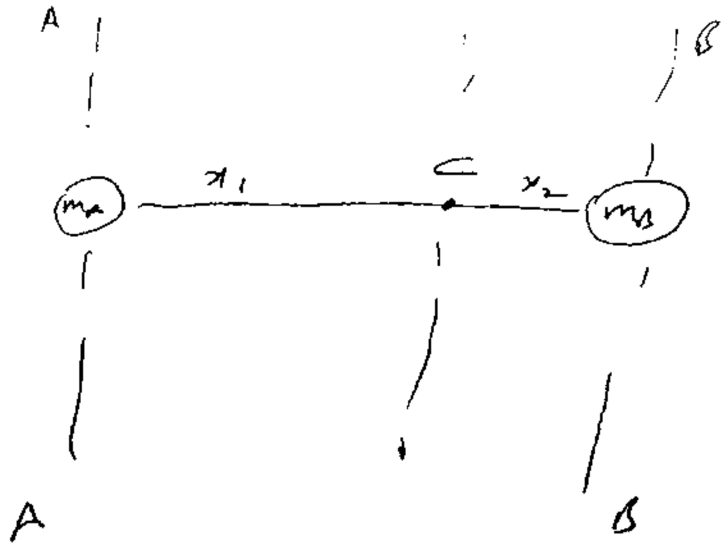
ACP

ROTATIONAL DYNAMICS

SOLVED SAMCQ

SSP Sir

Q.1



$(x_1 + x_2) = l$

$m_B x_2 = m_A x_1$

$$I_{BB} = m_A l^2$$

$$I_{AA} = m_B l^2$$

$$\frac{I_{BB}}{I_{AA}} = 3 = \frac{m_A l^2}{m_B l^2} \Rightarrow 3m_B = m_A$$

$$\therefore \frac{x_2}{x_1} = \frac{m_A}{m_B} = 3 \quad \text{or} \quad x_2 = 3x_1$$

$$x_1 + x_2 = l \quad \text{or} \quad x_1 + 3x_1 = l$$

$$\Rightarrow x_1 = \frac{l}{4}$$

Q.2

$$I_A = \frac{Ma^2}{2}$$

$$I_B = Ma^2$$

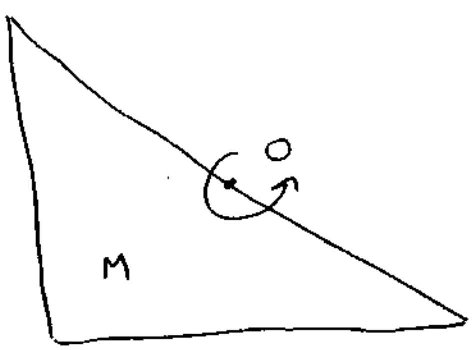
$$I_C = \frac{M(2a)^2}{12} = \frac{Ma^2}{3}$$

$$I_D = 4\left(\left(\frac{M}{4}\right)\frac{a^2}{12} + \left(\frac{M}{4}\right)\left(\frac{a}{\cancel{2}}\right)^2\right)$$

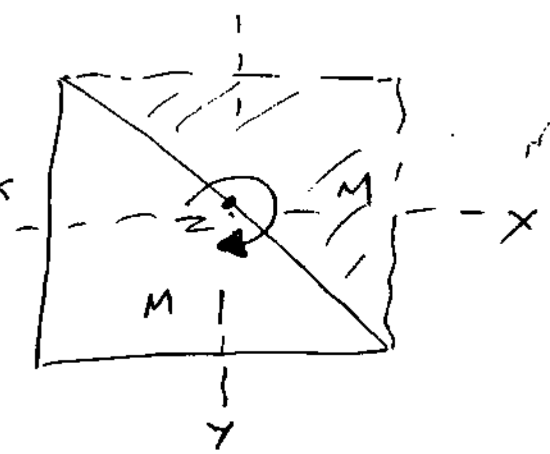
$$= \frac{Ma^2}{12} + Ma^2$$

I_D is largest.

Q.3

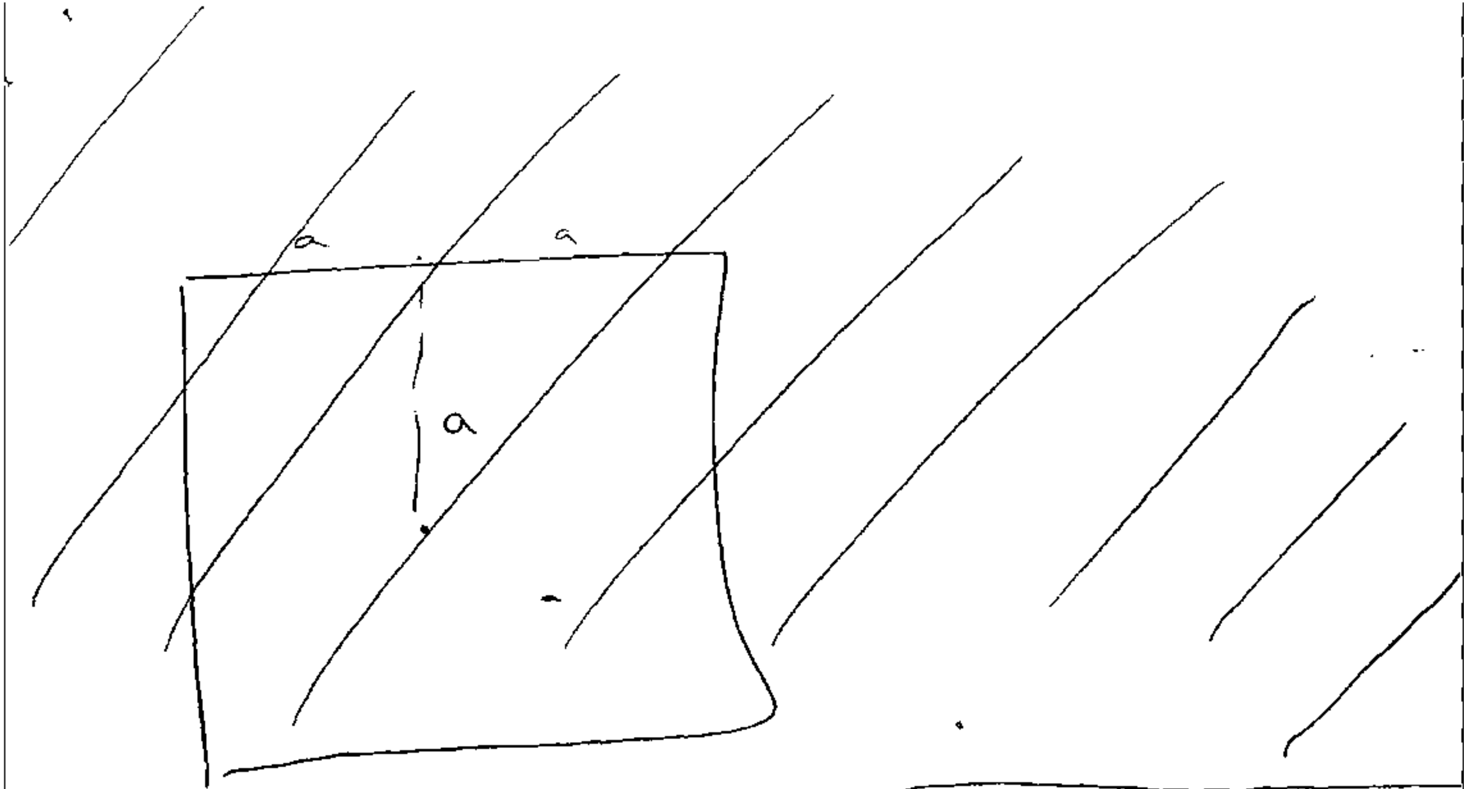


$= \frac{1}{2} \times$

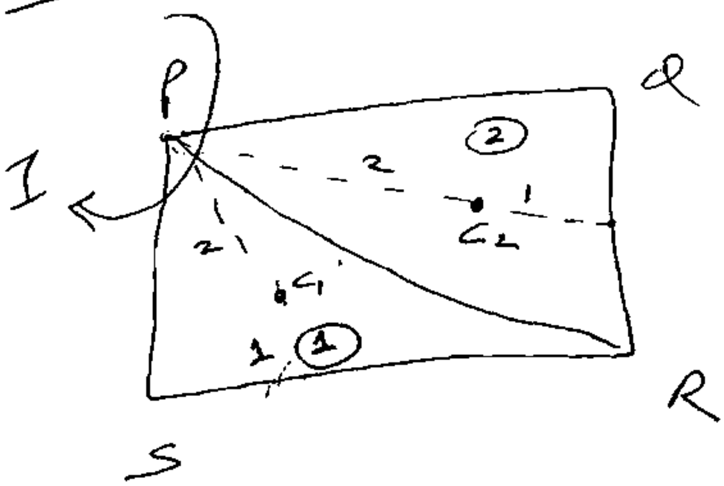


$$= \frac{1}{2} \times (I_x + I_y)$$

$$= I_x = (2M)\left(\frac{a^2}{12}\right)$$



Q.5



Let ① + ② be Δs PRS + PQR respectively.

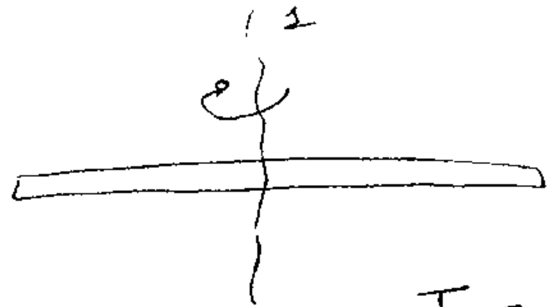
$$I = I_{①} + I_{②}$$

$$I_{①} = I_{c_1} + M(PC_2)^2$$

$$I_{②} = I_{c_2} + M(PC_1)^2$$

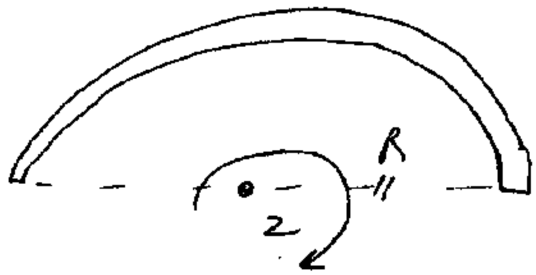
$$\text{but } PC_2 > PC_1 \Rightarrow I_{②} > I_{①}$$

Q.7



M, L

$$I_1 = \frac{ML^2}{12}$$



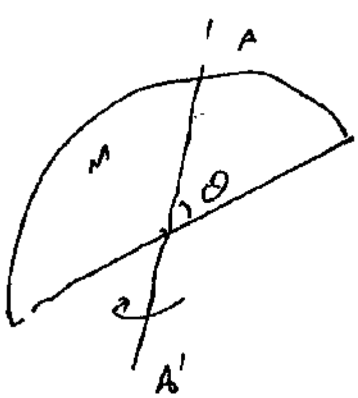
$$\pi R = L$$

$$I_2 = MR^2 = M\left(\frac{L}{\pi}\right)^2$$

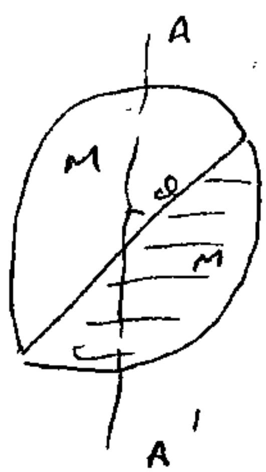
$$I_2 = \frac{ML^2}{\pi}$$

$$\frac{I_1}{I_2} = \frac{ML^2}{12} \div \frac{ML^2}{\pi} = \frac{\pi}{12} < 1$$

Q.8

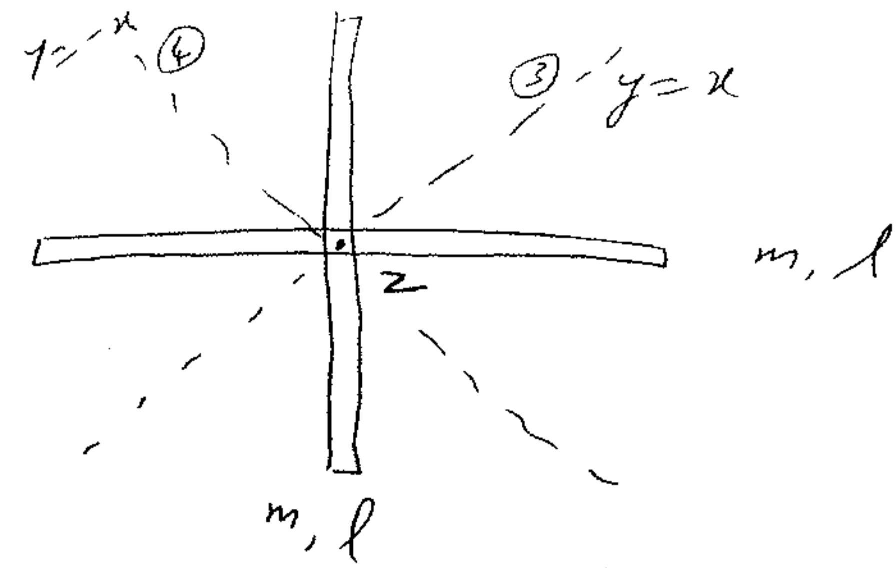


$$= \frac{1}{2} \times$$



$$= \frac{1}{2} \times \left[\frac{2M \cdot R^2}{4}\right] = \frac{MR^2}{4}$$

Q.4



$$I_z = I_x + I_y \quad (\perp^r \text{ axes theorem})$$

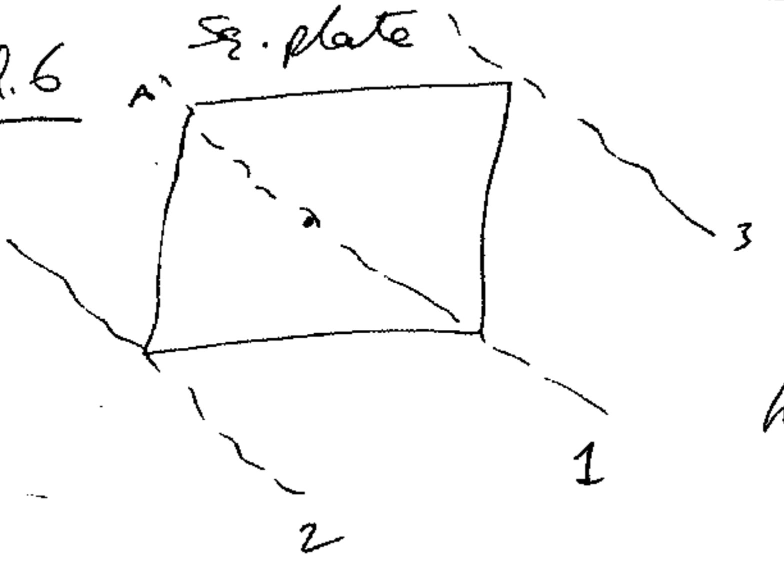
$$I_z = \frac{ml^2}{12} + \frac{ml^2}{12}$$

Also, $I_z = I_3 + I_4 = 2I_3$

$$\Rightarrow I_3 = \frac{I_z}{2} = \frac{1}{2}\left(\frac{ml^2}{12} + \frac{ml^2}{12}\right)$$

$$= \frac{ml^2}{12}$$

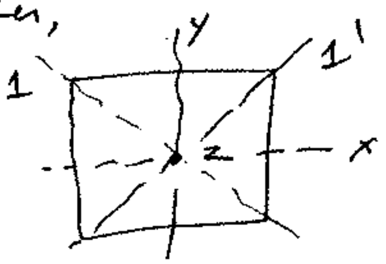
Q.6



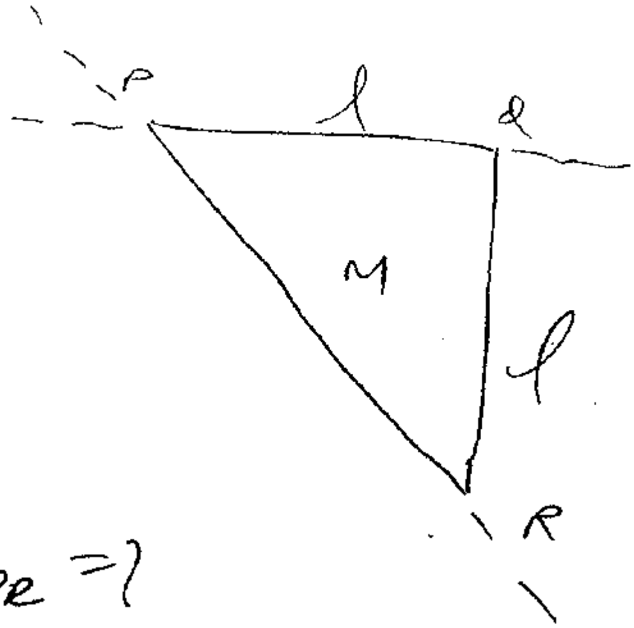
$$I_2 = I_3 = I_1 + M\left(\frac{l\sqrt{2}}{2}\right)^2$$

$$I_2 = I_3 = I_1 + \frac{Ml^2}{2}$$

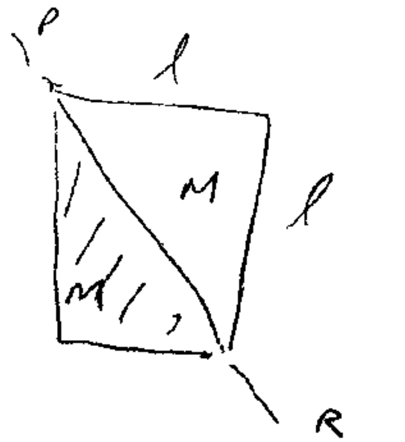
Further,



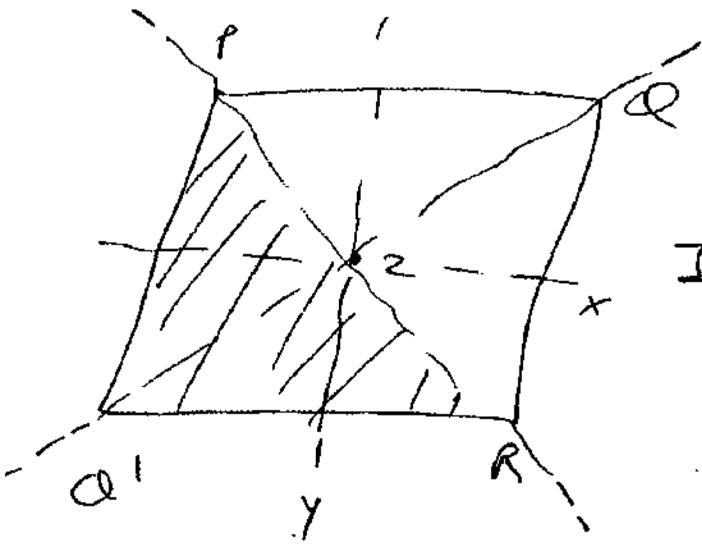
Q. 9



$= \frac{1}{2} \times$



$I_{PR} = ?$



$$I_{PR} + I_{QQ'} = I_z = I_x + I_y$$

$$2 I_{PR} = I_z = 2 I_x$$

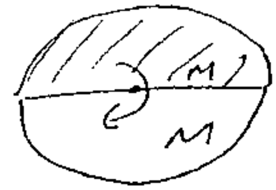
$$\underset{\text{of full plate}}{I_{PR}} = I_x$$

$$= \frac{2Ml^2}{12}$$

$$I_{PR}(\text{Reqd}) = \frac{1}{2}\left(\frac{2Ml^2}{12}\right) = \frac{Ml^2}{12}$$

Q.10

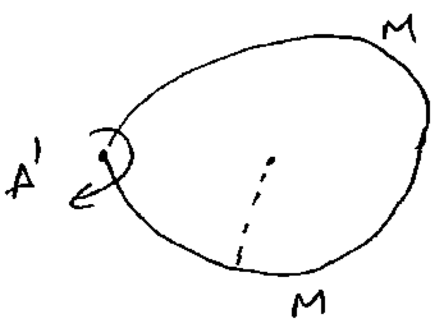
$= \frac{1}{2} \times$



$$= \frac{1}{2} \times \left(\frac{2MR^2}{2}\right) = \frac{MR^2}{2}$$

Q.11

$I_A = ?$



$$I_A = \frac{I_{A'}}{2} = \frac{I_c + (2M)(R^2)}{2}$$

$$= \frac{(2M)R^2 + (2M)R^2}{2}$$

$$= 2MR^2$$

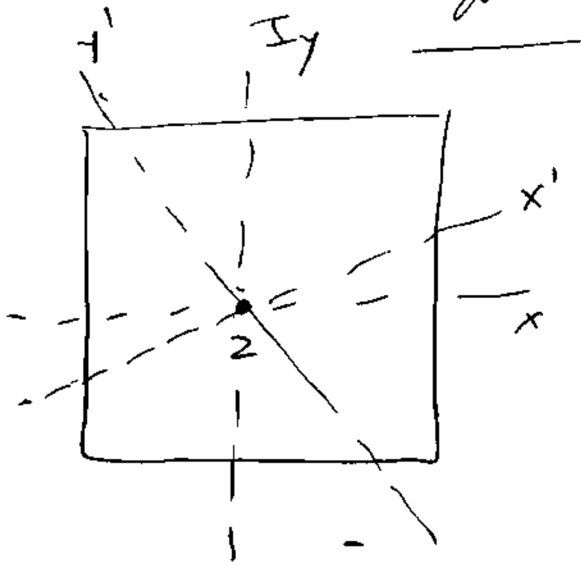
Q.12 $I_{cyl} = MR^2$ where $2\pi R = L$

$M = \sigma L^2$

$$\Rightarrow I_{cyl} = (\sigma L^2)\left(\frac{L}{2\pi}\right)^2$$

$$= \sigma L^4$$

Q.13 Square Plate : M, L

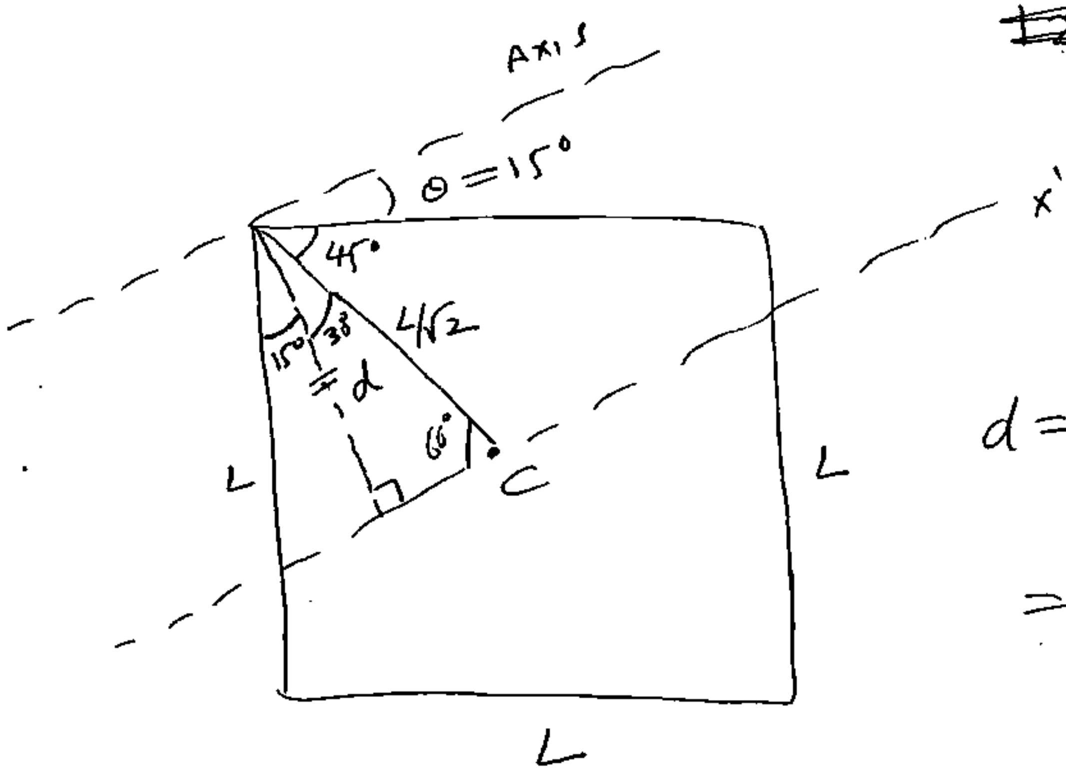


$$I_z = I_x + I_y = 2I_x$$

$$= I_{x'} + I_{y'} = 2I_{x'}$$

$$I_{x'} = \frac{I_z}{2} = I_x$$

$$I_{x'} = \cancel{\frac{ML^2}{12}} \; \frac{ML^2}{12}$$



$$d = \frac{L}{\sqrt{2}} \cdot \sin 60$$

$$= \frac{L}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{L\sqrt{3}}{2\sqrt{2}}$$

$$I_{Axis} = I_{x'} + Md^2$$

$$= \frac{ML^2}{12} + M\left(\frac{3L^2}{8}\right)$$

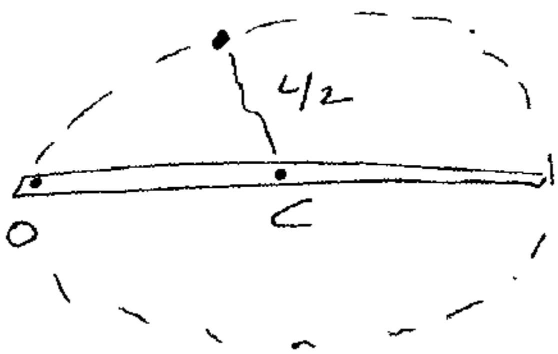
$$= \quad 11ML^2$$

9.14 $2\pi R = L$

$$R = \frac{L}{2\pi}$$

$$I_{RING} = MR^2 = \frac{ML^2}{4\pi^2}$$

9.15



LOCUS = CIRCLE

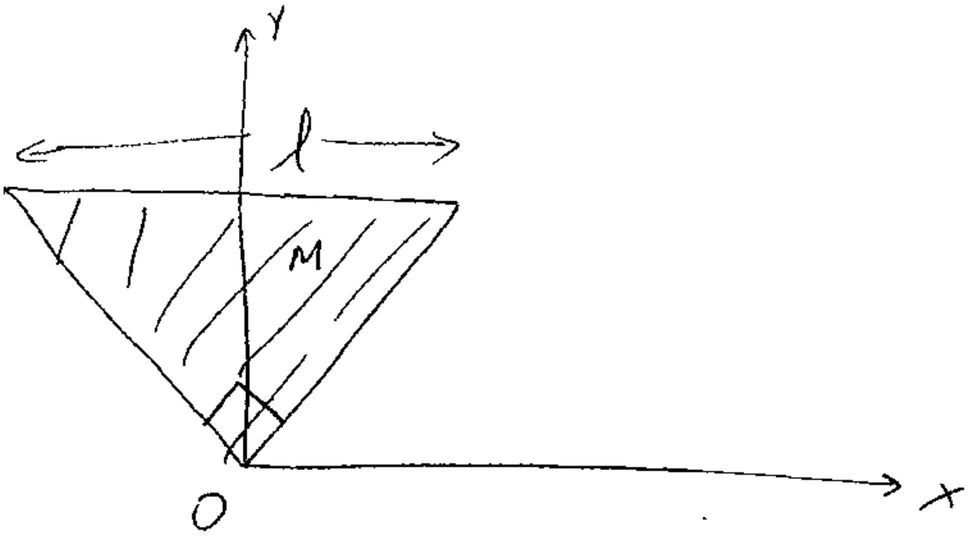
$$I_O = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$I_P = I_C + Md^2$$

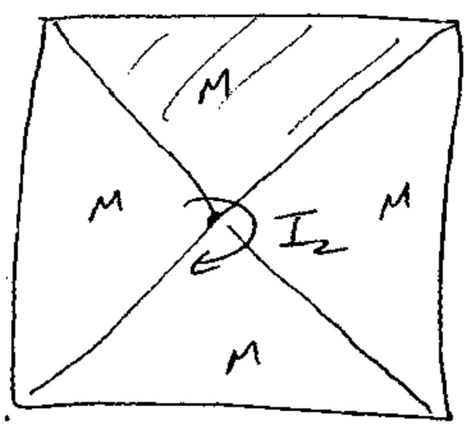
---

16 $I_P = I_C + Md^2$

I_P is minimum when $d = 0$



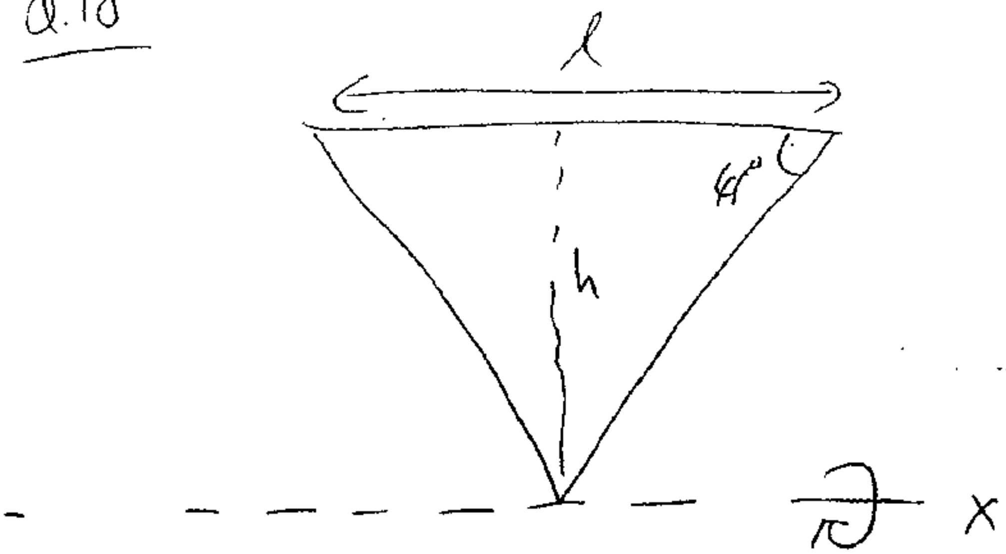
(17)



$$I_z = I_x + I_y$$
$$= 2I_x$$
$$= 2\left(\frac{4Ml^2}{12}\right)$$
$$= \frac{8Ml^2}{12} = \frac{8Ml^2}{12}$$

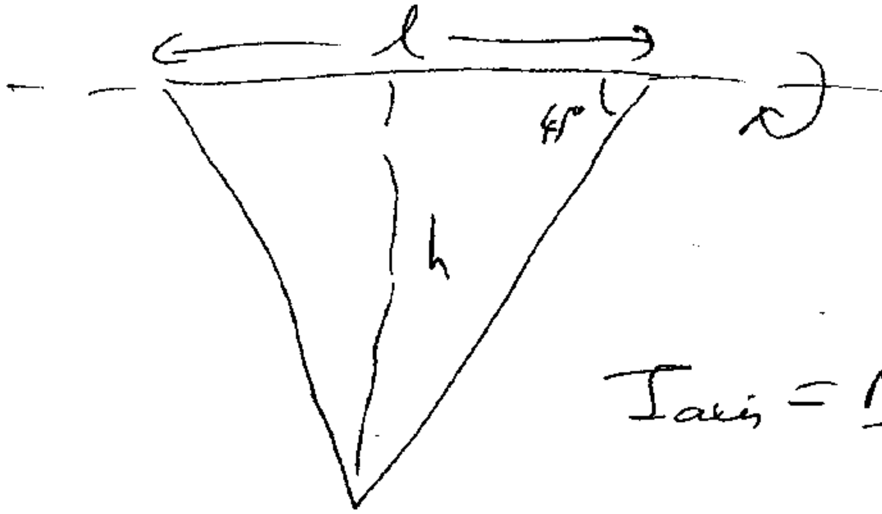
$$I_{req'd} = \frac{1}{4}\left(\frac{8Ml^2}{12}\right)$$
$$= \frac{Ml^2}{6}$$

Q.18



$$I_x = \frac{Mh^2}{2} = \frac{M}{2}\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{8}$$

Q.19

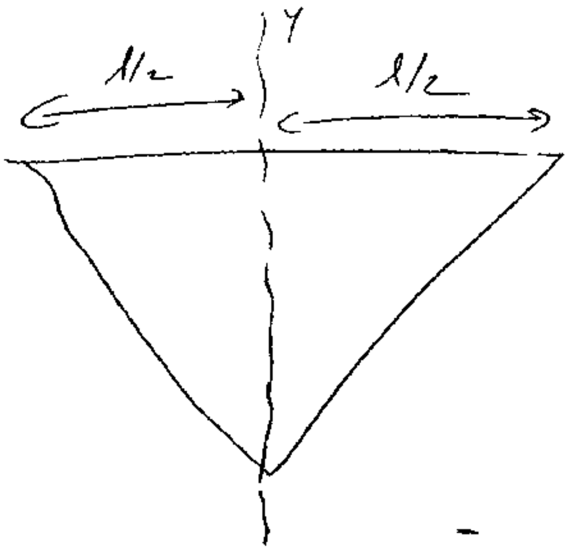


$$I_{axis} = \frac{Mh^2}{6}$$

$$= \frac{M}{6}\left(\frac{l}{2}\right)^2$$

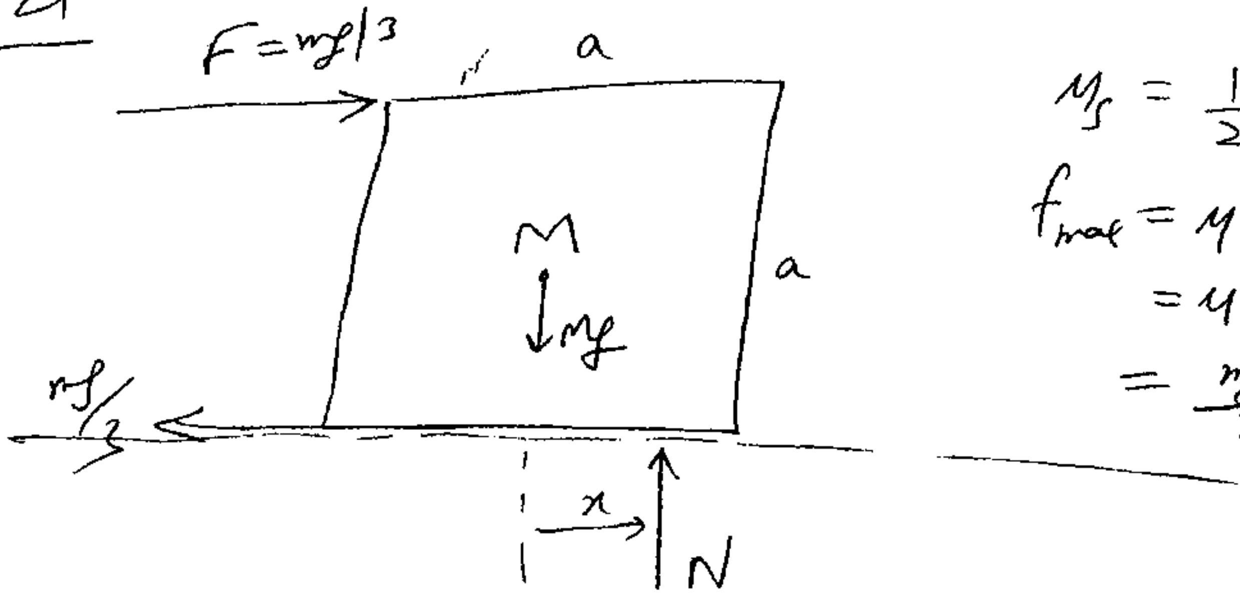
$$= \frac{Ml^2}{24}$$

Q.20



$$I_y = \frac{M\left(\frac{l}{2}\right)^2}{6} = \frac{Ml^2}{24}$$

Q.21



$\mu_s = \frac{1}{2}$

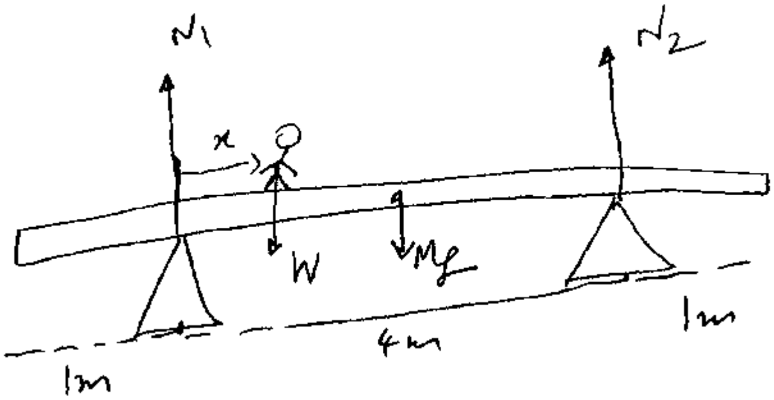
$f_{max} = \mu N$

$= \mu mg$

$= \frac{mg}{2}$

$$N.x = \left(mg\right)\left(\frac{a}{[illegible]}\right) + \left(\frac{mg}{[illegible]}\right)\left(\frac{a}{2}\right) = \frac{mga}{[illegible]}$$

Q.22



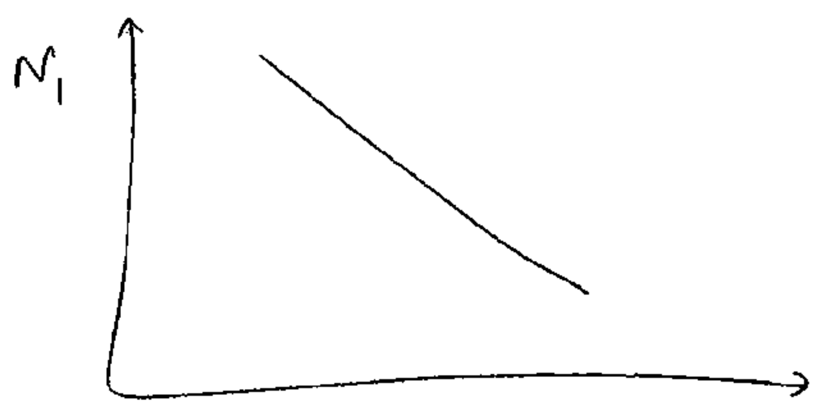
Torque about N_2

$$(Mg)(2) + W(4-x) = N_1 \cdot 4$$

$$\Rightarrow \quad N_1 = \frac{2Mg + 4W}{4} - \frac{Wx}{4}$$

"Slope is −ve."

