}C_2$ matches.
Hence ${}^{n-2}C_2 + 6 = 84$ or $n^2 - 5n - 150 = 0$.
Therefore n = 15.

PERMUTATIONS & COMBINATIONS

Exercise 3

Q.1

Number of ways to choose n people out of 2n = ${}^{2n}C_n$.
Number of ways to arrange these n people in a circle = (n – 1)!
Number of ways to arrange rest in a row = n!

Total number of arrangements = ${}^{2n}C_n \times (n-1)! \times n!$ i.e. $\frac{(2n)!}{n}$.

Q.2

Three cards must be from three of the four suits and two card must be from any one suit.
Number of ways to choose two cards from any one suit = ${}^{4}C_1 \times {}^{13}C_2$

Number of ways to choose three remaining cards from other three suits = $\left({}^{13}C_1\right)^3$

Total number of ways to choose 5 cards = ${}^{4}C_1 \times {}^{13}C_2 \times \left({}^{13}C_1\right)^3$ i.e. 685464.

Q.3

Number of ways to choose 12 people including A but not B or B but not A= $2 \times {}^{15}C_{11}$.

Number of ways to choose 12 people not including any of A & B = ${}^{15}C_{12}$.

Total number of ways to choose 12 people = $2 \times {}^{15}C_{11} + {}^{15}C_{12}$ i.e. 3185.

Q.4

For sum of numbers to be even we need to select
(i) two numbers odd & one even – ${}^{15}C_2 \times {}^{15}C_1$

(ii) all three numbers even – ${}^{15}C_3$

Hence total number of selections = ${}^{15}C_2 \times {}^{15}C_1 + {}^{15}C_3$ i.e. 2030.

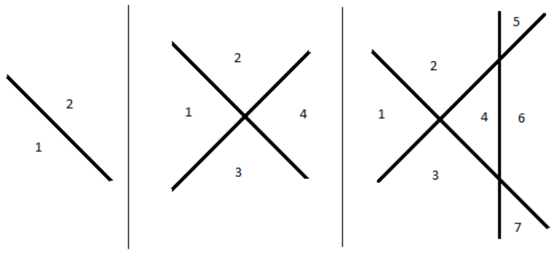
Q.5

Let A & B get x & y number of books respectively, then C will get $\frac{x+y}{2}$ number of books.

Now $x + y + \frac{x+y}{2} = 27 \Rightarrow x + y = 18$.

Now to distribute books, chose 9 books for C and then for rest of the 18 books we have 2 ways (A or B) for each book.
Number of ways to distribute = ${}^{27}C_9 \times 2^{18}$.

Q.6



As shown 1 line divides a plane in two parts, 2 lines in 4 parts, 3 lines in 7 parts…
Let An denote the number of parts in which n lines divide a plane, then
$A_1 - A_0 = 1, A_2 - A_1 = 2, A_3 - A_2 = 3$ implies
$A_n - A_{n-1} = n$.

Now $\sum_{n=2}^{20} A_n - \sum_{n=2}^{20} A_{n-1} = \sum_{n=2}^{20} n \;\Rightarrow\; A_{20} - A_1 = \dfrac{20\times 21}{2} - 1$ or $A_{20} = 211$.

Q.7

Let number of ways to climb n stairs by taking single or double steps be A_n.

If first step is a single step then number of ways to climb remaining stairs = A_{n-1}

If first step is a double step then number of ways to climb remaining stairs = A_{n-2}

Now $A_n = A_{n-1} + A_{n-2},\ n \ge 3$.

Also $A_1 = 1$ & $A_2 = 2$

Hence $A_3 = A_2 + A_1 = 3$,

$A_4 = A_3 + A_2 = A_1 + 2A_2 = 5$,

$A_5 = A_4 + A_3 = 2A_1 + 3A_2 = 8$,

$A_6 = A_5 + A_4 = 3A_1 + 5A_2 = 13$,

Therefore $A_{12} = 233$.

