<u>**In chapter Exercise - 1**</u>

1.(a) $(x+2)^2-4(x+1)+1=0$

$\Rightarrow\quad x^2+4x+4-4x-4+1=0$

$\Rightarrow\quad x^2+1=0 \quad\Rightarrow\quad x=\pm 1$

(b) $x^2+14x+45=0$

$x^2+5x+5x+45=0$

$(x+9)(x+5)=0 \quad\Rightarrow\quad x=-5,-9$

(c) $x^4-8x^2-9=0$

$\Rightarrow\quad x^4-9x^2+x^2-9=0$

$\Rightarrow\quad x^2\left(x^2-9\right)+1\left(x^2-9\right)=0$

$\Rightarrow\quad \left(x^2-9\right)\left(x^2+1\right)=0 \quad\Rightarrow\quad x^2=9$ or $x^2=-1$

$\Rightarrow\quad x=\pm 3,\pm 1$

(d) $x^2-\left(\sqrt{5}+\sqrt{2}\right)x+\sqrt{10}=0$

$\Rightarrow\quad x^2-\sqrt{5}x-\sqrt{2}x+\sqrt{10}=0$

$\Rightarrow\quad x\left(x-\sqrt{5}\right)-\sqrt{2}\left(x-\sqrt{5}\right)=0$

$\Rightarrow\quad \left(x-\sqrt{5}\right)\left(x-\sqrt{2}\right)=0 \quad x=\sqrt{2},\sqrt{5}$

2.(a) $x^4-5x+6=0$

$x^4-2x^2-3x^2+6=0$

$x^2\left(x^2-2\right)-3\left(x^2-2\right)=0$

$\left(x^2-3\right)\left(x^2-2\right)=0 \quad\Rightarrow\quad x=\pm\sqrt{2},\pm\sqrt{3}$

(b) $x^{2/3}+x^{1/3}-2=0$

$x^{2/3}+2x^{1/3}+x^{1/3}-2=0$

$x^{1/3}\left(x^{1/3+2}\right)-1\left(x^{1/3}+2\right)=0$

$\left(x^{1/3}+2\right)\left(x^{1/3}-1\right)=0 \quad\Rightarrow\quad x^{1/3}=-2 \quad\Rightarrow\quad x=-8$

$x^{1/3}=+1 \quad\Rightarrow\quad x=1$

(c) $4^{(x-1)}-2.2^{x-1}-2^{x-1}+2=0$

$\Rightarrow\quad 2^{(x-1)}\left(2^{x-1}-2\right)-1\left(2^{x-1}-2\right)=0$

$\Rightarrow\quad \left(2^{x-1}-2\right)\left(2^{x-1}-1\right)=0$

$\Rightarrow\quad 2^{x-1}=2$, or $2^{x-1}=1$

$\Rightarrow\quad x-1=0$ or $x-1=0$

$\Rightarrow\quad x=2$ or $x=1$

(d) $7^{1+x}+7^{1-x}=50$

$\Rightarrow\quad 7.7^x+\dfrac{7}{7^x}=50=\left(7^x=y\,\text{say}\right)$

$\Rightarrow\quad 7.y+\dfrac{7}{y}=50 \quad\Rightarrow\quad 7y^2-50y+7=0$

$\Rightarrow \quad 7y^2 - 49y - y + 7 = 0\,7 \qquad \Rightarrow \quad 7y(y-7) - 1(y-7) = 0$

$\Rightarrow \quad (7y-1)(y-7) = 0 \qquad \Rightarrow \quad y = 7, 1/7$

$\Rightarrow \quad 7^x = 7 \quad \Rightarrow \quad x = 1 \qquad 7^x = 1/7 \quad \Rightarrow \quad x = -1$

3.(a) $\sqrt{x+1} + \sqrt{2x-5} = 3$

$x + 1 \ge 0 \quad \Rightarrow \quad x >, -1, \; 2x - 5 \ge 0 \quad \Rightarrow \quad x \ge 5/2$

$\Rightarrow \quad x \ge 5/2$

Square $(x+1) + (2x-5) + 2\sqrt{(x+1)(2x-5)} = 9$

$\Rightarrow \quad 3x - 4 + 2\sqrt{2x^2 - 3x - 5} = 9$

$\Rightarrow \quad 2\sqrt{2x^2 - 3x - 5} = 13 - 3x$

$\Rightarrow \quad 4(2x^2 - 3x - 5) = 9x^2 - 78x + 169$

$\Rightarrow \quad x^2 - 66x + 189 = 0 \quad \Rightarrow \quad (x-63)(x-3) = 0 \quad \Rightarrow \quad x = 3, 63$

Only $x = 3$ satisfy

(b) $\sqrt{x} = y \quad \Rightarrow \quad y^2 - y - 2 = 0 \quad \Rightarrow \quad y^2 - 2y + y - 2 = 0$

$\Rightarrow \quad y(y-2) + 1(y-2) = 0 \quad \Rightarrow \quad (y-2)(y+1) = 0$

$y = 2, -1$ but $\sqrt{x} = y$

$\sqrt{x} = 2, \quad \Rightarrow \quad x = 4, \; \sqrt{x} \ne -1$

(c) $\sqrt{3x+1} - \sqrt{x-1} = 2$

$\Rightarrow \quad (3x+1) + (x-1) + 2\sqrt{3x+1}\sqrt{x-1} = 4$

$\Rightarrow \quad 2\sqrt{(3x+1)(x-1)} = 4 - 4x$

$\Rightarrow \quad 3x^2 - 3x + x - 1 = (2-2x)^2$

$\Rightarrow \quad 3x^2 - 2x - 1 = 4x^2 - 8x + 4$

$\Rightarrow \quad x^2 - 6x + 5 = 0 \quad \Rightarrow \quad (x-1)(x-5) = 0$

$\Rightarrow \quad x = 1, 5$

(d) $\sqrt{x+20} + \sqrt{x+4} = 4\sqrt{x-1}$

$x + 20 + x + 4 + 2\sqrt{(x+20)(x+4)} = 16(x-1)$

$24 + 24 + 2\sqrt{x^2 + 24x + 80} = 16x - 16$

$2\sqrt{x^2 + 24x + 80} = 14x - 40$

$x^2 + 24x + 80 = (7x-20)^2 = 49x^2 - 280x + 400$

$48x^2 - 304x + 320 = 0$

$3x^2 - 19x + 20 = 0$

$3x^2 - 15x - 4x + 30 = 0$

$3x(x-5) - 4(4-5) = 0$

$(3x-4)(x-5) = 0 \quad \Rightarrow \quad x = 5, x = 4/3$

4.(a) $\sqrt{\frac{x}{1-x}} = y$

$y + \frac{1}{y} = \frac{13}{6} \Rightarrow 6y^2 - 13y + 6 = 0$

$\Rightarrow 6y^2 - 4y - 9y + 6 = 0$

$\Rightarrow 2y(3y-2) - 3(3y-2) = 0 \Rightarrow (3y-2)(2y-3) = 0$

$\Rightarrow y = \frac{2}{3}, \frac{3}{2}$

If $\sqrt{\frac{x}{1.x}} = \frac{2}{3} \Rightarrow \frac{x}{1-x} = \frac{4}{9} \Rightarrow 9x = 4 - 4x \Rightarrow x = \frac{4}{13}$

If $\sqrt{\frac{x}{1-x}} = \frac{3}{2} \Rightarrow \frac{x}{1-x} = \frac{9}{4} \Rightarrow 4x = 9 - 9x = 5x = \frac{9}{13}$

(b) $x - 1/x = y \Rightarrow (x + 1/x)^2 = (x - 1/x)^2 + 4$

$y^2 - \frac{3}{2}y = 0 \Rightarrow 2y^2 - 3y = 0$

$y(2y-3) = 0 \Rightarrow y = 0, 3/2$

$x - \frac{1}{x} = 0 \Rightarrow x = \pm 1,$ $\quad x - \frac{1}{x} = \frac{3}{2}x \Rightarrow \frac{x^2 - 1}{x} = 3/2$

$\Rightarrow 2x^2 - 2 = 3x$

$\Rightarrow 2x^2 - 3x - 2 = 0$

$\Rightarrow 2x^2 - 4x + x - 2 = 0$

$\Rightarrow 2x(x-2) + 1(x-2) = 0$

$\Rightarrow x = 2, -1/2$

(c) $\left(x + \frac{1}{x}\right) = y \Rightarrow x^2 + 1/x^2 = y^2 - 2$

$\Rightarrow 9(y^2 - 2) - 27y + 8 = 0 \Rightarrow 9y^2 - 27y - 10 = 0$

$\Rightarrow 9y^2 - 30y + 3y - 10 = 0 \Rightarrow 3y(3y-10) + 1(3y-10) = 0$

$\Rightarrow (3y-10)(3y+1) = 0 \quad y = \frac{10}{3}, -1/3$

$x + \frac{1}{x} = \frac{10}{3} \Rightarrow \frac{x^2+1}{x} = \frac{10}{x} \Rightarrow 3x^2 + 3 = 10x$

$\Rightarrow 3x^2 - 10x + 3 = 0 \Rightarrow 3x^2 - 9x + 3 = 0$

$\Rightarrow 3x(x-3) - 1(x-3) = 0 \quad x = 3, 1/3$

If $x + 1/x = -1/3 \Rightarrow \frac{x^2+1}{x} = -1/3 \Rightarrow 3x^2 + 3 = -x$

$\Rightarrow 3x^2 + x + 3 = 0$ no real roots.

(d) $\left(4 + \sqrt{15}\right)^x = y$

$y + \frac{1}{y} = 8 \Rightarrow y^2 - 8y + 1 = 0$

$\Rightarrow \quad y = \frac{8 \pm \sqrt{64-4}}{2} = \frac{8 \pm \sqrt{60}}{2} = 4 \pm \sqrt{15}$

If $\quad y = 4 \pm \sqrt{15} \quad \Rightarrow \quad \left(4+\sqrt{15}\right)^x = 4+\sqrt{15} \quad \Rightarrow \quad x = 1$

If $\quad y = 4 - \sqrt{15} \quad \Rightarrow \quad \left(4+\sqrt{15}\right)^x = \left(4-\sqrt{15}\right) \quad \Rightarrow \quad x = -1$

5.(a) $\quad x^2 + 5x = y$

$y^2 - 2y - 24 = 0 \quad \Rightarrow \quad y^2 - 6y + 4y - 24 = 0$

$\Rightarrow \quad y(y-6) + 4 = 0 \quad \Rightarrow \quad y = 6, -4$

If $\quad y = 6 \Rightarrow x^2 + 5x - 6 = 0 \Rightarrow x^2 + 6x - x - 6 = 0 \Rightarrow (x+6)(x-1) = 0 \Rightarrow x = 1, -6$

If $\quad y = -4 \Rightarrow x^2 + 5x + 5 = 0 \Rightarrow (x+4)(x+1) = 0 \Rightarrow x = -4, -1$

(b) dividing x^2

$\Rightarrow \quad 2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$

$\Rightarrow \quad 2\left(x^2 + 1/x^2\right) + x + 1/x - 11 = 0$

Let $\quad x + 1/x = y$

$\Rightarrow \quad 2\left(y^2 - 2\right) + y - 11 = 0$

$\Rightarrow \quad 2y^2 + y - 15 = 0 \quad \Rightarrow \quad 2y^2 + 6y - 5y - 15 = 0$

$\Rightarrow \quad y = -3, 5/2$

If $\quad x + 1/x = \frac{5}{2} \quad \Rightarrow \quad 2x^2 - 5x + 2 = 0 \quad \Rightarrow \quad 2x^2 - 4x - x + 2 = 0$

$\Rightarrow \quad 2x(x-2) - 1(x-2) = 0 \quad x = 2, 1/2$

(c) $\quad (x+2)(x+4)(x+6)(x+12) = -4x^2$

$\Rightarrow \quad \left(\frac{x^2 + 14x + 24}{x}\right)\left(\frac{x^2 + 10x + 8y}{x}\right) = -y$

Let $\quad x + \frac{24}{x} = y$

$\Rightarrow \quad (y+14)(y+10) + 4 = 0$

$\Rightarrow \quad y^2 + 24y + 14y = 0 \quad \Rightarrow \quad (y+12)^2 = 0 \quad \Rightarrow \quad y = -12$

$x + \frac{24}{x} = -12 \quad \Rightarrow \quad x^2 + 12x + 24 = 0$

$\Rightarrow \quad x = \frac{-12 \pm \sqrt{144-96}}{2} = \frac{-12 \pm \sqrt{48}}{2} = -6 \pm 2\sqrt{3}$

(d) $\quad (6-x)^4 + (8-x)^4 = 16$

$\Rightarrow \quad \frac{6-x+8-x}{2} y = 7 - x \quad \Rightarrow \quad x = 7 - y$

$\Rightarrow \quad (6-7+y)^4 + (8-7+y)^4 = 16$

$\Rightarrow \quad (y-1)^4 + (y+1)^4 = 16$

$\Rightarrow \quad y^4 + 6y^2 - 7 = 0 \quad \Rightarrow \quad \left(y^2+7\right)\left(y^2-1\right) = 0$

$y^2-1=0 \quad \Rightarrow \quad y=1,-1$

$y=1 \quad \Rightarrow \quad x=6$

$y=-1 \quad \Rightarrow \quad x=8$

$y^2+7=0 \quad \Rightarrow \quad y\pm i\sqrt{7}$

In chapter Exercise – 2

1.(a) $x^2-\left(5+2\sqrt{3}-5-2\sqrt{3}\right)x+\left(5+2\sqrt{3}\right)\left(5-2\sqrt{3}\right)=0$

$\Rightarrow \quad x^2+10x+\left(25-12\right)=0$

$\Rightarrow \quad x^2+10x+13=0$

(b) $x^2-\left(1+i+1+i\right)x+\left(1+i\right)\left(1-i\right)=0$

$x^2-2x+2=0$

(c) $x^2-\left(2+i\sqrt{3}+2-i\sqrt{3}\right)x+\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)=0$

$x^2-4x+7=0$

(d) $x^3-\left(2i+5i+2\right)x^2+\left(2i\times 5i+2i\times 2+5i\times 2\right)x-5i\times 2i\times 2=0$

$x^3-\left(7i+2\right)x^2+\left(14i-10\right)x+20=0$

2. (a) $\alpha+\beta=\frac{-b}{a},\alpha,\beta=c/a \text{ g}$

$\left(1+\alpha\right)\left(1+\beta\right)=1+\alpha+\beta+\alpha\beta=1\frac{-b}{a}+\frac{c}{a}=\frac{a-b|c}{a}$

(b) $\alpha^3\beta+\alpha\beta^3=\alpha\beta\left(\alpha^2+\beta^2\right)=\left(\frac{c}{a}\right)\left(\left(\alpha+\beta\right)^2-2\alpha\beta\right)$

$=\left(\frac{c}{a}\right)\left(\frac{b^2-2ac}{a^2}\right)=\frac{cb^2-2ac^2}{a^3}$

(c) $\frac{\alpha+\beta}{\alpha\beta}=\frac{-b}{c}$

(d) $\frac{1}{a\alpha+\beta}+\frac{1}{a\beta+b}=\frac{\alpha}{a\alpha^2+b\alpha}+\frac{\beta}{a\beta^2+b\beta}=\frac{\alpha+\beta}{-c}$

$=\frac{-b}{a\times\left(-c\right)}=\frac{b}{ac}$

3. by equation transformation

(a) $-2\alpha=y \quad \Rightarrow \quad \alpha=\frac{y}{-2}$

$\Rightarrow \quad a\left(\frac{y}{-2}\right)^3+b\left(\frac{y}{-2}\right)+c=0$

$\Rightarrow \quad \frac{ay^3}{-8}-\frac{by}{2}+c=0 \quad \Rightarrow \quad ay^3+4by-8c=0$

(b) $\frac{1}{\alpha}=y \quad \Rightarrow \quad \alpha=\frac{1}{y}$

$a\left(\frac{1}{y}\right)^3+b\left(\frac{1}{y}\right)+c \quad \Rightarrow \quad a+by^2+cy^3=0$

(c) $\alpha+2=y \quad \Rightarrow \quad \alpha=y-2$

$\Rightarrow \quad (y-2)^3+b(y-2)+c=0$

$\Rightarrow \quad a\left(y^3-3y^2.2+3y.2^2-2^3\right)+by-2b+c=0$

$\Rightarrow \quad ay^3-6ay^2+(12a+b)y-8a+c-2b=0$

(d) $\alpha+\beta+\gamma=0 \quad \Rightarrow \quad \alpha+\beta=-x\,,\ \beta+x=-\alpha\,,\ x+\alpha=-\beta$

$\Rightarrow \quad -\alpha=y \quad \Rightarrow \quad \alpha=-y$

$a(-y)^3+b(-y)+c=0 \quad \Rightarrow \quad ay^3+by-c=0\,ay$

4. (a) $(x-1)(x-3)(x-2)^2<0$

+ 1 − 2 − 3 +

$\Rightarrow \quad (1,2)(2,3)$

(b) $\frac{6x-5}{4x+5}<0 \quad \Rightarrow \quad (6x-5)(4x+1)<0$

$\Rightarrow \quad \frac{-1}{4}<x<5/6 \quad \Rightarrow \quad \left(\frac{-1}{4},\frac{5}{6}\right)$

(c) $\frac{0.5}{-\left(x^2-x+1\right)}<0 \quad \Rightarrow \quad \frac{0.5}{x^2-x+1}>0$

$\because \quad x^2-x+1>0 \quad x\in R \quad \Rightarrow \quad x\in R$

(d) $\frac{\left(x^2-5x+6\right)}{x^2+x+1}<0 \qquad x^2+x+1>0$

$\Rightarrow \quad x^2-5x+6<0 \quad \Rightarrow \quad 2<x<3$

(e) $\frac{(x-1)(x+2)^2}{-(1+x)}<0 \quad \Rightarrow \quad (x-1)(x+2)^2(1+x)>0$

$\Rightarrow$ 5

+ −2 + −1 − 1 +

$(-\infty,-2)\cup(-2,-1)\cup(1,\infty)$

(f) $\frac{x^2+4x+4}{2x^2-x-1}>0 \quad \Rightarrow \quad \frac{(x+2)^2}{2x^2-2x+x-1}>0$

$\frac{(x+2)^2}{2x(x-1)+1(x-1)}>0 \quad \Rightarrow \quad \frac{(x+2)^2}{(x-1)(2x+1)}>0$

$(x+2)^2(x-1)(2x+1)>0$

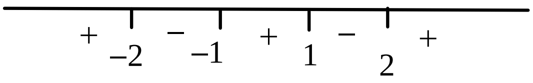
$\Rightarrow \quad (-\infty,-2)(-2,-1/2)(1,\infty)$

+ −2 + −1/2 − 1 +

(g) $x^4-5x^2+4<0$

$\Rightarrow \quad x^4-4x^2-x+4<0 \quad \Rightarrow \quad x^2\left(x^2-4\right)-1\left(x^2-4\right)<0$

$\Rightarrow \quad \left(x^2-4\right)\left(x^2-1\right)<0 \quad \Rightarrow \quad (x-2)(x+2)(x-1)(x+1)<0$



$(-2,-1)(1,2)$

(h) $x^4-2x^2-63\le 3 \quad \Rightarrow \quad x^4-9x+7x^2-63\le 0$

$\Rightarrow \quad x^2\left(x^2-9\right)+7\left(x^2-9\right)\le 0$

$\Rightarrow \quad \left(x^2+7\right)\left(x^2-9\right)\le 0 \quad \Rightarrow \quad x^2-9\le 0$

$\Rightarrow \quad -3\le x\le 3$

5.(a) $x^4-2x^2-63\le 0 \quad \Rightarrow \quad x^4-9x^2+7x^2-63\le 0$

$\Rightarrow \quad x^2\left(x^2-9\right)+7\left(x^2-9\right)\le 0$

$\Rightarrow \quad \left(x^2+7\right)\left(x^2-9\right)\le 0 \quad \Rightarrow \quad x^2-9\le 0$

(b) $\dfrac{1}{x-1}-2\le 0 \quad \Rightarrow \quad \dfrac{1-2x+2}{x-1}\le 0$

$\Rightarrow \quad (3-2x)(x-1)\le 0 \quad \Rightarrow \quad (x-1)(2x-3)\ge 0$

$x\ge 3/2, x\le 1$

(c) $\dfrac{x-2}{x^2+1}+1/2<0 \quad \Rightarrow \quad \dfrac{2x-4+x^2+1}{2\left(x^2+1\right)}<0$

$\Rightarrow \quad x^2+2x-3<0 \quad \Rightarrow \quad x^2+3x-x-3<0$

$\Rightarrow \quad x(x+3)-1(x+3)<0 \Rightarrow \quad (x+3)(x-1)<0$

$\Rightarrow \quad -3<x<1$

(d) $\dfrac{x+1}{(x-1)^2}-1<0 \qquad \dfrac{x+1-(x-1)^2}{(x-1)^2}<0$

$\Rightarrow \quad x+1-x^2-1+2x<0$

$\Rightarrow \quad -x^2+3x<0 \quad \Rightarrow \quad x^2-3x>0 \quad \Rightarrow \quad x>3, x<0$

(e) $\dfrac{x^2-7x+12}{2x^2+4x+5}-1/2>0$

$\dfrac{2x^2-14x+24-2x^2-4x-5}{\left(2x^2+4x+5\right)2}>0$

$\dfrac{-18x+19}{\left(2x^2+4x+5\right)}->0 \quad \because \quad 2x^2+4x+5>0 \quad \forall x$

(f) $\dfrac{x^2+6x-7-2x^2-2}{x^2+1}\le 0$

$x^2+6x-9\le 0 \quad \Rightarrow \quad x^2-6x+9\ge 0$

$\Rightarrow \quad (x-3)^2\ge 0 \quad \Rightarrow \quad x\in R$

(g) $\dfrac{1-2x-3x^2-9x+3x^2+15}{3x-x^2-5}\ge 0$

$\dfrac{-11x+16}{-\left(x^2-3x-5\right)}\ge 0 \quad \Rightarrow \quad \dfrac{11x-16}{x^2-3x+5}\ge 0$

$\Rightarrow \quad x \le \frac{16}{11}$

(h) $\frac{5x^2 - 2 - 4x^2 + x - 3}{4x^2 - x + 3} < 0$

$\frac{x^2 + x - 5}{4x^2 - x + 3} < 0 \quad \Rightarrow \quad x^2 + x - 5 < 0$

$\Rightarrow \quad x \in \left(\frac{-1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2} \right)$

In chapter Exercise - 3

2. (a) $D = (-1)^2 - 4(-3) = 1 + 12 > 0$

(b) $D = \left(2\sqrt{3}\right)^2 - 4 \times 3 = 0$

(c) put $x = 1 \Rightarrow \quad a + b - c - 2a + a - b + c = 0$

$\Rightarrow \quad$ 1is one roots of equation

So, real and distinct roots.