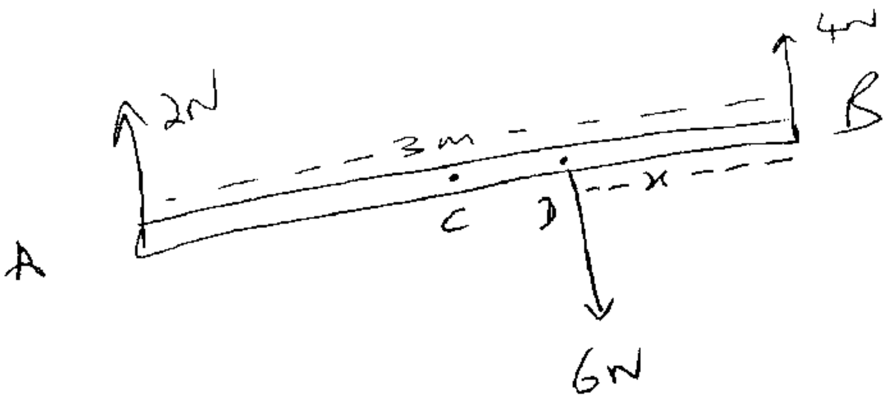
$\Rightarrow$



Q.23 $\quad 6x_1 = Wx_2$

$Wx_1 = 24x_2$

4



Torque about C = 0

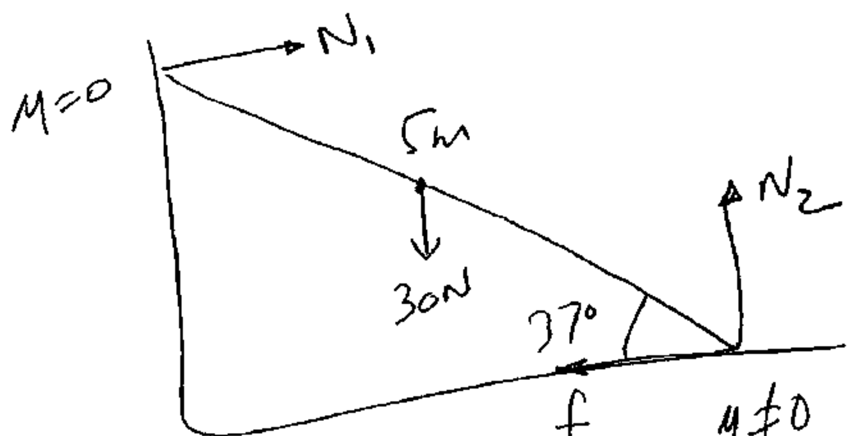
$$\Rightarrow 2(1.5) + 6(1.5 - x) = 4(1.5)$$

$$3 + 9 - 6x = 6$$

$$6x = 12 - 6 = 6$$

$$\Rightarrow x = 1\text{m}$$ 1

Q.25



$N_1 = f$

for equilibrium, torque about base = 0

or

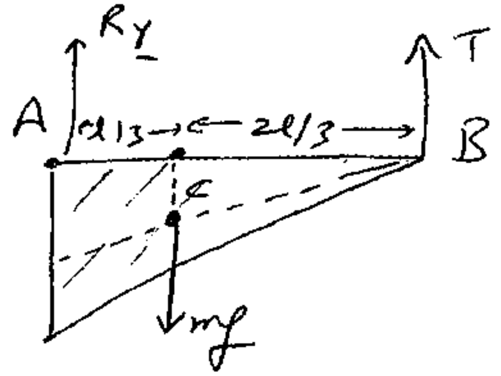
$\ldots) = 30(5\cos 37°)$

26

$$m\left(\frac{l}{2}\right)\sin 60^\circ = M\left(\frac{l}{2}\right)\sin 30$$

$$\frac{M}{m} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3}$$

2.21



$$\tau_B = 0 \Rightarrow mg\left(\frac{2l}{3}\right) = R_y(l)$$

$$\text{or } R_y = \frac{2mg}{3}$$

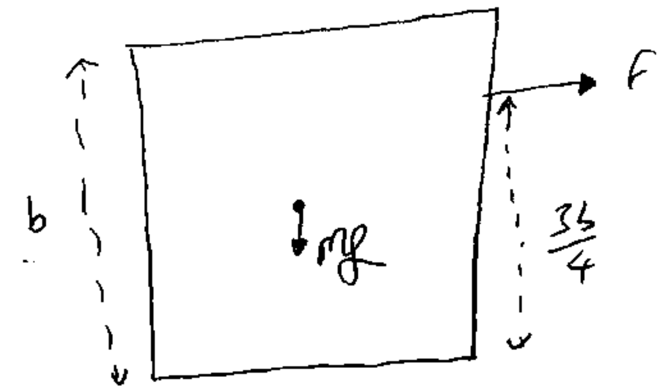
2.28 $\quad 16\, l_1 = m\, l_2$

$$m l_1 = 4\, l_2$$

dividing $\Rightarrow \dfrac{16}{m} = \dfrac{m}{4}$ or $m^2 = 16 \times 4$

$$\Rightarrow m = 8\ \text{kg}$$

Q. 29



For Toppling: $F_1 \left(\frac{3b}{4} \right) = Mg \left(\frac{b}{2} \right)$

$$F_1 = \frac{2Mg}{3}$$

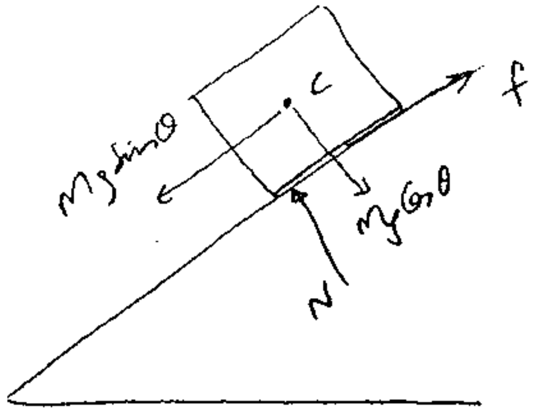
For slipping: $F_2 = \mu N = \mu Mg$

If Toppling happens before slipping

$\Rightarrow \quad F_1 < F_2$ or $\frac{2}{3} Mg < \mu Mg$

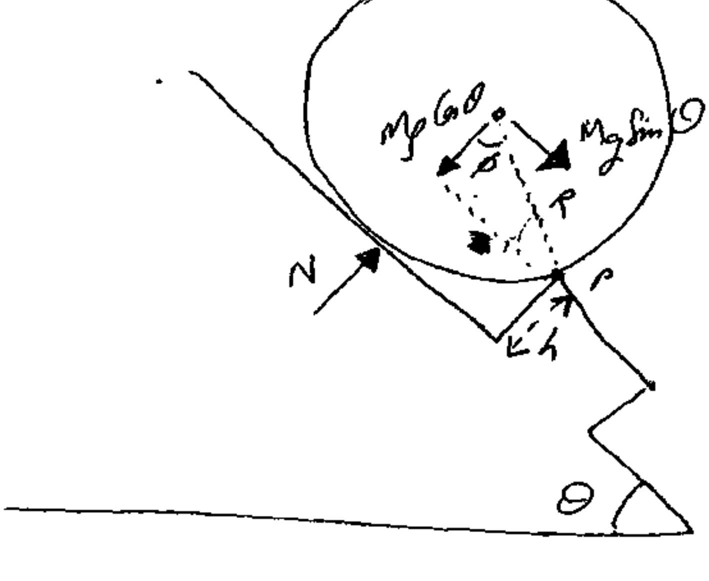
or $\mu > \frac{2}{3}$

Q.30



For balancing torque about C, the resultant N, must be to the left of the $Mg\cos\theta$ line of action.

Q.31



$\cancel{R^2 = x^2 + (R-h)^2}$

$\cancel{R^2 = x^2 + R^2 - 2Rh + h^2}$

$\cancel{x^2 = 2Rh - h^2}$

$\cancel{x^2 = h(2R-h)}$

About P, for equilibrium:

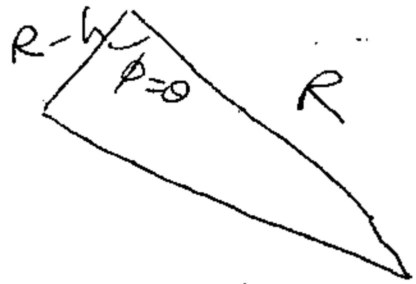
$$(Mg\cos\theta - N)\cancel{x}\,R\sin\phi = Mg\sin\theta\,\cancel{(R-h)}\,R\cos\phi$$

~~At toppling~~ On verge of toppling $N=0$

$\ldots \sin\theta\cos\phi$

$$\sin(\theta - \phi) = 0$$

$$\theta = \phi$$



$$R\cos\theta = R - h$$

$$h = R(1-\cos\theta)$$

Q.32 CM of hollow cone $= \frac{h}{3}$ from Base

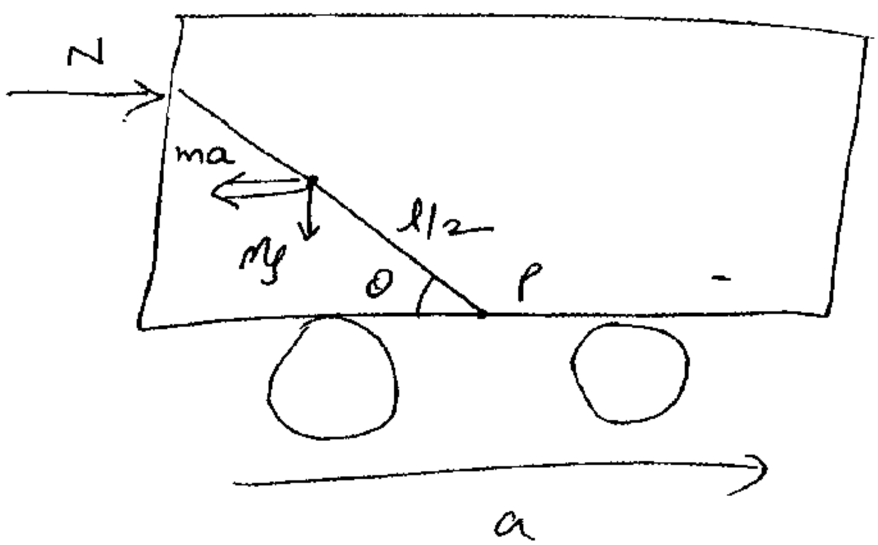
$$mg\cos\theta . R = mg\sin\theta\left(\frac{h}{3}\right)$$

$$\cancel{mg}\cos\theta . \cancel{R} = \cancel{mg}\sin\theta . \left(\frac{2\cancel{R}}{3}\right)$$

$$\tan\theta = \frac{3}{2}$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\right)$$

Q.33



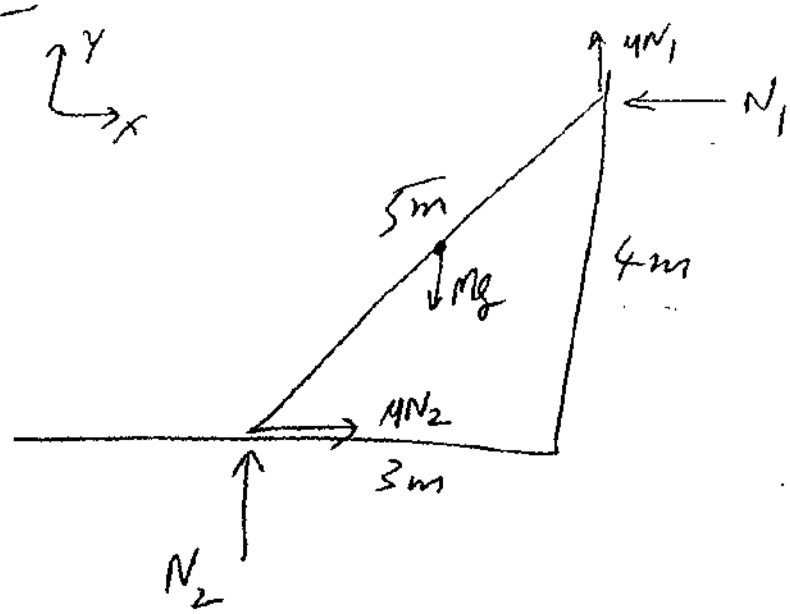
for eqlbm: about P, $\quad N = ma$

$$N(l\sin\theta) - ma\left(\frac{l}{2}\sin\theta\right) - mg\frac{l}{2}\cos\theta = 0$$

$$\Rightarrow \quad ma\left(\frac{l}{2}\sin\theta\right) = mg\frac{l}{2}\cos\theta$$

$$a = g\cot\theta$$

Q.34



$\Sigma F_x = 0$ or $N_1 = \mu N_2$

Torque about base:

$$Mg(1.5) = \mu N_1(3) + N_1(4)$$

$$N_1 = \frac{1.5Mg}{3\mu + 4}$$

Torque about point on the wall

$$N_2(3) - \mu N_2(4) = Mg(1.5)$$

$$N_2 = \frac{1.5Mg}{3 - 4\mu}$$

$$\Rightarrow \frac{\cancel{1.5Mg}}{3\mu + 4} = \mu\left(\frac{\cancel{1.5Mg}}{3-4\mu}\right)$$

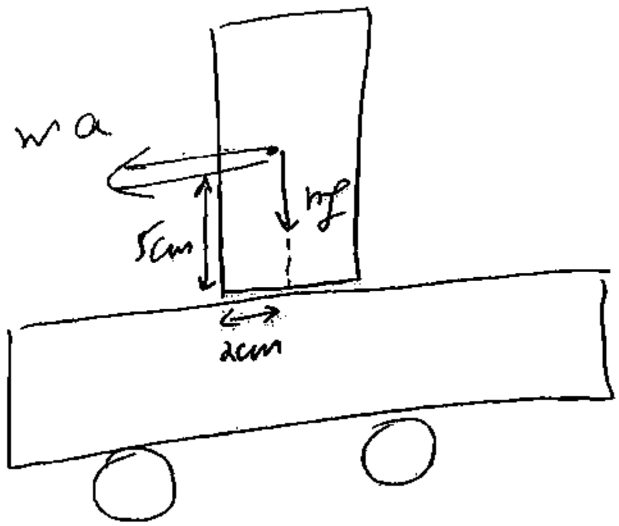
$$3\mu^2 + 8\mu - 3 = 0$$

$$3\mu^2 + 9\mu - \mu - 3 = 0$$

$$3\mu(\mu + 3) - 1(\mu+3) = 0$$

$$(\mu+3)(3\mu-1) = 0$$

Q. 35

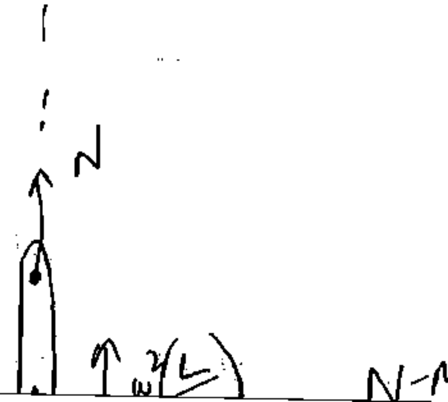


$$(ma)(5) \geq mg(2)$$

$$a \geq \frac{2g}{5}$$

$$a \geq 4\,m/s^2$$

Q. 36



$$\frac{1}{2} I\omega^2 = Mg\left(\frac{L}{2}\right)$$

$$\frac{1}{2}\left(\frac{M\cancel{L^2}}{3}\right)\omega^2 = \cancel{M}g\left(\cancel{\frac{L}{2}}\right)$$

$$\omega^2 = \frac{3g}{L}$$

$$N - Mg = M\omega^2 \frac{L}{2}$$

Q.37

$$\tau = I\alpha$$

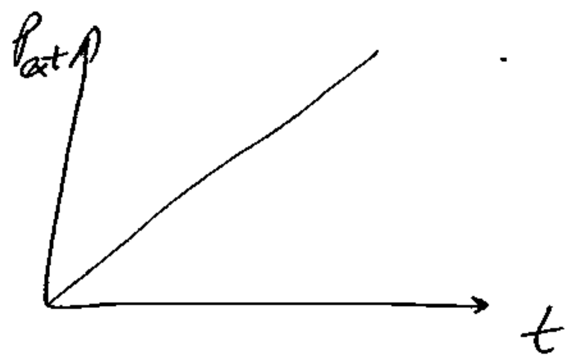
As $\tau =$ constant, $\alpha =$ constant

$$\Rightarrow \omega = \alpha t$$

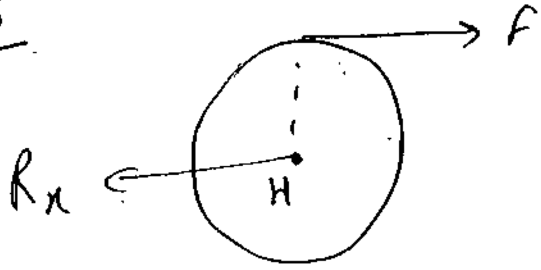
$$P = \tau . \omega = (I\alpha)(\omega) =$$

$$P = I\alpha^2 \cdot t$$

$\Rightarrow$



Q.38



$$R_x = F$$

F will give a constant torque or constant angular acceleration. Thus ω

Q.39

$$\tau_{av} = I\,\frac{\Delta\omega}{\Delta t}$$

$$= \cancel{0.4\,kg\,m^2}\;\frac{\left[30-20\right]\frac{kg\,m^2}{s}}{2s}$$

$$= 5\ Nm$$

Q.40

$$\Delta U = -(Mg)\left(\frac{L}{4}\right)$$

$$I = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16}$$

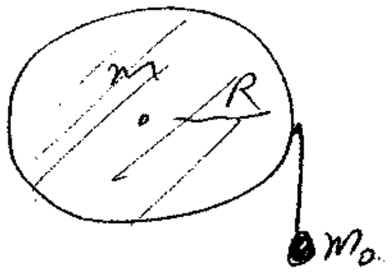
$$= \frac{4ML^2 + 3ML^2}{48} = \frac{7ML^2}{48}$$

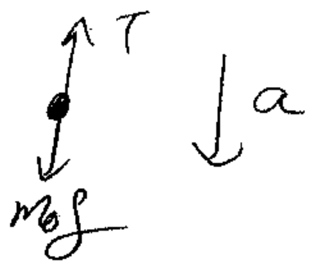
$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}I\omega^2 \cancel{\;\;} - Mg\left(\frac{L}{4}\right) = 0$$

$$\Rightarrow \frac{1}{}\left(\frac{7ML^2}{}\right)\omega^2 = Mg\left(\frac{L}{4}\right)$$

d.41





① $m_0 g - T = m_0 a$

② $T.R = \left(\frac{mR^2}{2}\right)\left(\frac{a}{R}\right)$

$$T.\cancel{R} = \frac{m\cancel{R}\,a}{2}$$

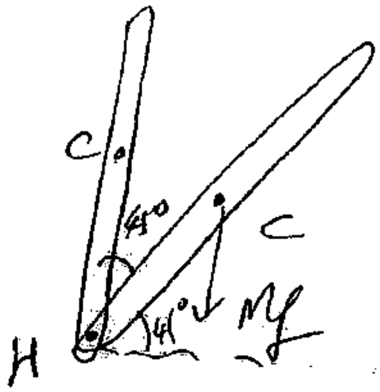
$$\Rightarrow \quad m_0 g - \frac{ma}{2} = m_0 a$$

$$m_0 g = a\left(m_0 + \frac{m}{2}\right)$$

$$a = \frac{m_0 g}{\left(m_0 + \frac{m}{2}\right)} = \frac{2m_0 g}{m+2m_0}$$

Q.42

m, l



$$I_H = \frac{ml^2}{3}$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{Mg\left(\frac{l}{2\sqrt{2}}\right)}{\frac{Ml^2}{3}}$$

$$= \frac{3g}{2\sqrt{2}\,l}$$ 2

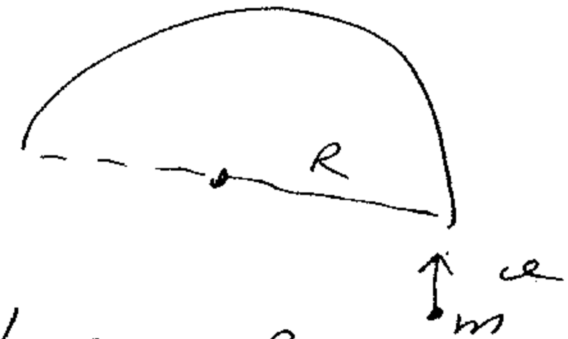
Q.43 $L_0 = L_f$

$$(m.5)(r) = (mu.r) + \left(\frac{mr^2}{2}\right)\left(\frac{u}{r}\right)$$

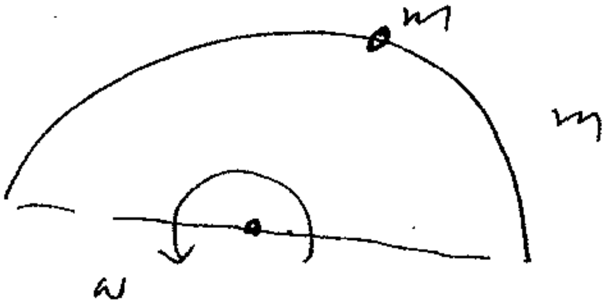
$$5\cancel{mr} = \left(\frac{3\cancel{mr}}{2}\right).u$$

$u = \underline{10}$ m/s 2

Q.44



$L_0 = mvR$



$$\cancel{L_f} \; I\cancel{\omega} = mR^2 + mR^2 = 2mR^2$$

$$L_f = (\cancel{2mR})\; I\omega = (2mR^2)\,\omega$$

$$L_0 = L_f$$

$$\cancel{m} v \cancel{R} = (2\cancel{m}R^{\cancel{2}})\,\omega$$

or $\omega = \dfrac{v}{2R}$ ①

Q.45

$$KE = \frac{L^2}{2I}$$

$$\frac{KE_2}{KE_1} = \frac{L_2^2}{2I_2} \div \frac{L_1^2}{2I_1}$$

$$= \frac{I_1}{I_2} \quad (\text{As } L_2 = L_1)$$

$$= \frac{m r_1^2}{m r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2$$

Q.46 Centripetal force $= m\omega^2 r$

$$m\omega^2 r = 9$$

$$(0.5\,\text{kg})(\omega^2)(2\,\text{m}) = 9$$

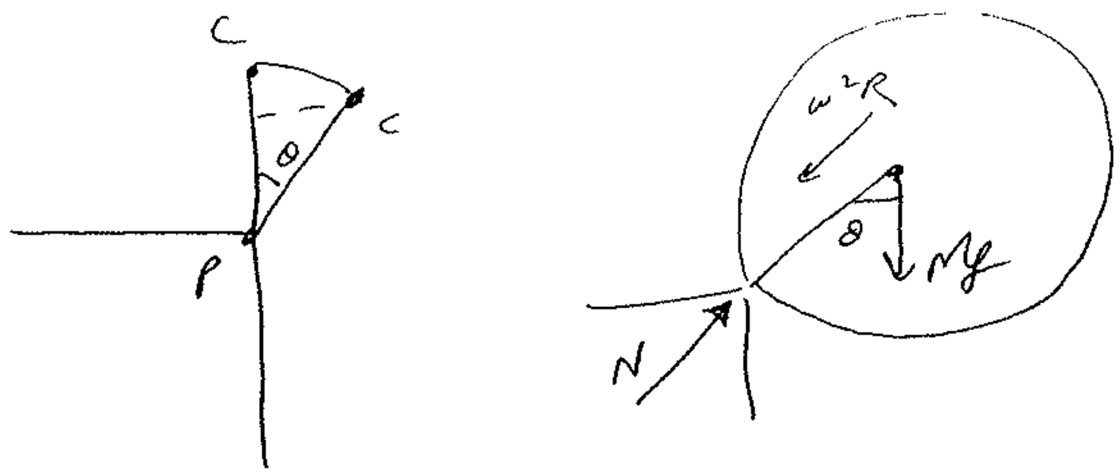
rad

Angular Momentum

$$= I\omega$$

$$= (mr^2)\omega$$

Q.47



$$\frac{1}{2} I_P \omega^2 = MgR(1-\cos\theta)$$

$$\frac{1}{2}\left(\frac{3MR^2}{2}\right)\omega^2 = Mg\cancel{R}(1-\cos\theta)$$

$$\omega^2 = \frac{4g(1-\cos\theta)}{3R}$$

As the cylinder leaves contact with the edge, $N = 0$.

$$\Rightarrow \quad Mg\cos\theta - \underbrace{N}_{0} = M\omega^2 R$$

$$Mg\cos\theta = M\left(\frac{4g(1-\cos\theta)}{3\cancel{R}}\right)\cancel{R}$$

$$3\cos\theta = 4 - 4\cos\theta$$

1

(1)

Q.48

$$V_{COM} = \omega R$$

$$= \sqrt{\frac{4g}{3R}\left(1 - \cos\theta_c\right)} \cdot R$$

$$= \sqrt{\frac{4g}{3}\left(1 - \frac{4}{7}\right) R}$$

$$= \sqrt{\frac{4gR}{7}}$$

Q.49 Rot. KE $= \frac{1}{2} I_c \omega^2$

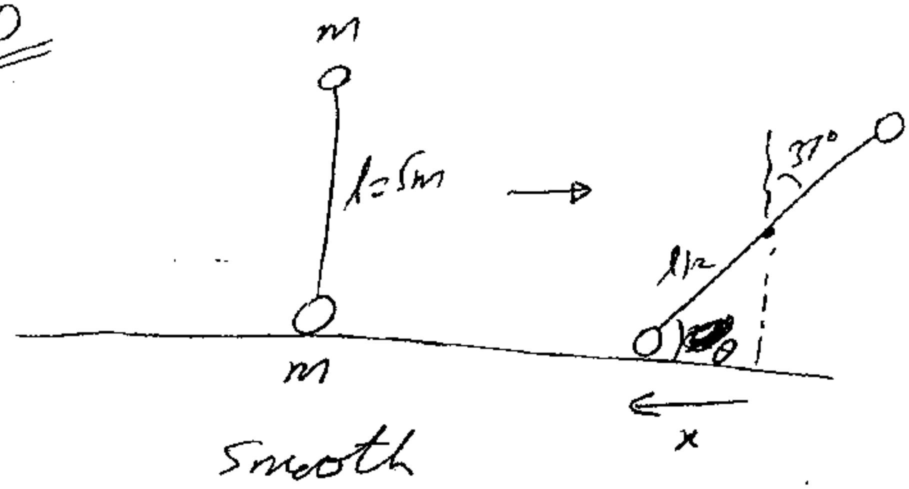
$$= \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{4g}{3R}\left(1 - \frac{4}{7}\right)\right)$$

$$= \frac{MgR}{7}$$

At the HORIZONTAL LEVEL

Translational KE = (Loss in PE) − Rot KE

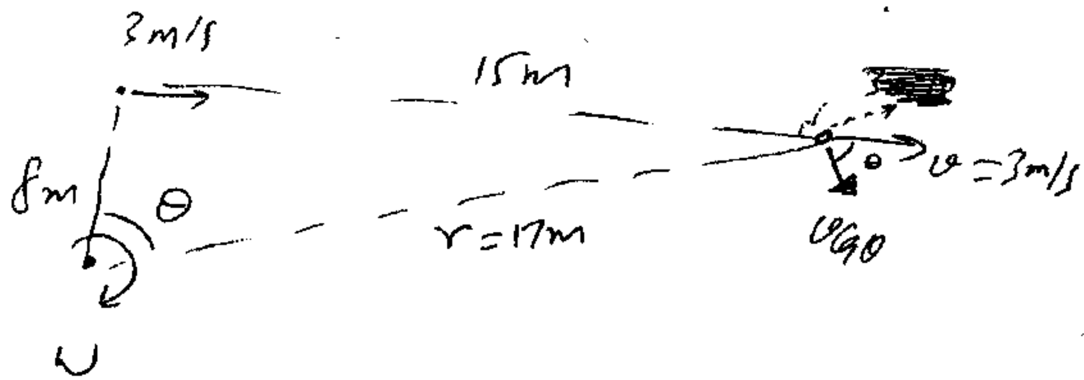
Q.50



$$x = \frac{l}{2}\cos\theta = \frac{l}{2}\cos 53^\circ$$

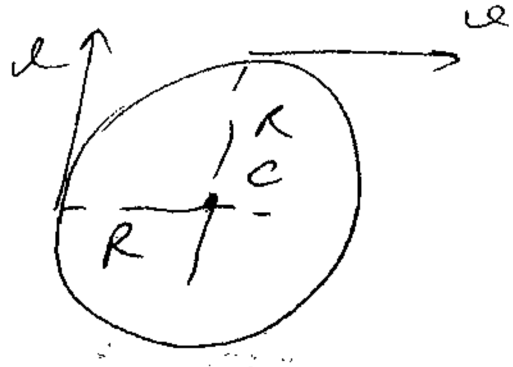
$$= \frac{5}{2} \times \frac{3}{5} = \frac{3}{2} = 1.5\,m$$

Q.51



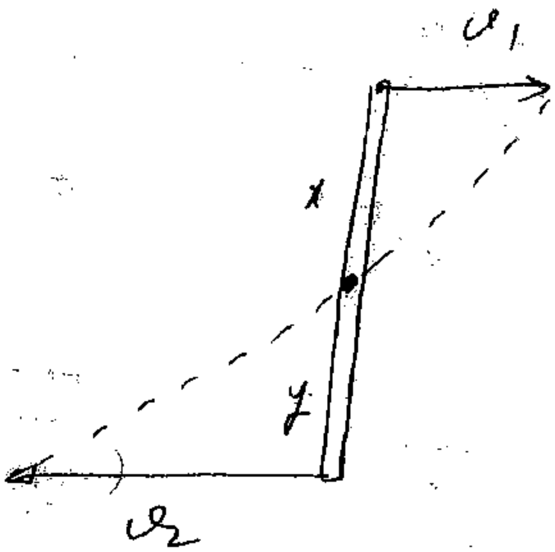
$$\omega = \frac{v\cos\theta}{r} = \frac{3\times 8}{17\times 17} = \frac{24}{289}\,\frac{m}{s}$$

Q.52



$$\omega = \frac{v}{R}$$

Q.53



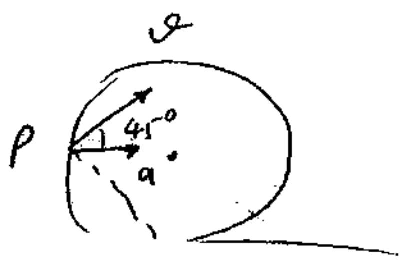
$$\frac{x}{v_1} = \frac{y}{v_2}$$

$$\cancel{x = \frac{v_1}{v_2} \cdot y}$$

$$\frac{x}{x+y} = \frac{v_1}{v_1+v_2}$$

$$x = \frac{v_1 \cdot l}{v_1+v_2}$$

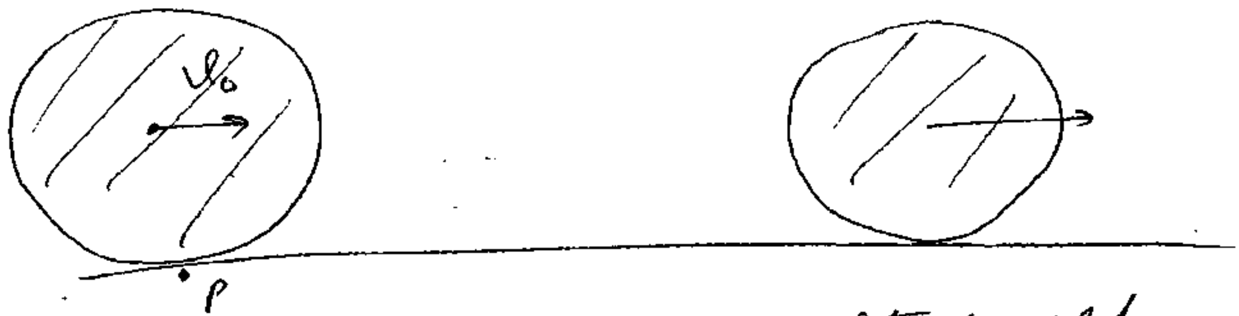
Q.54



$$\bar{a}_{PG} = \bar{a}_{PC} + \cancel{\bar{a}_{CG}} \to 0$$

Q.55 Disc m, R

$t = t_0$



Slides   Starts Rolling.

