EXERCISE 1 (A)

ONLY ONE OPTION IS CORRECT

1.

[Sol. $5 \begin{matrix} / 3M \\ \backslash 2W \end{matrix}$; $n(S) = {}^5C_3 = 10$

$n(A) = {}^3C_1 \cdot {}^2C_2 = 3$

$\therefore \quad P(2W \text{ and } 1M) = 3/10$

So, P(2W and 1M & chair person is woman) = $\frac{3}{10} \cdot \frac{2}{3} = \frac{1}{5}$

2.

[Hint: $n(S) = {}^{12}C_4 = 55 \times 9 = 495$

$n(A) = {}^6C_1 . {}^5C_2 . 2^2 = 6 \times 10 \times 4$

$P(E) = \frac{6\times10\times4}{55\times9} = \frac{2.2.4}{11.3} = \frac{16}{33}$ Ans]

3.

[Hint: $p^2 + 2p + 4p - 1 = 1$ (exhaustive)

$p^2 + 6p - 2 = 0 \quad \Rightarrow \quad$ (A)]

4.

[Hint : $\frac{{}^7C_3 + {}^7C_5}{{}^9C_5}$]

5.

[Hint: n(S) = number of ways in which 20 people can be divided into two equal groups = $\frac{20!}{10!\,10!\,2!}$

n(A) = 18 people can be divided into groups of 10 and 8 = $\frac{18!}{10!\,8!}$

$P(E) = \frac{18!}{10!\,8!} \cdot \frac{10!\,10!\,2}{20!} = \frac{10.9.2}{20.19} = \frac{9}{19}$ Ans]

6.

[Sol. $n(S) = 9 \cdot 10 = 90$

$n(A) = 1 + 2 + 3 + ... + 7 = \frac{7\cdot8}{2} = 28$

$p = \frac{28}{90} = \frac{14}{45}$ Ans.]

x (Tens)	y(units)
1 or 2 is not possible at ten's place	
3	0
4	0,1
5	0,1, 2
⋮	
9	0,1, 2, 3, 4, 5, 6

7.

[Hint: Number should be divisible by 2 and 3.

$n(S) = 5 \cdot 5!$; n (A) : reject '0' $= 2 \cdot 4!$

reject 3, $\quad 4! + 2 \cdot 3 \cdot 3!$

Total $n(A) = 3 \cdot 4! + 6 \cdot 3! = 18 \cdot 3!$

$\therefore \quad p = \frac{18\cdot3!}{5\cdot5!} = 18\%$]

8.

[Hint: Check whether $p_1 + p_2 + p_3 + p_4 = 1$ or not

9

[Hint : P(E) = 1 – P(all different) = 1 – (6/6) · (5/6) · (4/6) = 1 – (120/216) = 4/9]

10.

[Hint: $10 \left\langle \begin{matrix} 4D \\ 6G \end{matrix} \right. \xrightarrow[5]{} \left\langle \begin{matrix} 2D \\ 3G \end{matrix} \right.$

$$p = \frac{{}^4C_2 \cdot {}^6C_3}{{}^{10}C_5} = \frac{10}{21} \Rightarrow \text{(D)} \;]$$

11.

[Hint: $52 \xrightarrow[\text{10's}]{\text{face cards}} 36 \left\langle \begin{matrix} 9\text{ H} \\ 9\text{ S} \\ 9\text{ D} \\ 9\text{ C} \end{matrix} \right.$;

$$P(A) = \frac{1}{9} \;;\; P(H) = \frac{1}{4} \;;\; P(S) = \frac{1}{4} \;;\; P(A \cap H) = \frac{1}{36} \;;\; P(A \cap S) = \frac{1}{36} \;;\; P(A \cup S) = \frac{1}{3} \;]$$

12.

[Sol. P(a) = 0.3 ; P(b) = 0.5 ; P(c) = 0.2 $\Rightarrow$ a, b, c are exhaustive

P(same horse wins all the three races) = P(aaa or bbb or ccc)

$$= (0.3)^3 + (0.5)^3 + (0.2)^3 = \frac{27+125+8}{1000} = \frac{160}{1000} = \frac{4}{25}$$

P(each horse wins exactly one race)

$$= \text{P(abc or acb or bca or bac or cab or cba)} = 0.3 \times 0.5 \times 0.2 \times 6 = 0.18 = \frac{9}{50} \;]$$

13

[Sol. $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$0.7 = 0.4 + p - 0.4p$

$\therefore \quad 0.6p = 0.3 \Rightarrow p = 1/2$]

14.

[Hint: $0.8 = 0.3 + (1 - a) - 0.3(1 - a) \Rightarrow a = 2/7$]

15.

[Hint : $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^7C_1}{{}^8C_2 + 1} = \frac{7}{29}$; A : 11 is picked , B : sum is even]

16.

[Hint : $P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$

$$= \frac{P(A)}{P(A) + P(B)} = \frac{0.10}{0.10 + 0.32} \;]$$

17.

[Hint : 1 – P (Determinant has negative value)

$$1-\frac{3}{16}=\frac{13}{16}\left(\begin{vmatrix}1 & 1\\1 & 0\end{vmatrix};\begin{vmatrix}0 & 1\\1 & 1\end{vmatrix};\begin{vmatrix}0 & 1\\1 & 0\end{vmatrix}\right)\ ]$$

18.

[Hint: n(S) = × × × × × × × = 15^7; n (A) = $9^7 - 8^7$]

19.

[Hint: Even chance means probability is half. Suppose *n* cards are drawn

P(E) = P(S or FS or FFS...... n terms)

$$=\frac{P(S)[1-(PF)^n]}{1-P(F)}=1-\left(\frac{3}{4}\right)^n\ge\frac{1}{2}\quad\Rightarrow\quad\left(\frac{3}{4}\right)^n\le\frac{1}{2}$$

$n_{min} = 3$ Ans.]

]

20.

[Sol. Let A : event that the three letters are palindrome

B : event that the three digits are palindrome

$$P(A)=\frac{26^2}{26^3}=\frac{1}{26}\qquad (L_1L_2L_3)\ ;\ |||ly\ \ P(B)=\frac{10}{10\cdot 10}=\frac{1}{10}\qquad \underset{\text{(there are 10 digits 0 – 9)}}{\text{abc}}$$

hence, $P\ (A\text{ or }B) = P(A) + P(B) - P(A\cap B) = \frac{1}{26}+\frac{1}{10}-\frac{1}{26\cdot 10}=\frac{10+26-1}{260}=\frac{7}{52}$ Ans.]

21.

[Sol. From the given data n (S) = N^n ; n (A) = n ! $\Rightarrow$ $P_1=\frac{n!}{N^n}$

$P_1=\frac{n!}{N^n}$ Since the n different gift coupans can be placed in the n definite (Out of N) envelope in ${}^nP_n$ = n ! ways

$P_2=\frac{N!}{(N-n)!\,N^n}$ As n arbitrary envelopes out of N given envelopes can be chosen in ${}^NC_n$ ways and the n gift coupans can occupy these envelopes in n ! ways.]

$$=\frac{N!}{N^n\,(N-n)!}\ ]$$

22.

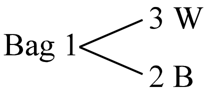
[Hint: P (S $\cap$ F) = 0.0006, where S : moter cycle is stolen ; F : moter cycle found

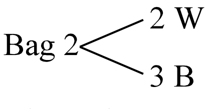
P (S) = 0.0015

$$P\ (F/S)=\frac{P(F\cap S)}{P(S)}=\frac{6\times 10^{-4}}{15\times 10^{-4}}=\frac{2}{5}\quad\Rightarrow\quad (B)\ ]$$

23

[Hint:





$$P(E) = P\ [W\ B\ W\ B]=\frac{2}{5}\cdot\frac{2}{6}\cdot\frac{1}{5}\cdot\frac{1}{6}=\frac{1}{225}\qquad]$$

24.

[Hint: P (6) = { (51, 15, 24, 42) } or { 11 & (22 or 13 or 31) or (22 & 11) }]

25.

[Hint: P $(A \cap C) = P(A) \cdot P(C)$

$$\frac{1}{20} = \frac{1}{5} \cdot P(C) \Rightarrow P(C) = \frac{1}{4}$$

now $P(B \cup C) = \frac{1}{6} + \frac{1}{4} - P(B \cap C)$, hence $P(B \cap C) = \frac{3}{8} - \frac{1}{3} = \frac{1}{24} = P(B) \cdot P(C) \Rightarrow$ (A)]

26.

[Hint: E : $B_1\ B_2\ B_3 \times \times \times$ where × means B or G

$$P(E) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = \frac{8}{64}$$ Ans.]

27.

[Sol. P (number chosen is odd) = 3/5

P (number chosen is even) = 3/5

E: (ab + c) is even ;

note that event E can occus in two cares

E_1 : all the three number a, b and c are odd; $P(E_1) = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$

E_2 : c is even and atleast one of a or b is even

$$P(E_2) = \frac{2}{5} \cdot \left(\frac{2}{5} + \frac{2}{5} - \left(\frac{2}{5}\right)^2\right) = \frac{2}{5} \cdot \left(\frac{4}{5} - \frac{4}{25}\right) = \frac{2}{5} \cdot \frac{16}{25} = \frac{32}{125}$$

$$P(E) = P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \frac{59}{125}$$ Ans.]

28.

[Hint: $p = \frac{1}{5} = 0.2$; $q = 0.8$; P (E) = 1 – P (0 or 1)]

29.

[Sol. We want to fail the first try, so we have $\frac{5}{6} \cdot \frac{1}{5} = \frac{1}{6}$ for the probability. The odds are therefore 1 : 5.]

30.

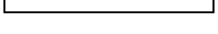
[Hint: $9 \begin{cases} 6R \\ 3G \end{cases}$

E : Event that the 2nd drawn marble is red; R : 1st drawn is red; G = 1st drawn is green

$P(E) = P(E \cap R) + P(E \cap G)$

$= P(R) \cdot P(E/R) + P(G) \cdot P(E/G)$

$$= \frac{6}{9} \cdot \frac{5}{8} + \frac{3}{9} \cdot \frac{6}{8} = \frac{48}{72} = \frac{2}{3}$$]



31.

[Hint: 6 persons $\begin{cases} 5 \text{ coffee} \\ 1 \text{ tea} \end{cases}$

Total number of ways in which 3 persons one of which drinks tea and 2 others can be selected

$= {}^1C_1 \cdot {}^5C_2$ ways

number of ways any 3 can be selected ${}^6C_3$

$$\therefore \quad P(E) = \frac{{}^5C_2}{{}^6C_3} = \frac{10}{20} = \frac{1}{2} \;]$$

32.

[Sol. A = Rose bush has withered

B_1 = Gardener did not water the rose bush $P(B_1) = 2/3$

B_2 = Gardener watered the rose bush $P(B_2) = 1/3$

$$P(A/B_1) = \frac{3}{4} \;\; ; \;\; P(A/B_2) = \frac{1}{2}$$

$$P(B_1/A) = \frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1) + P(B_2).P(A/B_2)} = \frac{\frac{2}{3}.\frac{3}{4}}{\frac{2}{3}.\frac{3}{4} + \frac{1}{3}.\frac{1}{2}} = \frac{6}{6+2} = \frac{3}{4} \text{ Ans]}$$

33.

[Hint: P (A) = p

p (object is not detected in one cycle) = 1 – p

p (object is not detected in *n* cycle) = $(1 - p)^n$

p (object will be detected) = $1 - (1 - p)^n$]

34.

[Sol. n (S) = number of ways in which two numbers are drawn in a definite order

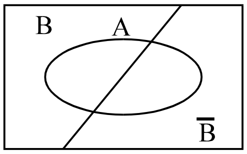
$= 9 \times 8 = 72$

n (A) = any one number from 0, 2, 4, 6, 8 can be taken in ${}^5C_1$ ways and any one can be taken from the remaining 8 in ${}^8C_1$ ways.

Hence total ways = $8 \times 5 = 40$; $p = \frac{40}{72} = \frac{5}{9}$]

35.

[Sol. $P(A) = P\{(B \cap A) \cup (\overline{B} \cap A)\} = P(B \cap A) + P(\overline{B} \cap A)$



$$= P(B)\,P(A/B) + P(\overline{B})\,P(A/\overline{B}) = \frac{4}{52} \cdot \frac{3}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{1}{13}$$

also, $P(B) = \frac{1}{13}$]

36.

[Sol. So leah can start watching 3 seconds before the green-to-yellow change, or start watching right at the yellow-to-red change. She can also start watching any time during that interval, and still see a colour change. That interval is 9 seconds long, while the entire cycle is 63.

$$P = \frac{9}{63} = \frac{1}{7} \text{ Ans.]}$$

37.

[Sol. Let E : B obtains more head than A

P (E) = P[B throws 3H and A throws 0 or 1 or 2 H

or B throws exactly 2H and A throws 1H or no head or B throws exactly 1 H and A throws no head

$$= \frac{1}{8}\cdot 1 + \frac{3}{8}\left(\frac{1}{2}+\frac{1}{4}\right) + \frac{3}{8}\left(\frac{1}{4}\right)$$

$$= \frac{4}{32} + \frac{9}{32} + \frac{3}{32} = \frac{16}{32} \text{ Ans.}$$

Note that in case of one extra coin the probability is independent of the number of coins held by the players.]

38.

[Sol. Method-1 : since the 2^{nd} is known to be W, there are only 3 ways of the remaining 8 in which the 1^{st} can be white, so that probability = 3/8

Method-2 : A – 2^{nd} drawen found to be white

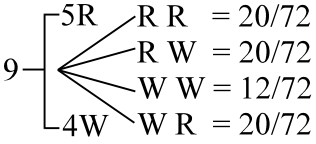
B_1 - 1^{st} drawn is W ; B_2 – 1^{st} drawn is R

$$P(B_1) = \frac{4}{9} \quad ; \quad P(B_2) = \frac{5}{9}$$

$$P(A / B_1) = \frac{3}{8} \ ; \quad P(A / B_2) = \frac{4}{8}$$

$$P(B_1 / A) = \frac{\frac{4}{9}\cdot\frac{3}{8}}{\frac{4}{9}\cdot\frac{3}{8}+\frac{5}{9}\cdot\frac{4}{8}} = \frac{12}{12+20} = \frac{12}{32} = \frac{3}{8}$$

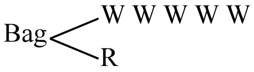
Alternatively :



$$P[(W W) / (R W \text{ or } W W)] = \frac{12}{32} = \frac{3}{8} \text{ Ans.]}$$

39

[Sol.



P(A) = P(R or W W R or W W W W R)

$$= \frac{1}{6} + \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} + \frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}; \qquad \therefore \qquad P(B) = \frac{1}{2} \ ]$$

40.

[Hint: $p(S) = P\big(A \text{ and } (B \text{ or } C)\big) = p\cdot\frac{1}{2}\left(q+\frac{1}{2}\right)$

$$\frac{1}{2} = \frac{p}{2}\left(q+\frac{1}{2}\right); \qquad 1 = p\left(q+\frac{1}{2}\right) \Rightarrow (D)\ ]$$

EXERCISE 1 (B)

MORE THAN ONE OPTIONS MAY BE CORRECT

1

[Sol. P(E) = P (at least one of the dice shows up the face 5)

= 1 – P(none shows up the face five)

$$= 1 - \frac{25}{36} = \frac{11}{36}$$

|||ly P(F) = 1 – P(none of the dice shows up the face one)

$$= 1 - \frac{25}{36} = \frac{11}{36}$$

E = {15, 25, 35, 45, 55, 65, 51, 52, 53, 54, 56}

F = {11, 12, 13, 14, 15, 16, 21, 31, 41, 51, 51, 61}

obviously E and F are neither mutually exceusively nor independent

but they are equiprobable ⇒ **(C)** and **(D)**]

2

[Sol. (B) $1 \geq P(A) + P(B) - P(A \cap B)$ or $P(A \cap B) \geq P(A) + P(B) - 1 \Rightarrow$ (B)

(C) Let $P(A) > P(A/B)$

or $P(A) > \dfrac{P(A \cap B)}{P(B)}$

$P(A).\ P(B) > P(A \cap B)$ ....(1)

TPT $P(A/B^C) > P(A)$

$\dfrac{P(A \cap B^c)}{P(B^c)} > P(A)$

$P(A) - P(A \cap B) > P(A)\ [1 - P(B)]$

$- P(A \cap B) > - P(A).\ P(B)$

or $P(A).\ P(B) > P(A \cap B)$ ....(2)

from (1) and (2) $P(A) > P(A/B) \Leftrightarrow P(A/B^c) > P(A)$

3

[Hint: For (A) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

⇒ T.P.T. $P(A \cap B) \geq P(A) + P(B) - 1$

or T.P.T. , $1 \geq P(A) + P(B) - P(A \cap B)$

or T.P.T. , $1 \geq P(A \cup B)$

which is true ⇒ (A) is correct

(B) and (C) are obvious]

4 [Hint : $P\left(E_2/E_1\right) = \dfrac{P\left(E_1 \cap E_2\right)}{P\left(E_1\right)}$

$$\frac{1}{2} = \frac{P\left(E_1 \cap E_2\right)}{1/4} \Rightarrow P\left(E_1 \cap E_2\right) = \frac{1}{8} = P\left(E_2\right).\ P\left(E_1/E_2\right)$$

$= P(E_2).\ \frac{1}{4} \quad \Rightarrow \quad P(E_2) = \frac{1}{2}$

Since $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1).\ P(E_2) \quad \Rightarrow$ events are independent

Also $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8} \quad \Rightarrow \quad E_1$ & E_2 are non exhaustive]

5

[Sol. Given $P(A \cup B \cup C) = \frac{3}{4}$ ...(1)

$$P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) + P(A \cap B \cap C) = \frac{1}{2} \qquad .....(2)$$

And $P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) = \frac{2}{5}$ ....(3)

$\Rightarrow$ **(D) is incorrect**

$\therefore$ From equation (2) - equation (3)

$P(A \cap B \cap C) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} = P(ABC)$ $\Rightarrow$ **(A) is correct.**

$\therefore P(AB) + P(BC) + P(CA) = P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) + 3\,P(A \cap B \cap C)$

$= \frac{2}{5} + 3 \times \frac{1}{10} = \frac{7}{10}$ $\Rightarrow$ **(B) is correct**

$\therefore P(A) + P(B) + P(C) = P(A \cup B \cup C) + P(AB) + P(BC) + P(CA) - P(ABC)$

$= \frac{3}{4} + \frac{7}{10} - \frac{1}{10} = \frac{27}{20}$ $\Rightarrow$ **(C) is correct**

Also $P(A\,\overline{B}\,\overline{C}) + P(\overline{A}\,\overline{B}\,C) + P(\overline{A}\,B\,\overline{C}) = P(A) + P(B) + P(C) - 2\,[P(AB) + P(BC) + P(CA)] + 3\,P(ABC)$

$= \frac{27}{20} - \frac{14}{10} + \frac{3}{10} = \frac{5}{20} = \frac{1}{4}$ $\Rightarrow$ **(D) is correct**]

6

[Sol.

1R	1R	1R	1R	1R	1R	→	red balls in the box.
B_1	B_2	B_3........B_k		B_{2009}	B_{2010}	→	boxes.
1W	2W	3W	kW	200W	2010W	→	white balls in the box.

Now P(n) = probability that child stops after drawing exactly n marbles.
i.e. at the nth position red marble must be drawn.

$$\therefore \quad P(n) = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)......\left(\frac{n-2}{n-1}\right)\left(\frac{n-1}{n}\right)\underbrace{\left(\frac{1}{n+1}\right)}_{\text{red}} = \frac{1}{n(n+1)}$$

$$\therefore \quad \frac{1}{n(n+1)} < \frac{1}{2010}$$

$$\Rightarrow \quad \frac{2}{n(n+1)} < \frac{1}{1005}$$

$$\Rightarrow \quad \frac{n(n+1)}{2} > 1005$$

$$\Rightarrow \quad n \geq 45 \Rightarrow$$ **B, C, D**]

7

[Sol**.**

(A) Given $P(A \cap B) = \frac{1}{12}$ $\Rightarrow P(A) \cdot P(B) = \frac{1}{12}$ ...(1)

and $P(\overline{A} \cap \overline{B}) = P(\overline{A}) \cdot P(\overline{B}) = \frac{1}{2}$ $\Rightarrow (1 - P(A))\,(1 - P(B)) = \frac{1}{2}$ ...(2)

Solving equation (1) and equation (2) and using P(A) < P(B)

we have $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{3}$

$\therefore\ P(A - B) = P(A) - P(A \cap B)$

$= \frac{1}{4} - \frac{1}{4} \times \frac{1}{3} = \frac{2}{12} = \frac{1}{6}$ ⇒ **(A) is correct**

(B) **Method-I :**

After three heads, tail can occur in fourth toss with probability = $\frac{1}{2}$ ⇒ **(B) is correct**

Method-II :

Required probability = $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

where B is event of occurrence of first 3 heads and B is event of occurrence of tail on fourth trial.

(D) As $0 \leq P(A \cup B) \leq 1$

$0 \leq P(A) + P(B) - P(A \cap B) \leq 1$

Hence $P(A) + P(B) - 1 \leq P(A \cap B) \leq P(A) + P(B)$ ⇒ **(D) is correct]**

$$\therefore\ P(A/B) = \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2}$$ ⇒ **(B) is correct**

(C) Numbers of ways in which two E's do not occur adjacently

$= \text{Total number of ways} - \text{when two E's occur simultaneously} = \frac{7!}{2!} - 6! = \left(\frac{7}{2} - 1\right)6! = \frac{5}{2} \times 6!$

$\therefore$ Required Probability $= \frac{\frac{5}{2} \times 6!}{\frac{7!}{2!}} = \frac{5}{2} \times \frac{6! \times 2}{7 \cdot 6!} = \frac{5}{7}$ ⇒ **(C) is incorrect**

(D) As $0 \leq P(A \cup B) \leq 1$

$0 \leq P(A) + P(B) - P(A \cap B) \leq 1$

Hence $P(A) + P(B) - 1 \leq P(A \cap B) \leq P(A) + P(B)$ ⇒ **(D) is correct]**

$$\therefore\ P(A/B) = \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2}$$ ⇒ **(B) is correct**

8

[Sol. $P(A) + P(B) - P(A \cap B) = 0.8$

∴ $P(A \cap B) = 0.5 + 0.4 - 0.8 = 0.1$

$P\left(\overline{A} \cap B\right) = P(B) - P(A \cap B) = 0.5 - 0.1 = 0.4$ ⇒ **(A)** is correct

$P\left(B/\overline{A}\right) = \dfrac{P(B \cap \overline{A})}{P(\overline{A})} = \dfrac{0.4}{1 - P(A)} = \dfrac{0.4}{0.6} = \dfrac{2}{3}$ ⇒ **(B)** is correct

$P(A) \cdot P(B) = 0.2$ ⇒ $P(A \cap B) < P(A) \cdot P(B)$ ⇒ **(C)** is correct

$P(A \text{ or } B \text{ but not both}) = 0.9 - 2 \times 0.1 = 0.7$]

9

[Hint: $P(0H \text{ or } 2H \text{ or } 4H)$

$Q = p^4 + 6p^2(1 - p)^2 + (1 - p)^4 = 8p^4 - 16p^3 + 12p^2 - 4p + 1 = \dfrac{(2p-1)^4 + 1}{2}$. Now interpret]

10

Sol. $E_1 = \{(2, 2), (2, 3), (3, 2), (2, 5), (5, 2), (3, 5), (5, 3), (3, 3), (5, 5)\}$

$E_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$E_3 = \{(1, 3), (3, 1), (2, 2)\}$

Now $P(E_1) = \dfrac{9}{36}$; $P(E_2) = \dfrac{6}{36}$; $P(E_3) = \dfrac{3}{36}$

Clearly $P(E_1), P(E_2), P(E_3)$ are in A.P. ⇒ **(A)** is correct.

