SEQUENCE SERIES

EXERCISE – 1 [A]

Q.1 **[A]**

$\Rightarrow 5t_5 8t$

$\Rightarrow 5(a+4d)=8(a+7d)$

$\Rightarrow 3a=-36d$

$\Rightarrow d=-\frac{1}{12}a$

Now, $T_{13}=a+12d$

$\Rightarrow a+12\left(-\frac{1}{12}a\right)=0$

Q.2 **[C]**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d+34$ ………..(1)

$\Rightarrow T_{13}=a+12d$

$\Rightarrow a+12d=64$ ………..(2)

Solving (1) and (2)

$\Rightarrow a=4$ and $d=5$

$\Rightarrow \therefore T_{18}=a+17d$

$\Rightarrow 4+17\times 5=89$

Q.3 **[B]**

$\Rightarrow T_7=a+6d$

$\Rightarrow a+6d=40$

$\Rightarrow a=40-6d$

$\Rightarrow S_{13}=\frac{12}{2}[2a+12d]$

$\Rightarrow \frac{13}{2}[2(40-6d)+12d]$

$\Rightarrow \frac{13}{2}[80]=13\times 40=520$

Q.4 **[A]**

$\Rightarrow S_{40}=\frac{40}{2}[2a+39d]$

$\Rightarrow 20[2(2)+39(4)]=3200$

Q.5 **[C]**

Let the terms of A. P are $(a-d), a, (a+d)$

Now, $\left(a-d\right)+\left(a+d\right)=12$

$\Rightarrow 2a=12$
$\Rightarrow a=6$
$\Rightarrow \text{and}(a-d)a=24$
$\Rightarrow \left(6-d\right)=6=24$
$\Rightarrow d=2$
$\Rightarrow \therefore \text{first tern } a-d=6-2=4$

Q.6 [C]

$\Rightarrow S_{2m}=\frac{2n}{2}\left[2\left(2\right)+\left(2n-1\right)\left(3\right)\right]=n\left[a+6n\right]$ ……….(1)

$\Rightarrow S_{n}=\frac{n}{2}\left[2\left(57\right)+\left(n-1\right)\left(2\right)\right]=n\left[56+n\right]$ ……….(2)

Solving (1) and (2)

$\Rightarrow 1+6n=56+n$
$\Rightarrow n=11$

Q.7 [A]

$\Rightarrow S_{10}=4S_{5}$
$\Rightarrow \frac{10}{2}=\left[2a+9d\right]=4\times\frac{5}{2}\left[2a+4d\right]$
$\Rightarrow 2a=d$
$\Rightarrow \frac{a}{d}=\frac{1}{2}$

Q.8 [A]

$\Rightarrow Sp=\frac{p}{2}\left[2a+\left(p-1\right)d\right]=x$ ………(1)

$\Rightarrow Sq=\frac{q}{2}\left[2a+\left(q-1\right)d\right]=y$ …..….(2)

$\Rightarrow Sr=\frac{r}{2}\left[2a+\left(q-1\right)d\right]=z$ ………(3)

Now substituting value from (1), (2), (3) in

$\Rightarrow \frac{x}{p}=\left(q-r\right)+\frac{y}{q}\left(r-p\right)+\frac{z}{r}\left(p-q\right)$
$\Rightarrow 2\left[2a-\left(p-1\right)d\right]\left(q-r\right)+2\left[2a-\left(q-1\right)d\right]\left(r-p\right)+2\left[2a-\left(r-1\right)d\right]\left(p-q\right)=0$

Q.9. [A]

Odd two digit number will be 11, 13, 15, ………99 – total 45 numbers

$$\Rightarrow S=\frac{45}{2}\left[2(11)+(45-1)2\right]$$

$$\Rightarrow \frac{45}{2}[22+88]=2475$$

Q.10 [D]

$$\Rightarrow S=\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+.......+\frac{1}{\sqrt{2n}+\sqrt{2n+1}}$$

$$\Rightarrow \frac{\sqrt{3}-1}{2}+\frac{\sqrt{5}=\sqrt{3}}{2}+\frac{\sqrt{7}-\sqrt{5}}{2}+.......+\frac{\sqrt{2n+1}-\sqrt{2n}}{2}$$

$$\Rightarrow \frac{1}{2}\left(2n\sqrt{2n+1}-1\right)$$

Q.11 [D]

$\Rightarrow a_1, a_2.........a_{n+1}$ are in A. P.

Let $a_1 = a$ and common difference be d

Then, $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+........+\frac{1}{a_na_{n+1}}$

$$\Rightarrow \frac{1}{a(a+d)}+\frac{1}{(a+d)(a+2d)}+........+\frac{1}{(a+nd)(a+(n-1)d)}$$

$$\Rightarrow \frac{1}{d}\left[\frac{d}{a(a+d)}+\frac{d}{(a+d)(a+2d)}+........+\frac{d}{(a+nd)(a+(n-1)d)}\right]$$

$$\Rightarrow \frac{1}{d}\left[\left(\frac{1}{d}-\frac{1}{a+d}\right)+\left(\frac{1}{a+d}-\frac{1}{a+2d}\right)+........+\left(\frac{2}{a+(n-1)d}-\frac{1}{a+nd}\right)\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a}-\frac{1}{a+nd}\right]$$

$$\Rightarrow \frac{1}{d}\left[\frac{a+nd-a}{a(a+nd)}\right]$$

$$\Rightarrow \frac{n}{a_1a_{n+1}}$$

Q.12 [B]

$$\Rightarrow d=\frac{b-a}{n+1}=\frac{38-2}{n+1}=\frac{36}{n+1}$$

If n A. M. are inserted between 2 and 38 then total numbers of terms A. P. is n + 2

$$\Rightarrow S_{n+2}=\frac{n+2}{2}\left[2a+(n+2-1)d\right]$$

$$\Rightarrow \frac{n+2}{2}\left[2(2)+(n+1)\frac{36}{n+1}\right]=200$$

$$\Rightarrow \frac{n+2}{2}[4+36]=200$$

$$\Rightarrow n+2=10$$

$$\Rightarrow n=8$$

Q. 13 [A]

$$\Rightarrow d=\frac{19-3}{3+1}=\frac{16}{4}=4$$

$$\Rightarrow A_1=a+d=3+4=7$$

$$\Rightarrow A_2=a+2d=11$$

$$\Rightarrow A_3=a+3d=15$$

Q.14 [B]

A, b, c, d, e, f i.e. A. M. 's between 2 and 12

$$\Rightarrow d=\frac{b-a}{n+1}=\frac{12-2}{6+1}=\frac{10}{7}$$

$$\Rightarrow S=\frac{8}{2}[2a+7d]=4[4+10]=56$$

$$\Rightarrow \therefore a+b+c+d+e+f=200-a-b$$

$$\Rightarrow 56-2-12=42$$

Q.15 [D]

Let first be a and common difference be d.

$$a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$$

$$\Rightarrow 6a+23d=75$$

Now, $S_{24}=\frac{24}{2}[2a+23d]=12[75]=900$

Q.16. [A]

a, b, c are in A. P.

$$\Rightarrow \frac{a+c}{2}=b$$

$$\Rightarrow \frac{a+c}{2abc}=\frac{1}{ac}$$

$$\Rightarrow \frac{ab+cb}{2abc}=\frac{1}{ac}$$

$$\Rightarrow \frac{\frac{1}{ab}+\frac{1}{bc}}{2}=\frac{1}{ac}$$

$\Rightarrow \therefore \frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ are in A. P.

Q.17 [B]

$\Rightarrow \log 2 \log\left(2^n - 1\right), \log\left(2^n + 3\right)$ are in A.P.

$\Rightarrow \therefore \log\left(2^n - 1\right) = \dfrac{\log 2 + \log 2^n + 3}{2}$

$\Rightarrow 2\log\left(2^n - 1\right) = \log\left(2 \times \left(2^n + 3\right)\right)$

$\Rightarrow \log\left(2^n - 1\right)^2 = \log\left(2^{n+1} + 6\right)$

$\Rightarrow \left(2^n - 1\right)^2 = 2^{n+1} + 6$

$\Rightarrow 2^{2n} + 1 - 2^{n+1} = 2^{n+1} + 6$

$\Rightarrow 2^{2n} - 4.2^n - 5 = 0$

Let $^n = t$

$\Rightarrow t^2 - 4t - 5 = 0$

$\left(t - 5\right)\left(t + 1\right) = 0$

$\Rightarrow t = 5$ or $t = -1$

$\Rightarrow 2^n = 5$ or $2^n = -1$ (not possible)

$\Rightarrow \log_2^5 = n$

Q.18 [B]

Let $x = \sqrt{2} + 1$

$\Rightarrow y = 1$

$\Rightarrow z = \sqrt{2} - 1$

$\Rightarrow \dfrac{x}{y} = \dfrac{1}{\sqrt{2} + 1} = \dfrac{1}{\sqrt{2} + 1} \times \dfrac{\sqrt{2} - 1}{\sqrt{2} - 1} = \dfrac{\sqrt{2} - 1}{1} = \dfrac{y}{z}$

$\Rightarrow \therefore x, y, z$ are in G. P.

Q. 19 [B]

Let first term of G. P be A and common ratio be R.

$\Rightarrow T_p = AR^{p-1} = a$

$\Rightarrow T_q = AR^{-q-1} = b$

$\Rightarrow T_r = AR^{r-1} = c$

Now, $a^{q-r}.b^{r-p}.c^{p-q} = \left(AR^{(p-1)^{(q-r)}}\right).\left(AR^{(q-1)^{(r-1)}}\right).\left(AR^{(r-1)}\right)^{(p-q)}$

$\Rightarrow = A^0 R^0 = 1$

Q.20 [C]

Let the first term of G.P. be $\frac{a}{r^2}, \frac{1}{r}, a, ar, ar^2$

If third term is 4

$\Rightarrow a = 4$

$\therefore$ their product $= (a)^5 = (4)^5$

Q.21 [D]

$\Rightarrow x, 2x+2, 3x+3$ are in G. P.

then $(2x+2)^2 = x(3x+3)$

$\Rightarrow 4x^2 + 4 + 8x = 3x^2 + 3x$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+1)(a+4) = 0$

$\Rightarrow x = -4$ or $x = -1$

if $x = -1$ then term will be – 1, 0, 0 Not possible

if $x = -4$

Then term will be -4, -6, -8

$\Rightarrow a = -4$

$\Rightarrow r = \frac{-6}{-4} = \frac{3}{2}$

$\Rightarrow T_4 = ar^3 = -4 \times \left(\frac{3^3}{2}\right) = -4 \times \frac{27}{8} = -13.5$

Q.22 [B]

$a = x$ let common ratio be r.