Conserving Angular momentum about P

$$mv_0R = \left(\frac{mR^2}{2}\right)\omega \to \left(\frac{v}{R}\right) + mvR$$

$$mv_0R = \frac{3mvR}{2}$$

$$v = \frac{2}{3}v_0$$

~~Q.27~~

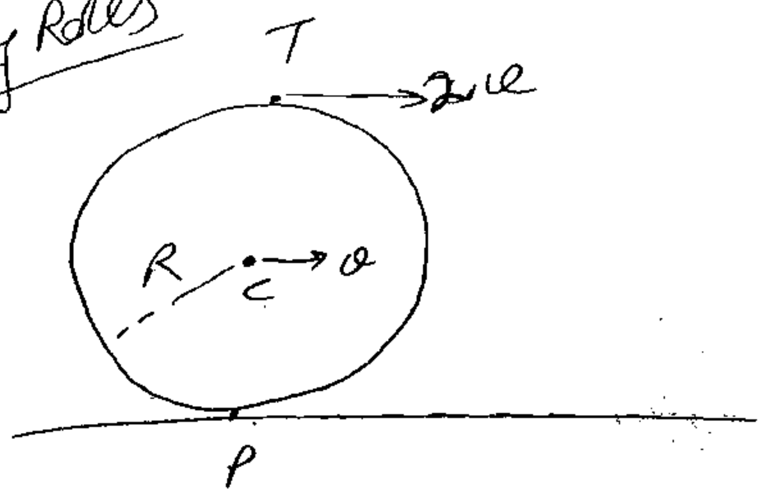
Q.56

$$v = v_0 - (\mu g)t$$

$$\frac{2}{3}v_0 = v_0 - (\mu g)t$$

$$\mu g t = \frac{v_0}{3}$$

Q.57 Ring Rolls



$$\bar{a}_{TG} = \bar{a}_{TC} + \underbrace{\bar{a}_{CG}}_{0}$$

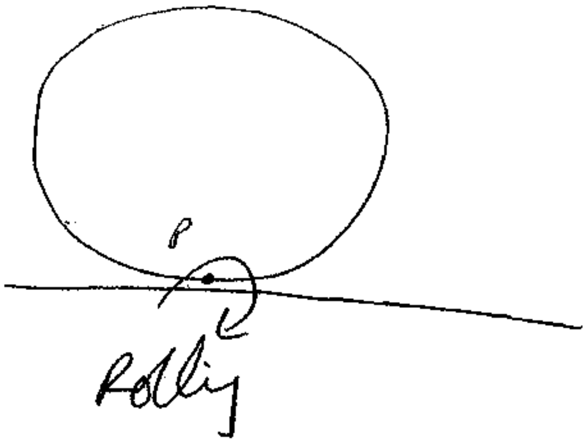
$$= \frac{v^2}{R}$$

But centripetal acceleration $= \dfrac{(2v)^2}{R_c} = \dfrac{4v^2}{R_c}$

$$\Rightarrow \frac{4v^2}{R_c} = \frac{v^2}{R}$$

or $R_c = 4R$

Q.58 Sphere



$$KE = \frac{1}{2} I_P \omega^2$$

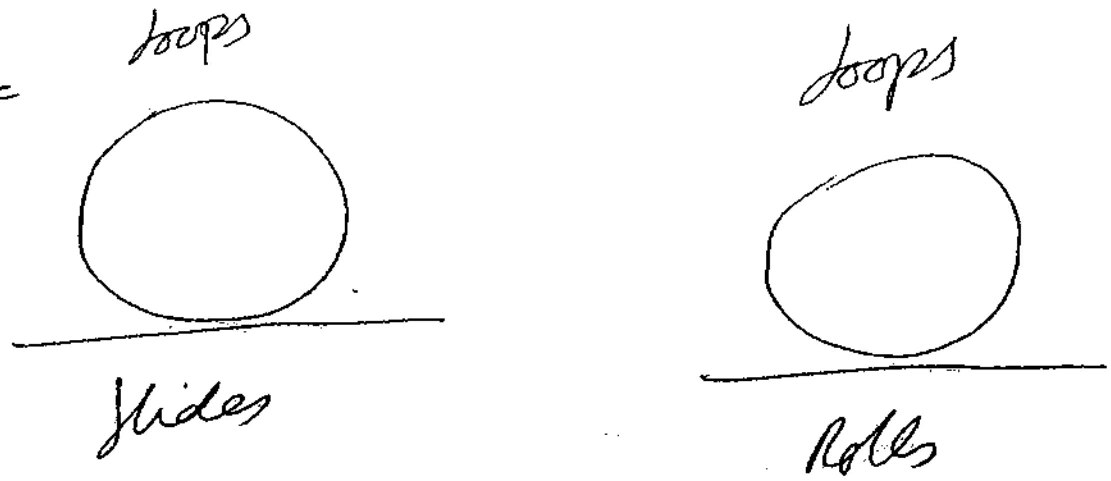
$$= \frac{1}{2}\left(\frac{2}{5}MR^2 + MR^2\right)\omega^2$$

$$KE = \frac{7}{10}MR^2\omega^2 = \frac{7}{10}Mv^2$$

$$\frac{KE_1}{KE_2} = \frac{\frac{7}{10}M_1v^2}{\frac{7}{10}M_2v^2} = \frac{2}{1}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{2}{1} \quad 2$$

Q. 59



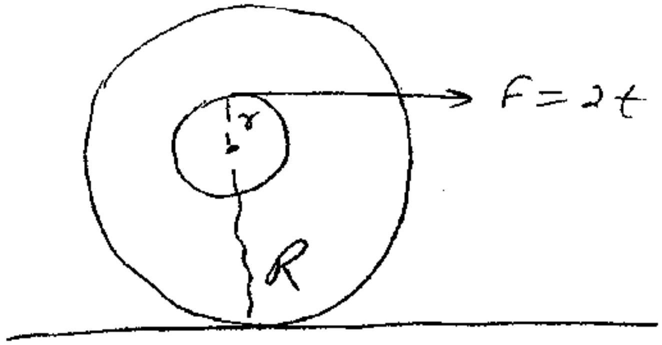
$$\frac{KE_{rolls}}{KE_{slides}} = \frac{\cancel{\frac{1}{2}}\left(MR^2+MR^2\right)\omega^2}{\cancel{\frac{1}{2}}Mv^2}$$

$$= \frac{2(\omega R)^2}{v^2}$$

$$= 2$$

Q. 60 All statements are correct.

Q.61

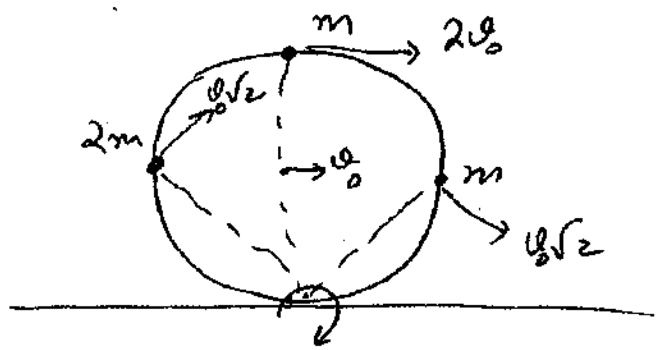


$$\Delta L = \int \tau\, dt \quad \text{where } \tau = F.(R+r)$$
$$= 2t.(R+r)$$

$$= \int_0^t 2t(R+r)\, dt = \frac{2(R+r).t^2}{2}$$

$$= (R+r)t^2$$

Q.62



$$KE_{System} = \frac{1}{2}(2m)\left(v_0\sqrt{2}\right)^2 + \frac{1}{2}(m)\left(2v_0\right)^2 + \frac{1}{2}(m)\left(v_0\sqrt{2}\right)^2$$

$$= \frac{1}{2}mv_0^2\left[4 + 4 + 2 + 2\right] + \underbrace{\frac{1}{2}\left(mR^2 + mR^2\right)}_{RING}$$

$$I_{shell}\text{ about cm} = \frac{2}{3}MR^2$$

Q.63

$$\frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\left(\frac{1}{3}+\frac{1}{2}\right)mv^2 = mgh$$

$$\frac{5}{6}mv^2 = mgh$$

$$\cancel{m}v^2 = \frac{6}{5}\cancel{m}gh$$

$$\cancel{\frac{1}{2}mv^2 =}$$

$$v = \sqrt{\frac{6gh}{5}}$$

Q.64

$$f.R = I_{disc}\,\alpha$$

$$fR = \left(\frac{MR^2}{2}\right)\alpha$$

$$f = \frac{Ma}{2}$$

Q.65

$$F \cdot R = I\alpha$$

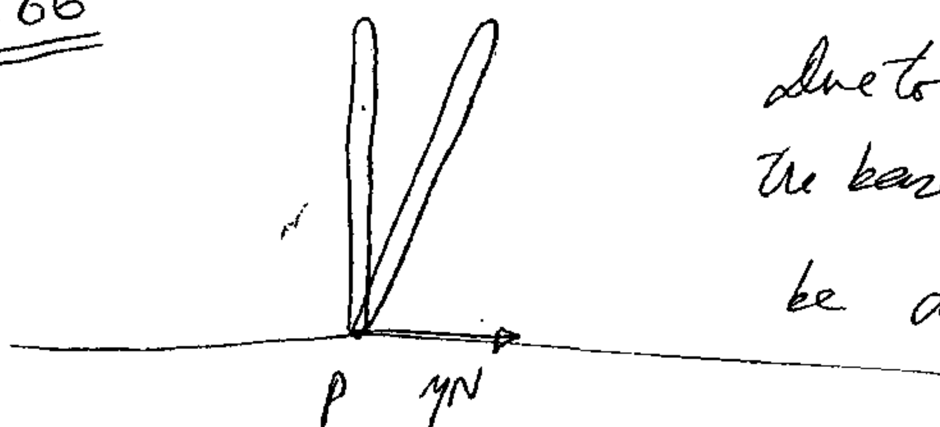
$$F = Ma$$

dividing: $R = \dfrac{I\alpha}{M \cancel{a} \alpha R}$

$$I = MR^2$$

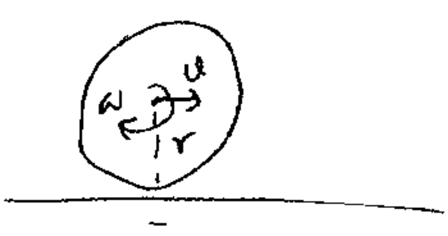
$\Rightarrow$ Body is a thin-pipe.

Q.66



due to friction at the base, COM will be displaced to the right.

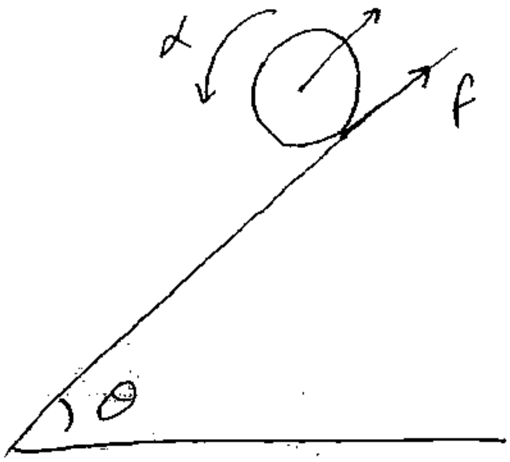
.67



$V_p = V - \omega r$

V_p is + or − or 0, depends

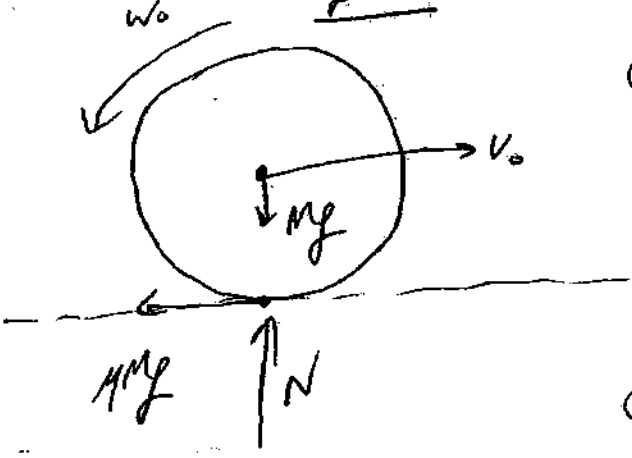
2.68



As α is always as indicated, whether cylinder is going up or down, f will be up the plane.

2.69

Disc



① $0 = V_0 - \mu g t$

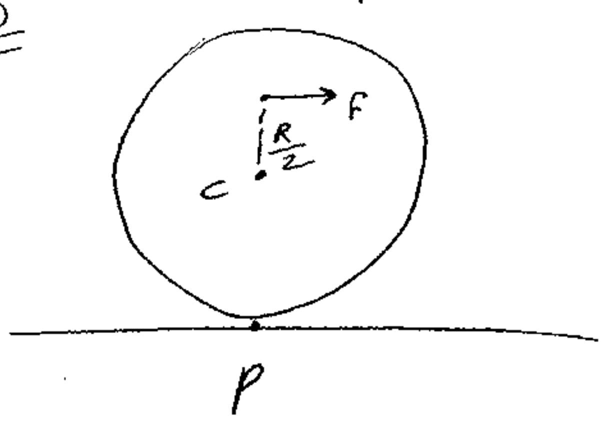
② $0 = w_0 - \left(\frac{\mu M g R}{I}\right) t$

②' $0 = w_0 - \left(\frac{\mu M g R}{\frac{MR^2}{2}}\right) t$

from ① $V_0 = \mu g t$

②' $w_0 R = 2 \mu g t$

$\underline{\underline{1.70}}$ Solid Sphere



$$\tau_P = I_P \cdot \alpha$$

$$\cancel{\frac{FR}{2} = \left(\frac{2}{5}MR^2\right)\alpha}$$

or $\cancel{\alpha = \frac{5F}{4M}}$

$$F\left(\frac{R}{2}+R\right) = \left(\frac{2}{5}MR^2 + MR^2\right)\alpha$$

$$\frac{F \cdot 3R}{2} = \left(\frac{7}{5}MR^2\right)\alpha$$

$$\alpha = \frac{15F}{14MR}$$

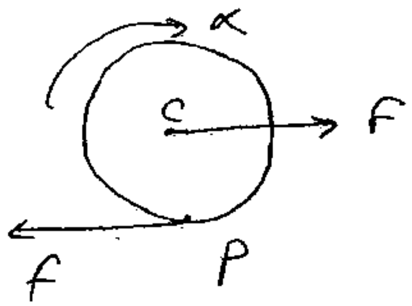
$$a_{\text{Top-most}} = \alpha \cdot (2R) = \frac{15F}{7M}$$

Q. 71

$$\Delta K_P = mgh$$

$$\Delta K_Q = mg(2h)$$

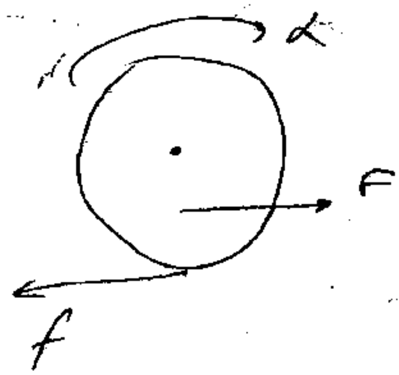
Q. 72



Rolls forward.

Only a Backward friction can provide a clockwise torque.

Q. 73

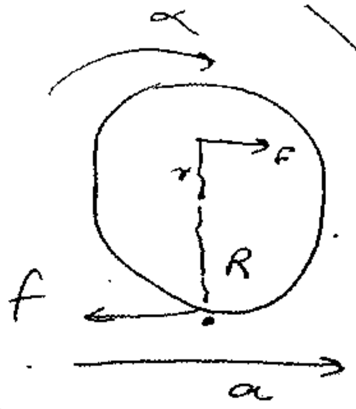


Rolls forward.

f has to be backward to provide a clockwise torque, that is greater than the anti-clockwise torque

... F

Q.14



rolls forward

$$F(R+r) = (\overset{I}{\cancel{MR^2}} + MR^2)\alpha$$

1 $$F(R+r) = \cancel{2MR(R\alpha)a} \left(\frac{I}{R} + MR\right) \cancel{R\alpha} \to a$$

$$F - f = Ma$$

$$f = F - Ma = \left[\frac{I}{R(R+r)} + \frac{MR}{R+r}\right] a - Ma$$

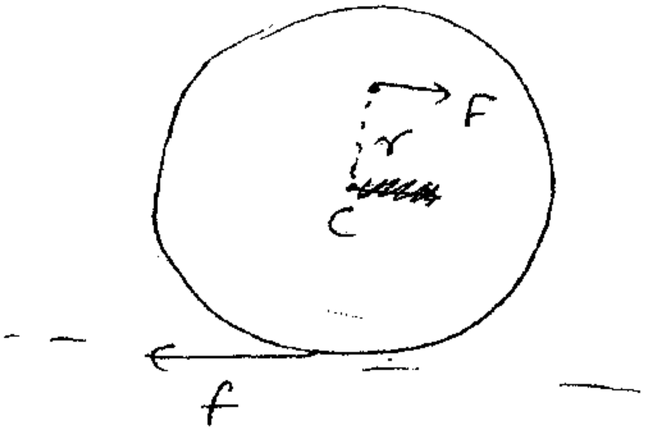
$$= \cancel{\frac{2M(2R)a}{R+r} - 2Ma}$$

$$= \cancel{Ma\left[\frac{2R}{R+r} - 1\right]}$$

$$f = \frac{I}{R(R+r)} + Ma\left[\frac{R}{R+r} - 1\right]$$

$$f = \frac{I}{R(R+r)} + \frac{Ma(-r)}{R+r}$$

Q.74



$I = kMR^2$

(1) $F - f = Ma$

(2) $Fr + fr = (kMR^2)\left(\frac{a}{R}\right)$

$\cancel{F = (kMR)a}$

Do (2) $- r.$(1)

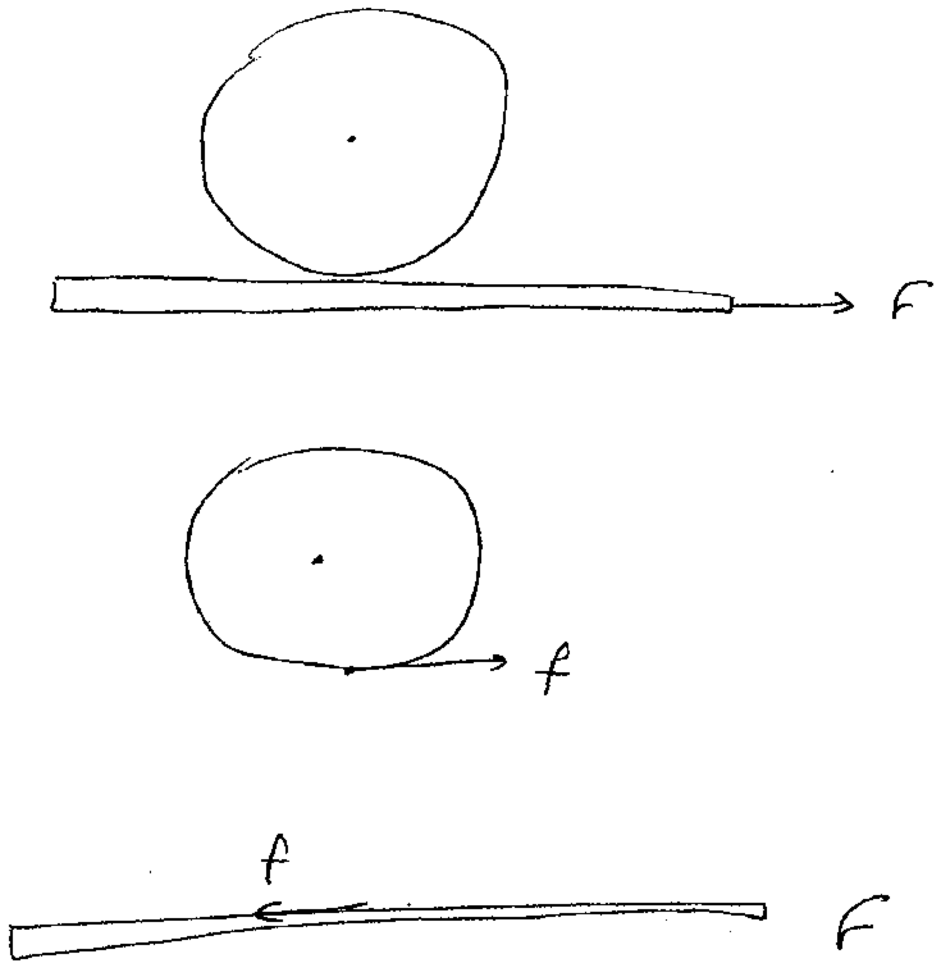
$\Rightarrow$ $2fr = (kMR)a - (Ma).r$

$2fr = Ma(kR - r)$

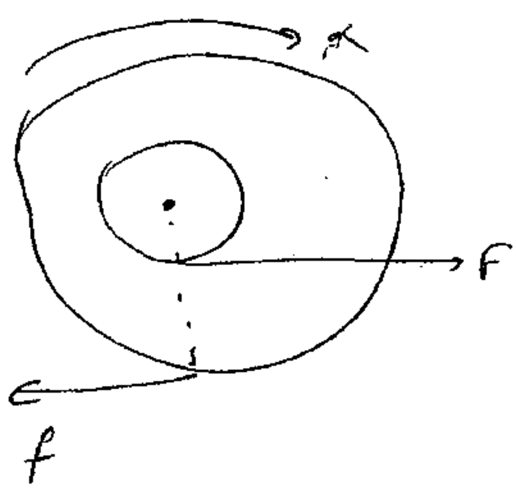
$$f = \frac{Ma(kR - r)}{2r}$$

f can be +ve or -ve, as r can

Q.75



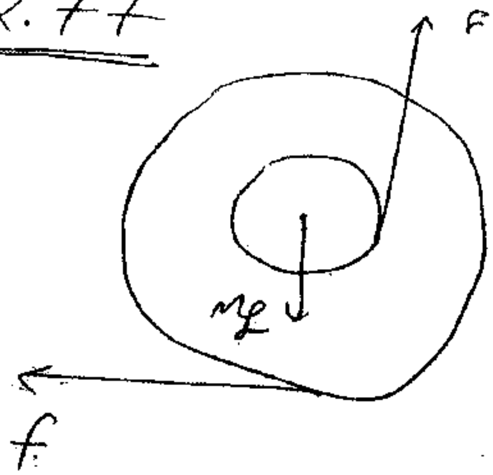
Q.76



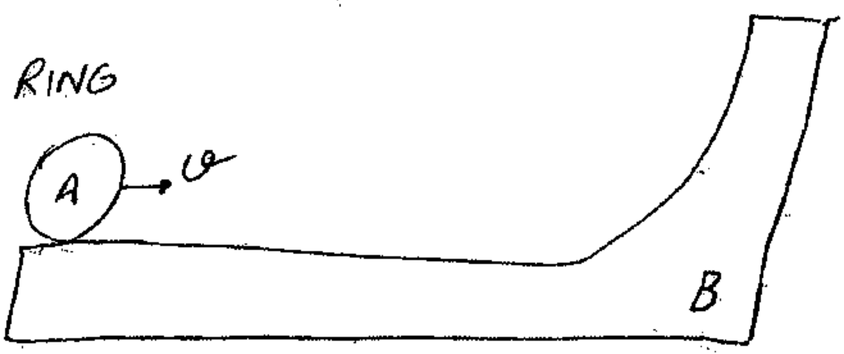
Only a backward f can give a

torque

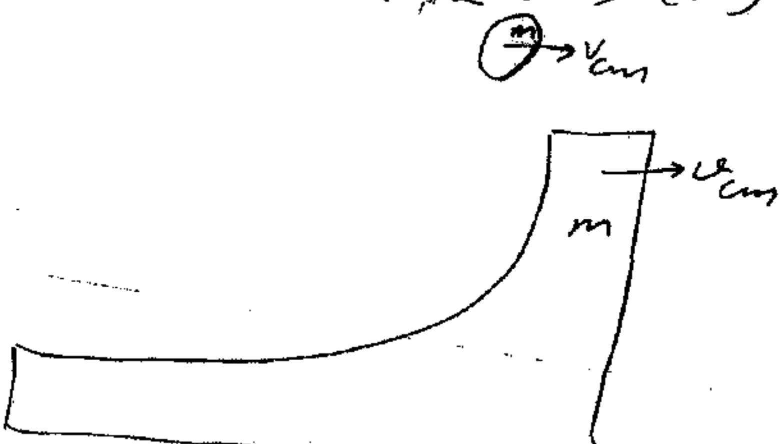
Q.77



Q.78



$$(KE)_i = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v}{R}\right)^2 = Mv^2$$



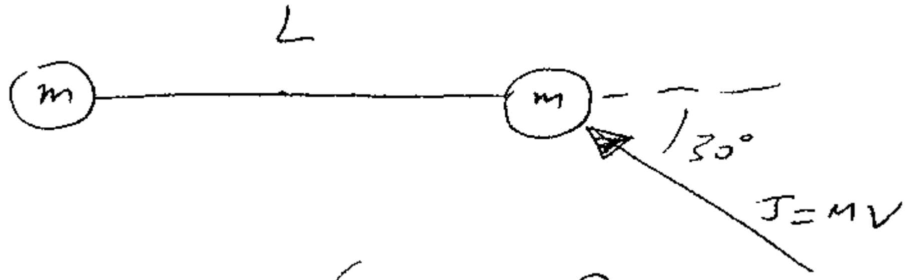
$P_f = P_i$

$2m v_{cm} = m v$

$v_{cm} = \frac{v}{2}$

$$(KE)_f = \frac{1}{2}I\omega^2 + \frac{(2m)}{2}\left(\frac{v}{2}\right)^2 = \frac{Mv^2}{2} + \frac{Mv^2}{4} = \frac{3Mv^2}{4}$$

Q.79



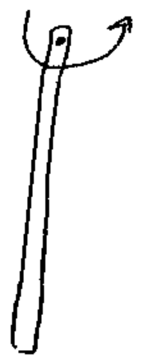
$$I\omega = (J \sin 30^\circ)\left(\frac{L}{2}\right)$$

$$\left[M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2\right]\omega = MV \cdot \frac{L}{4}$$

$$\omega\left(\frac{\cancel{M}L^{\cancel{2}}}{2}\right) = \cancel{M}V\frac{\cancel{L}}{4}$$

$$\omega = \frac{V}{2L}$$ ✓

Q.80



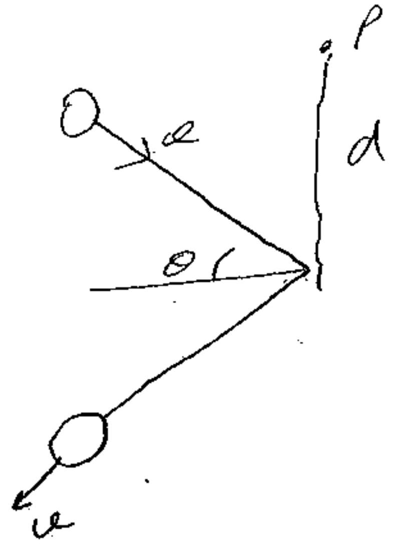
$$J = I\omega = \frac{ML^2}{3}\cdot\omega$$

$$(10\,N s) = \left(\frac{2\,kg\cdot(1\,m)^2}{3}\right)\cdot\omega$$

$$\omega = \frac{30}{2} = 15\,\frac{rad}{s}$$

$$KE_i = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}\right)(15)^2 = \frac{225}{3}$$

81



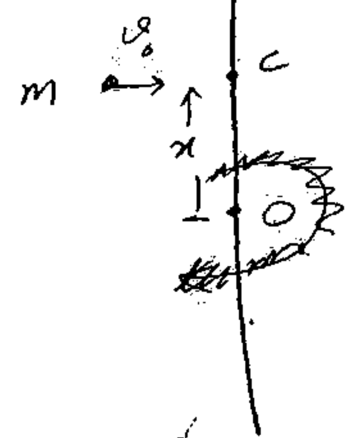
$$\Delta J = (-mvd\cos\theta) - (mvd\cos\theta)$$

$$|\Delta J| = 2mvd\cos\theta$$

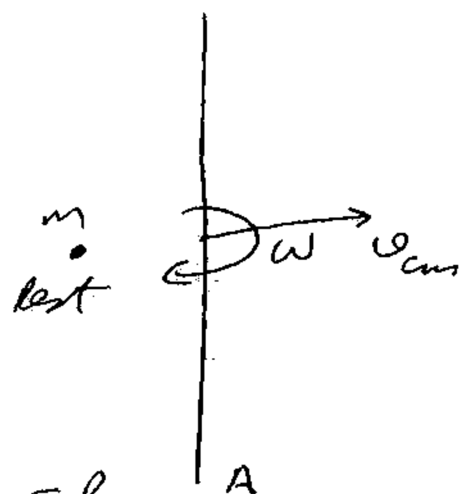
(4)

Q.P2

Before M, L



After



(1) $P_i = P_f$

$$Mv_{cm} = mv_0$$

$$v_{cm} = \left(\frac{mv_0}{M}\right)$$

(2) About O

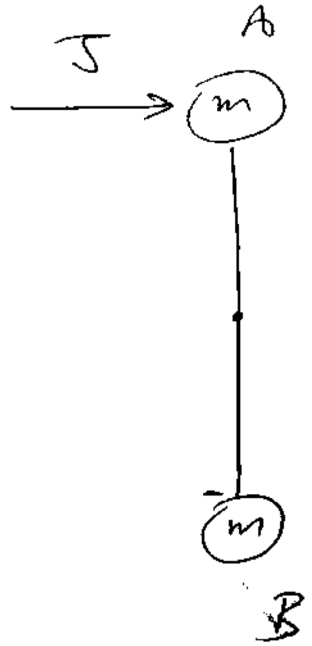
$$L_i = L_f$$

$$mv_0 x = \left(\frac{ML^2}{12}\right)\omega$$

If $V_A = 0 \Rightarrow \frac{\omega L}{2} = v$

$$\frac{12\, mv_0 x}{ML^2} \cdot \frac{L}{2} = \frac{mv_0}{M}$$

Q.83



① $J = (2m)\, v_{cm}$

② $J\left(\frac{l}{2}\right) = \left[m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2\right]\omega$

$$\Rightarrow v_{cm} = \frac{J}{2m}$$

$$\omega = \frac{\frac{Jl}{2}}{\frac{ml^2}{2}}$$

$$\omega = \frac{J}{2ml}$$

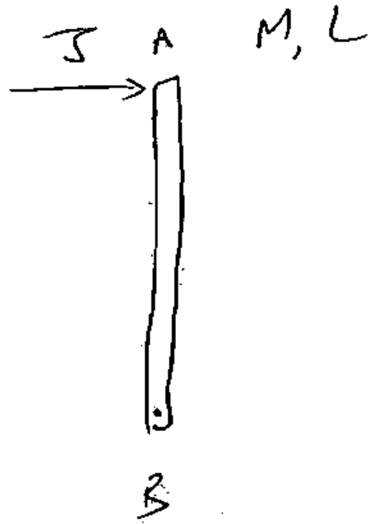
$$\omega l = \frac{J}{2m}$$

$$V_A = V_{cm} + \omega l$$

$$= \frac{J}{2m} + \frac{J}{2m} = \frac{J}{m}$$

Q.~~84~~

Q.85



$$J = M v_{cm}$$

$$\frac{J L}{2} = \left(\frac{ML^2}{12}\right)\omega$$

$$\Rightarrow \omega = \frac{6J}{M.L}$$

$$|v_B| = V = |v_{cm} - \omega L|$$

$$= \left|\frac{J}{M} - \frac{6J}{M}\right|$$

$$V = \frac{+5J}{M}$$

$$J = \frac{MV}{5}$$

Q. 86

m, l

① $J = m v_{cm}$

② $J \cdot \frac{l}{2} = \left(\frac{ml^2}{12}\right)\omega$

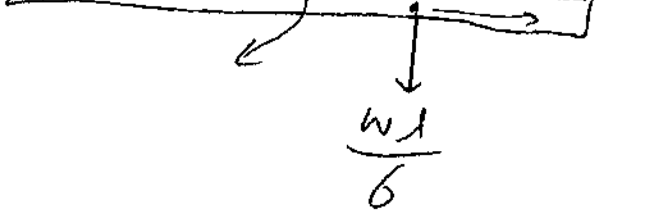
$\frac{J}{2} = \frac{ml}{12}\omega$

$\Rightarrow \omega = \frac{6J}{ml}$

After $t = \frac{\pi m l}{12 J}$,

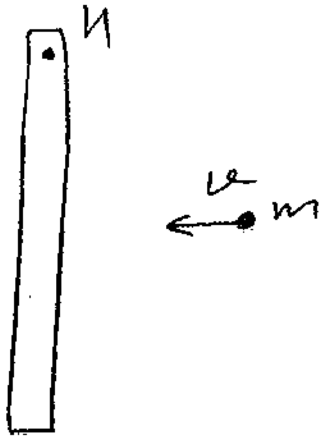
$\theta = \omega t = \left(\frac{6J}{ml}\right)\left(\frac{\pi m l}{12J}\right)$

$= \frac{\pi}{2}$



$v_P = \sqrt{\left(\frac{\omega l}{6}\right)^2 + (v_{cm})^2} = \sqrt{\left(\frac{J}{m}\right)^2 + \left(\frac{J}{m}\right)^2}$

Q. 87



Angular momentum is conserved about Hinge

$$\frac{mvL}{2} = \left(\frac{ML^2}{3}\right)\omega$$

(1) $$\frac{M}{m} = \frac{3vL}{2L^2\omega} = \frac{3v}{2\omega L}$$

(2) Elastic collision ⇒ KE is conserve

$$\cancel{\tfrac{1}{2}}mv^2 = \cancel{\tfrac{1}{2}}\left(\frac{ML^2}{3}\right)\omega^2$$

(2') $$\left(\frac{v}{\omega L}\right)^2 = \frac{M}{3m}$$

from (1) & (2'), $\left(\frac{v}{\cdot}\right)^2 = \frac{1}{3}\left(\frac{3v}{2\omega L}\right)$

QUESTIONS

~~QUESTION~~ TO PONDER

Q.1 Should there be any matter at the center of gravity of an object?

Q.2 The torque exerted by a force about some axis depends on the choice of axis. How can the condition, $\sum \tau_{z,ext} = 0$, be satisfied for any choice of axis?

Q.3 What is the path of a particle in a rigid object rotating about a fixed axis?

Q.4 If a rigid object has only translational motion (for example, the body of a car traveling in a straight line on a flat road), are there any points within the object that always have the same velocity as the center of mass? If so, which ones?

Q.5 If a rigid object has only rotational motion about a fixed axis are there any points within the object that always have the same velocity as the center of mass? If so which ones?

Q.6 If a rigid object moves with both transition and rotation about an axis with a fixed orientation (for example, a rolling wheel), are there any points within the object that always have the same velocity as the center of mass? If so, which ones?

Q.7 What is the direction of the angular velocity of a rigid object rotating about a fixed axis? What is the direction of the linear velocity of a particle in a rigid object rotating about a fixed axis?

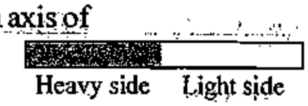
Q.8 A rigid object rotating about a fixed axis has nonzero angular velocity and angular acceleration. Particle A in the object is twice as far from the axis of rotation as particle B. What is the ratio of the following quantities for A and B:

(a) the angular speeds
(b) the linear speeds
(c) the magnitudes of the angular accelerations
(d) the tangential components of the accelerations
(e) the radial components of the accelerations
(f) the magnitudes of the linear accelerations?

Q.9 Do the angular velocities of the hands of a wall clock point into the wall or out of the wall? At the instant the clock is unplugged, do the angular accelerations of the hands point into the wall or out of the wall?

Q.10 A car is moving forward and slowing down. Is the direction of te angular velocity of the wheels toward the driver's left or right? What is the direction of the angular acceleration of the wheels?

Q.11 One side of a door (figure) is made of material with a larger mass density than the other side. To minimize the moment of inertia about an axis of rotation along the hinges, should the hinges be placed at the heavier side or the lighter side? Explain.



Q.12 Consider three rods made of the same material and with the same length and mass, but with different cross-sectional shapes (figure). Which of the three has the largest moment of inertia about an axis through the center of mass and along the rod's long axis? Which rod has the smallest moment of inertia about that axis?

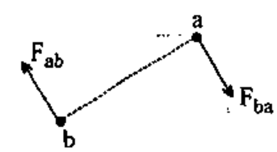
Q.13 Is it possible to find an axis of rotation (call the axis A) about which the moment of inertia for an object is smaller than the moment of inertia about an axis through the center of mass and parallel to A?

Q.14 Suppose you are designing a cart for coasting down a hill. To maximize your coasting speed, should you design the wheels so that their moments of inertia about their rotation axes are large or small, or does it matter? Keeping the moment of inertia of the wheels fixed, will the cart's speed be increased or decreased by increasing the mass of the cart's body? Assume that mechanical energy is conserved.