$P(E_1 \cap E_2) = \dfrac{3}{36} = \dfrac{1}{12} \neq P(E_1)\,P(E_2)$ ⇒ **(B)** is incorrect.

Now $P(E_3/E_1) = \dfrac{P(E_3 \cap E_1)}{P(E_1)} = \dfrac{1}{9}$ ⇒ **(C)** is incorrect.

Also $P(E_1 + E_2) + P(E_2 - E_3) = [P(E_1) + P(E_2) - P(E_1 \cap E_2)] + [P(E_2) - P(E_2 \cap E_3)$

$= \dfrac{9}{36} + \dfrac{12}{36} - \dfrac{3}{36} - \dfrac{1}{36} = \dfrac{17}{36}$ ⇒ **(D)** is correct.]

11

[Sol.

(A) Given $P(A \cap B) = P(A)\,P(B)$ ....(1)

$P\left(A \cap (B \cup C)\right) = P(A)\,P(B \cup C)$ ....(2)

$P\left(A \cap (B \cap C)\right) = P(A)\,P(B \cap C)$ ....(3)

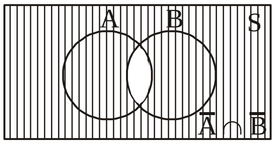
To prove that $P(A \cap C) = P(A)\,P(C)$

From (2), we get $P\left[(A \cap B) \cup (A \cap C)\right] = P(A)\left[P(B) + P(C) - P(B \cap C)\right]$

⇒ $\underline{\underline{P(A \cap B)}} + P(A \cap C) - \underline{P(A \cap B \cap C)} = \underline{\underline{P(A)\,P(B)}} + P(A)\,P(C) - \underline{P(A)\,P(B \cap C)}$

⇒ $P(A \cap C) = P(A)\,P(C)$ ⇒ **(A)** is correct

(B) See by Venn diagram



(C) If nine letters are in the correct envelopes, the tenth must be also in the correct envelope, so the probability is zero.

(D) Let P(S) = probability of successful flight

Let $P(\overline{S})$ = probability of unsuccessful flight

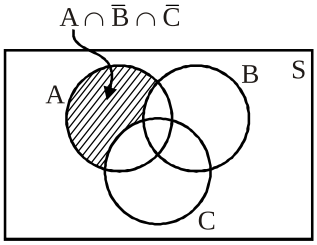
$P(S) = 1 - P(\overline{S}) = 1 - (0.01)^3 = 1 - 0.000001 = 0.999999$ **Ans.**]

12

[Sol. $$P\left((A \cap \overline{B})/\overline{C}\right) = \frac{P(A \cap \overline{B} \cap \overline{C})}{P(\overline{C})} = \frac{P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)}{1 - P(C)} \quad \ldots\ldots (1)$$

(MM), (TT), (AA), H, E, I, C, S

A : $\boxed{MM}$, T, T, A, A, H, E, I, C, S



$$P(A) = \frac{\frac{(10)!}{2!\,2!}}{\frac{(11)!}{2!\,2!\,2!}} = \frac{2}{11} = P(B) = P(C) \Rightarrow$$ **(A)** is correct

$A \cap B$: $\boxed{MM}$, $\boxed{TT}$, A, A, M, E, I, C, S

$$P(A \cap B) = \frac{\frac{9!}{2!}}{\frac{(11)!}{2!2!2!}} = \frac{2}{55} = P(A \cap C) = P(B \cap C) \Rightarrow$$ **(B)** is correct

$A \cap B \cap C$: $\boxed{MM}$, $\boxed{TT}$, $\boxed{AA}$, H, E, I, C, S

$$P(A \cap B \cap C) = \frac{8!}{\frac{(11)!}{2!\,2!\,2!}} = \frac{4}{495} \Rightarrow$$ **(C)** is correct

From (1)

$$P\left((A \cap \overline{B})/\overline{C}\right) = \frac{\frac{2}{11} - \frac{2}{55} - \frac{2}{55} + \frac{4}{495}}{1 - \frac{2}{11}} = \frac{58}{405} \Rightarrow$$ **(D)** is correct]

13

[Sol. Clearly as $n \to \infty$ the fraction $\frac{m}{n}$ is equal to $\frac{1}{2}$.

(A) We have n(S) = 6!
Note that 1 + 2 + 3 + 4 + 5 + 6 = 21 ⇒ divisible by 3
For divisible by 6, N must be even, so $n(A) = 3 \cdot 5!$

Hence $P(A) = \frac{3 \cdot 5!}{6!} = \frac{1}{2}$ **Ans.**

(B) We have n(S) = 216
Product is prime only when two rolls are 1 and the third is prime are 2, 3, 5.

$\therefore$ Favourable cases are 2 11 (3) ; 3 11 (3) and 5 11 (3) i.e. 9 cases

Hence $P(A) = \frac{9}{216} = \frac{1}{24}$ **Ans.**

(C) $$\lim_{n\to\infty} n\left(\frac{e}{\left(1+\frac{1}{n}\right)^n}-1\right) = \lim_{n\to\infty} -n\left(1-\frac{e}{\left(1+\frac{1}{n}\right)^n}\right) = \lim_{n\to\infty} -n\left(\frac{\left(1+\frac{1}{n}\right)^n-e}{\left(1+\frac{1}{n}\right)^n}\right);\ \text{Put } n = \frac{1}{y}$$

$$= -\frac{1}{e}\lim_{y\to 0}\left(\frac{(1+y)^{1/y}-e}{y}\right) = -1\cdot\lim_{y\to 0}\left(\frac{e^{\frac{\ln(1+y)}{y}-1}-1}{y}\right) = -\lim_{y\to 0}\left(\frac{e^{\frac{\ln(1+y)}{y}-1}-1}{\frac{\ln(1+y)}{y}-1}\right)\cdot\left(\frac{\frac{\ln(1+y)}{y}-1}{y}\right)$$

$$= -\lim_{y\to 0}\left(\frac{\ln(1+y)-y}{y^2}\right) = -\lim_{y\to 0}\frac{\left(\cancel{y}-\frac{y^2}{2}+\frac{y^3}{3}-\ldots\ldots\right)-\cancel{y}}{y^2} = \frac{1}{2}\ \textbf{Ans.}$$

(D) $$I = \int_0^1\left[\sqrt{4x}\,\right]dx\text{ ; Put } x = t^2\text{ ; } I = 2\int_0^1[2t\,]t\,dt = 2\left[\int_0^{1/2}0\,dt+\int_{1/2}^1 t\,dt\right] = 2\left[\left.\frac{t^2}{2}\right|_{1/2}^1\right] = \frac{2}{2}\left[1-\frac{1}{4}\right] = \frac{3}{4}\ \textbf{Ans.]}$$

14

[Sol. $P(E) = P(F) = \frac{1}{2}$ $\Rightarrow$ A, B, C

$$\begin{array}{l}\text{H T H H}\\ \text{T H H H}\\ \text{T T H H}\\ \text{H H H H}\end{array}\qquad\left.\begin{array}{l}\text{H T T H}\\ \text{T H T H}\\ \text{T T T H}\\ \text{H H T H}\end{array}\right\}\quad\left\{\begin{array}{l}\text{H T H T}\\ \text{T H H T}\\ \text{T T H T}\\ \text{H H H T}\end{array}\right.\qquad\begin{array}{l}\text{H T T T}\\ \text{T H T T}\\ \text{T T T T}\\ \text{H H T T}\end{array}$$

$P(E) = \frac{8}{16} = \frac{1}{2} = P(F)$]

15

[Sol. $\boxed{R}$ $\boxed{G}$ $\boxed{Y}$ $\boxed{R / G / Y}$

$P(R) = \frac{1}{2} = P(G) = P(Y)$ $\Rightarrow$ **(D)**

$P(R \cap G) = \frac{1}{4} = P(R)\cdot P(G)$; $P(G \cap Y) = \frac{1}{4}$

$= P(G)\cdot P(Y)$; $P(Y \cap R) = \frac{1}{4}$

$= P(Y)\cdot P(R) \Rightarrow$ **(A)**]

16

(A*) $p = 1, q = 0$ (B*) $p = \frac{2}{3}, q = \frac{1}{2}$

(C*) $p = \frac{3}{5}, q = \frac{2}{3}$ (D*) there are infinitely many values of p and q.

[Sol. $P(I) = p;\ P(II) = q;\ P(III) = \frac{1}{2}$

$P(S) = P(I\ \&\ II\ \text{ or }\ I\ \&\ III)$

$\frac{1}{2} = P(I \text{ and } II) + P(I \text{ and } III) - P(I\ \&\ II \text{ and } III)$

$\frac{1}{2} = pq + p \cdot \frac{1}{2} - \frac{pq}{2}$

$\frac{1}{2} = \frac{pq}{2} + \frac{p}{2} = \frac{p}{2}(q + 1)$

$\therefore \quad p(q + 1) = 1$]

17

[Sol. (A) $P_1 = \frac{1}{6},\ P_2 = {}^4C_3\left(\frac{6}{6^3}\right) \cdot \frac{5}{6} = \frac{20}{216} \Rightarrow P_1 > P_2.$

(B) P(B gets 10 or more/ A gets 9) = P(B gets 10 or more) = $\frac{6}{36} = \frac{1}{6}$.

(C) $P(A \cup B) = 1 - (1 - P(A)\ (1 - P(B)) = 1 - \left(1 - P(A) - P(B) + P(A)\,P(B)\right)$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\Rightarrow \quad P(A \cap B) = P(A) \cdot P(B)$

Hence A and B are independent events.

(D) It is sure event, so its probability is 1.]

18

[Sol. $A = \{HHT,\ HTH, HHH, HTT\} \rightarrow P(A) = \frac{1}{2}$

$B = \{HHT,\ THH, HHH, THT\} \rightarrow P(B) = \frac{1}{2}$

$C = \{HHT,\ THH\} \rightarrow P(C) = \frac{1}{4}$

$P(A \cap B) = P\{HHT, HHH\} = \frac{1}{4} = P(A)\ P(B)$

$P(C \cap A) = P(HHT) = \frac{1}{8} = P(C) \cdot P(A)$

hence A and B are independent and A and C are independent

$P(B \cap C) = P(HHT, THH) = \frac{1}{4} \neq P(B) \cdot P(C)$

$$P(A \cap B \cap C) = P(HHT) = \frac{1}{8} \neq P(A) \cdot P(B) \cdot P(C) \quad \Rightarrow \quad \text{A, C are correct.]}$$

COMPREHENSION TYPE

19

20

21 The value of P_3 equals

(A) $2P_2$ (B) $10P_2$ (C) $11P_2$ (D*) $20P_2$

[Sol. $52 \xrightarrow[40]{\text{face cards}} \left\langle \begin{matrix} 4 \text{ aces} \\ 36 \text{ non aces} \end{matrix} \right.$

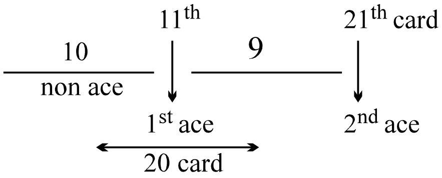
(i) 10 cards are drawn before the 1st ace

– first 10 cards are all non aces and 11th card is an ace.

$$\therefore \qquad P_1 = \frac{{}^{36}C_{10}}{{}^{40}C_{10}} \cdot \frac{4}{30} = \frac{(36)!}{(10!)(26!)} \; \frac{(30!)(10!)}{40!} \cdot \frac{4}{30}$$

$$P_1 = \frac{(30)(29)(28)(27)}{(40)(39)(38)(37)} \cdot \frac{4}{30} = \frac{(27)(28)(29)}{(10)(37)(38)(39)} \;\; \textbf{Ans.}$$

(ii) Position of pack now



$$29 \left\langle \begin{matrix} 3 \text{ aces} \\ 26 \text{ non aces} \end{matrix} \right.$$

$$P_2 = \frac{{}^{26}C_9}{{}^{29}C_9} \cdot \frac{3}{20} P_1 = \frac{26!}{(9!)(17!)} \cdot \frac{(9!)(20!)}{29!} \cdot \frac{3}{20} \cdot P_1 = \frac{(20)(19)(18)}{(29)(28)(27)} \left(\frac{(27)(28)(29)}{(37)(38)(39)} \cdot \frac{1}{10} \right) \frac{3}{20}$$

$$= \frac{(18)(3)}{(37)(39)} \frac{1}{20} = \frac{9}{(10)(13)(37)} \;\; \textbf{Ans.}$$

(iii) $P_3 = 20P_2 = (20)\left(\frac{9}{(10)(13)(37)} \right) = \frac{18}{(13)(37)}$]

22

23

24

[Sol. $P(S/T) = \frac{P(S \cap T)}{P(T)} \quad \Rightarrow \quad 0.5 = \frac{P(S \cap T)}{0.69} \quad \Rightarrow \quad P(S \cap T) = 0.5 \times 0.69 = P(S)P(T)$

$\Rightarrow$ S and T are independent **Ans.(22)**

$\therefore$ P (S and T) = P (S) · P(T) = 0.69 × 0.5 = 0.345 **Ans.(23)**

P (S or T) = P (S) + P (T) – P ($S \cap T$) = 0.5 + 0.69 – 0.345 = 0.8450 **Ans. (24)**

25

26

27

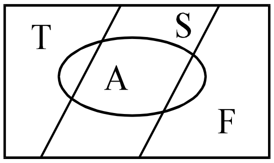
[Sol. A : She get a success

T : She studies 10 hrs : $P(T) = 0.1$

S : She studies 7 hrs : $P(S) = 0.2$

F : She studies 4 hrs : $P(F) = 0.7$

$P(A/T) = 0.8$; $P(A/S) = 0.6$; $P(A/F) = 0.4$



$$\begin{aligned} P(A) &= P(A \cap T) + P(A \cap S) + P(A \cap F) \\ &= P(T) \cdot P(A/T) + P(S) \cdot P(A/S) + P(F) \cdot P(A/F) \\ &= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4) \\ &= 0.08 + 0.12 + 0.28 = 0.48 \quad \textbf{Ans.(25)} \end{aligned}$$

$$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{(0.7)(0.4)}{0.48} = \frac{0.28}{0.48} = \frac{7}{12} \quad \textbf{Ans.(26)}$$

$$P(F/\overline{A}) = \frac{P(F \cap \overline{A})}{P(\overline{A})} = \frac{P(F) - P(F \cap A)}{0.52} = \frac{(0.7) - 0.28}{0.52} = \frac{0.42}{0.52} = \frac{21}{26} \quad \textbf{Ans.(27)}$$

28

29

30

[Sol. A's set {1, 2, 3, 4, 5, 6, 7, 8, 9}

B's set {1, 2, 3, 4, 5, 6, 7, 8}

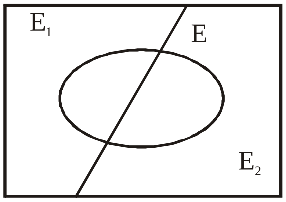
Let E_1 : event that A picks 9 as one of his 3 numbers. In this case A's number will always be greater than B's 3 digit number.

$$P(E_1) = \frac{{}^8C_2}{{}^9C_3} = \frac{28}{84} = \frac{1}{3} \quad \textbf{Ans.(i)}$$

Let E_2 : event that A does not pick 9

$$P(E_2) = \frac{{}^8C_3}{{}^9C_3} = \frac{2}{3}$$

Let E: event that 3 numbers picked up by A and arranged in descending order is greater than the corresponding number of B.



$$\begin{aligned} P(E) &= P(E \cap E_1) + P(E \cap E_2) \\ &= P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) \end{aligned}$$

$$P(E) = \frac{1}{3} \cdot \underbrace{1}_{think} + \frac{2}{3} \cdot P(E/E_2) = \frac{1}{3} + \frac{2p}{3}$$

To find $P(E/E_2)$

Suppose A picks 3 number say a, b, c from {1, 2, 3, 4, 5, 6, 7, 8}. Now B must also pick the same 3 numbers in any order which can be done in six ways.

Hence Probability that both A and B picks the same three number = $6\left[\frac{1}{8}\cdot\frac{1}{7}\cdot\frac{1}{6}\right] = \frac{1}{56}$ **Ans.(ii)**

Hence Probability that either A or B's pick is higher number with both having equal probability of picking a higher number.

Hence Probability that A's pick is higher = $\frac{1}{2}\left(1-\frac{1}{56}\right) = \frac{55}{112}$

$\therefore$ $P(E/E_2) = \frac{55}{112}$

$\therefore$ $P(E) = \frac{1}{3}\cdot 1+\frac{2}{3}\cdot\frac{55}{112}$

$= \frac{1}{3}+\frac{55}{168}$

$= \frac{56+55}{168}$

$= \frac{111}{168} = \frac{37}{56}$ **Ans.(iii)**]

ASSERTION / REASON TYPE

31

[Sol. $P(E) = P(14, 41, 32, 23) = 1/9$

$P(F) = P(2, 4, 6) = 1/2$

$P(E \cap F) = P(41, 23) = \frac{1}{18} = P(E) \cdot P(F)$

Hence E and F are independent]

32 From a well shuffled pack of 52 cards. A card is drawn, outcome is noted and the card is replaced in the

[Hint : Mean = $\mu = np = 16\left(\frac{1}{4}\right) = 4$ and variance = $\sigma^2 = npq = 16\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 3$]

33

[Sol. If p_1, p_2, p_3, p_4 are the probabilities of success in a single throw for A, B, C and D then

$P(A) = p_1 = \frac{1}{2}$ ☐

$P(B) = p_2 = \frac{1}{8} + {}^3C_1\left(\frac{1}{8}\right)$ ☐ ☐ ☐

(All even + Exactly one even)

$P(C) = p_3 = \frac{1}{2^5}+\frac{{}^5C_1}{2^5}+\frac{{}^5C_2}{2^5} = \frac{16}{2^5} = \frac{1}{2}$ ☐ ☐ ☐ ☐ ☐

[(i) All even; (ii) one even & 4 odd

(iii) 3 even and 2 odd]

$$P(D) = p_4 = \underbrace{\frac{1}{2^7}}_{\text{all even}} + \underbrace{\frac{{}^7C_1}{2^7}}_{\substack{\text{exactly}\\ \text{one even}}} + \underbrace{\frac{{}^7C_3}{2^7}}_{\substack{\text{exactly}\\ \text{3 even}}} + \underbrace{\frac{{}^7C_5}{2^7}}_{\substack{\text{exactly}\\ \text{5 even}}}$$

☐ ☐ ☐ ☐ ☐ ☐ ☐

$$= \frac{1+7+35+21}{2^7} = \frac{64}{2^7} = \frac{1}{2}$$

Hence probability of success is same for all in the single throw.

All equiprobable to win.

If they thow is succession i.e. A, B, C and D then

P(A) = P(S or F F F F S or ....)

$$= \frac{P(S)}{1-(P(F))^4} = \frac{1}{2}\cdot\frac{1}{1-\frac{1}{16}} = \left(\frac{1}{2}\right)\left(\frac{16}{15}\right) = \frac{8}{15}$$

|||ly $P(B) = \frac{4}{15}$; $P(C) = \frac{2}{15}$; $P(D) = \frac{1}{15}$

Hence both the statements are correct and S-2 is not the correct explanation.]

34

[Sol. We have $f'(x) = 3x^2 - 2ax + b$

Now $y = f(x)$ is increasing $\Rightarrow f'(x) \geq 0 \ \forall\ x$ and for $f'(x) = 0$ should not form an interval.

$\Rightarrow (2a)^2 - 4 \times 3 \times b \leq 0 \qquad \Rightarrow a^2 - 3b \leq 0$

This is possible for exactly 16 ordered pairs (a, b), $1 \leq a, b \leq 6$ namely (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6) & (4, 6)

Thus, required probability $= \frac{16}{36} = \frac{4}{9}$]

35

[Sol. $P_n = P_{n-1}\,P(T) + P_{n-2}\,P(T)\,P(H)$; $P_n = \frac{P_{n-1}}{2} + \frac{P_{n-2}}{4}$; $P_1 = 1$, $P_2 = \frac{3}{4}$; $P_3 = \frac{3}{8} + \frac{1}{4}$

$P_3 = \frac{5}{8}$; $P_4 = \frac{P_3}{2} + \frac{P_2}{4}$; $= \frac{8}{16}$

Alternatively: Clearly $p_1 = 1$ and $p_2 = 1 - P(H\,H) = 1 - \frac{1}{4} = \frac{3}{4}$

Now for $n \geq 3$,

Compute $P_3 = \frac{5}{8}$; $P_4 = \frac{1}{2}$

Hence $P_2 = \frac{12}{16}$; $P_3 = \frac{10}{16}$; $P_4 = \frac{8}{16} \Rightarrow$ P_2, P_3, P_4 are in A.P.

$$P_n = \underbrace{(1-p)}_{T}p_{n-1} + \underbrace{p}_{H}\underbrace{(1-p)}_{T}p_{n-2}$$]

36.

Sol Let the two non-negative integers by x and y

Then $x = 5a + \alpha$ and $y = 5b + \beta$ where $0 \le \alpha \le 4, 0 \le \beta \le 4$

Now $x^2 + y^2 = (5a+\alpha)^2 + (5b+\beta)^2 = 25(a^2+b^2) + 10(a\alpha + b\beta) + \alpha^2 + \beta^2$

$\therefore x^2 + y^2$ is divisible by 5 if and only if 5 divides $\alpha^2 + \beta^2$

The total numbers of ways of choosing α and $\beta = 5 \times 5 = 25.$

Further, $\alpha^2 + \beta^2$ will be divisible by 5 if

$(\alpha_1 \beta) \in \{(0,0),(1,2),(1,3),(2,1),(2,4),(3,1),(3,4),(4,2),(4,3)\}$

$\therefore$ Favourable number of ways of chossing α and $\beta = 9$

$\therefore$ Required probability $= \frac{9}{25}.$

37

[Sol Here, n(s) = 1 length of the interval $[0,5] = 5n(E) =$ length of the interval $\le [0,5]$ in which P belongs such that the given equation has real root.