$\Rightarrow S_\infty = 5$

$\Rightarrow \frac{x}{1-r} = 5$

$\Rightarrow r = \frac{5-x}{5}$

Or $r \in (-1,1)$ for an infinite G. P.

$\Rightarrow -1 < \frac{5-x}{5} < 1$

$\Rightarrow 10 > x > 0$

Q.23 [B]

$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow n = 2$

$$\Rightarrow r = \left(\frac{64}{1}\right)^{\frac{1}{n+1}} = 4$$

$\Rightarrow G_1 = ar = 4$

$\Rightarrow G_2 = ar^2 = 16$

Q. 24 [B]

$\Rightarrow 3, 3^2 3^3, = 16$

$$\Rightarrow G.M = \sqrt{3^n \times 3} = 3^{\frac{n+1}{2}}$$

Q.25 [A]

A, b, c are in A.P.

$\Rightarrow \therefore \frac{a+c}{2} = b$ …….(1)

and $c - b = b - a$ ……..(2)

and $b - a, c - b$ a ar in G. P.

then $(c-b)^2 = a(b-a)$

form (2)

$\Rightarrow (b-a)^2 = a(b-a)$

$\Rightarrow b - a = a$ ……..(3)

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{b}{a} = 2$

From (3)

$\Rightarrow b = 2a$

$\Rightarrow \frac{a+c}{2} = 2a$

$\Rightarrow \frac{c}{a} = 3$

$\Rightarrow \therefore a : b : c = 1 : 2 : 3$

Q.26 [C]

$\Rightarrow a, b, c$ are in H. P.

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$ and $\frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

Now, $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)=\left(\frac{1}{b}+\frac{2}{b}-\frac{1}{a}-\frac{1}{a}\right)\left(\frac{2}{b}-\frac{1}{b}\right)$

$\Rightarrow\left(\frac{3}{2}-\frac{2}{a}\right)\left(\frac{1}{b}\right)=\frac{3}{b^2}-\frac{2}{ab}$

Q.27 [D]

$\Rightarrow(a)\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{b}$

$\Rightarrow\frac{b(b-c)+b(b-a)}{b(b-a)(b-c)}=(b-a)(b-c)$

$\Rightarrow b^2+bc+b^2-ab$

$\Rightarrow b^2+ac-ab-bc$

$\Rightarrow b^2=ac$

a, b, c are in G.P.

but a, b, c are in H. P. so not correct

(b) as a, b, c, are in H. P.

$\Rightarrow b=\frac{2ac}{a+c}$

But $b=\frac{2ac}{a+c}$ is given so not correct

$\Rightarrow(c)\frac{b+a}{b-a}+\frac{b+c}{b-c}=1$

$\Rightarrow(b+a)(b-c)+(b+c)(b-a)=(b-a)(b-c)$

$\Rightarrow b^2+bc+ab+ac+b^2-ab+bc-ac$

$\Rightarrow b^2-bc-ab+ac$

$\Rightarrow b^2+bc+ab=3ac$

No result

$\therefore$ Answer is none.

Q. 28 [B]

Let $S=1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+.......$ …….(1)

$\Rightarrow\frac{1}{2}s=\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^2}+......$ ……..(2)

(1) – (2)

$\Rightarrow\frac{1}{2}s=1+\frac{2}{2}+\frac{2}{2^2}+\frac{2}{2^3}+.......$

$$\Rightarrow 1+2\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....\right)$$

$$\Rightarrow 1+2\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)=1+2=3$$

$$\Rightarrow S=6$$

Q. 29 [B]

Let $S=1+2.2+3.2^2+4.2^3+.....+100.2^{99}$ …….(1)

$\Rightarrow 2S=2+2.2+3.2^2+.......+99.2^{99}+100.2^{100}$ …….(2)

(1) - (2)

$$\Rightarrow -1S=1+2+2^2+2^3+.......2^{99}-100.2^{100}$$

$$\Rightarrow 1\frac{\left(2^{100}-1\right)}{2-1}-100.2^{100}$$

$$\Rightarrow 2^{1000}-1-100.2^{100}$$

$$\Rightarrow S=99.2^{100}+1$$

Q.30 [B]

Let $S=3+33+333+......33.....33$

$$\Rightarrow S=3\left(1+11+111+......+111.....111\right)$$

$$\Rightarrow 3\left(1+\left(10+1\right)+\left(10^2+10+1\right)+........+\left(10^n+10^{a-1}+10^{a-2}+.......+10+1\right)\right)$$

$\Rightarrow 3\left(n+10\left(n-1\right)+10^2\left(n-2\right)+........10^n\right)$ …..(1)

Let $S^1=n+10\left(n-1\right)+10^2\left(n-2\right)+.......+10^n$ …...(2)

$\Rightarrow 10S^1=10n+10^2\left(n-1\right)+......210^n+10^{n+1}$ ……(3)

$(2)-(3)$

$$\Rightarrow -9S^1=n-10-10^2-10^3......10^{a+1}$$

$$\Rightarrow n-\left(10+10^2+10^3+.....10^n+10^{n+1}\right)$$

$$\Rightarrow n-\frac{10\left(10^n-1\right)}{10-1}=n-\frac{10^{n+1}-10}{9}$$

$$\Rightarrow \frac{9n-10^{n+1}+10}{9}$$

$$\Rightarrow S^1 = \frac{10^{n+1} - 10 - 9n}{81}$$

$\Rightarrow \therefore$ From (1)

$$\Rightarrow S = \frac{10^{n+1} - 10 - 9n}{27}$$

Q. 31 [B]

1234, 2345, 3456 ……….

d = 1111

$$T_n = 1234 + (n-1)1111$$

$$= 123 + 1111n$$

Q. 32 [A]

$$a = 2 + d$$
$$b = 2 + 2d$$
$$c = (2 + 2d)d$$
$$2(a + d)d.d = 160$$
$$\Rightarrow d.d(1 + d) = 4.4.5$$
$$\Rightarrow d = 4$$
$$a = 6, b = 10$$
$$c = 40$$
$$a + b + c = 56$$

Q.33 [C]

$$\Rightarrow a^3 + 2^3 + 3^3 + 4^3 + ...15^3 \text{ is}$$

$$\Rightarrow S = \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{15(15+1)}{2}\right)^2 = (120)^2 = 14400$$

Q.34

$$\Rightarrow (1^2 - t_1) + (2^2 - t_2) + ... + (n^2 - t_n) = \frac{1}{3}n(n^2 - 1)$$

$$\Rightarrow 1^2 + 2^2 + ...n^2 - (t_1 + t_2 + ...t_n) = \frac{1}{3}n(n^2 - 1)$$

$$\Rightarrow \frac{n(n+1)}{2}$$

$$\Rightarrow t_n = n$$

Q.35 [D]

$$\Rightarrow \frac{1}{3\times7} + \frac{1}{7\times11} + \frac{1}{11\times15} + ...$$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{3} - \frac{1}{7}\right] + \frac{1}{4}\left[\frac{1}{7} - \frac{1}{11}\right] + \frac{1}{4}\left[\frac{1}{11} - \frac{1}{15}\right]$$

$$\Rightarrow = \frac{1}{4}\left[\frac{1}{3}-\frac{1}{\infty}\right]=\frac{1}{12}$$

Q. 36 [A]

$$\Rightarrow \frac{a+b}{2}ab=\frac{2ab}{a+b}$$

$$\Rightarrow a=b$$

Q. 37 [C]

$$\Rightarrow \frac{a}{b},\frac{b}{c},\frac{c}{a}=\text{H.P.}$$

$$\Rightarrow \frac{b}{c}=\frac{2\frac{a}{b}\frac{c}{a}}{\frac{a}{b}+\frac{c}{a}}$$

$$\Rightarrow \frac{b}{c}=\frac{2\frac{c}{b}}{\frac{a^2+c^2}{ab}}$$

$$\Rightarrow a^2b+b^2c=2ac^2$$

Q.38 [C]

$$\Rightarrow a,b,c=\text{G.P.}$$

$$\Rightarrow b^2=ac$$

Now, $\frac{1}{\log_a^x}+\frac{1}{\log_b^x}=\log_x^a+\log_x^b=\log_x^{ab}=\log_x^{b^2}$

$$\Rightarrow 2\log_x^b=2\frac{1}{\log_b^x}$$

$$\Rightarrow \therefore \log_a^x,\log_b^x,\log_c^x=\text{H.P}$$

Q.39 [A]

$$\Rightarrow A_2+A_2=a+b, G_1G_2=ab$$

$$\Rightarrow \frac{1}{H_1}+\frac{1}{H_2}=\frac{a+b}{ab}$$

$$\Rightarrow \therefore \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$$

$$\Rightarrow \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$$

Q. 40 [D]

Let the two number be a and b, then

$$\Rightarrow \frac{\frac{2ab}{a+b}}{\sqrt{ab}}=\frac{12}{13}$$

$$\Rightarrow \frac{2\sqrt{ab}}{a+b} = \frac{12}{13}$$

$$\Rightarrow \frac{(a+b)^2 - \left(2\sqrt{ab}\right)^2}{a+b} = \frac{5}{13}$$

$$\Rightarrow \frac{a-b}{a+b} = \frac{5}{13}$$

$$\Rightarrow 13a - 13b - 5a + 5b$$

$$\Rightarrow \frac{a}{b} = \frac{9}{4}$$

Q.41. [C]

$$\Rightarrow \frac{a+b}{2} - \sqrt{ab} = 2$$

$$\Rightarrow \frac{a}{b} = \frac{4}{1} = a = 4b.$$

$$\Rightarrow \frac{4b+b}{2} - \sqrt{4b^2} = 2$$

$$\Rightarrow \frac{5}{2}b - 2b = 2$$

$$\Rightarrow b = 4 \text{ and } a = 16$$

Q.42 [C]

$$\frac{\frac{a+b}{2}}{\frac{2ab}{a+b}} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{4ab} = \frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{(a+b)^2 - 4ab} = \frac{m}{m-n}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\sqrt{m}}{\sqrt{m-n}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{m} + \sqrt{m-n}}{\sqrt{m} - \sqrt{m-n}}$$