Q.8

If each side is divided in n parts then let number of required triangles be A_n.

Now $A_1 = 1, A_2 = 4, A_3 = 10, A_4 = 20, \ldots$

Explanation :
For one point on each side there will be 5 triangles out of which one will be facing down hence 4 triangles including ABC itself have same orientation.
For two points on each side there will be 14 triangles out of which 4 will be facing downwards hence 10 triangles including ABC itself have same orientation.

Now $A_2 - A_1 = 3, A_3 - A_2 = 6, A_4 - A_3 = 10$

Clearly $A_5 - A_4 = 15$ & $A_6 - A_5 = 21$.

Hence $A_5 = A_4 + 15 = 35$ & $A_6 = A_5 + 21 = 56$.

Q.9

We have N = $10^6 = 2^6 \times 5^6$.

Let $d_{i,j} = 2^i \times 5^j$ be a divisor of N where $0 \le i, j \le 6$.

Now Total number of factors = (6 + 1)(6 + 1) = 49.

Product of all the divisors = $\left(10^6\right)^{\frac{49}{2}} = 10^{147}$.

Q.10

x, y, z can be put such that y is between x & z in ${}^9C_3$ ways.

remaining 6 letters can be arranged in 6! ways.

Total number of ways = ${}^9C_3 \times 6!$.

Q.11

Number of ways to select two men = ${}^8C_2$.

Number of ways to select women = ${}^6C_2$ (As spouses can't be included)

Number of ways to form two mixed pair = 2.

Total number of ways = ${}^6C_2 \times {}^6C_2 \times 2 = 840$.

Q.12

n = $2^2 \times 3^2 \times 5$.

Number of factors of n = (2 + 1)(2 + 1)(1 + 1) = 18,

n^2 = $2^4 \times 3^4 \times 5^2$.

Number of factors of n^2 = (4 + 1)(4 + 1)(2 + 1) = 75

Number of positive integral divisors of n^2, which do not divide n

= number of factors of n^2 – number of factors of n = 75 – 18 = 57.

Q.13

In the word MISSISSIPPI, other than 4 S we have M, P, P, I, I, I, I.

These can be arranged in $\frac{7!}{4!\times 2!}$.

Now we can put 4 S in 8 gaps in ${}^{8}C_{4}$ ways.

Hence total number of ways = $\frac{7!}{4!\times 2!}\times {}^{8}C_{4}$ i.e. 7350.

Q.14

Total number ways to select two adjacent squares =

$(7 \text{ in each row})\times(8 \text{ rows})+(7 \text{ in each column})\times(8 \text{ columns})=112$

Number ways to choose two squares at random = ${}^{64}C_{2}$.

the number of ways of keeping 2 identical kings on an 8 × 8 chess-board so that they are not in adjacent squares = ${}^{64}C_{2}-112=1904$.

Q.15

For no two HEADs being consecutive, number of times a HEAD appears must be less than or equal to 5. Now

0 HEAD & 10 TAILs – 1 way

1 HEAD & 9 TAILs – 10 ways

2 HEADs & 8 TAILs – ${}^{9}C_{2}$ ways (9 gaps created by 8 TAILs to put 2 HEADS)

3 HEADs & 7 TAILs – ${}^{8}C_{3}$ ways (8 gaps created by 7 TAILs to put 3 HEADS)

4 HEADs & 6 TAILs – ${}^{7}C_{4}$ ways (7 gaps created by 6 TAILs to put 4 HEADS)

5 HEADs & 5 TAILs – ${}^{6}C_{5}$ ways (6 gaps created by 5 TAILs to put 5 HEADS)

Total number of ways = $1+10+{}^{9}C_{2}+{}^{8}C_{3}+{}^{7}C_{4}+{}^{6}C_{5}$ = 144.

Q.16

(i) Keeping the central seat vacant we have 10 seats out of which 5 are to be occupied so number of arrangements will be ${}^{10}C_{5}\times 5!$.