Q.15 If a particle is in uniform circular motion, is either the direction or the magnitude of the angular momentum about the center of its motion constant? If the particle's speed is changing as it travels in a circle, is either the direction or the magnitude of the angular momentum constant?

Q.16 If the net torque exerted on a particle is in the same direction as the particle's angular momentum, is there a change in the direction of the particle's angular momentum? Is there a change in the magnitude of the particle's angular momentum?

Q.17 Consider an isolated system of two particles a and b that interact with each other such that $F_{ab} = -F_{ba}$ but the direction of the forces is perpendicular to the line joining the particles, as shown in figure. What happens to this system as time goes on? Is total linear momentum conserved? Is total angular momentum conserved? Is such a system possible? Explain.



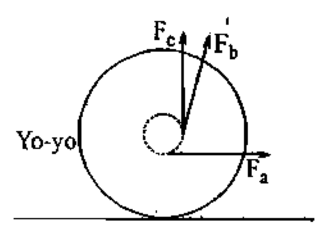
Q.18 When a billiard ball rolls down a slope without sliding, what force is responsible for the torque that causes the angular acceleration about an axis through the center of mass? What force is responsible for the torque that causes the angular acceleration about an axis through the point of contact with the surface?

Q.19 Legend has it that a cat always lands on its feet. High-speed cameras have shown that when a cat begins a fall with its feet up, its tail rotates rapidly and the cat's body also rotates, so that it does, infact, land on its feet. Explain the motion in terms of conservation of angular momentum. Include in your explanation a comparison of the sense of the rotation of the cat's body with that of its tail. How do you think a bobtailed cat might do in a fall that begins with its feet up?

Q.20 A small satellite orbiting the earth has only one window for the astronaut, and the window is facing away from the earth. Explain how the astronaut can rotate the satellite so he can view the earth and not use any rocket fuel in the process.

Q.21 A spinning ice skater rapidly extends his arms. [Neglect friction during the time interval the arms are extended]. Is his kinetic energy conserved? Is his potential energy conserved? Is his mechanical energy conserved? Is his angular momentum conserved? If any of these quantities are not conserved, tell whether they increase or decrease.

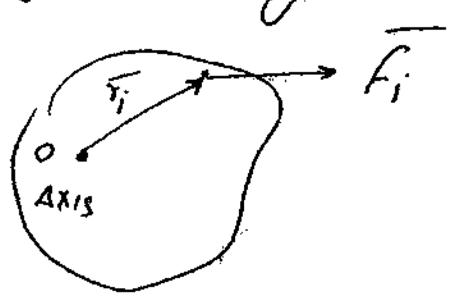
Q.22 A yo-yo with half the string wound on its axle is placed on its edge on the floor, as shown in figure. Consider pulling gently on the string in the three different directions indicated by F_a, F_b and F_c in the figure. The force in each case is gentle enough so that the yo-yo does not slide. In which case, if any, does string wind onto the yo-yo, and in which case, if any, does the string wind off the yo-yo?



Questions to Ponder

A1 Not necessary, think of a ring or a spherical shell.

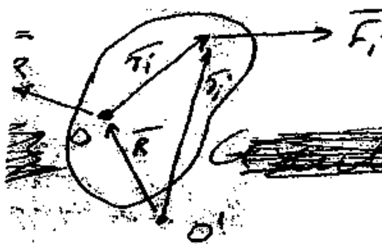
A.2 Consider a Rigid body in equilibrium about an axis



Several forces act on it such that

$$\sum \bar r_i \times \bar F_i = 0 \quad \text{and} \quad \sum \bar F_i + \bar F_R = 0$$

where $\bar F_R$ is the reaction force acting at the axis. If now, we take the axis elsewhere,



$$\sum \bar\tau = \sum \bar r_i' \times \bar F_i + \sum \bar R \times \bar F_R$$
$$= \sum \bar r_i' \times \bar F_i - \sum \bar R \times \bar F_i$$
$$= \sum (\bar r_i' - \bar R) \times \bar F_i = \sum \bar r_i \times \bar F_i = 0$$

A3 Circular

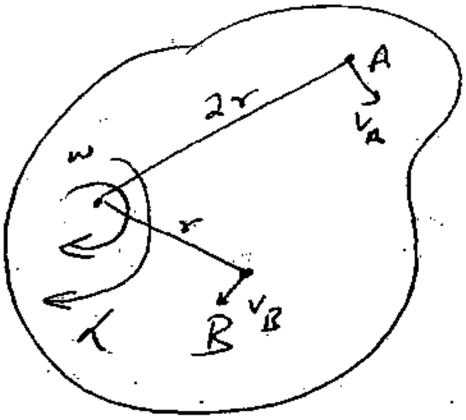
4 All those points which do not have a circular motion about its centre of mass

A.5 All points lying on its axis.

A.6 All points lying on its axis

A.7 Use Right-hand rule to get direction of ω. Further, $\bar{v} = \bar{\omega} \times \bar{r}$

A.8



a) $\omega_A = \omega_B = \omega$

b) $v_A = \cancel{\omega}\ \omega(2r)$

$v_B = \omega r$

$\Rightarrow v_B = \dfrac{v_A}{2}$

c) $\alpha_A = \alpha_B$

d) $(a_T)_A = \alpha(2r)$

$(a_T)_B = \alpha r$ or $(a_T)_A = 2(a_T)_B$

(e) $(a_r)_A = \omega^2(2r)$

$(a_r)_B = \omega^2 r$ or $(a_r)_A = 2(a_r)_B$

(f) $|\bar{a}| = \sqrt{(a_T)^2 + (a_r)^2}$

A.9 Angular velocities of a clock's hands point into the wall.
When clock is unplugged, the angular accelerations point out of the wall.

A.10 As car is moving forward, direction of the angular velocity of the wheels are towards the driver's left.
If the car is slowing down, the angular acceleration of the wheels is towards ~~right~~ right.

A.11 Axis should be placed at the heavier side to minimize I

A.12 ~~As the shapes have the same mass, it means they have to~~

A.12

$$3l = 2\pi r = 4L = K$$

where l = length of the side of Δ

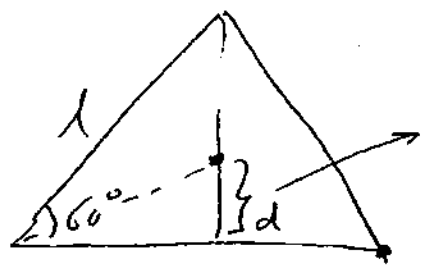
r = radius of circle

L = length of the side of square

For Δ, mass of side $= \frac{M}{3}$

Square, mass of side $= \frac{M}{4}$

$$I_\Delta = 3\left(\left(\frac{M}{3}\right)\left(\frac{l^2}{12}\right) + \left(\frac{M}{3}\right)\left(\frac{l}{2\sqrt{3}}\right)^2\right)$$



$$d = \frac{l}{2}\tan 30^\circ = \frac{l}{2\sqrt{3}}$$

$$= \frac{Ml^2}{12} + \frac{Ml^2}{12} = \frac{Ml^2}{6}$$

$$I_O = MR^2$$

$$I_\square = 4\left[\left(\frac{M}{4}\right)\left(\frac{L^2}{12}\right) + \frac{M}{4}\left(\frac{L}{2}\right)^2\right] = \frac{ML^2}{12} + \frac{ML^2}{4}$$

$$I_\triangle : I_\circ : I_\square = \frac{l^2}{6} : R^2 : \frac{L^2}{3}$$

Now $3l = 2\pi R = 4L = K$

$$\text{or } I_\triangle : I_\circ : I_\square = \frac{(K/3)^2}{6} : \left(\frac{K}{2\pi}\right)^2 : \frac{(K/4)^2}{3}$$

$$= \frac{K^2}{9\times6} : \frac{K^2}{4\pi^2} : \frac{K^2}{16\times3}$$

$$\Rightarrow I_\triangle < I_\square < I_\circ$$

A.13 Parallel Axis theorem:

$$I_p = I_c + Md^2$$

The least value of $I_p = I_c$ if $d = 0$

Hence not possible.

1.14 $a = \frac{g\sin\theta}{}$ for a body rolling

If I is less, acceleration will be more.

Q.15 $\bar{L} = I\bar{\omega}$

$|\bar{L}| = I|\bar{\omega}|$

If particle is in uniform circular motion, its angular momentum is constant, both in direction and magnitude

If the particle's speed is changing, then magnitude of angular momentum will change but its direction will remain constant as long as the ~~b~~ particle does not change its sense of rotation.

A.16

$$\overline{\tau}_{Av} = \frac{\Delta \overline{L}}{\Delta t} \quad \text{or} \quad \overline{L}_f = \overline{L}_i + \int \overline{\tau}\, dt$$

Under the given condition, there is no change in the direction of angular momentum but the magnitude does change.

A.17 As $\Sigma \overline{F} = \overline{F}_{ab} + \overline{F}_{ba} = 0$, linear Momentum is conserved.

Torque is acting on the system, so angular momentum goes on increasing, so does its rotational kinetic energy.

This means the system has a source of energy e.g. a battery operated fan

A.18 About the centre of mass, friction provides the torque.

About an axis, passing through the ~~centre~~ point of contact with the surface, "$mg \sin\theta$" component provides the torque.

A.19 Tail spins one way, the cat spins the other way.

A.20 Astronaut can rotate clockwise, the satellite will rotate anticlockwise to conserve angular momentum.

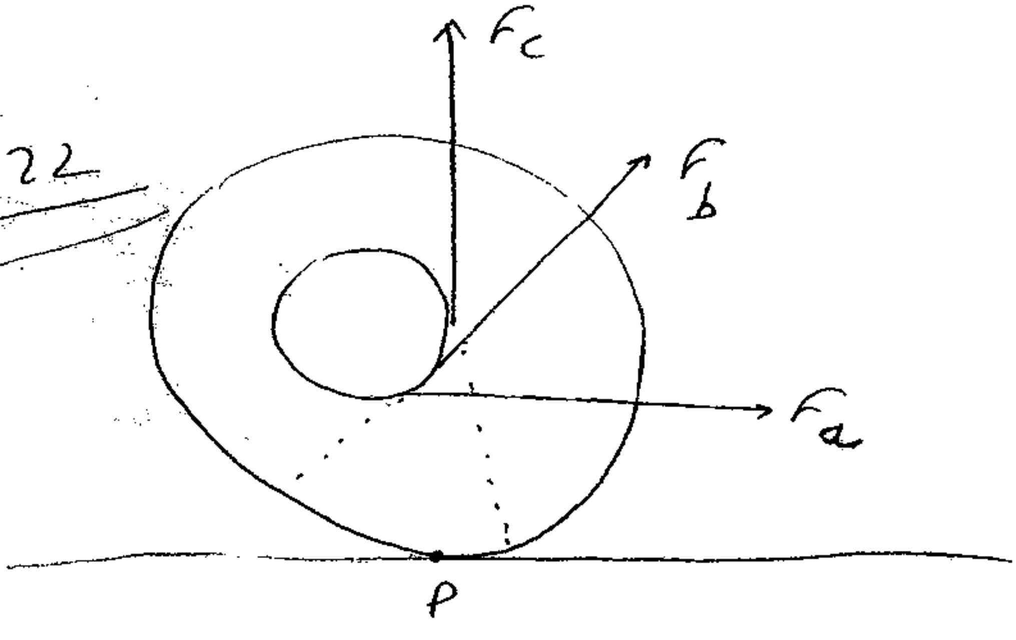
A.21 Angular momentum is conserved. ... [illegible] changes.

$$KE = \frac{L^2}{2I} = \cancel{\text{constant}}. \text{ As } I \downarrow,$$

$KE \uparrow$.

The KE is supplied the PE of the skator.

1.22

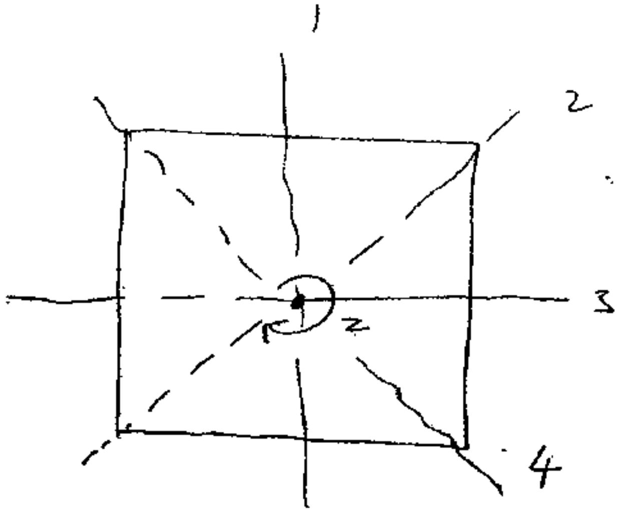


In case of F_a & F_b, torque about the point of contact is clockwise, the YO-YO will move to right and string will wind on to

ROTATIONAL DYNAMICS

(SOLVED MAMCQ QUESTIONS)

A.1 (ABCD)



$$I_1 + I_3 = I_z \quad (\perp^r \text{ axes theorem})$$

Also, $I_1 = I_3$ (Symmetry)

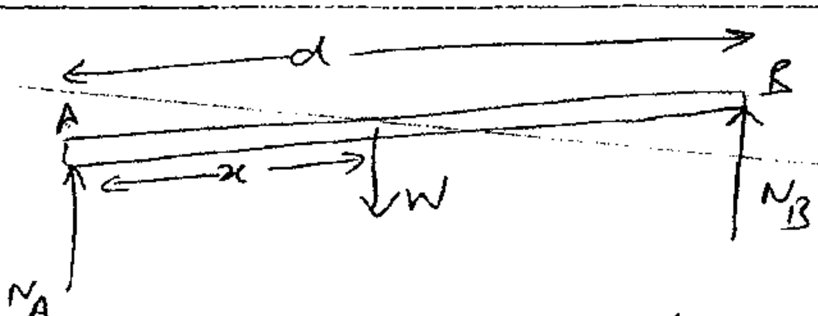
$$\Rightarrow \quad I_1 = I_3 = \frac{I_z}{2}$$

Similarly, $I_2 + I_4 = I_z$

Also $I_2 = I_4$ (Symmetry)

$$\Rightarrow \quad I_2 = I_4 = \frac{I_z}{2}$$

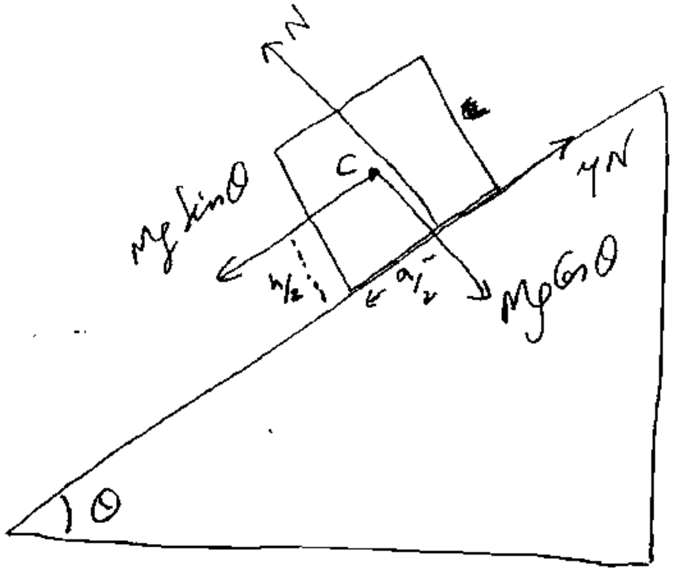
2 (B, D)



Taking τ_A : $W\cancel{x} = N_B \cdot d$ or $N_B = \frac{W.x}{d}$

τ . $W(d-x) = N_A \cdot d$ or $N_A = W(d-x)$

Q.3



For sliding:

~~$f \leq \mu mg\cos\theta$~~

$mg\sin\theta \geq \mu mg\cos\theta$

$\tan\theta > \mu$ (1)

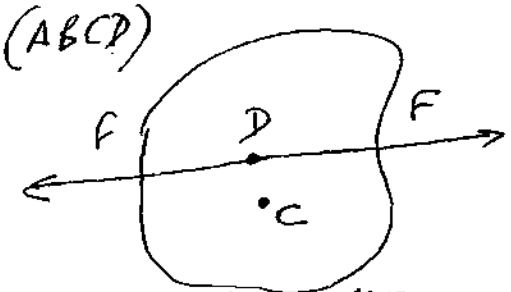
For toppling: $(mg\sin\theta)\frac{h}{2} > (mg\cos\theta)\frac{a}{2}$

$\Rightarrow \tan\theta > \left(\frac{a}{h}\right)$ (2)

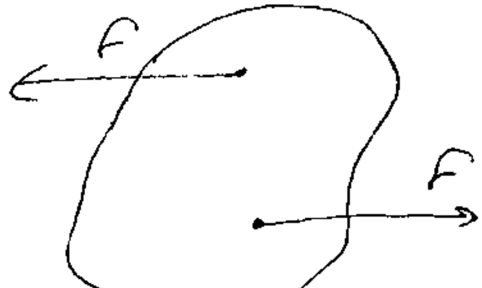
If $\mu > \frac{a}{h}$ (toppling will precede sliding.)

Else $\frac{a}{h} > \mu$ (sliding will precede toppling.)

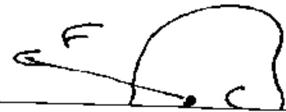
Q.4 (ABCD)



D ≠ Centre of mass
Equilibrium of 2 forces

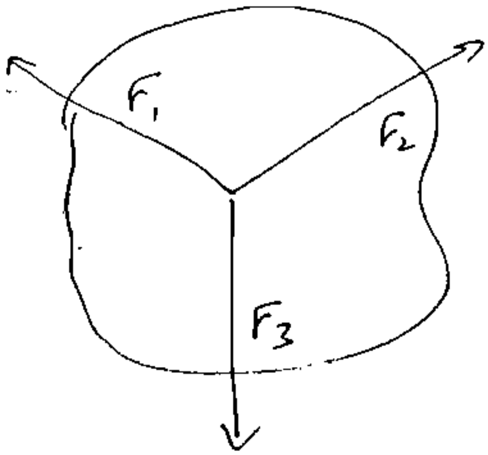


Will constitute a couple
so eqlbm. is not possible

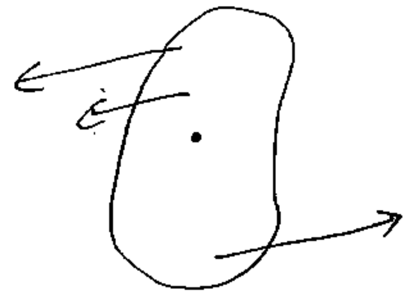


A.4 (contd)

(B)

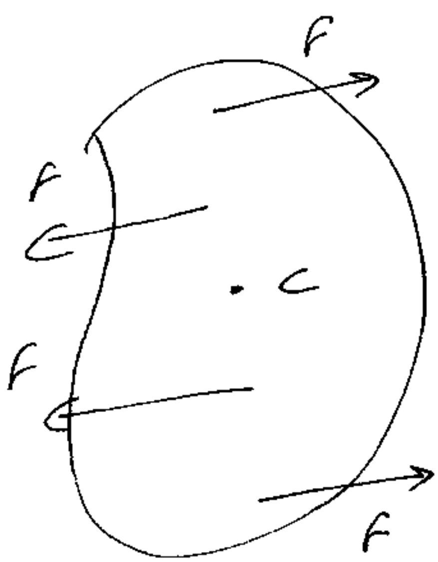


(C)



Minimum 3 forces are required to create equilibrium if their lines of action are not parallel.

(D)



Minimum 4 forces are required, if their lines of action are parallel and all the forces have the same magnitude.

A.5 (B) $I = \Sigma M_i r_i^2$

Here r_i decreases, hence I decreases.

Angular Momentum is conserved.

$KE = \dfrac{L^2}{2I}$, as I decreases, KE increases

As Angular momentum $= I\omega$ is constant.

~~..~~ ω increases, as I decreases.

A.6 (ABC) Rotational $KE = \frac{1}{2} I\omega^2$

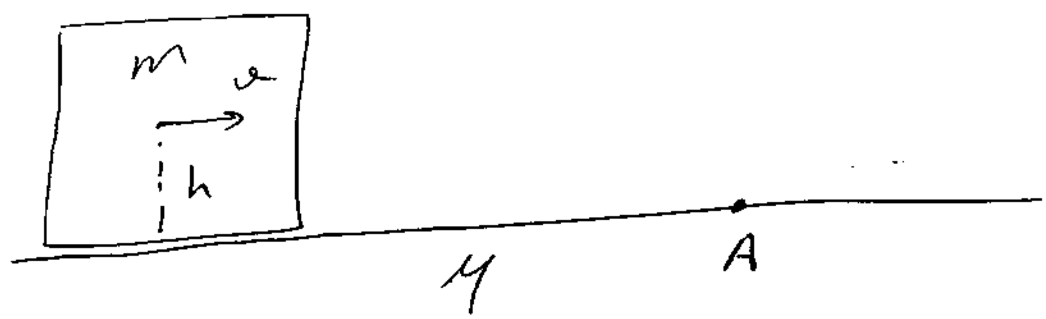
R.K.E. depends on the value of I.

$$I_2 = \frac{1}{2}\left[mb^2 + mb^2 + Ma^2 + Ma^2\right]\omega^2$$

$$= (Ma^2 + mb^2)\omega^2$$

Hence (D) is incorrect.

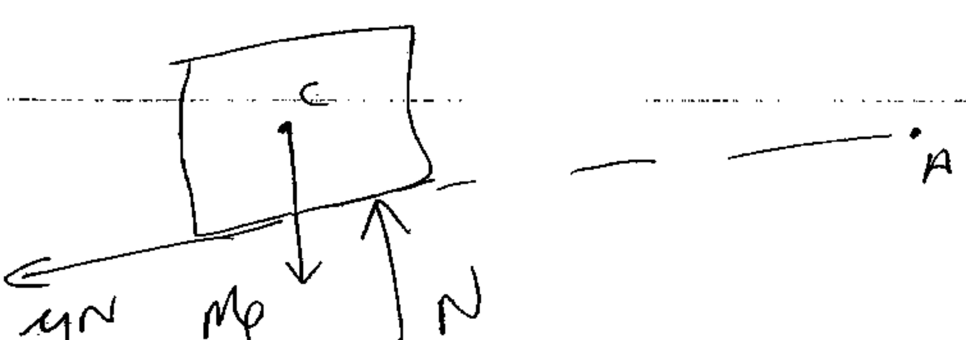
A.7



About A.

Angular = momentum × momentum arm
Momentum = $(mv).(h)$

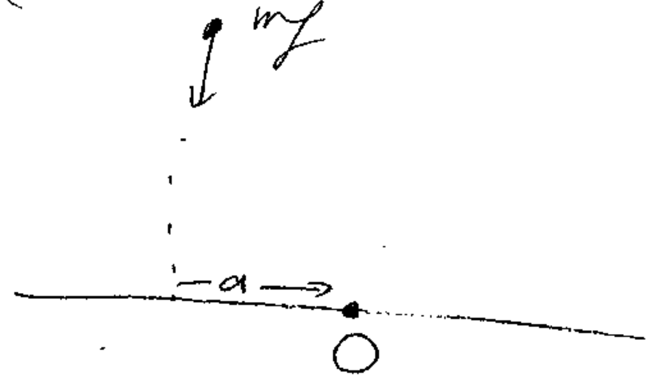
- velocity of block decreases, as it decelerates due to the sliding friction force μmg.
- ~~About A. weight Aste bally decelerates, the normal~~



Normal force does not pass C, otherwise block will topple.

although $Mg = N$ their torques

A.8 (ACD)



$$\tau = \frac{d}{dt}(I\omega) = \frac{dL}{dt}, \quad \tau = (mg)(a)$$

As τ due to gravity is constant, ~~Iω increases~~ angular momentum increases.

Further I decreases and ω increases.

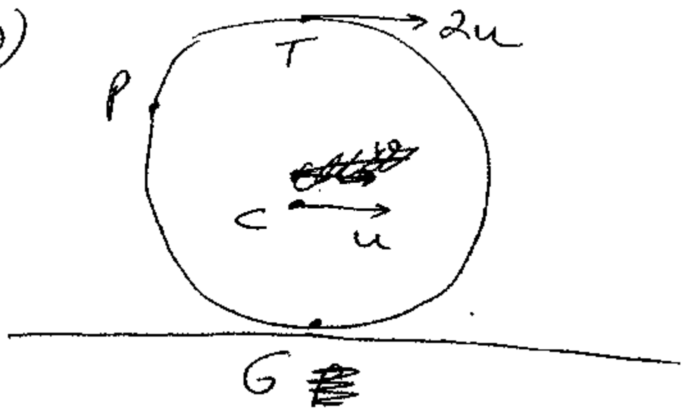
7.9 (ACD)

External torque on the system is zero, so angular momentum is conserved.

Here I decreases, so ω increases.

$RKE = \frac{L^2}{2I}$, as decreases RKE increase

A.10 (ACD) RING ROLLS

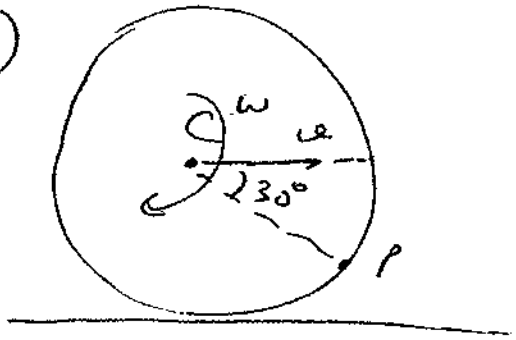


G – Ground

T – Topmost point

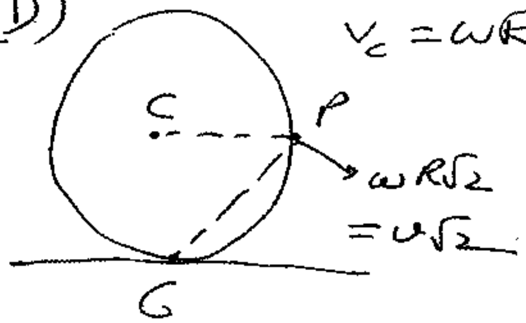
(A) Any point P, has a speed: $0 \leq v < 2u$

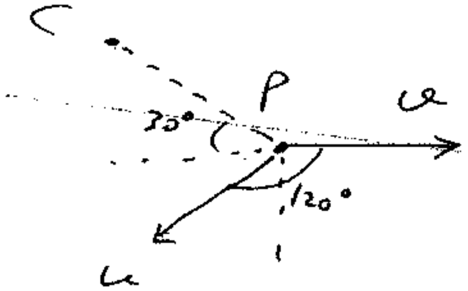
(C)



(B), (D)

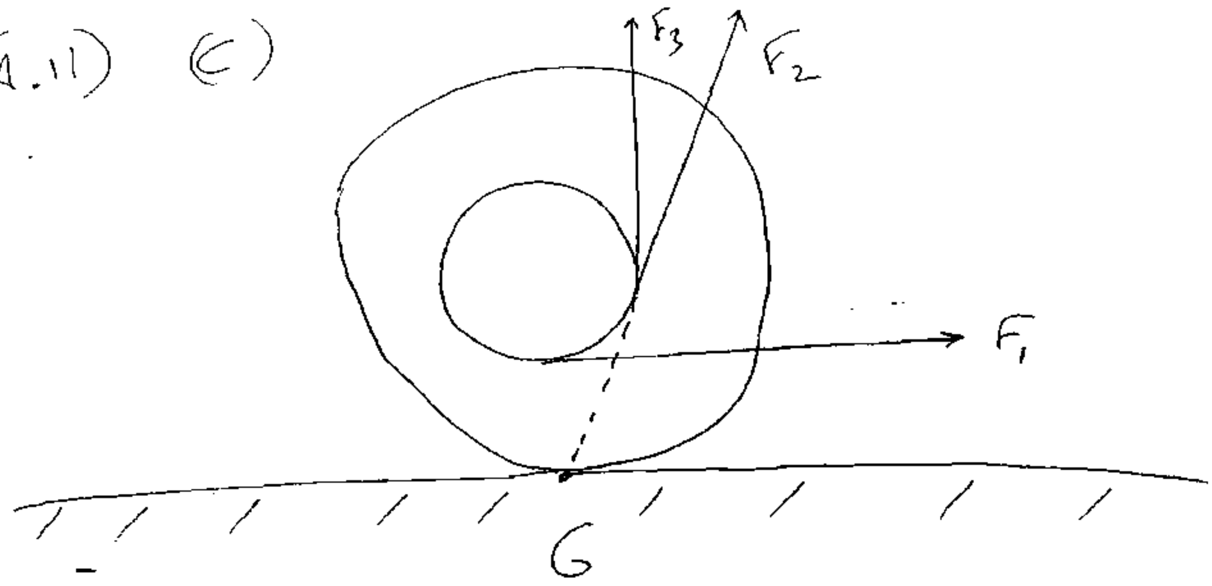
$V_c = \omega R$





$v_{resultant} = v$

(A.11) (C)



Consider torques about G,

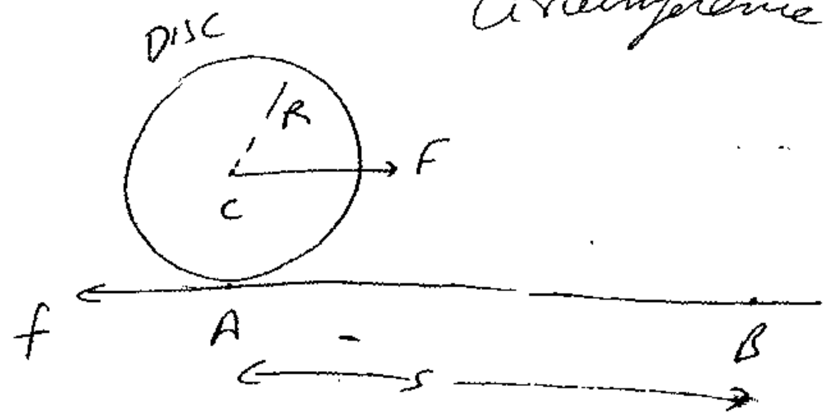
· When F_3 is applied, torque is counter-clockwise, cm will move left.

· ~~tttt~~ When F_1 is applied, torque is clockwise, cm will move to right.

· When F_2 is applied, torque is 0, cm will not move.

A.12 (ABCD)

Circumference = S



Beyond B, f will not stop acting so no torque about C, hence angular acceleration will disappear.

Earlier $a_1 = \frac{F-f}{M}$, later $a_2 = \frac{F}{M}$, hence linear acceleration increases.

As α disappears, ω remains constant after B.

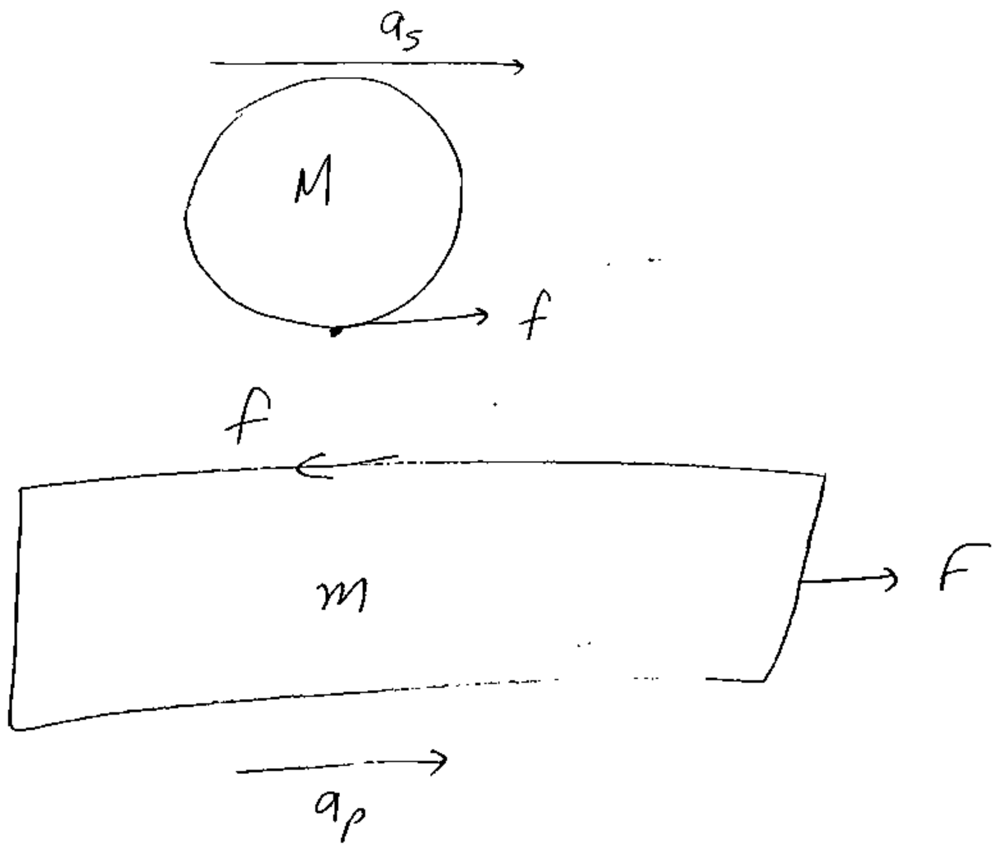
$\omega = \alpha . T$

$2\pi = \frac{1}{2}\alpha T^2 = \frac{1}{2}\left(\frac{\omega}{T}\right)T^2 = \frac{\omega T}{2}$

or $T = \frac{4\pi}{\omega}$

After B, $T' = \frac{2\pi}{\omega} = \frac{T}{2}$

A.13



$$F - f = m a_p$$

$$f = M a_s$$

$$\Rightarrow a_s < a_p$$

---

hollow sphere $\equiv m, R$, water $\equiv m$

.14

$L = L_{sphere} + L_{water}$; $L_{sphere} = \left(\frac{2}{3} MR^2\right)\omega + MvR$

$= 5MvR$

A(14) contd.