Q. 43 [C]

$$x = \frac{\log 3}{\log 5} + \frac{\log 5}{\log 7} + \frac{\log 7}{\log 9}$$

$$\frac{x}{3} \geq \left(\frac{\log 3}{\log 5} . \frac{\log 5}{\log 7} . \frac{\log 7}{2og3}\right)$$

(By Am $\geq$ am)

$$\Rightarrow \frac{x}{3} \geq \left(\frac{1}{2}\right)^{1/3}$$

$$\Rightarrow x \geq \frac{3}{3\sqrt{2}}$$

Q.44 [B]

$$\Rightarrow \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ...$$

$$\Rightarrow 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} .... = 1$$

Q.45 [A]

$T_6 = 8T_3$

$\Rightarrow ar^5 = 8ar^2$

$\Rightarrow \ell^3 = 8$

$\Rightarrow r = 2$

$T_7 + t_8 = 192$

$\Rightarrow ar^6 + ar^7 = 192$

$\Rightarrow a(64 + 128) = 192$

$\Rightarrow a = 1$ ……..(1)

$\Rightarrow a = 1$ …….(2)

$$T_5 + T_6 + ...T_{11} = \frac{2^4\left(2^7 - 1\right)}{2 - 1} = 2032$$

$T_6 + T_9 = 2^5 + 2^8 = 288$

Q.46.

$A_1 = G_1 = H_1 = A$

$A_{3n-1} = G_{2n-1} = H_{2n-1} = 8$

So, $An = \frac{A + B}{2}, Gn = \sqrt{AB}, Hn = \frac{2AB}{A + B}$

$\Rightarrow \boxed{b^2 = ac}$

Q.47

Let $a = \frac{1}{\frac{1}{b} - d}$ and $c\frac{1}{\frac{1}{b} + d}$

$\Rightarrow a = \frac{b}{1 - bd}$ and $c = \frac{b}{1 + bd}$

Now, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{4}$

$\Rightarrow \frac{1 - bd}{b} + \frac{1}{b} + \frac{1 + bd}{b} = \frac{1}{4}$ hence b = 12

Now, a + b + c = 37

$$\Rightarrow \frac{12}{1 - 12d} + 12 + \frac{12}{1 + 12d} = 37$$

$\Rightarrow \frac{24}{1-144d^2} = 25$

$\Rightarrow d = \frac{1}{60}$. Hence numbers are 15, 12, 10

Q.48 [C]

$x, |x+1|, |x-1| = A.P$

For x < - 1

$\Rightarrow x, -x-1, -x+1 = A.P$

$\Rightarrow \therefore -x-1-x = -2x-1$

$\Rightarrow -x+1+x+1 = 2$

From (1) and (2)

$\Rightarrow -2x-1 = 2$

$\Rightarrow -2x = 3x = \frac{-3}{2}$

$\Rightarrow \therefore S_{20} = \frac{20}{2}\left[2\left(\frac{-3}{2}\right)+(19)2\right] = 350$

Q.49 [D]

If A. M. are inserted between two given number then product of rth A.M. from being and rth H.M. form and is equal to the product 0 there numbers.

Hence, $a_4 \times h_7 = 2 \times 3$ i.e. 6

Q.50 [A]

$\Rightarrow y = \frac{2ab}{a+b}, x = \frac{2ay}{a+y}, z = \frac{2by}{b+y}$

or $y = \frac{2ab}{a+b}, x = \frac{4ab}{a+3b}, z = \frac{4ab}{3a+b}$

now, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{a+b}{2ab}+\frac{a+3b}{4ab}+\frac{3a+b}{4ab}$

$\Rightarrow \frac{1}{a}+\frac{1}{b} = \frac{10}{9}$

Q.51 [B]

$\Rightarrow a_1 + a_{2n} = a_2 + a_{2n-1} = a_3 + a_{2n-2} = a+b$

$\Rightarrow g_1 g_{2n} = g_2 g_{2n-1} = g_3 g_{2n-2} = ... = ab$

Hence, $\frac{a_1+b_{2n}}{g_1 g_{2n}} + \frac{a_2+a_{2n-1}}{g_2 g_{2n-1}} + \frac{a_3+a_{2n-2}}{g_3 g_{2n-3}} + ... = n\frac{a+b}{ab}$

But $\frac{2ab}{a+b} = h$, therefore $\frac{a_1+a_{2n}}{g_1 g_{2n}} + \frac{a_2+a_{2n-1}}{g_2 g_{2n-1}} + \frac{a_3+a_{2n-2}}{g_3 g_{2n-3}} + ... = \frac{2n}{h}$

Q.52 [B]

$\Rightarrow a = 2n-1$

$$\Rightarrow n=\frac{a+1}{2}$$

$$\Rightarrow (1+3+5+.......p)+(1+3+5+......+q)=(1+3+5+.....+r)$$

$$\Rightarrow \left(\frac{p+1}{2}\right)^2+\left(\frac{q+1}{2}\right)^2=\left(\frac{r+1}{2}\right)^2$$

$$\Rightarrow (p+1)^2+(q+1)^2=(r+1)^2$$

P > hence smallest pythagaion to put will be 6, 8, 10.

Therefore p = 7, q = 5, r = 9

Least value $p+q+r=21$

Q.53 [A]

$$\Rightarrow S=\frac{(1+2+3+.......+n)^2-(1^2+2^2+3^2+.......+n^2)}{2}$$

$$\Rightarrow \frac{n^2(n+1)^2}{8}-\frac{n(n+1)(2n+1)}{12}$$

$$\Rightarrow \frac{n(n+1)(n-1)(3n+1)}{24}$$

Q.54 [C]

$$\Rightarrow \sum_{n=1}^{\infty}\sin^{2n}\theta=\frac{1}{1-\sin^2\theta}\Rightarrow x=\frac{1}{\cos^2\theta}$$

$$\Rightarrow \sum_{n=1}^{\infty}\cos^{2n}\phi=\frac{1}{1-\cos^2\phi}\Rightarrow y=\frac{1}{\sin^2\phi}$$

$$\Rightarrow \sum_{n=1}^{\infty}\cos^{n}(\theta+\phi)\cos^{n}(\theta-\phi)=\frac{1}{1-\cos(\theta+\phi)\cos(\theta-\phi)}$$

$$\Rightarrow 2=\frac{1}{1-\cos^2\theta+\sin^2\phi}$$

Now, $z=\dfrac{1}{1-\frac{1}{x}+\frac{1}{y}}$ or $z(xy-y+x)=xy$

$$\Rightarrow xyz-xy=yz-zx$$

Q.55

$$\Rightarrow \frac{1}{3}+\frac{1}{3^n}+\frac{1}{3^3}+.....\infty=\frac{1}{2}$$

Hence $y=(0.64)^{\log_{0.25}^{0.5}}$

$$\Rightarrow y=(0.64)^{\frac{1}{2}}=0.8$$

Q.56 [C]

$$\Rightarrow b=\frac{2ac}{a+c}$$

Now, $\frac{b+a}{b-a}+\frac{b+c}{b-c}=\frac{\frac{b}{a}+1}{\frac{b}{a}-1}+\frac{\frac{b}{c}+1}{\frac{b}{a}-1}$

$$\Rightarrow \frac{\frac{2c}{a+c}+1}{\frac{2c}{a+c}-1}+\frac{\frac{2a}{a+c}+1}{\frac{2a}{a+c}-1}=\frac{3c+a}{c-a}+\frac{3a+c}{c-a}=2$$

Q.57 [B]

$\Rightarrow \frac{a+b}{2}=\frac{3}{2}$

$\Rightarrow a+b=3$

$\Rightarrow \frac{2ab}{a+b}=\frac{4}{3}$

$\Rightarrow 2ab=4$

$\Rightarrow ab=2$

$\Rightarrow \therefore x^2=3x+2$

Q.58. [B]

$$\Rightarrow \frac{\pm\sqrt{\frac{c}{a}}}{\pm\sqrt{\frac{n}{1}}}=\pm\sqrt{\frac{cl}{an}}$$

Q.59 [B]

$$\Rightarrow \frac{1}{xy-x^2}+\frac{1}{xy-y^2}=\frac{1}{x(y-x)}-\frac{1}{y(y-x)}$$

$$\frac{y-x}{(y-x)xy}=\frac{1}{xy}=\frac{1}{G^2}$$

Q.60 [A]

$S=1.3^2+2.5^2 3.7^2+......$

$\Rightarrow T_n=n.(2n+1)^2$

$\Rightarrow T_n=4n^3+4n^2+n$

$\Rightarrow S_n \sum 4n^3+4n^2+n$

$$\Rightarrow 4\left(n\frac{n+1}{2}\right)^2+4\left(\frac{n(n+1)(2n+1)}{6}\right)+\frac{n(n+1)}{2}\Rightarrow 188090$$

SEQUENCE SERIES

EXERCISE – 1 [B]

Q.1 **[B]**

-4, -1, +2, +5 + ….
Is an A.P. with
First term a = -4
And common difference d = 3
Therefore

$T_n = a + (n-1)d$

$\Rightarrow T_{10} = -4 + (10-1)*3$

$\Rightarrow T_{10} = 23$

Q.2 **[A]**

First term a = 2
Common difference d = 4
n = 40

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{40} = \frac{40}{2}\left[4 + (40-1)4\right]$

=1600

Q.3 **[D]**

4, 9, 14, ….., 104
First term a = 4
Common difference d = 5
n^{th} tem is $T_n = 104$