Now $x^2 + Px + \frac{1}{4}(P+2) = 0$ will have real roots

if $P^2 - 4.1(P+2) \ge 0 \Rightarrow P^2 - P - 2 \ge 0$

$\Rightarrow (P+1)(P-2) \ge 0 \Rightarrow P \le -1$ or $P \ge 2$

But $P \in [0,5]$. So, $E = [2,5]$

$\therefore$ n(E) = length of the interval [2,5] = 3

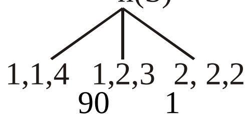
$\therefore$ Required Probability $= \frac{3}{5}.$

MATRIX MATCH TYPE

38 ball(s) in atleast 8 boxes, is

[Ans. (A) S, (B) Q, (C) R; (D) P]

[Sol. **(A)** n(S) = 540; n (A) = 222 $\Rightarrow \frac{6!}{2!2!2!3!}\cdot 3!$



$\therefore$ p = $\frac{90}{540} = \frac{1}{6}$ **Ans.**]

(B) n (S) = $^{15}C_2$; n (A) = $^5C_1 \cdot {}^5C_1 + {}^5C_2 = 35$

p = $\frac{35}{15 \cdot 14} \cdot 2 = \frac{5 \cdot 2}{15 \cdot 2} = \frac{1}{3}$ **Ans.**

$$\left.\begin{matrix} S-1: & 1 & 4 & 7 & 10 & 13 \\ S-2: & 2 & 5 & 8 & 11 & 14 \\ S-3: & 3 & 6 & 9 & 12 & 15 \end{matrix}\right] \textit{disjoint three sets}$$

(C) $10 \begin{matrix} \diagup 5R \\ \diagdown 5B \end{matrix}$

P(E) = P(RRR or RBR or BBB or BRB) = $\left(\frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8} + \frac{5}{10}\cdot\frac{5}{9}\cdot\frac{4}{8}\right)\cdot 2 = \frac{160 \cdot 2}{10 \cdot 9 \cdot 8} = \frac{4}{9}$ **Ans.**

(D) Let A = event that there is a ball in atleast 8 boxes.
P(A) = P(all 10 boxes have a ball + exactly 9 boxes have a ball + exactly 8 boxes have a ball)

$$p = \underbrace{\frac{1}{{}^{10}C_5}}_{*} + \underbrace{\frac{{}^5C_1 \cdot {}^5C_4}{{}^{10}C_5}}_{**} + \underbrace{\frac{{}^5C_2 \cdot {}^5C_3}{{}^{10}C_5}}_{***}$$

* All 10 boxes have one ball each. Ø Ø Ø Ø Ø Ø Ø Ø Ø Ø

** Exactly 1 box is empty and one has two balls. $\boxed{Ø\ Ø}$ Ø Ø Ø Ø Ø Ø Ø Ø $\boxed{}$

*** Exactly 2 boxes empty and exactly 2 boxes have 2 balls each.

$\boxed{Ø\ Ø}$ $\boxed{Ø\ Ø}$ Ø Ø Ø Ø Ø Ø $\boxed{}$ $\boxed{}$

[**Think:** 1st person put one ball in any of the five boxes (have exactly one ball in 5 boxes) then the 2nd person has only one option of the remaining 5 out of ${}^{10}C_5$ choices.]

$$= \frac{1}{{}^{10}C_5}[1 + 25 + 100] = \frac{126}{252} = \frac{1}{2}$$

$\underset{B_1}{\boxed{}}\ \underset{B_2}{\boxed{}}\ldots\ldots\underset{B_3}{\boxed{}}$]

39

[Sol.

(A) S = {2, 3, 5, 7, 11}

Total ways in which A and B can be chosen = $({}^5C_4 \cdot 4!)^2 = (5!)^2$

P(E) = 1 – P(A and B does not contain the element 2)

$$1 - \frac{(4!)^2}{(5!)^2} = 1 - \frac{1}{25} = \frac{24}{25} = 96\% \text{ Ans.}$$

(B) $A \lt \begin{matrix}3W\\2R\end{matrix}$; $B \lt \begin{matrix}2W\\4R\end{matrix}$; $\dfrac{{}^3C_2 . {}^2C_2}{{}^5C_2 . {}^6C_2} = \dfrac{1}{50}$]

(C) A: blood result says positive about the disease

B_1: Person suffers from the disease = $\frac{1}{100}$; B_2 : person does not suffer = $\frac{99}{100}$

$P(A/B_1) = \frac{99}{100}$; $P(A/B_2) = \frac{1}{100}$

$$P(B_1/A) = \frac{P(B_1)\cdot P(A/B_1)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)} = \frac{\frac{1}{100}\cdot\frac{99}{100}}{\frac{1}{100}\cdot\frac{99}{100} + \frac{99}{100}\cdot\frac{1}{100}} = \frac{99}{2\cdot 99} = \frac{1}{2} = 50\% \text{ Ans.]}$$

40

[Ans. (A) S ; (B) R; (C) P]

[Sol.

(A) P(R) = P(T ; HTT ; HTHTT ; HHTTT) ⇒ 11/16

(B) $P(A \cup B) = 0.6$; $P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ $P(A) + P(B) = 0.6 + 0.2 = 0.8$

$$\frac{1}{2}\left(P(\bar{A})+P(\bar{B})\right) = 2 - 0.8 = 1.2 \;\Rightarrow\; (R)$$

(C) $P(X) = \frac{2}{3}$; $P(Y) = \frac{3}{4}$; $P(Z) = p$

E : exactly two bullets hit

$P(E) = P(X\,Y\,\bar{Z}) + P(Y\,Z\,\bar{X}) + P(Z\,X\,\bar{Y})$

$$\frac{11}{24} = \frac{2}{3}\cdot\frac{3}{4}(1-p) + \frac{3}{4}\cdot\frac{1}{3}p + p\frac{2}{3}\cdot\frac{1}{4} \quad\Rightarrow\quad p = \frac{1}{2}$$

EXERCISE 1 (C)

INTEGER TYPE

1.

[Sol. What matter is that the last two cards are the same as the first one. the probability for the second is $\frac{3}{51}$; for the third is $\frac{2}{50}$.

$\frac{1}{17}\cdot\frac{2}{50} = \frac{1}{425}$

There are 4 total kings, so there are ${}^4C_3 = 4$ combinations of 3 cards that have the property that all 3 cards are kings. Since there are 13 ranks, there are $13 \cdot 4 = 52$ combinations of 3 cards where all 3 cards are the same rank.

There are 52 total cards, so there are ${}^{52}C_3 = 22100$ total possible combinations of 3 cards. Therefore, the probability of drawing 3 kings is $\frac{4}{22100}$ and the probability of drawing 3 cards of the same rank is $\frac{52}{22100}$.

2.

[Hint: $P(W/W) = \frac{2}{3}$; $P(L/W) = \frac{1}{3}$; $P(W/L) = \frac{1}{3}$; $P(L/L) = \frac{2}{3}$]

3.

[Hint: $P(E) = \frac{{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\ldots.}{2^n} = \frac{2^{n-1}}{2^n} = \frac{1}{2}$]

4.

[Hint: $P(1\ 2\ \overset{(6\text{ cases})}{3}\ \text{or}\ 2\ 2\ 2) = \frac{6+1}{3^3} = \frac{7}{27}$]

5.

[Sol. $p = P(H \text{ or } T\,H \text{ or } T\,T\,H)$; $\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{7}{8}$

$q = T\,H \text{ or } T\,T\,T\,H \text{ or } \ldots\ldots.$; $P(E) = \frac{1/4}{1-1/4} = \frac{1}{4}\times\frac{4}{3} = \frac{1}{3}$

6.

Hint: [P [(2&2) or (3&3) or (4&4) ...]

7.

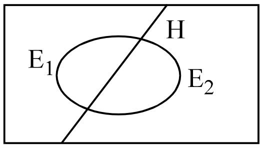
[Hint: 'A' is horse; B and C joceky

H: Horse 'A' wins the race

E_1: 'B' rides 'A' ; $P(E_1) = 2/3$

E_2: 'C' rides 'A' ; $P(E_2) = 1/3$

$P(H / E_1) = 1/6$; $P(H / E_2) = 3/6$



$$P(H) = P(H \cap E_1) + P(H \cap E_2) = \frac{2}{3}\times\frac{1}{6} + \frac{1}{3}\times\frac{3}{6}\ ]$$

8.

[Hint: A : they obtained the same result

$$B_1 : A \cap \overline{B}\ ; P(B_1) = \frac{1}{8}\cdot\frac{11}{12} \qquad \text{Now} \quad P(A/B_1) = 0$$

$$B_2 : \overline{A} \cap B\ ; P(B_1) = \frac{7}{8}\cdot\frac{1}{12} \qquad P(A/B_2) = 0$$

$$B_3 : A \cap B\ ; P(B_3) = \frac{1}{8}\cdot\frac{1}{12} \qquad P(A/B_3) = 1$$

$$B_4 : \overline{A} \cap \overline{B}\ ; P(B_4) = \frac{7}{8}\cdot\frac{11}{12} \qquad P(A/B_4) = \frac{7}{8}\cdot\frac{11}{12}\cdot\frac{1}{1001}$$

$$\text{Now} \quad P(B_3/A) = \frac{\frac{1}{8}\cdot\frac{1}{12}}{\frac{1}{8}\cdot\frac{1}{12} + \frac{7}{8}\cdot\frac{11}{12}\cdot\frac{1}{1001}} = \frac{13}{14}\ ]$$

9.

[Sol. Let C_1, C_2, C_3, C_4 are coins.

4 coins tossed twice → each coin is tossed twice.

Let S : denotes the success that a coin is discarded

P(S) = 1 – coin is not discarded

$$= 1 - P(HH) = 1 - \frac{1}{4} = \frac{3}{4}$$

Hence S can take value 0, 1, 2, 3, 4

P(S = 3 or 4) = P(S = 3) + P(S = 4)

$$= {}^4C_1\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) + {}^4C_4\left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^3\left(1+\frac{3}{4}\right)$$

$$= \frac{(27)(7)}{256} = \frac{189}{256} = \frac{m}{n}$$

∴ m + n = 445 **Ans.**

Alternatively: Different possibilities are as follows:

1. 0 coin discard in first toss and (3 or 4 coins of the remaining 4 coins discards in the second toss)

2. 1 coin discard in first toss and (2 or 3 coins of the remaining three coins discards in the second toss)
3. 2 coins discard in first toss and (1 or 2 coins of the remaining two coins discards in the second toss)
4. 3 coins discard in first toss and (1 or 0 coin of the remaining one coin discards in the second toss)
5. All 4 coins discard in first toss.

$\therefore$ Required probability = $\frac{1}{16}\left[\frac{4}{16}+\frac{1}{16}\right] + \frac{4}{16}\left[\frac{3}{8}+\frac{1}{8}\right] + \frac{6}{16}\left[\frac{2}{4}+\frac{1}{4}\right] + \frac{4}{16}\left[\frac{1}{2}+\frac{1}{2}\right] + \left(\frac{1}{16}\right)(1)$

$= \frac{189}{256}$]

10.

[Sol. $P(E) = \frac{2}{5}$; $P(F) = P(\text{plumbing}) = 1 - \frac{4}{7} = \frac{3}{7}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\frac{2}{3} = \frac{2}{5} + \frac{3}{7} - x$

$\Rightarrow \quad x = \frac{17}{105}$ Ans.]

[Sol. $\left.\begin{array}{l} n(S) = {}^5C_2 = 10 \\ n(A) = 1 \cdot {}^4C_1 = 4 \end{array}\right] \Rightarrow p = \frac{2}{5}$

12.

[Sol. $P(S_1) = P(S_2) = P(S_3) = 1/2$

E : event that the current will flow.

$P(E) = P\left((S_2 \cap S_3) \text{ or } S_1\right) = P(S_2 \cap S_3) + P(S_3) - P(S_1 \cap S_2 \cap S_3) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$]

13. A certain team wins with probability 0.7, loses with probability 0.2 and ties with probability 0.1. The team plays three games. If the probability that the team wins at least two of the games, but lose none is $\frac{p}{100}$ and that the team wins at least one game is $\frac{q}{1000}$, where $p, q \in N$, then find the value of $\frac{7q}{p}$.

[**Ans.** 139]

[Sol. $P(W) = 0.7$; $P(L) = 0.2$; $P(T) = 0.1$

E : winning at least 2 games but lose none

P (E) = P (W W T or W T W or T W W or W W W)

$= 3 \times 0.7 \times 0.7 \times 0.1 + (0.7)^3 = 0.7 \times 0.7 [0.3 + 0.7] = 0.49$

F : wining at least 1 game

A = L or T $\Rightarrow$ P (A) = 0.3 ; P (F) = 1 – P (A A A) $= 1 - (0.3)^3 = 1 - 0.027 = 0.973$

14. If the probability of at most two tails or at least two heads in a toss of three coins is $\frac{p}{q}$, where p and q are coprimes then find the value of p + q.

[**Ans.** 15]

[Sol. A = at most two tails n (S) – {T T T}

B = at least two heads H H H, T H H, H T H, H H T

$P(A) = \frac{7}{8}$; $P(B) = \frac{4}{8}$; $P(A \cap B) = \frac{4}{8}$

$$P(A \cup B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$$

15.

[Sol. Let the number of red faces on the 2^{nd} cube = x
number of blue faces = (6 – x)
P (R R or B B) = 1/2

$$\frac{5}{6}\cdot\frac{x}{6} + \frac{1}{6}\cdot\frac{6-x}{6} = \frac{1}{2}$$

$5x + 6 - x = 18$

$4x = 12 \quad \Rightarrow \quad x = 3$ Ans.]

16.

[Sol. $12 \left\langle \begin{matrix} 6G \\ 6R \end{matrix} \right. \rightarrow 5$ drawn

Red Box		Green Box	
5R	0G	1R	6G
4R	1G	2R	5G
3R	2G	3R	4G
2R	3G	4R	3G
1R	4G	5R	2G
0R	5G	6R	1G

Let E is event as desired then

$$P(E) = \frac{{}^6C_0 \cdot {}^6C_5 + {}^6C_4 \cdot {}^6C_1}{{}^{12}C_5} = \frac{{}^6C_1 + {}^6C_4 \cdot {}^6C_1}{11 \cdot 9 \cdot 8} = \frac{6+90}{11 \cdot 9 \cdot 8} = \frac{96}{11 \cdot 9 \cdot 8} = \frac{4}{33}$$

hence p + q = 4 + 33 = 37 Ans.]

17.

[Sol. $P(0 \text{ heads}) = \left(\frac{1}{2}\right)^N$

$$P(1 \text{ head}) = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^{N-1} \binom{N}{1} = N\left(\frac{1}{2}\right)^N$$

$$P(2 \text{ head}) = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{N-2} \binom{N}{2} = \frac{N(N-1)}{2}\left(\frac{1}{2}\right)^N$$

$\therefore$ P (at most 2 heads) = P(0 heads) + P(1 head) + P(2 heads)

$$= \left(\frac{1}{2}\right)^N + N\left(\frac{1}{2}\right)^N + \frac{N(N-1)}{2}\left(\frac{1}{2}\right)^N$$

$$= \left(\frac{1}{2}\right)^N \left(1 + N + \frac{N(N-1)}{2}\right)$$

$$= \left(\frac{1}{2}\right)^N \left(\frac{N^2 + N + 2}{2}\right) = \frac{1}{2}$$

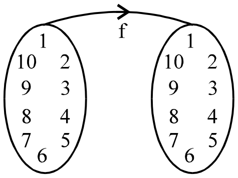
$= N^2 + N + 2 = 2^N$

$\Rightarrow N = 5$ **Ans.**]

18.

[Sol. Given f (i) + f (11 – i) = 11
f (10) = 11 – f (1)

$f(9) = 11 - f(2)$
$f(8) = 11 - f(3)$
$f(7) = 11 - f(4)$
$f(6) = 11 - f(5)$



Hence once we have chosen f (1), f(2), f (3), f (4) and f (5), the five remaining values f (6), f (7), f (8), f (9) and f (10) are determined automatically and will have a unique mapping.

Now each of the values from 1 to 5 can be chosen in 10 ways.

Hence total ways = 10^5

remaining values are automatically mapped uniquely.

Hence $625k = 100000$

$\Rightarrow$ $k = 160$ **Ans.**]

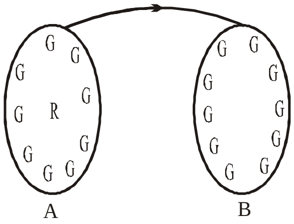
19.

[Sol. E_1: event that 9 Green balls are transferred from A to B and 9 Green balls return from B to A.

E_2 : (8G + 1R) from A to B and (8G + 1R) return from B to A.

E_3 : (8G + 1R) from A to B and 9G return from B to A.

E : event that red ball is still there in purse A



$$P(E) = 1 - P(E_3) = 1 - \frac{{}^9C_8 \cdot {}^1C_1}{{}^{10}C_9}\,\frac{{}^{18}C_9}{{}^{19}C_9}$$

$$= 1 - \frac{9}{19} = \frac{10}{19}$$

$\Rightarrow$ p + q = 29 **Ans.**]

20.

[Sol. Probability of T_1 winning $= {}^6C_4 \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} = \frac{160}{729}$

[Winning 4 out of first six game & winning the 7th]

Probability of T_2 winning $= {}^6C_4 \cdot \left(\frac{1}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\right) = \frac{20}{729}$

$\therefore$ Summing up $= \frac{180}{729} = \frac{20}{81}$

$\therefore$ (a + b) = 101

EXERCISE 2 (A)

ONLY ONE OPTION IS CORRECT

1.

[Sol. R R G G G B B B B when counters are alike

$$n(S) = \frac{9!}{2!3!4!}$$

$$n(A) = \frac{5!}{3!2!} \cdot {}^6C_4 \qquad | R | R | G | G | G |$$

$$\therefore \quad P(A) = \frac{5! \cdot 15}{3!2!} \cdot \frac{2!3!4!}{9!} = \frac{6! \cdot 60}{9 \cdot 8 \cdot 7 \cdot 6!} = \frac{60}{7 \cdot 8 \cdot 9} = \frac{15}{7 \cdot 2 \cdot 9} = \frac{5}{42}$$

Alternatively : n (S) = 9! $\qquad R_1R_1G_1G_2G_3B_1B_2B_3B_4$

$n(A) = 5! \cdot {}^6C_4 \cdot 4!$ when counters are different

$$p = \frac{5! \cdot 6 \cdot 5 \cdot 4 \cdot 3}{9!} = \frac{5 \cdot 4 \cdot 3}{9 \cdot 8 \cdot 7} = \frac{5}{42} \qquad]$$

2.

[Hint: L and W can be filled at 14 places in 2^{14} ways.

$\therefore \quad n(S) = 2^{14}$.

Now 13 L's and 1W can be arranged at 14 places in 14 ways.

Hence n(A) = 14

$$\therefore \quad p = \frac{14}{2^{14}} = \frac{7}{2^{13}} \quad]$$

3.

[Hint: $n(S) = {}^{10}C_7 = 120$

$n(A) = {}^5C_4 \cdot {}^3C_2 \cdot {}^2C_1$

$$P(E) = \frac{5 \cdot 3 \cdot 2}{120} = \frac{1}{4} \text{ Ans.]}$$

4.

[Hint: 1 , 2 , 2 , 3 , 3 , 3 (thrown 3 times)

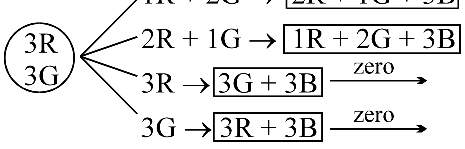
$$P(1) = \frac{1}{6} \;;\; P(2) = \frac{2}{6} \;;\; P(3) = \frac{3}{6}$$

P(S) = P(4 or 6) = P(112 (3 cases) or 123 (6 cases) or 222)

$$= 3.\frac{1}{6}.\frac{1}{6}.\frac{2}{1} + 6\frac{1}{6}.\frac{2}{6}.\frac{3}{1} + \frac{2}{6}.\frac{2}{6}.\frac{2}{6} = \frac{6+36+8}{216} = \frac{50}{216} = \frac{25}{108} \quad]$$

5.

[Sol.



$$; \quad \frac{{}^3C_1 \cdot {}^3C_2}{{}^6C_2} \cdot \frac{{}^2C_1 \cdot {}^1C_1 \cdot {}^3C_1}{{}^6C_3} + \frac{{}^3C_2 \cdot {}^3C_1}{{}^6C_3} \cdot \frac{{}^1C_1 \cdot {}^2C_1 \cdot {}^3C_1}{{}^6C_3} \quad]$$

6.

[Hint: P (T H or T T T H or T T T T T H or ....) = $\frac{2}{5}$

$$\frac{p(1-p)}{1-(1-p)^2} = \frac{2}{5} \quad \Rightarrow \quad \frac{\cancel{p}(1-p)}{(2-p)\cancel{p}} = \frac{2}{5} \quad \Rightarrow \quad 5(1-p) = 2(2-p) \;\Rightarrow\; 3p = 1 \;\Rightarrow\; p = \frac{1}{3}\,]$$

7.

[Hint: Square of a number ends in 0, 1, 4, 5, 6 and 9 favourable ordered pairs of
(a^2, b^2) can be (0, 0) ; (0, 5), (5, 0), (5, 5) ; (1, 4) , (4, 1) ; (1, 9) , (9, 1) ; (4, 6) , (6, 4) ;
(6, 9) , (9, 6) and P(0) = 1/10 = P(5) ; P(1) = P(4) = P(6) = P(9) = 2/10]

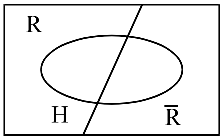
8. (A)

9.

[Sol. R: Missile is intercepted

$P(R) = \frac{1}{3}$; $P(\overline{R}) = \frac{2}{3}$

H: Missile hits the target



$P(H) = P(H \cap R) + P(H \cap \overline{R}) = P(R) \cdot P(H/R) + P(\overline{R}) \cdot P(H/R)$

$$= \frac{1}{3} \cdot (0) + \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$$

Hence $P(H\,H\,H) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ Ans.]

10.

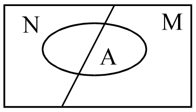
[Sol. Let P(S) = P(1 or 2) = 1/3
P(F) = P(3 or 4 or 5 or 6) = 2/3
P(A wins) = P[(S S or S F S S or S F S F S S or ....) or (F S S or F S F S S or ....)]

$$= \frac{\frac{1}{9}}{1-\frac{2}{9}} + \frac{\frac{2}{27}}{1-\frac{2}{9}} = \frac{1}{9} \times \frac{9}{7} + \frac{2}{27} \times \frac{9}{7} = \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$$

P (A winning) = $\frac{5}{21}$; P (B winning) = $\frac{16}{21}$ Ans.]

11.

[Sol. N = Normal die ; P(N) = 1/4
M = magnetic die ; P(M) = 3/4
A = die shows up 3



$P(A) = P(A \cap N) + P(A \cap M)$
$= P(N)\, P(A/N) + P(M) \cdot P(A/M)$

$$= \frac{1}{4} \cdot \frac{1}{6} + \frac{3}{4} \cdot \frac{2}{6} = \frac{7}{24}$$

$$P(N/A) = \frac{P(N \cap A)}{P(A)} = \frac{(1/4)\cdot(1/6)}{7/24} = \frac{1}{7} \text{ Ans.]}$$

12.