(d) $(x-p)^2 + (q+r)^2 = 0 \Rightarrow \quad x = p \pm \sqrt{(q+r)^2}$

$= p \pm i(q+r)$

Complex roots

3.(a) $x^2 - \frac{x}{2}\left(1+\sqrt{3}i + 1 - i\sqrt{3}\right) + \frac{1}{2}\left(1+\sqrt{3}i\right)\frac{\left(1-\sqrt{3}i\right)}{2} = 0$

$\Rightarrow \quad x^2 - x + \frac{1}{4}(1+3) = 0$

$\Rightarrow \quad x^2 - x + 1 = 0$

(b) $\frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1+i} = \frac{1-i}{2} = \frac{1-i}{2}$

$\Rightarrow \quad x^2 - x\left(\frac{1-i}{2} + \frac{1+i}{2}\right) + \left(\frac{1+i}{2}\right)\left(\frac{1-i}{2}\right) = 0$

$\Rightarrow \quad x^2 - x + \frac{1}{4}(1+1) = 0$

$\Rightarrow \quad x^2 - x + \frac{1}{2} = 0 \Rightarrow \quad 2x^2 - 2x + 1$

(c) $x^2 - x\left(3+\sqrt{5} + 3 - \sqrt{5}\right) + \left(3^2 - 5\right) = 0$

$x^2 - 6x + 4 = 0$

(d) $\left(x^2 + 1\right)\left(x^2 - 5\right) = 0$

$x^4 - 4x^2 - 5 = 0$

4.(a) $x^2 + x + 1 = 0 \quad D < 0$

Both roots are common

$\Rightarrow \quad \frac{a}{1}=\frac{b}{1}=\frac{c}{1} \quad \Rightarrow \quad a=b=c$

(b) $x^2+7x+1=0 \quad \Rightarrow \quad D=49-4=45$

$x=\frac{-7\pm\sqrt{45}}{2} \quad\quad a=c=1$

$b=7$ both roots are common

$\frac{x^2}{ab-c^2}=\frac{-x}{a^2-bc}=\frac{1}{ac=b^2}$

$x=\frac{ab-c^2}{a^2-bc}=\frac{a^2-bc}{ac-b^2} \quad \Rightarrow \quad \left(ac-b^2\right)\left(ab-c^2\right)=\left(a^2-bc\right)^2$

(c) $x^2-2x-3=0 \quad\quad \frac{x^2}{-2c-3b}=\frac{-x}{c+3a}=\frac{1}{b+2a}$

$ax^2+bx+c=0$

$x=\frac{-(2c+3b)}{c+3a}=\frac{c+3a}{b+2a}$

$\Rightarrow \quad (c+3a)^2+(b+2a)(2c+3b)=0$

5.(a) $x^2-3x+2=0 \quad \Rightarrow \quad x=1,2$

$f(1)=1-p+q=0$

$f(2)=16-4p+q=0$

$\quad -\quad\quad +\quad\quad -$

$-15+3p=0 \quad \Rightarrow \quad p=5 \quad \Rightarrow \quad q=p-1=4$

(b) $f(x)=x^4-px^2+q-3x-4$

$x^2-3x+2=0 \quad \Rightarrow \quad x=1,2$

$f(1)=1-p+q-3-4=0 \quad \Rightarrow \quad q-p-6=0$

$f(2)=16-4p+q-6-4=0 \quad \Rightarrow \quad q-4p+6=0$

$\quad -\quad\quad +\quad\quad -$

$q=p+6=4+6=10 \quad\quad 3p-12=0$

$p=4$

(c) $\left(x^2-3(x)+2\right)\left(x^2+3x+2\right)=0$

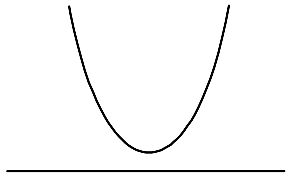
$\left(x^2+2\right)^2(3x)^2=0 \quad \Rightarrow \quad x^4+4x^2+4-9x^2=0$

$\Rightarrow \quad x^4-5x^2+4=0 \quad \Rightarrow \quad p=5, q=4$

<u>In chapter Exercise – 4</u>

1.(a) $f\left(x\right) = x^2 + 2x + 3$

$D = 4 - 12 < 0$

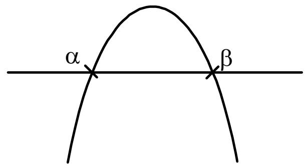


(b) $f\left(x\right) = -4x^2 - 6x + 2$

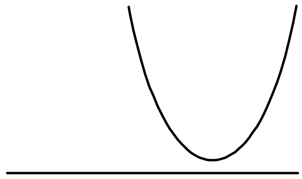
$D = 36 + 32 > 0$

$x = \dfrac{6 \pm \sqrt{68}}{-2\times4} = \dfrac{6 \pm \sqrt{68}}{-8}$

$\alpha = -\left(\dfrac{6+\sqrt{68}}{8}\right), \beta = -\left(\dfrac{6-\sqrt{68}}{8}\right)$



(c) $f\left(x\right) = ax^2 + bx + c, b^2 - 4ac < 0, a > 0$

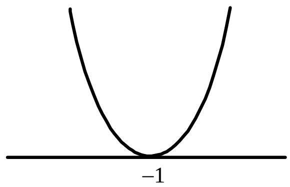


(d) $f\left(x\right) = ax^2 + bx + a$, $D = 0$

$4 - 4a^2 = 0$

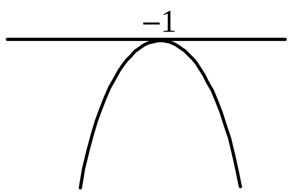
$\Rightarrow \quad a^2 = 1 \quad \Rightarrow \quad a = \pm1$

(i) $a = 1 \quad f\left(x\right) = x^2 + 2x + 1 = \left(x+1\right)^2$



(ii) $a = 1 \quad f\left(x\right) = -x^2 + 2x - 1$

$= -\left(x+1\right)^2$



2.(a) $x^2 - 5x + 14 > 0 \qquad D = 254\times14 < 0$

$\Rightarrow \quad x^2 - 5x + 14 > 0 \qquad \forall x \in R$

(b) $f\left(x\right)x^2 - 4x - 32$

$= x^2 - 8x + 4 - 32 = \left(x-8\right)\left(x+4\right)$

$f\left(x\right) > 0 \qquad \Rightarrow \qquad x > 8, x < -4$

$\left(-\infty, -4\right)\left(8, \infty\right)$

(c) $f\left(x\right) = x^2\left(2a+1\right)x + a^2 + a \ge 0$

$\Rightarrow \quad x^2 2ax - x + a^2 + a \ge 0$

$\Rightarrow \quad \left(x-a\right)^2 - \left(x-a\right) \ge 0$

$\Rightarrow \quad \left(x-a\right)\left(x-a-1\right) \ge 0 \qquad a+1, x \le a$

(d) $f\left(x\right) = x^2 - \left(a+b\right)x + ab \le 0$

$\Rightarrow \quad \left(x-a\right)\left(a-b\right) \le 0$

$\Rightarrow \quad b \le x \le a$

3.(a) $y = ax^2 + 10x + 25$

$D = 0 \quad \Rightarrow \quad 100 - 4a\times25 = 0$

$\Rightarrow \quad a = 1$

(b) $D < 0 \quad \Rightarrow \quad 100 - 4\times a\times25 < 0 \quad \Rightarrow \quad a > 1$

(c) $a < 0$ & $D < 0 \qquad \Rightarrow \qquad a < 0$ & $100 - 100\ a < 0 \quad a > 1$

So far no value of a

(d)

4.(a) $f(x)=3a^2+2x+11$ $\qquad D=4-4\times3\times11$

$f(x)\in\left[-\frac{D}{4a},\infty\right)$ $\qquad =4-132=-128$

$f(x)\in\left[\frac{128}{12},\infty\right)=\left[\frac{32}{3},\infty\right)$

(b) $f(x)=\sin^2 x+\sin x+2$

$f(y)=y^2+y+2$ $\qquad f(y)$ is minimum at $y=\frac{-1}{2}=-1/2$

i.e. $f(y)_{mini}=\frac{-D}{4}=\frac{-(1-8)}{4}=\frac{7}{4}$

$f(1)_{max}=1+1+2=4$

$\frac{7}{4}\le f(x)\le 4$

$y>0$

(c) $f(x)=y^2-y+1$

$\min i=\frac{-D}{4a}=\frac{-(1-4)}{4}=\frac{3}{4}$

5.(a) $y=\frac{x+2}{2x^2+3x+6}$

$\Rightarrow \quad 2yx^2+3yx+6y=x+2$

$\Rightarrow \quad 2yx^2+x(3y-1)+6y-2=0$

$\Rightarrow \quad D\ge0 \quad \Rightarrow \quad (3y-1)^2-8y(6y-2)\ge0$

$\Rightarrow \quad 9y^2-6y+1-48y^2+16y\ge0$

$\Rightarrow \quad -39y^2+10y+1\ge0$

$\Rightarrow \quad 39y^2-10y-1\le0$

$\Rightarrow \quad 39y^2-13y+3y-1\le0$

$\Rightarrow \quad 13(3y-1)+1(3y-1)\le0$

$\frac{-1}{13}\le y\le 1/3 \quad \Rightarrow \quad \left[\frac{-1}{13},1/3\right]$

(b) $y=\frac{x+2}{x-2} \quad \Rightarrow \quad yx-y2=x+2$

$\Rightarrow \quad x(y-1)=2y+2$

$\Rightarrow \quad x=\frac{2(y+1)}{y-1} \quad y\ne1$

$\in(-\infty,\infty)-\{1\}$

(c) $x^2-4x+1=yxh2+4yx+y$

$x^2(y-1)+x(4y+4)+y-1=0$

$D \ge 0 \quad \Rightarrow \quad 16(y+1)^2 - 4(y-1)^2 \ge 0$

$\Rightarrow \quad (2y+2)^2 - (y-1)^2 \ge 0$

$\Rightarrow \quad (2y+2+y-1)(2y+2-y+1) \ge 0$

$\Rightarrow \quad (3y+1)(y+3) \ge 0 \quad \Rightarrow \quad y \ge -1/3, y \le -3$

$(-\infty, -3] \cup [-1/3, \infty)$

(d) $x^2 - 2x + 2 = xy - y$

$x^2 - x(y+2) + 2 + y = 0$

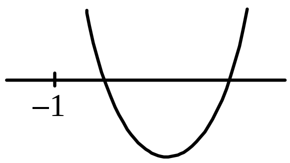
$D \ge 0 \quad \Rightarrow \quad (y+2)^2 - 4(2+y) \ge 0$

$\Rightarrow \quad (y+2)(y+2-4) \ge 0$

$\Rightarrow \quad (y+2)(y-2) \ge 0 \quad \Rightarrow \quad y \ge 2, y \le -2$

$(2, \infty] \cup [2, \infty)$

In chapter Exercise – 5



1.(a) $f(x) > 0$ (1)

$\frac{-b}{2a} > 0$ (2)

$D \ge 0$ (3)

$D \Rightarrow \quad (a+5)^2 + 1.2(a^2 + a - 2) \ge 0$

$\Rightarrow \quad a^2 + 10a + 25 + 8a^2 + 8a - 16 \ge 0$

$\Rightarrow \quad 9a^2 + 18a + 9 \ge 0 \quad \Rightarrow \quad a^2 + 2a + 1 \ge 0$

$\Rightarrow \quad (a+1)^2 \ge 0$ always true

$\frac{-b}{2a} > -1 \quad \Rightarrow \quad \frac{a+5}{2(a^2+a-2)} > -1$

$\Rightarrow \quad \frac{a+5}{2a^2+2a-4} + a > 0 \quad \Rightarrow \quad \frac{a+5+2a^2+2a-4}{2a^2+2a-4} > 0$

$\Rightarrow \quad \frac{2a^2+3a+1}{2(a^2+2a-2)} > 0 \quad \Rightarrow \quad \frac{(2a+1)(a+1)}{2(a+2)(a-1)} > 0$

$\Rightarrow \quad (2a+1)(a+1)(a+2)(a-1) > 0$

$+ \quad -2 \quad - \quad -1 \quad + \quad -1/2 \quad - \quad 1 \quad +$

$(-\infty, -2) \cup (-1, -1/2)(1, \infty)$

$af(-1) > 0 \quad \Rightarrow \quad [(a^2+a-2)(1) + (a+5) - 2](a^2+a-2) > 0$

$\Rightarrow \quad (a^2+a-2)(a^2+a-2+a+5-2) > 0$

$\Rightarrow \quad (a+2)(a-1)(a^2+2a+1) > 0$

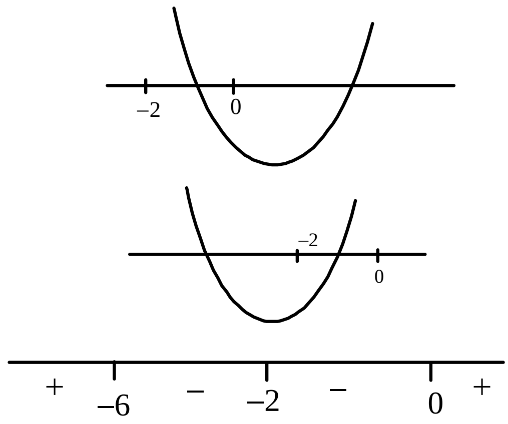
$\Rightarrow \quad (a+2)(a-1)(a+1)^2 > 0$

$+ \quad -2 \quad - \quad -1 \quad - \quad 1 \quad +$

$(-\infty, -2) \cup (1, \infty)$

2. $f(x)=(a^2+a+1)x^2+(a-1)x+a^2$

3. $f(x)=x^2+mx+m^2+6m=0$



$f(-2)f(0)<0$

$\Rightarrow \quad (4-2m=m^2+6m)(m^2+6m)<0$

$\Rightarrow \quad (m^2+4m+4)(m^2+6m)<0$

$\Rightarrow \quad (m+2)^2(m+6)m<0$

$m=(-6,-2)(-2,0)$

4. $f(0)f(3)<0$

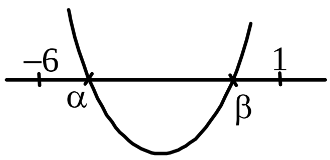
$\Rightarrow \quad (2a)(9-3(a+1)+2a)<0$

$\Rightarrow \quad a(9-3a-3+2a)<a$

$\Rightarrow \quad a(6-a)<0 \quad \Rightarrow \quad a(a-6)>0$

$\Rightarrow \quad a>6, a<0 \quad \Rightarrow \quad (-\infty,0)\cup(6,\infty)$

5. $f(-6)>0 \quad \& \quad f(1)>0 \quad \& \quad D\geq 0$



$f(-6)=36-12(k-3)+9>0$

$\Rightarrow \quad 36-12k+36+9>0 \quad \Rightarrow \quad 81>12k$

$\Rightarrow \quad k<\frac{27}{4}$

$f(1)>0 \quad \Rightarrow \quad 1+2k-6+9>0 \quad \Rightarrow \quad k>2$

$D\geq 0 \quad \Rightarrow \quad (k-3)^2-9\geq 0 \quad \Rightarrow \quad (k-3)^2-3^2\geq 0$

$\Rightarrow \quad (k-3+3)(k-3-3)\geq 0 \quad \Rightarrow \quad k(k-6)\geq 0$

$\Rightarrow \quad k\geq 6, k\leq 0$

QUADRATIC EQUATION BOOKLET SOLUTION

EX – 1 (A)

1. **(B)**

Required equation

$$x^2 - (1-2)x + (1x-2) = 0$$

$$x^2 + x - 2 = 0$$

2. **(B)**

Let $3^x = t$

$t + \frac{1}{t} = \frac{10}{3} \qquad \therefore \frac{t^2+1}{t} = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$3t^2 - 9t - t + 3 = 0$

$3t(t-3) - (t-3) = 0$

$(3t-1)(t-3) = 0$

$t = \frac{1}{3}, 3 \qquad \therefore 3^x = 3^{-1} \qquad \therefore x = -1$

$3^x = 3^1 \qquad \therefore x = 1$

3. **(B)**

Given $16 + 4m - 26 + x = 0$

$4m + x = 10$ .... (i)

$54 + 9m - 39 + x = 0$

$9m + x = -15$ .... (ii)

From (i) & (ii) $m = -5, x = 30$

4. **(D)**

$8\sec^2\theta - 4\sec\theta - 2\sec\theta + 1 = 0$

$4\sec\theta(2\sec\theta - 1) - (2\sec\theta - 1) = 0$

$(4\sec\theta - 1)(2\sec\theta - 1) = 0$

$\sec\theta = \frac{1}{4}$

$\sec\theta = \frac{1}{2}$

We know $\sec\theta$ does not lie bet^x $-1 \& 1$ hence no solution.

5. **(D)**

Given $4(a+b)^2 - 4a(a+2b+c) < 0$

$\cancel{a^2} + b^2 + 2\cancel{a}b - \cancel{a^2} - 2\cancel{a}b - ac < 0$

$b^2 - ac < 0$

$D = 4b^2 - 4ac = 4(b^2 - ac) < 0$

Hence roots are complex

6. **(A)**

$D = (2a+b)^2 - 4.2a.b$

$= (2a-b)^2$

Discriminate is perfect square hence roots are rational

7. **(D)**

$D = 49 - 4.6.K$

$= 49 - 24.K$

If $K = 1$ or 2 D is a perfect square hence roots are rational

8. **(B)**

Given $D = 4(bc+ad)^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\therefore b^2\cancel{c^2} + \cancel{a^2}d^2 + 2abcd - a^2c^2 - \cancel{a^2}d^2 - b^2\cancel{c^2} - b^2d^2 = 0$

$(ac-bd)^2$

$\therefore ac = bd$

9. **(A)**

$D < 0$

$1 - 4m < 0$

$4m > 1$

$m > \frac{1}{4}$

$m \in \left(\frac{1}{4}, \infty\right)$

10. **(D)**

Given $D = 1 - 4ab \geq 0$

$4ab \leq 1$ .... (i)

$D = 16ab - 4$

$= 4(ab-1) \leq 0$

Roots of equation $x^2 - 4\sqrt{ab}x + 1 = 0$ will be imaginary

11. **(A)**

Given $16 + 4P + 12 = 0$

$4 + P + 3 = 0$

$P = -7$

$P^2 - 4q = 0$

$49 = 4q$

$\therefore q = \frac{49}{4}$

12. **(C)**

Given $4q^2 - 4\Pr \geq 0$

$q^2 \geq Pr$ .... (i)

$4Pr - q^2 \geq 0$

$Pr \geq q^2$ .... (ii)

From (i) & (ii)

$q^2 = Pr$

$\frac{q}{r} = \frac{P}{q}$

13. (C)

$|\alpha - \beta| > 0$

$(\alpha + \beta)^2 - 4\alpha\beta > 0$

$(P+2)^2 - 4x2P > 0$

$P^2 + 4 + 4P - 8P > 0$

$(P-2)^2 > 0$

$\therefore P \in I - \{2\}$

14. (C)

$D = 100 - 4(21 - m) = 0$

$25 - 21 - m$

$m = -4$

15. (A)

Given $|\alpha - \beta| = 3$

$(\alpha - \beta)^2 = 9$

$(\alpha + \beta)^2 - 4\alpha\beta = 9$

$\left(\frac{a-4}{a-2}\right)^2 + \frac{8}{(a-2)} = 9$

$(a-4)^2 + 8(a-2) = 9(a-2)^2$

$a^2 - \cancel{8a} + \cancel{16} + \cancel{8a} + \cancel{16} = 9a^2 + 36 - 36a$

$8a^2 - 36a + 36 = 0$

$2a^2 - 9a + 9 = 0$

$2a^2 - 3a - 6a + 9 = 0$

$a(2a-3) - 3(2a-3) = 0$

$(a-3)(2a-3) = 0$

$a = \frac{3}{2}, 3$

16. (D)

Given $\alpha + \beta = -p$

$\alpha\beta = q$

$r+\delta=-p$

$r\delta=r$

$(\alpha-r)(\alpha-\delta)=\alpha^2-\alpha\delta-\alpha r+r\delta$

$=\alpha^2-\alpha(\delta+r)+r\delta$

$=\alpha^2+p\alpha+r$

$=q+r$

(Given $\alpha^2+p\alpha-q=0$)

$\alpha^2+p\alpha=q$

17. **(C)**

Given $\alpha+\beta=\dfrac{35}{2}$

$\alpha\beta=1$

$[(2\alpha-35)(2\beta-35)]^3=[4\alpha\beta-70(\alpha+\beta)+35^2]^3$

$=\left[4-\cancel{70}\times\dfrac{35}{\cancel{2}}+35^2\right]^3$

$=\left(4-\cancel{35}^2+\cancel{35}^2\right)^3=64$

18. **(B)**

Given $\alpha+\beta=\dfrac{-q}{p}$

$\alpha\beta=\dfrac{-r}{p}$

$\dfrac{\alpha}{\beta^2}+\dfrac{\beta}{\alpha^2}=\dfrac{\alpha^3+\beta^3}{(\alpha\beta)^2}=\dfrac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2}$