(ii) For keeping symmetrically opposite seats vacant seats can be occupied in the following ways

Case 1 : 0 seat on left 5 on right as shown(shaded seats are occupied) - ${}^{5}C_{0}\times 5!$ ways

OR similarly

0 seat on right and 5 on left - ${}^{5}C_0 \times 5!$ ways

Case 2 : 1 seat on left 4 on right as shown(shaded seats are occupied) - ${}^{5}C_1 \times 5!$ ways

OR similarly

1 seat on right and 4 on left - ${}^{5}C_1 \times 5!$ ways

Case 3 : 2 seat on left 3 on right as shown(shaded seats are occupied) - ${}^{5}C_2 \times 5!$ ways

OR similarly

2 seat on right and 3 on left - ${}^{5}C_2 \times 5!$ ways

Total number of ways = $\left({}^{5}C_0 + {}^{5}C_1 + {}^{5}C_2\right) \times 5! \times 2 = 3840$.

Q.17

A selection of any four vertices gives one point of intersection of diagonals lying inside the octagon.(diagonals of a quadrilateral intersect inside it).

Hence required number of points = ${}^{8}C_4$ = 70.

Q.18

The gaps between selected points sum up to 9 so the required number of selections will be the number of ways to divide 9 points in 6 groups where 1st and 6th groups (not simultaneously) may be empty but rest must have at least one point.

If the 1st group is empty, then number of positive integral solutions of $a_2 + a_3 + a_4 + a_5 + a_6 = 9$ will be ${}^{9-1}C_{5-1}$ i.e. 70.

If the 6th group is empty, then number of positive integral solutions of $a_1 + a_2 + a_3 + a_4 + a_5 = 9$ will be ${}^{9-1}C_{5-1}$ i.e. 70.

Total number of solutions = 140.

Alternately

Required number is number of ways to choose 6 points out of 15 points in a circular arrangement such that no two of the selected points are adjacent.

Required number of ways = ${}^{10}C_6 - {}^{8}C_4 = 140$
(Number of ways to choose r objects out of n objects arranged in a circle such that no two of the selected objects are adjacent = ${}^{n-r+1}C_r - {}^{n-r-1}C_{r-2}$)

Q.19

We can give each person a glove for right hand then distribute gloves for left hand such that nobody gets a pair.

Required number of ways = (Number of ways to permute 5 objects) $\times$ (number of ways to derange 5 objects) = $\left(\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)\times(5!)^2 = 5280$.

Q.20

Required number of ways = $3^6 - {}^{3}C_1 \times 2^6 + {}^{3}C_2 \times 1^6$ = 540.
(By Principle on Inclusion & Exclusion)

Q.21

Gaps created by m 0s = m + 1

Number of ways to put n 1s in these gaps = ${}^{m+1}C_n$.

Q.22

All 7 tiles of size 1×1 : 1 way.

5 tiles of size 1×1 & 1 of size 1×2 : 6 ways

3 tiles of size 1×1 & 2 of size 1×2 : 10 ways

1 tile of size 1×1 & 3 of size 1×2 : 4 ways

4 tiles of size 1×1 & 1 of size 1×3 : 5 ways

1 tile of size 1×1 & 2 of size 1×3 : 3 ways

2 tiles of size 1×1, 1 of size 1×2 & 1 of size 1×3 : 12 ways

2 tiles of size 1×2 & 1 of size 1×3 : 3 ways

Total = 44 ways.

Q.23

Seating arrangements can be made in following manner

Table 1	Table 2	Table 3	Arrangements
1	1	4	$\frac{6!}{4!2!}\times3!$ = 90

1	2	3	$\frac{6!}{2!3!}\times 2! = 120$
2	2	2	$\frac{6!}{2!2!2!3!} = 15$

Total number of arrangements = 225.

Q.24

(i) Numbers with 1 & 2 at first place = $2\times 4! = 48$.

Numbers with 31, 32, 34 at first two places = $3\times 3! = 18$

Numbers with 351, 352 at first three places = $2\times 2! = 4$

Number 35412, then comes 35421.