$T_n = a + (n-1)d$

$\Rightarrow 104 = 4 + (n-1)5$

$\Rightarrow n = 21$

Therefore, middle term will be 11^{th} term

$T_{10} = 4 + (11-1)5$

= 54

Q.4 **[B]**

$T_9 = 0$

$\Rightarrow a + (9-1)d = 0$

$\Rightarrow a = -8d$

Now,

$T_{29/}T_{19} = \frac{a + (29-1)d}{a + (19-1)d} = \frac{a + 28d}{a + 18d} \qquad = \frac{8d + 28d}{-8d + 18d} = \frac{2}{1}$

$T_{29} : T_{19} = 2:1$

Number lying between 10 and 200 are the numbers which are multiple of 7

14, 21, 28, ……, 196

a = 14

d = 7

$T_n = 196$

$T_n = a + (n-1)d$

$\Rightarrow 196 = 14 + (n-1)7$

$\Rightarrow n = 27$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{27}{2}\left[2*14 + (27-1)7\right]$

= 2835

Q.6 [B]

Let first term = a

Common difference = d

Then, A.P. be

$a, (a+d), (a+2d), (a+3d), ...$

$T_4 = a + 3d$

$\Rightarrow a + 3d + 3a$

$\Rightarrow a = \frac{3}{2}d$ ……(1)

$T_7 - 2(T_3) = 1$

$\Rightarrow a + 6d - 2(a + 2d) = 1$

$\Rightarrow 2d - a = 1$

Substituting value of a from (1)

$2d - \frac{3}{2}d = 1$

$\Rightarrow d = 2$

Q.7 [A]

Let the term of A.P. is a

And common difference is d

So,

$T_p = a + (p-1)d = A$

$T_Q = a + (Q-1)d = B$

$T_r = a + (r-1)d = C$

Therefore,

$A(Q-r) + B(r-p) + C(p-Q)$

$= a\left(a + (p-1)d\right)(Q-r) + \left(a + (Q-1)d\right)\left(a + (r-1)d\right)(p-Q)$

= 0

Q.8 [B]

$$\frac{Sn}{Sn} = \left(\frac{n}{2}\right)\left(2a + (n-1)d\right) / \left(\frac{n}{2}\right)\left(2a' + (n-1)d'\right)$$

$$\frac{Sn}{Sn'} = \frac{2a + (n-1)d}{2a + (n-1)d'}$$

$$\frac{2a + (n-1)d}{2a + (n-1)d'} = \frac{3n+8}{7n+15}$$

Let, substituting n = 23

$$\frac{2a + (23-1)d}{2a + (23-1)d'} = \frac{3*23+8}{7*23+15}$$

$$\frac{a + 11d}{a + 11d'} = \frac{77}{176}$$

$$T_{12/} T_{12'} = 7/16$$

Q.9 [B]

$$S_n : n/2\left(2a + (n-1)d\right) = 2n^2 + 5n$$

$$S_1 : \tfrac{1}{2}(2a) = 2 + 5 = 7$$

$$\Rightarrow a = 7$$

$$S_2 : (14 + d) = 18$$

$$\Rightarrow d = 4$$

$$T_n = a + (n-1)d = 7 + (n-1)4 = 4n + 3$$

10. [B]

Let the three terms of A.P. are a – d, a, a + d

Sum of first terns

$$a - d + a + a + d = 3a = 51$$

$$\Rightarrow a = 17$$

Product of first and third term

$$(a-d)(a+d) = a^a - d^a$$

$$\Rightarrow 17^2 - d^2 = 273$$

$$\Rightarrow d^2 = 16$$

$$\Rightarrow d = 4$$

So, third term

$$a + d = 17 + 4 = 21$$

Q.11 [B]

Let the four terms of A.P. are

$$a - 3d, a - d, a + d, a + 3d$$

Then,

$$a - 3d + a - d + a + d + a + 3d = 20$$

$$\Rightarrow 4a = 20$$

$$\Rightarrow 4 = 5$$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{2}{3}$$

$$\Rightarrow \frac{3(a^2-9d^2)=2(a^a-d^a)}{(a-d)(a+d)}=\frac{2}{3}$$

$$\Rightarrow 3(a^2-9d^2)=2(a^a-d^a)$$

$\Rightarrow a^2 25d^2$

$\Rightarrow a=5d$

$\Rightarrow d=1$

Smallest term

$a-3d=5-3=2$

Q.12 [A]

Let the three numbers are

$a-d, a, a+d$

$(a-d)(a+d)=5a$

$\Rightarrow (a^2-d^2)=5a$ (1)

$a+a+d=8(a-d)$

$\Rightarrow 6a=9d$

$\Rightarrow 2a=3d$ (2)

Solving (1) and (2), we get

$a=9, d=6$

So, the numbers are 3, 9, 15

Q.13 [B]

Let the first term is a

Common difference is d

Then,

$T_2=a$

$T_3=a+d$

$T_6=a+4d$

T_2, T_3 and T_6 are in G.P., Then

$(a+d)^2=a(a+4d)$

$\Rightarrow a^2+2ad+d^2=a^2+4ad$

$\Rightarrow d^2=2nd$

$\Rightarrow d=2a$

Common ratio

$T_{3/}T_2=(3a/a)=3$

Q.14 [A]

18, -12, 8, …….. - is in G.P.

Common ratio

$$r = -\frac{12}{18} = -\frac{2}{3}$$

$$T_r = ar^a$$

$$\Rightarrow \frac{512}{729} = 18\left(-\frac{2}{3}\right)^n$$

$$\Rightarrow n - 1 = 8$$

$$\Rightarrow n = 9$$

Q.15 [C]

Let the first term of G.P. is a

And the common ratio is r

Then, the five consecutive terms of G.P. are

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

$$\Rightarrow a = 4$$

Then,

$$\frac{ar^2}{} * \frac{a}{r} * a * ar * ar^2 = a^5 = 4^5$$

Q.16 [C]

Let the first term of G.P. is a

And the common ratio is r

Then,

$T_3 = ar^2 = 15$ (1)

$T_7 = ar^6 = 135$ (2)

Solving (1) and (2), we get

$$r^4 = 9$$

$$a = 5$$

Therefore,

$$T_5 = ar^4 = 5 * 9 = 45$$

Q.17 [B]

$1, x^2, 6 - x^2$ are in G.P. then

$$\frac{x^2}{1} = \frac{6 - x^2}{x^2}$$

$$\Rightarrow x^4 = 6 - x^2$$

$$\Rightarrow x^4 + x^2 = 6 = 0$$

$$\Rightarrow x^2 = \sqrt{2} \text{ or } x^2 = -\sqrt{2}$$

Q.18 [A]

$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + ...$ are in G.P.

With common ratio $-\frac{1}{3}$

Then, the sum infinite G.P. is

$$S_n = \frac{a}{a-r} = \frac{1}{\left(1+\frac{1}{3}\right)} = \frac{3}{4}$$

Q.19 [B]

$$1+\frac{2}{x}+\frac{4}{x^3}+\frac{8}{x^3}+....$$

Sum of infinite term is finite when common ratio is less than 1

i.e. $\left|\frac{2}{x}\right|<1$

$\Rightarrow |x|$

Q.20 [B]

$$96+48+24+12+....+\frac{3}{16}$$

Then, the common ratio $\frac{48}{96}=\frac{1}{2}$

$T_n = ar^n$

$\Rightarrow \frac{3}{16} = 96\left(\frac{1}{2}\right)^{n-1}$

$\Rightarrow \frac{3}{2^{n-6}} = \frac{3}{16}$

$\Rightarrow n = 10$

Q.21 [C]

$3+3a+3a^2+... = \frac{45}{8}$ is a G.P.

$S_n = \frac{a}{1-r}$

$\Rightarrow \frac{3}{1-a} = \frac{45}{8}$

$\Rightarrow 24 = 45(1-a)$

$\Rightarrow 45a = 45-24 = \frac{21}{45} = \frac{7}{15}$

Q.22 [C]

Let the number be a, ar, ar^2

Then,

$a+ar+ar^2=155$

$\Rightarrow a\left(1+r+r^2\right)=155$ (1)

And,

$ar^2-a=120$

$\Rightarrow a\left(r^2-1\right)=120$ (2)

Solving (1) and (2), we get

$r=5$ and $a=5$

Q.23 [D]

Let the numbers be a, ar, ar^2

Then there sum

$a+ar+ar^2=14$ (1)

And sum of their sQ.uare

$a^2+a^2r^2+a^2r^4=84$ (2)

SQ.uaring (1) and subtracting (2), we get

$\left(a+ar+ar^2\right)^2-a^2+a^2r^2+a^2r^4=196-84$

$\Rightarrow 2ar\left(a+ar+ar^2\right)=12$

$\Rightarrow ar=4$

Subtracting this in (1) and solving, we get

$r=2$ and $a=2$

Therefore three numbers are 3, 4, 8

Q.24 [B]

Let the four tern be a, ar, ar^2, ar^3

Then,

$a+ar^2=40$

$\Rightarrow a\left(1+r^2\right)=40$

$\Rightarrow \left(1+r^2\right)=\frac{40}{a}$ (1)

And

$ar+Ar^2=80$

$\Rightarrow ar\left(1+r^2\right)=80$

From (1)

$ar\left(\frac{40}{a}\right)=80$

$\Rightarrow r=2$ and $a=8$

Q.25 [B]

a, b, c are in G.P.

Let the common ration is r

i.e. $\frac{b}{a}=\frac{c}{a}=r$

Then, for a^{-1}, b^{-1}, c^{-1}

$\frac{b^{-1}}{a^{-1}}=\frac{a}{b}=\frac{1}{r}$ and $\frac{c^{-1}}{b^{-1}}=\frac{b}{c}=\frac{1}{r}$

Therefore, a^{-1}, b^{-1}, c^{-1} are also in G.P.

Q.26 [B]

P, Q., r are in A.P.

$\Rightarrow Q.,-p = r-p$ (1)

$T_p = ar^{(p-1)}$

$T_Q = ar^{(Q-1)}$

$Tr = ar^{(r-1)}$

$\frac{ar^{Q-1}}{ar^{p-1}} = r^{Q.-p}$

And

$\frac{ar^{r-1}}{ar^{Q.-1}} = r^{r-Q.}$

From (1) we get

Common ratio is same

Then T_p, T_Q, T_r are in G.P.