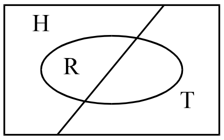
[Hint: $A\begin{cases}3\text{ R}\\2\text{ B}\end{cases}$; $B\begin{cases}2\text{ R}\\8\text{ B}\end{cases}$

R = event that a red marble is drawn

$P(R) = P(R \cap H) + P(R \cap T)$

$= P(H)\,P(R/H) + P(T)\cdot P(R/T)$

$$= \frac{1}{2}\left[\frac{3}{5}+\frac{2}{10}\right] = \frac{8}{10}\cdot\frac{1}{2} = \frac{2}{5} \qquad]$$



13.

14.

[Hint: A : the instrument has failed

B_1 : first unit fails and second is healthy

B_2 : first unit healthy and second unit fails

B_3 : both fails

B_4 : both healthy

$P(B_1) = 0.1 \times 0.8 = 0.08$

$P(B_2) = 0.2 \times 0.9 = 0.18$

$P(B_3) = 0.1 \times 0.2 = 0.02$

$P(B_4) = 0.9 \times 0.8 = 0.72$

$P(A/B_1) = P(A/B_2) = P(A/B_3) = 1$

$P(A/B_4) = 0$

Now compute $P(B_1/A)$]

15.

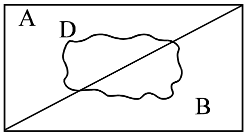
[Hint : A = event that the item came from lot A ; $P(A) = \frac{3}{3+2} = \frac{3}{5}$

B = item came from B ; P (B) = 2/5

D = item from mixed lot ' C ' is defective

$P(D) = P(D \cap A) + P(D \cap B)$

$$= P(A).\ P(D/A) + P(B).\ P(D/A) = \frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5}=\frac{8}{25} \text{ Ans.]}$$



16.

[Sol. $D[(R \cap R) + (B \cap B) + (G \cap G) + (O \cap O) + (V \cap V)]$

$P(R)\cdot P(R/R) + P(B)\cdot P(B/B) + \ldots\ldots\ldots$

$$\frac{1}{5}\left[\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}+\frac{2}{6}\right] = \frac{1}{5}\cdot\frac{5}{3} = \frac{1}{3}$$

17

[Sol. $4\begin{cases}3\ \textit{normal die}\\1\ \textit{rigged die}\end{cases}$

A : die shows up the face 5

B_1 : it is a rigged die ; $P(B_1) = 1/4$

B_2 : it is a normal die ; $P(B_2) = 3/4$

$P(A/B_1) = 1$; $P(A/B_2) = \frac{1}{216}$; $\qquad P(B_1/A) = \dfrac{\frac{1}{4}\cdot 1}{\frac{1}{4}\cdot 1 + \frac{3}{4}\cdot\frac{1}{216}} = \frac{216}{219}$ Ans.]

18.

[Hint: A : car met with an accident

B_1: driver was alcoholic, $P(B_1) = 1/5$

B_2: driver was sober, $P(B_2) = 4/5$

$P(A/B_1) = 0.001$; $P(A/B_2) = 0.0001$

$P(B_1/A) = \dfrac{(.2)(.001)}{(.2)(.001)+(.8)(.0001)} = 5/7$ **Ans.**]

19.

[Hint: $P(E) = \frac{1}{6}+\frac{1}{10}+\frac{1}{8} = \frac{47}{120}$ Ans.]

20.

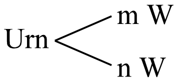
[Hint: $P(H) = \frac{2}{3}$; $P(T) = \frac{1}{3}$; $P(H\ T \text{ or } T\ H) = \frac{4}{9}$; $P(H\ H) = \frac{4}{9}$; $P(T\ T) = \frac{1}{9}$

$$\frac{{}^2C_1\cdot{}^3C_1\cdot{}^1C_1}{{}^6C_3}\left[\frac{4}{9}+\frac{4}{9}+\frac{1}{9}\right] = \frac{6}{20} = \frac{3}{10} = 30\% \text{ Ans.]}$$

21.

[Hint: $n(S) = {}^mC_{m-1}\cdot{}^nC_n = m$

$n(A) = {}^{m+n}C_{m+n-1} = m + n$



$\Rightarrow P = \dfrac{m}{m+n}$

This is equivalent to having all (m + n) balls are keeping one white ball in the given Urn]

22.

[Hint: E : event that 2nd drawn is white

$$P(E) = P(B\ W \text{ or } W\ W) = \frac{n}{m+n}\cdot\frac{m}{m+n-1} + \frac{m}{m+n}\cdot\frac{m-1}{m+n-1}$$

$$= \frac{m(m+n-1)}{(m+n)(m+n-1)} = \frac{m}{m+n} \quad \Rightarrow \quad \text{(A)]}$$

23.

[Sol. $P(F/F) = 0.9$; $P(C/F) = 0.1$; $P(C/C) = 0.8$; $P(F/C) = 0.2$

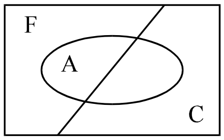
$P(F) = \frac{3}{10}$; $P(C) = \frac{7}{10}$

A : Wine tasted was French

B_1 : It is a Californian wine ; $P(B_1) = \frac{7}{10}$

B_2 : It is a French wine ; $P(B_2) = \frac{3}{10}$

$P(A/B_1) = 0.2$; $\qquad P(A/B_2) = 0.9$



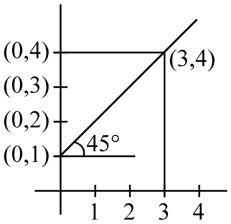
$P(B_1/A) = \dfrac{0.7\times0.2}{0.7\times0.2+0.3\times0.9} = \dfrac{0.14}{0.14+0.27} = \dfrac{14}{41}$ Ans.]

24.

[Hint : $\dfrac{K}{7^6} = {}^7C_3.\left(\dfrac{1}{7}\right)^3 \cdot {}^4C_2\left(\dfrac{1}{7}\right)^2 \cdot \left(\dfrac{1}{7}\right)^2 \Rightarrow K = 30$]

25.

[Sol. The y value of the point P on the line from (0, 1) to (3, 4) must be greater than the y calculated,



$\dfrac{3\cdot y}{2} = 2 \Rightarrow y = \dfrac{4}{3}$

hence favour length $3 - \dfrac{1}{3} = \dfrac{8}{3}$

Total length = 3 $\Rightarrow$ probability = $\dfrac{8}{9}$ **Ans.**]

26.

[Hint: n (S) = B G G (3) ; B B G (3) ; B B B (1) ; hence n(S) = 7

n (A) = B B G (3) $\Rightarrow$ $p = \dfrac{3}{7}$]

27.

[Hint: $P(A \cap B)$, P(A), P(B), $P(A \cup B)$ are in A.P. with d = P(A)

$\therefore$ $P(A) - P(A \cap B) = P(A)$ $\Rightarrow$ $P(A \cap B) = 0$ $\Rightarrow$ A & B are ME or incompatible

also $P(B) - P(A) = P(A)$ $\Rightarrow$ $2P(A) = P(B)$

$\therefore$ if P(A) = p ; P(B) = 2p $\Rightarrow$ (D) compatible means whcih can happen simultaneously]

28.

[Sol. Total number of possible cases = 3 (either 2 or 4 or 6 are drawn)

Hence required probability = $\dfrac{1}{3}\left(\dfrac{{}^3C_1\times{}^3C_1}{{}^6C_2}+\dfrac{{}^3C_2\times{}^3C_2}{{}^6C_4}+\dfrac{{}^3C_3\times{}^3C_3}{{}^6C_6}\right) = \dfrac{11}{15}$ $\Rightarrow$ (B)]

29.

[Hint: N = {1, 2,.......10} $\rightarrow$ 3 are drawn

A = minimum of the chosen number is 3

B = maximum number of the chosen number is 7.

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = \dfrac{{}^7C_2 + {}^6C_2 - {}^3C_1}{{}^{10}C_3}$]

30.

[Hint: N = {1, 2, ...5, ...10, ...100}

two tickets are drawn

A : maximum number on the two chosen ticket is ≤ 10 $\Rightarrow$ n(S) = 10

B : minimum number on the two chosen ticket is 5

$$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^{5}C_1}{{}^{10}C_2} = \frac{5}{45} = \frac{1}{9}$$ [one of the ticket is 5 and one is frm 6, 7, 8, 9, 10]]

31.

[Hint: 7 players (leaving S_1 & S_2) out of 14 can be selected in ${}^{14}C_7$ and the 8th player can be chosen in two ways i.e. either s_1 or s_2. Hence the total ways = ${}^{14}C_7 . 2$

Therefore $$p = \frac{2 . {}^{14}C_7}{{}^{16}C_8} = \frac{8}{15}$$]

[Alternatively: Let E_1 : S_1 and S_2 are in the same group

E_2 : S_1 and S_2 are in the different group

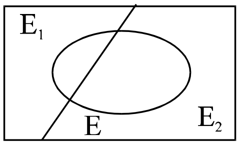
E : exactly one of the two players S_1 & S_2 is among the eight winners.

$E = (E \cap E_1) + (E \cap E_2)$

$P(E) = P(E \cap E_1) + p\,(E \cap E_2)$

$P(E) = P(E_1).P(E/E_1) + p(E_2).P(E/E_2)$ ....(1)

Now $$P(E_1) = \frac{\dfrac{(14)!}{(2)^7.7!}}{\dfrac{16!}{2^8.8!}} = \frac{1}{15}$$



$$P(E_2) = 1 - \frac{1}{15} = \frac{14}{15}$$

$$P(E) = \frac{1}{15}.1 + \frac{14}{15} \cdot P\text{ (exactly one of either } S_1 \text{ \& } S_2 \text{ wins)}$$

$$= \frac{1}{15} + \frac{14}{15} . \left(\frac{1}{2} . \frac{1}{2} + \frac{1}{2} . \frac{1}{2}\right) = \frac{1}{15} + \frac{1}{14} . \frac{1}{2} = \frac{1}{15} + \frac{7}{15} = \frac{8}{15}$$ Ans]

32.

[Hint:

3^a ends in → 7^b ends in ↓	1	3	7	9
1			8	
3				
7	8			
9				8

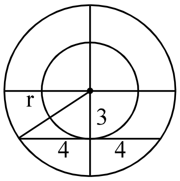
Out of 16 case 3 are favorable ⇒ $p = \frac{3}{16}$]

33.

[Sol. r = 5 $[\pi r^2 - 9\pi]$

annular region = 16

$p = \frac{16}{25}$ Ans.]



34.

[Sol. E_1 : event that the dot is removed from an odd face

E_2 : dot is removed from the even face

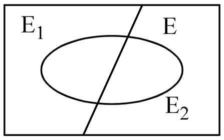
E : die throws has an odd number of dots on its top face

$P(E) = P(E \cap E_1) + P(E \cap E_2)$

$$= P(E_1) \cdot P(E / E_1) + P(E_2) \cdot P(E / E_2)$$

$$= \left(\frac{1+3+5}{21}\right)\cdot\frac{2}{6} + \left(\frac{2+4+6}{21}\right)\cdot\frac{4}{6}$$

$$= \frac{9}{21}\cdot\frac{1}{3} + \frac{12}{21}\cdot\frac{2}{3} = \frac{3}{21} + \frac{8}{21} = \frac{11}{21} \text{ Ans.]}$$



35.

[Sol. The n strings have a total of 2n ends. One boy picks up one end, this leaves (2n – 1) ends for the second boy to choose, of which only one is correct.

$$\therefore \quad p = \frac{1}{2n-1} \Rightarrow \frac{1}{2n-1} = \frac{1}{101} \Rightarrow 2n - 1 = 101 \Rightarrow n = 51 \text{]}$$

36.

[Hint:
A : card is of heart but not king 12
B : king but not heart 3
C : heart and king 1

$$P(E) = \frac{{}^{12}C_1 . {}^{3}C_1 + {}^{3}C_1 . {}^{1}C_1 + {}^{1}C_1 . {}^{12}C_1}{{}^{52}C_2} = \frac{2}{52} \text{]}$$

37.

[Sol. Determinant = ad – bc

probability that randomly chosen product (xy) will be odd = P(both odd) = p^2

$\therefore$ Probability (xy) is even = $1 - p^2$

Now (ad – bc) is even $\Rightarrow$ both odd or both even = $p^4 + (1 - p^2)^2$

Hence $p^4 + (1 - p^2)^2 = \frac{1}{2}$

put $p^2 = t$; $t^2 + (1 - t)^2 = \frac{1}{2} \Rightarrow 2t^2 - 2t + \frac{1}{2} = 0 \Rightarrow 4t^2 - 4t + 1 = 0$

$(2t - 1)^2 = 0 \Rightarrow t = \frac{1}{2}$; $\therefore$ $p^2 = \frac{1}{2}$; Hence $p = \frac{1}{\sqrt{2}}$ **Ans.**]

38.

[Sol. Given $P(a) = \frac{1}{16}$; $P(b) = \frac{1}{16}$; $P(c) = \frac{2}{16}$; $P(d) = \frac{3}{16}$; $P(e) = \frac{4}{16}$; $P(f) = \frac{5}{16}$;

$$P(a, c, e) = P(A) = \frac{1}{16} + \frac{2}{16} + \frac{4}{16} = \frac{7}{16} \ ; \ A^c = \{b, d, f\}$$

$$P(c, d, e, f) = P(B) = \frac{2}{16} + \frac{3}{16} + \frac{4}{16} + \frac{5}{16} = \frac{14}{16}$$

$$P(b, c, f) = P(C) = \frac{1}{16} + \frac{2}{16} + \frac{5}{16} = \frac{8}{16}$$

$$p_1 = P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{P(c, e)}{P(B)} = \frac{6}{14} = \frac{3}{7}$$

$$p_2 = P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{P(c, f)}{P(C)} = \frac{7}{8} = \frac{7}{8}$$

p_3 = P(C/A^c) = $\dfrac{P(C \cap A^c)}{P(A^c)} = \dfrac{P(b,f)}{P(b,d,f)} = \dfrac{6}{9} = \dfrac{2}{3}$

p_4 = P(A^c/C) = $\dfrac{P(A^c \cap C)}{P(C)} = \dfrac{P(b,f)}{P(b,c,f)} = \dfrac{6}{8} = \dfrac{3}{4}$

Hence $p_1 = \dfrac{72}{168}$; $p_2 = \dfrac{147}{168}$; $p_3 = \dfrac{112}{168}$; $p_4 = \dfrac{126}{168}$;

$p_1 < p_3 < p_4 < p_2 \Rightarrow$ (C)]

39.

[Sol. P(1,3,5,7,9) = $\left(\dfrac{5}{9}\right)^n$; P(1,3,7,9) = $\left(\dfrac{4}{9}\right)^n$; P(5) = $\left(\dfrac{5}{9}\right)^n - \left(\dfrac{4}{9}\right)^n$

P(1,2,3,4,6,7,8,9) = $\left(\dfrac{8}{9}\right)^n$; P(1,2,3,4,5,6,7,8,9) = $\left(\dfrac{8}{9}\right)^n + \left(\dfrac{5}{9}\right)^n - \left(\dfrac{4}{9}\right)^n$

P(div. by 10) = 1 – $\left[\left(\dfrac{8}{9}\right)^n + \left(\dfrac{5}{9}\right)^n - \left(\dfrac{4}{9}\right)^n\right]$]

40.

[Hint: Let F = {1 or 2 or 3 } and S = {4} ; P(F) = 3/6 ; P(S) = 1/6

P (Highest number thrown is 4) = P(FFS or FSS or SSS)

$= \dfrac{3}{6}.\dfrac{3}{6}.\dfrac{1}{6}.3 + \dfrac{3}{6}.\dfrac{1}{6}.\dfrac{1}{6}.3 + \dfrac{1}{216} = \dfrac{27}{216} + \dfrac{9}{216} + \dfrac{1}{216}$]

[Alternative: Highest number in three throws 4 $\Rightarrow$ atleast one of the throws must be equal to 4.

Number of ways when three blocks are filled from {1,2,3,4} = 4^3 $\boxed{X \;\; X \;\; X}$

$\therefore$ number of ways when filled from {1,2,3} = 3^3

$\therefore$ required number of ways = $4^3 - 3^3$

$\therefore$ Probability = $\dfrac{4^3 - 3^3}{6^3} = \dfrac{37}{216}$]

41.

[Sol. A : Coin randomly selected tossed 10 times, fell head wise 7 times

B_1 : coin was a fair coin P(B_1) = 1/2

B_2 : Coin was a weighted coin P(B_2) = 1/2

P(A/B_1) = ${}^{10}C_7 \cdot \left(\dfrac{1}{2}\right)^7 \cdot \left(\dfrac{1}{2}\right)^3 = {}^{10}C_3 \cdot \dfrac{1}{2^{10}}$

P(A/B_2) = ${}^{10}C_7 \cdot \left(\dfrac{4}{5}\right)^7 \cdot \left(\dfrac{1}{5}\right)^3 = {}^{10}C_3 \cdot \dfrac{4^7}{5^{10}}$

$$P(B_1/A) = \frac{\frac{1}{2^{10}}}{\frac{1}{2^{10}}+\frac{4^7}{5^{10}}} = \frac{1}{1+\frac{4^7\cdot 2^{10}}{5^{10}}} = \frac{5^{10}}{5^{10}+8^8} \text{ Ans.]}$$

42.

[Sol. We have P(E) = P(T T T T H + T T T T T) = $\frac{1}{32} + \frac{1}{32} = \frac{1}{16} = \frac{p}{q} \Rightarrow$ p + q = 17 **Ans.**]

43.

[Sol. We have $^8C_3\, p^3(1-p)^5 = \frac{1}{25}\, {}^8C_5\, p^5(1-p)^3$ or $1 - p = \frac{(1)(p)}{5}$

So, we find $p = \frac{5}{6} \Rightarrow$ m + n = 11 **Ans.**]

44.

[Sol. To get atleast 3 marks, atleast 4 must be right.

Now, n = 5; $p = \frac{1}{4}$; $q = \frac{3}{4}$

r = 4 or 5

$$P(r\geq 4) = {}^5C_4\left(\frac{1}{4}\right)^4\cdot\left(\frac{3}{4}\right) + {}^5C_5\left(\frac{1}{4}\right)^5$$

$$= 5\cdot\frac{1}{256}\cdot\frac{3}{4}+\frac{1}{45} = \frac{16}{4\cdot 256} = \frac{1}{64}$$ **Ans.**]

45.

[Sol. $P(S) = \frac{1}{2} = P(F)$

E : series would go to a full seven games.

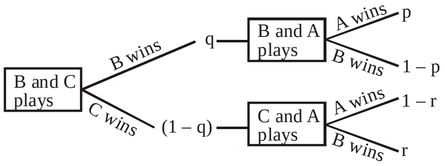
$$P(E) = P(\underbrace{\times\ \times\ \times\ \times\ \times\ \times}_{3S\ \&\ 3F}\ (S \text{ or } F)) = {}^6C_3\cdot\frac{1}{2^6} = \frac{20}{64} = \frac{5}{16}$$ **Ans.**]

46.

[Sol. H_1 can occur in 4 ways depending on the suit of the two.
H_2 can occur in 4 ways depending on the suit of the straight.
H_3 can occur in 4 ways depending on the suit of the missing king.
Hence all the hands are equally likely.
$(n(S) = {}^{52}C_5$ for all.) **Ans.**]

47.

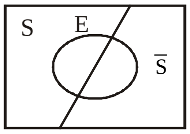
[Sol.



P(E) = qp + (1 – q)(1 – r)]

48.

[Sol. E : student completed the course



$$P(S/E) = \frac{P(S\cap E)}{P(E)} = \frac{P(S)\cdot P(E/S)}{P(S)\cdot P(E/S)+P(\overline{S})\cdot P(E/\overline{S})}$$

$$= \frac{(0.4)(0.8)}{(0.4)(0.8)+(0.6)(0.4)}$$

$$= \frac{0.32}{0.32+0.24} = \frac{32}{56} = \frac{4}{7}$$ **Ans.**]

49.

[Sol. Kota $\underline{\times\times\times\times\times\times}$ Bombay

$n(S) = 7^5$ (First passenger can get down at any one of the 7 stations in 7 ways etc.)

$n(A) = {}^7C_5 \cdot 5!$

$\Rightarrow$ Probability = $\frac{{}^7P_5}{7^5}$ $\Rightarrow$ C **Ans.**]

50.

[Hint: T_1 and T_2 plays and |||ly T_2 and T_3 and T_3 and T_1

Total games played is 3

P(game ends in tie) i.e. every team wins exactly one game

Case-1: T_1 v/s T_2 $\Rightarrow$ T_1 wins
T_2 v/s T_3 $\Rightarrow$ T_2 wins
T_3 v/s T_1 $\Rightarrow$ T_3 wins

$$P(\text{ties}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

Case-2: T_1 v/s T_2 $\Rightarrow$ T_2 wins
T_2 v/s T_3 $\Rightarrow$ T_3 wins
T_3 v/s T_1 $\Rightarrow$ T_1 wins

$$P(\text{ties}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

Hence P(ties) = $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ **Ans.**]

51.

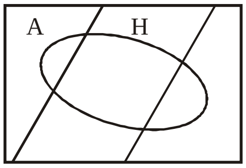
[Sol. H : Victim was hit

A : Event that Mr. A was given the live bullet ; $P(A) = \frac{1}{3}$

B : Mr. B had live bullet ; $P(B) = \frac{1}{3}$

C : Mr. C has live bullet ; $P(C) = \frac{1}{3}$

$$P(C/H) = \frac{P(C \cap H)}{P(H)} = \frac{P(C) \cdot P(H/C)}{P(H)}$$



$$P(H) = P(H \cap C) + P(H \cap B) + P(H \cap A)$$

$$= \frac{1}{3}\ [P(H/C) + P(H/B) + P(H/A)]$$

$$= \frac{1}{3}\ [0.8 + 0.7 + 0.6] = \frac{0.21}{3}$$

$P(C/H) = \frac{0.8}{0.21} = \frac{8}{21}$ **Ans.**]

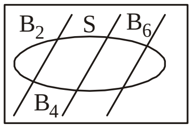
52.
53.

54.
[Sol.

(i) $P(A/B_E) = \frac{P(A \cap B_E)}{P(B_E)}$, where A = sum is 4.

And $P(B_E) = P(B_2) + P(B_4) + ...$

$$= \frac{1}{4}+\frac{1}{16}+\frac{1}{64}+........\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$



$$P(A \cap B_E) = P(A \cap B_2) + P(A \cap B_4) = P(B_2)\ P(A/B_2) + P(B_4)\ P(A/B_4)$$

$$= \frac{1}{4}\left(\frac{3}{36}\right)+\frac{1}{16}\left(\frac{1}{6^4}\right) = \frac{\left(4\cdot3\cdot6^2+1\right)}{16\cdot6^4} = \frac{433}{16\cdot6^4}$$

Hence $P(A/B_E) = \frac{433}{16\cdot6^4}\cdot3 = \frac{433}{32\cdot216} \cong \frac{1}{16}$ **Ans.**

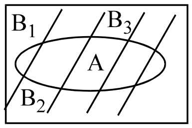
(ii) $n(S) = 6 \cdot 6 \cdot 6 = 6^3 = 216$

Now greatest number is 4, so atleast one of the dice shows up 4.