Rank of 35421 = 72^{nd}.

(ii) Total numbers = 5! = 120

Numbers with 5 at first place = 4! = 24

Numbers before those with 5 at first place = 96

Numbers after that 51234, 51243, 51324, 51342…

100^{th} number = 52143.

Q.25

Number 111…11 (2010 digits) is not a perfect square hence number of divisors is even.

Q.26

Number of divisors of given number is (6 + 1)(4 + 1)(2 + 1)(4 + 1) = 525.

Out of these divisors $2^3.3^2.5.7^2 = \sqrt{n}$, hence 262 divisors are less than $\sqrt{n}$ & 262 are greater than $\sqrt{n}$.

Q.27

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{100}\Rightarrow 100x+100y=xy \text{ or } (x-100)(y-100)=10000$$

$\Rightarrow (x-100)(y-100) = 2^4.5^4$

Number of divisors of $2^4.5^4$ = (4 + 1)(4 + 1) = 25.

Also each value of (x – 100) gives a unique value of (y – 100).

Hence number of solutions = 25.

Q.28

Let the selected terms be a, b, c.

Now a + c = 2b implies we need to select a & c such that their sum is even.

Number of ways to select two odd numbers = ${}^{50}C_2$

Number of ways to select two even numbers = ${}^{50}C_2$

Total number of selections = $2 \times {}^{50}C_2$ = 2450.

Q.29

126000 = $2^4.3^2.5^3.7$

Now LCM(a, b) = 2^4 in 9 ways (a can take any value from $2^0, 2^1, 2^2, 2^3, 2^4$. Similarly b can take any value from $2^0, 2^1, 2^2, 2^3, 2^4$. hence 5 + 5 – 1 = 9)

Similarly LCM(a, b) = 3^2 in 5 ways.

LCM(a, b) = 5^3 in 7 ways.

LCM(a, b) = 7 in 3 ways.

Hence LCM(a, b) = $2^4.3^2.5^3.7$ in $9 \times 5 \times 7 \times 3$ i.e. 945 ways.

Now number of unordered pairs (a, b) = $\frac{945+1}{2} = 473$ ways.

Q.30

Let the vertices of the given polygon be $A_1, A_2, ..., A_{21}$.

For isosceles triangles : choose any one vertex say A_1 and then take two other vertices at equal gap, one from left and other from right of A_1. This gives 10 triangles for each vertex (From $\Delta A_{21}A_1A_2$ till $\Delta A_{11}A_1A_{10}$).

Hence supposed number of triangles = 21×10.

Now in above counting equilateral triangles have been counted more than once as explained below –

To form equilateral triangle we need to choose three points dividing remaining 18 points in equal groups so by joining every 7^{th} point we get an equilateral triangle e.g. $\Delta A_1A_8A_{15}$.

Now each vertex will be a part of three equilateral triangles so actually there will be 7 equilateral triangles but in our counting of isosceles triangles this has been taken as $7 \times 3 = 21$ triangles.

Hence required number of triangles = 210 – 14 = 196.

Q.31

To form a five digit number each of the ten digits except 0 can be taken 5 times.

Required number of such numbers is coefficient of x^5 in $\left(1 + x + x^2 + ... + x^5\right)^{10} - 1$

i.e. coefficient of x^5 in $\left(1 - x^5\right)^{10}\left(1 - x\right)^{-10} - 1$

Hence total number of such numbers is ${}^{10+5-1}C_5 - 1$ or ${}^{14}C_5 - 1$.