Q.27 [C]

Let the sum is S

$S = 1+3x+5x^2+7x^3+....$ (1)

$xS = \quad x+3x^2+5x^2+....$ (2)

(1) – (2)

$(1-x)S = 1+2x+2x^2+2x^3$

$(1-x)S = 1+2(x+x^2+x^3+......)$

$(1-x)S = 1+2\left(\frac{x}{1-x}\right)$

$S = \frac{1+x}{(1-x)^2}$

Q.28 [D]

$S = 1+2\left(1+\frac{1}{n}\right)+3\left(1-\frac{1}{n}\right)^2+.....$ (1)

$\left(1-\frac{1}{n}\right)S = \left(1-\frac{1}{n}\right)+2\left(1-\frac{1}{n}\right)^2+3\left(1-\frac{1}{n}\right)^2+.....$ (2)

(1) – (2)

$\frac{1}{n}S = 1+\left(1-\frac{1}{n}\right)+\left(1-\frac{1}{n}\right)^2+\left(1-\frac{1}{n}\right)^3+......$

$\frac{1}{n}S = \frac{1}{1-\frac{1}{n}} = \frac{n}{n-1}$

$S = \frac{n^2}{n-1}$

Q.29 [C]

$$T_m = \frac{1}{a+(m-1)d} = n$$

$$\Rightarrow n\left(a+(m-1)d\right)=1 \qquad (1)$$

$$T_n = \frac{1}{a+(n-1)d} = m$$

$$\Rightarrow m\left(a+(n-1)d\right)=1 \qquad (2)$$

From (1) and (2)

$$n\left(a+(m-1)d\right) = m\left(a+(n-1)d\right)$$

$$na+(m-1)nd = ma+m(n-1)d$$

$$(n-m)a = (n-m)d$$

$$a = d$$

$$T_m = \frac{1}{a+(m-1)a} = n$$

$$\Rightarrow a = \frac{1}{mn}$$

$$T_r = \frac{1}{a+(r-1)d} = \frac{1}{a+(r-1)a} = \frac{mn}{r}$$

Q.30 [D]

First term is 1

N A.M's are inserted between the 1 and 51 then it become a A.P. of n+2 terms

Let the common difference is d

Then,

4^{th} A.M. will be the 5^{th} term of the A.P.

And 7^{th} A.M. will be the 8^{th} term of the A.P.

$$T_5 = 1+(5-1)d = 1+4d$$

$$T_8 = 1+(8-1)d = 1+7d$$

$$\frac{1+4d}{1+7d} = \frac{3}{5}$$

$$\Rightarrow d = 2$$

So, $T_{(n+2)} = 1+(n+2-1)d = 51$

$$\Rightarrow (n+1)2 = 50$$

$$\Rightarrow n = 24$$

Q.31 [B]

x, y, z are in A.P.

a is the A.M. of x and y

$$\Rightarrow a = \frac{x+y}{2} \qquad (1)$$

b is the A.M. of y and z

$\Rightarrow b = \frac{y+z}{2}$ (2)

Adding (1) and (2)

$\frac{a+b}{2} = y$

Q.32 [B]

Let the common difference is d

Then,

$\frac{1}{3}, \frac{1}{3}+d, \frac{1}{4}+2d, \frac{1}{24}$ are in A.P.

$d = \frac{1}{24} = \frac{1}{3} - 2d$

$\Rightarrow d = \frac{-7}{72}$

$A_1 = \frac{1}{3} + \left(\frac{-7}{24}\right) = \frac{17}{72}$

$A_2 = \frac{1}{3} + 2\left(\frac{-7}{24}\right) = \frac{5}{36}$

Q.33 [C]

H.M. between $\frac{a}{b}, \frac{b}{a}$ is

$H = \frac{2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)}{\left(\frac{a}{b}\right)+\left(\frac{b}{a}\right)} = \frac{2ab}{a^2b^2}$

Q.34 [B]

$\frac{2}{3}, a, b, c, d, \frac{2}{13}$ are in H.P

Then,

$\frac{3}{2}, \frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{13}{2}$ are in A.P.

So, then Second H.M is the second A.M and it will be the 3^{rd} term of the A.P.

$T_6 = \frac{3}{2} + (6-1)d = \frac{13}{2}$

$\Rightarrow d = 1$

Therefore,

$\frac{1}{b} = \frac{3}{2} + (3-1)d$

$\Rightarrow b = \frac{2}{7}$

Q.35 [B]

Let the one number be a the other number will be 4a

Then,

$AM+2=GM$

$\Rightarrow \frac{a+4a}{2}+2=\sqrt{a4a}$

$\Rightarrow \frac{5a}{2}+2=2a$

$\Rightarrow a=4$

Q.36 [C]

Let the two numbers is a, b

Then,

$\frac{a+b}{2}=34$ (1)

And,

$16^2=ab$ (2)

Solving (1) and (2)

$a=4, b=64$

Q.37 [C]

Let the two numbers is a, b

$\frac{a+b}{2}=A$

$ab=G^2$

$\frac{2ab}{a+b}=4$

$\Rightarrow 8\left(\frac{a+b}{2}\right)=2ab$

$\Rightarrow 4(A)=G^2$

$2A+G^2=27$

$\Rightarrow A=4.5$ (1)

$\Rightarrow ab=18$ (2)

Solving (1) and (2) we get

$a=6$

$b=3$

Q.38 [C]

Let the two numbers is a, b

Then,

$\frac{a+b}{2}=x \quad ab=y$

$\frac{2ab}{a+b}=Z$

So, $z<y<x$

Q.39 [A]

Let the two numbers is a, b

$AM = GM + 5$

$\frac{a+b}{2} = \sqrt{ab} + 5$ (1)

$GM = HM + 4$

$\sqrt{ab} = \frac{2ab}{a+b} + 4$ (2)

From (1), subtracting the value of $\sqrt{ab}$

$\frac{a+b}{2} = \frac{2ab}{a+b} + 4$ (3)

From (1)

$ab = \left(\frac{a+b}{2} - 5\right)^2$ (4)

Subtracting value of ab from (4) in (3) we get

$\frac{a+b}{2} - 5 = \frac{2}{a+b}\left(\frac{a+b}{2} - 5\right)^2 + 4$

Solving this we get

$a = 10$

$b = 40$

Q.40 [B]

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right)^2$$

Q.41 [C]

Sum of interior angles of an $n = gon = (n-2) \times 180^o$

Sum of n terms of A.P. $(a = 120^0, d = 5^o) = \frac{n}{2}\{2 \times 120^o + (n-1) \times 5^o\}$.

Hence $\frac{n}{2}\{2 \times 120^o + (n-1) \times 5^o\} = (n-2) \times 180^o$

$\Rightarrow n^2 - 25 + 144 = 0 \Rightarrow n = 9$ or 16.

But for $n = 16$, greatest angle exceeds 180^o hence only 9 is correct.

Q.42 [B]

$S = (1+3+5+...20 \text{ terms}) + (2+4+8+...20 \text{ terms})$

$\Rightarrow S = 20^2 + \frac{2(2^{20}-1)}{2-1}$ or $398+2^{21}$

Q.43

Common difference of the two A.P.s are 4 & 5, hence common difference of A.P. formed by common terms will be 20. Also the first common term is 21. Now

$S = 100(2\times 21\times 20) = 402200$.

Q.44 [C]

m^{th} term of first series $= 2m+61$, m^{th} term of second series $= 7m-4$. $7m-4 = 2m+61 \Rightarrow m = 13$.

Q.45 [C]

$d_1 = 3 \;\&\; d_2 = 2 \Rightarrow d(\text{common terms}) = 6$

First common term = 5

Hence common term are 5, 11, 17,…

Now general term = 6n – 1.

60^{th} term of first A.P. = 179

50^{th} term of second A.P. = 101

Comparing $6n-1$ with 101 gives n = 17

Q.46 [A]

$a+e = b+d = 2c \Rightarrow a-4b+6c-4d+2 = 0$.

Q.47 [B]

Given $11+11+d+11+2d+11+3d = 56 \;\&$

$11+(n-4)d+11+(n-3)d+11+(n-2)d+11+(n-1)d = 12$

$\Rightarrow d = 2 \;\&\; (2n-5)d = 34$ or $n = 11$.

Q.48 [C]

$\frac{2n}{2}\{2\times 2+(2n-1)\times 3\} = \frac{n}{2}\{2\times 57+(n-1)\times 2\} \Rightarrow n = 11.$

Q.49 [C]

$(a+6d)-(a+d) = 20 \Rightarrow d = 4 \;\&\; a+2d = 9 \Rightarrow a = 1$.

Now n^{th} term $= 4n-3 = 2001 \Rightarrow n = 501.$

Q.50 [A]

$(1+3+5+...p \text{ terms}) + (1+3+5+...q \text{ terms}) = (1+3+5+...r \text{ terms}) \Rightarrow p^2+q^2 = r^2$

Now smallest pythagorian triplet will be 3, 4 ,5, hence least value of $p+q+r = 12.$

Q.51 [B]

As a, x, y, z, b are in A.P. therefore $x + z = a + b \,\&\, y = \dfrac{a+b}{2}$

$\Rightarrow x + y + z = \dfrac{3}{2}(a+b)$. Hence $a + b = 10$

Q.52 [A]

Let the number be $a - d, a, a + d$.

Now $a - d + a + a + d = 15 \Rightarrow a = 5$

As given $a - d + 1, a + 4, a + d + 19$ are in G.P. hence

$(a+4)^2 = (a-d+1)(a+d+19) \Rightarrow 81 = 16 = (6-d)(24+d) \Rightarrow d = 3$.

Numbers are 2, 5, 8.

Q.53 [A]

Given $a \times ar \times ar^2 = 216 \,\&\, a \times ar + ar \times ar^2 + ar^2 \times a = 126$.

Or $(ar)^3 = 216 \,\&\, a^2 r(1 + r + r^2) = 126$.

$\Rightarrow 2r^2 - 5r + 2 = 0$. hence $r = \dfrac{1}{2} \,\&\, a = 12$.

Now $a = 12, b = 6, c = 3$.