$\therefore$ $n(A) = 4^3 - 3^3 = 37$

Hence $P(A) = \frac{37}{216}$ **Ans.**

(iii) $P(A) = P(B_1 \cap A) + P(B_2 \cap A) + ... + P(B_n \cap A)$



$$= \frac{1}{2}\left(\frac{1}{6}\right)+\frac{1}{2^2}\left(\frac{2}{36}\right)+\frac{1}{2^3}\left(\frac{1}{6^3}\right)$$

$$= \frac{1}{12}+\frac{1}{72}+\frac{1}{(8)(216)} = \frac{1}{(8)(216)}[144 + 24 + 1] = \frac{169}{(8)(216)}$$

Now $P(B_2/A) = \frac{P(B_2 \cap A)}{P(A)}$.

Hence $P(B_2/A) = \frac{1}{72}\cdot\frac{(8)(216)}{169} = \frac{169}{(8)(216)} = \frac{24}{169}$ **Ans.**]

55.

56.

57.

[So. Bag A $\left\langle \begin{matrix} 1W \\ 5B \end{matrix}\right.$; Bag B $\left\langle \begin{matrix} 2W \\ 4B \end{matrix}\right.$; Bag C $\left\langle \begin{matrix} 3W \\ 3B \end{matrix}\right.$

Let E : Event of drawing 1Black marble and 1White marble from any 2 selected bags.

E_1 : Event of selecting the bags B & C

E_2 : Event of selecting the bags C & A

E_3 : Event of selecting the bags A & B

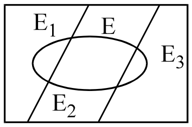
A : Event of drawing 1White marble from bag A

B : Event of drawing 1White marble from bag B

C : Event of drawing 1White marble from bag C

Now $E = (E \cap E_1) + (E \cap E_2) + (E \cap E_3)$

$$P(E \cap E_1) = P(E_1)\cdot P(E/E_1) = \frac{1}{3}\left(\frac{4\cdot3+2\cdot3}{6\cdot6}\right) = \frac{18}{108}$$



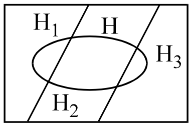
$$P(E \cap E_2) = P(E_2)\cdot P(E/E_2) = \frac{1}{3}\cdot\frac{3\cdot1+3\cdot5}{6\cdot6} = \frac{18}{108}$$

$$P(E \cap E_3) = P(E_3)\cdot P(E/E_3) = \frac{1}{3}\cdot\frac{5\cdot2+1\cdot4}{6\cdot6} = \frac{14}{108}$$

$\therefore$ $P(E) = P(E \cap E_1) + P(E \cap E_2) + P(E \cap E_3) = \frac{18}{108}+\frac{18}{108}+\frac{14}{108} = \frac{50}{108} = \frac{25}{54}$ **Ans.(i)**

$\therefore$ $P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)} = \frac{18/108}{50/108} = \frac{9}{25}$; let $E_1/E = H_1$

$P(E_2/E) = \frac{P(E_2 \cap E)}{P(E)} = \frac{18/108}{50/108} = \frac{9}{25}$; let $E_2/E = H_2$



$P(E_3/E) = \frac{P(E_3 \cap E)}{P(E)} = \frac{14/108}{50/108} = \frac{7}{25}$ **Ans.(ii)** ; let $E_3/E = H_3$

Let H : drawing 1 white marble from third bag i.e. $P(H) \to P$

$P(H) = P(H \cap H_1) + P(H \cap H_2) + P(H \cap H_3)$

$= P(H_1)\cdot P(H/H_1) + P(H_2)\cdot P(H/H_2) + P(H_3)\cdot P(H/H_3)$

$= P(H_1)\ P(A) + P(H_2)\ P(B) + P(H_3)\ P(C)$

$$= \frac{9}{25}\cdot\frac{1}{6} + \frac{9}{25}\cdot\frac{2}{6} + \frac{7}{25}\cdot\frac{3}{6}$$

$$= \frac{48}{25\cdot 6} = \frac{8}{25} = \frac{m}{n}$$

$\Rightarrow$ $(m + n) = 33$ **Ans.(iii)**]

58.

59.

60.

61.

[Sol. P (2^{nd} card is J of H) = $\frac{51}{52}\cdot\frac{1}{51} = \frac{1}{52}$

There are 16 ways to get a Jack or a hearts : get one of the thirteen hearts (Ace through King of hearts), or get one of the Jack of clubs, Jack of spades, or Jack of diamonds.

Hence, probability (Jack or hearts) = 16/52.

P (H / J) = $\frac{P(H \cap J)}{P(J)} = \frac{1}{4}$

P(win) = P(any 2's or Ace) = $\frac{4}{48} + \frac{4}{48} = \frac{1}{6}$

Q.6

[Hint: $n(s) = {}^{16}C_3$; $\frac{n(A)}{n(s)} = \frac{{}^{5}C_3}{{}^{16}C_3} = \frac{1}{56}$ Ans.]

Q.7

[Hint: H H T T; $n(s) = 2^4$; $n(A) = \frac{4!}{2!2!} = 6$; $p = \frac{6}{16}$; $P(A) = P(B) = \frac{1}{2}$]

Q.9 [Ans. $\frac{(13)^4}{{}^{52}C_4} = \frac{2197}{20825}$]

Q.10 [Ans. 952 to 715]

Q.11 [Ans. $\frac{3}{140}$]

[Hint: $V_1\ V_2\ V_3 \mid V_4\ V_5\ V_6\ V_7 \mid V_8$; $n(s) = 81$; $\frac{3!3!4!}{8!} = \frac{36}{8\cdot 7\cdot 6\cdot 5} = \frac{3}{140}$ Ans.]

Q.12

[Hint: $n(S) = 8^5$; $n(A) = {}^{8}C_5 \cdot 5!$]

Q.13

[Sol. The zeroes, for the $f(x^2)$ are ± 2 and ± 3 i.e. four zeroes. [Ans. $\frac{4}{21}$]

In the set of integers from [–10, 10]

There are 21 elements.
Four of these are the zeroes.
Therefore, the probability is P = $\frac{4}{21}$ Ans.]

Q.14 [Ans. $\frac{n(n-1)}{(m+n)(m+n-1)}$]

Q.15 [Ans. 2/3]

Q.16 [Ans. 1/2]
[Hint: H H T (3) ; H T T (3) $\Rightarrow$ n (s) = 6]

Q.17 [Ans. 1/5, 1/10]

Q.18
[Hint: P (T) = p; P (H) = 3p; p = 1/4] [**Ans.** 3/4, 1/4]

Q.20 [Ans. $\frac{{}^{4}C_{4} \cdot {}^{48}C_{9}}{{}^{52}C_{13}}$]

Q.4

[Sol. (a) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6} \Rightarrow P(A \cap B) = \frac{P(B)}{6} = \frac{1}{18}$ Ans.]

(b) $P(A \cup B) = \frac{1}{5} + \frac{1}{3} - \frac{1}{18} = \frac{18+30-5}{90} = \frac{43}{90}$ Ans.

(c) $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{1}{18} \cdot \frac{5}{1} = \frac{5}{18}$ Ans.

(d) $P(A) \cdot P(B) = \frac{1}{5} \cdot \frac{1}{3} = 15 \neq P(A \cap B)$. A & B are not independent]

Q.5 [**Ans.** (i) $\frac{5}{8}$, (ii) $\frac{3}{8}$]

EXERCISE 2 (B)

1.

Sol. m cards can be placed into n boxes independently in n^m ways.

$\therefore$ $n(S) = n^m$

Now m adjacent boxes can be chosen in n – m + 1 ways. In each of these the cards can be placed into m! ways.

total number of ways in which the cards can be placed in m consecutive boxes = (n – m + 1). m!

Required probability = $\frac{m!(n-m+1)}{n^m}$.

2.

Sol. Any three tickets out of 21 tickets can be chosen is ${}^{21}C_3$ ways. For the favourable choice if the chosen numbers are a, b and c, a < b < c, then we should have $\frac{a+c}{2} = b$. Obviously either both a and c are even or both are odd and then b is fixed. Hence for the favourable choice we have to chose two numbers from 1 to 21, which are either both even or both odd. This can be done in ${}^{11}C_2 + {}^{10}C_2$ ways.

Hence required probability = $\frac{{}^{11}C_2 + {}^{10}C_2}{{}^{21}C_3} = \frac{10}{133}$.

3.

Sol. Let E_1 be the event of success of A and let E_2 be the event of success of B

Since A has 3 shares in a lottery containing 3 prizes and 9-blanks, A will draw 3 tickets out of 12 tickets (containing 3 prizes and 9 blanks). A will get success if he draws atleast one prize out of 3 draws.

$\therefore$ $P(E_1') = \frac{{}^9C_3}{{}^{12}C_3} = \frac{21}{55}$

$\therefore$ $P(E_1) = 1 - \frac{21}{55} = \frac{34}{55}$

Again, $P(E_2') = \frac{{}^6C_2}{{}^8C_2} = \frac{15}{28}$

$\therefore$ $P(E_2) = 1 - \frac{15}{28} = \frac{13}{28}$

$\therefore$ $\frac{P(E_1)}{P(E_2)} = \frac{34}{55} \times \frac{28}{13} = \frac{952}{715}$

$\therefore$ $P(E_1) : P(E_2) = 952 : 715$.

4.

Sol. Let H, T and S be the events “head turns up”, “tail turns up” and “head or tail turns up”

Then $P(H) = P(T) = \frac{1}{2}$ and $P(S) = 1$

Since the given event is “at least m consecutive heads turn up”, therefore in any favorable out come there are m consecutive heads and the rest are any of head or tail

Consider the events

$$A_1 = \left\{\underbrace{H, H, H, \ldots, H}_{m\ \text{times}}, \underbrace{S, S, S, \ldots, S}_{n\ \text{times}}\right\} \quad \text{with } P(A_1) = \frac{1}{2^m}.1^n = \frac{1}{2^m}$$

$$A_2 = \left\{T, \underbrace{H, H, H, \ldots, H}_{m\ \text{times}}, \underbrace{S, S, S, \ldots, S}_{n-1\ \text{times}}\right\} \text{ with } P(A_2) = \frac{1}{2}.\frac{1}{2^m}.1^{n-1} = \frac{1}{2^{m+1}}$$

$$A_3 = \left\{S, T, \underbrace{H, H, H, \ldots, H}_{m\ \text{times}}, \underbrace{S, S, S, \ldots, S}_{n-2\ \text{times}}\right\} \text{ with } P(A_3) = 1.\ \frac{1}{2}.\frac{1}{2^m}.1^{n-2} = \frac{1}{2^{m+1}}$$

$$\ldots\ A_{n+1} = \left\{\underbrace{S, S, S, \ldots, S}_{n-1\ \text{times}}, T, \underbrace{H, H, H, \ldots, H}_{m\ \text{times}}\right\} \text{ With } P(A_{n+1}) = 1^{n-1}.\ \frac{1}{2}.\frac{1}{2^m} = \frac{1}{2^{m+1}}$$

The given event is $A_1 \cup A_2 \cup A_3 \cup A_{n+1}$. As $A_1, A_2, A_3, \ldots, A_{n+1}$ are pair – wise mutually exclusive.

The required probability

$$= P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_{n+1})$$

$$= \frac{1}{2^m} + \underbrace{\frac{1}{2^{m+1}} + \frac{1}{2^{m+1}} + \ldots + \frac{1}{2^{m+1}}}_{n-\text{times}} = \frac{1}{2^m} + \frac{n}{2^{m+1}} = \frac{2+n}{2^{m+1}}.$$

5.

Sol. Total number of sample points in the sample space = $6^4 = 1296$

Number of sample points in favour of the event

= Coefficient of x^{10} in the expansion of $(1 + x + x^2 + \ldots + x^5)^2 (x + x^2 + \ldots + x^6)^2$

= Coefficient of x^{10} in the expansion of $x^2(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansion of $(1 + x + x^2 + \ldots + x^5)^4$

= Coefficient of x^8 in the expansionof $\left(\frac{1-x^6}{1-x}\right)^4$

= Coefficient of x^8 in the expansion of $(1 - x^6)^4 (1 - x)^{-4}$

= Coefficient of x^8 in the expansion of $(1 - 4x^6)\left(1 + 4x + \frac{4\times5}{2!}x^2 + \frac{4\times5\times6}{3!}x^3 + \ldots\right)$

$= 1\ \times {}^{11}C_8 - 4\times {}^5C_2 = 125.$

$\therefore$ Required probability = $\frac{125}{1296}$.

6.

Sol. Probability of success = $\frac{3}{6} = \frac{1}{2} \Rightarrow \quad p = \frac{1}{2}\ ,\ q = \frac{1}{2}$

(i) For exactly four successes, required probability = ${}^7C_4 . \left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^3 = \frac{35}{128}$

(ii) For atleast four successes, required probability

$$= {}^7C_4\ \left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5 . \left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6 . \left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1}{2}\right)^7$$

$$= \frac{35}{128} + \frac{21}{128} + \frac{7}{128} + \frac{1}{128} = \frac{64}{128} = \frac{1}{2}.$$

7.

Sol. A particular thing is received by a man with probability $p = \frac{a}{a+b}$ and by a woman with probability $q = \frac{b}{a+b}$. If distributing a single object is an experiment, then this experiment is repeated m time. The required probability

$= {}^mC_1 .\ p.\ q^{m-1} + {}^mC_3 . p^3 . q^{m-3} + {}^mC_5 . p^5 . q^{m-5} + \ldots$

$$= \frac{(q+p)^m - (q-p)^m}{2} = \frac{1}{2}\left[1 - \left(\frac{b-a}{b+a}\right)^m\right] = \frac{1}{2}\left\{\frac{(b+a)^m - (b-a)^m}{(b+a)^m}\right\}.$$

8.

Sol. Let E denote the event that the target is hit when x shells are fired at point A. Let E_1 (E_2) denote the event that the artillery target is at point $A(B)$.

We have $P(E_1) = \frac{8}{9}$, $P(E_2) = \frac{1}{9}$.

$$\Rightarrow \quad P\left(\frac{E}{E_1}\right) = 1 - \left(\frac{1}{2}\right)^x \text{ and } P\left(\frac{E}{E_2}\right) = 1 - \left(\frac{1}{2}\right)^{21-x}$$

Now $P(E) = \frac{8}{9}\left[1 - \left(\frac{1}{2}\right)^x\right] + \frac{1}{9}\left[1 - \left(\frac{1}{2}\right)^{21-x}\right]$

$$\Rightarrow \quad \frac{d}{dx}(P(E)) = \frac{8}{9}\left(\frac{1}{2}\right)^x \ell n\, 2 + \frac{1}{9}\left(-\left(\frac{1}{2}\right)^{21-x} \ell n 2\right)$$

Now we must have $\frac{d}{dx}(P(E)) = 0 \qquad \Rightarrow \qquad x = 12$, also $\frac{d^2}{dx^2}(P(E)) < 0$

Hence P(E) is maximum, when x = 12.

9.

Sol. P(no one among $A_1, A_2 \ldots, A_n$ dies within a year) = $(1 - p)^n$

P (at least one among $A_1, A_2, \ldots, A_n$ dies within a year) = $1 - (1 - p)^n$

P(A_1dies within a year and is first to die) = $\frac{1}{n}\left[1-\left(1-p\right)^{n}\right]$.

10.

Sol. Let A_1, A_2 and A_3 be the events that the bag picked is A, B and C respectively.
Let E be the event that a white ball is drawn.
We are supposed to find P(A_1/E), P(A_2/E), P(A_3/E).

$$P\left(\frac{A_1}{E}\right)=\frac{P\left(A_1\cap E\right)}{P(E)}=\left(\frac{\text{Probability that bag Ais chosen and white ball is drawn}}{\text{Probability that a bag is chosen at random and a white ball is drawn}}\right)$$

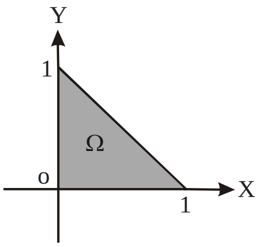
$$=\frac{P\left(A_1\right).P\left(E/A_1\right)}{P\left(A_1\right).P\left(E/A_1\right)+P\left(A_2\right).P\left(E/A_2\right)+P\left(A_3\right).P\left(E/A_3\right)}$$

$$=\frac{\frac{1}{3}.\frac{a_1}{a_1+b_1}}{\frac{1}{3}.\left[\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\frac{a_3}{a_3+b_3}\right]}=\frac{p_1}{p_1+p_2+p_3}\text{, } p_k=\frac{a_k}{a_k+b_k}\text{, k = 1, 2, 3.}$$

Similarly, p(A_2/E) = $\frac{p_2}{p_1+p_2+p_3}$, p(A_3/E) = $\frac{p_3}{p_1+p_2+p_3}$.

11.

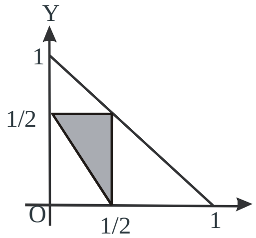
Sol. The sample space is the collection of points (x, y) in the x – y plane satisfying x > 0, y > 0 and x + y < 1 (z = (1 – (x + y)).
Thus measure of the sample space is the area of the region Ω = 1/2.



The required event A is the collection of points (x, y) in the sample space for which sum of any two numbers x, y and 1 – (x + y) is greater than the third i.e., x + y > ½, y < ½ and x < ½.
Hence measure of the favorable region is S_A, given by

$$S_A=\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)=\frac{1}{8}$$

$$\therefore \quad P(A)=\frac{S_A}{S_\Omega}=\frac{(1/8)}{(1/2)}=\frac{1}{4}.$$



12.

Sol. Let E_λ be the event that exactly λ out of n pass the examinations and let A be the event that a student selected randomly pass the examination.

$\therefore \quad P\left(E_\lambda\right)\propto\lambda^2$

$\Rightarrow \quad P\left(E_\lambda\right)=k\lambda^2$ (k is proportionality constant)

Since $E_0, E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events.

$\Rightarrow \quad P(E_0) + P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

$\Rightarrow \quad 0 + k\,(1)^2 + k\,(2)^2 + \ldots + k\,(n^2) = 1$

$\Rightarrow \quad k = \dfrac{6}{n(n+1)(2n+1)}$ . . . (i)

(i) $P(A) = \sum\limits_{\lambda=0}^{n} P(E_\lambda)P(A/E_\lambda) = \sum\limits_{\lambda=1}^{n} k\lambda^2 \times \dfrac{\lambda}{n} = \dfrac{k}{n}\sum\limits_{\lambda=1}^{n}\lambda^3 = \dfrac{3(n+1)}{2(2n+1)}$

(ii) $P\left(\dfrac{E_1}{A}\right) = \dfrac{P(E_1).P\left(\dfrac{A}{E_1}\right)}{P(A)} = \left(\dfrac{2}{n(n+1)}\right)^2$

$P\left(\dfrac{A}{\lambda}\right) = \dfrac{P(A \cap E_\lambda)}{P(E_\lambda)} = \dfrac{\lambda}{n}$.

13.

Sol. Let E_1 be the event that both A and B speak the truth, E_2 be the event that both A and B tell a lie and E be the event that A and B agree in a certain statement. Let C be the event that A speaks the truth and D be the event that B speaks the truth.

$\therefore \quad E_1 = C \cap D$ and $E_2 = C' \cap D'$. $P(E_1) = P(C \cap D) = P(C)\,P(D) = xy$ and

$P(E_2) = P(C' \cap D') = P(C')\,P(D') = (1-x)(1-y) = 1 - x - y + xy$

Now $P\left(\dfrac{E}{E_1}\right)$ = probability that A and B will agree when both of them speak the truth = 1 and

$P\left(\dfrac{E}{E_2}\right)$ = probability that A and B will agree when both of them tell a lie = 1.

Clearly, $\left(\dfrac{E_1}{E}\right)$ be the event that the statement is true.

$$\therefore \quad P\left(\frac{E_1}{E}\right) = \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2)P(E/E_2)}$$

$$= \frac{xy.1}{xy.1+(1-x-y+xy).1} = \frac{xy}{1-x-y+2xy}.$$

14.

Sol. n(S) = 36

Let E be the event of getting the sum of digits on the dice equal to 7, then n(E) = 6.

$P(E) = \dfrac{6}{36} = \dfrac{1}{6} = p$, then $P(E') = q = \dfrac{5}{6}$

probability of not throwing the sum 7 in first m trails = q^m.

$\therefore \quad$ P(at least one 7 in m throws) = $1 - q^m = 1 - \left(\dfrac{5}{6}\right)^m$.

According to the question $1 - \left(\dfrac{5}{6}\right)^m > 0.95$

$\Rightarrow \quad \left(\dfrac{5}{6}\right)^m > 0.05$

$\Rightarrow \quad m\{\log_{10} 5 - \log_{10} 6\} < \log_{10} 1 - \log_{10} 20$

$\therefore \quad m > 16.44$

Hence, the least number of trails = 17.

15.

Sol. A wins the series, if out of first n + r games A wins n games, $0 \le r \le n$ and wins the (n + r + 1)th game.

$$\therefore \quad P(A) = \left(\sum_{r=0}^{n} {}^{n+r}C_n . q^r . p^n\right) . p \text{ (where } p + q = 1)$$

Similarly, $P(B) = \sum_{r=0}^{n} \left({}^{n+r}C_n\right) . q^{n+1} . p^r$

Now P(A) + P(B) = 1

$$\Rightarrow \quad \sum_{r=0}^{n} \left[q^r . p^{n+1} + q^{n+1} . p^r\right] \; {}^{n+r}C_n = 1 \qquad \text{. . . (i)}$$

Now put $p = q = \frac{1}{2}$

from (i), $\sum_{r=0}^{n} \left({}^{n+r}C_n\right) . \frac{1}{2^{n+r}} = 1$.

16.

Sol. From the wavy curve method, given inequality is satisfied for x < 20 or 30 < x < 40.

$\therefore$ Number of favourable outcomes = 28

Required probability = $\frac{28}{100} = \frac{7}{25}$.

17.

Sol. We have

$0 \le \frac{1+3p}{3}, \frac{1-p}{2}$ and $\frac{1-2p}{2} \le 1 \Rightarrow p \in \left[-\frac{1}{3}, \frac{1}{2}\right]$. Further if the events

(say E_1, E_2 and E_3) are exclusive, then its necessary and sufficient condition is

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) \Rightarrow P(E_1 \cup E_2 \cup E_3) = \frac{8-3p}{6}$$

$$\Rightarrow 0 \le \frac{8-3p}{6} \le 1 \qquad \Rightarrow p \in \left[\frac{2}{3}, \frac{8}{3}\right].$$

Hence the required set is ϕ.

18. [Ans. 1/(n + 1)]

Sol. P(non occurrence of (A_i)) = 1 – (i + 1) = i/(i + 1).

$\therefore$ P(non occurrence of any of events)

$$= \left(\frac{1}{2}\right).\left(\frac{2}{3}\right)...\left\{\frac{n}{(n+1)}\right\} = \frac{1}{(n+1)}.$$

19.