Alternately

We can chose 5 digits in the following way

(i) all distinct : ${}^{10}C_5$

(ii) two alike and 3 distinct : $10 \times {}^{9}C_3$

(iii) two alike, two alike one distinct : ${}^{10}C_2 \times 8$

(iv) three alike two distinct : $10 \times {}^{9}C_2$

(v) three alike two alike : 10×9

(vi) four alike one distinct : 10×9

(vii) all five alike : 9

As no permutation is needed, so total number of numbers

$$= {}^{10}C_5 + 10\times{}^{9}C_3 + {}^{10}C_2\times 8 + 10\times{}^{9}C_2 + 10\times 9 + 10\times 9 + 9 \text{ i.e. } 2001 \text{ or } {}^{14}C_5 - 1.$$

Q.32

Consider $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r \quad \ldots(i)$

Differentiate both the sides with respect to x to get

$$n(1+x)^{n-1} = \sum_{r=0}^{n} r\times{}^nC_r\, x^{r-1} \quad \ldots(ii)$$

Multiply (i) & (ii) to get

$$n(1+x)^{2n-1} = \left(\sum_{r=0}^{n} r\times{}^nC_r\, x^{r-1}\right)\left(\sum_{r=0}^{n} {}^nC_r\, x^{r-1}\right) \quad \ldots(iii)$$

Coefficient of x^n in R.H.S. of (iii) is $\sum_{r=1}^{n} r\times{}^nC_r\times{}^nC_{n-r}$ or $\sum_{r=1}^{n} r\times\left({}^nC_r\right)^2$ &

Coefficient of x^n in L.H.S. of (iii) is $n\times{}^{2n-1}C_n$, hence

$$({}^nC_1)^2 + 2({}^nC_2)^2 + 3({}^nC_3)^2 + \ldots + n({}^nC_n)^2 = n\times{}^{2n-1}C_n \text{ i.e. } \frac{(2n-1)!}{(n-1)!(n-1)!}.$$

Q.33

Let S_p be the set of ordered triplets (a, b, c) such that

$c \ge b \ge a \ge 1$, $a + b + c = p$, and $a + b > c$ (the triangle inequality).

For example, $S_3 = \{(1,1,1)\}$.

Each triplet in S_{p-2} gives rise to a triplet in S_p under the transformation

$(a, b, c) \to (a, b + 1, c + 1)$,

and all triplets in S_p are produced by this transformation except those of the form (a, a, c).

These latter triplets can be counted directly as follows:

there is one for each value of a for which $3a \le p < 4a$,

and so there are exactly $I_p = \left\lfloor \frac{p}{3} \right\rfloor - \left\lceil \frac{p+1}{4} \right\rceil + 1$ such triangles, where $\lfloor\ \rfloor$ & $\lceil\ \rceil$ denote greatest integer & least integer function respectively.

So we have the recurrence $S_p = S_{p-2} + I_p$, with initial conditions $S_0 = S_1 = 0$.

From this follows

$$T(n) = \begin{cases} \left[\dfrac{(n+3)^2}{48}\right], & \text{if } n \text{ is odd} \\ \left[\dfrac{n^2}{48}\right], & \text{if } n \text{ is even} \end{cases}$$, where [.] denotes greatest integer function.

Q.34

A cube can be rotated into 6 × 4 = 24 configurations (i.e. the red face can be any one of the 6, and then there are 4 ways to rotate it that keep that face red),

so the number of different colourings (counting rotations, but not mirror reflections, as the same) is 6!/24 = 30.

Q.35

By joining n points on AB to C will divide the triangle into n + 1 triangles.

Now n line joining n points on AC to be will divide each of these triangles into n + 1 parts.

Hence total number of parts = (n + 1) × (n + 1).

Q.36

Least number 1 & greatest number $\underbrace{99...9}_{9 \text{ digits}}$.

(i) Number of ascending numbers of r – digits = coefficient of x^r in $\left(1+x+x^2+...+x^r\right)^9$

$\Rightarrow$ coefficient of x^r in $\left(1-x^{r+1}\right)^9 (1-x)^{-9}$ i.e. ${}^{9+r-1}C_r$ or ${}^{8+r}C_8$.

Total numbers of type (a) = ${}^9C_8 + {}^{10}C_8 + {}^{11}C_8 + ... + {}^{17}C_8$ or ${}^{18}C_9 - 1$.