$=\dfrac{\dfrac{-q^3}{p^3},\dfrac{3r}{p}.\dfrac{q}{p}}{r^2/p^2}$

$=-\dfrac{\dfrac{q^3-3pqr}{p^3}}{\dfrac{r^2}{p^2}}$

$=\dfrac{-q}{pr^2}\left(q^2+3pr\right)$

19. **(C)**

Product of roots $=\dfrac{(2m-1)}{m}=-1$

$2m-1=-m$

$3m=1$

$$m=\frac{1}{3}$$

20. **(B)**

$$\alpha+\beta=0$$

$$-2\frac{\left(2-a-a^2\right)}{1}=0$$

$$a^2+a-2=0$$

$$a^2+2a-a-2=0$$

$$a(a+2)-(a+2)=0$$

$$a=1,-2$$

21. **(B)**

$$|\alpha-\beta|=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$=\sqrt{49+4\times9}$$

$$=\sqrt{49+36} \qquad =\sqrt{85}$$

22. **(D)**

Given $\alpha+\beta=\frac{|\alpha-\beta|}{2}$

$$(\alpha+\beta)^2=\frac{(\alpha-\beta)^2}{4}$$

$$4(\alpha+\beta)^2=(\alpha+\beta)^2-4\alpha\beta$$

$$3(\alpha+\beta)^2=-4\alpha\beta$$

$$3\times\frac{16}{a^{\not{2}}}=-4\times\frac{c}{\not{a}}$$

$\therefore ac=-12$

$$\frac{3\times16}{-4}=ac$$

23. **(C)**

Given $\alpha+\beta=-1$

$$-\frac{(2a+3)}{(a+1)}=-1$$

$$2a+3=a+1$$

$$a=-2$$

$$\alpha\beta=\frac{3a+4}{a+1}=\frac{-6+4}{-2+1}=2$$

24. **(B)**

If $ax^2+bx+c=0$ is dx identify the

$$a=b=c=0$$

$\therefore \quad \lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - \lambda - 2\lambda + 2 = 0$

$\lambda(\lambda - 1) - 2(\lambda - 1) = 0$, $(\lambda - 1)(\lambda - 2) = 0$

$\lambda = 1, 2$

$\lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$\lambda(\lambda - 2) - 3(\lambda - 2) = 0 \qquad \lambda = 2, 3$

And $\lambda^2 = 4$

$\lambda = \pm 2$

$\therefore \quad \lambda = 2$ only cammon value

25. **(C)**

$(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ this is possible only when

$x - 1 = 0$

$x - 2 = 0$

$x - 3 = 0$

But no single value of x for all are zero at a time hence no of solution $= 0$

26. **(D)**

Replace x by $x - 2$ we get required equation

$a(x-2)^2 + b(x-2) + c = 0$

$ax^2 + 4a - 4ax + bx - 2b + c = 0$

$ax^2 + (b - 4a)x + 4a - 2b + c = 0$

27. **(B)**

Let $3^x = t$

$\frac{t}{3} + \frac{3}{t} = 2$

$\frac{t^2 + 9}{3t} = 2$

$t^2 - 6t + 9 = 0$

$(t-3)^2 = 0$

$t = 3$

$\therefore \quad 3^x = 3^1$

$x = 1$

28. **(C)**

$D = 0$

$(k+2)^2 - 4.2K = 0$

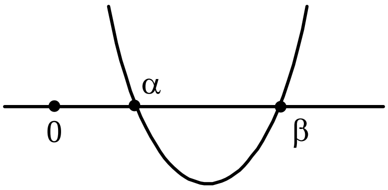
$k^2 + 4 + 4k - 8k = 0$

$(k-2)^2 = 0$

$k = 2$

29. **(A)**

Given $D \geq 0$



$1 - 4.2.K \geq 0$

$8K \leq 1$

$K \leq 1/8$

$f(0) > 0 \therefore K > 0$

$\frac{+1}{2.2} > 0 \qquad \therefore \frac{1}{a} > 0$

$\therefore$ roots are the there fore $K > 0$

$K \in \left(0, \frac{1}{8}\right]$

30. **(C)**

Given $\frac{\alpha}{\beta} = \frac{3}{2}$

$$\frac{\alpha+\beta}{\alpha-\beta} = \frac{5}{1}$$

$$\frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = 25$$

$$\frac{(\alpha+\beta)^2}{(\alpha+\beta)^2 - 4\alpha\beta} = 25$$

$$\frac{\frac{m^2}{144}}{\frac{m^2}{144} - 4 \times \frac{5}{12}} = 25$$

$$\frac{\frac{m^2}{\cancel{144}}}{\frac{m^2 - 20\times 12}{\cancel{144}}} = 25$$

$m^2 = 25m^2 - 240 \times 25$

$24m^2 = 240 \times 25$

$m^2 = 250 = 25 \times 10$

$m = 5\sqrt{10}$

31. **()**

Given $\alpha + \alpha^2 = 30$

$\alpha.\alpha^2 = p$

$\left(\alpha + \alpha^2\right)^3 = (30)^3$

$$\alpha^3+\alpha^6+3\alpha^3\left(\alpha+\alpha^2\right)=(30)^3$$

$$p+p^2+3.p\times 30=30^3$$

$$p^2+91p-27000=0$$

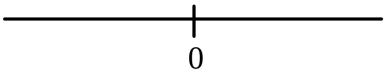
32. **(D)**

$$y=3\sec^2 x-4\sec x+1$$

$$\frac{dy}{dx}=6\sec^2 x\,\text{tn}-4\sec x\,\text{tn}$$

For maximum or minimum $y, \frac{dy}{dx}=0$

$$2\sec x\,\text{tn}\left(3\sec x-2\right)=0$$



Y minimum

When $\sec x=1$

33. **(B)**

$$\alpha+\beta=-b/a$$

$$\alpha\beta=c/a$$

$$\frac{\beta}{a\left(\alpha+\frac{b}{a}\right)}+\frac{\alpha}{a\left(\beta+\frac{b}{a}\right)}=\frac{\beta}{a\left(\not{\alpha}-\not{\alpha}-\beta\right)}+\frac{\alpha}{a\left(\not{\beta}-\alpha-\not{\beta}\right)}$$

$$=\frac{-\beta}{a\beta}-\frac{\alpha}{a\alpha}$$

$$=-\left(\frac{1}{a}+\frac{1}{a}\right)=\frac{-2}{a}$$

34. **(C)**

If α,β be the roots of equation $2x^2-3x+5=0$

Then equation whose roots are $\frac{1}{\alpha}\&\frac{1}{\beta}$

$5x^2-3x+2=0$ .... (i)

Given $ax^2+bx+2=0$ .... (ii)

Equation (i) & (ii) are identical

$$\frac{a}{5}=\frac{b}{-3}=\frac{2}{2}$$

$$a=5, b=-3$$

35. **(A)**

$$\alpha+\beta=\frac{-b}{a}$$

$$\alpha\beta=c/a$$

Given $\alpha+\beta=\alpha^2+\beta^2$

$$\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$$

$$\frac{-b}{a}=\frac{b^2}{a^2}-2.\frac{c}{a}$$

$$\frac{-b}{a}=\frac{b^2-2ac}{a^2}$$

$$b^2-2ac=-ab$$

$$b^2+ab=2ac$$

36. **()**

37. **(A)**

$$x^2-\left(22^0--22^0\right)x+\left(22^0\right)+\left(22^0\right)=0$$

$$x^2+4=0$$

38. **(D)**

$$\alpha+\beta=z$$

$$\alpha^3+\beta^2=98$$

$$(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=98$$

$$8-3.\alpha\beta.z=98$$

$$-90=6.\alpha\beta$$

$$\alpha\beta=\frac{-30}{2}=-15$$

Revered equation $x^2-2x-15=0$

39. **(C)**

$$y=x^2+2$$

$$\therefore x=\sqrt{y-2}$$

Replace x by $\sqrt{x-2}$ in the given equation we get revered equation whose roots are

$$\alpha^2+2\ \&\ \beta^2+2$$

$$2(x-2)-3\sqrt{x-2}-6=0$$

$$2x-4-6=3\sqrt{x-2}$$

$$2x-10=3\sqrt{x-2}$$

$$4x^2+100-40x=9(x-2)$$

$$4x^2-40x+100=9x-18$$

$$4x^2-49x+118=0$$

40. **(A)**

$$y=\frac{-2}{x}\ \therefore x=\frac{-2}{y}$$

Replace x by $-2/x$ we get revered equation whose roots are $\frac{-2}{\alpha},\frac{-2}{\beta}$

$$2\times\frac{4}{x^2}+\frac{7\times 2}{x}+6=0$$

$$8+14x+6x^2=0$$

41. **(A)**

$$y=\frac{x-1}{x+1} \qquad \therefore yx+y=x-1$$

$$y+1=x(1-y)$$

$$x=\frac{1+y}{1-y}$$

Replace x by $\frac{1+x}{1-x}$ we get revered equation

$$\left(\frac{1+x}{1-x}\right)^2-2\frac{(1+x)}{(1-x)}+3=0$$

$$(1+x)^2-2(1+x)(1-x)+3(1-x)^2=0$$

$$x^2+1+2x-2(1-x^2)+3(1+x^2-2x)=0$$

$$x^2+1+2x-2+2x^2+3+3x^2-6x=0$$

$$6x^2-4x+2=0$$

$$3x^2-2x+1=0$$

42. **(B)**

Given $\alpha+\beta=\frac{-2(a+b)}{2}$

$$\alpha\beta=\frac{a^2+b^2}{2}$$

We have to find equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

Sum of roots $=(\alpha+\beta)^2+(\alpha-\beta)^2=2(\alpha^2+\beta^2)$

$$=2\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$=2\left[(a+b)^2-(a^2+b^2)\right]$$

$$=2\left(\cancel{a}^2+\cancel{b}^2+2ab-\cancel{a}^2-\cancel{b}^2\right)$$

$$=4ab$$

Product of roots $=(\alpha+\beta)^2(\alpha-\beta)^2=(\alpha+\beta)^2\left((\alpha+\beta)^2-4\alpha\beta\right)$

$$=(a+b)^2\left[(a+b)^2-\cancel{4}^2\frac{(a^2+b^2)}{\cancel{2}}\right]$$

$$=(a+b)^2(a^2+b^2+2ab-2a^2-2b^2)$$

$$=-(a+b)^2(a^2+b^2-2ab)$$

$$=-(a+b)^2(a-b)^2=-(a^2-b^2)^2$$

Revered equation

$$x^2 - 4abx - \left(a^2 - b^2\right)^2 = 0$$

43. **(C)**

$$x^2 + bx - a = 0$$

Sum of roots $= -b < 0$

Product of roots $= -a < 0$

Both roots have difference sign and these root is –ve whose magnitude is greater.

44. **(B)**

Sum of roots $= P > 0$

Product of roots $= -q < 0$

Both roots are red and opposite sign

45. **(B)**

Given $D = b^2 - 4ac < 0$

If $b = 0, a > 0, c > 0$

Then $D < 0$

46. **(B)**

Product of roots $= 1$

$$\frac{-2}{\ell} = 1 \qquad \therefore \ell = -2$$

47. **(A)**

$$a + \frac{b}{x} + \frac{c}{x^2} = 0$$

If $x \to \infty \therefore a \to 0$

48. **(C)**

Let α be common root

$$\alpha^2 - 11\alpha + a = 0$$

$$\alpha^2 - 14x + 2a = 0$$

$$\frac{\alpha^2}{-22a + 14a} = \frac{\alpha}{a - 2a} = \frac{1}{-14 + 11}$$

$$\frac{\alpha^2}{-8a} = \frac{\alpha}{-a} = \frac{1}{-3}$$

$$\therefore \alpha = a/3$$

$\therefore a = 24 \qquad \alpha = 8$

If $a = 0$ then $x = 0$ is common root

49. **(B)**

$$x^3 - 2x^2 + 2x - 1 = 0$$

$$x^3 - x^2 - x^2 + x + x - 1 = 0$$

$$x^2(x - 1) - x(x - 1) + (x - 1) = 0$$

$$(x-1)(x^2-x+1)=0 \text{ .... (i)}$$

From (i) two common roots are imaginary

$$\frac{a}{1}=\frac{b}{-1}=\frac{a}{1}$$

$$a=-b$$

$$a+b=0$$

50. **(D)**

$$x^2-x-2x+2=0$$

$$x(x-1)-2(x-1)=0$$

$$x=1,2$$

$$x^2-x+3x-3=0$$

$$x(x-1)+3(x-1)=0$$

$$x=1,-3$$

Common root $=1$

$f(1)=4+3-7=0$

51. **()**

52. **(A)**

$$y=\frac{x}{x^2-5x+9}$$

$$yx^2-5yx+9y=x$$

$$yx^2-x(5y+1)+9y=0$$

For read x, $d\geq 0$

$$(5y+1)^2-4.y.9y\geq 0$$

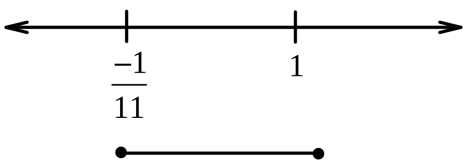
$$25y^2+1+10y-36y^2\geq 0$$

$$11y^2-10y-1\leq 0$$

$$11y^2-11y+y-1\leq 0$$

$$11y(y-1)+(y-1)\leq 0$$

$$(11y+1)(y-1)\leq 0$$



$$y_{\max m}=1$$

53. **(D)**

$$y=\frac{x^2+34x-71}{x^2+2x-7}$$

$$yx^2+2xy-7y=x^2+34x-71$$

$$(y-1)x^2+2x(y-17)+(71-7y)=0$$

For real x,

$$D \geq 0$$

$$4(y-17)^2 - 4(y-1)(71-7y) \geq 0$$

$$y^2 - 34y + 289 - (-7y^2 + 78y - 71) \geq 0$$

$$y^2 - 34y + 289 + 7y^2 - 78y + 71 \geq 0$$

$$8y^2 - 112y + 360 \geq 0$$

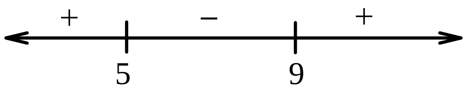
$$2y^2 - 289 + 90 \geq 0$$

$$y^2 - 14y + 45 \geq 0$$

$$y^2 - 5y - 9y + 45 \geq 0$$

$$y(y-5) - 9(y-5) \geq 0$$

$$(y-5)(y-9) \geq 0$$



$$y \in (-\infty + 5) \cup [9, \infty)$$

54. **(D)**

$$D = 1 - 4.2.P$$

$$= (1 - 8P)$$

For ration root D must be perfect square and $P = -6$

$$D = 1 + 48 = 49$$

Hence $P = -6$

55. **(C)**

$$a\alpha^2 + b\alpha + c = 0$$

$$b\alpha^2 + c\alpha + a = 0$$

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

$$\therefore \frac{ab - c^2}{bc - a^2} = \frac{bc - a^2}{ac - b^2}$$

$$(bc - a^2)^2 = (ab - c^2)(ac - b^2)$$

$$\cancel{b^2c^2} + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + \cancel{b^2c^2}$$

$$a(a^3 + b^3 + c^3 - 3abc) = 0$$

Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$

56. **(C)**

Given $D = 0$

$$x^2 + 2x(2 + k) + 3k + 8 + 3k - 5 = 0$$

$$x^2 + 2(2 + k)x + 6k + 3 = 0$$

$$\cancel{4}(2 + k)^2 - \cancel{4}.3(2k + 1) = 0$$

$$4 + k^2 + 4k - 6k - 3 = 0$$

$$k^2 - 2k + 1 = 0$$
$$(k-1)^2 = 0$$
$$k = 1$$

57. **(B)**

$x = \alpha$ is the root of equation $x^2 - ax + b = 0$

$$\alpha^2 - a\alpha + b = 0$$
$$\alpha(\alpha - a) + b = 0$$
$$b = \alpha(a - \alpha)$$

58. **(D)**

Given $x = -1$ is the root of given equation

$$1 - (P-3) - (3P-5) - (2P-9) + 6 = 0$$
$$7 - P + 3 - 3P + 5 - 2P + 9 = 0$$
$$24 = 6P$$
$$P = 4$$

59. **(B)**

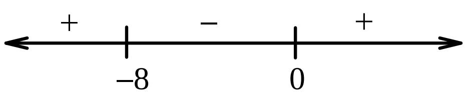
$$y_{min} = \frac{-D}{4a} = -\frac{(12^2 - 4.40)}{4}$$
$$y_{min} = -\frac{4 \times \cancel{4}(3^2 - 10)}{\cancel{4}}$$
$$= -4 \times -1 = 4$$

60. **(C)**

$$y = \frac{x^2 - 2x + 1}{x+1}$$
$$yx + y = x^2 - 2x + 1$$
$$x^2 - x(2+y) + (1-y) = 0$$

For real $x, D \geq 0$

$$(2+y)^2 - 4(1-y) \geq 0$$
$$\cancel{4} + y^2 + 4y - \cancel{4} + 4y \geq 0$$
$$y^2 + 8y \geq 0$$
$$y(y+8) \geq 0$$



$$y \in (-\infty, -8] \cup [0, \infty)$$

EXERCISE – 1B

31. **(B)**

Sum of roots $= i(a-b) - i(a-b) = 0$

$$\frac{-B}{A} = 0$$

$$B = 0$$

$$\therefore \quad \frac{AB}{C} = 0$$

32. **(C)**

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\alpha + \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\frac{-b}{a} = \frac{\frac{b^2}{a^2} - \frac{2.c}{a}}{\frac{c^2}{a^2}}$$

$$\frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$-bc^2 = b^2a - 2a^2c$$

$$b^2a + bc^2 = 2a^2c$$

$$\frac{b^2 a}{a^2 c} + \frac{bc^2}{a^2 c} = 2$$

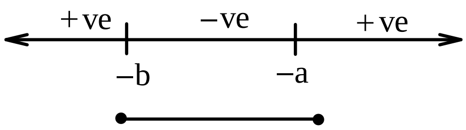
$$\frac{b^2}{ac} + \frac{bc}{a^2} = 2$$

33. **(D)**

$$x^2 + ax + bx + ab < 0$$

$$x(x+a) + b(x+a) < 0$$

$$(x+a)(x+b) < 0$$



34. **(B)**

$$D < 0$$

$$b^2 - 4ac < 0$$

$$b^2 < 4ac$$

35. **(C)**

$$f(1) < 0$$

$ax^2 + bx + c < 0 \qquad \forall x \in R$

$c < 0$

36. **(A)**

Replace x by $1/x$ we get

$cx^2 + bx + a = 0$

37. **(A)**

Given $\alpha + \beta = -b/a$

$\alpha\beta = c/a$

$$\frac{1}{a^2(\alpha + b/a)^2} + \frac{1}{a^2(\beta + b/a)^2} = \frac{1}{a^2.\beta^2} + \frac{1}{a^2\alpha^2}$$

$$= \frac{1}{a^2}\frac{(\alpha^2 + \beta^2)}{(\alpha\beta)^2} = \frac{1}{a^2}\frac{\left[(\alpha+\beta)^2 - 2\alpha\beta\right]}{(\alpha\beta)^2}$$

$$= \frac{1}{\cancel{a}^2}\frac{(b^2/a^2 - 2c/a)}{c^2/\cancel{a}^2}$$

$$= \frac{(b^2 - 2ac)}{a^2c^2}$$

38. **(B)**

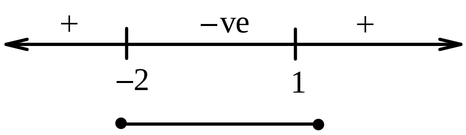
(i) $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x - 1 < 0 \qquad \forall x \in R$

$\lambda^2 + \lambda - 2 < 0$

$\lambda^2 + 2\lambda - \lambda - 2 < 0$

$\lambda(\lambda + 2) - (\lambda + 2) < 0$

$(\lambda - 1)(\lambda + 2) < 0$



$\lambda \in (-2, 1)$ .... (i)

(ii) $(\lambda + 2)^2 + 4(\lambda^2 + \lambda - 2) < 0$

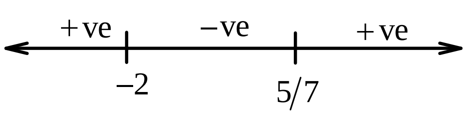
$\lambda^2 + 4\lambda + 4 + 4\lambda^2 + 4\lambda - 8 < 0$

$5\lambda^2 + 8\lambda - 4 < 0$

$5\lambda^2 + 10\lambda - 2\lambda - 4 < 0$

$5\lambda(\lambda + 2) - 2(\lambda + 2) < 0$

$(5\lambda - 2)(\lambda + 2) < 0$



$-2 < \lambda < 5/2$

From (i) and (ii)

$\lambda \in (-2,1)$

39. **()**

40. **(B)**

Replace x by $x-h$ is equation $ax^2+bx+c=0$

$$a(x-h)^2+b(x-h)+c=0$$

$$ax^2+(6-2ah)x+ah^2-bh+c=0 \quad \text{....... (i)}$$

Given $\alpha+h, \beta+h$ are roots of

$$px^2+qx+r=0 \quad \text{.......... (ii)}$$

(i) & (ii) are identical

$$\frac{a}{p}=\frac{b-2a-h}{q}=\frac{ah^2-bh+c}{r}$$

$$\therefore \quad b-2ah=\frac{aq}{p}$$

$$2ah= \ b-\frac{aq}{p}$$

$$h=\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$$

41. **(B)**

Let α be the common root

$$\alpha^2+2\alpha+3\lambda=0$$

$$2\alpha^2+3\alpha+5\lambda=0$$

$$\frac{\alpha^2}{10\lambda-9\lambda}=\frac{\alpha}{6\lambda-5\lambda}=\frac{1}{3-4}$$

$$\frac{\alpha^2}{\lambda}=\frac{\alpha}{\lambda}=-1$$

$$\alpha=1$$

42. **(C)**

We know $x=-\frac{b}{2a}$ lies between roots of equation there fore one root must be exceed $\frac{-b}{2a}$

43. **(B)**

If a, b, c are rational then irrational root occur in pair lie if

$m+\sqrt{n}$ or root then other root must be

$m-\sqrt{n}$

44. **()**

45. **(A)**

$$\frac{ax-ab+bx-ab}{(x-a)(x-b)}=1$$

$(a+b)x-2ab=x^2-(a+b)x+ab$

$x^2-2(a+b)x+3ab=0$ .... (i)

Given

sum of roots $=0$

$2(a+b)=0$

$\therefore a+b=0$

46. **(A)**

Given

$$\sin\theta+\cos\theta=-\frac{m}{\ell}$$

$$\sin\theta\cos\theta=\frac{n}{\ell}$$

$\sin\theta+\cos^2\theta=1$

$(\sin\theta+\cos\theta)^2-2\sin\theta\cos\theta=1$

$\frac{m^2}{\ell^2}-\frac{2.x}{\ell}=1$

$m^2-2x\ell=\ell^2$

$\ell^2-m^2+2x\ell=0$

47. **(A)**

Sum of roots $=2-\sqrt{3}+2+\sqrt{3}=4$

Product of roots $=(2-\sqrt{3})(2+\sqrt{3})$

$=4-3=1$

Required equation $x^2-4x+1=0$

48. **(C)**

Given $\alpha_1-\beta_1=\alpha_2-\beta_2$

$(\alpha_1-\beta_1)^2=(\alpha_2-\beta_2)^2$

$(\alpha_1+\beta_1)^2-4\alpha_1\beta_1=(\alpha_2+\beta_2)^2-4\alpha_2\beta_2$

$p^2-4q=q^2-4p$

$p^2-q^2+4(p-q)=0$

$(p-q)(p+q+4)=0$

$\therefore p+q=-4 \quad \because (p\neq a)$

49. **(B)**

Given $\alpha_1\beta$ be the roots of

$ax^2+bx+c=0$

$ax^2+bx+c=0$ .... (i)

$a\beta^2+b\beta+c=0$ .... (ii)

From (i) & (ii) $2\alpha, 2\beta$ satisfy equation

$ax^2 + 2bx + 4c = 0$

50. **(B)**

coeff of x^2 + coeff of x + constant term $= 0$

$\therefore 1$ is a root of equation hence both roots are real and distinct because $(D \neq 0)$

51. **(A)**

$$(\alpha - \beta)^2 = 16$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 16 \qquad \therefore 9 - 4\alpha\beta = 16$$

$$\frac{-7}{4} = \alpha\beta$$

Required equation

$$x^2 - 3x - 7/4 = 0$$

52. **(A)**

$$yx^2 + yx + y = x^2 + 3x + 1$$

$$(y-1)x^2 + (y-3)x + y - 1 = 0$$

For real x, $D \geq 0$

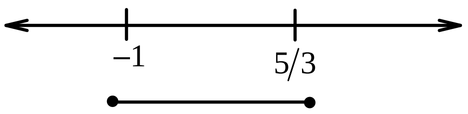
$$(y-3)^2 - 4(y-1)(y-1) \geq 0$$

$$(y-3)^2 - (2y-2)^2 \geq 0$$

$$(y-3+2y-2)(y-3-2y+2) \geq 0$$

$$(3y-5)(-y-1) \geq 0$$

$$(3y-5)(y+1) \leq 0$$



$$y \in [-1, 5/3]$$

53. **()**

54. **(A)**

(Same as Q. No. - 52)

55. **(B)**

Product of roots < 0 then roots are real and opposite in sign

56. **(B)**

Both equation are identical because roots are imaginary

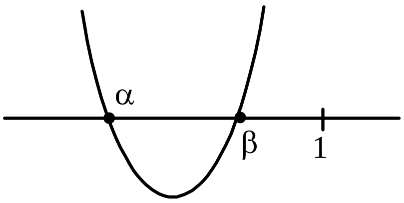
$$\frac{a}{1} = \frac{b}{3} = \frac{c}{5} = \lambda$$

$$a = \lambda, b = 3\lambda, c = 5\lambda$$

$$a + b + c = \lambda.9$$

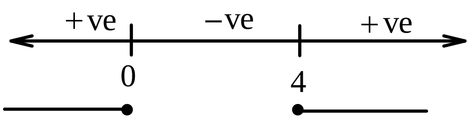
$(a+b+c)_{min} = 9$

57.