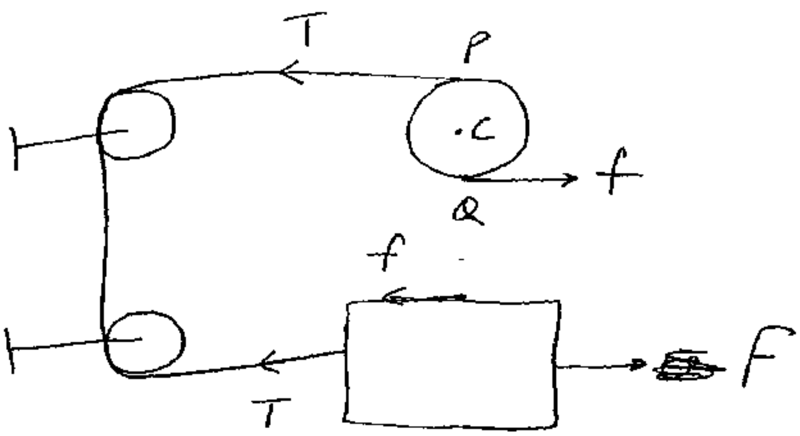
$$(KE)_{\text{water}} = \frac{1}{2} m v^2$$

$$(KE)_{\text{sphere}} = \frac{1}{2}\left(\frac{2}{5} mR^2\right)\omega^2$$

$$= \frac{1}{5} m v^2$$

$$\text{Total } KE = \frac{7}{10} m v^2$$

A.15



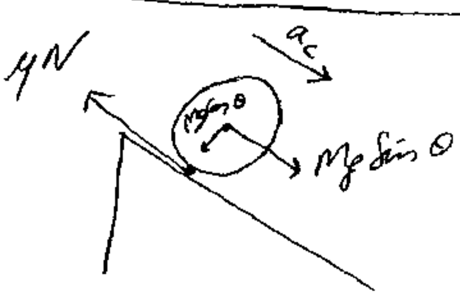
$v_P = v$ (left)

$v_Q = v$ (right)

$\Rightarrow V_c = 0$

$$\omega_c = \frac{v - (-v)}{2R} = \frac{v}{R}$$

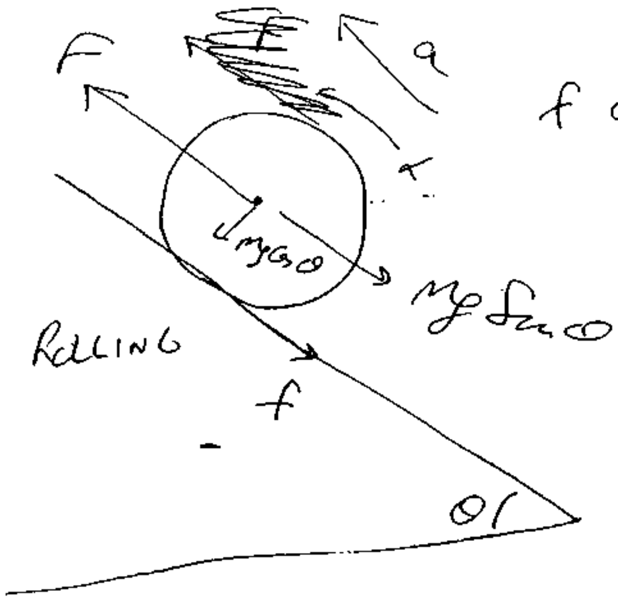
.16



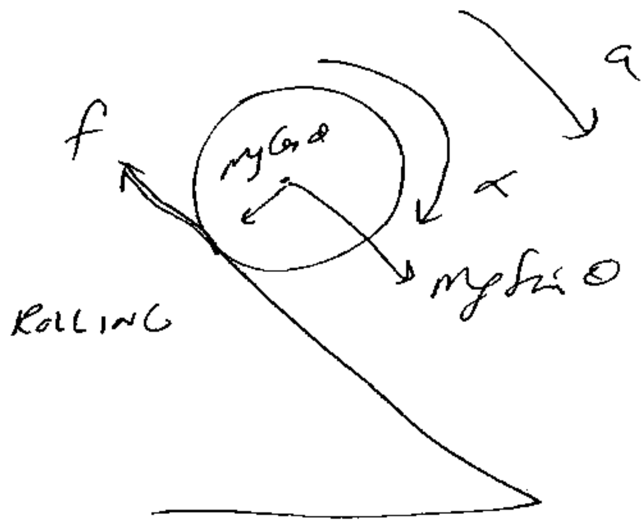
$$a_c = \left(\frac{Mg\sin\theta - \mu Mg\cos\theta}{M}\right)$$

$$V_c = a_c \cdot t$$

A.17



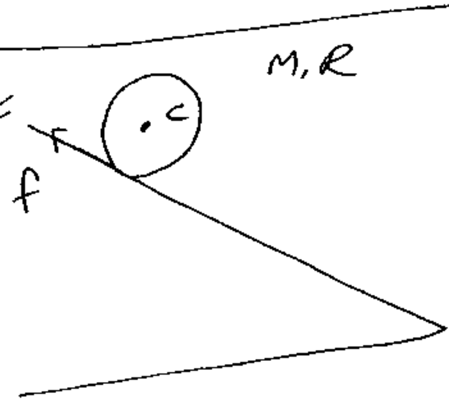
f can be downward.



f can be upwards.

$f \le \mu N$

..18

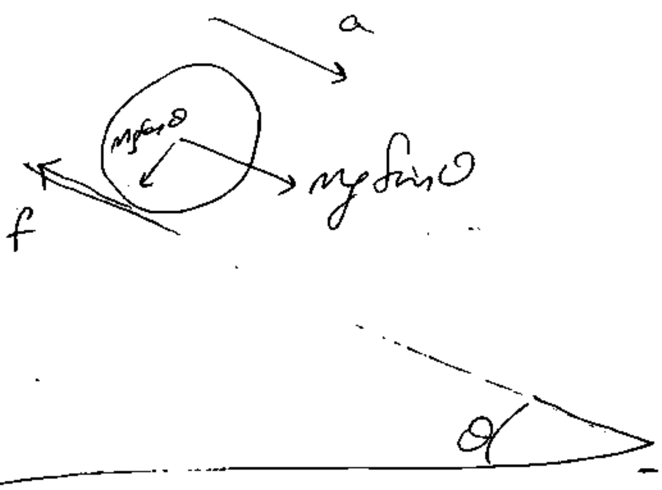


$I_{cylinder} = \frac{MR^2}{2}$, $I_{RING} = MR^2$

The body will roll if friction is present.

f provides torque about C

7.19



① $Mg\sin\theta - f = Ma$

② $f.R = I\alpha = I\left(\frac{a}{R}\right)$

① ÷ ② $\Rightarrow$

$$\frac{Mg\sin\theta - f}{fR} = \frac{M}{I/R}$$

$$Mg\sin\theta - f = \left(\frac{MR^2}{I}\right)f$$

$$Mg\sin\theta = \left(\frac{MR^2}{I}+1\right)f$$

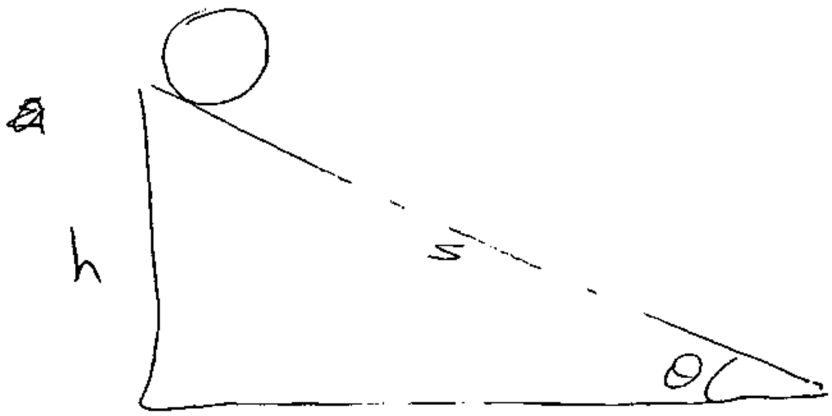
$$f = \frac{Mg\sin\theta}{1+\left(\frac{MR^2}{I}\right)}$$ and $f \leq \mu N$ or $f \leq \mu Mg\cos\theta$

f is more for Ring than cylinder as

$I_{ring} > I_{cylinder}$

A.20

$$a = \frac{g\sin\theta}{1+\left(\frac{I}{MR^2}\right)}$$



$$v^2 = 2aS = 2\left(\frac{g\sin\theta}{1+\left(\frac{I}{MR^2}\right)}\right)s$$

$$v^2_{cyl} = \frac{2gh}{\left(1+\frac{1}{2}\right)} = \frac{4gh}{3} \text{ or } v_{cyl} = \sqrt{\frac{4gh}{3}}$$

$$v_{ring} = \frac{2gh}{1+1} = gh \text{ or } v_{Ring} = \sqrt{gh}$$

A.21 M, R

Rolling will start after

$$\omega = \frac{v}{R}.$$

A.22 Line of action of friction passes through the point on the horizontal surface. Torques due to weight and normal force cancel off. Net torque is 0. Hence Angular momentum is conserved about

horizontal force

A.23

$$a = -\mu g$$

$$V = V_0 - \mu g . t$$

$$\omega = \underset{0}{\omega_0} + \alpha t$$

$$\alpha = \frac{\mu M g R}{(MR^2)} = \frac{\mu g}{R}$$

$$\omega = \left(\frac{\mu g}{R}\right) t$$

As at rolling,

$V = \omega R$ or $V_0 - \mu g t = \left(\frac{\mu g}{R}\right) t . R$

$V_0 = 2\mu g t$ or $t = \dfrac{V_0}{2\mu g}$ ✓

1.24

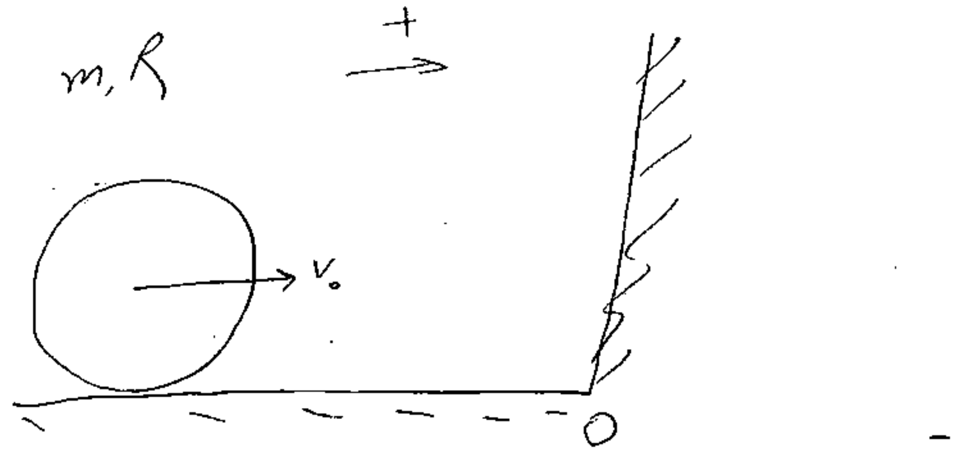
$$S = ut + \frac{1}{2} a t^2$$

$$= V_0\left(\frac{V_0}{2\mu g}\right) + \frac{1}{2}(-\mu g)\left(\frac{V_0}{2\mu g}\right)^2$$

$$= \frac{V_0^2}{2\mu g} - \frac{V_0^2}{8\mu g} = \frac{3V_0^2}{8\mu g}$$

A.25   m, R

Sphere



About O

~~$\Delta p =$~~

Before collision   After collision

$$\bar{L}_i = +mv_0R + I\omega \quad , \quad \bar{L}_f = -mv_0R + I\omega$$

$$|\Delta \bar{L}| = 2mv_0R = \text{Angular Impulse of wall about O.}$$

Immediately after collision

$$|L_f| = mv_0R - \left(\frac{2}{5}mR^2\right)\cdot\left(\frac{v}{R}\right)$$

$$= \frac{3}{5}mv_0R$$

$$L = mvR + I\left(\frac{v}{R}\right) = mvR + \left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)$$

A (Q4 contd.)

$$v = v_0 - (\mu g)t = v_0 - (\mu g)\left(\frac{v_0}{2\mu g}\right)$$

$$= \frac{v_0}{2}$$

$$\Delta K = \left[\frac{1}{2}(m)\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}(mR^2)\left(\frac{v_0}{2R}\right)^2\right]$$

(Translational KE) (Rotational KE)

$$- \frac{1}{2}mv_0^2$$

$$= \left[\frac{mv_0^2}{8} + \frac{mv_0^2}{8}\right] - \frac{1}{2}mv_0^2$$

$$= -\frac{mv_0^2}{4} = W_{friction}$$

ROTATIONAL DYNAMICS

PASSAGE (1-3)

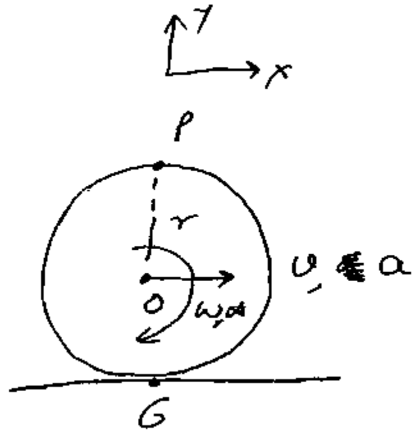
&

COLUMN-MATCHING (1-4)

SOLUTIONS

PASSAGE—1

1.(a) $\bar{a}_{PG} = \bar{a}_{PO} + \bar{a}_{OG}$



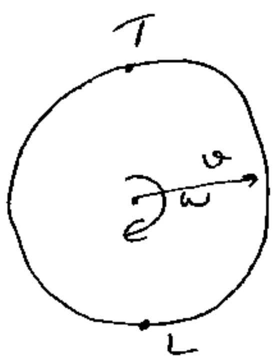
$\bar{a}_{PO} = (\alpha r)\hat{i} + (\omega^2 r)\hat{j}$

$\bar{a}_{OG} = a\hat{i}$

$\bar{a}_{PG} = (\alpha r + a)\hat{i} + (\omega^2 r)\hat{j}$

$a_{PG} = \sqrt{(a+\alpha r)^2 + (\omega^2 r)^2}$

2.(a, b)



$\omega = 20$ rad/s

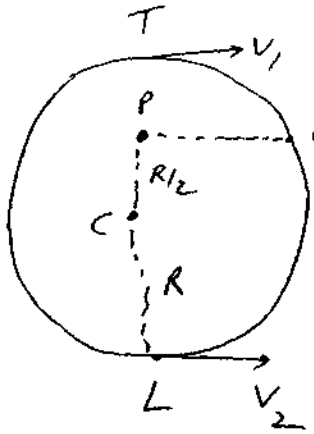
$V = 30$ m/s

$r = 20$ m

$V_T = V + \omega r = 30 + (20)(20) = 430$ m/s forward

$V_L = V - \omega r = 30 - (20)(20) = -370$ m/s $= 370$ m/s backward

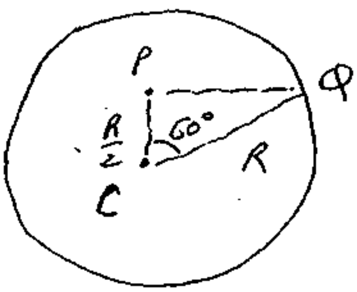
(3) (a,b,c,d)



$$\omega = \frac{V_T - V_L}{2R} = \frac{v_1 - v_2}{2R}$$

$$V_{PL} = \omega \cdot \frac{3R}{2} + V_2 = \frac{3(v_1 - v_2)}{4R} + v_2$$

$$= \frac{3v_1 + v_2}{4R}$$



$$PQ = PC \cdot \tan 60° = \frac{\sqrt{3}R}{2}$$

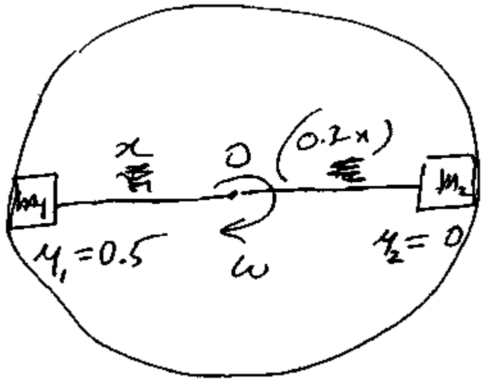
$$\overline{V_Q} = \overline{V_P} + \overline{V_{QP}}$$

$$= \overline{V_P} - \frac{\omega\sqrt{3}R}{2}\hat{j}$$

$$\overline{V_c} = \overline{V_L} + \omega R\,\hat{i}$$

$$= V_2\hat{i} + \left(\frac{v_1 - v_2}{2}\right)\hat{i} = \left(\frac{V_1 + V_2}{2}\right)\hat{i}$$

PASSAGE-2

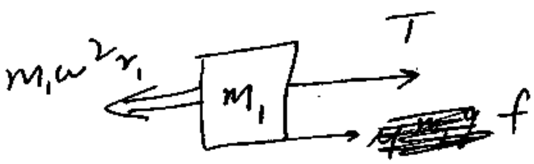


$\omega = 10\,rad/s$

$m_1 = 10\,kg,\ m_2 = 5\,kg$

1) $r_1 = x = 0.124\,m$

$r_2 = 0.3 - x = 0.3 - 0.124 = 0.176\,m$



$T = 5(10)^2(0.176) = 88N$

$m_1\omega^2 r_1 = (10)(10)^2(0.124)$
$= 124N$

$f = m_1\omega^2 r_1 - T$
$= 124 - 88$
$= 36N$

$f \le \mu m_1 g$

2) $f = m_1 \omega^2 r_1 - T$

$f = (m_1 r_1 - m_2 r_2)\omega^2 \le \mu N$ (for No slipping)

for slipping,

$(m_1 r_1 - m_2 r_2)\,\omega^2 \geq \mu m_1 g$

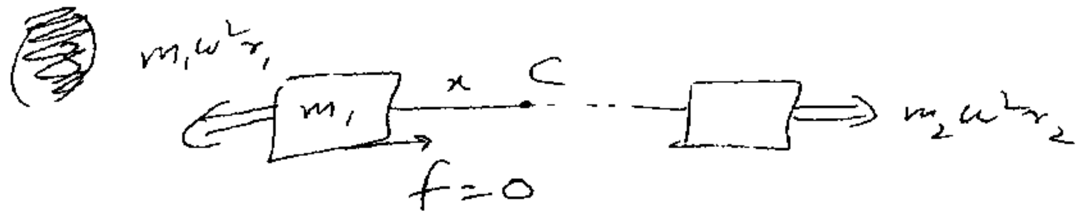
$$\omega^2 \geq \frac{\mu m_1 g}{m_1 r_1 - m_2 r_2}$$

$$\geq \frac{(0.5)(10)(10)}{(10)(0.124) - (5)(0.176)}$$

$$\omega^2 \geq \frac{50}{0.36}$$

$\Rightarrow \quad \omega > 11.67 \ \frac{\text{rad}}{\text{s}}$

3) (a)



$$m_1\omega^2 r_1 = m_2\omega^2 r_2$$

$$(10)\cancel{(10^2)}x = (5)\cancel{(10^2)}(0.3-x)$$

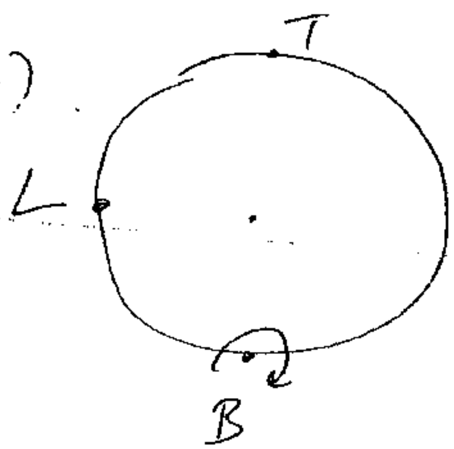
$$10x = 1.5 - 5x$$

$$15x = 1.5$$

$$x = 0.1\,m$$

Passage – 3

1) (b)

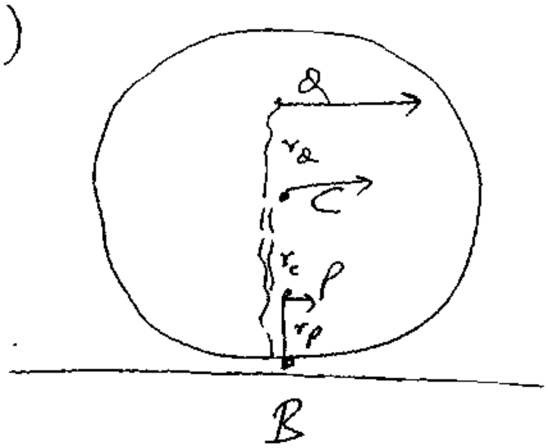


$$v_T = \omega(2R) = 2v$$

$$v_L = \omega(R\sqrt{2}) = v\sqrt{2}$$

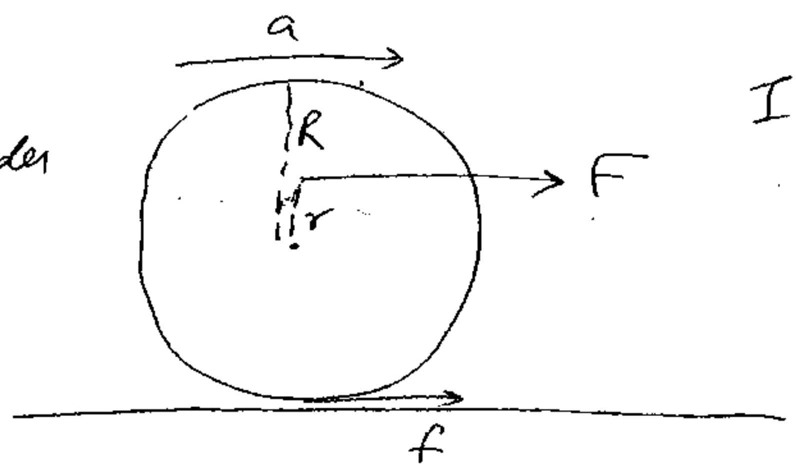
$$v_B = \omega(0) = 0$$

2) (a)



$$\left.\begin{aligned} V_Q &= \omega . r_Q \\ V_C &= \omega . r_c \\ V_P &= \omega . r_P \end{aligned}\right\} \Rightarrow \quad V_Q > V_C > V_P$$

3) Cylinder



$I_{cylinder} = \frac{MR^2}{2}$

If cylinder rolls, $F + f = Ma$ (1)

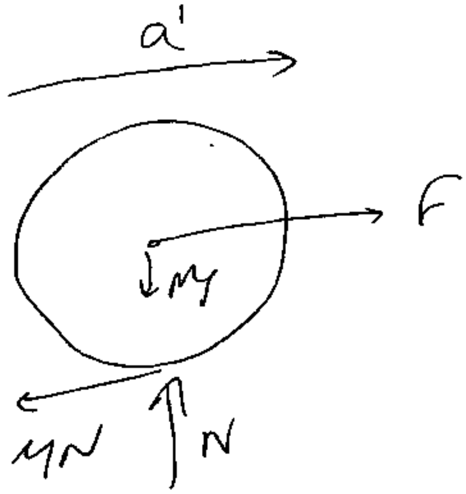
$$F.R - f.r = \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right) \quad (2)$$

Do (1) $\times R$ − (2) $\Rightarrow$ $fr + fr = MaR - \frac{MaR}{2}$

MaR

(3) contd.

Cylinder could also slip, whereby,



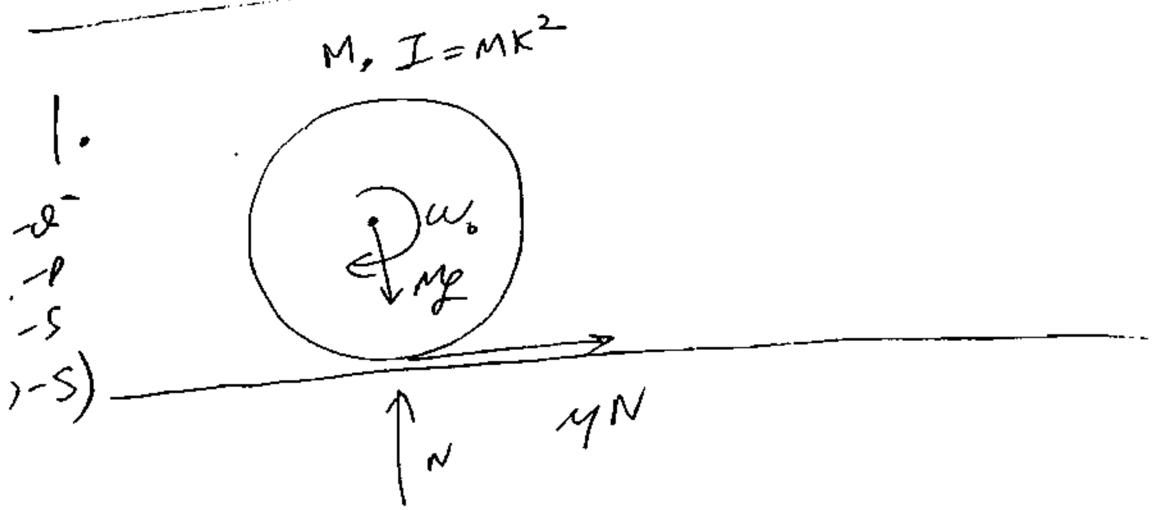
$$F - \mu(Mg) = Ma'$$

Thus, friction could be backward or forward. Data given is insufficient to interpret.

[illegible]

COLUMN MATCHING Qs.

1.



$$v = (\mu g)\cdot t$$

$$\omega = \omega_0 - \left(\frac{\mu M g R}{M K^2}\right) t = \omega_0 - \left(\frac{\mu g R}{K^2}\right)\cdot t$$

At rolling, $v = \omega r$

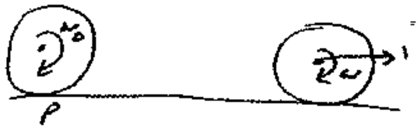
$$(\mu g) t = \left[\omega_0 - \left(\frac{\mu g R}{K^2}\right) t\right]\cdot R$$

$$(\mu g) t = \omega_0 R - \left(\frac{\mu g R^2}{K^2}\right) t$$

$$t = \frac{\omega_0 R}{\dots},$$ As $I\uparrow, K\uparrow, t\uparrow_s$.

Friction acts in the same direction as the motion of the circular body. Hence, Translational work done by friction is +ve.

The frictional force reduces the rotational kinetic energy, hence rotational work done is −ve.

Also, Conserving Angular momentum about initial point of contact: 

$$(MK^2)\omega_0 = (MVR) + (MK^2)\omega$$

$$(MK^2)\omega_0 = M(R^2+K^2)\omega$$

$$\omega = \left(\frac{K^2}{R^2+K^2}\right)\omega_0$$

$$2 \qquad 1\ (MK^2)\,\omega_0^2$$

$$\Delta KE = \frac{1}{2}M(R^2+K^2)\left(\frac{K^2}{R^2+K^2}\right)^2 \omega_0^2 - \frac{1}{2}MK^2\omega_0^2$$

$$= \frac{1}{2}M\left(\frac{K^4}{R^2+K^2}\right)\omega_0^2 - \frac{1}{2}MK^2\omega_0^2$$

$$KE = \frac{1}{2}MK^2\omega_0^2\left[\frac{K^2}{R^2+K^2} - 1\right]$$

~~As ΔKE is −ve, work~~

$$KE = \frac{1}{2}MK^2\omega_0^2\left[\frac{1}{\frac{R^2}{K^2}+1} - 1\right]$$

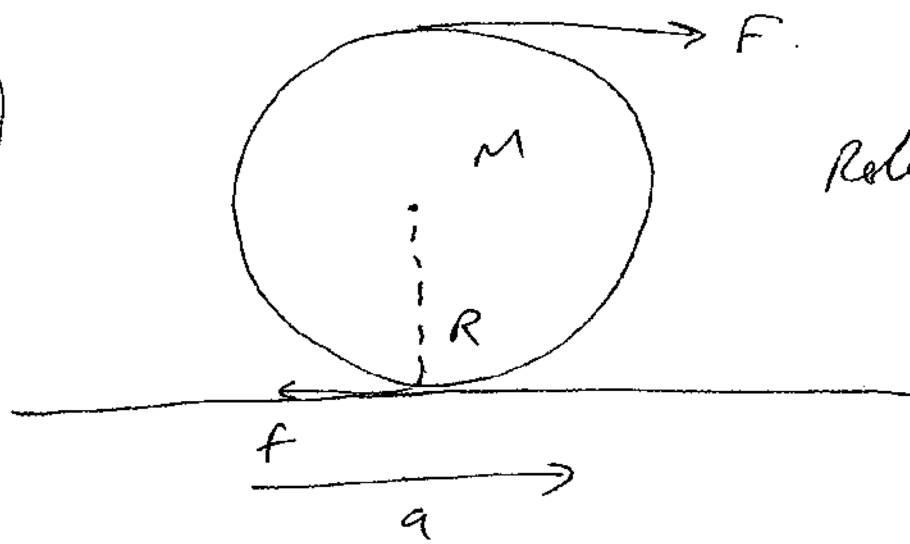
As I ↑, K ↑, ΔKE ↑, work done by friction is more.

~~Passage~~

Column – Matching

2.

A – R
B – P
C – S
D – Q



Rolls.

① $F - f = Ma$

② $FR + fR = I\left(\frac{a}{R}\right)$

Do ② $- R \times$ ①

$$fR + fR = \frac{Ia}{R} - MaR$$

$$f = \left(\frac{I}{R^2} - M\right)a$$

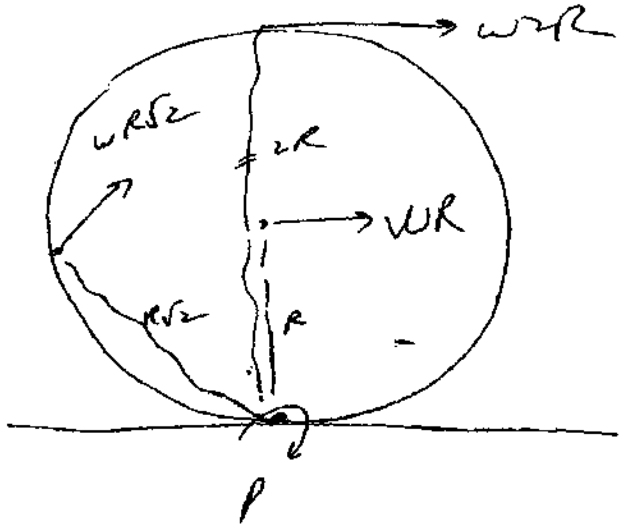
Value of I, decides direction of f.

Column Matching

4)

A - q
B - P
C - S
D - R

SPHERE ROLLS



$V = \omega R$

$V_P = 0$

1, 3, 5, 7,

8, 9, 10,

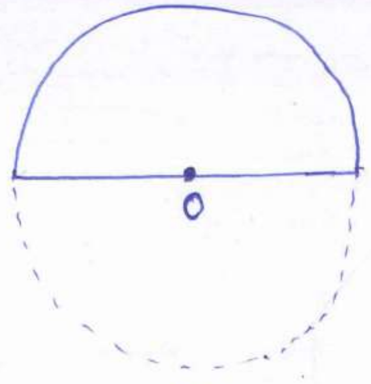
12, 26,

33,

EX-5

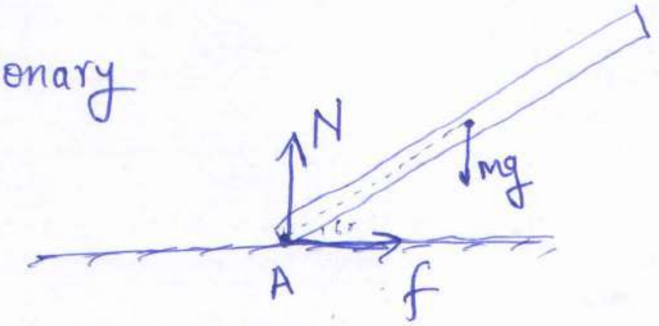
(1) $\frac{1}{2}(2M)R^2 = I_{axis,\ complete}$

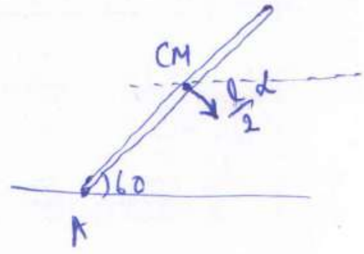
$I_{axis,\ half} = \dfrac{\frac{1}{2}(2M)R^2}{2} = \dfrac{MR^2}{2}$



axis ⊥r to plane of disk

(2) point A is stationary





(a)

$\tau_A = Mg\frac{l}{2} = I_A\alpha$

$mg\frac{l}{2}\cdot\frac{1}{2} = \frac{ml^2}{3}\cdot\alpha$

$l\alpha = \frac{3g}{2\times2}$

$\alpha = \frac{3g}{2\times2l} = \frac{3g}{4l}$

$a_{cmx} = \frac{l\alpha}{2}\cos 30$

$a_{cmy} = \frac{l\alpha}{2}\sin 30$

(B)

$f = m\cdot a_{cmx}$

$= m\cdot\frac{l}{2}\cdot\frac{3g}{4l}\cdot\frac{\sqrt{3}}{2}$

$f = \frac{3\sqrt{3}}{16}mg$

$mg - N = m a_{cm,y}$

$N = mg - m\cdot\frac{l}{2}\cdot\frac{3g}{4l}\cdot\frac{1}{2}$

$N = mg - \frac{3}{16}mg = \frac{13}{16}mg$

(C)

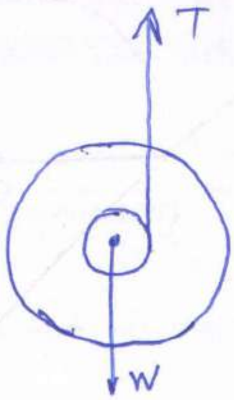
$f \leq \mu N$

$\frac{3\sqrt{3}}{\cancel{16}}\cancel{mg} \leq \mu\cdot\frac{13}{\cancel{16}}mg$

$\mu \geq \frac{3\sqrt{3}}{13}$

(3)

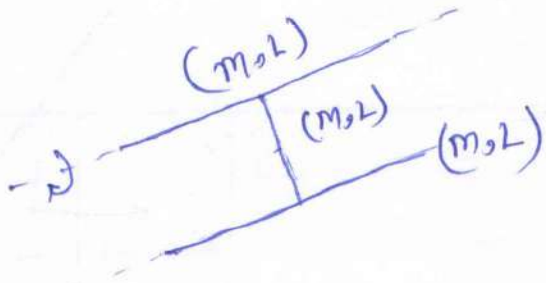
(a) $T = W$ { because height from floor is constant }



(B) $T \times r = \omega \times r = mK^2 \cdot \alpha$

$$\alpha = \frac{\omega r}{mK^2} = \frac{Wr}{mK^2} = \frac{mg \cdot r}{mK^2}$$

$$\alpha = \frac{gr}{K^2}$$



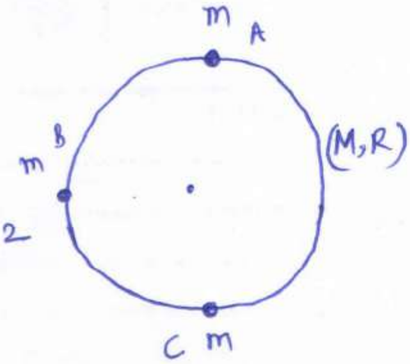
(4) Apply energy conservation

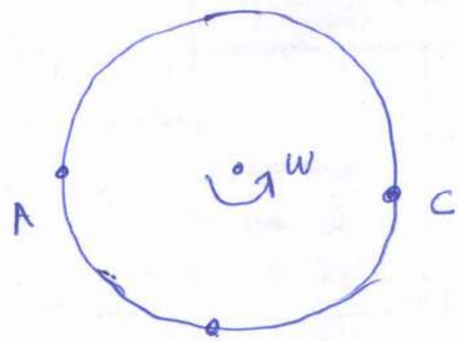
$$mgl + mg\frac{l}{2} = \frac{1}{2}\left(\frac{ml^2}{3} + ml^2\right)\omega^2$$

$$\frac{3}{2}\cancel{m}g\cancel{l} = \frac{1}{6} \cdot 4\cancel{m}\omega^2 l^{\cancel{2}}$$

$$\sqrt{\frac{9}{4}\frac{g}{l}} = \omega = \frac{3}{2}\sqrt{\frac{g}{l}}$$

(5) Energy Conservation





$$mgR = \frac{1}{2}\left\{3mR^2 + \frac{MR^2}{2}\right\}\omega^2$$

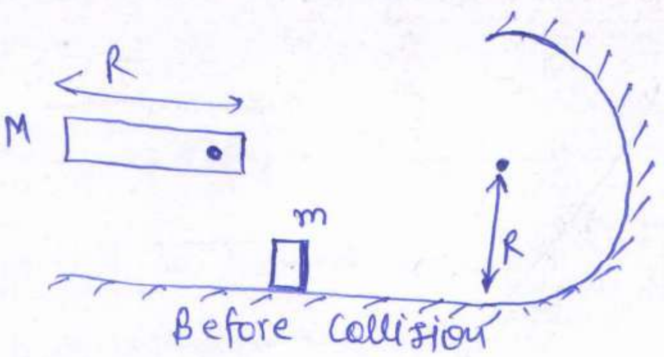
$$\frac{4mgR}{(6m+M)R^2} = \omega^2$$

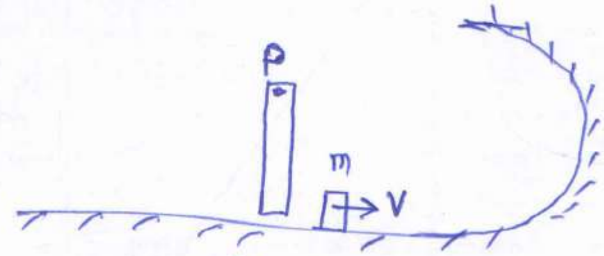
$t = 0$

$$\omega^2 = \frac{4 \times 0.1 \times 10}{(6 \times 0.1 + 1) \times 0.5} = \frac{4}{0.8} = \frac{1}{0.2} = 5$$

$\omega = \sqrt{5}$ rad/sec.