(ii) Number of strictly increasing numbers of r – digits = ${}^9C_r$.

Total numbers of type (b) = ${}^9C_1 + {}^9C_2 + {}^9C_3 + ... + {}^9C_9$ or $2^9 - 1$.

Q.37

$$\sum_{r=1}^{n} {}^{n}P_r = \frac{n!}{(n-1)!} + \frac{n!}{(n-2)!} + \frac{n!}{(n-3)!} + ... + n!$$

$$e = \sum_{r=0}^{\infty} \frac{1}{r!} \Rightarrow e \times n! = \left\{ n! + \frac{n!}{1!} + \frac{n!}{2!} + \frac{n!}{3!} + ... + \frac{n!}{(n-1)!} \right\} + \frac{n!}{n!} + \frac{n!}{(n+1)!} + \frac{n!}{(n+2)!} + \ldots \infty \text{ terms}$$

$$\Rightarrow e \times n! = \sum_{r=1}^{n} {}^{n}P_r + \frac{n!}{n!} + \frac{n!}{(n+1)!} + \frac{n!}{(n+2)!} + \ldots \infty \text{ terms}$$

$$\Rightarrow e \times n! - 1 = \sum_{r=1}^{n} {}^{n}P_r + \frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms}$$

$$\Rightarrow [e \times n! - 1] = \sum_{r=1}^{n} {}^{n}P_r + \left[\frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms} \right]$$

Now,

$$n + r > n + 1 \Rightarrow \frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms} < \frac{1}{(n+1)} + \frac{1}{(n+1)^2} + \ldots \infty \text{ terms}$$

$$\Rightarrow \frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms} < \frac{\frac{1}{(n+1)}}{1 - \frac{1}{(n+1)}}$$

$$\Rightarrow \frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms} < \frac{1}{n}$$

$$\Rightarrow \left[\frac{1}{(n+1)} + \frac{1}{(n+2)(n+1)} + \ldots \infty \text{ terms} \right] = 0$$

$$\Rightarrow [e \times n! - 1] = \sum_{r=1}^{n} {}^{n}P_r$$

Q.38

$${}^{2n}C_n = \frac{(2n)!}{n!n!} \Rightarrow {}^{2n}C_n = 2^n \frac{1.3.5...(2n-1)}{1.2.3...n}.$$

Q.39

Let $a(n) = 2^{3n}3^{n}$ & $b(n) = (4n)!$

$a(1) = 24$ & $b(1) = 24 \Rightarrow a(1)$ divides $b(1)$.

Now $a(n+1) = 2^{3n+3}3^{n+1}$ & $b(n+1) = (4n+4)!$

or $a(n+1) = 24a(n)$ & $b(n+1) = (4n+4)(4n+3)(4n+2)(4n+1)b(n)$

$$\frac{b(n+1)}{a(n+1)} = \frac{(4n+4)(4n+3)(4n+2)(4n+1)}{4!}\frac{b(n)}{a(n)}$$

As $r!$ divides product of r consecutive numbers, hence if $a(n)$ divides $b(n)$, then $a(n + 1)$ divides $b(n + 1)$. By mathematical induction $a(n)$ divides $b(n)$.

Q.40

Let $a(n) = 2^{n}$ & $b(n) = (n+1)(n+2)...2n$

$a(1) = 2$ & $b(1) = 2 \Rightarrow a(1)$ divides $b(1)$.

Now $a(n+1) = 2^{n+1}$ & $b(n+1) = (n+2)(n+3)...(2n+2)$

or $a(n+1) = 2a(n)$ & $b(n+1) = 2(2n+1)b(n)$

$$\frac{b(n+1)}{a(n+1)} = (2n+1)\frac{b(n)}{a(n)}$$

hence if $a(n)$ divides $b(n)$, then $a(n + 1)$ divides $b(n + 1)$.
By mathematical induction $a(n)$ divides $b(n)$.