(i) $D \geq 0 \quad \therefore a^2 - 4a \geq 0$

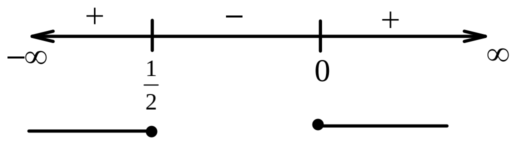
$a(a-4) \geq 0$



$a \in (-\infty, 0] \cup [4, \infty)$ .... (i)

(ii) $a(a+a+1) > 0$

$a(2a+1) > 0$



$a \in \left(-\infty, \frac{-1}{2}\right) \cup (0, \infty)$ .... (ii)

$a \in \left(-\infty, \frac{1}{2}\right) \cup [4, \infty)$ (i) & (ii)

58. (A)

Product of roots $= 30$

Sum of roots $= 11$

$\therefore$ Required equation $x^2 - 11x + 30 = 0$

$x^2 - 5x - 6x + 30 = 0$

$x(x-5) - 6(x-5) = 0$

$x = 5, 6$

59. (B)

$y_{min} = \frac{-D}{4a}$

$= -\frac{(36 + 4.8.3)}{4 \times -8}$

$= \frac{-\cancel{12}^{3}(3+8)}{-\cancel{4} \times 8}$

$= \frac{3 \times 11}{8} = 33/8$

60. (A)

(Same as Q. No. 57)

QUADRATIC EQUATION

EXERCISE-1-C

1. (A)

Given $\alpha+\beta=-2a$

$\alpha\beta=b$

$|\alpha-\beta|\le 2m$

$(\alpha+\beta)^2-4\alpha\beta\le 4m^2$

$4a^2-4b\le 4m^2$

$a^2-m^2\le b$ …….(i)

$D>0$

$4a^2-4b>0$

$\therefore\ a^2>b$ ……..(ii)

From (i) & (ii)

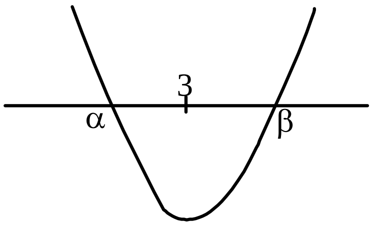
$b\in\left[a^2-m^2,a^2\right]$

2. $D>0$

$(1-2k)^2-4\left(k^2-k-2\right)>0$

$4k^2-4k+1-4k^2+4k+8>0+\ 9>0$

$f(3)<0$



$9+3(1-2k)+\left(k^2-k-2\right)<0$

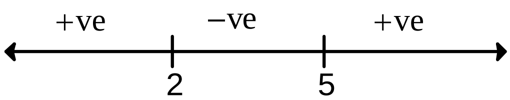
$9+3-6k+k^2-k-2<0$

$k^2-7k+10<0$

$k^2-2k-5k+10<0$

$k(k-2)-5(k-2)<0$

$(k-2)(k-5)<0$



$k\in(2,5)$

3. **No solution**

4. In both equation coefficient of x^2 + coefficient of x + constant term $=0$

$\therefore$ 1 is a root of both equation hence only one root common

5. $\alpha+\beta=-\dfrac{b}{a}$

$\alpha\beta=\dfrac{c}{a}$

We have to find equation whose roots are $2\alpha+3\beta$ and $3\alpha+2\beta$

Sum of roots $= 5\alpha + 5\beta = 5x - \frac{b}{a}$

Product of roots $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$$= 6\alpha^2 + 6\beta^2 + 13\alpha\beta$$

$$= 6\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + B\alpha\beta$$

$$= 6(\alpha+\beta)^2 + \alpha\beta$$

$$= 6.\frac{b^2}{a^2} + \frac{c}{a} = \frac{6b^2 + ac}{a^2}$$

Required equation

$$x^2 + \frac{5b}{a}x + \frac{6b^2 + ac}{a^2} = 0$$

6. $\alpha + \beta = a$

$\alpha\beta = b$

$$\therefore \left.\begin{matrix} \alpha^2 - a\alpha + b = 0 \\ \beta^2 - a\beta + b = 0 \end{matrix}\right\} \qquad \text{\_\_\_\_\_\_\_\_\_\_(i)}$$

$A_n = \alpha^n + \beta^n$

$$A_{n+1} = \alpha^{n+1} + \beta^{n+1} = \alpha^{n-1}.\alpha^2 + \beta^{n-1} + \beta^2$$

$$= \alpha^{n-1}(a\alpha - b) + \beta^{n-1}(a\beta - b)$$

$$= a(\alpha^n + \beta^n) - b(\alpha^{n-1} + \beta^{n-1})$$

$$A_{n+1} = a.A_n - bA_{n-1}$$

7. (C)

$3x^2 - 2x(a+b+c) + (ab+bc+ac) = 0$

$$D = 4(a+b+c)^2 - 4.3(ab+bc+ac)$$

$$= 4(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$= 2\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right] \ge 0$$

8. (A)

Product of roots < 0

$$-\frac{(a^2+1)}{1} < 0$$

$a^2 + a > 0$

9. (D)

$f(n) = (x-a)(x-c) + (x-b)(x-d)$

$f(d) = (b-a)(b-) < 0$

$f(d) = (d-a)(d-c) > 0$

$f(b).f(d) < 0$

One root between $b \,\&\, d$

10. Given

$\alpha+\beta=\alpha^2+\beta^2$ ________(i)

$\alpha\beta=\alpha^2\beta^2$ _________(ii)

$\alpha\beta(\alpha\beta-1)=0$

Either $\alpha=0$ or $\beta=0$

$\therefore\ \beta=0\,\text{or}\,1$ $\alpha=0,\text{or}\,1$

$\alpha\beta=1$

$\therefore\alpha=\dfrac{1}{\beta}$

$\alpha+\dfrac{1}{\alpha}=\alpha^2+\dfrac{1}{\alpha^2}$

$\dfrac{\alpha^2+1}{\alpha}=\dfrac{\alpha^4+1}{\alpha^2}$

$\alpha^3+\alpha=\alpha^4+1$

$\therefore\ \alpha^4-\alpha^3-\alpha+1=0$

$\alpha^3(\alpha-1)-(\alpha-1)=0$

$(\alpha^3-1)(\alpha-1)=0$

$\therefore\alpha=1$

$\therefore\beta=1$

$\left.\begin{matrix}\alpha=0\\\beta=0\end{matrix}\right]$ $\left.\begin{matrix}\alpha=0\\\beta=1\end{matrix}\right]$ $\left.\begin{matrix}\alpha=1\\\beta=0\end{matrix}\right]$ $\left.\begin{matrix}\alpha=1\\\beta=1\end{matrix}\right]$

Total four equation possible

11. (C)

$\alpha+\beta=-P$ $\therefore\alpha\beta=-2$

$\beta+r=-3P$ $\beta r=-4$

$\gamma+\alpha=-6P$ $\gamma\alpha=8$

$2(\alpha+\beta+\gamma)=-10P$

$\alpha+\beta+\gamma=-5P$

$\therefore\ \gamma=-4P$ $\therefore\alpha\beta=-2$

$\alpha=-2P$ $-2P^2=-2$

$\beta=P$ $P=\pm1$

12. (C)

$D_1+D_2=P^2-4q+r^2-4S$

$=p^2+r^2-4^2.\dfrac{pr}{2}$

$D_1+D_2=(p-r)^2\geq0$

$\therefore$ At least one of $D_1\,\&\,D_2\geq0$

13. (B)

$n-2>0$ $\therefore n>2$ _________(i)

$64-4(n-2)(n+4)<0$

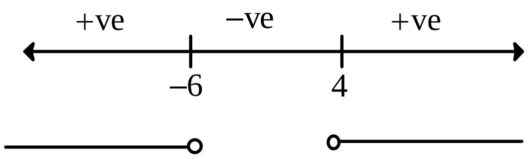
$16-n^2-2n+8<0$

$n^2+2n-24>0$

$n^2 + 6n - 4n - 24 > 0$

$n(n+6) - 4(n+6) > 0$

$(n-4)(n+6) > 0$



$n \in (-\infty, -6) \cup (4, \infty)$ ________(ii)

From (i) & (ii) $n > 4$

14. (A)

Let α be common root

$a\alpha^2 + b\alpha + c = 0$

$c\alpha^2 + b\alpha + a = 0$

$$\frac{\alpha^2}{ab - bc} = \frac{\alpha}{c^2 - a^2} = \frac{1}{ab - bc}$$

$$\alpha = \frac{(a-c)b}{(c-a)(c+a)} = \frac{-b}{a+c}$$

$$\alpha = \frac{(c-a)(c+a)}{-b(c-a)} = \frac{c+a}{-b}$$

$$\therefore \frac{-b}{a+c} = \frac{(c+a)}{-b}$$

$(a+c)^2 = b^2$

$(a+c)^2 - b^2 = 0$

$(a+b+c)(a-b+c) = 0$

15. (A)

$$\gamma = \frac{\alpha}{\beta}$$

$$\frac{\gamma+1}{\gamma-1} = \frac{\alpha+\beta}{\alpha-\beta} = \frac{\alpha+\beta}{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}$$

$$\frac{\gamma+1}{\gamma-1} = \frac{-\frac{b}{a}}{\sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}} = \frac{-\frac{b}{a}}{\frac{\sqrt{b^2 - 4ac}}{a}}$$

$$\frac{(\gamma+1)^2}{(\gamma-1)^2} = \frac{b^2}{b^2 - 4ac}$$

$$(b^2 - 4ac)\gamma^2 + 2\gamma(b^2 - 4ac) + (b^2 - 4ac) = b^2\gamma^2 - 2\gamma.b^2 + b^2$$

$4ac - \gamma^2 - 2\gamma(b^2 + b^2 - 4ac) + 4ac = 0$

$ac\gamma^2 - \gamma(b^2 - 2ac) + ac = 0$

$acx^2 + (2ac - b^2)x + ac = 0$

16. **No solution**

17. $y = \dfrac{1}{4\left(x^2 + 2.\frac{1}{4}x + 1\right)}$

$= \dfrac{1}{4\left(x^2 + 2.\frac{1}{4}x + \frac{1}{16} - \frac{1}{16}\right) + 1}$

$y = \dfrac{1}{4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}}$

$y_{maximum} = \dfrac{4}{3}$

18. (D)

No solution

19. **No solution**

20. (A)

$D = b^2 - 4ac$

$= \dfrac{(4ac + c)^2}{4} - 4ac = \dfrac{16a^2 + c^2 + 8ac - 16ac}{4}$

$D = \dfrac{(4a - c)^2}{4} \geq 0$

Hence roots are real

21. (D)

Let $y = \dfrac{x^2 - x}{1 - ax}$

$y - ayx = x^2 - x$

$x^2 + (ay - 1)x - y = 0$

For real x, $D \geq 0$

$(ay - 1)^2 + 4y \geq 0$

$a^2y^2 - 2ay + 1 + 4y \geq 0$

$a^2y^2 + 2y(2 - a) + 1 \geq 0$ __________(i)

Inequation (i) hold for all real if $D \leq 0$

$4(2 - a)^2 - 4a^2 \leq 0$

$4 + a^2 - 4a - a^2 \leq 0$

$4a \geq 4$

$a \geq 1$

22. (B)

$$y=\frac{x^2-(b+c)x+bc}{(x-a)}$$

$yx-ay=x^2-(b+c)x+bc$

$x^2-(b+c+y)x+bc+ay=0$

For real x, $D\ge 0$

$(b+c+y)^2-4(bc+ay)\ge 0$

$y^2+2y(b+c)+(b+c)^2-4bc-4ay\ge 0$

$y^2+2y(b+c-2a)+(b-c)^2\ge 0$ ________ (i)

Inequation (i) valid only when

$4(b+c-2a)^2-4(b-c)^2\le 0$

$(b+c)^2+4a^2-4a(b+c)-(b-c)^2\le 0$

$4bc+4a^2-4a(b+c)\le 0$

$bc+a^2-ab-ac\le 0$

$b(c-a)-a(c-a)\le 0$

$(b-a)(c-a)\le 0$ _________(ii)

From (ii) $b-a\ge 0$ & $c-a\le 0$

$c\le a\le b$

$b-a\le 0$ & $c-a\ge 0$

$b\le a\le c$

23. (C)

$f(x)=(x+2)^2-3$

$f(-4)=1$

$f(x)\ge 1$ When $x\le -4$

24. (B)

$$y=\frac{x^2-x+c}{x^2+x+2c}$$

$yx^2+xy+2cy=x^2-x+c$

$(y-1)x^2+(y+1)x+(2y-1)c=0$

For real x, $(y+1)^2-4(y-1)(2y-1)c\ge 0$

$y^2+2y+1-4c(2y^2-3y+1)\ge 0$

$(1-8c)y^2+2y(1+6c)+(1-4c)\ge 0$ _______(ii)

(ii) Is valid for all y if

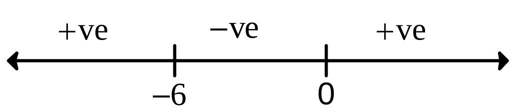
$D\le 0$

$4(1+6C)^2-4(1-8C)(1-4C)\le 0$

$1+36C^2+12C-1+12C-32C^2\le 0$

$4C^2+24C\le 0$

$4C(C+6)\le 0$



$C \in [-6, 0]$

25. (A)

Sum of roots $t - 1 - t - 1 = -\frac{2a}{a+2}$

$-2 = \frac{-2a}{a+2}$

$a + 2 = a$

Not possible for any value of a.

26. $f(n) = ax^2 + 2bx - 3c$

Given $4a + 4b - 3c < 0$

Clearly $f(2) > 0$

It has no real roots and $f(2) > 0$

$\therefore f(n) > 0 \quad \forall x \in R$

$f(0) > 0 \quad \Rightarrow \quad -3C > 0$

$C < 0$

Product of roots $= \frac{-3C}{a} > 0$

$\therefore C < 0$

$a > 0$

27. (B)

Given $mn^2 - mx + 5m + 1 > 0$

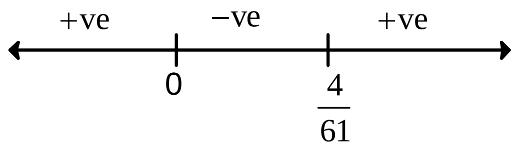
$\forall x \in R$

If $m > 0$

& $81m^2 - 4m(5m+1) < 0$

$81m^2 - 20m^2 - 4m < 0$

$m(61m - 4) < 0$



$m \in \left(0, \frac{4}{61}\right)$

28. (D)

$\tan 3 - \frac{1}{\sqrt{3}}$, $\quad \tan 15^\circ = 2 - \sqrt{3}$

Sum of roots $= \frac{1}{\sqrt{3}} + 2 - \sqrt{3} = -P$

Product of roots $= \frac{2}{\sqrt{3}} - 1 = -q$

$\therefore 2 - p - q = 2 + \frac{1}{\sqrt{3}} + 2 - \sqrt{3} + \frac{2}{\sqrt{3}} - 1$

$= 3 + \sqrt{3} - \sqrt{3} = 3$

29. Let $\alpha \,\&\, 2\alpha$ be the roots of given equation

Sum of roots $= 3\alpha = \dfrac{-(3a-1)}{(a^2-5a+3)}$ ________(i)

Product of roots $= 2\alpha^2 = \dfrac{2}{(a^2-5a+3)}$ ________(ii)

From (i) & (ii) eliminate α we get $a = \dfrac{2}{3}$

30. Let $\alpha \,\&\, \alpha^2$ be the roots of given equation $\alpha + \alpha^2 = \dfrac{6}{8} = \dfrac{3}{4}$

$$\alpha^3 = -\frac{(k+3)}{8}$$

$$(\alpha + \alpha^2) = \frac{27}{64}$$

$$\alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2) = \frac{27}{64}$$

$$-\frac{(k+3)}{8} + \frac{(k+3)^2}{8^2} - 3\frac{(k+3)}{8}.\frac{3}{4} = \frac{27}{8}$$

$$\frac{(k+3)}{8}\left(\frac{k+3}{8} - 1 - \frac{9}{4}\right) = \frac{27}{8}$$

$$(k+3)\left(\frac{k+3}{8} - \frac{13}{4}\right) = 27$$

$$\frac{(k+3)}{4}\frac{(k+3-26)}{2} = 27$$

$$(k+3)(k-23) = 216$$

$$k^2 - 20k - 285 = 0$$

31. (c)

Given $\alpha^2 - 5\alpha + 3 = 0$

$\beta^2 - 5\beta + 3 = 0$

$\therefore \alpha, \beta$ are the roots of equation

$x^2 - 5x + 3 = 0$

$\therefore \alpha + \beta = 5$

$\alpha\beta = 3$

We have to find equation roots are $\dfrac{\alpha}{\beta} \,\&\, \dfrac{\beta}{\alpha}$

Sum of roots $= \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} - 2$$

$$= \frac{25}{3} - 2 = \frac{19}{3}$$

Product of roots $=\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=1$

Required equation $x^2-\frac{19}{3}x+1=0$

$3x^2-19x+3=0$

32. (B)

Coefficient of x^2 + Coefficient of x + Constant term $=0$

$\therefore$ 1 is the roots of given equation let other roots be β

$\therefore$ Product of roots $=\beta.1=\frac{a-b}{b-c}$

$\therefore \beta=\frac{a-b}{b-c}$

33. (B)

Let α,β be the roots of equation

$a_1x^2+b_1x+c_1=0$ and $\frac{1}{\alpha},\frac{1}{\beta}$ are the roots of $c_1x^2+b_1x+a_1=0$ ______(i)

Given $a_2x^2+b_2x+c_2=0$ _______(ii)

$\therefore \frac{c_1}{a_2}=\frac{b_1}{b_2}=\frac{a_1}{c_2}$

34. (A)

Let α be the common root of given equation

$3\alpha^2+a\alpha+1=0$

$2\alpha^2+b\alpha+1=0$

$\frac{\alpha^2}{a-b}=\frac{\alpha}{2-3}=\frac{1}{3b-2a}$

$\therefore \alpha=b-a$

$\& \alpha=2a-3b$

$\therefore 2a-3b=b-a$

$3a=3b$ $\therefore a=b$

$\therefore 5ab-2a^2-3b^2=5a^2-5a^2=0$

35. (C)

Equation whose roots are x_1+d & x_2+d

Replace x by $x-d$ we get required equation

$a(x-d)^2+b(x-d)+c=0$

$a(x^2-2dx+d^2)+bx-bd+c=0$

$ax^2+x(b-2ad)+ad^2+c-bd=0$

Given $px^2+qx+r=0$

$\frac{q}{p}=\frac{b-2ad}{q}=\frac{ad^2+c-bd}{r}$

$\therefore \frac{aq}{p}=b-2ad$

$$2ad = b - \frac{aq}{p}$$

$$\therefore d = \frac{1}{2ap}(pb - aq)$$

36. (B)

$f(-1) = a - b + c < 0$

$a + c < b$ (Given)

And roots are imaginary then

$f(n) = ax^2 + bx + c < 0 \quad \forall x \in R$

$f(-2) = 4a - 2b + c < 0$

$\therefore 4a + c < 2b$

37. (A)