(6)





taking Block and rod as a system

Angular momentum conservation about P

$$\frac{mR^2}{3}\cdot\omega = mV \times R$$

$$V = \frac{\omega R}{3}\cdot\frac{m}{M}$$

Energy conservation

$$mg\frac{R}{2} = \frac{1}{2}\cdot\frac{mR^2}{3}\cdot\omega^2$$

$$\omega^2 = \frac{3g}{R}$$

$$\omega = \sqrt{\frac{3g}{R}}$$

for complete circular

$$V \geq \sqrt{5gR}$$

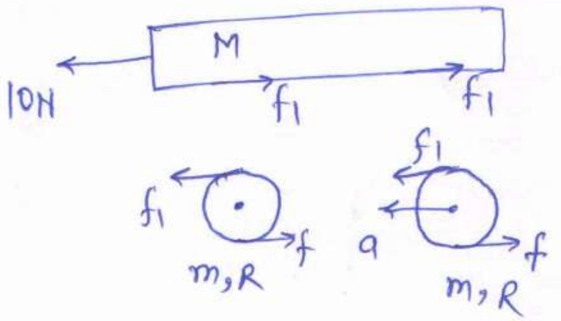
$$\frac{m}{M}\cdot\frac{\omega R}{3} \geq \sqrt{5gR}$$

$$\frac{m}{M}\cdot\frac{1}{3}\cdot\sqrt{\frac{3g}{R}}\cdot R \geq \sqrt{5gR}$$

$$\frac{m}{M} \geq \sqrt{5}\cdot\sqrt{3}$$

$$\frac{m}{M} \geq \sqrt{15}$$

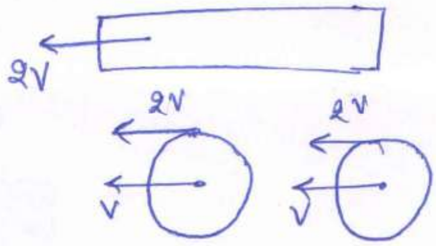
(7)



$$10 - 2f_1 = M\cdot(2a) \quad \text{—(1)}$$

$$2f_1 - 2f = 2ma \quad \text{—(2)}$$

$$f_1 \times 2R = \frac{3}{2} mR^2 \cdot \alpha = \frac{3}{2} mR^2 \cdot \frac{a}{R}$$



$$f_1 = \frac{3}{4} ma \quad \text{—(3)}$$

(1) & (3)

$$10 = 2Ma + \frac{3}{2} ma$$

$$a = \frac{10}{2M + \frac{3}{2}\cdot m} = \frac{10}{12+6} = \frac{10}{18}\ m/sec^2$$

$$v = u + at$$

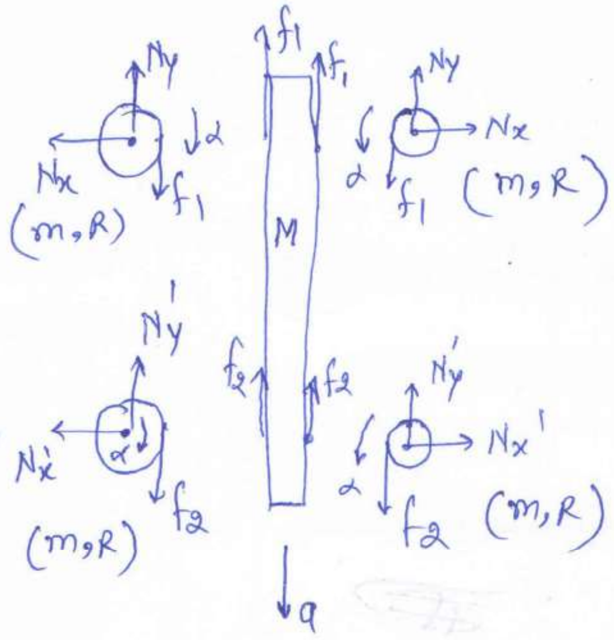
$$= v = 0 + \frac{10}{18} \times \frac{5}{2} = \frac{25}{18}\ m/sec.$$

Velocity of carriage is $2v$

$$= \frac{25}{9}\ m/sec,$$

(8)

Due to no slipping



$$a = R\alpha$$

$$f_2 \times R = \frac{mR^2}{2} \cdot \alpha$$

$$f_2 = \frac{m \cdot R\alpha}{2} = \frac{ma}{2}$$

$$f_1 \times R = \frac{mR^2}{2} \cdot \alpha$$

$$f_1 = \frac{ma}{2}$$

$$Mg - 2f_1 - 2f_2 = M \cdot a$$

$$Mg - 2ma = Ma$$

$$a = \frac{Mg}{(2m + M)}$$

(A) $a = \frac{5}{9} \times 10 = \frac{50}{9}$ m/sec^2

(B) $a \simeq 10$ m/sec^2

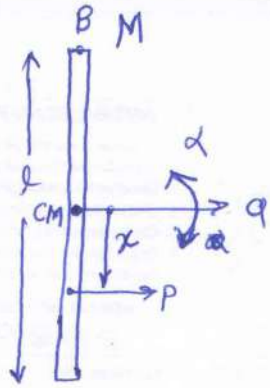
(C) $a \simeq 0$ m/sec^2

(9)

$$Px = \frac{ml^2}{12} \cdot \alpha$$

$$l\alpha = \frac{12Px}{ml} \quad \text{— (1)}$$

$$a = \frac{P}{m} \quad \text{—— (2)}$$



Acceleration of point B

$$a - \frac{l}{2}\alpha = 0$$

$$\cancel{\frac{P}{m}} = \frac{\cancel{l}}{2} \cdot \frac{12\cancel{P}x}{\cancel{m}l^{\cancel{2}}}$$

$$1 = \frac{6x}{l}$$

$$x = \frac{l}{6} = \frac{0.9}{6} m = \frac{90}{6} cm$$

By (2)

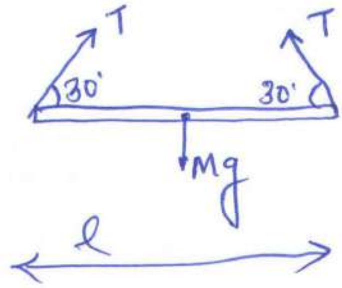
$$a = \frac{2.7}{1.5} = \frac{9}{5} m/sec^2$$

By (1)

$$\alpha = \frac{12 \times 2.7 \times (l/6)}{1.5 \times l^2} = \frac{\overset{4}{\cancel{12}} \times \overset{9}{\cancel{27}}}{\underset{5}{\cancel{15}}} \times \frac{1}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{9}} = 4 \text{ rad}/sec^2$$

$$\alpha = 4 \text{ rad}/sec^2$$

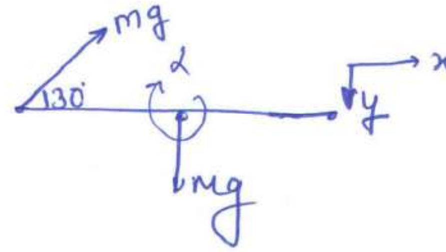
10 Initially rod in equilibirium



$$2T\sin 30 = mg$$

$$T = mg$$

Now, spring 2 is break

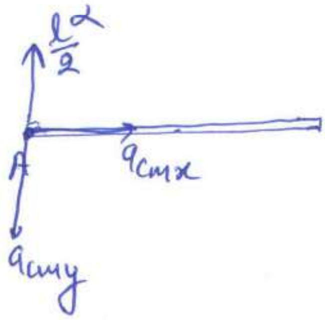


(a) $\cancel{m}g\sin 30 \times \frac{l}{2} = \frac{\cancel{m}l^2}{12} \cdot \alpha$

$$\frac{3g}{l} = \alpha$$

(b) $a_{cm_x} = g\cos 30$

$a_{cmy} = \frac{g}{2}$



$$a_{net.A} = \sqrt{\left(\frac{g}{2} - \frac{3g}{2}\right)^2 + (g\cos 30)^2} = \frac{\sqrt{7}}{2}g$$

$=$

(c)

$$a_{net.B} = \frac{\sqrt{17}}{2} \cdot g$$

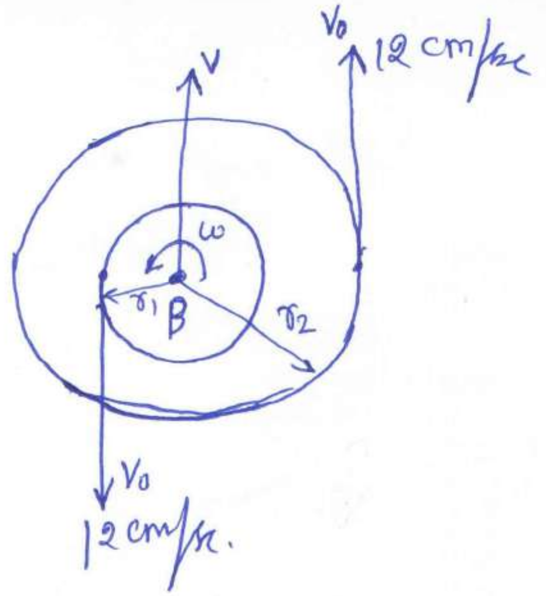
(11)

$\omega r_1 - v = v_0$ — (1)

$\omega r_2 + v = v_0$ — (2)

By (1) & (2)

$\omega = \frac{2v_0}{r_1 + r_2}$



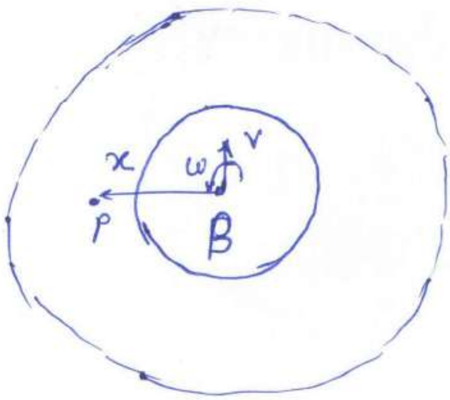
$\omega = \frac{2 \times 12}{6} = 4 \text{ rad/sec}$

Now from (1)

$4 \times 2 - v = 12$

$v = -4$ cm/sec.

For some point P
if net velocity is zero.
Then it point will be
instantaneous ~~axis~~ center of
rotation



$\Rightarrow v = \omega x$

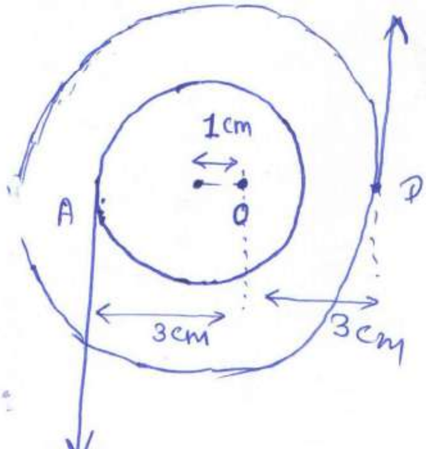
$\Rightarrow (-4) = 4 \times x$

$x = -1$ cm

1 cm right to point B.

(c)

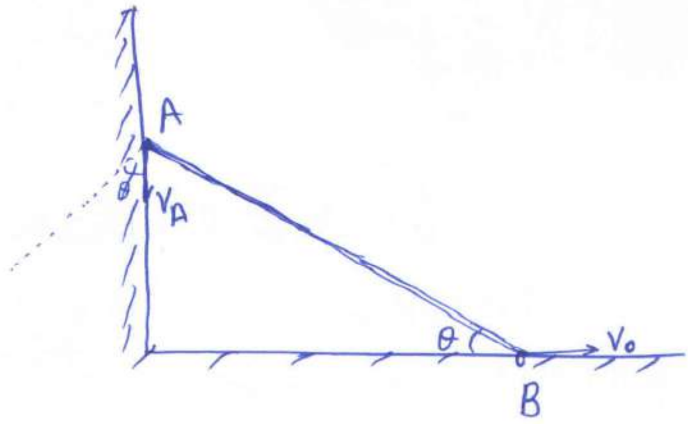
At point A $= 4 \times 4 = 16$ cm/sec.



At point D $= 4 \times 2 = 8$ cm/sec.

12 string constrained

$$V_A \sin\theta = V_0 \cos\theta$$



$$V_A = V_0 \cdot \cot\theta$$

$$= V_0 \times \frac{4}{3}$$

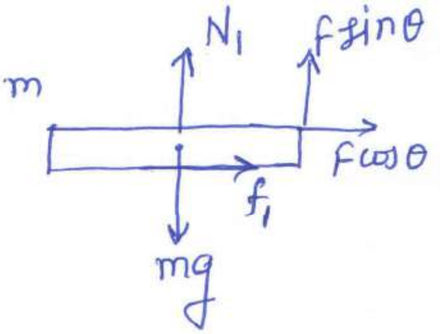
$$\omega = \frac{V_0 \sin\theta + V_A \cos\theta}{l}$$

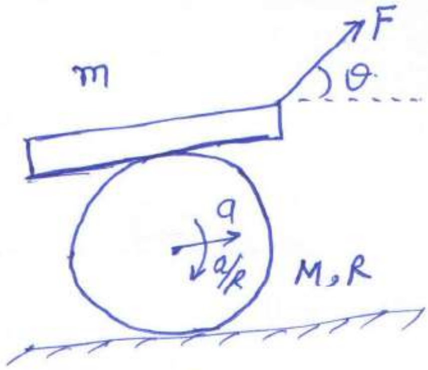
$$= \frac{V_0 \cdot \frac{3}{5} + V_0 \cdot \frac{4}{3} \cdot \frac{4}{5}}{l} = \frac{25}{15} \cdot \frac{V_0}{l} = \frac{5}{3} \frac{V_0}{l}$$

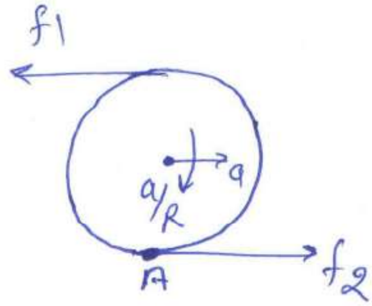
$$V_{cm\,x} = \frac{V_B}{2} = \frac{V_0}{2}$$

$$V_{cm\,y} = \frac{V_A}{2} = \frac{2V_0}{3}$$

(13)







$f_1 + F\cos\theta = 2ma$ —— (1)

$\tau_A = I\alpha$

$f_1 \times 2R = -2MR^2 \cdot \alpha$

$F_1 = -MR\alpha = -Ma$ —— (2)

from (1) & (2)

$\frac{F\cos\theta}{2m+M} = a$

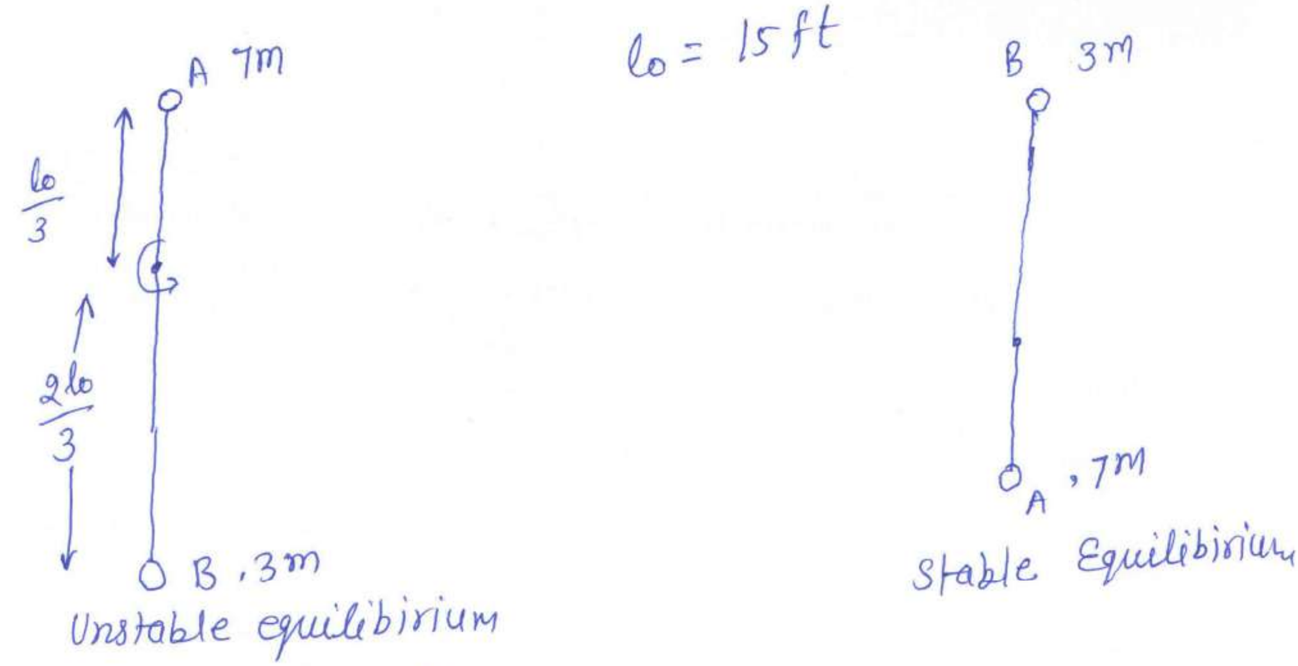
$f_1 = \frac{M \cdot}{2m+M} \cdot F\cos\theta$

$f_2 + F\cos\theta = Ma + 2m \cdot a$

$f_2 + F\cos\theta = (M+2M)\frac{(F\cos\theta)}{(2m+M)}$

$f_2 = 0$

14)



Energy conservation

$$4mgl_0 = \frac{1}{2}\left\{7m\cdot\frac{l_0^2}{9} + 3m\cdot\frac{4}{9}l_0^2\right\}\cdot\omega^2$$

$$4\cancel{m}g\cancel{l_0} = \frac{19}{18}\cancel{m}l_0^{\cancel{2}}\cdot\omega^2$$

$$\sqrt{\frac{18\times4}{19l_0}\cdot g} = \omega$$

$$\omega = \sqrt{\frac{72}{19l_0}\cdot g}$$

MODULE - III

Rotational Dynamics

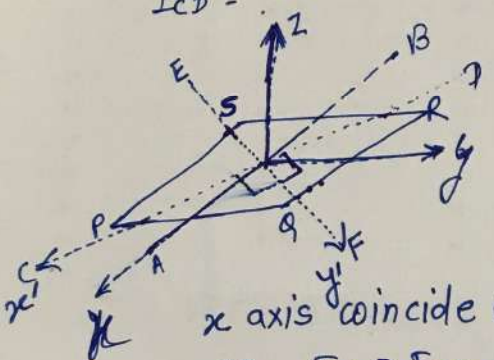
Window to IIT-JEE (Q.1 to 14)

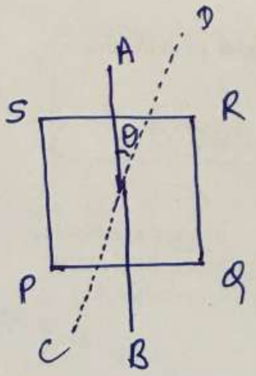
(1)

$I_{AB} = I$

$I_{CD} = ?$

PQRS ≡ Uniform square plate in x-y plane (x'-y')





x axis coincide with axis AB

so, $I_x = I_{AB} = I$

$I_x = I_y$ {Due to symmetry}

$I_z = I_x + I_y = 2I$ --- (1) {$\perp^r$ axis theorem}

Consider two axis

CD is x' & EF is y', in plane of plate

$I_{x'} = I_{CD}$

$I_{x'} = I_{y'}$ {due to symmetry}

$I_z = I_{x'} + I_{y'} = 2I_{CD}$ —— (2)

from equation (1) & (2)

$2I_{CD} = 2I$

$I_{CD} = I$

→ Conclusion:- Consider many axis in the plane of plate and passing through center of plate, all axis have same moment of intertia of square plate.

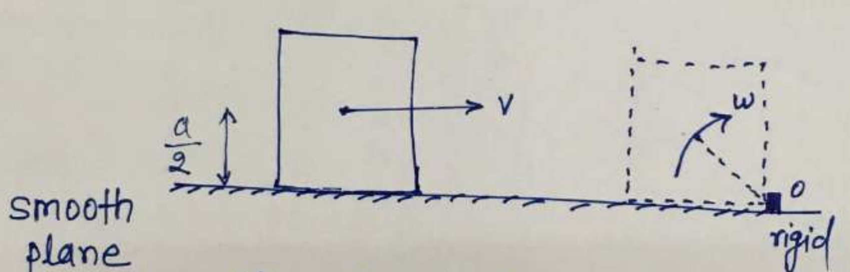
(2)

$$\vec{\tau} = \vec{A} \times \vec{L}$$

$$\frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

- $\frac{d\vec{L}}{dt}$ is always $\perp^r$ to $\vec{L}$, A - correct
- $\frac{d\vec{L}}{dt}$ is always $\perp^r$ to $\vec{A}$ also, so. angular momentum along $\vec{A}$ is constant
  B - correct
- $\vec{\tau}$ $\perp^r$ to $\vec{L}$ so, C - correct.

(3)



All external force which will act due to collision will pass through point A.
so, Angular momentum conservation about o,

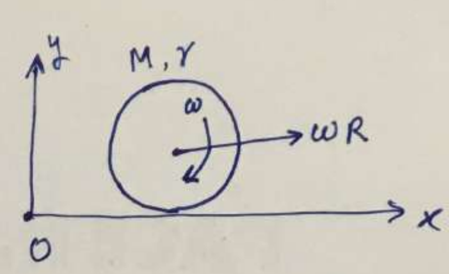
$$\Rightarrow mv \cdot \frac{a}{2} = \left(\frac{2ma^2}{12} + \frac{ma^2}{2}\right)\omega$$

$$\Rightarrow \cancel{m}v \cdot \frac{\cancel{a}}{2} = \frac{2}{3}\cancel{m}a^{\cancel{2}}\omega$$

$$\frac{3v}{4a} = \omega$$

(4)

$$\vec{L}_k = I_{cm}\vec{\omega} + \vec{r}_{cm,k} \times \vec{P}_{cm,k}$$



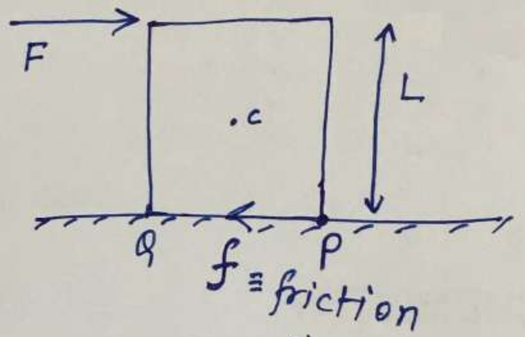
$$\vec{L}_O = \frac{mr^2}{2}\omega(-\hat{k}) + m\omega r^2(-\hat{k})$$

$$\vec{L}_O = \frac{3}{2}mr^2\omega(-\hat{k})$$

(C) ✓

(5) Given

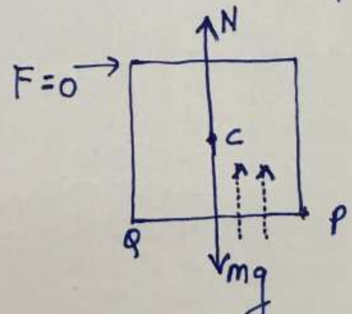
Not slide before topple.



$$f \leq \mu N$$

P, Q, two corner point of block.

When F = 0, Normal passing through center.



As you increase F, Normal shifted towards point P.

At maximum F for which block is still in equilibirium, Normal is passing through point P

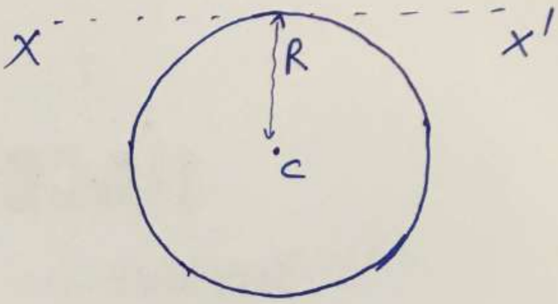
$$F_M \times a = mg \times \frac{a}{2}$$

$$F_M = \frac{Mg}{2}$$

If F is just large $\{F - F_M \to 0\}$ than F_m then block will topple.

6.

Mass of wire $\{m\} = L \cdot P$

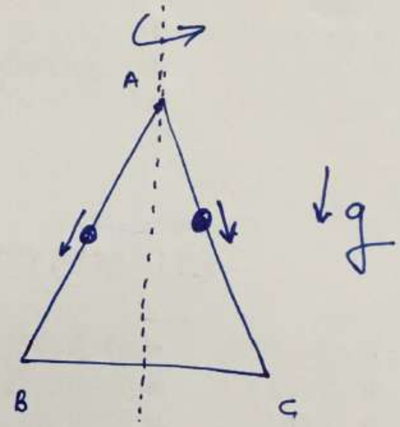


$I_{xx'} = \frac{3}{2} m R^2$ $\quad L = 2\pi R$

$= \frac{3}{2}(LP)\left(\frac{L}{2\pi}\right)^2$

$I_{xx'} = \frac{3}{8} \cdot \frac{L^3 P}{\pi^2}$ (D) ✓

7. Net torque of external forces about axis of rotation is zero

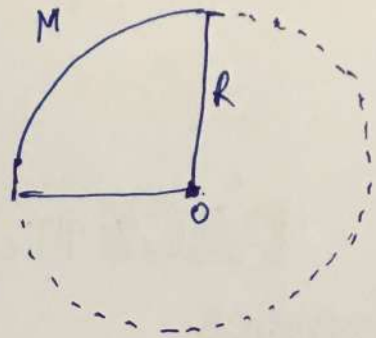


so, Angular momentum will be conserved

& No friction for energy loss, so, total energy is conserve.

⑧

$$I_{axis} = \frac{mR^2}{2}$$



If this disk was complete, it's mass were 4M

so, $I_{axis, \text{complete disk}} = \frac{(4M)(R^2)}{2}$ — axis passing through O & $\perp^r$ to plane.

each Quarter part is identical for axis.

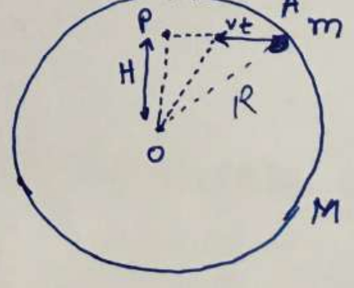
so, moment of inertia of each part

$$= \frac{I_{axis, \text{complete disk}}}{4}$$

$$I_{axis, part} = \frac{(4m)R^2/2}{4} = \frac{mR^2}{2}$$

⑨ $\tau_{axis} = 0$

$\vec{L}_{axis} = \text{constant}$



Particle leaving from point A & moving towards P.

$$\left(\frac{MR^2}{2}\omega_0 + mR^2\omega_0 + mvH\right) = \left(m(H^2+(R-vt)^2)+\frac{MR^2}{2}\right)\omega + mvH$$

· Axis passing through O & $\perp^r$ to plane

$$\frac{\left(\frac{MR^2}{2}+mR^2\right)\omega_0}{\frac{MR^2}{2}+m((R-vt)^2+H^2)} = \omega$$

By angular momentum conservation

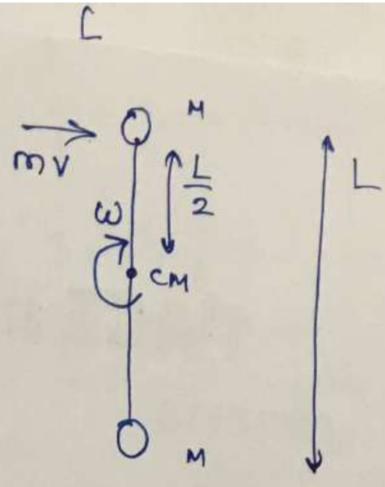
ω decreases.

then due to symmetry after crossing P

ω increases.

10)

impulsive torque about c.o.m



$= mv \times \frac{l}{2}$

$= \int \tau\, dt = L_f - L_i =$

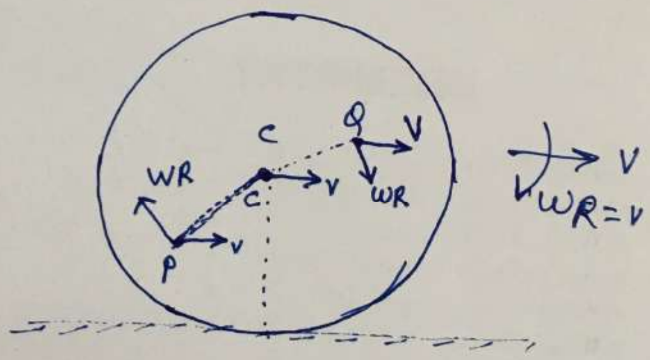
$\Rightarrow mv \cdot \frac{l}{2} = \frac{2ml^2}{4}\omega$

$\omega = \frac{4v}{4l} = \frac{v}{l}$

11) $\vec{v_c} = \longrightarrow v$

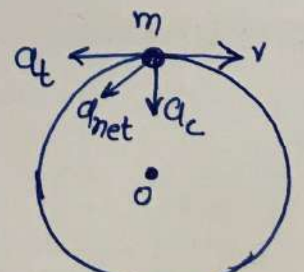
$\vec{v_Q} =$

$\vec{v_P} =$



$|\vec{v_Q}| > |\vec{v_c}| > |\vec{v_P}|$

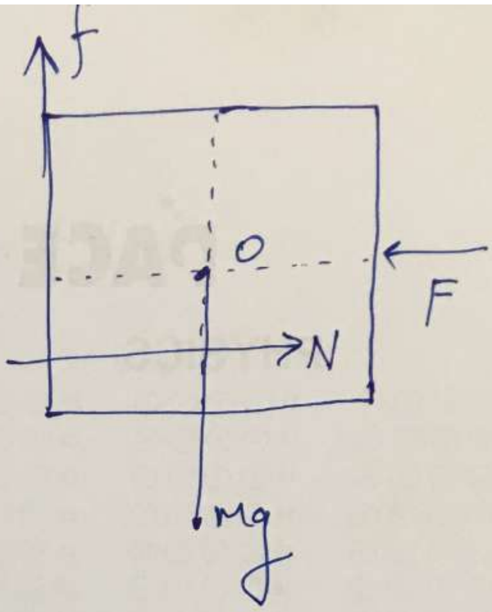
12) $L_O = mvR$
v is decreasing
so, $|\vec{L_O}|$ is decreasing



till velocity become zero angular momentum will remains unchanged in direction, not in magnitude

(D) ✓

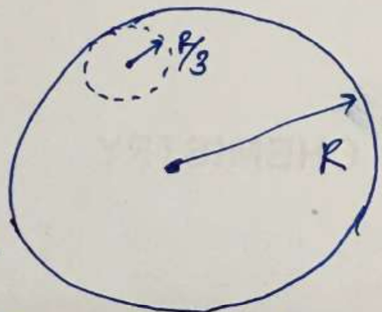
(13)



$f = mg$

$N = F$

τ_0 Torque about point 'O' of F is zero but of N is non-zero.



14) Assume positive and -ve mass at hole of disk. thas mass will be $\frac{1}{9}$th of total disk mass.

so, $I_0 = M.I_{(+m)} + M.I_{(-m)} + M.I$ of remaing disk

$= 9m \cdot \frac{R^2}{2} + M.I_{(-m)}$

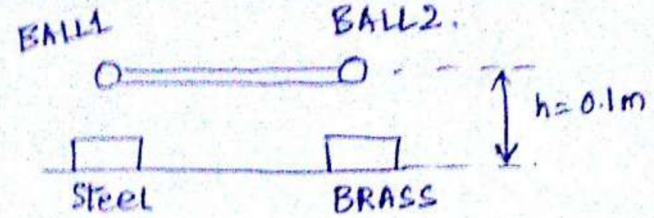
$= \frac{9mR^2}{2} + \frac{(-m)(R/3)^2}{2} + (-m)\left(\frac{2R}{3}\right)^2$

$= \left(\frac{9}{2} - \frac{1}{18} - \frac{4}{9}\right) mR^2 = \frac{81-1-8}{18} MR^2 = \frac{72}{18} MR^2 = 4MR^2$

Exercise-5
(Q.15 to Q.54).

ROTATIONAL DYNAMIC
MODULE - III (JEE ADVANCED)

Q15)



THE HEIGHT OF FREE FALL OF THE ROD AND BALLS SYSTEM IS $h = 0.1m$.

BEFORE COLLISION OF BALLS WITH THE PLATES

u_{BALL1} = SPEED OF BALL1 BEFORE COLLISION

$= \sqrt{2 \times g \times h} = \sqrt{2 \times 9.8 \times 0.1} = 1.4 \ m/s$.

u_{BALL2} = SPEED OF BALL2 BEFORE COLLISION

$= \sqrt{2gh} = 1.4 \ m/s$.

AFTER COLLISION OF BALL WITH THE PLATES

V_{BALL1} = SPEED OF BALL1 AFTER COLLISION

$= e_{STEEL} \times u_{BALL1} = 0.6 \times 1.4$
$= 0.84 \ m/s$

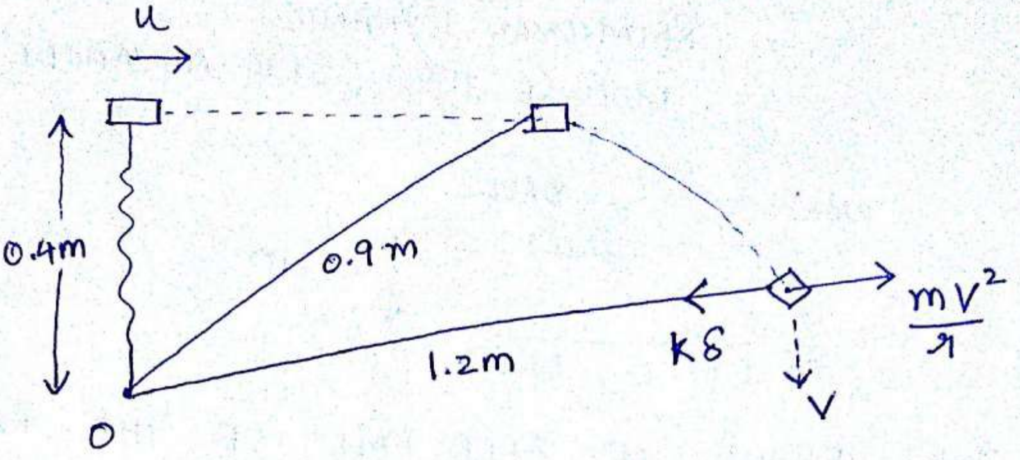
V_{BALL2} = SPEED OF BALL2 AFTER COLLISION

$= e_{BRASS} \times u_{BALL2} = 0.4 \times 1.4 = 0.56 \ m/s$

ANGULAR VELOCITY OF BAR = $\dfrac{\text{RELATIVE VELOCITY OF BALL1 W.R.T BALL2}}{\text{SEPERATION BETWEEN BALL1 AND BALL2.}}$

$= \dfrac{0.84 - 0.56}{1} = 0.28 \ rad/sec$ (CLOCKWISE).

Q16)



WHEN OP is 0.4m

THE ELASTIC STRING IS SLACK, ~~HENCE~~ ALSO THERE IS NO EXTERNAL FORCE ACTING ON THE BLOCK SO ITS MOMENTUM IS CONSERVED. IT TRAVELS IN A STRAIGHT LINE TILL OP is 0.9M.