Q.54 [A]

$$x = \log_{0.4}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\infty \text{ terms}\right) \Rightarrow c = \log_{0.4}\left(\frac{1}{2}\right)$$

Hence $(0.16)^x = (0.16)^{-\log_{0.4}{}^{2}} = 2^{-\log_{0.4}{}^{0.16}}$

Therefore $(0.16)^x = 2^{-2} = \dfrac{1}{4}$.

Q.55 [C]

$t_n = 3 \times 2^{n-1}$. Now $12288 = 3 \times 2^{12}$.

Hence $m = 13$.

Q.56 [A]

$$S_{10} = \frac{a(r^{10}-1)}{r-1} \,\&\, S_5 = \frac{a(r^5-1)}{r-1}. \text{ Now } \frac{S_{10}}{S_5} = 244 \Rightarrow \frac{r^{10}-1}{r^5-1} = 244 \text{ or } r = 3.$$

Q.57 [A]

Let the first term a and common ratio be b, then

$$x = ab^{p-1}, y = ab^{q-1}, z = ab^{r-1} \Rightarrow \frac{y}{x} = b^{q-p}, \frac{z}{y} = b^{r-q}, \frac{x}{z} = b^{p-r}$$

$$\text{Now } x^{q-r} y^{r-p} z^{p-q} = \left(\frac{y}{z}\right)^r \left(\frac{z}{y}\right)^p \left(\frac{x}{z}\right)^q = b^{r(q-p)+p(r-p)+q(p-r)}$$

Or $x^{q-r}y^{r-p}z^{p-q} = b^0 = 1$.

Q.58 [34]

$$9^{1/3}.9^{1/9}.9^{1/27}...\infty \text{ terms} = 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...\infty \text{ terms}}$$

$$= 9^{\frac{1/3}{9^{1-1/3}}} = 9^{1/2} = 3.$$

Q.59 [D]

$$x = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...\infty \text{ terms} = \frac{1}{2}$$

Now $x^{\log_b a} = \left(\frac{1}{2}\right)^{\log_{\sqrt{5}} 0.2} = 4.$

Q.60 [C]

Given $a + ar + ar^2 + ... + ar^9 = S_1$ & $ar^{10} + ar^{11} + ar^{12} + ...ar^{19} = S_2$

$$\Rightarrow \frac{a\left(1-r^{10}\right)}{1-r} = S_1 \;\&\; \frac{ar^{10}\left(1-r^{10}\right)}{1-r} = S_2$$

Or $\frac{S_2}{S_1} = r^{10}$.

SEQUENCE SERIES

EXERCISE – 1 [C]

Q.1 [C]

All number divisible by 6 are 6, 12, 18, …., 198

Sum $=\dfrac{33(6+198)}{2}=3366$

Now sum of all the even numbers less than $200=\dfrac{99(2+198)}{2}=9900$

Hence required Sum $=9900-3366=6534$.

Q.2 [C]

$$S=\log a+\log\frac{a^3}{b^2}+\ldots n \text{ terms}$$

$$\Rightarrow S=(1+2+3+..n \text{ terms})\log a-(1+2+3+\ldots n-1 \text{ terms})\log b$$

$$\Rightarrow S=\frac{n(n+1)}{2}\log a-\frac{n(n-1)}{2}\log b$$

$$\Rightarrow S=\frac{n^2}{2}\log\frac{a}{b}+\frac{n}{2}\log ab.$$

Q.3 [C]

First term = a, second term = b, last term = c.

Common difference, $d=b-a\Rightarrow c=a+(n-1)(b-a)$

Hence $n=\dfrac{b+c-2a}{b-a}$. Now sum of n terms will be

$$S_n=\frac{a+c}{2}\left(\frac{b+c-2a}{b-a}\right).$$

Q.4 [B]

$\ell=a+(n-1)d\Rightarrow d=\dfrac{\ell-a}{n-1}$. Also $S=\dfrac{n}{2}(a+\ell)\Rightarrow n=\dfrac{2S}{a+\ell}$.

Hence $d=\dfrac{\ell-a}{\dfrac{2S}{a+\ell}}$ i.e. $d=\dfrac{\ell^2-a^2}{2S-\ell-a}$

Q.5 [B]

$2d=z-x$, also $2y=z+x\Rightarrow 4y^2=(z-x)^2+4zx$

Hence $4y^2=4d^2+4zx$ i.e. $d^2=y^2-zx$

Q.6 [B]

$$S_p=\frac{p}{2}\{2a+(p-1)d\}=0\Rightarrow d=-\frac{2a}{p-1}$$

Now sum of next q term $=S_{p+q}-S_p$

$$=\frac{p+q}{2}\{2a+(p+q-1)d\}$$

$\Rightarrow$ Now Sum of next q term $=\frac{p+q}{2}\left\{2a-(p+1-1)\frac{2a}{p-1}\right\}=\frac{-(p+q)q}{p-1}$.

Q.7 [A]

Total number of terms by the end of n^{th} group $=1+2+3+...+n=\frac{n(n+1)}{2}$

Sum of all the terms till n^{th} group $=\frac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}$

Sum of all the terms till $(n-1)^{th}$ group $=\frac{\frac{n(n-1)}{2}\left(\frac{n(n-1)}{2}+1\right)}{2}$

Sum of n^{th} group $=\frac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}-\frac{\frac{n(n-1)}{2}\left(\frac{n(n-1)}{2}+1\right)}{2}$

$\Rightarrow$ Sum of n^{th} group $=\frac{n(n^2+1)}{2}$

Q.8 [D]

Let the numbers be $a-3d, a-d, a+d, a+3d.$

As given $a-3d+a-d+a+d+a+3d=48 \Rightarrow a=12$

Also $\frac{a^2-9d^2}{a^2-d^2}=\frac{27}{35} \Rightarrow \frac{144-9d^2}{144-d^2}=\frac{27}{35} \Rightarrow d=\pm 2$

Hence the numbers are 6, 10, 14, 18.

Q.9

$S=\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+...n$ terms $=1-\frac{1}{2}+1-\frac{1}{4}+1-\frac{1}{8}+1-\frac{1}{16}+...n$ terms

$\Rightarrow S=n-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+....n\text{ terms}\right)$ Or $S=n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}=n-1+2^{-n}$.

As given $S=9+2^{-10}$ hence n = 10.

Q.10 [C]

$S=1+n+n^2+...+n^{127}=\frac{n^{128}-1}{n-1}$

$\Rightarrow S=(n^2+1)(n^4+1)(n^8+1)(n^{1+}+1)(n^{32}+1)(n^{64}+1)$

Hence $n^m + 1$ will divides s for $n = 2, 4, 8, 16, 32, 64$.

Q.11 [A]

$$S = \frac{1}{2} + \frac{1}{4} + ...\infty \text{ terms} = 2 \ \& \ S_n = 1 + \frac{1}{2} + \frac{1}{4} + ...n \text{ terms} = 2\left(1 - \frac{1}{2^n}\right)$$

Now $2 - 2\left(1 - \frac{1}{2^n}\right) < \frac{1}{1000} \Rightarrow \frac{1}{2^n} < \frac{1}{2000}$ or $2^n > 2000$.

Hence ≥ 11.

Q.12 [A]

$$S = \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + ... + 10 \text{ terms}$$

$$\Rightarrow S = \left(x^2 + x^4 + x^6 + ... + 10 \text{ terms}\right) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + ... + 10 \text{ terms}\right) + 20$$

Or $S = \frac{x^2\left(x^{20} - 1\right)}{x^2 - 1} + \frac{\frac{1}{x^2}\left(1 - \frac{1}{x^{20}}\right)}{1 - \frac{1}{x^2}} + 20$ i.e. $S = \left(\frac{x^{20} - 1}{x^2 - 1}\right)\left(\frac{x^{22} + 1}{x^{20}}\right) + 20.$

Q.13 [D]

$$S = \sqrt{ax}\left(1 + \sqrt{b} + b + b\sqrt{b} + ...\infty \text{ terms}\right) + x\left(1 + \sqrt{y} + y + y\sqrt{y} + ...\infty \text{ terms}\right)$$

$$\Rightarrow S = \frac{\sqrt{ax}}{1 - \sqrt{b}} + \frac{x}{1 - \sqrt{y}}.$$

Q.14 [B]

$$a + ar + ar^2 = S, a \times ar \times ar^2 = P \ \& \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = R.$$

$$\Rightarrow a\left(1 + r + r^2\right) = S, a^3a^3 = P \ \& \frac{a + r + r^2}{ar^2} = R$$

$$\Rightarrow P^2R^3 \times \frac{\left(1 + r + r^2\right)^3}{a^3r^6} \text{ or } P^2R^3 = a^3\left(1 + r + r^2\right)^3 = S^3$$

Q.15 [D]

In first rebound ball will travel $2 \times 100 \times \frac{4}{5}$, in second rebound ball will travel $2 \times 100 \times \left(\frac{4}{5}\right)^2$,

in second rebound will travel $2 \times 100 \times \left(\frac{4}{5}\right)^3$, and so on inf initely.

Hence total distance traveled $= 100 + 200 \times \left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + ...\infty \text{ terms}\right) = 900$ mts.

Q.16 [C]

$S = 1 + r + r^2 + ...\infty \text{ terms} = \frac{1}{1-r} \Rightarrow r = \frac{S-1}{S}$.

Also $1 + r^2 + r^4 + ...\infty \text{ terms} = \frac{1}{1-r^2}$

Hence $1 + r^2 + r^4 + ...\infty \text{ terms} = \frac{S^2}{S^2 - (S-1)^2}$ i.e. $\frac{S^2}{2S-1}$.

Q.17 [B]

As given $a + ar + ar^2 + ... + ar^{2n-1} = 3\left(a + ar^2 + ar^4 + ... + ar^{2n-2}\right)$

$\Rightarrow \frac{a\left(1-r^{2n}\right)}{1-r} = 3\frac{a\left(1-r^{2n}\right)}{1-r^2} \Rightarrow r = 2$.

Q. 18 [B]

As given $x = a + ar + ar^2 + ...\infty \text{ terms} = \frac{a}{1-r}$

and $y = a^2 + a^2r^2 + a^2r^4 + ...\infty \text{ terms} \frac{a^2}{1-r^2}$

$\Rightarrow r = \frac{x^2 - y}{x^2 + y}$.