Sol. Let the number of marble be 2n (where n is large)

$$\text{Required probability} = \lim_{n\to\infty} \frac{n \times {}^{n}C_4}{{}^{2n}C_5} \times \frac{{}^{n}C_3 \times {}^{n}C_2}{{}^{2n}C_5}$$

$$= \lim_{n\to\infty} \frac{n \times n(n-1)(n-2)(n-3)}{4!} \times \frac{n(n-1)(n-2)}{3!} \times \frac{n(n-1)}{2!} \times \frac{(5)^2((2n-5)!)^2}{(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{n^4(n-1)^3(n-2)^2(n-3)((2n-5)!)^2 \times 5\times5\times4\times3!}{3!2!(2n!)^2}$$

$$= \lim_{n\to\infty} \frac{50.n^4(n-1)^3(n-2)^2(n-3)}{(2n(2n-1)(2n-2)(2n-3)(2n-4))^2} = \frac{50}{1024} = \frac{25}{512}.$$

20.

Sol. $$P(B/A\cup B') = \frac{P(B\cap(A\cup B'))}{P(A\cup B')}$$

$$= -\frac{P(A\cap B)}{P(A)+P(B')-P(A\cap B')} = \frac{P(A)-P(A\cap B')}{0.7+0.6-0.5}$$

$$= \frac{0.7-0.5}{0.8} = \frac{1}{4}.$$

21.

Sol. Any element of A has four possibilities : element belongs to (i) both P_1 and P_2 (ii) neither P_1 nor P_2 (iii) P_1 but not to P_2 (iv) P_2 but not to P_1. Thus n(S) = 4^n. For the favourable cases, we choose one element in n ways and this element has three choices as (i), (iii) and (iv), while the remaining n – 1 elements have one choice each, namely (ii).

Hence required probability = $\frac{3n}{4^n}$.

22.

Sol. Let $\overline{A}$ be the event of different birthdays. Each can have birthday in 365 ways, so N persons can have their birthdays in 365^N ways. Number of ways in which all have different birthdays = ${}^{365}P_N$

$$\therefore \quad P(A) = 1 - P(\overline{A}) = 1 - \frac{{}^{365}P_N}{(365)^N} = 1 - \frac{(365)!}{(365)^N(365-N)!}.$$

23.

Sol. $P(E \cap F) = P(E)\,P(F) = \frac{1}{12}$ . . . (i)

$P(E^c \cap F^c) = P(E^c).P(F^c) = \frac{1}{2}$

$\Rightarrow (1-P(E))(1-P(F)) = \frac{1}{2}$ . . . (ii)

Solving (i) and (ii), we get $P(E) = \frac{1}{3}$ & $P(F) = \frac{1}{4}$, as $P(E) > P(F)$.

24.

Sol. Let S be the sample space and let E be the required event, then $n(S) = \left({}^{52}C_2\right)^2$. For the number of elements in E, we first choose a card (which we want common) and then from the remaining cards (51 in numbers) we choose two cards and distribute them among A and B in 2! ways. Hence $n(E) = {}^{52}C_1 . {}^{51}C_2 .$ 2!. Thus $P(E) = \frac{50}{663}$.

25.

Sol. Let E be the event that a television chosen randomly is of standard quality. We have to find

$$P(II/E) = \frac{P(E/II).P(II)}{P(E/I).P(I)+P(E/II).P(II)}$$

$$= \frac{(9/10)(3/10)}{(4/5)(7/10)+(9/10)(3/10)} = 27/83$$

26.

Sol. $n(S) = {}^{50}C_5$ = Total number of ways

$n(E) = {}^{30}C_2 \times {}^{19}C_2$ = Number of favourable ways

$$P(E) = \frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}.$$

27.

Sol. $P = \frac{3}{4}, q = \frac{1}{4}, n = 5$

Required probability = ${}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4.\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5 = \frac{459}{512}$.

28.

Sol. For at least 4 successes, required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + {}^7C_7\left(\frac{1}{2}^7\right) = \frac{1}{2}.$$

EXERCISE 3

1 The probabilities that three men hit a target are, respectively, 0.3, 0.5 and 0.4. Each fires once at the target. (As usual, assume that the three events that each hits the target are independent)

(a) Find the probability that they all : **(i)** hit the target ; **(ii)** miss the target

(b) Find the probability that the target is hit : **(i)** at least once, **(ii)** exactly once.

(c) If only one hits the target, what is the probability that it was the first man?

Sol. a) i) P(all hit the target)

= P(1st one hits it) $\times$ P(2nd one hits it) $\times$ P(3rd person hits it) ($\because$ all events are independent)

$= 0.3 \times 0.4 \times 0.5$

$= 0.06$

$= 6\%$

ii) P(1st one misses the target)

= 1-P(he hits the target)

$= 1 - 0.3$

$= 0.7$

$|||^{ly}$,

P(2nd one misses) $= 1 - 0.5$

$= 0.5.$

& P(3rd one misses) $= 1 - 0.4$

$= 0.6.$

$\therefore$ P(all miss it) = P(1st misses) $\times$ P(2nd misses) $\times$ P(3rd misses)

($\because$ they are independent events)

$= 0.7 \times 0.5 \times 0.6$

$= 0.21$

$= 21\%$

b) i) P(target is hit atleast once)

= 1-P(target is never hit)

= 1-P(all miss the target)

= 1-0.21 (from previous question)

= 0.79.

($\because$ Hitting the target at least once or never hitting it are mutually exclusive & exhaustive events)

ii) P(target is hit exactly once)

= P(1st hits) $\times$ P(2nd misses) $\times$ P(3rd misses) + P(1st misses) $\times$ P(2nd hits) $\times$ P(3rd misses) + P(1st misses) $\times$ P(2nd misses) $\times$ P(3rd hits)

$= 0.3 \times 0.5 \times 0.6 + 0.7 \times 0.5 \times 0.6 + 0.7 \times 0.5 \times 0.4$

$= 0.090 + 0.210 + 0.140$

$= 0.44$

$= 44\ \%$

c) Here, we use Bayes theorem of reverse probability.

Accordingly,

P(1st man hits / only 1 hits the target)

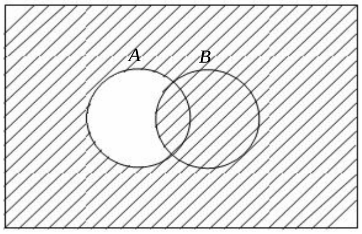
$$= \frac{\text{P(1st man hits) P(only 1 hits the target 1st man hits)}}{\text{P}\left(\text{only 1 hits the target}\right)}$$

$$=\frac{0.3\times0.5\times0.6}{0.44}$$

$$=\frac{0.09}{0.44}=\frac{9}{44}\cdot$$

2

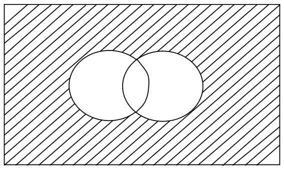
Sol.



Obviously $\left(\overline{A}\cup B\right)$ Is the shaded region.

Now, $P\left(\frac{\overline{A}}{\overline{B}}\right)=\frac{P\left(\overline{A}\cap\overline{B}\right)}{P\left(\overline{B}\right)}$

$$=\frac{1-P\left(A\cup B\right)}{1-P\left(B\right)}$$

[$\therefore$  The shaded region is $\overline{A}\cap\overline{B}$, for which $A\cup B$ is the compliment]

$$\Rightarrow 0.10=\frac{1-\left[P\left(A\right)+P\left(B\right)-P\left(A\cap B\right)\right]}{1-P\left(B\right)}$$

$$\Rightarrow 0.10=\frac{1-\left[0.4+0.8-P\left(A\cap B\right)\right]}{1-0.8}$$

$$\Rightarrow 0.1\times0.2=P\left(A\cap B\right)-0.2$$

$$\Rightarrow P\left(A\cap B\right)=0.22$$

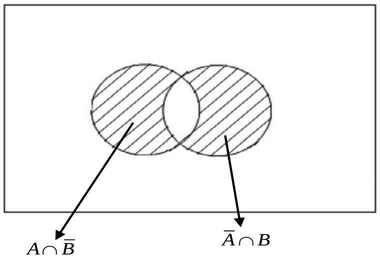
$$\therefore P\left(\overline{A}\cup\overline{B}\right)=1-P\left(\text{only } A\right)$$

$$=1-\left[P\left(A\right)-P\left(A\cap B\right)\right]$$

$$=1-\left[0.4-0.22\right]$$

$$=0.82\cdot$$

ii) $P\left[\left(\overline{A}\cap\overline{B}\right)\cup\left(A\cap\overline{B}\right)\right]$



The shaded region is $\left[\left(\overline{A}\cap B\right)\cup\left(A\cap\overline{B}\right)\right]$ from figure it is evident that

$$\therefore P\left[\left(\overline{A}\cap B\right)\cup\left(A\cap\overline{B}\right)\right]=P\left(A\right)+P\left(B\right)-2P\left(A\cap B\right)$$

$$=0.4+0.8-2\times0.22$$

$$=0.2-0.44$$

$$=0.76.$$

3

Sol. P(target is destroyed)

= P(exactly 1 hits) × P(target is destroyed / exactly 1 hits)

+ P(exactly 2 hits) × P(target is destroyed / exactly 2 hits)

+ P(exactly 3 hits) × P(target is destroyed / exactly 3 hits)

P(exactly 1 hit)

= P(hit in 1st shot) × P(miss in 2nd shot) × P(miss in 3rd shot)

+ P(miss in 1st one) × P(hit in 2nd shot) × P(miss in 3rd shot)

+ P(miss in 1st) × P(miss in 2nd) × P(hit in 3rd)

$$=\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}\ =\frac{1}{4}\cdot$$

|||ly , P(exactly 2 hits)

= P(1st hit) × P(2nd hit) × P(3rd miss) + P(1st hit) × P(2nd miss) × P(3rd hit)

+ P(1st miss) × P(2nd hit) × P(3rd hit)

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}+\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\ =\frac{11}{24}\cdot$$

P(exactly 3 hit) = P(hit in 1st shot) × P(hit in 2nd shot) × P(hit in 3rd shot)

$$=\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\ =\frac{1}{4}\cdot$$

$\therefore$ P(destroying the target)

$$=\frac{1}{4}\times\frac{1}{3}+\frac{11}{24}\times\frac{7}{11}+\frac{1}{4}\times1$$

$$=\frac{1}{12}+\frac{7}{24}+\frac{1}{4}$$

$$=\frac{2+7+6}{24}$$

$$=\frac{15}{24}$$

$$=\frac{5}{8}\cdot$$

4

Sol. The difference ranges from o (in case of equal nos.) to 5 (in case of 1 & 6).

$\therefore$ A player wins if the scores are 0 & 4 or 4 & 0 or 1 & 5 or 5 & 1 or 0 & 5 or 5 & 0.

For any player,

P(score is ‘0’) $=\frac{\text{favorable out comes }(=6)}{\text{Total outcomes }(6\times 6)}$ ($\because$ 6 out comes on each die)

((1,1) , (2,2) ...... (6,6) are the 6 favorable outcomes)

$$=\frac{6}{36} \quad =\frac{1}{6}\cdot$$

P(score is 1) $=\frac{\text{Favorable outcomes}}{36}$

Favorable outcomes $=(1,2),(2,3),(3,4),(4,5),(5,6)$ & the reverse of all these i.e. (2,10 , (3,2) .....

= 10 outcomes

$\therefore$ P(score is 1) $=\frac{10}{36}=\frac{5}{18}\cdot$

P(score is 4) $=\frac{\text{Favorable outcomes}}{36}$

Favorable outcomes = (1,5) , (2,6) , (5,1) , (6,2) = 4 outcomes.

$\therefore$ P(score is 4) $=\frac{4}{36}=\frac{1}{9}\cdot$

P(score is 5) ,

Favorable outcomes = (1,6) , (6,1) = 2 outcomes.

$\therefore$ P(score is 5) $=\frac{2}{36} =\frac{1}{18}\cdot$

P(a player wins)

= P(1st player scores 0 & 2nd scores 4) + P(1st score 4 & 2nd scores 0)

+ P(1st score 1 & 2nd score 5) + P(1st scores 5 & 2nd scores1)

+ P(1st scores 0 & 2nd scores 5) + P(1st scores 5 & 2nd scores 0)

$$=\frac{1}{6}\times\frac{1}{9}+\frac{1}{9}\times\frac{1}{6}+\frac{5}{18}\times\frac{1}{18}+\frac{1}{18}\times\frac{5}{18}+\frac{1}{6}\times\frac{1}{18}+\frac{1}{18}\times\frac{1}{6}\cdot$$

$$=\frac{1}{27}+\frac{5}{162}+\frac{1}{54}$$

$$=\frac{14}{162}=\frac{7}{81}\cdot$$

$\therefore$ P(neither player wins) = 1-P(either player wins) ($\because$ draw is hot possible)

$$=1-\frac{7}{81}$$

$$=\frac{74}{81}\cdot$$

5

Sol. T.P.T. $P\left(\bar{H}/S\right)$ is independent of a $P\left(\bar{H}/S\right)=\frac{P\left(\bar{H}\cap S\right)}{P(S)}$

Now, note that,

$P\left(S/\bar{H}\right)+P\left(\bar{S}/\bar{H}\right)=1$

$\therefore P\left(S/\bar{H}\right)=1-(1-a)=a.$

$\therefore$ B Bayes thm.on revers probability.

$$P\left(\bar{H}/S\right)=\frac{P\left(S/\bar{H}\right)\times P\left(\bar{H}\right)}{P\left(S/\bar{H}\right)\times P\left(\bar{H}\right)+P\left(S/H\right)\times P\left(H\right)}$$

$$=\frac{a\times(1-a)}{a\times(1-a)+(1-a)\times a}$$

$$=\frac{1}{2}\cdot$$

which is free of a.

6

Sol. P(low cloud ceiling) $=\frac{k}{100}\cdot$

$\therefore$ P(favourable weather) $=1-\frac{k}{100}\cdot$

P(safe landing)
= P(favorable weather) × P(safe landing / favorable weather)
+ P(low cloud ceiling) × P(blind landing instruments working proparly)
× P(safe landing / proper functioning of blind landing instruments)
+ P(low cloud ceiling) × P(bilnd landing instruments fail)
× P(safe landing / failing of blind landing instuments)

$$=\left(\frac{100-k}{100}\right)\times P_1+\frac{k}{100}\times P\times P_1+\frac{k}{100}(1-P)P_2$$

Here, we use Bayes theorem of reverse probability :
P(pilot used blind landing instrument / Plane landed safely)

$$=\frac{\text{P}\left(\text{pilot used blind land. instru.}\right)\times\text{P}\left(\text{plane landed safly / he used it}\right)}{\left[\begin{array}{l}\text{P}\left(\text{pilot used blind landing instru.}\right)\times\text{P}\left(\text{plane landed safly / he used blind landing instru.}\right)\\+\text{P}\left(\text{pilot did not used blind landing instru.}\right)\times\text{P}\left(\text{plane laned safly / he did not use it}\right)\end{array}\right]}$$

$$= \frac{\frac{k}{100}\left[PP_1+\left(1-P\right)P_2\right]}{\frac{k}{100}\left[PP_1+\left(1-P\right)P_2\right]+\left(1-\frac{k}{100}\right)P_1}\cdot$$

7

Sol. Let the candidate be allowed n chances.

P(answering correctly)

= P(answering correctly in 1st chances) + P(answering worngly in 1st & correctly in 2nd chance)

+ ..... + P(answering wrongly in 1st (n-1) chances but correctly in nth chance)

Now,total no.of ways of answering the quation ;

eh may tick any option or may not tick it, therefore 2 ways to go about avrey option.

$\therefore \left(2^5-1\right)$ ways, subtracting 1 for the case when no option is selected.

$\therefore$ 31 total ways.

Now, there is only one combination.

which corresponds to the correct answer $\Rightarrow$ only 1 way of answering correctly correspond to wrong answer.

$\therefore$ P(answering correclty in 1st chance)

$$=\frac{1}{31}\cdot$$

P(correct answer only in 2nd chance)

$$= \underset{P\left(\begin{array}{c}\text{wrong answer}\\ \text{in } 1^{st}\text{ chance}\end{array}\right)}{\underset{\downarrow}{\left(\frac{30}{31}\right)}} \times \underset{\begin{array}{c}\text{conditional probability of}\\ \text{correct ans. in } 2^{nd}\text{chance}\end{array}}{\underset{\downarrow}{\left(\frac{1}{30}\right)}} = \frac{1}{31}\cdot$$

$|||^{ly}$ P(correct answer only in 3rd chance)

$$=\left(\frac{30}{31}\right)\left(\frac{29}{30}\right)\times\frac{1}{29}=\frac{1}{31}\cdot$$

$\therefore$ P(correctly answer only in 4th chance)

$$=\frac{30}{31}\cdot\frac{29}{30}\times\cdots\frac{1}{\left(32-4\right)}=\frac{1}{31}\cdot$$

$\therefore$ P(answering correclty)

$$=\frac{1}{31}+\frac{1}{31}\cdots\text{n times}$$

$$=\frac{n}{31}\geq\frac{1}{3}$$

$$\Rightarrow n\geq\frac{31}{3}=10.33$$

$\Rightarrow n=11$ (an integer).

8

Sol. a) The first person to be asked may be randomly selected in n ways ; of which (n-2) are favorable ways in which all except the 2 people knowing the answer are selected.

$|||^{ly}$, the second person may be selected in a total of 9n-1) ways.

($\because$ the 1 person already asked cannot be re-selected) ; of which (n-3) are favorable ones.

$|||^{ly}$, the 3rd & 4th people may be selected in (n-2) & (n-3) ways respectively, of which (n-4) & (n-5) ways ways are respectively favorable.

$\therefore$ P(1st 4 do not know the answer)

$$=\left(\frac{n-2}{n}\right)\times\frac{n-3}{(n-1)}\times\frac{(n-4)}{(n-2)}\times\frac{(n-5)}{(n-3)}$$

$$=\frac{(n-4)(n-5)}{n(n-1)}$$

b) If we continue similarly so that the 1st (r-1) people do not know the answer , then we have, P(1st (r-1) poeple do not know the answer)

$$=\left(\frac{n-2}{n}\right)\left(\frac{n-3}{n-1}\right)\left(\frac{n-4}{n-2}\right)\cdots\left(\frac{n-r}{n-(r-2)}\right)$$

$$=\frac{(n-r+1)(n-r)}{n(n-1)}$$

we multiply this by the conditional probability that the 4th personknows the answer. Now total ways of selecting the rth person is $(n-(r-1)),\because(r-1)$ people have alrady been tested.

of these, only 2 ways of favorable in which , either of the 2 people knowing the answer are selected.

$\therefore$ P(rth person i the 1st one to know the answer)

$$=\frac{(n-r+1)(n-r)}{n(n-1)}\times\frac{2}{(n-r+1)}$$

$$=\frac{2(n-r)}{n(n-1)}.$$

9

Sol. For head to appear twice, obvioulsy the forst time, a head should appear, after which the same coin is tossed again & head should appear in this 2nd toss also.

$\therefore$ If a 2 headed coin is selected. a head is sure to appear. followed by another one.

$\therefore$ P(head appears twice)

= P(two-headed coin is selected) $\times$ 1 + P(head coin is selected) $\times$ P(head appears in 1st throw) $\times$ P(head appears in 2nd throw)

P(2 headed coin is selected) $=\frac{1}{3}$ & P(fair coin is selected) $=\frac{2}{3}$]

$\therefore$ P(head appears twice) $=\frac{1}{3}\times1+\frac{2}{3}\times\frac{1}{2}\times\frac{1}{2}$

$$=\frac{1}{2}.$$

ii) The same coin is tossed twice

$\Leftrightarrow$ a head appears in 1^{st} throw.

Let,

A : event that head appears in 1^{st} throw.

B : event that the 2 headed coin is selected.

C : event that fair coin is selected.

By bayes theorem.

$$P(B/A)=\frac{P(B)\times P(A/B)}{P(A/B)\times P(B)+P(A/C)\times P(C)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{2}{3}\times\frac{1}{2}}$$

$$=\frac{1}{2}.$$

iii) For tail to appear, a fair coin should be 1^{st} tossed with proabbility $\frac{2}{3}$, on which tail should appear with probability $\frac{1}{2}$.

Now, one of the 2 remaining coins is selected. The selection of fair coin, with probability $=\frac{1}{2}$

Favorurs our event.Thereby a tail should occur on this coin.

with probability $=\frac{1}{2}$.

$\therefore$ P(tail appears twice) is a

product of all these abv. mentioned probabilities, $\because$ all events shouldoccur simultaneously.

$\therefore$ P(tail appears twice) $=\left(\frac{2}{3}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$

$$=\frac{1}{12}.$$

10

Sol. Let us say, that we observe the highway for a very long time, in which we notice 3n is a very large number. 1 to 30 trucks stops at the pump.

$\therefore$ During our observation piriod, $\frac{3n}{30}$ trucks stops at the the petrol pump.

$\therefore$ out of 2n cars, $\frac{2n\times 2}{50}$

will stop at the petrol pump.
To find the probaility that the vehicle stopped at the petrol pump is a car :
the total no. of possibilities, i.e. total no.of vehicles ,stopping at the pump.over a long time
is $\left(\frac{3n}{30}+\frac{4n}{50}\right)$.

of these, the favorable possibilities, i.e. of the vehicle being a car. are only $\frac{4n}{50}\cdot$

$\therefore$ P(the vehicle stopped is a car)

$$=\frac{\frac{4n}{50}}{\frac{3n}{30}+\frac{4n}{50}}$$

$$=\frac{\frac{4}{5}}{1+\frac{4}{5}} =\frac{4}{9}\cdot$$

11

Sol. Let A’,B’,C’, be the events that a radioset choosen at random, is manufactured from A,B,C respectively.

$\therefore P\left(A'\right)=\frac{18}{50},P\left(B'\right)=\frac{20}{50},P\left(C'\right)=\frac{12}{50}\cdot$

Let E be the event that a radio set chosen at random is of excelent quality.
$\therefore$ As given,

$P\left(E/A'\right)=0.9,$

$P\left(E/C'\right)=0.9,$

$P\left(E/B'\right)=0.6.$

To find :
P(B’/E),
which can be done quite simoly, by Bayes theorem ; as;

$$P\left(B'/E\right)=\frac{P\left(E/B'\right)\times P\left(B'\right)}{P\left(E/B'\right)\times P\left(B'\right)+P\left(E/A'\right)\times P\left(A'\right)+P\left(E/C'\right)\times P\left(C'\right)}$$

$$=\frac{0.6\times\frac{20}{50}}{0.6\times\frac{20}{50}+0.9\times\frac{18}{50}+0.9\times\frac{12}{50}}$$

$$=\frac{12}{12+16.2+10.8}=\frac{12}{39}=\frac{4}{13}\cdot$$

12

Sol. Red box : 5 balls

contains 0-5 green balls.

Green box : 9 balls.

cantains 1-6 red balls.

Total no.of ways of filling the boxes.

Select 5 from the 14 balls in ${}^{14}C_5$ ways. Put them in the red box. Put the remaining balls in the green box in only 1 ways.

Now, let us look for the total no.of favorable ways.