WHEN OP IS 0.9M

THE ELASTIC SRING BECOMES TAUT, AS STRING EXTENDS FURTHER, RESTORING FORCE (Kx) ACTS IN RADIAL DIRECTION, TO OPPOSE THE RESTORING FORCE, THE BLOCK STARTS GOING IN CIRCULAR MOTION, SO THE CENTRIFUGAL FORCE OPPOSESS RESTORING FORCE.

WHEN OP IS 1.2M

THE STRING IS IN MAX ELONGATION, SO THE RADIAL VELOCITY OF BLOCK BECONES 0. THERE IS ONLY TANGENTIAL VELOCITY 'V'.

CONSERVATION OF ANGULAR MOMENTUM ABOUT 'O' SINCE THERE IS NO EXTERNAL TORQUE

$$\underbrace{m \times u \times (0.4)}_{\text{INITIAL}} = \underbrace{m \times V \times (1.2)}_{\text{FINAL}} \quad \text{--- (1)}$$

AT MAX ELONGATION, THERE ARE ONLY TWO RADIAL FORCES HENCE THEY ARE EQUAL ACCORDING TO NEWTONS LAWS

$$K(\delta) = \frac{mV^2}{r} \quad \text{--- (2)}$$

NOTE: HERE 'r' is THE RADIUS OF CURVATURE OF THE PATH OF THE BLOCK. AND IT IS NOT EQUAL TO STRING LENGTH (1.2m)

THERE IS NO EXTERNAL FORCE ON THE SYSTEM, SO ENERGY IS CONSERVED

$$\frac{1}{2} m u^2 = \frac{1}{2} K \delta^2 + \frac{1}{2} m V^2 \quad \text{--- (3)}$$

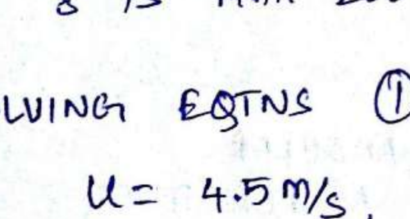
'δ' is MAX ELONGATION = $1.2 - 0.9 = 0.3$ m.

SOLVING EQTNS (1), (2) & (3) for 3 UNKNOWNS, U, V, r.

$u = 4.5$ m/s, $V = 1.5$ m/s, $r = 3.75$ m.

Q17) a) THE SYSTM IS IN EQUILIBRIUM SO THE TORQUE ABOUT ANY POINT ON THE BAR IS ZERO.

IF WE LOOK AT TORQUE ABOUT PIVOT.



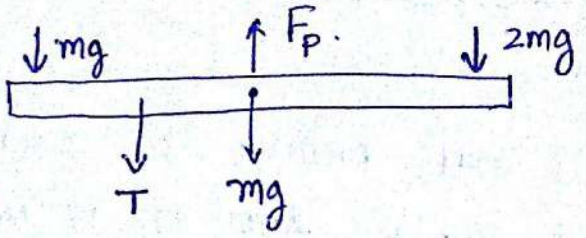
ANTI CLOCKWISE = CLOCKWISE

$$mg \times \frac{L}{2} + T \times \frac{L}{4} = 2mg \times \frac{L}{2}.$$

$$\underline{T = 2mg}$$

NOTE: TORQUE DUE TO PIVOT AND THE WEIGHT OF BAR ARE NOT CONSIDERER SINCE THEIR TORQUE IS ZERO ABOUT PIVOT.

b) TO SOLVE FOR ANY FORCE, WE NEED TO LOOK AT THE FREE BODY DIAGRAM OF THE BAR.



'F_P' is the FORCE DUE TO PIVOT, IT ACTS UPWARDS AS ALL OTHER FORCES ACTING ON BAR ARE DOWNWARD

$$F_P = mg + mg + 2mg + T \quad T = 2mg$$
$$= 6mg.$$

c) WHEN CORD IS CUT, THE TORQUE DUE TO TENSION IN THE STRING BECOMES ZERO AS TENSION BECOMES ZERO.

HENCE, NET TORQUE = $I\alpha$.

Moment of INERTIA OF BAR AND MASSES

ANGULAR ACCELERATION

$$I = \frac{ml^2}{12} + \frac{ml^2}{4} + (2m)\frac{l^2}{4} = \frac{5ml^2}{6}.$$

$$\text{NET TORQUE} = 2mg \times \frac{L}{2} - mg \times \frac{L}{2} = mg\frac{L}{2}.$$

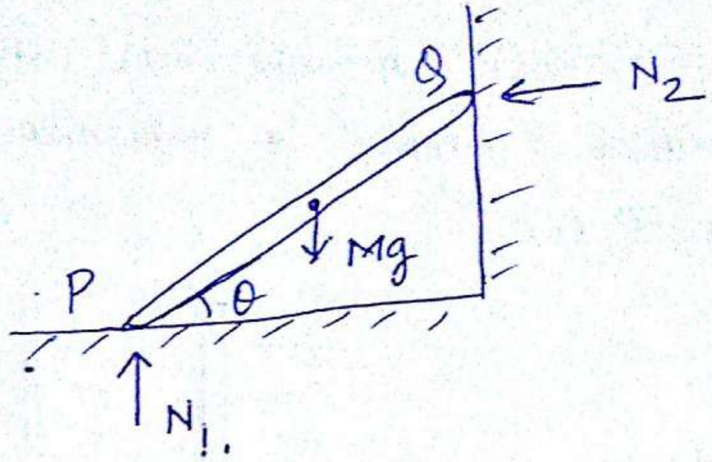
$$\alpha = \frac{mgL/2}{\frac{5mL^2}{6}} = \frac{3g}{5L}.$$

d) CONSERVATION OF ENERY: GAIN IN ROTATIONAL ENERGY IS EQUAL TO LOSS IN POTENTIAL ENERGY.

$$\frac{1}{2}I\omega^2 = 2mg\left(\frac{L}{2}\right) - mg\left(\frac{L}{2}\right).$$

$$v = \sqrt{\frac{3gL}{10}}$$

Q18)



THE WALL AND THE FLOOR ARE SMOOTH. HENCE THERE ARE NO FRICTIONAL FORCES.

FROM THE ABOVE FREE BODY DIAGRAM.

$$N_1 = Mg \quad -(1).$$
$$N_2 = 0. \quad -(2).$$

Net TORQUE ABOUT POINT 'P' IS.

$$Mg \times \frac{l}{2} \cos\theta.$$

$$\text{NET TORQUE} = I\alpha.$$

$$I = \frac{Ml^2}{3}.$$

$$\Rightarrow \quad \alpha = \frac{3g\cos\theta}{2l}.$$

NOTE: TORQUE DUE TO N_1 ABOUT 'P' IS ZERO AND $N_2 = 0$ SO TORQUE DUE TO N_2 IS ZERO.

Q19)

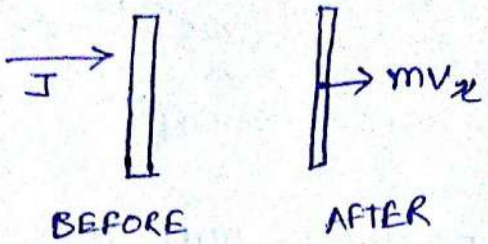
THE ROD IS STRUCK A HORIZONTAL IMPULSIVE BLOW.

THE HORIZONTAL IMPULSE IMPARTS A HORIZONTAL MOMENTUM TO THE C.O.M. OF ROD.

$J = mV_x.$

$6. = 0.6\, V_x$

$V_x = 10\ m/s.$



IN T=1sec, C.O.M. TRAVELS ALONG X,Y DIRECTION LIKE A SIMPLE PROJECTILE WITH INITIAL SPEED 10 m/s.

SO, DISTANCE COVERED IN X $S_x = V_x \times t$

$= 10m.$

DISTANCE COVERED IN Y $S_y = \frac{1}{2} g t^2 = 5m.$

W.R.T. THE EDGE OF THE TABLE

S_y OF C.O.M. $= 5m - \frac{0.30}{2} = 4.85m.$

ROTATIONAL MOTION

IMPULSE IMPARTS AN ANGULAR MOMENTUM TO THE ROD, AND IT IS CONSERVED.

$J \times 0.2 = I \times \omega.$

(MOMENT ABOUT EDGE OF TABLE

$I \to ml^2/3$

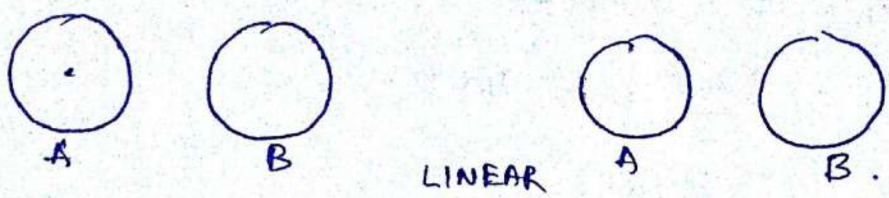
$$6 \times 0.2 = \frac{0.6 \times 0.09}{3}\,\omega.$$

$$\omega = \frac{200}{3}\ rad/sec.$$

AFTER 1SEC, $\theta = \frac{200}{3}$ rad w.r.t. INITIAL POSITION

20)

a) BEFORE COLLISION   AFTER COLLISION

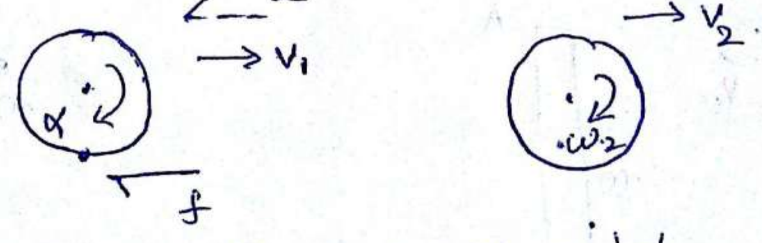


INITIAL VELOCITY $= V_1$ $= 0$ LINEAR VELOCITY AFTER COLLISION $= 0$ $= V_1$

ANGULAR VELOCITY $= \omega_1$ $= 0$.   ANGULAR VELOCITY AFTER COLLISION $= \omega_1$ $= 0$.

b) ROLLING: SPHERE A & SPHERE B IMMIDIATELY AFTER COLLISION WILL UNDERGO SLIPPING. AFTER A WHILE THEY WILL START ROLLING

SPHERE 'B':   IMMIDIATELY AFTER COLLISION.   STARTS ROLLING



TAKING MOMENT OF FRICTION ABOUT C.O.M. OF 'B'.

$$f \times r = I\alpha \quad — ①$$

FROM N.L.M., PSUEDO FORCE 'ma'

$$f = ma \quad — ②$$

WE ALSO KNOW FROM ROLLING THAT $V_1 = r\omega_1$ — ③

AFTER SPHERE 'B' STARTS ROLLING: $V_2 = r\omega_2$ — ④.

AND FROM KINEMATICS, $V_2 = V_1 - at$ — ⑤

$$\alpha t = \omega_2 \quad — ⑥$$

SOLVING THE EQUATIONS,

$$V_1 - at = r(\alpha t)$$

$$V_1 - \frac{2r\alpha}{5} t = r\alpha t.$$

$$V_1 = \frac{7r}{5}(\alpha t) = \frac{7r\,\omega_2}{5}.$$

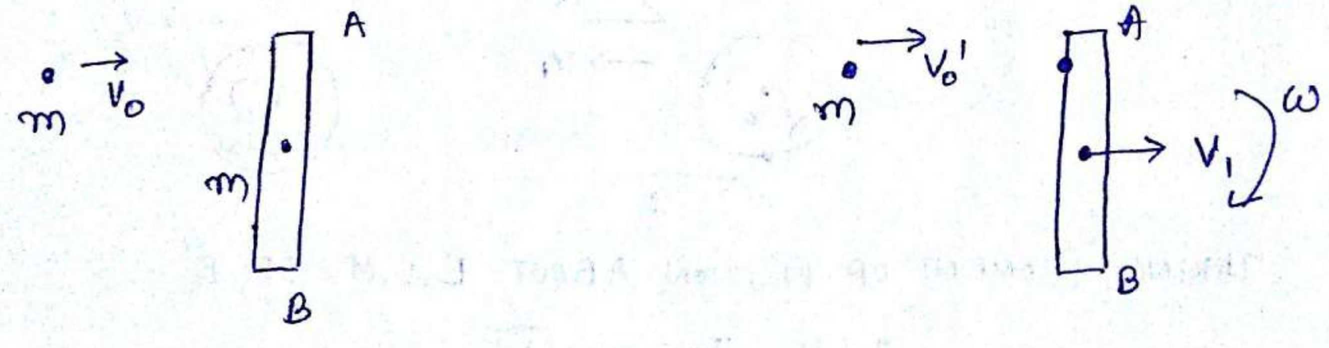
$$\omega_2 = \frac{5V_1}{7r}.$$

$$V_2 = \frac{5V_1}{7}.$$

c) WHEN $\mu_k = 0$, THE SPHERES WILL CONTINUE WITH THE SAME LINEAR AND ANGULAR VELOCITIES IMMIDIATELY AFTER COLLISION.

21)

BEFORE COLLISION AFTER COLLISION



COEFFICIENT OF RESTITUTION IS APPLIED AT THE POINT OF IMPACT. SO VELOCITY OF POINT 'A' AFTER COLLISION IS $\left[\underset{(C.O.M.)}{V_1} + \frac{l}{2}\omega.\right]$

$$\therefore \quad e = \frac{\left(V_1 + \frac{l}{2}\omega\right) - V_0'}{V_0 - 0}$$

$$\Rightarrow \quad V_0' = V_1 + \frac{l\omega}{2} - \frac{V_0}{2} \quad \text{--- (1)}$$

SINCE THERE IS NO EXTERNAL FORCE ACTING ON THE SYSTEM, MOMENTUM IS CONSERVED.

BEFORE COLLISION AFTER COLLISION

$$m v_0 = m v_1 + m v_0' \quad --- (2)$$

SINCE THERE IS NO EXTERNAL TORQUE, WE CAN CONSERVE ANGULAR MOMENTUM ABOUT CENTRE OF ROD.

$$m v_0 \times \frac{l}{2} = \frac{m l^2}{12} \omega + m v_0' \times \frac{l}{2} \quad --- (3)$$

Before After.

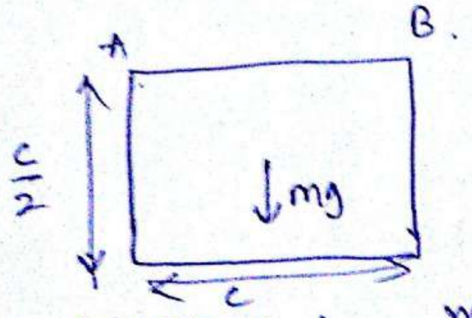
SOLVING EQTNS (1), (2) & (3).

WE GET $\omega = \frac{9 v_0}{5L}$ and $V_1 = \frac{3 V_0}{10}$.

X and Y COORDINATES OF END 'B' ARE OBTAINED BY COMBINING LINEAR AND ROTATIONAL MOTION.

$$x_B = v_f t - \frac{L}{2} \sin \omega t \; ; \; y_B = -\frac{L}{2} \cos \omega t.$$

22) i, a)



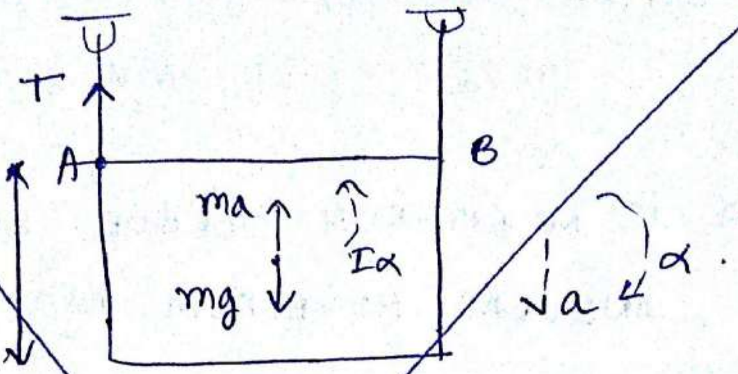
- TORQUE EQUATION ! $mg \times \frac{c}{2} = \frac{m}{3}\left(c^2 + \frac{c^2}{4}\right) \alpha$

$$\alpha = \frac{1.2 g}{c}$$

b) ACCELERATION OF COM $= R\alpha$.

WHERE 'R' $= \sqrt{\left(\frac{c}{4}\right)^2 + \left(\frac{c}{2}\right)^2}$

ii)



AFTER 'B' IS RELEASED WIRE-'A' WILL DEVELOP TENSION 'T'

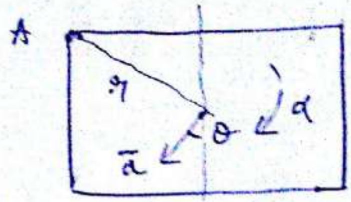
$$T = mg - ma$$

TORQUE EQUATION ABOUT 'A'

$$mg\left(\frac{c}{2}\right) = I\alpha + ma\left(\frac{c}{2}\right)$$

$$mg\frac{c}{2} = \frac{m}{12}\left(c^2 + \frac{c^2}{4}\right)\alpha$$

22) i, PLATE ATTACHED TO PINS.



$$\vec{a} = r\alpha \measuredangle \theta$$

$$\Rightarrow a_x = r\alpha \sin\theta = (r\sin\theta)\alpha$$

$$a_y = r\alpha \cos\theta = (r\cos\theta)\alpha$$

$$\Rightarrow a_x = \frac{1}{4}c\alpha \qquad a_y = \frac{1}{2}c\alpha.$$

$$I = \frac{1}{12} m \left[c^2 + \frac{c^2}{4} \right] = \frac{5}{48} mc^2$$

a) TORQUE EQUATION

$$W\left(\frac{c}{2}\right) = I\alpha + ma_x\left(\frac{c}{4}\right) + ma_y\left(\frac{c}{2}\right)$$

$$\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + m\left(\frac{1}{4}c\alpha\right)\left(\frac{c}{4}\right) + m\left(\frac{1}{2}c\alpha\right)\left(\frac{c}{2}\right)$$

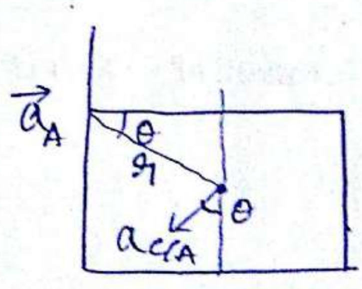
ON SOLVING, $\alpha = \dfrac{1.2g}{c}$.

b)

$$a_x = \frac{1}{4}c\alpha = \frac{1}{4}(1.2g) = 0.3g$$

$$a_y = \frac{1}{2}c\alpha = \frac{1}{2}(1.2g) = 0.6g$$

ii, PLATE SUSPENDED FROM WIRES.



$$a_c = a_A + a_{c/A}$$

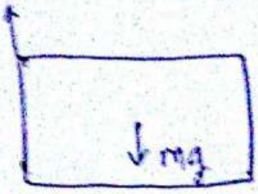
$$= a_A + r\alpha \measuredangle \theta$$

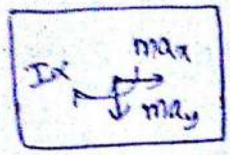
$$\Rightarrow a_y = 0 - r\alpha\cos\theta$$

$$= -(r\cos\theta)\alpha.$$

$$r\cos\theta = \frac{1}{2}c.$$

$$\Rightarrow a_y = \frac{1}{2}c\alpha \qquad \vec{a}_y = \frac{1}{2}c\alpha \downarrow$$





$\Sigma F_x = 0$

$\Rightarrow a_x = 0.$

TORQUE EQUATION: $W\left(\frac{c}{2}\right) = I\alpha + ma_y\left(\frac{c}{2}\right)$

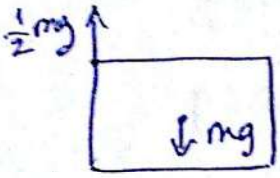
$$\frac{1}{2}mg\frac{c}{2} = \frac{5}{48}mc^2\alpha + \left(\frac{1}{2}mc\alpha\right)\left(\frac{c}{2}\right)$$

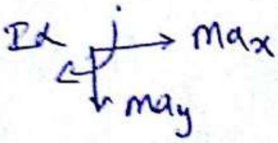
$$\frac{1}{2}mgc = \frac{17}{48}mc^2\alpha \quad \alpha = \frac{24g}{17c}.$$

$$a_y = \frac{1}{2}c\alpha = \frac{12g}{17}.$$

iii, PLATE SUSPENDED FROM SPRING

IMMIDIATELY AFTER SPRING 'B' IS RELEASED, THE TENSION IN SPRING 'A' IS STILL $\frac{1}{2}mg$ SINCE ITS ELONGATION IS UNCHANGED.





a) ANGULAR ACCELERATION

$$\left(\frac{1}{2}mg\right)\left(\frac{1}{2}c\right) = I\alpha.$$

$$\frac{1}{4}mgc = \frac{5}{48}mc^2\alpha \Rightarrow \alpha = \frac{2.4g}{c}.$$

b) $\Sigma F_x = 0 \Rightarrow a_x = 0.$

$$\Sigma F_y : \quad mg - \frac{1}{2}mg = ma_y$$

$$a_y = g/2.$$

23)

MOMENT OF INERTIA $I_{xx} = \int (dm) y^2$

NOW WHEN WE EXPRESS THIS INTEGRAL IN VARIABLE 'x'

THE AREA UNDER THE GRAPH IS DOUBLED SINCE

'y' IS VARYING FROM $-x^2$ to x^2.

HENCE $I_{xx} = 2 \times \left[\int_0^a (a-x) \rho x^4 \, dy \right]$

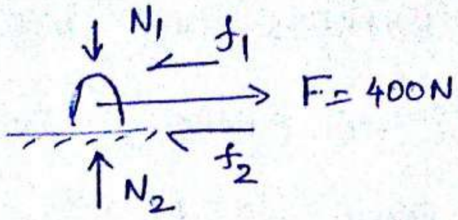
$$= 2 \left[\int_0^a (a-x) \rho x^4 (2x) dx \right]$$

ON SOLVING,

WE OBTAIN $I_{xx} = \frac{2a^7 \rho}{21}$

25)

ON BLOCK B, FORCES ARE

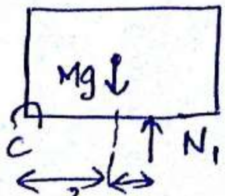


$$N_2 = N_1 + 25$$

$$f_2 = \mu N_2.$$

ON BLOCK A,

TORQUE EQUILIBRIUM ABOUT 'C'.



$$N_1(3+x) = 25 \times 10 \times 3$$

$$f_1 = \mu N_1$$

$$\text{WORK DONE} = \int_0^{3/2} [400 - f_1 - f_2] \cdot dx$$

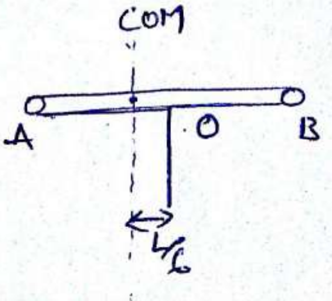
$$= 404\,J$$

CHANGE IN KINETIC ENERGY $= \frac{1}{2} \times 2.5 \times v^2 = 404.$

$$v \cong 18 \text{ m/s}$$

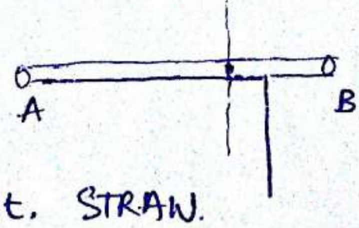
26) THERE IS NO FRICTION BETWEEN STRAW AND TABLE SO C.O.M REMAINS UNALTERED.

FOR THE FLY + STRAW SYSTEM.



x_{COM} w.r.t EDGE OF TABLE (O)

$$\frac{m\left(-\frac{L}{2}\right) + 2m(0)}{3m} = -L/6$$



NOW FOR FLY TO MOVE

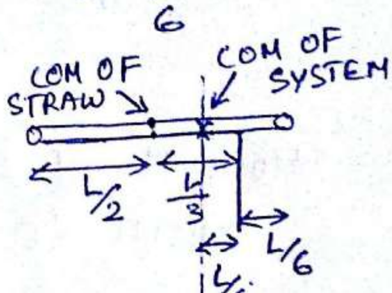
TO END 'B', IT WALKS 'L' w.r.t. STRAW.

SO THE C.O.M. OF ONLY THE STRAW HAS MOVED TO LEFT BY $\dfrac{mL}{m+2m} = \dfrac{L}{3}$ to LEFT.

SO, THE FLY IS AT END 'B' WITH SOME PART OF STRAW EXTENDS OVER THE EDGE.

$$\text{EXTENSION OVER THE EDGE} = L - \left(\frac{L}{2} + \frac{L}{3}\right) = \frac{L}{6}.$$

NOW AFTER SECOND FLY LANDS,

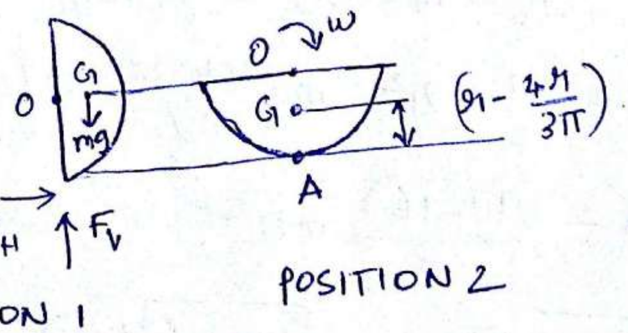


TORQUE.

$$(m+2m)g \times \frac{L}{6} = Mg \times \frac{L}{6}.$$

$$\underline{3m = M}.$$

27)



WHEN THE SEMICIRCULAR DISK IS IN POSITION1, ITS WEIGHT HAS POSITIVE POTENTIAL ENERGY. THUS,

$$V_1 = mgr.$$

WHEN THE DISK IN POSITION 2, THE P.E. IS

$$V_2 = mg\left(r - \frac{4r}{3\pi}\right).$$

K.E. IN POSITION 1 IS 'ZERO'.

K.E. OF SEMICIRCULAR DISK IN POSITION 2, IS

$$T_2 = \frac{1}{2} I_A \omega^2$$

ENERGY IS CONSERVED, $V_1 + T_1 = V_2 + T_2$.

$$mgr = mg\left(r - \frac{4r}{3\pi}\right) + \frac{1}{2} I_A \omega^2$$

$$\omega = \sqrt{\frac{2mg(4r/3\pi)}{I_A}}$$

MOMENT OF INERTIA ABOUT 'O' - $I_0 = \frac{1}{2} mr^2$

$$I_G = I_0 - m(OG)^2$$

$$I_A = I_G + m(AG)^2$$

$$= I_0 - m(OG)^2 + m(AG)^2.$$

$$I_A = \frac{1}{2} mr^2 - m\left(\frac{4r}{3\pi}\right)^2 + m\left(r - \frac{4r}{3\pi}\right)^2$$

$$I_A = \frac{(9\pi - 16)}{6\pi} mr^2$$

$$\Rightarrow \omega = \sqrt{\frac{2mg\left(\frac{4r}{3\pi}\right)}{I_A}} = \sqrt{\frac{2mg(4r/3\pi)}{\frac{(9\pi - 16)}{6\pi} mr^2}}$$

$$\omega = \sqrt{\frac{16g}{(9\pi - 16)r}}$$

28) IF THE MOMENT OF INERTIA ABOUT A CERTAIN AXIS IS 'J', THE MOMENT ABOUT AN OBLIQUE AXIS INCLINED AT ANGLE 'A' TO THE FIRST IS $J(\sin A)^2$.

THIS IS BECAUSE $r = y \sin A$

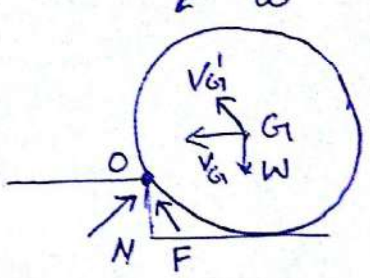
AND WHILE INTEGRATION $\sin^2 A$ REMAINS.

SO IN THIS CASE. $J = \frac{ml^2}{3}$.

$$A = 45^\circ$$

$$I_{OBLIQUE} = J\sin^2 A = \frac{ml^2}{3} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{ml^2}{6}.$$

29) BEFORE THE IMPACT,



VELOCITY OF COM is V_G.

$V_G = \omega R$.

AFTER STRIKING THE STEP, IT STARTS TURNING AROUND POINT 'O'.

THEREFORE, THE VELOCITY OF THE C.O.M. (V_G), MUST BE PERPENDICULAR TO 'OG'. THIS CHANGE IN DIRECTION HAPPENS OVER A VERY SHORT INTERVAL.

ANGULAR MOMENTUM BEFORE IMPACT:

$$H_O = I_G \omega + mR \sin\theta \, V_G$$

$$= \left[\frac{I_G}{R} + m(R-h)\right] V_G.$$

ANGULAR MOMENTUM AFTER IMPACT

$$H_O' = I_O \omega' = I_O \frac{V_G'}{R} = \left[\frac{I_G}{R} + mR\right] V_G'$$

ω, ω' ARE ANGULAR VELOCITIES FOR BEFORE AND AFTER

$$H_0 = H_0'.$$

$$\Rightarrow \quad V_G' = \left(\frac{I_G + mR(R-h)}{I_G + mR^2}\right) V_G = \left(1 - \frac{Rh}{k_0^2}\right) V_G$$

WHERE k_0 = RADIUS OF GYRATION ABOUT 'O'.

ROLLING ABOUT POINT 'O'

AFTER THE IMPACT, THE CHANGE IN POTENTIAL ENERGY mgh, IS OBTAINED BY A DECREASE IN THE KINETIC ENERGY,

$$\frac{1}{2} I_0 (\omega')^2 = mgh + \frac{1}{2} I_0 (\omega'')^2$$

OR,

$$\left(V_G''\right)^2 = \left(V_G'\right)^2 - 2gh\frac{R^2}{k_0^2}.$$

(V_G'') IS THE VELOCITY AFTER CLIMBING THE STEP, WHICH IS 'ZERO' FOR MINIMUM V_G.

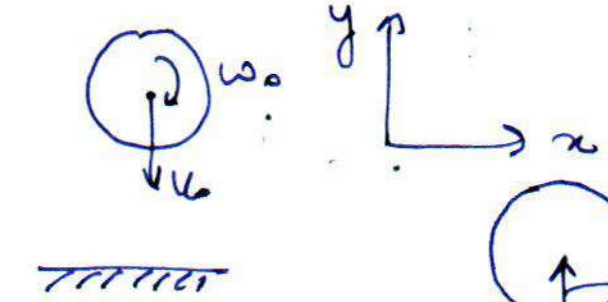
$$\Rightarrow \quad 0 = \left(1 - \frac{Rh}{k_0^2}\right)^2 (V_G)^2_{min} - 2gh\frac{R^2}{k_0^2}.$$

$$(V_G)^2_{min} = \frac{2gh\, R^2/k_0^2}{\left(1 - Rh/k_0^2\right)^2}$$

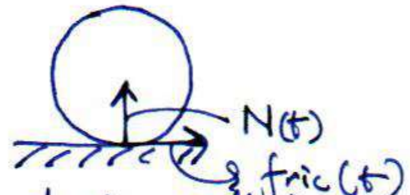
IN CASE OF RING $k_0 = \sqrt{2}\cdot R$

$$\Rightarrow (V_G)_{min} = \frac{2}{1.7}\sqrt{0.3\, gR}$$

(31)



before collision



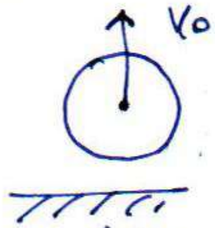
during collision
a time dependent Normal force & friction force acts on body

$fric(t) = \mu N(t)$

Impulse due to time varying friction in x-axis $= \int fric(t).dt$

$= \int \mu N(t).dt$

$J_{(x)} = \mu \int N(t).dt$



(after collision)

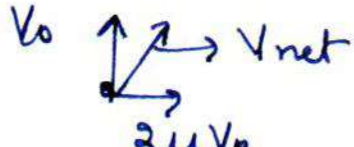
Impulse due to time varying Normal Reaction N(t)

$= \int N(t).dt =$ momentum change along y-axis

$\boxed{J_{(y)} = 2mV_0}$

So $\boxed{J_{(x)} = 2\mu m V_0}$

& $V_{(x)} = 2\mu V_0$



$\boxed{V_{net} = V_0\sqrt{1+4\mu^2}}$

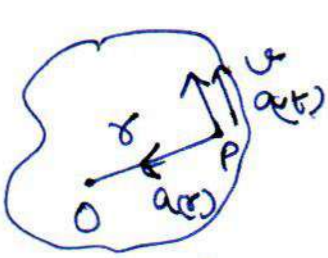
$J_{(x)}$ also changes angular momentum of the body

$I\omega_f = I\omega_0 - J_{(x)} \times R$

$\omega_f = \omega_0 - \frac{J_{(x)} \times R}{I} = \omega_0 - \frac{2\mu m V_0 R}{\frac{2}{5} m R^2}$

$\Rightarrow \boxed{\omega_f = \omega_0 - \frac{5\mu V_0}{R}}$

(82)



$\beta = at \qquad a = 2 \times 10^{-2}\ \text{rad/sec}^2$

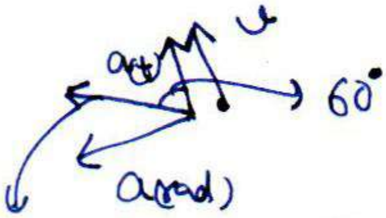
$a_{(t)} = r\beta = rat$

$\frac{d\omega}{dt} = at \Rightarrow \int_0^{\omega} d\omega = a\int_0^{t} t\,dt$

$$\omega = \frac{at^2}{2}$$

$$v = \omega r = \frac{at^2 r}{2}$$

$$a_{rad} = \frac{v^2}{r} = \frac{a^2 t^4 r^{\cancel{2}}}{4\cancel{r}} = \frac{a^2 t^4 r}{4}$$



a_{net}

$$\tan 60^\circ = \frac{a_{(rad)}}{a_{(t)}}$$

$$\sqrt{3} = \frac{a^{\cancel{2}} t^{\cancel{4}\,3} \cancel{r}}{4 \times \cancel{r}\,\cancel{a}\,t}$$

$$t^3 = \frac{4\sqrt{3}}{a}$$

$$t^3 = \frac{\cancel{4}^{\,2}\sqrt{3} \times 10^2}{\cancel{2}}$$

$$t^3 = 200 \times \sqrt{3} = 200 \times 1.732$$

$$t^3 = 346.4$$

$$\Rightarrow t \simeq 7\ \text{sec}$$

(33). angular deceleration $\beta \propto \sqrt{\omega}$

$$\beta = k\sqrt{\omega}$$

$$-\frac{d\omega}{dt} = k\sqrt{\omega}$$

$$-\int_{\omega_0}^{0} \frac{d\omega}{\sqrt{\omega}} = k\int_0^t dt$$

$$-\left[2\sqrt{\omega}\right]_{\omega_0}^{0} = kt$$

$$2\sqrt{\omega_0} = kt \qquad \underbrace{t}_{\text{timetaken}} = \frac{2\sqrt{\omega_0}}{k}$$

$$-\frac{d\omega}{dt} = k\sqrt{\omega}$$

$$-\frac{d\omega}{d\theta} \times \omega = k\sqrt{\omega}$$

$$-\int_{\omega_0}^{0} \sqrt{\omega}\, d\omega = k\int_{\theta_1}^{\theta_2} d\theta$$

$$-\left(\frac{2}{3}\omega^{3/2}\right|_{\omega_0}^{0} = k(\theta_2 - \theta_1)$$

$$\Rightarrow \underbrace{(\theta_2 - \theta_1)}_{\text{angular displacement}} = \frac{2}{3}\frac{\omega_0^{3/2}}{k}$$

$$\omega_{mean} = \frac{\text{angular displacement}}{\text{time taken}}$$

$$= \frac{\cancel{2}}{3}\frac{\omega_0^{3/2}}{\cancel{k} \times \cancel{2}\,\omega_0^{1/2}}\cancel{k}$$

$$\omega_{mean} = \frac{\omega_0}{3}$$

34.

$$\omega = \omega_0 - a\phi \qquad \phi_{(0)} = 0$$

$$\frac{d\phi}{dt} = \omega_0 - a\phi$$

$$\int_0^{\phi} \frac{d\phi}{(\omega_0 - a\phi)} = \int_0^{t} dt$$

$$-\frac{1}{a} \ln|\omega_0 - a\phi| \Big|_0^{\phi} = t$$

$$\ln\left|\frac{\omega_0 - a\phi}{\omega_0}\right| = -at$$

$$1 - \frac{a\phi}{\omega_0} = e^{-at}$$

$$1 - e^{-at} = \frac{a\phi}{\omega_0}$$

$$\Rightarrow \boxed{\phi = \frac{\omega_0}{a}(1 - e^{-at})}$$

$$\omega = \omega_0 - \cancel{a}\frac{\omega_0}{\cancel{a}}(1 - e^{-at})$$

$$\boxed{\omega = \omega_0 e^{-at}}$$

35. angular acceleration $\beta = \beta_0 \cos\phi$

$$\frac{d\omega}{dt} = \beta_0 \cos\phi$$

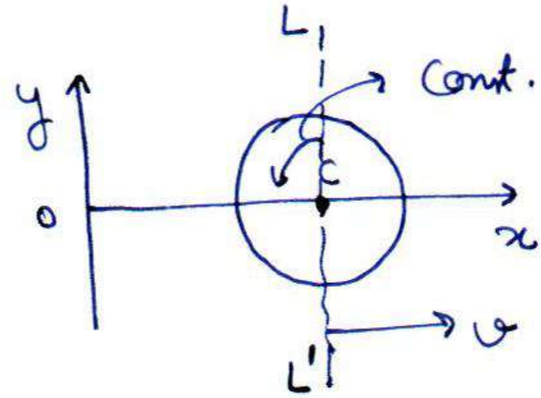
$$\frac{d\omega}{d\phi} \cdot \omega = \beta_0 \cos\phi$$

$$\int_0^{\omega} \omega \, d\omega = \beta_0 \int_0^{\phi} \cos\phi \, d\phi$$

$$\frac{\omega^2}{2} = \beta_0 \sin\phi$$

$$\boxed{\omega = \pm(2\beta_0 \sin\phi)^{1/2}}$$

36. (a)



Const. angular acceleration $= \beta$

$$\omega_{(t)} = \beta t$$

velocity of any particle at a distance y on line LL' (due to rotation only)

$$v_{(rot)} = \omega \times y = \beta t y$$

(for particles on +ve y-axis, this $v_{(rot)}$ will be towards −ve x-axis) and the pt. where magnitude of this rotational velocity equals translational velocity v will be at rest or our required instantaneous axis of rotation)

so $\beta t y = v$

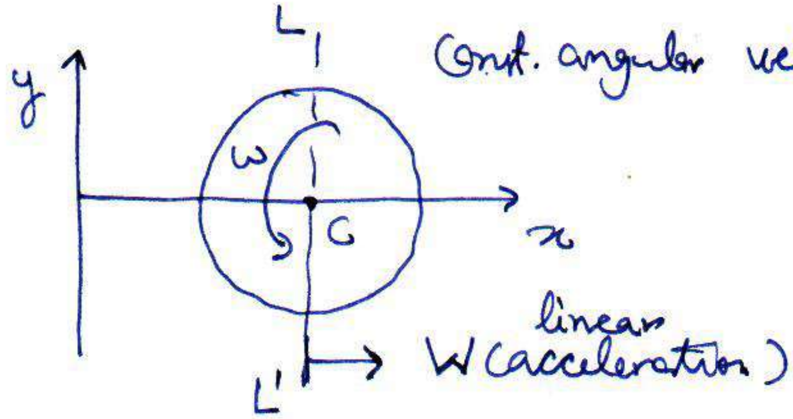
$$y = \frac{v}{\beta t}$$

also $x = v \times t$

or $t = x/v$

$$\Rightarrow y = \frac{v^2}{\beta x}$$

36(b).