Q.19 [D]

$(1+r)\left(1+r^2\right)\left(1+r^4\right)\left(1+r^8\right) = \frac{1-r^{16}}{1-r} \Rightarrow n = 16.$

Q.20 [B]

$0.7 + 0.77 + 0.777 + ... = \frac{7}{9}(0.9 + 0.99. + 0.999 + ...)$

$= \frac{7}{9}\left(10 - \frac{1}{10} - \frac{1}{10^2} - \frac{1}{10^3} - ...\right) = \frac{7}{81}\left(89 + \frac{1}{10^{10}}\right)$

Q.21 [C]

Sum of n terms after first n terms $= S_{2n} - s_n = 2S_n \Rightarrow S_{2n} \Rightarrow S_n = 3S_n$

$\Rightarrow \frac{2n}{2}\left\{2a + (2n-1)d\right\} = 3 \times \frac{n}{3}\left\{2a + (n-1)d\right\}$

$\Rightarrow a = (n+1)\frac{d}{2}$.

Now $\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}\left\{2a + (3n-1)d\right\}}{\frac{n}{2}\left\{2a + (n-1)d\right\}} \Rightarrow \frac{S_n}{S_n} = \frac{3\left\{(n+1)d + (3n-1)d\right\}}{\left\{(n+1)d + (n-1)d\right\}}$

$\Rightarrow \frac{S_{3n}}{S_n} = 6$.

Q.22 [D]

For integral roots discrmin ant must be a perfect square, hence $9 + 4a_i = k^2$. The values of a_i for which it's a perfect square are $4, 10, 18, 28, 40....., 270$.

Now Let

$S_n = 4 + 10 + 18 + 28 + ... + t_n$

$S_n = \quad 4 + 10 + 18 + ... + t_{n-1} + t_n$

- -

$0 = 4 + 6 + 8 + 10 + ...n \text{ terms - } t_n$

$\Rightarrow t_n = \frac{n(n+3)}{2}$. Also 270 is 15th term.

Now $S_{15} = \frac{1}{2}\sum_{r=1}^{15} r^2 + \frac{3}{2}\sum_{r=1}^{15} r$ or $S_{15} = \frac{15 \times 16 \times 31}{12} + \frac{3 \times 15 \times 16}{4} = 1600$

Q.23 [A]

$f(2n) = \sum_{r=1}^{2n} r^4 \Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + \sum_{r=1}^{n} (2r)^4$

$\Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + 16 \times \sum_{r=1}^{n} r^4$

$\Rightarrow f(2n) = \sum_{r=1}^{n} (2r-1)^4 + 16f(n)$

$\Rightarrow f(2n) - 26f(n) = \sum_{r=1}^{n} (2r-1)^4$

Q.24 [A] & [B]

For roots to be real $q^2 \ge 4pr$. But as p, q, r are in A.P. hence $q = \frac{p+r}{2}$.

Thus $(p+r)^2 \ge 16pr$ or $p^2 - 14pr + r^2 \ge 0 \Rightarrow \left(\frac{p}{r} - 7\right)^2 \ge 48$

$\Rightarrow \left|\frac{p}{r} - 7\right| \ge 4\sqrt{3}$. Similarly $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$.

Q.25 [B]

$\log_{5.2^x+1} 2, \log_{2^{1-x}+1} 4, 1$ are in H.P. hence $\log_2(5.2^x + 1), \log_2(2^{1-x} + 1)^{1/2}, \log_2 2$ are in A.P.

$\Rightarrow \left(5.2^x+1\right),\left(2^{1-x}+1\right)^{1/2},2$ are in G.P.

$\Rightarrow 2^{1-x}+1=\left(5.2^x+1\right)\times 2$ or $\frac{2}{2^x}+1=10.2^x+2$

$\Rightarrow 10\left(2^x\right)^2+2^x-2=0$

$\Rightarrow 2^x=\frac{4}{5}$

Hence x is a negative real number.

Q.26 [A]

$$\frac{S_{kx}}{S_x}=\frac{\frac{kx}{2}\left\{2a+\left(kx-1\right)d\right\}}{\frac{x}{2}\left\{2a+\left(x-1\right)d\right\}}\Rightarrow \frac{S_{kx}}{S_x}=\frac{k\left\{2a-d+kxd\right\}}{\left\{2a-d+xd\right\}}$$

Clearly if $2a=d$, then $\frac{S_{kx}}{S_x}=k^2$ i.e. independent of x.

Q.27 [D]

p, q, r are in H.P. hence q is H.M. of p & r. Also $\sqrt{pr}$ is G.M. of p & r.

Now H.M. < G.M. $\Rightarrow q<\sqrt{pr}$ or $q^2<pr$.

As discriminant is $4\left(q^2-pr\right)$ hence roots must be imaginary.

Q.28 [B]

$$S_n=\frac{a\left(r^n-1\right)}{r-1},S_{2n}=\frac{a\left(r^{2n}-1\right)}{r-1}\ \&\ S_{3n}=\frac{a\left(r^{3n}-1\right)}{r-1}$$

$$S_{2n}=\left(1+r^n\right)S_n\ \&\ S_{3n}=\left(1-r^n+r^{2n}\right)S_n$$

$$S_{2n}-S_n=\left(r^n\right)S_n,S_{3n}-S_{2n}=r^n\left(r^n-2\right)S_n,S_{3n}-S_n=r^n\left(r^n-1\right)S_n$$

Q.29 [A]

$$S=\frac{1}{2.4}+\frac{1.3}{2.4.6}+\frac{1.3.5}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1.(4-3)}{2.4}+\frac{1.3.(6-5)}{2.4.6}+\frac{1.3.5.(8-7)}{2.4.6.8}+\frac{1.3.5.7.(10-9)}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1}{4}-\frac{1.3}{2.4}+\frac{1.3}{2.4}-\frac{1.3.5}{2.4.6}+\frac{1.3.5}{2.4.6}-\frac{1.3.5.7}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8}-\frac{1.3.5.7.9}{2.4.6.8.10}+...\infty \text{ terms}$$

$$S=\frac{1}{4}.$$

Q30. [B]

$$\sum_{k=1}^{\infty}\frac{2^{k+2}}{3^k}=2^2\times\sum_{r=1}^{\infty}\left(\frac{2}{3}\right)^k \Rightarrow s=4\times\left(\frac{\frac{3}{2}}{1-\frac{2}{3}}\right)=8.$$

Q.31 [C]

$$S=1^2-\frac{2^2}{5}+\frac{3^2}{5^2}-\frac{4^2}{5^3}-\frac{5^2}{5^4}+....\infty \text{ terms}$$
$$\frac{1}{5}S=\quad \frac{1^2}{5}-\frac{2^2}{5^2}+\frac{3^2}{5^3}-\frac{4^2}{5^4}-...\infty \text{ terms}$$

$$\frac{6}{5}S=1-\frac{3}{5}+\frac{3}{5^2}-\frac{7}{5^3}+\frac{9}{5^4}-...\infty \text{ terms}$$

$$\frac{6}{5}S=1-\frac{3}{5}+\frac{3}{5^2}-\frac{7}{5^3}+\frac{9}{5^4}-...\infty \text{ terms}$$
$$\frac{6}{25}S=\quad \frac{3}{5}-\frac{3}{5^2}+\frac{7}{5^3}-\frac{9}{5^4}+...\infty \text{ terms}$$

$$\frac{36}{25}S=1-2\left(\frac{1}{5}-\frac{1}{5^2}+\frac{1}{5^3}-\frac{1}{5^4}+...\infty \text{ terms}\right)$$

Hence $S=\frac{25}{54}$.

Q. 32 [B]

According to Cauchy – Schwarz inequality

$$\left(x_1^{\,2}+x_2^{\,2}+x_3^{\,2}+...+x_{n-1}^{\,2}\right)\left(x_2^{\,2}+x_3^{\,2}+x_4^{\,2}+...+x_3^{\,2}\right)\ge\left(x_1x_2+x_2x_3+x_3x_4+...+x_{n-1}x_n\right)$$

Hence as given in question

$$\left(x_1^{\,2}+x_2^{\,2}+x_3^{\,2}+...+x_{n-1}^{\,2}\right)\left(x_2^{\,2}+x_3^{\,2}+x_4^{\,2}+...+x_3^{\,2}\right)=\left(x_1x_2+x_2x_3+x_3x_4+...+x_{n-1}x_n\right)$$

Now in Cauchy – Schwarz inequality equality occurs when

$\frac{x_1}{x_2}=\frac{x_2}{x_3}=\frac{x_3}{x_4}=...\frac{x_{n-1}}{x_n}$. Hence numbers are in G.P.

Q.33 [B]

$$S_n=1+\frac{3}{2}+\frac{5}{3}+...\frac{2n-1}{n}\Rightarrow S_n=2-1+2-\frac{1}{2}+2-\frac{1}{3}+...+2-\frac{1}{n}$$

$$\Rightarrow S_n=2n-\left(1+\frac{1}{2}+\frac{1}{3}+...\frac{1}{n}\right) \text{ or } S_n=2n-H_n.$$

Q.25 [B]

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+...\infty \text{ terms }=\frac{1}{2}$$

Hence $y=(0.64)^{\log_{0.25}0.25}$ or $y=0.64^{1/2}=0.8$

Q.34 [C]

$$4x^2+9y^2+16z^2=6xy-12yz-8zx=0$$

$$\Rightarrow 8x^2+18y^2+32x^2-12xy-34yz-16zx=0$$

$$\Rightarrow(2x-3y)^2+(3y-4z)^2(3y-4z)^2+(4z-2x)^2=0$$

$\Rightarrow 2x = 3y = 4z$ or $\frac{x}{6} = \frac{y}{4} = \frac{z}{3}$.

Hence x, y, z are in H.P.

Q.35 [C]

$$a_n = 1 + 10 + 10^2 + ... + 10^{n-1} \Rightarrow a_n = \frac{10^n - 1}{9}$$

Now $a_{124} = \frac{10^{224} - 1}{9}$. Observe that $271 \times 369 = 10^5 - 1$

Rewrite a_{124} as $\frac{\left(\left(10^5\right)^{124} - 1\right)10^4 + 10^4 - 1}{9}$ or $\frac{27\ln}{9} + 1111$

Now remainder when 1111 is divided by 27 is 27.