Given $x^2 + 2ax + a < 0$

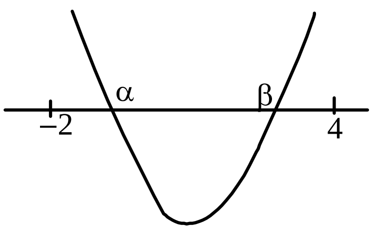
$\forall x \in [1, 2]$

$f(1) < 0 \quad \Rightarrow a < -\frac{1}{3}$ ________(i)

$f(2) < 0 \quad \Rightarrow a < -\frac{4}{5}$ _________(ii)

From (i) & (ii) $a \in \left(-\infty, -\frac{4}{5}\right)$

38. (A)



(i) $D \geq 0$

$4m^2 - 4(m^2 - 1) \geq 0$

$1 \geq 0$

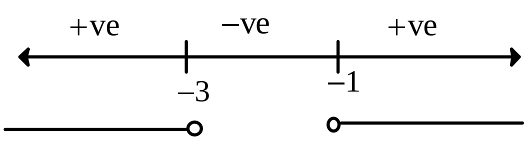
(ii) $f(-2) > 0$

$4 + 4m + m^2 - 1 > 0$

$m^2 + m + 3m + 3 > 0$

$m(m+1) + 3(m+1) > 0$

$(m+1)(m+3) > 0$



$f(4) > 0$

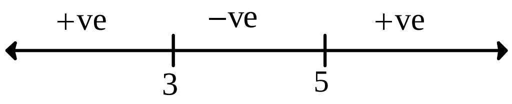
$16 - 8m + m^2 - 1 > 0$

$m^2 - 8m + 15 > 0$

$m^2 - 3m - 5m + 15 > 0$

$m(m-3) - 5(m-3) > 0$

$(m-3)(m-5) > 0$



$m \in (-\infty, 3) \cup (5, \infty)$ __________(ii)

$-2 < +\frac{2m}{2} < 4$

$-2 < m < 4$ __________(iii)

$m \in (-1, 3)$

From (i), (ii), & (iii)

39. (C)

Let $p = 2k + 1$

$q = 2m + 1$

$D = 4p^2 - 4.2q$

$= 4\left[(2k+1)^2 - 2(2m+1)\right]$

$= 4\left(4k^2 + 4k + 1 - 4m - 2\right)$

$= 4\left(4k^2 + 4(k-m) - 1\right)$

Clearly $4k^2 + 4(k-m) - 1$ is odd if D is a perfect square $4k^2 + 4(k-m) - 1 = (2n+1)^2$

$4k^2 + 4(k-m) - 4n^2 - 4n = 2$

Which is not possible

$\therefore$ D can not be a perfect square hence root can not be a rational

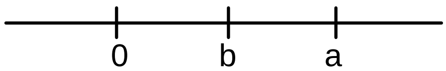
40. (A)

Given $a + b + 2c = 0$

$D = b^2 - 4ac = a^2 + 8ac + 36c^2$

$= (a+4c)^2 + 20c^2 > 0$

41. (A)



$f(n) = 2n^2 - an - b^2$

$f(a) = 2a^2 - a^2 - b^2 = a^2 - b^2 > 0$

$f\left(-\frac{a}{2}\right) = 2.\frac{a^2}{4} + \frac{a^2}{2} - b^2$

$= \left(a^2 - b^2\right) > 0$

$f(0) < 0$

$\therefore$ Both roots lies between $-\frac{a}{2}$ & a

42. (C)

Let $y=\frac{(x+m)^2-4mn}{2(x-n)}$

$2xy-2ny=x^2+m^2+2mx-4mn$

$x^2+2x(m-y)+2n+m^2(y-2m)=0$

For real x,

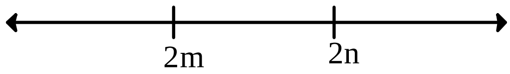
$D\ge 0$

$4(m-y)^2-4\left(2n(y-2m)+m^2\right)\ge 0$

$m^2+y^2-2my-2ny+4mn\ge 0$

$y^2-2y(m+n)+4mn\ge 0$ ______(i)

$y=2m,2n$



If $m<4$

(i) Satisfy if y can not lies between 2m and 2n

43. (A)

Given $\alpha+\beta=-\frac{b}{a}$

$\alpha\beta=\frac{c}{a}$

Product of roots $=\frac{\alpha}{\beta}.\frac{\beta}{\alpha}=\frac{\gamma}{p}$

$\therefore p=\gamma$

44. (C)

$y+yx^2=2x$

$yx^2-2x+y=0$

For real x

$D\ge 0$

$4-4.y^2\ge 0$ $\therefore y^2\le 1$

$|y|\le 1$

$y\in[-1,1]$

Let $\lambda=y^2+y-2=y^2+2.\frac{1}{2}y+\frac{1}{4}-\frac{1}{4}-2$

$=\left(y+\frac{1}{2}\right)^2-\frac{9}{4}$

$\therefore -1\le y\le 1$

$-\frac{1}{2}\le y+\frac{1}{2}\le\frac{3}{2}$

$0\le\left(y+\frac{1}{2}\right)^2\le\frac{9}{4}$

$-\frac{9}{4}\le\left(y+\frac{1}{2}\right)^2-\frac{9}{4}\le 0$

$\therefore y^2+y-2\in\left[-\frac{9}{4},0\right]$

45. $D = a^2 - 4(a+1)$

$= a^2 - 4a + 4 - 8$

$= (a-2)^2 - 8$

46. (C)

$D \geq 0$

$(a-3)^2 - 4a \geq 0$

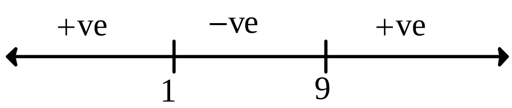
$a^2 + 9 - 6a - 4a \geq 0$

$a^2 - 10a + 9 \geq 0$

$a^2 - a - 9a + 9 \geq 0$

$a(a-1) - 9(a-1) \geq 0$

$(a-1)(a-9) \geq 0$



$a \in (-\infty, 1] \cup [9, \infty)$ ________(i)

$f(2) > 0$

$\therefore 4 - 2(a-3) + a > 6$

$4 - 2a + 6 + a > 0$

$10 > a$

$2 > \frac{(a-3)}{2}$ $\therefore a < 7$

Condition for both root less than 2

$m \in (-\infty, 1]$

Required condition for at least one root one roots greater than 2

$m \in [9, \infty)$

47. **Not Solution**

48. (D)

$\alpha + \beta = m$

$\alpha\beta = n$

$(\alpha^2 + \alpha + 1)(\beta^2 + \beta + 1) = (m\alpha - n + \alpha + 1)(m\beta - n + \beta + 1)$

$= [(1-n) + \alpha(1+m)][(1-n) + \beta(1+m)]$

$= (1-n)^2 + (1+m)(1-n)(\alpha+\beta) + (1+m)^2 \alpha\beta$

$= (1-n)^2 + (1+m)(1-n).m + (1+m)^2 .n$

$= (1-n)^2 + (1+m)(m - mn + n + mn)$

$= 1 + n^2 - 2n + m + n + m^2 + mn$

$= 1 + (m-n) + m^2 + n^2 + mn$

49. (A)

Let root of equation be $\alpha \,\&\, \alpha^2$

$\alpha + \alpha^2 = -\frac{b}{a}$

$\alpha^3 = \frac{c}{a}$

$\left(\alpha+\alpha^2\right)^3 = \left(-\frac{b}{a}\right)^3$

$\alpha^3+\alpha^6+3\alpha^3\left(\alpha+\alpha^2\right) = -\frac{b^3}{a^3}$

$\frac{c}{a}+\frac{c^2}{a^2}-3\frac{c}{a}.\frac{b}{a} = -\frac{b^3}{a^3}$

$\frac{ac+c^2-3bc}{a^2} = -\frac{b^3}{a^3}$

$\therefore a^2x+ac^2-3abc = -b^3$

$\therefore b^3+ac^2+a^2c = 3abc$

50. $\alpha+\beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$

$x = \frac{-abc \pm \sqrt{a^2b^2c^2-4a^3c^3}}{2a^3}$

$= \frac{-abc \pm ac\sqrt{b^2-4ac}}{2a^3}$

$= \frac{c}{a}\left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$

$x = \alpha\beta.\alpha, \alpha\beta.\beta$

Required roots $\alpha^2\beta$ & $\beta^2\alpha$

51. (C)

Given

$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$

$\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1} = \frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}$

$\frac{\left(\alpha_1+\beta_1\right)^2}{\left(\alpha_1+\beta_1\right)^2-4\alpha_1\beta_1} = \frac{\left(\alpha_1+\beta_2\right)^2}{\left(\alpha_2+\beta_2\right)^2-4\alpha_2\beta^2}$

$\frac{b^2}{b^2-4c} = \frac{q^2}{q^2-4r}$

$\therefore b^2q^2-4rb^2 = b^2q^2-4cq^2$

$rb^2 = cq^2$

52. (A)

Given

$a > 0$

$a+b+c > 0$

$4a+2b+c > 0$

Or $a < 0$

$a+b+c < 0$

$4a+2b+c < 0$

$a\left(a+4a+2b+c\right) = a^2+a\left(4a+2b+c\right) > 0$

$(\forall a)$

$\therefore a \,\&\, (5a+2b+c)$ have opposite sign.

53. **Not solution**

54. (B)

Let roots of equation $x^2 - px + q = 0$ be $\alpha \,\&\, \beta$ & roots of equation $x^2 - an + b = 0$ be $\alpha \,\&\, \alpha$

$\alpha + \alpha = a$

$\therefore \frac{a}{2}$ __________(i)

$a^2 = 4b$ _________(ii)

$\frac{a^2}{4} - p.\frac{q}{2} + q = 0$

$a^2 - 2ap + 4q = 0$

$4b - 2ap + 4q = 0$ (from (ii))

$4(b+q) = 2ap$

$\therefore ap = 2(b+q)$

55. $x^2 - 8x + 4y^2 + 12 = 0$

For real $x, D \ge 0$

$64 - 4(4y^2 + 12) \ge 0$

$16 - 4y^2 - 12 \ge 0$

$4 \ge 4y^2$

$\therefore y^2 \le 1$

$|y| \le 1$

$-1 \le y \le 1$

56. (C)

Given $\frac{a}{2} = \frac{b}{-3} = \frac{c}{4}$

$\therefore 6a = -4b = 3c$

57. (D)

Given $\alpha + \beta = p, \alpha\beta = q$

$$\alpha + \beta = \left(\alpha^{\frac{1}{2}}\right)^2 + \left(\beta^{\frac{1}{2}}\right)^2 = \left(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}\right)^2 - 2\alpha^{\frac{1}{2}}.\beta^{\frac{1}{2}}$$

$$p = \left(\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}}\right)^2 - 2\sqrt{q}$$

$$\alpha^{\frac{1}{2}} + \beta^{\frac{1}{2}} = \left(\alpha^{\frac{1}{4}}\right)^2 + \left(\beta^{\frac{1}{4}}\right)^2 = \left(\alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}}\right)^2 - 2\alpha^{\frac{1}{4}}\beta^{\frac{1}{4}}$$

$$\left(\sqrt{p + 2\sqrt{q}} + 2.q^{\frac{1}{4}}\right)^{\frac{1}{2}} = \alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}}$$

$$\therefore\left(\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}\right)=\left(p+2\sqrt{q}+4q^{\frac{1}{2}}+4q^{\frac{1}{4}}\left(p+2\sqrt{q}\right)\right)^{\frac{1}{4}}$$

$$\alpha^{\frac{1}{4}}+\beta^{\frac{1}{4}}=\left[p+6\sqrt{q}+4q^{\frac{1}{4}}\left(p+2\sqrt{q}\right)\right]^{\frac{1}{4}}$$

$$\therefore k=\frac{1}{4}$$

58. No solution

59. (A)

Let $x^2+n+1=t$

$$\left(t+1\right)^2-\left(a-3\right)+\left(t+1\right)+\left(a-4\right)t^2=0$$

$$t^2\left(1+a-4-a+3\right)+t\left(2-a+3\right)+1=0$$

$$t\left(5-a\right)+1=0$$

$$x^2+n+1=\frac{1}{a-5}$$

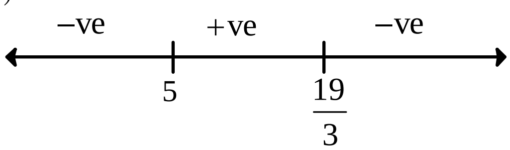
We know minimum value of x^2+n+1 equals $=\frac{3}{4}$

$$\therefore\frac{1}{a-5}>\frac{3}{4}$$

$$\frac{1}{a-5}-\frac{3}{4}>0$$

$$\frac{4-3a+15}{4\left(a-4\right)}>0$$

$$\frac{\left(19-3a\right)}{4\left(a-5\right)}>0$$



$$a\in\left(5,\frac{19}{3}\right)$$

60. (C)

$$-1\le\frac{x^2+nx-2}{x^2-3x+4}\le 2$$

$$x^2+nx-2\ge -x^2+3n-4$$

$$2x^2+\left(n-3\right)x+2\ge 0$$

$$\therefore D\le 0$$

$$\left(n-3\right)^2-4.2.2\le 0$$

$$\left(n-3\right)^2\le 4^2$$

$|n-3| \le 4$

$-4 \le n-3 \le 4$

$-1 \le n \le 7$ ________(i)

Similarly

$x^2 + nx - 2 \le 2x^2 - 6x + 8$

$x^2 - x(6+n) + 10 \ge 0$

$D \le 0$

$(6+n)^2 - 4.10 \le 0$

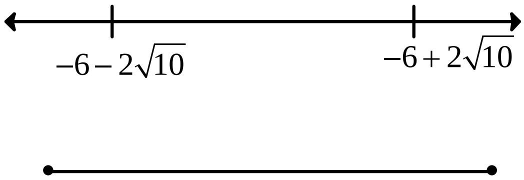
$36 + n^2 + 12n - 40 \le 0$

$n^2 + 12n - 4 \le 0$

$$n = \frac{-12 \pm \sqrt{144+16}}{2}$$

$$n = \frac{-12 \pm 4\sqrt{10}}{2}$$

$n = -6 \pm 2\sqrt{10}$



$n \in \left[-6, -2\sqrt{10}, -6+2\sqrt{10}\right]$ __________(ii)

From (i) & (ii)

$n \in \left[-1, \sqrt{40}-6\right]$

QUADRATIC EQUATION

EXERCISE – 2(A)

1. x – 2 is a factor

$\Rightarrow\ x = 2$ is a root

$\Rightarrow\ 8 - 4k + k^2 - 5 = 0$

$\Rightarrow\ k^2 - 4k + 3 = 0$

$k = 1,\ k = 3$

2. Let

$p(x) = (x+1)Q(x) + 6$

(where 6 is the remainder)

$\Rightarrow p(-1) = 6 \quad \Rightarrow \quad -a + b + 12 - 5 = 6$

$\Rightarrow \quad b - a = -1$ ….(1)

Also $p(1) = 0$ (x – 1 factor)

$a + b - 12 - 5 = 0$

$\Rightarrow a + b = 17$ ….(2)

Solve (1) & (2)

b = 8 , a = 9

3. (a) 43 a is non divisible by 4

But it is even no.

$\Rightarrow$ either x & y both even

Or x & y both odd

In either care $x^2 - y^2$ is a multiple

Of 4 $\therefore$ not possible

(b) $2^a - 2^b = 31$

$b \neq 0$

$\therefore$ Bothe no. even

$\therefore$ all hence must also be even

But if given 31 $\Rightarrow$ contradiction

Hence over a = 5, b = 0 solution

(c) $(2k+1)^2 = 4k^2 + 4k + 1$

$= 4k(k-1) + 1$

$= 8I + 1$

(k (k+1) has to be even 2 I)

(d) 2017 when divided by 8 leaves remainder 1 $\therefore (2017)^{2016}$ also leaves remainder $1^{2016} = 1$

4. **(AC)**

According to question

$t(0)f(2) < 0$

$\Rightarrow\ (k^2+5)(k^2+2k-3) < 0$

$(k+3)(k-1) < 0$

$-3 < k < 1$

Option (A) & (C)

5. $\sin^2\theta = \frac{4}{\sin^3\theta - 1} = 1 - \frac{4}{\sin^3\theta - 1}$

$\Rightarrow \sin^2\theta = 1$

Or $\sin\theta = 1$ or $\sin\theta = -1$

Discard

Does not satisfy

Original equation

$\Rightarrow$ A, B, D

6. By putting x = 0, x = p & x = q

In Expiration we get LHS = k of s

$\Rightarrow$ expression is an identify in x

$\Rightarrow$ A, B, C

7. (a) $f(x) \geq 0$

$\Rightarrow D \leq 0$

$64k^2 - 16k \leq 0$

$k(4k-1) \leq 0$

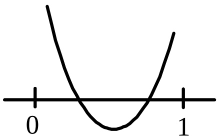
$\left[0, \frac{1}{4}\right]$

(b) $t(0) = k$

$\therefore k < 0 \quad \Rightarrow \quad t(0) < 0$

$\Rightarrow$ 0 lies between roots

(c) Both roots between 0,1)



(i) $D > 0 \Rightarrow k \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$

(ii) $t(0) > 0 \Rightarrow k > 0$

(iii) $t(1) > 0 \Rightarrow k < \frac{4}{7}$

$\therefore (i) \cap (ii) \cap (iii) \Rightarrow \left(\frac{1}{4}, \frac{4}{7}\right)$

(d) min value is $\frac{-D}{4a} \Rightarrow k - \frac{64k^2}{16}$

$k(1-4k)$

8. $x = 2 = \frac{-b}{2a}$

$10 = -\frac{D}{4a} = 8 - \frac{b^2}{4a}$

Solve to get $a = \frac{-1}{2}$, $b = 2$

9. (a) $y = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$

$yx^2 + 2yx + 3y = x^2 + 14x + 9$

$(y-1)x^2 + 2(y-7)x + 3y - 9 = 0$

$x \in R \Rightarrow D \geq 0$

$4(y-7)^2 - 4(g-1)(3y-9) \geq 0$

$y^2 - 14y + 49 - (39^2 - 12y + 9) \geq 0$

$-2y^2 - 2y + 40 \geq 0$

$y^2 + y - 20 \leq 0$

y can 10 interval values

$y \in [-5, 4]$

(b) there are 17 integral values

By graph

(c) put y = – 5 we get

$x^2 + 4x + 4 = 0$

$\Rightarrow x = -2$

(d) 2017 is beyond value of expiration

10. **(AB)**

If y Real $D \geq 0$

$16x^2 - 4 \times 4(x+6) \geq 0$

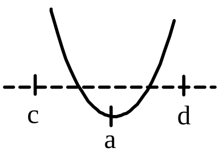
$x^2 - x - 6 \geq 0$

$(x-3)(x+2) \geq 0$

$x \in [-2, 3]$

11. vector is $(a, b - a^2)$

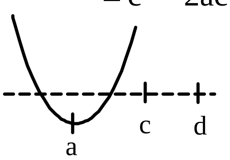
(a) of c < a < d $\Rightarrow$ min is $b - a^2$



(b) of a < c < d

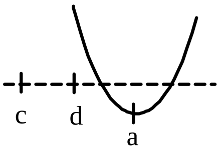
$\therefore$ min is f (c)

$= c^2 - 2ac + b$



(c) $\therefore f(\min) = f(d)$

$d^2 - 2ad + b$



(d) No comment

12. $x^3+1=0 \Rightarrow -1,-\omega,-\omega^2$

$\therefore p(-1)=0 \quad \& \quad p(-\omega)=0 \quad \& \quad p(-\omega^2)=0$

$\Rightarrow f_1(+1)-f_2(+1)+f_3(1)=0$

$\& \quad f_1(1)-\omega f_2(1)+\omega^2 f_2(1)=0$

$\& \quad f_1(1)-\omega^2 f_2(1)+\omega f_2(1)=0$

$\Rightarrow f_1(1)=f_2(1)=f_3(1)=0$

13. **(AC)**

α,β roots $\Rightarrow x^2-px+9=0$

Of $\frac{1}{\alpha},\frac{1}{\beta}$ roots $\Rightarrow 9x^2-px+1=0$

$\Rightarrow (x^2-px+9)(9x^2-px+1)=0$

has roots $\frac{1}{\alpha},\frac{1}{\beta},\alpha,\beta$

but equation is given as $x^4-ax^3+bx^2-ax+1=0$

comparing

$$\frac{q}{1}=\frac{-p(q+1)}{-a}=\frac{p^2+q^2+1}{b}=\frac{p(q+1)}{a}=\frac{q}{1}$$

$\Rightarrow a=\frac{p(q+1)}{q}$

$\& \quad b=\frac{p^2+q^2+1}{q}$

14. Clearly $a<0$ & $c>0$ (y intercept)

Also $-\frac{b}{2a}>0$

$\Rightarrow b>0$

(a) correct

Hence $t(2)>0 \Rightarrow 4a+2b+c>0$ …(1)

$\Rightarrow$ (c) correct

Also $f(1)>0 \Rightarrow a+b+c>0$ …(ii)

Add (i) & (ii)

$\Rightarrow 4a+3b+2c>0$ (c)

15. $t(1)+t(2)+t(3)=0$

$\Rightarrow$ at least one out of t (1), t(2) or t(3) is negative or even

$\Rightarrow$ (a) option correct

(b) $b+2a=0 \Rightarrow t(2)=c=f(0)$

$\Rightarrow$ by symmetry about vector $t(1)=t(3)$

$t(2)=0 \qquad \Rightarrow t(2)+f(3)=0$

$t(2)\,\&\,t(3)$ opp signed

$\therefore$ one root between 2 & 3

(c) wrong

(d) $t(3)=0 \Rightarrow t(1)\,\&\,t(2)$ opp signed

$\therefore$ other root between 1 & 2
(d) correct

16. Apply one root common condition & get the answer

17. $x^2-(b+1)x+b-2=0$

$D=(b+1)^2-4(b-2)$

$D=(b-1)^2+8\ >0$

$\therefore$ roots real
Now, if b > 2
$\therefore$ S_2 is positive & $S_1>0$
$\Rightarrow$ both roots positive
Of b = 2 are root is 0
Of b < 2 $\therefore$ $S_2<0$
$\Rightarrow$ roots opp singed
$\therefore$ Any value of b at least one root is non-negative
$\Rightarrow$ A & B also correct