Sr No.	G balls in R box	R balls in R box	R balls in G box	$(R \text{ balls in } G \text{ box})+(G \text{ balls in } R \text{ box})$
$1\cdot\rangle$	0	5	1	1
$2\cdot\rangle$	1	4	2	3
$3\cdot\rangle$	2	3	3	5
$4\cdot\rangle$	3	2	4	7
$5\cdot\rangle$	4	1	5	9
$6\cdot\rangle$	5	0	6	11

This table is prepared easily, by 1st filling the 1st column, then the others, by considering the fact that total no.of balls in red box is 5 & that total no.of red balls is 6.

$\therefore$ Only the 1st row (Sum = 1) & 5th row (Sum = 9) are fovarable ones.

No. of ways , satisfying 1st row : select 5 red balls from 6 in ${}^{6}C_5$ ways. select 0 form 8 green balls in ${}^{8}C_0$ ways. & put all remaining balls in green box.

$\therefore {}^{6}C_5 \times {}^{8}C_0 = 6$ ways.

No.of ways satisfying the 5th row, sekec 1 frim 6 red balls in ${}^{6}C_1$ ways. select 4 from 8 green balls in ${}^{8}C_4$ ways.& put them in red box; put the remaining balls in green box.

$\therefore$ No.of ways $= {}^{6}C_1 \times {}^{8}C_4$

$\therefore$ No.of favorabel cases $= 6 + {}^{6}C_1 \times {}^{8}C_4$

$= 6 \times 71$

As seen earlier total cases $= {}^{14}C_5$.

$\therefore$ P(the sum is not prime) $= \dfrac{6\times 71}{{}^{14}C_5}$

$$= \frac{213}{1001}\cdot$$

13

Sol. Let us first try to simplify the equation.

$$\log_3(x+y) - \log_3 x + \log_3 y + 1 = 0$$

$$\Rightarrow \log_3(x+y) - \log_3 xy = -1$$

$$\Rightarrow \log_3\left(\frac{x+y}{xy}\right) = -1 \qquad (\because \log a + \log b = \log ab)$$

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{3}\cdot$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{3}\cdot$$

By inspection, (or trial & error)

$(x, y) = (6, 6)$ or $(4, 12)$ or $(12, 4)$.

Total no. of ways : The 1st card can be chosen in 52 ways, & the 2nd one, can be subsequently chosen in 51 ways.

$\therefore 51\times 52$ ways.

Favorable ways : For $(x, y) = (6, 6)$, Select 1st card. a 6 from any of the 4 units in 4 ways.

Next, select the 2nd one a 6 from the remaining 3 suits in 3 ways.

$\therefore 4\times 3 = 12$ ways.

$(x, y) = (12, 4)$

Select x - a queen from any of the 4 suits in 4 ways & y - a 4 from any of the 4 suits in 4 ways. Thereby, v ways.

Also, for $(x, y) = (12, 4)$.

Select x & y in 4 ways each.

$\therefore$ 16 move ways.

$\therefore$ Total favorable ways $= 12 + 16 + 16 = 44.$

$\therefore$ P(x & y satisfy the equation)

$$= \frac{44}{52\times 51}$$

$$= \frac{11}{663}\cdot$$

14

Sol. a) Choose any 2 nos. from {1,2,...3n} in $^{3n}C_2 = \frac{3n.(3n-1)}{2}$ ways.

Now, $x^2 - y^2 = (x+y)(x-y)$ should be divisible by 3. now, consider :

a	b	c
1	2	3
4	5	6
7	8	9
10	11	12
⋮	⋮	⋮

Let us all the nos. 1,4,7,.... of type 'a' : Which are of the from $3k+1 \;\; k \in w$, 2,5,8,... of type b, of from $3k+2 \;\; k \in w$ & 3,6,9,.... of type c, of from 3k, $k \in N$, For any 2 nos. of the same type the difference is a multiple of 3 : $\therefore x^2 - y^2$ is divisible by 3.

Also, if 1 no. is of the type a, & another of type b, them the sum is a multiple of 3, as.

$(3m+1)+(3n+2) = 3(m+n+1)$.

$\therefore x^2 - y^2$ is divisible by 3.

$\therefore$ Favorable cases : Select any 2 nos. of type a, which has in all n nos. in $^nC_2$ ways $= \frac{n(n-1)}{2}\cdot$

$|||^{ly}$, for type b & type c, $\frac{n(n-1)}{2}\cdot$

cases each.

OR select 1 no. of type a in n ways & another one of type b, again in n ways. So that $n \times n = n^2$ cases.

$\therefore$ Total favorable cases $= \frac{3 \times (n)(n-1) + n^2}{2}\cdot$

$\therefore$ P($x^2 - y^2$ is divisible by 3)

$$= \frac{\frac{3}{2}n(n-1)+n^2}{\frac{2n(3n-1)}{2}}$$

$$= \frac{\frac{3}{2}(n-1)+n}{\frac{3}{2}(3n-1)} = \frac{(5n-3)}{3(3n-1)}\cdot$$

b) Case I : If x is odd. y is even (or x is even, y is odd) obviously, $x^3 + y^3 =$ odd.

$\because$ x is odd $\Rightarrow x^3$ is odd & y is even $\Rightarrow y^3$ is even. $\therefore x^3 + y^3$ is obviously not divisible by 8.

Case II : x,y both even

Let $x = 2n, y = 2m.$

$\therefore x^3+y^2=(2n)^3+(2m)^3$

$=8(m^2+n^2)$, divisible by 8 in all such cases.

Case II : x odd, y odd.
consider 4 types of odd nos. of the from :
$8k+1, 8k+3, 8k+5, 8k+7.$

$(8k+1)^3=8^3k^3+3.8^2k^2+3.8.k+1,$

leaves remainder = 1, when divided by 8.

$|||^{ly}$,

$(8k+3)^3=8^3k^3+3.8^2k^2\cdot 3+3.8k.3^2+27,$

$=8^3k^3+3.8^2k^2.3+3.8k.3^2+3.8+3$

(writing 27 as $3.8+3$)

Leaves remainder 3 when divide by 8.

$(8k+5)^3=8^3k^3+3.8^2k^2\cdot 5+3.8k.5^2+125.$

Leaves remainder 5 on dividing by 8.

$(8k+7)^3=8^3k^3+3.8^2k^2.7+3.8k.7^2+343.$

Leaves remeaindеr 7 on dividing by 8.

$\therefore$ When x,y are of form $(8k+1)$ & $(8k+7)$, x^3+y^3 is divdible by 8

($\because$ remainders are 1 & 7, which odd up to make a multiple of 8)

$|||^{ly}$, if x,y are of form $(8k+3)$ & $(8k+5)$, x^3+y^3 is divisible by 8.

$\therefore$ Consider whole nos. {0,1,2,....8n},

we will find probability for 1st $(8n+1)$ whole nos. & tyhen let $n\to\infty$.

$\therefore$ Total cases : Select any 2 out of $8n+1$ whole nos. in ${}^{8n+1}C_2$ ways $=\dfrac{(8n+1)(8n)}{2}$ ways.

Favorable cases :

1) Select any 2 even nos. out of the $4n+1$ Evevn nos. (inluding 0) in ${}^{4n+1}C_2$ ways ; which will favour our event as x^3+y^3 will be divisible by 8.
2) From odd nos :

Observe that there are n nos. of the from $(8k+1)$. n nos. of the from $(8k+3)$ & no nos. of the from $(8k+5)$ & $(8k+7)$, each.

$\therefore$ Select any one ono. of the from $(8k+1)$ & simultaneously, 1 no. of from $(8k+7)$, in ${}^nC_1$ ways each.

$\therefore$ Nolof seections $={}^nC_1\times{}^nC_1=n^2.$

In other cases,

Select any one no.of from $(8k+3)$ in n ways & simultaneously select 1 no.of the from $(8k+5)$, also in n ways.

$\therefore$ No. of selections $=n\times n=n^2.$

$\therefore$ Total favorable cases

$$= {}^{4n}C_2 + n^2 + n^2$$

$$= (4n+1)\frac{4n}{2} + 2n^2$$

$$= 2n(4n+1) + 2n^2$$

$$= 2n(5n+1).$$

$\therefore$ P($x^3 + y^3$ is divisible by 5)

$$= \frac{2n(5n+1)}{{}^{8n+1}C_2}$$

$$= \frac{2n(5n+1)}{(8n+1)4n}$$

$$= \frac{(5n+1)}{2(8n+1)}$$

we have to find this probability as $n \to \infty$

$\therefore$ probability $= \lim_{n\to\infty} \frac{(5n+1)}{2(8n+1)}$

$$= \lim_{n\to\infty} \frac{\left(5+\frac{1}{n}\right)}{2\left(8+\frac{1}{n}\right)}$$

$$= \frac{5}{16}\cdot$$

15

Sol. Let us find the probability that the animal escapes,since it is simplest to find.

P(animal escapes) , is simply, the probability that the hunter misses each shot.

P(hunter misses a shot at distance r) $= 1 - \frac{a^2}{r^2}$

$\therefore$ P(animal escapes)
P(hunter misses when r = 2a)
$\times$ P(hunter misses when r = 3a)
$\times$ ... $\times$ P(hunter misses when r = na)

$$= \left(1 - \frac{a^2}{(2a)^2}\right)\left(1 - \frac{a^2}{(3a)^2}\right)\cdots\left(1 - \frac{a^2}{(na)^2}\right)$$

$$= \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{n^2}\right)$$

$$= \prod_{r=2}^{n}\left(1 - \frac{1}{r^2}\right)$$ (π standes for product)

$$=\prod_{r=2}^{n}\frac{r^2-1}{r^2}$$

$$=\prod_{r=2}^{n}\frac{(r+1)(r-1)}{r^2}$$

$\therefore$ many terms will cancel out :

$$=\frac{3.1}{2^2}\cdot\frac{4.2}{3^2}\cdot\frac{5.3}{4^2}\cdots\quad\frac{(n+1)(n-1)}{n^2}$$

$$=\frac{1}{2}\cdot\frac{(n+1)}{n}$$

$\because$ each r^2 in denominator cancels with an r in numerator of previous term & another r in numerator of next term.

$\therefore$ only $\frac{n+1}{2n}$ remains.

$\therefore$ P(animal escapes) $=\frac{n+1}{2n}\cdot$

$\therefore$ P(hunter succeeds) $=1-\frac{n+1}{2n}$

$$=\frac{n-1}{2n}\cdot$$

$\therefore$ odds in favour of hunter $=\frac{n-1}{2n-(n-1)}$

$$=\frac{n-1}{n+1}.$$

$\therefore$ odds against hunter $=n+1:n-1$

[Note : If probability of $E=\frac{a}{b}$, odds in favour of E are : $\frac{a}{b-a}$ & odds against ot are $\frac{b-a}{a}$]

16

Sol. E_1 : both H & T are present in n tosses.

The compliment of this event is that either H or T is absent, i.e. the n tosses comprise of all heads or of all tails.

$\therefore$ P(E_1) = 1-[P(all heads) + P(all tails)]

$$=1-\left[\left(\frac{1}{2}\right)^n+\left(\frac{1}{2}\right)^n\right]$$

$$=1-\frac{1}{2^{n-1}}\cdot$$

E_2 : atmost 1 h occurs

$\Rightarrow$ either no H occurs or just 1 H occurs.

P(no H occurs) = P(all tails)

$$=\left(\frac{1}{2}\right)^n$$

P(exactly 1 H) $={}^nC_1\left(\frac{1}{2}\right)^{n-1}\cdot\frac{1}{2}$

$=n.\left(\frac{1}{2}\right)^n$, by binomial probabillity distribution.

$\therefore$ P(E_2) = P(no H) + P(exactly 1 H)

$$=\left(\frac{1}{2}\right)^n+n\cdot\left(\frac{1}{2}\right)^n$$

$$=\left(\frac{1}{2}\right)^n(n+1).$$

$P\left(E_1\cap E_2\right)$:

$$E_2:\underbrace{T\ T\ \ldots T}_{\substack{\text{n times}\\ \downarrow\\ E_1\text{ dose not}\\ \text{occur here}}}\quad\text{or}\quad E_2:\underbrace{T\ T\ \ldots T}_{\substack{\text{just 1 H}\\ \downarrow\\ \text{In this case}\\ E_1\text{ also occur}}}$$

$\therefore \mathrm{P}\left(\mathrm{E}_1\cap \mathrm{E}_2\right)=\mathrm{P}\left(\text{Exactly 1 H}\right)$

$$={}^nC_1\left(\frac{1}{2}\right)^n=\frac{n}{2^n}$$

Now, for E_1 & E_2 to be independent the necessary & sufficient condition is that

$P\left(E_1\cap E_2\right)=P\left(E_1\right)\times P\left(E_2\right)$

i.e. $\frac{n}{2^n}=\left[1-\frac{1}{2^{n-1}}\right]\times\left(\frac{1}{2}\right)^n(n+1)$

$\Rightarrow n=\frac{\left(2^{n-1}-1\right)}{2^{n-1}}(n+1)$

$\Rightarrow n\left(2^{n-1}\right)=(n+1)\left(2^{n-1}-1\right)$

$\Rightarrow n.2^{n-1}=n.2^{n-1}+2^{n-1}-n-1$

$\Rightarrow 2^{n-1}=n+1.$

By observation , $n=3$ is a solution.

Also, it is the only solution

$\because$ L.H.S increases faster than R.H.S , on increasing.

17

Sol. P(an article from the new lot is defective)

= P(this article comes originally from first lot) × P(it is defective / it comes from 1st lot)

+ P(it comes originally from 2nd lot) × P(it is defective / it comes from 2nd lot)

P(the article comes from 1st lot)

$$=\frac{K}{K+L}$$

P(it is defective / it comes from 1st lot)

$$=\frac{n}{N}.$$

($\because$ out of N articles of 1st lot, n are defective).

|||^{ly} , P(the article comes from 2nd lot)

$$=\frac{L}{K+L}.$$

& P(it is defective / it comes from 2nd lot)

$$=\frac{m}{M}.$$

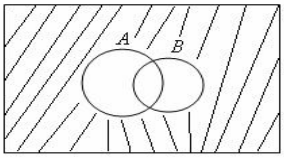
$\therefore$ P(the article is defective)

$$=\frac{K}{K+L}\cdot\frac{n}{N}+\frac{L}{K+L}\cdot\frac{m}{M}$$

$$=\frac{KnM+LmN}{MN\left(K+L\right)}\cdot$$

18.

Sol



$A'\cap B'$ is the shaded region $P\left(A'\cap B'\right)=1-P\left(A\cup B\right)$

$$=1-\left[y+x\right)$$

$$P\left(A\cap B\,|\,A\cap B\right)=\frac{P\left(\left(A\cap B\right)\cap\left(A\cup B\right)\right)}{P\left(A\cup B\right)}=\frac{P\left(A\cap B\right)}{P\left(A\cup B\right)}=\frac{x}{x+y}$$

19.

Sol $P\left(x=4\right)={}^{n}C_{4}.\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{n-4}={}^{n}C_{4}\left(\frac{1}{2}\right)^{n}$

$$P\left(x=5\right)={}^{n}C_{5}\left(\frac{1}{2}\right)^{n}$$

$$P\left(x=6\right)={}^{n}C_{6}\left(\frac{1}{2}\right)^{n}$$

These are in A.P.

$$\Rightarrow 2\left({}^{n}C_{5}\left(\frac{1}{2}\right)^{n} \right) = {}^{n}C_{6}\left(\frac{1}{2}\right)^{n} + {}^{n}C_{4}\left(\frac{1}{2}\right)^{n}$$

$$\Rightarrow 2.{}^{n}C_{5} = {}^{n}C_{6} + {}^{n}C_{4}.$$

$$\Rightarrow 2 \cdot \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}$$

$$= \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!} + \frac{n(n-1)(n-2)(n-3)}{4!}$$

$$\Rightarrow \frac{2(n-4)}{5} = \frac{(n-4)(n-5)}{30} + 1$$

$$\Rightarrow 12n - 48 = n^2 - 9n + 20 + 30$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$n = 7$ or 14.

$\because n > 7, \ \therefore n = 14.$

20.

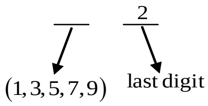
Sol Total no.of out comes $= 9!$

The no.should be divisible be 4 & by 9 to be divisible by 36.

It is divisible by 9, $\because$ the sum of digits = 45 is divisible by 9 (the divisibility test of 9).

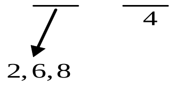
To be div. 4, the last 2 digits should be divisible by 4.

$\therefore$ Forourable outcomes



$\therefore$ No.of cases $= 5 \times 7!$

$|||^{ly}$ for last digit 6 no. of cases $= 5 \times 7!$



No.of cases $= 3 \times 7!$

$|||^{ly}$ last digit 8 , no of cases $= 3 \times 7!$

$\therefore$ Probability that the no. is div. by $= \dfrac{2 \times 5 \times 7! + 2 \times 3 \times 7!}{9!} = \dfrac{16}{8 \times 9} = \dfrac{2}{9}$

21.

Sol P (white ball in = P (black in 1^{st} & white in 2^{nd}) + P (white in 1^{st} & white in 2^{nd})

$$= \frac{n}{3n} \cdot \frac{2n}{3n} + \frac{2n}{3n} \cdot \frac{(2n-1)}{(3n-1)}$$

$$= \frac{2}{9} + \frac{2}{3}\frac{(2n-1)}{(3n-1)} = \frac{6n-2+12n-6}{9(3n-1)} = \frac{18n-8}{9(3n-1)}$$

22.

Sol P(A wins) = P (A wins in 1st throw) + P (A.B. & C lose in 1st throw & A wins in 2nd throw)

$$=\frac{1}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right)\cdots$$

$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^3+\left(\frac{5}{6}\right)^6\cdots\text{up to }\infty\right]$$

$$=\frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^3}\right]=\frac{36}{91}.$$

|||ly , P (B wins) $=\frac{5}{6}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}\cdots$

$$=\frac{5}{6}\cdot\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^3\cdots\right]$$

$$=\frac{5}{6}P(A)$$

$$=\frac{5}{6}\times\frac{36}{91}=\frac{30}{91}$$

$$P(C)=1-P(A)-P(B)$$

$$=\frac{25}{91}$$

$\therefore$ Proportion of the probabilities of their victores $=36:30:25$

23.

Sol For real roots, $b^2\geq 4c$

$$b^2\geq 4c$$

$$\therefore c=1, b=2,3,4,5,6$$

$$c=2, b=3,4,5,6$$

$$c=3, b=4,5,6$$

$$c=4, b=4,5,6$$

$$c=5, b=5,6$$

$$c=6, b=5,6$$

$\therefore$ No.of favorable out comes = 19.

No.of total outcomes $=6\times6=36.$

$\therefore$ Required probability $=\frac{19}{36}.$

24.

Sol i) P(Atleast one event) = 1 – P(no even) = $1 - P\left(A_1^{'} \cap A_2^{'} ....\right)$

$\because A_1^{'}, A_2^{'} ....$ are independent

$\therefore P\left(A_1^{'} \cap A_2^{'} \cap ....\right)$

$= P\left(A_1^{'}\right) \cdot P\left(A_2^{'}\right) ....$

$= (1-P)^n$

$\therefore$ P (Atleast 1 event) $= 1-(1-P)^n$

ii) P (Atleast m events occur)
= P(exactly m occur) + P (exactly m+1 occur)
$\vdots$

$= {}^nC_m.\ P^m (1-P)^{n-m} + {}^nC_{m+1}\ P^{m+1}(1-p)^{n-m-1} \cdots + {}^nC_n\ P^n$

iii) P(exactly m events occur) $= {}^nC_m.\ P^m (1-P)^{n-m}$

25.

Sol i) Total outcomes : Each person has 3 options to get down.

$\therefore$ Total outcomes $= 3 \times 3 = 9.$

Favourable outcomes : 1st person has 3 options the second has, correspondingly only 2 options.

$\therefore$ Favourable outcomes $= 3 \times 2 = 6.$ $\therefore$ Probability required $= \frac{6}{9} = \frac{2}{3}.$

ii) Total outcomes = 9 as before fav. outcomes only 1st person gets down at 1st floor - 2nd person has 2 option : 2nd or 3rd floor.

$\therefore$ 2 cases $|||^{ly}$, 2 cases in 2nd person gets down at 1st floor.

$\therefore$ Fav. outcomes = 4. $\therefore$ Probability $= \frac{4}{9}.$

26.

Sol No.of subsets (total) $= 2^5 = 32.$

$\therefore$ Total outcomes $= 32 \times 32 = 2^{10}.$

Favourable outcomes :

i) Both null sets (1)

ii) Both signleton sets : $5 \times 5 = 25$

iii) Both have 2 elements : $\left({}^5C_2\right)^2 = 100$

iv) Both have 3 elements : $\left({}^5C_3\right)^2 = 100$

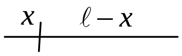
v) Both having 4 elements : $\left({}^5C_4\right)^2 = 25$

vi) Both having 5 elements : 1

$\therefore$ Total no.of favourable outcomes = 252.

$\therefore$ Probability $= \dfrac{252}{2^{10}}$

27.

Sol 

Let x be the smaller part.

$\therefore o < x \le \dfrac{\ell}{2}$

$\therefore$ Probability that $x > \dfrac{\ell}{3} = \dfrac{\text{Interval of length when } x > \frac{\ell}{3}}{\text{total interval of length of x}}$

$$= = \frac{\frac{\ell}{2} - \frac{\ell}{3}}{\frac{\ell}{2}} = \frac{1}{3}$$

28.

Sol Total no.of outcomes $= 10^4$.

Let us count no.of nos. with no.8

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

Each digit can be filled in 9 ways from 0-9. excluding 8.

0 in the thousands place will give a 3 digit no. & so on.

$\therefore$ No.of such has $= 9 \times 9 \times 9 \times 9 - 1 + 1$ where 1 is subtracted for the no. 0000, & 1 is also added for 10^4.

$\therefore$ Required probability

$= 1 - \text{P(no 8 occurs)}$

$= 1 - \dfrac{9^4}{10^4} = 1 - \left(\dfrac{9}{10}\right)^4$

29.

Sol Total outcomes : ${}^nC_2$

Favourable outcomes $= w \times r$.

$\therefore$ Probability that $= \dfrac{wr}{{}^nC_2} = \dfrac{1}{2}$

They have diff. colours

$\Rightarrow \dfrac{wr}{n(n-1)} = \dfrac{1}{4}$

$4wr = n^2 - n$

$= (w+r)^2 - n$

$\Rightarrow n = w^2 + r^2 - 2wr$

$\Rightarrow n = (w-r)^2$

Now, $|w-r| =$ an integer.

$\Rightarrow$ n is an integer square.

30. T

Sol $P(\text{hit}) = \dfrac{k}{d^2}$, d = distance of the fox from hunter.

$$P(100) = \frac{1}{2} = \frac{k}{100^2}$$

$$\Rightarrow k = 5000$$

$$\therefore P(150\,m) = \frac{5000}{150\times150} = \frac{2}{9}$$

$$P(200\,m) = \frac{5000}{200\times200} = \frac{1}{8}\cdot$$

$\therefore$ P(successful hit) = P (hit at 100 m) + P (miss at 100 & hit at 150) + P (miss at 100,150 & hit at 200)

$$= \frac{1}{2} + \frac{1}{2}\cdot\frac{2}{9} + \frac{1}{2}\cdot\frac{7}{9}\cdot\frac{1}{8} = \frac{95}{144}\cdot$$

31.