Const. angular velocity = ω

velocity of any particle P on line LL' due to rotation only

$$v_{rot} = y\omega$$

$$v_{tran} = W \times t$$

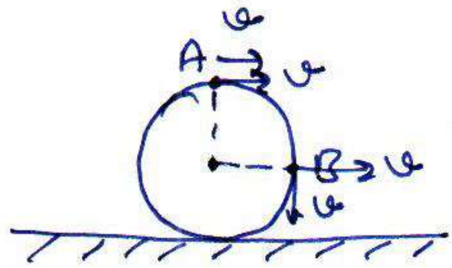
for instantaneous axis of rotation

$$y\omega = W \times t \qquad \text{also } x = \frac{1}{2} W t^2$$

$$y\omega = W \times \left(\frac{2x}{W}\right)^{1/2} \qquad t = \left(\frac{2x}{W}\right)^{1/2}$$

$$\boxed{y = \frac{\sqrt{2xW}}{\omega}}$$

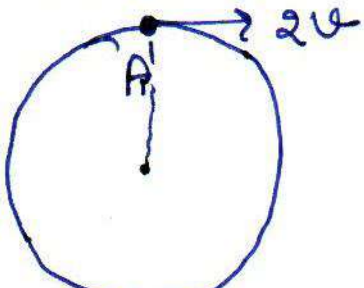
37.



Cyl. rolls without slipping so $v = r\omega$

& $v_A = 2v$ $\quad v_B = \sqrt{2}v$

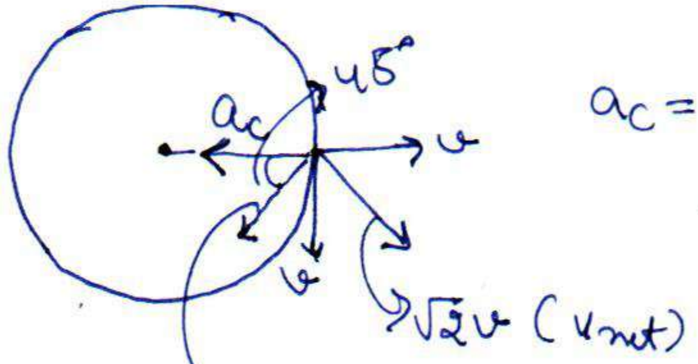
(a)



req. centripetal acc. $= \dfrac{(2v)^2}{r_{cur}} = \dfrac{v^2}{r}$

$\Rightarrow \boxed{r_{cur} = 4r}$ acc. ⊥ to $(2v)$ net velocity

37(b)



$$a_c = \frac{v^2}{r}$$

$a_c \perp$ to $v_{net} = \frac{v^2}{r} \times \frac{1}{\sqrt{2}}$

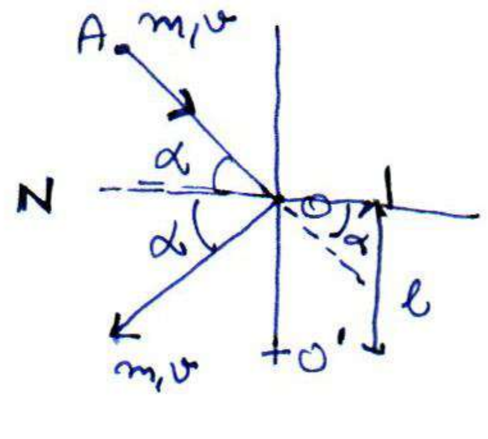
$$\& \quad \frac{(\sqrt{2}v)^2}{r_{cur}} = \underbrace{\frac{v^2}{\sqrt{2}r}}$$

acc. component $\perp$ to net. velocity.

$$\frac{2\cancel{v^2}}{r_{cur}} = \frac{\cancel{v^2}}{\sqrt{2}r}$$

$$\boxed{r_{cur} = 2\sqrt{2}r}$$

38



As the only force, body experienced during collision is along the Normal, so for any pt. lying on this normal line, there will be no Torque due to this Normal Force. So angular momentum (M) remains conserved about any pt. lying on this normal.

About pt. O'

$\vec{L}_{ini} = mvl\cos\alpha$ (going inside the paper)

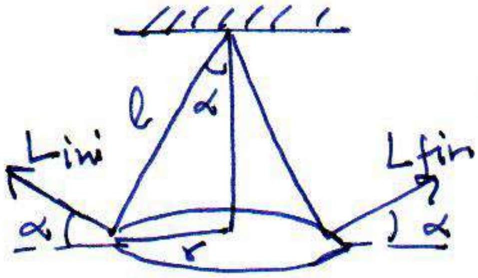
$\vec{L}_{fin} = mvl\cos\alpha$ (coming outwards from paper)

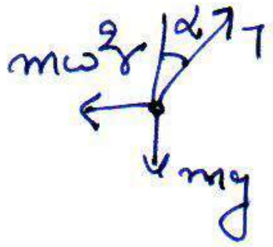
$|\Delta\vec{L}| = 2mlv\cos\alpha$

---

(39)

2.





$$r = l\sin\alpha$$

$$mg = T\cos\alpha$$

$$m\omega^2 r = T\sin\alpha$$

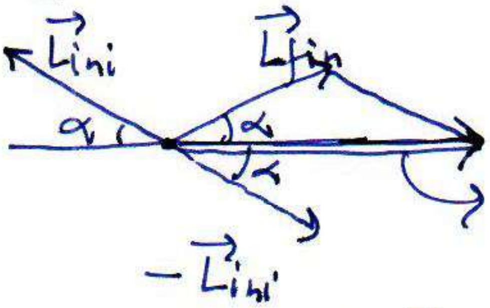
$$m\omega^2 l \cancel{\sin\alpha} = T\cancel{\sin\alpha}$$

$$T = m\omega^2 l$$

$$\cancel{m}g = \cancel{m}\omega^2 l\cos\alpha$$

$$\boxed{\cos\alpha = \frac{g}{\omega^2 l}}$$

$$|\vec{L}_{ini}| = m\omega r l = m\omega l^2\sin\alpha$$

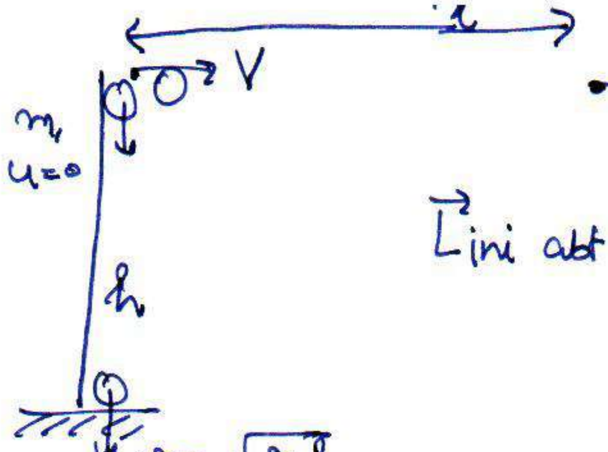


$$|\Delta L| = 2|L|\cos\alpha$$

$$|\Delta L| = 2m\cancel{\omega} l^2 \sin\alpha \times \frac{g}{\omega^{\cancel{2}}\cancel{l}}$$

$$= \frac{2mlg}{\omega}\left(\sqrt{1 - \frac{g^2}{(\omega^2 l)^2}}\right)$$

40.



at time $t = \sqrt{\frac{2h}{g}}$

$\vec{L}_{ini}$ abt O $= 0$

$v = \sqrt{2gh}$

$h = \frac{1}{2} g t^2$

$t = \sqrt{\frac{2h}{g}}$

As the body decends a height h, pt. O moves a distance x suchthat $x = V \times t$

$x = V \times \sqrt{\frac{2h}{g}}$

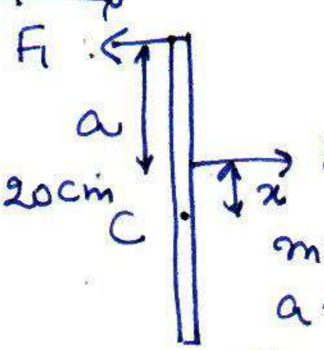
$\vec{L}_{fin}$ abt O at $t = \sqrt{\frac{2h}{g}}$ $= \underbrace{(m v \times x)}_{\text{Coming out of paper}} + \underbrace{(m V \times h)}_{\text{(going inside paper)}}$

$= \left(m \times \sqrt{2gh} \times V \times \sqrt{\frac{2h}{g}}\right) - mVh$

$= 2mVh - mVh$

$= mVh$

41.



$F_2 > F_1$ (for no net torque)

as $F_2 = 5N$

$F_1 = F_2 - F_{net}$

$F_1 = 3N$

$m = 1$ Kg

$a = 2 m/s^2$

$F_{net} = m \times a$

$= 2N$

as body is only translating

$F_1 \times (a + x) = F_2 (x)$

$3 \times (20 + x) = 5 (x)$

$60 + 3x = 5x$

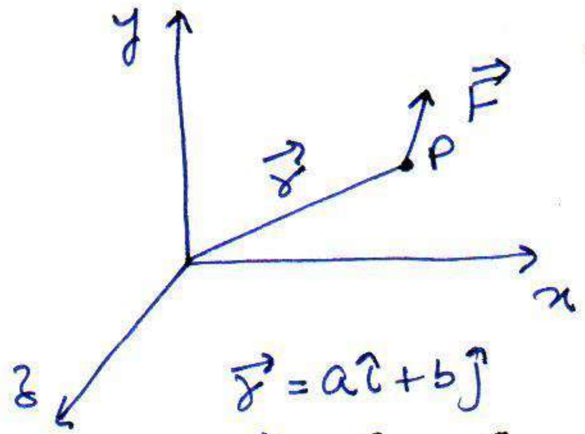
$x = 30$cm,

so length of rod $= 2(a + x)$

$= 2(20 + 30)$

$= 100$ cm

(42)



$$\vec{r} = a\hat{i} + b\hat{j}$$

$$\vec{F} = A\hat{i} + B\hat{j}$$

moment $\vec{N} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ A & B & 0 \end{vmatrix}$

$$\vec{N} = \hat{k}(aB - bA)$$

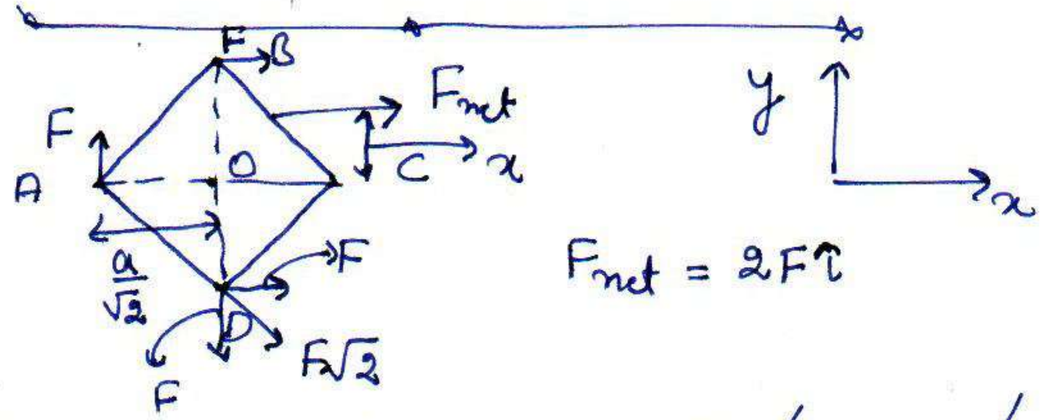
arm of the force $(l) = r\sin\theta$

$$|\vec{N}| = |r||\vec{F}|\sin\theta$$

$$\Rightarrow r\sin\theta = \frac{|\vec{N}|}{|\vec{F}|}$$

$$= \frac{|aB - bA|}{\sqrt{A^2 + B^2}}$$

(43)



$$F_{net} = 2F\hat{i}$$

$\vec{\tau}_0 = F_{net} \times x$
$= 2Fx$

(going inside)

$\vec{\tau}_0 = F \times \frac{a}{\sqrt{2}} + \cancel{F \times \frac{a}{\sqrt{2}}} - \cancel{F \times \frac{a}{\sqrt{2}}}$

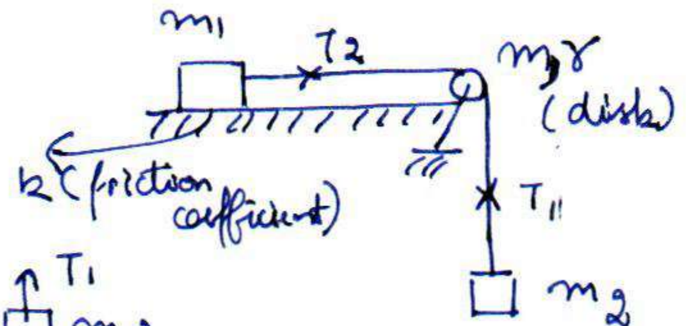
(due to forces at A, B + D)

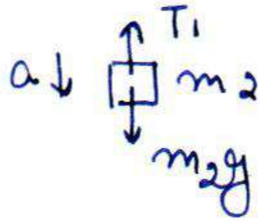
$= F \times \frac{a}{\sqrt{2}}$

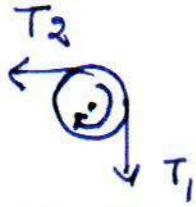
$$2\cancel{F}x = \cancel{F} \times \frac{a}{\sqrt{2}}$$

$x = \frac{a}{2\sqrt{2}}$ or midway b/w B + C

(44)







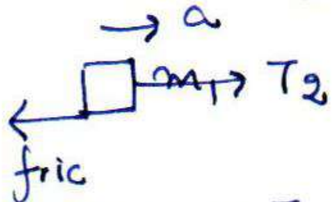
$$m_2 a = m_2 g - T_1$$

$$\underline{T_1 = m_2 g - m_2 a}$$

$$(T_1 - T_2) \times r = I\alpha$$

$$(T_1 - T_2) \times \cancel{r} = \frac{1}{2} m r^{\cancel{2}} \times \frac{a}{\cancel{r}}$$

$$\underline{(T_1 - T_2) = \frac{ma}{2}}$$



$$m_1 a = T_2 - \text{fric}$$

$$m_1 a = T_2 - k m_1 g$$

$$\underline{T_2 = m_1 a + k m_1 g}$$

$$m_2 g - m_2 a - m_1 a - k m_1 g = \frac{ma}{2}$$

$$g(m_2 - k m_1) = a\left(\frac{m}{2} + m_1 + m_2\right)$$

$$a = \frac{2g(m_2 - k m_1)}{(m + 2(m_1 + m_2))}$$

$$S = \frac{1}{2} a t^2$$

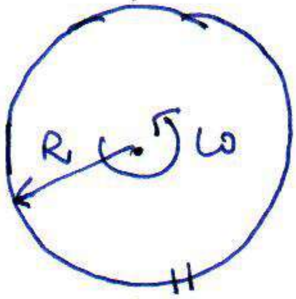
$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2} g(m_2 - k m_1)}{(m + 2(m_1 + m_2))} \times t^2$$

$$S = \frac{g(m_2 - k m_1) t^2}{(m + 2(m_1 + m_2))}$$

Work (friction) $= -\text{fric} \times S$

$$= -\mu m_1 g \times S = -\frac{k m_1 g^2 t^2 (m_2 - k m_1)}{(m + 2(m_1 + m_2))}$$

(45). (a) Ring



$$dM = \frac{M}{2\pi R}dx$$

$g\,dx$

$$d\text{fric} = k\,dM\,g$$

$$= \frac{kMg}{2\pi R}dx$$

$$d\tau = d\text{fric} \times R$$

$$d\tau = \frac{kMg}{2\pi}dx$$

all the $d\tau$'s add up in same direction

so $\tau_{net} = \frac{kMg}{\cancel{2\pi}} \times \cancel{2\pi} R$

$$\boxed{\tau_{net} = kMgR} \quad —(1)$$

frictional Torque due to ring of mass M + radius R

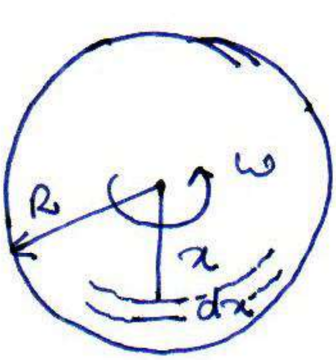
$$kMgR = I\alpha = MR^2\alpha$$

$$\Rightarrow \alpha = \frac{kg}{R}$$

$$\cancel{\omega_f}^{\,0} = \omega_0 - \alpha t$$

$$t = \frac{\omega_0}{\alpha} = \frac{\omega R}{kg}$$

(b) Disk



$dA = 2\pi x\,dx$ of ring at x

$$dM = \frac{M}{\cancel{\pi} R^2} \times 2\cancel{\pi} x\,dx$$

$$dM = \frac{2M}{R^2}x\,dx$$

45(b) cont.

from part(a)

frictional Torque due to ring of mass M + radius r

$$\tau = k M g R$$

IIIly for ring of mass dM + radius x

$$d\tau = k\, dM\, g\, x$$

$$= k \times g \times x \times \frac{2M}{R^2} x\,dx$$

$$d\tau = \frac{2M}{R^2} kg\, x^2 dx$$

$$\vec{\tau}_{net} = \int_0^R d\tau = \frac{2Mkg}{\cancel{R^2}} \times \frac{R^{\cancel{3}}}{3}$$

whole disk

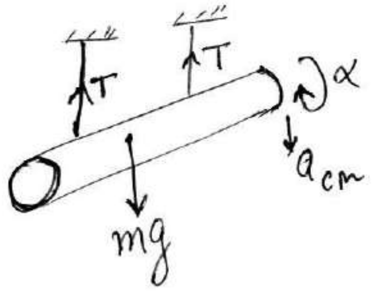
$$= \frac{2}{3} MkgR = I\alpha$$

$$\frac{2}{3} \cancel{M} kg \cancel{R} = \frac{\cancel{M} R^{\cancel{2}}}{2} \times \alpha$$

$$\alpha = \frac{4kg}{3R}$$

time to stop $= \frac{\omega_0}{\alpha} = \frac{3\omega R}{4kg}$

(46) By Symmetry we can say Tension in Both the string will be same.



$r = 1.3\,cm$

$m = 8\,kg$

So.
$$mg - 2T = m a_{cm} \quad \text{—(I)}$$

$$\Sigma \tau_{com} = I\alpha \Rightarrow Tr + Tr = I\alpha$$

$$2Tr = \frac{mr^2}{2}\alpha \Rightarrow T = \frac{mr\alpha}{4} \quad \text{—(II)}$$

for no slipping condition $a_{cm} = r\alpha$

so $$mg - \frac{2ma}{4} = ma \Rightarrow a_{cm} = \frac{2g}{3}$$

$$\& \ \alpha = \frac{a_{cm}}{r} = \frac{2\times10}{3\times1.3\times10^{-2}} = 512.82\,rad/sec^2$$

$$\& \ T = \frac{ma_{cm}}{4} = \frac{mg}{6} = \frac{8\times10}{6} = 13.33\,N$$

(b) Power due to gravitation force

$$P = \vec{F}.\vec{v} = Fv\cos0^\circ = mgv$$

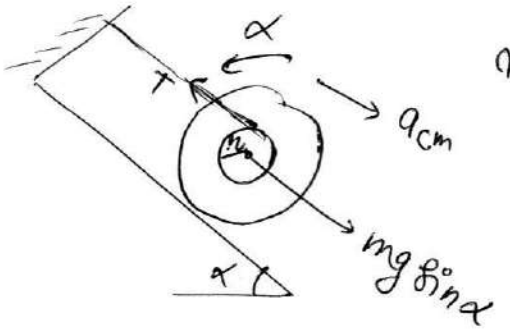
$$P = mga_{cm}t = mg\times\frac{2}{3}gt$$

$\left(\because a_{cm} \text{ is constant} \quad v = 0 + at\right)$

$v = at$

$$\boxed{P = \frac{2}{3}mg^2t}$$

(47)



$m = 200\,gm = 0.2\,kg$

$r = 3\,cm = 0.03\,m$

$I = 0.45\,g.m^2 = .45\times10^{-3}\,kg.m^2$

$$mg\sin\alpha - T = ma_{cm} \quad \text{--- (I)}$$

$$\Sigma\tau_{cm} = I\alpha \Rightarrow Tr = I\alpha \quad \text{--- (II)}$$

$$a_{cm} = r\alpha \quad \text{--- (III)}$$

By (I), (II) & (III)

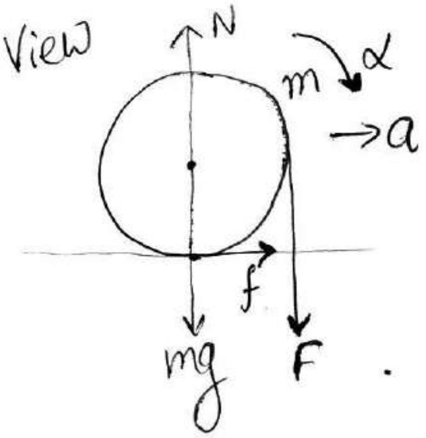
$$mg\sin\alpha = \left(m + \frac{I}{r^2}\right)a_{cm}$$

$$\boxed{a_{cm} = \frac{g\sin\alpha}{1 + \frac{I}{mr^2}}}$$

Put down the values

$$\boxed{a_{cm} = 1.4\,m/s^2}$$

(48) Top View



$$f = ma \quad \text{— (I)}$$

$$N = mg + F \quad \text{— (II)}$$

$$f = \mu N \quad \text{— (III)}$$

$$FR - fR = \frac{1}{2} mR^2 \alpha \quad \text{— (IV)}$$

$$a = R\alpha \quad \text{— (V)}$$

By eq (I) & (IV) [adding]

$$\left.\begin{aligned} F - f &= \frac{ma}{2} \\ f &= ma \end{aligned}\right] \Rightarrow F = \frac{3ma}{2}$$

$$f = \frac{2F}{3} = \mu N = \mu(mg + F)$$

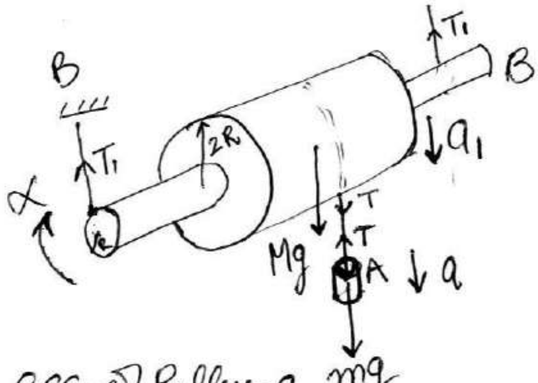
$$2F = 3\mu mg + 3\mu F$$

$$\boxed{F = \frac{3\mu mg}{2 - 3\mu}}$$

now $\quad a = \dfrac{2F}{3m} = \dfrac{2}{3m}\left(\dfrac{3\mu mg}{2-3\mu}\right) = \dfrac{2\mu g}{2-3\mu}$

$$\boxed{a = \frac{2\mu g}{2 - 3\mu}}$$

(49)



let the acc. of Pulley $= a_1$ & ang. acc. $= \alpha$

for block A

$$mg - T = ma \quad \text{—} (1)$$

for block B

$$Mg - 2T_1 + T = Ma_1 \quad \text{—} (11)$$

for no slipping condition

$$a_1 = r\alpha \quad \text{—} (111)$$

$$\& \quad a = a_1 + 2r\alpha \quad \text{—} (1V)$$

$$a = a_1 + 2a_1 = 3a_1$$

$$a = 3a_1$$

for Pulley B

$$\Sigma \tau_{COM} = I\alpha$$

$$T \times 2r + 2T_1 \times r = I\alpha \quad \text{—} (V)$$

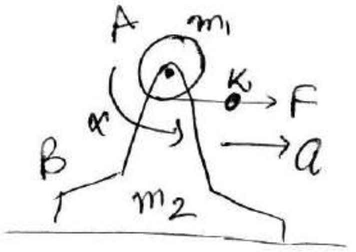
$$T + T_1 = \frac{I}{2r}\alpha \quad \text{—} (V)$$

$$[(1) \times 3] + [(V) \times 2] + (11)$$

$$3mg + Mg = a\left[\frac{I}{3r^2} + 3m + \frac{M}{3}\right]$$

$$\boxed{a = \frac{3g(M+3m)}{M + 9m + I/r^2}}$$

(50)



for translational motion

$$F = (m_1 + m_2)\,a \quad \text{— (I)}$$

for Rotational motion

$$F \times r = \frac{m_1 r^2}{2}\alpha \quad \text{— (II)}$$

acc. of point K,

$$a_K = a + r\alpha$$

$$= \frac{F}{m_1+m_2} + \frac{2F}{m_1}$$ (Put down the value of a & α)

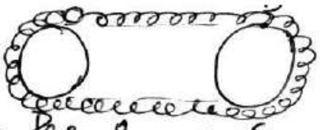
$$\boxed{a_K = \frac{F(3m_1+m_2)}{m_1(m_1+m_2)}}$$

(b) $$KE = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}\times\frac{m_1R^2}{2}\times\omega^2$$ [$v = at$ & $\omega = \alpha t$]

$$= \frac{1}{2}(m_1+m_2)\left(\frac{F}{m_1+m_2}\right)^2 t^2 + \frac{1}{2}\,\frac{m_1R^2}{2}\times\left(\frac{2F}{m_1 r}\right)^2 t^2$$

$$= F^2t^2\left[\frac{1}{2(m_1+m_2)} + \frac{1}{m_1}\right]$$

$$\boxed{KE = \frac{F^2t^2(3m_1+2m_2)}{2m_1(m_1+m_2)}}$$

(51) vel. of COM $= v$ 

& vel. of lower part of belt in Ground $= v - v = 0$

" " upper " " " " " $= v + v = 2v$

lets assume $m_{AB} = m_{CD} = m_1$

$m_{AD} = m_{BC} = m_2$

So total Mass $m = 2m_1 + 2m_2$

{Part AB & CD is in pure translation & Part BC & AD is in like a Rolling motion}

$$KE = KE_{AB} + KE_{BC} + KE_{CD} + KE_{AD}$$

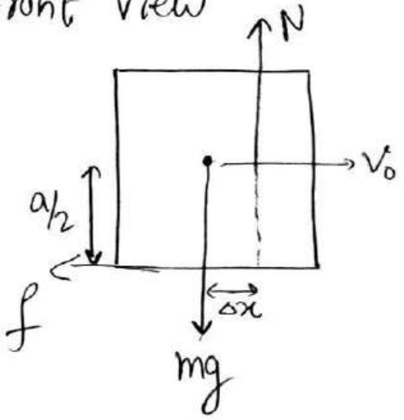
$$= \frac{1}{2}m_1(2v)^2 + \left(\frac{1}{2}(m_2R^2)\left(\frac{v}{R}\right)^2 + \frac{1}{2}m_2v^2\right) + 0 + \left(\frac{1}{2}(m_2R^2)\frac{v^2}{R^2} + \frac{1}{2}m_2v^2\right)$$

$$= 2m_1v^2 + 2m_2v^2$$

$$= (2m_1 + 2m_2)v^2$$

$$\boxed{KE = mv^2}$$

(52) front View



So $N = mg$

Since cube is in motion friction will act at its Maximum Value.

$f = \mu N$

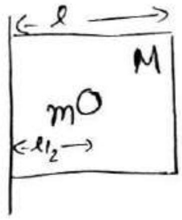
Cube is in Rotational Equilibrium

$$\Sigma \tau_{com} = 0$$

$$f \times \frac{a}{2} = N \Delta x \Rightarrow \mu \cancel{N} \frac{a}{2} = \cancel{N} \Delta x$$

$$\boxed{\Delta x = \frac{\mu a}{2}}$$

(53)



(a) initial angular Momentum

$$J_1 = mv\frac{l}{2}$$

final Ang. Momentum

$$J_2 = mv'\frac{l}{2} + \frac{Ml^2}{3}\omega \quad -$$

$J_1 = J_2$ (law of angular Momentum)

$$mv\frac{l}{2} = mv'\frac{l}{2} + \frac{Ml^2}{3}\omega \quad \text{— (I)}$$

Kintec Energy is conserved (Elastic collision)

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}\left(\frac{Ml^2}{3}\right)\omega^2 \quad \text{— (II)}$$

or $\quad e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{\omega l/2 - v'}{v} \quad$ — (III)

By using (I) & (III) or (I) & (II)

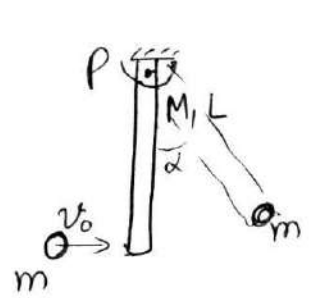
$$\boxed{v' = \left(\frac{3m-4M}{3m+4M}\right)v} \quad \& \quad \omega = \left(\frac{12mv}{3m+4M}\right)\frac{1}{l}$$

(b) $F = M\omega^2\frac{l}{2}$

$$= M\left(\frac{12mv}{(3m+4M)l}\right)^2 \times \frac{l}{2}$$

$$\boxed{F = \frac{8Mv^2}{l\left(1+\frac{4M}{3m}\right)^2}}$$

(54)



$m << M$ let final angular vel. $= \omega$

By angular Momentum Conservation about P.

$$m v_0 l = m l^2 \omega + \frac{M l^2}{3} \omega$$

$$\omega = \frac{m v_0}{m l + \frac{M l}{3}}$$

now Energy conservation at intial & final Position.

$$\frac{1}{2}\left(m l^2 + \frac{M l^2}{3}\right)\omega^2 = \left[M g \frac{l}{2}(1-\cos\alpha)\right] + m g l(1-\cos\alpha)$$

$$\frac{1}{2}\left(m l^2 + \frac{M l^2}{3}\right)\frac{(m v_0)^2}{\left(m l + \frac{M l}{3}\right)^2} = g l(1-\cos\alpha)\left(\frac{M}{2} + m\right)$$

$\cancel{v_0 = g l(1-\cos\alpha)}$ $v_0 = \left[g l(1-\cos\alpha)(5 M m + M^2 + 6 m^2)\right]^{1/2}$ / $m\sqrt{3}$

$$v_0 = \frac{M}{m\sqrt{3}}\left[g l \times 2\sin^2\alpha/2 \left(\cancel{5\frac{m}{M}} + 1 + \cancel{\frac{6 m^2}{M^2}}\right)\right]^{1/2}$$ now $m << M$, $\frac{m}{M} << 1$

$$\boxed{v_0 = \frac{M}{m}\sqrt{\frac{2 g l}{3}}\sin\alpha/2}$$

(b) $P_i = m v_0$ $P_f = m\omega l + M\frac{\omega l}{2} = \left(m + \frac{M}{2}\right)\omega l = \left(m + \frac{M}{2}\right)\frac{m v_0}{m l + \frac{M l}{3}} \cdot l$

$$\Delta P = P_f - P_i = \left[\left(m + \frac{M}{2}\right)\left(\frac{m v_0}{m l + \frac{m l}{3}}\right) l\right] - m v_0 = \frac{1}{2} m v_0 \frac{M}{3m + M}$$ (Put the values from above)

$$\boxed{\Delta P = M \sin\left(\frac{\alpha}{2}\right)\sqrt{\frac{1}{6} g l}}$$

PTO

(c) $m v_0 x = m x^2 \omega + \frac{ML^2}{3}\omega$

$$\omega = \frac{m v_0 x}{m x^2 + \frac{ML^2}{3}} \quad \text{—(1)}$$

$$\Delta P = \left[m \omega x + M \frac{\omega l}{2}\right] - m v_0$$

$$= \frac{1}{2}(m v_0 M l)\left(\frac{3x - 2l}{3mx^2 + ML^2}\right)$$ {Put the value of ω from eq. (1) & simplify it}

$$\Delta P = 0$$

$$3x - 2l = 0$$

$$\boxed{x = 2l/3}$$