18. clearly $\sin\theta$ and $\cos\theta$ roots $\Rightarrow$ they are real $\Rightarrow$ they are real $\Rightarrow$(a) correct
Now

$\sin\theta+\cos\theta=\dfrac{-b}{a}$ $\qquad\qquad$ $\sin\theta\cos\theta=\dfrac{c}{a}$

Or $1+2\sin\theta\cos\theta=\dfrac{b^2}{a^2}$ $\qquad\qquad$ $\sin^2\theta=\dfrac{2c}{a}$

$\Rightarrow$ b correct

$1+\dfrac{2c}{a}=\dfrac{b^2}{a^2}$

$\Rightarrow \dfrac{a+2c}{\sqrt{2}b}=\dfrac{b}{\sqrt{2}a}=\dfrac{(\sin\theta+\cos\theta)}{-\sqrt{2}}$

= C also correct

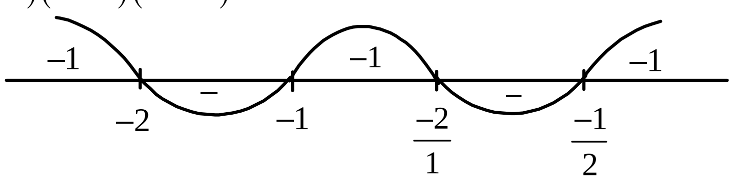
19. **(BC)**

$\dfrac{2x}{2x^2+5x+2}-\dfrac{1}{x+1}>0$

$\dfrac{2x^2+2x-2x^2-5x-2}{(x+1)(2x+1)(x+2)}>0$

$-\dfrac{(3x+2)}{(x+1)(x+2)(2x+1)}>0$

$\Rightarrow$



$\Rightarrow(-2,-1)\cup\left(-2/3,\ -1/2\right)$

20. obvious range question

21. **(ABC)**

$x^2+px+qr=0 \quad \ldots(1)$

$\alpha,\beta \quad \Rightarrow \alpha+\beta=-p$

$x^2+qx+rp=0 \quad \ldots(2)$

$\beta, y \quad \Rightarrow \beta+y=-g$

$x^2+rx+pq=0 \quad \ldots(3)$

$y,\alpha \quad \Rightarrow y+\alpha=-r$

Apply 1 root common condition

$(rp-qr)^2=(q-p)(p^2-q^2)r$

$\Rightarrow r^2(p-q)^2=-(p-q)^2(p+q)r$

$\Rightarrow p+q+r=0$

$\therefore \alpha+\beta+q=-\frac{(p+q+r)}{2}=0$

22. **(ABC)**

$(p^2-4p+5)x^2+(2p-1)x-3p=0$

Clearly well $x^2>0$ & $S_2<0$

$\Rightarrow$ Both roots opp signed

23. clearly sum of coefficient is 0

$\Rightarrow$ exactly are root is 1

Let other root be α

Here $b+c-2a<0$ $\quad [a>b>c]$

$\Rightarrow S_2<0 \quad \Rightarrow$ (A) correct & (D)

(B) let $-1<\alpha<0 \Rightarrow \alpha+1>0$

$\Rightarrow \frac{a+c-2b}{a+b-2c}>0 \quad (a+b-2c>0)$

$\Rightarrow a+c-2b>0$

(B) correct

(C) if $\alpha<-1$

$\Rightarrow \alpha+1<0 \Rightarrow a+c<2b$

(C) correct

24. As done is precious question are root is 1 & other root α

$\therefore$ given $\alpha+1>0 \Rightarrow a+c<b$

(A) D = $4b^2-4ac$

Given 2b > a + c

$\Rightarrow 4b^2-4ac>(a-c)^2>0 \Rightarrow$ A correct

(B) D = $4a^2-4bc$

Definitely (+) ve

(C) D = $4c^2-4ab$

Definitely (+) ve

Incorrect

$\therefore$ Also in A & B S_1 positive & $S_2>0$

$\Rightarrow$ roots both repative

25. $x^2-2ax+ab=0$

(roots real & positive)

$D = 4a^2 - 4ab$

$\Rightarrow 4a(a-b) > 0 \Rightarrow a > b$

$S_1 > 0$ & $S_2 > 0$ $\Rightarrow \frac{b}{a} < 1$

$\Rightarrow a > 0, \ b > 0$

$\Rightarrow$ option A correct

(B) $t(0)t(b) = ab(b^2 - ab) < 0$

B correct

(C) $t(2a) + (2a-b) = ab(b^2 - ab) < 0$

$\Rightarrow$ also correct

(D) since $a \neq b$ D is positive

Both roots definite

26. Let I be integral root

$(I+2010)(I+9) = -1$

$\Rightarrow$ I + 2010 = 1 & I + a = – 1 …(i)

Or I + 2010 = – 1 & I + a = 1 …(ii)

$\Rightarrow a = 2008$ or 2012

27. use transformation

28. **(AC)**

roots of equation as $\frac{3}{5} \& \frac{-4}{5}$

But given $-1 < x < 0 \Rightarrow$ only $\frac{-4}{5}$

$\therefore \cos\alpha = \frac{-4}{5} \Rightarrow \sin\alpha = 1\frac{3}{5}$

$\therefore \sin 2\alpha = 2 \sin\alpha \cos\alpha$

$= \pm\frac{24}{25}$

29. Put D = 0 & get answer

30. $y = \frac{x^2 + 2x - 11}{x - 3}$

$yx - 3y = x^2 + 2x - 11$

$x^2 + (2-y)x + 3y - 11 = 0$

$D \geq 0 \quad (y-2)^2 4(3y-11) \geq 0$

$y^2 - 4y + 4 - 12y + 44 \geq 0$

$y^2 - 16y + 48 \geq 0$

$(-\infty, 4] \cup [12, \infty)$

a = 4, b = 12

31. **(BC)**

$a + b + c = 0 \Rightarrow 1$ root is one

$\therefore$ if α is another roots

$\Rightarrow\ 1\cdot\alpha = \frac{c}{a} = \text{irrational}$

$\therefore\ \alpha$ irrational

32. Put values & verify

33. **(ABC)**

simplify the equation

$t(x) = a(x-b)(x+c) + b(x-a)(x+c) - c(x-a)(x-b)$

Here $S_2 = \frac{-3abc}{a+b+c} < 0$

$\Rightarrow$ roots real & opp signed

Also $t(b)\cdot t(a)$

$= b(b-a)(b+a)(a)(a-b)(a+c)$

Which is positive

34. **(AB)**

clearly a b c < d (q < b < c < d)

$\therefore S_2 = \frac{abc-d}{a} < 0$

(c) $t(a)t(b)$ no comment on if signed

(d) $t(b)t(c)$ no comment on if singed

$\Rightarrow$ cant say for sure C & D correct

35. $x^2 + (a-b-1)x - (a+b) = 0$

$x \in \text{real} \Rightarrow D > 0$ & destine

$(a-b-1)^2 + 4(a+b) > 0 \quad \forall \in R$

$a^2 + b^2 + 1 - 2ab - 2a + 2b + 4a + 4b > 0 \qquad \forall b \in R$

$b^2 - 2b(a-3) + a^2 + 2a + 1 > 0$

$\Rightarrow$ only possible of D < 0

$4(a-3)^2 - 4(a^2 + 2a + 1) < 0 \Rightarrow a > 2$

36. **(AC)**

$x^2 - 4x + 3 + 9y^2 = 0$

$\Rightarrow D \geq 0$ (x Real)

$16 - 4(3 + 9y^2) \geq 0$

$\Rightarrow -\frac{1}{3} \leq y \leq \frac{1}{3}$

Also, $y^2 = -\frac{(x^2 - 4x + 3)}{9}$

y real $x^2 - 4x + 3 \leq 0$

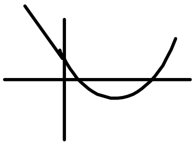
$x \in [1, 3]$

37. for all roots to be real

Both roots of $ax^2 + bx + c = 0$ must be positive

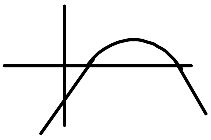
(i) $a > 0$

$\Rightarrow b < 0$ & $c > 0$



(ii) $a < 0$

$\Rightarrow b > 0$ & $C < 0$



38. **(BC)**

observe C > 0 (y intercept)

a < 0 (concave dorm)

& $\frac{-b}{2r} < 0$ or $\frac{b}{Er} > 0$ or $b < 0$

39. **(BD)**

$ax^2 + bx + c = 0$ has no real roots

$\Rightarrow$ it has same sign throughout

Now $t(1)$ & $t(0)$ and a have same sign

40. $x^2 - 3x + 2m = 0$ of $x^2 - x + m = 0$

Has 2α root has root α

$\Rightarrow 4\alpha^2 - 6\alpha + 2m = 0$ & $\alpha^2 - \alpha + m = 0$

$\therefore$ Apply are root common condition

m = 0 or – 2

QUADRATIC EQUATION
EXERCISE –2(B)

PASSAGE – I

1. (B) 2. (B)

Let α, β be the roots of equation $x^2 + ax + b = 0$

We have to find equation whose roots are $\alpha + k, \beta + k$ replace x by x – k we get

$(x - k)^2 + a(x - k) + b = 0$

$x^2 + x(a - 2k) + k^2 - ak + b = 0$ …(i)

Given,

$x^2 + bx + a = 0$ … (ii)

Equation (i) & (ii) are identical

$$\frac{1}{1} = \frac{a-2k}{b} = \frac{k^2 - ak + b}{a}$$

$$\therefore k = \frac{a-b}{2} \;\&\; k^2 - ak + b = a$$

$$\frac{(a-b)^2}{4} - a\frac{(a-b)}{2} = (a-b)$$

$(a - b)\,[(a - b) - 2a - 4] = 0$

For $a = b = a + b + 4 = 0$

$k = 0$

$ab = \lambda$

$\lambda = a(-4 - a)$

$\lambda = -4a - a^2$

$\lambda = 4 - (a+2)^2$

$(a + b)^2$ always positive values of $(a + 2)^2 = 0$

At $a = -2$

$\lambda_{max} = 4$

$\lambda \in (-\infty, 4)$

PASSAGE – 2

3. (C)

$D = 2^2 - 4.3 < 0$

Equation $x^2 + 2x + 3 = 0$ have both roots imaginary and hence both roots common

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

$\therefore$ a : b : c = 1 : 2 : 3

4. (C)

$x^3 + 3x^2 + 3x + 1 + 1 = 0$

$(x+1)^3 = -1$

$\therefore x = -2$

$\therefore x^3 + 2x^2 + x^2 + 2x + x + 2 = 0$

$x^2(x+2) + x(x+2) + (x+2) = 0$

$(x+2)(x^2+x+1)=0$

Equation $ax^2+bx+c=0$ and $x^2 + x + 1 = 0$ have both roots common.

$\therefore \frac{a}{1}=\frac{b}{1}=\frac{c}{1}$ $\therefore$ a = b = c

5. (A)

$(6k + 2)x^2 + rx + 3k - 1 = 0$ …(i)

$(12k + 4)x^2 + px + 6k - 2 = 0$ …(ii)

Equation (i) & (ii) have both roots common

$\therefore \frac{4(3k+1)}{2(3k+1)}=\frac{p}{r}=\frac{4(3k-1)}{2(3k-1)}$

P = 2r $\therefore$ 2r – P = 0

6. (D)

Let α be common root

$\alpha^2-a\alpha+b=0$

$\alpha^2+b\alpha-a=0$

$\frac{\alpha^2}{a^2-b^2}=\frac{\alpha}{b+a}=\frac{1}{b+a}$

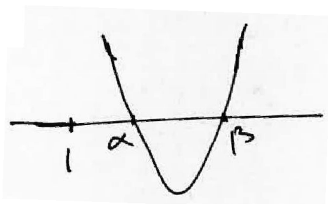
$\alpha=\frac{(a-b)(a+b)}{(a+b)}$ $\therefore \alpha=1$

$\alpha=a-b$

$\therefore$ a – b $\therefore$ a – b = 1

PASSAGE – 3

7. (A)



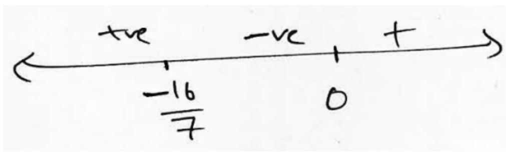
$D\geq 0$

$9a^2-4(a+1).4a\geq 0$

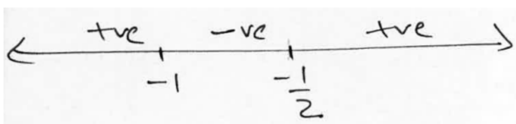
$9a^2-16a^2-16a\geq 0$

$-7a^2-16a\geq 0$

$a(7a+16)\leq 0$



$a\in\left[-\frac{16}{7},0\right]$



(a + 1) f (1) > 0

(a + 1) (a + 1 – 3a + 4a) > 0

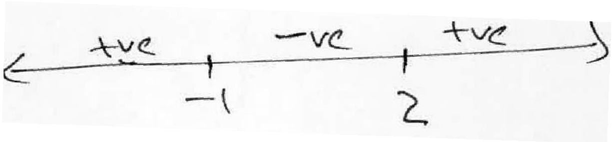
(a + 1) (2a + 1) > 0

$a\in(-\infty,-1)\cup\left(-\frac{1}{2},\infty\right)$ …(ii)

$\frac{3a}{2(a+1)}>1$ $\therefore\left(\frac{3a}{2a+2}-1\right)>0$

$$\frac{(3a-2a-2)}{2(a+1)} > 0$$

$$\frac{(a-2)}{2(a+1)} > 0$$

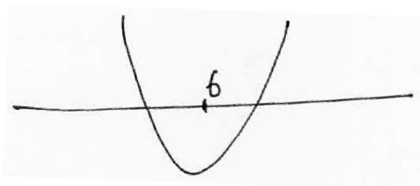


$a \in (-\infty, -1) \cup (2, \infty)$ … (iii)

From (i), (ii), (iii)

$$a \in \left[-\frac{16}{7}, -1\right)$$

8. (B)



1.f(6) < 0

$36 + 2(a-3).6 + 9 < 0$

$12 + 4(a-3) + 3 < 0$

$4a + 3 < 0$

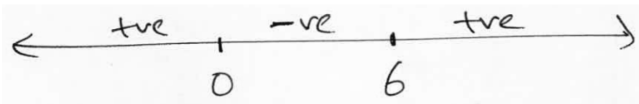
$a < -3/4$ … (i)

$D > 0$

$4(a-3)^2 - 4.9 > 0$

$a^2 + \cancel{9} - 6a - \cancel{9} > 0$

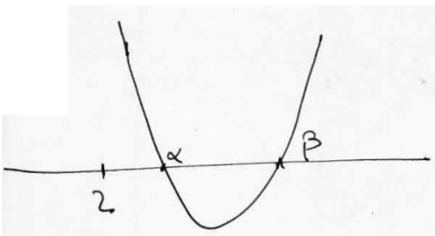
$a(a-6) > 0$



$a \in (-\infty, 0) \cup (6, \infty)$ … (ii)

From (i) & (ii) $a < -3/4$

9. (B)



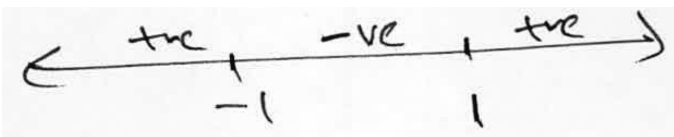
$D \geq 0$

$4a^2 + 4(1-a^2) \geq 0$

$\cancel{a^2} + 1 - \cancel{a^2} \geq 0$

$(1-a^2)f(0) > 0 \quad \therefore -(1-a)(1+a) > 0$

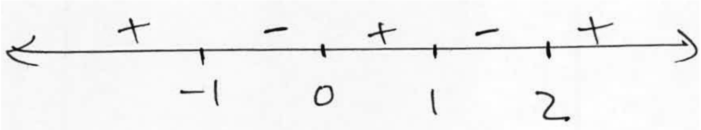
$(a-1)(a+1) > 0$



$a \in (-\infty, -1) \cup (1, \infty)$

$(1-a)(1+a)(\cancel{1} - a^2 + 2a - \cancel{1}) > 0$

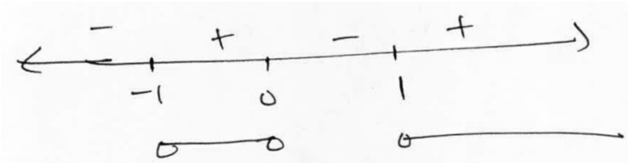
(1 – a) (1 + a) a (2 – a) > 0



$a(-\infty, -1) \cup (0,1) \cup (2, \infty)$ … (iii)

$0 < \frac{-\cancel{2}a}{\cancel{2}(1-a^2)} < 1$

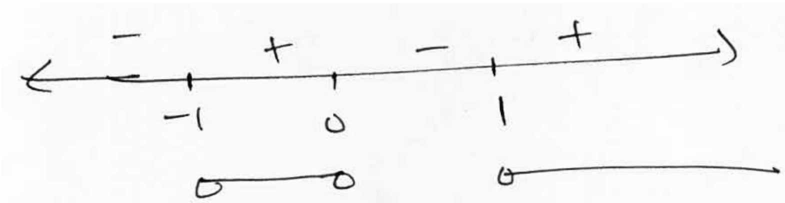
$\frac{a}{(a-1)(a+1)} > 0$ $\frac{a}{a^2-1} - 1 < 0$



$\frac{a - a^2 + 1}{a^2 - 1} < 0$

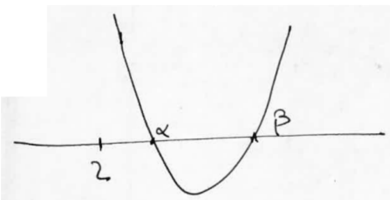
$a = \frac{1 \pm \sqrt{1+4}}{2}$ $\frac{a - a^2 + 1}{(a^2 - 1)} > 0$

$a = \frac{1 \pm \sqrt{5}}{2}$, $a = \pm 1$



a > 2

10. (D)



$D \geq 0$

$16a^2 + 4(2a^2 - 3a + 5) \geq 0$

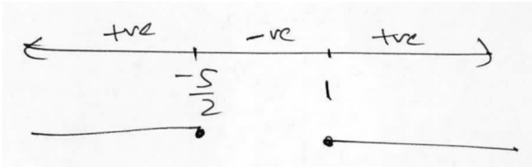
$4a^2 - 2a^2 + 3a - 5 \geq 0$

$2a^2 + 3a - 5 \geq 0$

$3a^2 + 5a - 2a - 5 \geq 0$

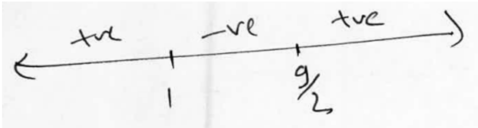
$a(2a+5) - (2a+5) \geq 0$

$(a-1)(2a+5) \geq 0$



f(2) > 0 ∴ $4 - 8a + 2a^2 - 3a + 5 > 0$

$2a^2 - 11a + 9 > 0$



$2a^2 - 2a - 9a + 9 > 0$

$2a(a-1) - 9(a-1) > 0$

$(2a-9)(a-1) > 0$

$a \in (-\infty, 1) \cup \left(9/2, \infty\right)$

$\frac{\not{4}^{2} a}{\not{2}} > 2$

$\boxed{a > 1}$

$\therefore$ intersection of all $\boxed{a > 9/2}$

PASSAGE – 4

$x^2 + ax + bc = 0$ roots α, β

$x^2 + bx + ac = 0$ root α, γ

$\alpha + \beta = -a$, $\alpha + \gamma = -b$

$\alpha\beta = bc$, $\alpha\gamma = ac$

$$\alpha^2 + a\alpha + bc = 0$$

$$\therefore \quad \underline{\alpha^2 + bx + ac = 0}$$

$$\frac{\alpha^2}{a^2c - b^2c} = \frac{\alpha}{bc - ac} = \frac{1}{b-a}$$

$$\alpha = \frac{\not{c}\left(a^2 - b^2\right)}{\not{c}\left(b-a\right)} = -\left(a+b\right)$$

$$\alpha = \frac{c\left(b - \not{a}\right)}{\left(b - \not{a}\right)}$$

$$\therefore \quad \frac{\alpha\beta}{\alpha\gamma} = \frac{b4}{a4}$$

$$\frac{\beta}{\gamma} = \frac{b}{a} \quad \ldots(i)$$

$$\beta - \gamma = b - a \quad \ldots(ii)$$

Form (i) & (ii) $\frac{b}{a}\gamma - \gamma = \left(b-a\right)$

$$\frac{\left(b-a\right)\gamma}{a} = \left(b-a\right)$$

$\therefore \gamma = a$

$\alpha = c$

$\beta = b$

12. (A)

$x^2 + cx + ab = 0$

$x^2 - (a+b)x + ab = 0$

$x^2 - (\alpha+\beta)x + \gamma\beta = 0$

$(x-\gamma)(x-\beta) = 0$

13. (A)

14. (B)

$D = (a+b+c)^2 - 4.3(ab+bc+ac)$

D = –12 (ab + bc + ac)

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2\sum ab$

$a^2 + b^2 + c^2 = -2\sum ab$

We know $a^2 + b^2 + c^2 > 0$

$-2\sum ab > 0$

$\therefore \quad \sum ab < 0$

$\therefore \quad D > 0$

15. (C)

a + b +c = 0

$\Rightarrow a^2 = -ab - ac$ … (i)

Also, $b^2 = -ab - bc$ … (ii)

Also, $c^2 = -ac - bc$ … (iii)

Substitute in equation

–ab –ac – ab – bc – ac – bc = 2

–ab – ac – ca

Passage – 5

let $x^2 - 5x + a = 0$ roots α, β

$\alpha + \beta = 5$, $\alpha\beta = a$

$x^2 - 6x + b = 0$ roots α, y

$\alpha + y = 6$, $\alpha y = b$

$x^2 - 8x + c = 0$ roots α, δ

$\alpha + \delta = 8$, $\alpha\delta = c$

$\alpha \in N \Rightarrow$

(i) let $\alpha = 1, \Rightarrow \beta = 4 \Rightarrow a = 4$

$\gamma = 5 \Rightarrow b = 5$

$\delta = 7 \Rightarrow c = 7$

(ii) Let $\alpha = 2 \Rightarrow \beta = 3, \gamma = 4, \delta = 6$

$\Rightarrow a = 6, b = 8, c = 12$

(iii) Let $\alpha = 3 \Rightarrow B = 2, \gamma = 3, \delta = 5$

$\Rightarrow\ a = 6, b = 9, c = 15$

(iv) Let $\alpha = 4 \Rightarrow\ \beta = 1, \gamma = 2, \delta = 4$

$\Rightarrow\ a = 4, b = 8, c = 16$

Now

16. 3b = c + 2a holds true for all

17. clearly A, B, C & D

18. where $\alpha = 3$ a + b + c is 3

19. where $\alpha = 1$ a + b + c is / 6

20. clearly $\alpha = 3$ or 4 b + c = 24

Passage – 6

(A) α, β be the roots of equation

$x^2 - p(x+1) - c = 0$, $(c \neq 1)$ we have to find equation whose roots are $\alpha+1, \beta+1$

Replace x by x – 1 we get required equation

$(x-1)^2 - px - c = 0$

$x^2 - x(p+2) + 1 - c = 0$

$\therefore \quad (\alpha+1)(\beta+1)$ = product of roots = (1 – c)

$$\frac{(\alpha+1)^2}{\alpha^2+2\alpha+c-1+1} + \frac{(\beta+1)^2}{\beta^2+2\beta+c-1+1}$$

$$= \frac{(\alpha+1)^2}{\alpha^2+2\alpha-(\alpha+1)(\beta+1)+1} + \frac{(\beta+1)^2}{\beta^2+2\beta-(\alpha+1)(\beta+1)+1}$$

$$= \frac{(\alpha+1)^2}{\alpha^2+2\alpha-\alpha-\beta} + \frac{(\beta+1)^2}{\beta^2+2\beta-\alpha\beta-\alpha-\beta}$$

$$= \frac{(\alpha+1)^2}{(\alpha+1)(\alpha-\beta)} - \frac{(\beta+1)^2}{(\beta+1)(\alpha+\beta)}$$

$$= \frac{(\alpha-\beta)}{(\alpha-\beta)} = 1$$

Passage – 7

(C) $a\left(\frac{1+t\theta}{1-t\theta}\right)^2 + b\left(\frac{1+\tan\theta}{1-\tan\theta}\right) + c = 0$

$\therefore\ \frac{\alpha}{1} = \frac{1+t\theta}{1-t\theta}$

$\frac{\alpha-1}{\alpha+1} = \frac{2+\theta}{2} \qquad \therefore\ t\theta = \frac{\alpha-1}{\alpha+1}$

$a\tan^2\theta - b\tan\theta + c = 0$

Or $\tan\theta = -\alpha$ or $-\beta$

$\therefore\ \theta = \tan^{-3}(-\lambda),\ \tan^{-1}(-\beta)$

But $\alpha > 0, \beta > 0$ so, roots cants be negative

No solution.