Sol P (he calls right person)

= P (he calls the right person 1st time) + P (he calls wrong person 1st but thereafter calls right person).

$$\text{P (right person } 1^{st} \text{ time) } = \frac{\textcircled{1} \longrightarrow \text{only 1 correct no.}}{\textcircled{10} \longrightarrow \text{no.of ways of choosing last digit}}$$

P (wrong person 1st time, but right next time) $= \dfrac{1}{10}\cdot\dfrac{9}{10}\cdot\dfrac{1}{9}$

($\because$ After once dialing the wrong last digit, he won 7 dial it again $\therefore$ only 9 possible cases, from which only 1 is correct)

$\therefore$ Probability $= \dfrac{1}{10} + \dfrac{9}{10}\cdot\dfrac{1}{9} = \dfrac{1}{5}\cdot$

32.

Sol P (B draws a bigger no. than A)

= P (A draws a bigger no. than B) (obviously)

= P say.

$\therefore$ P (A draws a bigger no.) + P (B draws a bigger no.) + P (they draw equal nos.) = 1

$\therefore$ 2P = 1- P (equal nos.)

$$= 1 - \frac{100}{100\times100}$$

$$P = \frac{1}{2}\times\frac{99}{100}$$

$$= \frac{99}{200}$$

OR

Meth. II

Total outcomes : $100 \times 100 = 100^2$

Favourable outcomes $= {}^{100}C_2$

(Select any 2 nos. from 100, & the bigger no. is choosen by B)

$\therefore$ Probability $= \frac{{}^{100}C_2}{100 \times 100} = \frac{100.99}{2.100.100} = \frac{99}{200}$.

33.

Sol $3^m + 2^n$

	3^1	3^2	3^3	3^4
Remainder when divided by 5	3	4	2	1

$\therefore$ This pattern continues, for $3^5, 3^6$.....

$\therefore$ The remainders should be {3,2} or {1,4} with {3^m , 3^n}.

$\therefore$ Total outcomes $= 100 \times 100.$

Favourable outcomes : for the pair (3,2)

outcomes $= 100 \times 100. = \underbrace{25 \times 25}_{\text{When remainder with n=2 \& n=3}} + \underbrace{25 \times 25}_{\text{Remainder with m=3 \& n=2}}$

$|||^{ly}$ for (1,4) pair, outcomes $= 25^2 \times 2.$

$\therefore$ Probability $= \frac{4 \times 25 \times 25}{100 \times 100}$

$$= \frac{1}{4}.$$

34.

Sol Total outcomes : no.of ways of arranging 2n different things in a row $= (2n)!$

Favourable outcomes :

Starting with white : $(n!) \times (n!)$

Because, arrange all white balls in n! ways & black ones in n! ways & then keep them as white black white ....
in one way.

$\therefore$ No.of cases $= (n!)^2$

$|||^{ly}$ with balck first, $(n!)^2$ cases.

$\therefore$ REq. probability $= \dfrac{(n!)^2 \times 2}{(2n)!}$

35.

Sol P (B bags move ducks than A)

= P (B bags 51 & A bags 0 or 1 or 2 mor ... 50 ducks) + P (B bags 50 & A bags 0-49 ducks)

$$= \left(\frac{1}{2}\right)^{51}\left[\left(\frac{1}{2}\right)^{50} + {}^{50}C_1\left(\frac{1}{2}\right)^{50} + {}^{50}C_2\left(\frac{1}{2}\right) + ... + {}^{50}C_{50}\left(\frac{1}{2}\right)^{50}\right]$$

$$+ {}^{51}C_{50}\left(\frac{1}{2}\right)^{51}\left[\left(\frac{1}{2}\right)^{50} + {}^{50}C_1\left(\frac{1}{2}\right)^{50} + ... + {}^{50}C_{49}\left(\frac{1}{2}\right)^{50}\right]$$

$$= \left(\frac{1}{2}\right)^{101}\left[1 + {}^{50}C_2 ... {}^{50}C_{50} + {}^{51}C_{50} + {}^{51}C_{50}.{}^{50}C_1 + {}^{51}C_{50}.{}^{50}C_2 .... + {}^{51}C_{50}.{}^{50}C_{49}\right].$$

36.

Sol $P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore$ P (1 throw is sufficient / 1 didnot appear)

$= \dfrac{\text{P (1 throw is sufficient \& 1 did not appear)}}{\text{P (1 did not appear)}}$

$$= \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}\cdot$$

P (2 throws sufficient / 1 didn't appear in 1st 2 throws)

$= \dfrac{\text{P } (\text{2 throws sufficient}) \cap (\text{l did not appear})}{\text{P}(\text{l did not appear in 1}^{\text{st}}\text{ 2 throws})}$

$$= \frac{\frac{4}{6}\cdot\frac{1}{6}}{\frac{5}{6}\cdot\frac{5}{6}} = \frac{4}{25}\cdot$$

$\therefore$ P (atleast 3 through necessry / l did not appear)

$= 1-$ (P (2 throww sufficient) + P (1 throw sufficient))

$$= 1 - \frac{1}{5} - \frac{4}{25} = \frac{16}{25}\cdot$$

37.

Sol Consider a box containing w white & b black balls such that w + b = n.

Let an experiment consist of : removing the balls one by one without replacement until a white ball comes out.

$\therefore$ P (white ball comes out)

= P (black at 1st draw) + P (black at 1st & white at 2nd draw)

$$\vdots$$

$$=\frac{w}{n}+\frac{bw}{n(n-1)}+\frac{b(b-1)w}{n(n-1)(n-2)}\cdots+\frac{b(b-1)(b-2)...1..w}{n(n-1)(n-2)....(n-b+1)(n-b)}$$

Now, a white ball has to ultimately comes out

$\therefore$ P (white ball comes out)

$$=1=\frac{w}{n}+\frac{bw}{n(n-1)}\cdots$$

$$\Rightarrow 1+\frac{n-w}{n-1}+\frac{(n-w)(n-w-1)}{(n-1)(n-2)}\cdots+\frac{(n-w)...2.1.}{(n-1).....(w+1)w}=\frac{n}{w}\cdot$$

38.

Sol P (even sixes) = P (0 sixes) + P (2 sixes) + P (4 sixes) ......

$$=\left(\frac{5}{6}\right)^n+{}^nC_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{n-2}+{}^nC_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{n-4}.....$$

Consider :

$$\left(\frac{5}{6}+\frac{1}{6}\right)^n=\left(\frac{5}{6}\right)^n+{}^nC_1\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)+{}^nC_2\left(\frac{5}{6}\right)^{n-2}\left(\frac{1}{6}\right)^2$$

$$\left(\frac{5}{6}-\frac{1}{6}\right)^n=\left(\frac{5}{6}\right)^n-{}^nC_1\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)+{}^nC_2\left(\frac{5}{6}\right)^{n-2}\left(\frac{1}{6}\right)^2....$$

Add

$$\Rightarrow 1+\left(\frac{4}{6}\right)^n=2\left(\left(\frac{5}{6}\right)^n+{}^nC_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{n-2}....\right)$$

$\Rightarrow$ P (even sixes) $=\frac{1}{2}\left(1+\left(\frac{2}{3}\right)^n\right)$

39.

Show that the conditional probability of choosing the integer 2, given that an even integer is chosen, is $\frac{\log 2}{n\log 2\ +\ \log(n!)}$.

Sol P (choosing k) = clogk , say.

P (choosing l) + P (choosing 2) + ... + P (choosing 2n) = 1

$$\therefore c\log 1+c\log 2\cdots+c\log 2n=1$$

$$\Rightarrow c\log 2n!=1$$

$$\Rightarrow c = \frac{1}{\log(2n!)}$$

P (choosing 2 / an even intger is chosen)

$$= \frac{\text{P}\ \left((2\text{ choosen}) \cap (\text{even integer choosen})\right)}{\text{P}\ (\text{ even integer choosen})}$$

$$= \frac{\frac{1}{\log(2n)!} \times \log 2}{\frac{1}{\log(2n!)} \times \log 2 + \frac{1}{\log(2n!)} \log 4......}$$

$$= \frac{\log 2}{\log 2.4.6.8...2n}$$

$$= \frac{\log 2}{\log 2^n (n!)}$$

$$= \frac{\log 2}{n \log 2 + \log(n!)}$$

40.

Sol i) P (l is white, the other black) $= \frac{{}^5C_1 . {}^7C_1}{{}^{12}C_2}$

$$= \frac{35 \times 2}{12.11} = \frac{35}{66}$$

ii) P (l white, l black) = P (1st white, 2nd black) + P (1st black , 2nd white)

$$= \frac{5}{12} \cdot \frac{7}{11} + \frac{7}{12} \cdot \frac{5}{11} = \frac{35}{66}$$

iii) P (l white, l balck) = P (1st white, 2nd black) + P (1st black , 2nd white)

$$= \frac{5}{12} \cdot \frac{7}{12} + \frac{7}{12} \cdot \frac{5}{12}$$

$$= \frac{35}{72}$$

41.

Sol P (A, B are not neighbours)

$= 1 -$ P (A , B are neightbours)

Total outcomes $= (n-1)!$

Favourable ones $= (n-2)! \times 2!$

(tie up A & B together , & also permut them internally in 2! ways)

$\therefore$ Required probability $= 1 - \frac{2-(n-2)!}{(n-1)!}$

$$=1-\frac{2}{(n-1)}$$

$$=\frac{n-3}{n-1}\cdot$$

42.

Sol Total outcomes $= {}^{6n}C_3.$

Favorable ones

Consider non-negative non-equal integral solution of : $x_1+x_2+x_3=6n.$

Total integral splution $= {}^{6n+2}C_2$

$$=\frac{(6n+2)(6n+1)}{2}$$

Subtract all solution, when

$x_1=x_2 \mid x_2=x_3 \mid x_3=x_1 \mid x_1=x_2=x_3$

For $x_1=x_2=x_3,\quad (2n,2n,2n)=1$solution

For $x_1=x_2\neq x_3,$

$$2x_1+x_3=6n.$$

$$x_1=\{0,1,.....3n\}-\{2n\}=(3n)\text{ Solution.}$$

$|||^{ly}$ for $x_2=x_3\neq x_1$ & $x_1=x_3\neq x_2$,3n solution each.

$\therefore$ No. of favourable solution

$$=(3n+1)(6n+1)-(9n+1)$$

$$=18n^2$$

But each solution repeats 3! times

$\therefore$ No.of solution $=\dfrac{18n^2}{6}$

$$=3n^2$$

$\therefore$ Probability $=\dfrac{3n^2}{6\,{}^{n}C_3}$

$$=\frac{3n^2 \quad 6}{6n(6n-1)(6n-2)}$$

$$=\frac{3n}{2(3n-1)(6n-1)}.$$

43.

[Sol. $n(S) = 120$

for n(A)

(a)	up up up up 5	${}^4C_4$
	(strictly increased)	
(b)	up up up 5 d	${}^4C_3$
(c)	up up 5 d d	${}^4C_2$
(d)	up 5 d d d	${}^4C_1$
(e)	5 d d d d	${}^4C_0$
	(strictly decreased)	
(f)	d 1 up up up	${}^4C_3$
(g)	d d 1 up up	${}^4C_2$
(h)	d d d 1 up	${}^4C_1$
(i)	d d d d 1	${}^4C_0$
	(strictly decreasing)	

Hence $n(A) = 2\,({}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_0 + {}^4C_4) = 30$

Hence $P = \frac{30}{120} = \frac{1}{4}$

$\Rightarrow$ $p + q = 5$ **Ans.**

Aliter: $n(A) = 2 \cdot ({}^5C_1) + 2\,({}^5C_2)$ (think !)

$= 10 + 20 = 30$

For each selection of one ball or two balls out of five, there will be two favourable ways.]

44.

[Sol. $N \begin{cases} K \text{ white} \\ (N-K) \text{ black balls} \end{cases}$

Given $\frac{{}^KC_2 + {}^{N-K}C_2}{{}^NC_2} = \frac{{}^KC_1\ {}^{N-K}C_1}{{}^NC_2}$

$K(K-1) + (N-K)(N-K-1) = 2K(N-K)$

$K^2 - K + N^2 + K^2 - 2NK - N + K = 2NK - 2K^2$

$4K^2 - 4NK + (N^2 - N) = 0$

$$k = \frac{4N \pm \sqrt{16N^2 - 16(N^2 - N)}}{8} = \frac{4N \pm 4\sqrt{N}}{8} = \frac{N + \sqrt{N}}{2} \text{ or } \frac{N - \sqrt{N}}{2}$$

Hence N must be a perfect square

$\Rightarrow$ $N = 196$ [As $N \in (180, 220)$]

$K = \frac{196 \pm 14}{2} = 105$ or 91

as $k > N - K \Rightarrow 2K > N \Rightarrow K > \frac{196}{2} \Rightarrow K > 98$

Hence $K = 105$ and $N = 196$

$\therefore$ $N + K = 301$ **Ans.**]

45.

[Sol. Let B_1 : pack A was selected $\Rightarrow P(B_1) = \frac{1}{2}$; Pack A $\xrightarrow[\downarrow]{\text{4 aces}}$ 48 cards in 12 different denominations

B_2 : pack B was selected $\Rightarrow P(B_2) = \frac{1}{2}$; Pack B $\xrightarrow[\text{A K Q J}]{\overset{1}{\downarrow}\overset{1}{\downarrow}\overset{1}{\downarrow}\overset{1}{\downarrow}}$ 48 cards

3 ace
3 kings
3 queen
3 jack
36 other cards

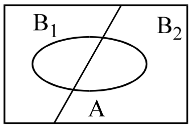
A : two cards drawn all of same rank.

Now A = $(A \cap B_1) + (A \cap B_2)$

$\therefore \quad P(A) = P(A \cap B_1) + P(A \cap B_2) = P(B_1)P(A/B_1) + P(B_2)P(A/B_2)$

$$P(A/B_1) = \frac{{}^{12}C_1 \cdot {}^{4}C_2}{{}^{48}C_2}$$

$$P(A/B_2) = \frac{{}^{9}C_1 \cdot {}^{4}C_2 + {}^{4}C_1 \cdot {}^{3}C_2}{{}^{48}C_2}$$



$$P(B_1/A) = \frac{P(A \cap B_1)}{P(A)} = \frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{{}^{12}C_1 \cdot {}^{4}C_2}{{}^{12}C_1 \cdot {}^{4}C_2 + {}^{9}C_1 \cdot {}^{4}C_2 + {}^{4}C_1 \cdot {}^{3}C_2} = \frac{(12)(6)}{(12)(6) + (9)(6) + (4)(3)} = \frac{12}{23} \Rightarrow m + n = 35 \textbf{ Ans.}]$$

46.

[Sol. By symmetry the probability of more wins than losses equals the probability of more losses than wins. We calculate the probability of the same number of wins and losses.

$\therefore \quad P(L) = P(W) = P(D) = 1/3$

Case-I: Probability of no wins and no losses = P(D D D D D D) = $\frac{1}{3^6}$

Case-II: Probability of 1 win, 1 loss and 4 draws = P(W L D D D D) = $\frac{6!}{4!} \cdot \frac{1}{3^6} = \frac{30}{3^6}$

Case-III: Probability of 2 wins, 2 losses and 2 draws = P(W W L L D D) = $\frac{6!}{2!2!2!} \cdot \frac{1}{3^6} = \frac{90}{3^6}$

Case-IV: Probability of 3 wins and 3 losses = P(W W W L L L) = $\frac{{}^{6}C_3}{3^6} = \frac{20}{3^6}$.

Hence probability of the same number of wins or losses = $\frac{(1 + 30 + 90 + 20)}{729} = \frac{141}{729} = \frac{47}{243}$.

Hence probability more wins than losses = probability more losses than wins

$$= \frac{1}{2}\left[1 - \frac{47}{243}\right] = \frac{1}{2}\left[\frac{196}{243}\right] = \frac{98}{243} \quad \Rightarrow \quad p + q = 341 \textbf{ Ans.}]$$

47.

is the probability that two more balls drawn, one from each bag are both green, find n ∈ N.

[Ans. 16]

[Sol. $P_1 \left< \begin{matrix} 3R \\ 1G \end{matrix} \right.$; $P_2 \left< \begin{matrix} R \\ 3G \end{matrix} \right.$

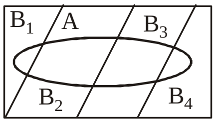
A : two balls drawn, one from each purse, found to be both green.

B_1 : R ball transferred from P_1 to P_2

B_2 : G ball transferred from P_1 to P_2

B_3 : R ball transferred from P_2 to P_1

B_4 : G ball transferred from P_2 to P_1



$$P(B_1) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8};\ P(B_2) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$$

$$P(B_3) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8} \quad ; \quad P(B_4) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$$

$$P(A/B_1) = \frac{1}{3}\cdot\frac{3}{5} = \frac{1}{5} \quad ; \quad P_1 \left< \begin{matrix} 2R \\ 1G \end{matrix} \right. \quad ; \quad P_2 \left< \begin{matrix} 2R \\ 3G \end{matrix} \right.$$

$$P(A/B_2) = 0 \quad ; \quad P_1 \left< \begin{matrix} 3R \\ 0G \end{matrix} \right. \quad ; \quad P_2 \left< \begin{matrix} R \\ 4G \end{matrix} \right.$$

$$P(A/B_3) = \frac{1}{5}\cdot\frac{3}{3} = \frac{1}{5} \quad ; \quad P_1 \left< \begin{matrix} 4R \\ 1G \end{matrix} \right. \quad ; \quad P_2 \left< \begin{matrix} 0R \\ 3G \end{matrix} \right.$$

$$P(A/B_4) = \frac{2}{5}\cdot\frac{2}{3} = \frac{4}{15} \quad ; \quad P_1 \left< \begin{matrix} 3R \\ 2G \end{matrix} \right. \quad ; \quad P_2 \left< \begin{matrix} 1R \\ 2G \end{matrix} \right.$$

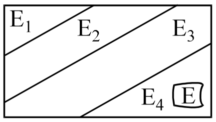
$$\underbrace{P(B_1 / A)}_{E_1} = \frac{P(B_1)\cdot P(A / B_1)}{P(B_1)\cdot P(A / B_1) + \ldots\ldots\ldots\ldots} = \frac{\frac{3}{8}\cdot\frac{1}{5}}{\underbrace{\frac{3}{8}\cdot\frac{1}{5} + \frac{1}{8}\cdot 0 + \frac{1}{8}\cdot\frac{1}{5} + \frac{3}{8}\cdot\frac{4}{15}}_{=\frac{8}{40}}} = \frac{3/40}{8/40} = \frac{3}{8}$$

$$\underbrace{P(B_2 / A)}_{E_2} = \frac{\frac{1}{8}\cdot 0}{8/40} = 0;\ \underbrace{P(B_3 / A)}_{E_3} = \frac{\frac{1}{8}\cdot\frac{1}{5}}{8/40} = \frac{1}{8}\ ;\ \underbrace{P(B_4 / A)}_{E_4} = \frac{\frac{3}{8}\cdot\frac{4}{15}}{8/40} = \frac{4}{8}$$

note that only in the last case two more green balls one from each purse can be drawn

$$P(E) = P(E \cap E_4)$$

$$= P(E_4) \cdot P((E/E_4) = \frac{4}{8}\cdot\frac{1}{4}\cdot\frac{1}{2} = \frac{1}{16}$$



Hence final answer is 16 **Ans.**]

48.
[Sol.

(i) A : both have the same date of birth

$$P(A) = \frac{1}{365}\cdot\frac{1}{365}+\frac{1}{365}\cdot\frac{1}{365}+.......365\,\text{times} = \frac{1}{365}\ \text{Ans.}$$

(ii) B : at least 2 in a group of 3 will have the same birthday

P (B) = 1 – P (all 3 have different birthday)

$$= 1 - 1 \times \frac{364}{365}\cdot\frac{363}{365} = 1 - \frac{364\times 363}{(365)^2}\ \text{Ans.]}$$

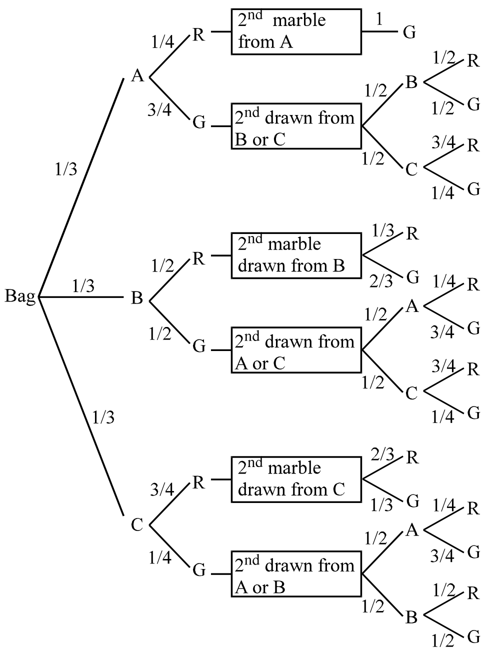
49.

[Sol. A: coin drawn found to be rupee [Ans. $\frac{2}{n(n+1)}$]

$$\left.\begin{array}{l} B_0\text{: 0R + n other} \\ B_1\text{: 1R + (n – 1) other} \\ B_2\text{: 2R + (n – 2) other} \\ \vdots \\ B_n\text{ : nR + 0 other} \end{array}\right\}\quad P(B_1) = \frac{1}{n+1}$$

$$P(B_1/A) = \frac{P(A\cap B_1)}{P(A)} = \frac{P(B_1)\cdot P(A/B_1)}{P(A)} = \frac{\frac{1}{n}}{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+.....+\frac{n}{n}} = \frac{1}{1+2+3+.....+n} = \frac{2}{n(n+1)}$$

50. [Sol.



Let E : second drawn marble is green.

$$P(E) = \frac{1}{3}\left[\left\{\frac{1}{4}\cdot 1+\frac{3}{4}\cdot\frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}\right)\right\}+\frac{1}{2}\left\{\frac{2}{3}+\frac{1}{2}\left(\frac{3}{4}+\frac{1}{4}\right)\right\}+\left\{\frac{3}{4}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{2}\left(\frac{3}{4}+\frac{1}{2}\right)\right\}\right]$$

$$= \frac{1}{3}\left[\frac{1}{4}+\frac{3}{8}\cdot\frac{3}{4}+\frac{1}{2}\left(\frac{2}{3}+\frac{1}{2}\right)+\left(\frac{3}{4}\cdot\frac{1}{3}+\frac{1}{8}\cdot\frac{5}{4}\right)\right] = \frac{1}{3}\left[\frac{1}{4}+\frac{9}{32}+\frac{7}{12}+\frac{13}{32}\right]$$

$$= \frac{1}{3}\left[\frac{24+27+56+39}{96}\right] = \frac{1}{3}\left[\frac{146}{96}\right] = \frac{1}{3}\left[\frac{73}{48}\right] = \frac{73}{144}\ \textbf{Ans.}\ ]$$