ASSERTION AND REASON

31.

Statement II connect: $(2k+1)^2 = 4k^2 + 4k + 1$

$= 4k(k+1) + 1$

$8I + 1$

Now statement I, $D = b^2 - 4ac$

$= (8I+1) - 4(2p+1)(2q+1)$

$= 8k - 3$

Remained when divided by 8 is not 1

$\Rightarrow$ Not a perfect square

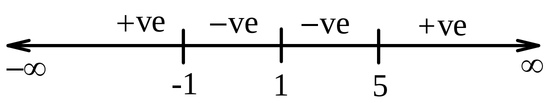
32. (D)

If $a = 4, b = 1$ then both equation are identical hence both roots common.

If irrational roots common then both equation identical hence $\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = \lambda$

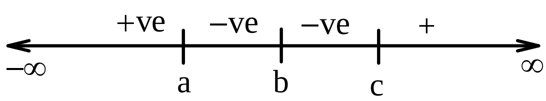
33. (A)

$$\frac{(x+1)(x-1)^2}{(x-5)} \geq 0$$



$x \in (-\infty, -1] \cup [5, \infty) \cup \{1\}$

$$\frac{(x-a)(x-b)^2}{(x-c)} \geq 0$$



$x \in (-\infty, a) \cup (c, \infty) \cup \{b\}$

34. (D)

35. (A)

$(\alpha_1 - \beta_1)^2 = (\alpha_2 - \beta_2)^2$

$(\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1 = (\alpha_2 + \beta_2)^2 - 4\alpha_2\beta_2$

$b^2 - 4c = c^2 - 4b$

$(b^2 - c^2) + 4(b - c) = 0$

$(b - c)(b + c + 4) = 0$

$\therefore b \neq c \therefore b + c = -4$

$|\alpha - \beta| = \sqrt{(\alpha - \beta)^2 - 4\alpha\beta}$

$$= \sqrt{\frac{\beta^2}{A^2} - \frac{4c}{A}} = \frac{\sqrt{\beta^2 - 4AC}}{|A|}$$

36. (D)

37. (A)

38. (A)

$x^2 + 4x + 7 > 0 \ \forall x \in R$

True because $D < 0$

$ax^2 + bx + c > 0 \quad \forall x \in R$ if $D < 0$, $a > 0$

39. (C)

$P(x)$ is divisible by $(x-3)$ then remainder is $P(3)$

$f(x) = (x-8)^3 (x+4)$

$f^1(x) = (x-8)^3 + (x+4).3(x-8)^2$

$f^1(x) = (x-8)^2 (x - 8 + 3x + 12)$

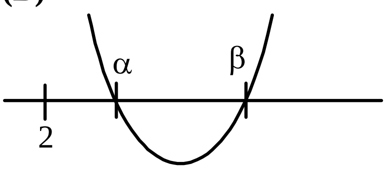
$f^1(x) = (x^2 - 16x + 64)(4x+4)$

$\therefore f^1(x)$ is divisible by $(x^2 - 16x + 64)$

40. (B)

$ax^2 + bx + c = 0$ has integral root if

41. (B)



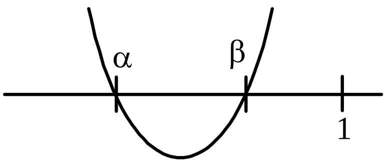
$b^2 - 4c > 0 \qquad \therefore b^2 > 4c$

$\frac{-b}{2a} > 2$

$-b > 4 \ \therefore b < -4$

$f(2) > 0 \Rightarrow 2b + c + 4 > 0$

Statement 1 is true



$4 - 4.c > 0$

$4c < 4$

$c < 1$ .... (i)

$1.f(1) > 0$

$1+2+c > 0 \quad \therefore c > -3$ .... (ii)

From (i) & (ii)

$c \in (-3,1)$

Statement (ii) is true but not correct explanation of 1

42. **(4)**

$y = x+2$

Equation whose roots are 2 more than given equation then replace x by $x-2$ we get required equation therefore statement 1 is false

$x^2 + (x) + 5 = 0$ has no real solution hence

Statement II is true

Matrix Match

(A) $\rightarrow P, Q, R, S$

$$\frac{(x-a)(x-b)}{(b+c)(c+a)} + \frac{(x-a)(x+c)}{(b+c)(b-a)} + \frac{(x-b)(x+c)}{(c+a)(a-b)} = 1 \quad \text{......... (i)}$$

$x = a, x = b, x = c$ satisfy (i) and maximum power of x is two then it is an identify not equation hence it satisfy all value of x

(B) $\rightarrow P, Q, R, S$

Same argument it is quadratic is x and

$x = a, x = bx = c$ satisfy then it is an identify

(C) $\rightarrow P, Q, R, S$

(D) $\rightarrow S$

$c(x-a)(x-b) + a(x-b)(x-c) + b(x-a)(x-c) = 3abc$ .... (i)

Only $x = 0$ satisfy (i)

EXERCISE – 2C

2. **(0)**

$$\left(x^2-4x-8x+32\right)(x-1)-10(x-8)>0$$

$$\left[x(x-4)-8(x-4)\right](x-1)-10(x-8)>0$$

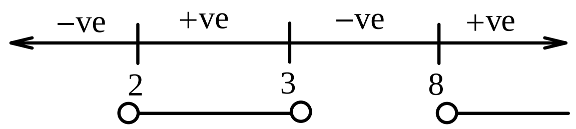
$$(x-8)(x-4)(x-1)-10(x-8)>0$$

$$(x-8)\left(x^2-5x-6\right)>0$$

$$(x-8)\left(x^2-2x-3x-6\right)>0$$

$$(x-8)\left[x(x-2)-3(x-2)\right]>0$$

$$(x-8)(x-2)(x-3)>0 \quad \text{........... (i)}$$



No negative integer satisfy equation (i)

3. **(2)**

Let $x+\frac{1}{x}=t \quad \therefore t\geq 2$

Or $t\leq -2$

$$t^2-7t+6=0$$

$$t^2-t-6t+6=0$$

$$t(t-1)-6(t-1)=0$$

$$(t-1)(t-6)=0 \qquad t\neq 1$$

$$\therefore t=6$$

$$x+\frac{1}{x}=6$$

$$x^2-6x+1=0$$

$$D>0$$

There fore two value of x satisfy given equation

4. **(0)**

$$\frac{\sqrt{x-1}+1}{\sqrt{x-1}-1}=3 \quad \therefore x\geq 1 \,\&\, x\neq 2$$

$$\sqrt{x-1}+=3\sqrt{x-1}-3$$

$$4=2\sqrt{x-1}$$

$$\therefore \sqrt{x-1}=2$$

$$x-1=4$$

$$x=5$$

$x=5$ do not satisfy equation (2) hence no common solution of given equation

5. **(0)**

$$D=4\cos^2\theta-4(3+\sin\theta)(2-\sin\theta)$$

$= 4\left[\cos^2\theta - 6 + \sin\theta + \sin^2\theta\right]$

$D = 4\left[\sin\theta - 5\right] < 0$

Hence no real value of x satisfy given equation

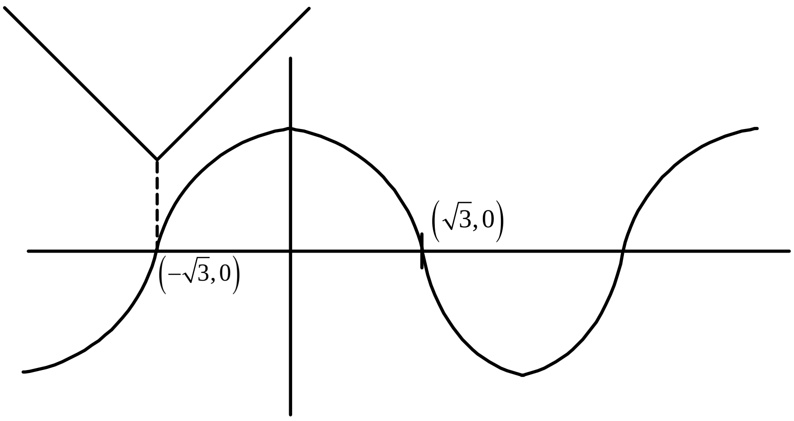
6. **(0)**

Draw the graph of $y = \cos\left(\dfrac{\pi x}{2\sqrt{3}}\right)$ &

$y = x^2 + 2\sqrt{3}x + 4$

$y = \left(x + \sqrt{3}\right)^2 + 1$

$\left(x + \sqrt{3}\right)^2 = \left(y - 1\right)$



From graph it is clear that two equation have no inter section point hence no common solution

7. **(1)**

$1 - \sin^2 x - \sin x + a = 0$

$\sin^2 x + 2.\dfrac{1}{2}\sin x + \dfrac{1}{4}\dfrac{-1}{4} = a + 1$

$\left(\sin x + \dfrac{1}{2}\right)^2 = a + \dfrac{5}{4}$ .... (i)

$0 < \sin x < 1 \quad \therefore x \in \left(0, \pi/2\right)$

$\dfrac{1}{2} < \sin x + \dfrac{1}{2} < 3/2$

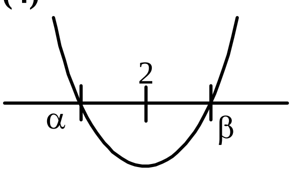
$\dfrac{1}{4} < \left(\sin x + \dfrac{1}{2}\right)^2 < 9/4$

$\dfrac{1}{4} < a + \dfrac{5}{4} < \dfrac{9}{4}$ (from (i))

$-1 < a < 1$

Hence only one integer between -1 & 1

8. **(4)**



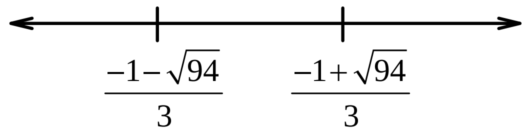
Given 2 lies between roots of given equation therefore

(i) $D > 0$

$(k+1)^2 - 4(k^2 + k - 8) > 0$

$k^2 + 2k + 1 - 4k^2 - 4k + 32 > 0$

$3k^2 + 2k - 31 < 0$



$k \in \left(\frac{-1-\sqrt{94}}{3}, \frac{-1+\sqrt{94}}{3}\right)$ .... (i)

(ii) $1.f(2) < 0$

$4 - (k+1).2 + (k^2 + k - 8) < 0$

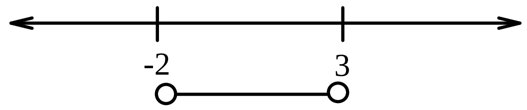
$4 - 2k - 2 + k^2 + k - 8 < 0$

$k^2 - k - 6 < 0$

$k^2 - 3k + 2k - 6 < 0$

$k(k-3) + 2(k-3) < 0$

$(k+2)(k-3) < 0$



$k \in (-2, 3)$ .... (ii)

From (i) & (ii)

$k \in \left(-2, \frac{-1+\sqrt{94}}{3}\right)$ .... (iii)

$k = -1, 0, 1, 2$ only satisfy equation (iii)

9. **()**

10. **(3)**

Let the roots of equation be

$x, x+1, x+2$

$\therefore x + x + 1 + x + 2 = -a$

$3(x+1) = -a$ .... (a)

$x(x+1) + x(x+2) + (x+1)(x+2) = b$

$3x^2 + 6x + 2 = b$ .... (ii)

$3(x^2 + 2x + 1 - 1) + 2 = b$

$3(x+1)^2 = b + 1$

$3\frac{a^2}{9} = b + 1$

$\therefore \frac{a^2}{b+1} = 3$

11. **(4)**

$yx^2 + 3xy + cy = x^2 - 3x + c$

$(y-1)x^2 + 3x(y+1) + c(y-1) = 0$

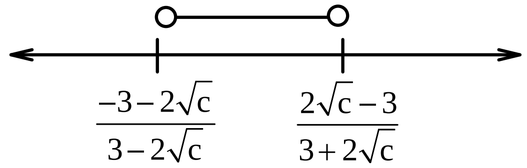
For real $x, D \geq 0$

$9(y+1)^2 - 4(y-1)^2.C \geq 0$

$(3y+3)^2 - \left(2\sqrt{c}(y-1)\right)^2 \geq 0$

$\left[3y+3+2\sqrt{c}(y-1)\right]\left[3y+3-2\sqrt{c}(y-1)\right] \geq 0$

$\left[\left(3+2\sqrt{c}\right)y+3-2\sqrt{c}\right]\left[\left(3-2\sqrt{c}\right)y+3+2\sqrt{c}\right] \geq 0$



Given $\frac{-3-2\sqrt{c}}{3-2\sqrt{c}} = 7$

$-3-2\sqrt{c} = 21 - 14\sqrt{c}$

$12\sqrt{c} = 24$

$c = 4$

12. **()**

Given $(a+4)x^2 - 2ax + (2a-6) \leq 0$

$D \leq 0$

$4a^2 - 4(a+4)(2a-6) \leq 0$

$a^2 - 2a^2 - 2a + 24 \leq 0$

$a^2 + 2a - 24 \geq 0$

$a^2 + 6a - 4a - 24 \geq 0$

$a(a+6) - 4(a+6) \geq 0$

$(a+6)(a-4) \geq 0$

$a \in (-\infty, -6] \cup [4, \infty)$ .... (ii)

$a + 4 < 0$

$a < -4$ .... (ii)

13. **()**

Same as Q. no – 14 in EX – 3

14. **(2)**

Given $\tan\alpha + \tan\beta = p$

$\tan\alpha \tan\beta = q$

$\cot\alpha + \cot\beta = r, \cot\alpha \cot\beta = s$

$$\frac{1}{\tan\alpha}+\frac{1}{\tan\beta}=r$$

$$\frac{\tan\alpha+\tan\beta}{\tan\alpha.\tan\beta}=r$$

$$\therefore \frac{p}{q}=r \qquad \text{...........} \text{ (i)}$$

$$+\alpha+\in+\beta=p$$

$$\frac{\cot\alpha+\cot\beta}{\cot\alpha\cot\beta}=p \qquad \therefore \frac{r}{3}=p \qquad \text{.........} \text{ (ii)}$$

Form (i) & (ii)

$$qs=1$$

$$\left(\frac{p}{r}+q\right)s=\frac{ps}{p}+qs=1+1=2$$

15. **(1)**

D

Given

(i) $D>0$

$$4k^2-4(k-5)(k-4)>0$$

$$\cancel{k}^2-\cancel{k}^2+9k-20>0$$

$$k>20/9 \qquad \text{..........} \text{ (i)}$$

(ii) $(k-5)f(1)<0$

$$(k-5)(\cancel{k}-5-\cancel{2k}+\cancel{k}-4)<0$$

$$(k-5)x-9<0 \qquad \therefore k-5>0$$

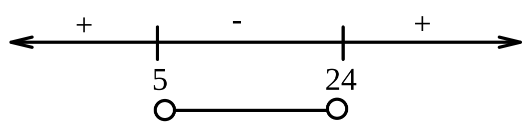
$$k>5 \quad \text{............} \text{ (ii)}$$

$$(k-5)f(2)<0$$

$$(k-5)(4(k-5)-4k+k-4)<0$$

$$(k-5)(\cancel{4k}-20-\cancel{4k}+k-4)<0$$

$$(k-5)(k-24)<0$$



$$k\in(5,24)$$

$$\therefore a=5$$

$$b=24$$

$$a^2-b=25-24=1$$

16. **(4)**

$$kx^2+(1-k)x+5=0 \quad \text{............} \text{ (i)}$$

Given $\alpha+\beta=\dfrac{k-1}{k}$

$$\alpha\beta = \frac{5}{k}$$

Given $\frac{\alpha^2+\beta^2}{\alpha\beta} = 4/5$

$$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = 4/5$$

$$\frac{\frac{(k-1)^2}{k^2} - \frac{2.5}{k}}{5/k} = 4/5$$

$$\frac{(k-1)^2 - 10k}{\cancel{5}k} = 4/\cancel{5}$$

$$k^2 - 2k - 10k + 1 = 4k$$

$$k^2 - 16k + 1 = 0$$

Given $k_1 + k_2 = 16$

$k_1k_2 = 1$

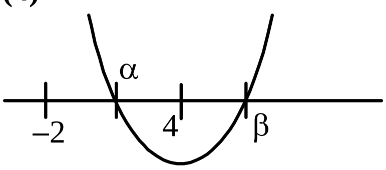
$$\left(\frac{k_1}{k_2} + \frac{k_2}{k_1} + 2\right)^{1/4} = \left(\frac{k_1^2 + k_2^2}{k_1k_2} + 2\right)^{1/4}$$

$$= \left(\frac{(k_1+k_2)^2 - 2k_1\cancel{k}_2 + 2\cancel{k}_1k_2}{k_1k_2}\right)^{1/4}$$

$$= (16\times16)^{1/4}$$

$$= \left(4^4\right)^{1/4} = 4$$

17. **(4)**



Given

(i) $D > 0$

$$4m^2 - 4(m^2 - 1) > 0$$

$$\cancel{4m^2} - \cancel{4m^2} + 4 > 0$$

(ii) $f(-2)f(4) < 0$

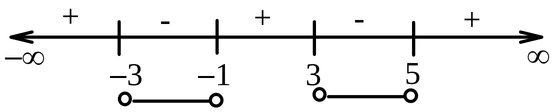
$$(4 + 4m + m^2 - 1)(16 - 8m + m^2 - 1) < 0$$

$$(m^2 + 4m + 3)(m^2 - 8m + 15) < 0$$

$$(m^2 + m + 3m + 3)(m^2 - 3m - 5m + 15) < 0$$

$$(m(m+1) + 3(m+1))(m(m-3) - 5(m-3)) < 0$$

$$(m+3)(m+1)(m-3)(m-5) < 0$$



$m \in (-3,-1) \cup (3,5)$

Largest $m = 4$

18. **(2)**

D

(i) $D \geq 0$

$4 - 4 \times 4a \geq 0$

$1 - 4a \geq 0 \quad \therefore 4a \leq 1$

$a \leq \frac{1}{4}$ .... (i)

(ii) $4xf(-1) > 0$

$(4 + 2 + a) > 0 \quad \therefore a > -6$ ...... (ii)

$4xf(1) > 0$

$4x(4 - 2 + a) > 0 \quad \therefore a > -2$ .... (iii)

Given from (i), (ii), (iii)

$a \in \left(-2, \frac{1}{4}\right]$

$a = -1 \text{ and } 0 \text{ only}$

19. **(6)**

$3x^2 - 7x + 8 \geq x^2 + 1$

$2x^2 - 7x + 7 \geq 0$

$x \in R$ .... (i)

$\therefore$ $3x^2 - 7x + 8 \leq 2x^2 + 2$

$x^2 - 7x + 6 \leq 0$

$x^2 - 7x + 6 \leq 0$

$x^2 - x - 6x + 6 \leq 0$

$x(x-1) - 6(x-1) \leq 0$

$(x-1)(x-6) \leq 0$

D

$x \in [1, 6]$

Total number of integral value of $x = 6$

20. **(2)**

Given α be the root of equation $x^2 - x - a = 0$ and

2α be the root of equation $x^2 - x - 3a = 0$

$$\alpha^2 - \alpha - a = 0 .....(i)$$

$$4\alpha^2 - 2\alpha - 3a = 0......(ii)$$

$$\frac{\alpha^2}{3a - 2a} = \frac{\alpha}{-4a + 3a} = \frac{1}{-2 + 4}$$

$$\frac{\alpha^2}{a} = \frac{\alpha}{-a} = \frac{1}{2}$$

$$\therefore \alpha = -a/2$$

$$4\alpha = -1$$

$$\therefore \frac{-a}{2} = -1$$

$$a = 2$$

21. **()**

Let $y = \dfrac{ax^2 + 3x - 4}{a + 3x - 4x^2}$

$$ay + 3xy - 4x^2y = ax^2 + 3x - 4$$

$$(a + 4y)x^2 + 3x(1 - y) - (4 + ay) = 0 \quad \text{........... (i)}$$

For real x, $D \geq 0$

$$9(1-y)^2 + 4(a+4y)(4+ay) \geq 0$$

$$9 + 9y^2 - 18y + 4\left(4a + a^2y + 16y + 4ay^2\right) \geq 0$$

$$(9 + 16a)y^2 + \left(4a^2 + 64\right)y + 16a + 9 - 18y \geq 0$$

$$(9 + 16a)y^2 + \left(4a^2 + 46\right)y + 16a + 9 \geq 0 \quad \text{....... (ii)}$$

(ii) Satisfy only when

$$9 + 16a > 0 \qquad \therefore a > \frac{-9}{16} \ \&$$

$$D \leq 0 \left(4a^2 + 46\right)^2 - 4(10a + 9)^2 \leq 0$$

$$\left(4a^2 + 46\right)^2 - 4(9 + 16a)(16a + 9) \leq 0$$

$$16a^4 + 2116 + 368a^2 - 4\left(256a^2 + 288a + 81\right) \leq 0$$

$$16a^4 + 2116 + 368a^2 - 1024a^2 - 1152a - 324 \leq 0$$

$$\cancel{16}a^4 - \underset{41}{\cancel{656}}\,a^2 - \underset{72}{\cancel{1152}}\,a + \underset{112}{\cancel{1792}} \leq 0$$

$$a^3(a-1) + a^2(a-1) - 40a(a-1) - 112(a-1) \leq 0$$

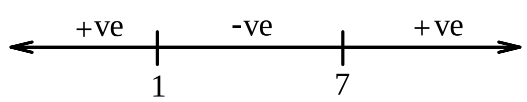
$$(a-1)\left(a^3 + a^2 - 40a - 112\right) \leq 0$$

$$(a-1)\left(a^3 - 7a^2 + 8a^2 - 56a + 16a - 112\right) \leq 0$$

$$(a-1)\left[a^2(a-7) + 8a(a-7)\right] \leq 0$$

$$(a-1)(a-7)\left(a^2 + 8a + 16\right) \leq 0$$

$$(a-1)(a-7)(a+4)^2 \leq 0$$



22. **(1)**

Given $\alpha, 2\alpha$ be the root of equation

$(\ell - m)x^2 + \ell x + 1 = 0$

$\therefore \qquad 3\alpha = \frac{-\ell}{(\ell - m)}$ .... (i)

$2\alpha^2 = \frac{1}{\ell - m}$ .... (ii)

From (i) & (ii)

$$\frac{1}{(\ell - m)} = 2.\frac{\ell^2}{-(\ell - m)^2}$$

$(\ell \pm m)$

$9(\ell - m) = 2\ell^2$

$2\ell^2 - 9\ell + 9m = 0$ .... (iii)

For real $\ell_1 \qquad D \geq 0$

$81 - 4.2.9m \geq 0$

$8m \leq 9$

$m \leq 9/8$

Greatest $[m] = 1$

23. (4)

If both roots are integer then sum of roots and product of roots are integer

$\alpha + \beta = \frac{-2(a+1)}{(a+2)}$ .... (i)

$\alpha\beta = \frac{a}{a+2}$ .... (ii)

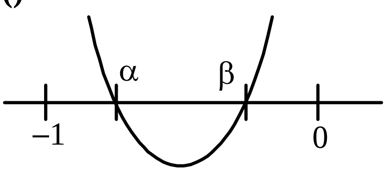
From (i) & (ii)

$a = 0, -3, -1, -4$ only for which both are integer

24. ()

25. ()

26. ()



$b^2 - 4ac \geq 0$ .... (i)

$a(a - b + c) > 0$ .... (ii)

$ac > 0$ .... (iii)

$-1 < \frac{-b}{2a} < 0$ .... (iv)

Given a, b, c, are natural number which satisfy all four condition above and whose product is least then only $a = 4, b = 4, c = 1$

Then $\sqrt{abc} = \sqrt{16} = 4$

27. **()**

28. **()**

Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$= (x-1)(x-2)(x-3)(x-4) + x^3$ ...... (i)

Given $P(1) = 1$

$P(2) = 2^3$

$P(3) = 3^3$

$P(4) = 4^3$

$P(10) = 9 \times 8 \times 7 \times 6 + 10^3 = 4024$

$$\frac{P(10)}{2012} = 2$$

29. **()**

30. **()**

Given

$a + b = k$

$ab = k$

Let $\lambda = a^2 + b^2 = (a+b)^2 - 2ab$

$\lambda = k^2 - 2k$

$$\frac{d\lambda}{dk} = (2k - 2)$$

For minimum $\lambda, \frac{d\lambda}{dk} = 0$ $\quad \therefore k = 1$

$x^2 - x + 1 = 0$

Sum of roots $= 1$

EXERCISE – 3

1. **(0)**

$P(x) = ax^2 + bx + c$

$Q(x) = -ax^2 + dx + c$

$D_1 + D_2 = b^2 - \cancel{4ac} + d^2 + \cancel{4ac} > 0$

Then at least are of D_1 and $D_2 \geq 0$

Hence at least two real roots of

Equation $P(x).Q(x) = 0$

Lie either $P(x) = 0$ or $Q(x) = 0$

2. **(0)**

$\lambda^2 x^2 + (2\lambda - \lambda^2)x + 3 = 0$

$$\alpha + \beta = \frac{\lambda^2 - 2\lambda}{\lambda^2} = \frac{\lambda - 2}{\lambda}$$

$$\alpha\beta = \frac{3}{\lambda^2}$$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = 4/3$$

$$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = 4/3$$

$$\frac{(\alpha + \beta)^2}{\alpha\beta} = 2 + 4/3$$

$$\frac{\frac{(\lambda - 2)^2}{\cancel{\lambda^2}}}{3/\cancel{\lambda^2}} = \frac{10}{3} \therefore \frac{(\lambda - 2)^2}{\cancel{3}} = \frac{10}{\cancel{3}}$$

$$\lambda^2 - 4\lambda + 4 = 10$$

$$\lambda^2 - 4\lambda - 6 = 0$$

$$\lambda_1 + \lambda_2 = 4$$

$$\lambda_1\lambda_2 = -6$$

$$\frac{\lambda_1^2}{\lambda_2} + \frac{\lambda_2^2}{\lambda_1} = \frac{\lambda_1^3 + \lambda_2^3}{\lambda_1\lambda_2} = \frac{(\lambda_1 + \lambda_2)^3 - 3\lambda_1\lambda_2(\lambda_1 + \lambda_2)}{\lambda_1\lambda_2}$$

$$= \frac{4.4.4 + 3 \times 6 \times 4}{-6}$$

$$= \frac{4 \times \cancel{2}[8 + 9]}{-\cancel{6}_3} = -\frac{68}{3}$$

$$\frac{\lambda_1^{\cancel{2}}}{\lambda_{\cancel{2}}} . \frac{\lambda_2^{\cancel{2}}}{\cancel{\lambda}_1} = -6$$

Required equation $x^2 + \frac{68}{3}x - 6 = 0$

$$3x^2+68x-18=0$$

3. **(0)**

Let $\alpha_1\beta$ be the roots of equation

$$\alpha+\beta=\frac{-x}{\ell}$$

$$\alpha\beta=\frac{x}{\ell}$$

Given $\frac{\alpha}{\beta}=\frac{P}{q}$

$$\therefore\sqrt{\frac{P}{q}}+\sqrt{\frac{q}{P}}+\sqrt{\frac{n}{\ell}}=\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{x}{\ell}}$$

$$=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}+\sqrt{\frac{x}{\ell}}$$

$$=\frac{-x/\ell}{\sqrt{x/\ell}}+\sqrt{\frac{x}{\ell}}$$

$$=-\sqrt{x/\ell}+\sqrt{\frac{x}{\ell}}=0$$

4. **()**

Sum of roots $=-P<0$

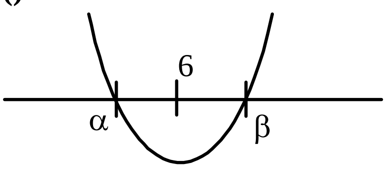
Product of root $=q>0$

This is possible only when both roots are –ve

5. **()**

6. **()**

7. **()**



(i) $D>0$

$$4(P-3)^2-4.9>0$$

$$(P-3)^2>9$$

$$|P-3|>3$$

$$\therefore P-3>3 \text{ or } P-3<-3$$

$$P>6 \text{ or } P<0$$

(ii) $1.f(6)<0$

$$36+2(P-3)\times 6+9<0$$

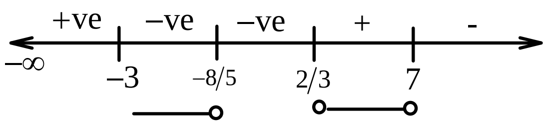
$$12+4(P-3)+3<0$$

$4P + 3 < 0 \qquad \therefore P < -3/4$

$\therefore P \in (-\infty, -3/4)$

8. **()**

$x = -3, 2/3, 7, -8/5$



$x \in (-\infty, -3] \cup \left[\frac{2}{3}, 7\right] \cup \{-8/5\}$

9. **()**

$2x + P + q = \frac{(x+p)(x+q)}{r}$

$2rx + r(p+q) = x^2 + (p+q)x + pq$

$x^2 + (P + q - 2r)x + pq - r(P+q) = 0$

Given sum of roots $= 0$

$P + q = 2r \qquad$ .... (i)

Product of roots $= Pq - \frac{(P+q)}{2}(P+q)$

$= \frac{2Pq - (P+q)^2}{2} = \frac{-(P^2 + q^2)}{2}$

10. **()**

Let other roots be β then

$\alpha + \beta = \frac{-2}{4}$

$\beta = -\frac{1}{2} - \alpha \qquad$ .... (i)

& $\qquad 4\alpha^2 + 2\alpha - 1 = 0 \qquad$ .... (ii)

$4\alpha^3 - 3\alpha = \alpha(4\alpha^2 - 3) = \alpha(1 - 2\alpha - 3)$

$= -2\alpha^2 - 2\alpha$

$= \frac{-1}{2}(4\alpha^2 + 4\alpha)$

$= \frac{-1}{2}(1 - 2\alpha + 4\alpha)$

$= \frac{-1}{2}(2\alpha + 1)$

$= \frac{-1}{2} - \alpha$

$= \beta\,(\text{from (i)})$

There fore $4\alpha^3 - 3\alpha$ be its other root.

11. **()**

Given $4b^2 - 4ac > 0$

$b^2 > ac$ .... (i)

$D = (a+c)^2 .4b^2 - 4\left[(a+c)a - 2\left(ac - b^2\right)\right]$

$\left[c(a+c) - 2\left(ac - b^2\right)\right]$

$= 4\left[(a+c)^2 b^2 - \left(a^2 + ac - 2ac + b^2\right)\left(ac + c^2 - 2ac + 2b^2\right)\right]$

$= 4\left[(a+c)^2 b^2 - \left(a^2 + b^2 - ac\right)\left(c^2 + 2b^2 - ac\right)\right]$

$= 4\left[a^2b^2 + \cancel{b^2c^2} + 2acb^2 - a^2c^2 - a^2.2b^2 + a^3c - \cancel{b^2c^2} - 2b^4 + b^2ac + ac^3 + ac2b^2 - a^2c^2\right]$

$= 4\left[-a^2b^2 + 4acb^2 - 2a^2c^2 + a^3c - 2b^4 + b^2ac + ac^3\right]$

12. **()**

$\alpha^2 + P\alpha + q = 0$ and $\alpha^{2x} + p^x x^x + q^x = 0$

$\beta^2 + P\beta + q = 0$ $\beta^{2x} + P^x\beta^x + q^x = 0$

$\alpha + \beta = -P$

$\alpha^{2x} - \beta^{2x} + P^x\left(\alpha^x - \beta^x\right) = 0$

$\left(\alpha^x - \beta^x\right)\left(x^x + \beta^x + P^x\right) = 0$ .... (i)

If $\alpha \neq \beta$

$\alpha^x + \beta^x = -P^x$

Given $\frac{x^n}{\beta^x} + 1 + \left(\frac{\alpha}{\beta} + 1\right)^x = 0$

$\alpha^x + \beta^x + (\alpha + \beta)^x = 0$

$\therefore -P^x + (-P)^x = 0$ .... (ii)

(ii) Satisfy is n is even integer

13. **()**

$D = 4^2 P^2 q^2 r^2 .s^2 - 4\left(P^4 + q^4\right)\left(r^4 + s^4\right)$

$D = 4\left(4P^2q^2r^2s^2 - P^4r^4 - P^4s^4 - q^4r^4 - q^4s^4\right)$

$= -4\left[\left(P^2r^2 - q^2s^2\right)^2 + \left(P^2s^2 - q^2r^2\right)^2\right] \geq 0$

$D > 0$ not possible only $D = 0$ possible lie if roots are real then it is equal.

14. **()**

$x^2 + ax + b = 0, \alpha, \beta$

$x^2 + cx + d = 0, \alpha, r$

$x^2 + ex + f = 0, \beta, r$

$(\alpha - \beta)^2 = a^2 - 4b$

$\left[(\alpha + v) - (\beta - v)\right]^2 = a^2 - 4b$

$(-c+e)^2 = a^2 - 4b$

$e^2 + c^2 - 2ec = a^2 - 4b$ .... (i)

Similarly $(\alpha - v)^2 = c^2 - 4d$

$a^2 + e^2 - 2ae = c^2 - 4d$ .... (ii)

$a^2 + c^2 - 2ac = e^2 - 4f$ .... (iii)

Adding (i), (ii) & (iii)

$2(a^2 + c^2 + e^2) - 2(ac + ce + ae) = a^2 + c^2 + e^2 - 4(b + d + f)$

$a^2 + c^2 + e^2 = 2(ac + ce + ae) - 4(b + d + f)$

Adding $2ac + 2ce + 2ae$

$a^2 + c^2 + e^2 + 2ac + 2ce + 2ae = 4(ac + ce + ae - b - d - f)$

$(a + c + e)^2 = 4(ac + ce + ae - b - d - f)$

15.

16.

17.

18.

19. **()**

$\alpha + \beta = -\frac{b}{a} < 0$

$\alpha\beta = c/a > 0$

This is possible only when both roots are – ve

20. **()**

Given $\alpha + \beta = -P$

$\alpha\beta = q$

$q.\frac{\alpha^2}{\beta^2} + (2q - p^2)\frac{\alpha}{\beta}q$

$= \frac{q(\alpha^2 + \beta^2) + (2q - p^2)\alpha\beta}{\beta^2}$

$= \frac{q\left[(\alpha + \beta)^2 - 2\alpha\beta\right] + (2q - p^2)\alpha\beta}{\beta^2}$

$= \frac{q(P^2 - 2q) + (2q - p^2)q}{\beta^2} = \frac{q}{\beta^2}\left(P^2 - 2q + 2q - p^2\right) = 0$

$\therefore \frac{\alpha}{\beta}$ is the roots $qx^2 + (2q + p^2)x + q = 0$

21. **()**

Let $y=\dfrac{4x^2+36x+9}{12x^2+8x+1}$

$$12yx^2+8yx+y=4x^2+36x+9$$

$$4(3y-1)x^2+(2y-9)x+(y-9)=0$$

For real x, $D\geq 0$

$$\cancel{16}(2y-9)^2-\cancel{4}-\cancel{4}(3y-1)(y-9)\geq 0$$

$$4y^2+81-36y-3y^2+28y-9\geq 0$$

$$y^2-8y+72\geq 0 \quad \text{............ (i)}$$

$$D<0$$

$$y^2-8y+72\geq 0 \qquad \forall y\in R$$

22. ()

23. ()

24. ()

$$\alpha+\beta=\frac{-b}{a} \quad , \quad \alpha_1-\beta=\frac{-b_1}{a_1}$$

$$\alpha\beta=c/a \quad , \quad -\alpha_1\beta=\frac{c_1}{a_1}$$

$$\therefore \quad \alpha+\alpha_1=\frac{-b}{a}-\frac{b_1}{a_1}=\frac{-(ba_1+ab_1)}{aa_1}$$

$$a\beta^2+b\beta+c=0 \qquad aa_1\beta^2+a_1b\beta+a_1c=0$$

$$a_1\beta^2-b_1\beta+c_1=0 \qquad aa_1\beta^2-ab_1\beta+ac_1=0$$

$$\beta(a_1b+ab_1)=ac_1-a_1c$$

$$\beta=\left(\frac{ac_1-a_1c}{a_1b+ab_1}\right)$$

$$-\alpha\alpha_1\beta^2=\frac{cc_1}{aa_1}$$

$$\alpha\alpha_1=\frac{-cc_1}{aa_1}\cdot\frac{(a_1b+ab_1)^2}{(ac_1-a_1c)^2}$$

$$\therefore \text{Required equation } x^2+\frac{(ba_1+ab_1)}{aa_1}x\frac{-cc_1}{aa_1}\frac{(a_1b+ab_1)^2}{(aa_1-a_1c)^2}=0$$

$$\frac{aa_1x^2}{ab_1+a_1b}+x-\frac{cc_1(ab_1+a_1b)}{(ac_1-a_1c)^2}=0$$

$$\frac{x^2}{\left(\frac{b_1}{a_1}+\frac{b}{a}\right)}+x+\frac{cc_1}{bc_1+b_1c}=0$$

$$\because \frac{(ac_1-a_1c)^2}{a_1b+ab_1}=bc_1+b_1c$$

25. **()**

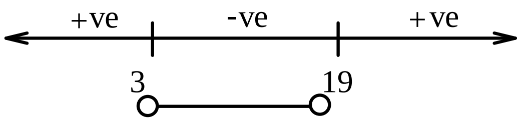
$$x^2+(m-3)x+4(m-3)>0 \quad \forall x \in R$$

$$D<0$$

$$(m-3)^2-4.4(m-3)<0$$

$$(m-3)(m-3-16)<0$$

$$(m-3)(m-19)<0$$



$$m \in (3,19)$$

26. **()**

Given $\alpha-\beta=(\alpha+\delta)-(\beta+\delta)$

$$(\alpha+\beta)^2-4\alpha\beta=(\alpha+\delta+\beta+\delta)^2-4(\alpha+\delta)(\beta+\delta)$$

$$\frac{b^2}{a^2}-4.\frac{c}{a}=\frac{B^2}{A^2}-4\frac{C}{A}$$

$$\frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$$

27. **()**

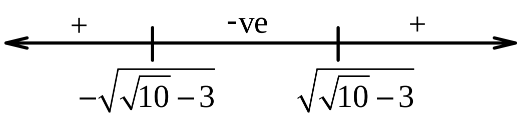
Let $\lambda=x+4$

$$(\lambda-1)^4+(\lambda+1)^4\ge 4$$

$$2(\lambda^4+6\lambda^2+1)\ge 4$$

$$\lambda^4+6\lambda-1\ge 0$$

$\therefore$ $\lambda^4+6\lambda-1=0$ $\quad\therefore \lambda=\pm\sqrt{\sqrt{10}-3}$



$$\lambda \le -\sqrt{\sqrt{10}-3} \text{ or } \lambda \ge \sqrt{\sqrt{10}-3}$$

$$x \le -4-\sqrt{\sqrt{10}-3} \text{ or } x \ge -4+\sqrt{\sqrt{10}-3}$$

28. **()**

29. **()**

30. **()**

31. **()**

32. **()**

33. **()**

34. **()**

35. **()**

36. **()**

37. **()**

38. **()**

$\alpha+\beta=-P, \quad \gamma+\delta=-q$

$\alpha\beta=1, \quad \gamma\delta=1$

$$R.H.S=(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$$

$$=\left(\alpha\beta-\gamma(\alpha+\beta)+\gamma^2\right)\left(\alpha\beta+\delta(\alpha+\beta)+\delta^2\right)$$

$$=\left(1+P\gamma+\gamma^2\right)\left(1+\delta^2-P\delta\right)$$

$$=(P\gamma-q\gamma)(-\delta P-q\delta)$$

$$=-\gamma\delta\left(P^2-q^2\right) \qquad \because \gamma^2+P\gamma+1=0$$

$$=\left(q^2-P^2\right) \qquad \& \qquad \delta^2+q\delta+1=0$$

39. **()**

Given $\alpha+\beta=-p \quad , \quad \gamma+\delta=-\gamma$

$\alpha\beta=q \quad , \quad \gamma\delta=s$

$$E=\left(\alpha\beta-r(\alpha+\beta)+\gamma^2\right)\left(\alpha\beta-\delta(\alpha+\beta)+\delta^2\right)$$

$$=\left(q+p\gamma+\gamma^2\right)\left(q+p\delta+\delta^2\right)$$

$$=(q+p\gamma-s-r\gamma)(q+p\delta-s-r\delta)$$

$$=\left[(q-s)+\gamma(p-r)\right]\left[(q-s)+\delta(p-r)\right]$$

$$=(q-s)^2+\gamma\delta(P-r)^2+(P-r)(q-s)(\gamma+\delta)$$

$$E=(q-s)^2+s(P-r)^2-r(P-r)(q-s)$$

40. **()**

Let root be α & α^2

$\alpha+\alpha^x=-b/a$ .... (i)

$\alpha.\alpha^x=c/a \qquad \therefore \alpha=(c/a)^{\frac{1}{x+1}}$

Put this value of α is (i)

$$\left(\frac{c}{a}\right)^{\frac{1}{x+1}}+\left(\frac{c}{a}\right)^{\frac{x}{x+1}}=-b/a$$

$$\left(a^{x+1}.\frac{c}{a}\right)^{\frac{1}{x+1}}+\left(\frac{c^x}{a^x}.a^{x+1}\right)^{\frac{1}{x+1}}=-b$$

$$\left(a^x c\right)^{\frac{1}{x+1}} + \left(a c^x\right)^{\frac{1}{x+1}} + b = 0$$