Q.36 [A]

$$a = \frac{a}{1-r}, y = \frac{b}{1+r} \ \& \ z = \frac{c}{1-r^2} \Rightarrow \frac{xy}{z} = \frac{ab}{c}.$$

Q.37 [C]

$1 + |\cos x| + |\cos x|^2 + ...\infty$ terms $= \frac{1}{1 - |\cos x|} \Rightarrow 8^{\frac{1}{1-|\cos x|}} = 4^3$

Or $2^{\frac{3}{1-|\cos x|}} = 2^6$ hence $\cos x = \pm\frac{1}{2}$

Now in $(-\pi, \pi)$, x=$-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$.

Q.38 [A]

Let $b = ar, c = ar^2 + d = ar^3$, then

$$(a-c)^2 + (b+c)^2 + (b-d)^2 - (a-d)^2 = \left(a - ar^2\right)^2 + \left(ar - ar^2\right)^2 + \left(ar - ar^3\right)^2 - \left(a - ar^3\right)^2$$

$$= a^2(1-r)^2(1+r)^2 + a^2r^2(1-r)^2 + a^2r^2(1-r)^2(1+r)^2 - a^2\left(1 + r + r^2\right)^2$$

$$= a^2(1-r)^2(1+r)^2\left(1+r^2\right) - a^2(1-r)^2\left(1+r^2\right)(1+r)^2 = 0.$$

Q.39 [C]

$$S = 1 + (1+x) + \left(1 + x + x^2\right) + \left(1 + x + x^2x^3\right) + ...n \text{ terms}$$

$$\Rightarrow (1-x)S = (1-x) + \left(1-x^2\right) + \left(1-x^3\right) + \left(1-x^4\right) + ...n \text{ terms}$$

$$\Rightarrow (1-x)S = n - \left(x + x^2 + x^3 + ...n \text{ terms}\right)$$

$$\Rightarrow S = \frac{n}{1-x} - \frac{\left(x^n - 1\right)}{(1-x)^2}.$$

Q.40 [A]

$2, \frac{7}{4}, \frac{14}{9}, \ldots$ is an H.P. hence $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \ldots.$ will be an A.P. with $d = \frac{1}{14}$

6^{th} term $= \frac{1}{2} + 5 \times \frac{1}{14}$ i.e. $\frac{6}{7}$.

Hence 6^{th} term of the given series will be $\frac{7}{6}$.

Q.41 [C]

$\frac{a+b}{2} + \frac{2ab}{a+b} = 25 \,\&\, ab = 144 \Rightarrow (a+b)^2 - 50(a+b) + 576 = 0.$ Hence $a+b = 18$ or 32.

Q.42 [D]

A.M. of n A.M.s between a & b is single A.M. between a & b, hence

Sum of n A.M.s between a & b, S = nA.

$\Rightarrow \frac{S}{A} = n$.

Q.43 [B]

$2b = a + c, x^2 = ab, y^2 = bc \Rightarrow \frac{x^2 + y^2}{2} = b^2$.

Q.44 [B]

Let the roots be $x_1 \,\&\, x_2$, then $x_1 + x_2 = 2A \,\&\, x_1 + x_2 = G^2$.

Required equation is $x^2 - 2Ax + G^2 = 0.$

Q.45 [C]

$\frac{p+q}{2} = 2\sqrt{pq} \Rightarrow (p+q)^2 = 16pq$

$\Rightarrow \left(\frac{p}{q}\right)^2 - 14\left(\frac{p}{q}\right) + 1 = 0.$ Hence $\frac{p}{q} = 7 + 2\sqrt{12}.$ Now $7 + 2\sqrt{12} = \left(2+\sqrt{3}\right)^2 = \frac{2+\sqrt{3}}{2-\sqrt{3}}$.

Q.46 [C]

Let $b = ar \,\&\, c = ar^2, 2p = a + ar, 2q = ar + ar^2 \Rightarrow 2p = a(1+r), 2q = ar(1+r)$

$\frac{a}{p} + \frac{c}{q} = \frac{2}{1+r} + \frac{2ar^2}{ar(1+r)} \Rightarrow \frac{a}{p} + \frac{c}{q} = 2.$

Q.47 [A]

G.M. of roots of $x^2 - 2ax + b^2 = 0$, $\sqrt{b^2} = |b|$

Now for $x^2 - 2bx + a^2 = 0, b = \frac{\text{sum of roots}}{2}$ i.e. AM.

Q.48 [B]

$$a+b+c=3A, abc=G^3 \& \frac{3abc}{ab+bc+ca}=H \Rightarrow ab+bc+ca=\frac{3G^3}{H}$$

Now required equation is $x^3-3Ax^2+\left(\frac{3G^3}{H}\right)x-G^3=0$.

Q.49 [A]

$$S=13^2+2.5^2+3.7^2+\ldots \Rightarrow t_n=n(2n+1)^2$$

$$S_{20}\sum_{r=1}^{20}\left(4r^3+4r^2+r\right) \Rightarrow S_{20}=4\times\frac{20^2\times21^2}{4}+4\times\frac{20\times21\times41}{6}+\frac{20\times21}{2}$$

Hence $S_{20}=188090.$

Q.50 [C]

Given $t_n=(2n-1)(2n+1)(2n+3).$ Let $T_n=(2n-1)(2n+1)(2n+3)(2n+5)$

$T_{n-1}=(2n=3)(2n-1)(2n+1)(2n+3).$

Now $T_n-T_{n-1}=8t_n$

$$S_n\sum_{r=1}^{n}t_r=\frac{1}{8}\times\sum_{r=1}^{n}\left(T_r-T_{r=1}\right)$$

Hence $S_n=\frac{T_n-T_0}{8}$ i.e. $S_n=\frac{(2n-1)(2n+1)(2n+3)(2n+5)+15}{8}$

Q.51 [B]

Given $t_n=\frac{1}{n(n+1)} \Rightarrow t_n=\frac{1}{n}-\frac{1}{n+1}$

Hence $S=\sum_{r=1}^{\infty}\left(\frac{1}{r}-\frac{1}{r+1}\right)=1$

Q.52 [B]

$S_n=1+3+6+10+15+21+\ldots+t_n$

$S_n=\quad 1+3+6+10+15+\ldots+t_{n-1}+t_n$

$0=1+2+3+4+5+6+\ldots n$ terms $-t_n$

$\Rightarrow t_n=\frac{n(n+1)}{2}$ Now $t_n=5050$ gives $n=100$.

Q.53 [A]

$S=1^2+2^2+3^3x^2+4^2x^2+\ldots\infty$ terms

$xS=\quad 1^2x+2^2x^2+3^2c^2+\ldots\infty$ terms

$(1-x)S=1+3x+5x^2+7x^3+\ldots\infty$ terms

$$x(1-x)S = \quad x+3x^2+5x^2+...\infty \text{ terms}$$

$$(1-x)^2 S = 1+2(x+x^2+x^3+...\infty \text{ terms})$$

$$\Rightarrow S = \frac{1}{(1-x)} + \frac{2x}{(1-x)^3}.$$

Q.54 [A]

$$S = 1-\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+...\infty \text{ terms}$$

$$\frac{1}{2}S = \frac{1}{2}-\frac{3}{4}+\frac{5}{8}...\infty \text{ terms}$$

$$\frac{3}{2}S = 1-2\left(\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}-\frac{1}{2^4}+...\infty \text{ terms}\right)$$

$$\Rightarrow S = \frac{2}{9}.$$

Q.55 [C]

$$S = 1+3+7+15+...n \text{ terms}$$

$$\Rightarrow S = 2-1+4-1+8-1+16-1+...$$

$$\Rightarrow S = (2+4+8+16+...n \text{ terms})-n$$

$$\Rightarrow S = 2^{n+1}-1-n.$$

Q.56 [D]

a, b, c, d are in A.P. $\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ will also be in A.P.

i.e. $\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are in A.P.

$\Rightarrow bcd, acd, abd, abc$ are in H.P.

Q.57 [A]

Roots are equal therefore Discrminant = 0.

$$(c-a)^2 = 4(b-c)(a-b) \Rightarrow a^2+4b^2+c^2-4ab-4bc+2ax = 0.$$

$$\Rightarrow (a+c+2b)^2 = 0$$

$$\Rightarrow a+c = 2b.$$

Q.58 [B]

Clearly the given expression is in form of sum of a G.P.

Q.59 [C]

$$x = \frac{1}{1-a}, y = \frac{1}{a-b}, z = \frac{1}{1-c}$$

Now a, b, c are in A.P. hence $1-a, 1-b, 1-c$ will also be in A.P.

Hence $x = \frac{1}{1-a}, y = \frac{1}{1-b}, z\frac{1}{1-c}$ will be in H.P.

Q.60 [B]

$$\left(a^2+b^2\right)x^2-2ab\left(a+c\right)x+\left(b^2+x^2\right)=0 \Rightarrow \left(ax-b\right)^2+\left(bx-c\right)^2=0$$

Hence $\frac{b}{a}=\frac{c}{}a=x$. Therefore a, b, c are in G.P.

EXERCISE – 2 [B]

Q.1

$$a = 2r, d = 2r - 2$$

$$Ar = \frac{r}{2}\left(4r + (r-1)(2r-2)\right)$$

$$= \frac{r}{2}\left(4r + 2r^2 - 2r - 2r + 2\right)$$

$$= \frac{r}{2}\left(2r^2 + 2\right)$$

$$= r^3 + r$$

$$\sum_{r=1}^{n} Ar = \sum r^3 + \sum r$$

$$= \left|\left[\frac{r(r+1)^2}{2}\right] + \frac{r(r+1)}{2}\right| r = n$$

$$= \left|\frac{r(r+1)}{2}\left(\frac{r^2 + r + 12}{2}\right)\right| r = n$$

$$= \frac{n}{4}(n+1)\left(n^2 + n + 2\right)$$

$$\boxed{\text{Ans : a}}$$

Q.2

$$B_{10} = A_{12} - A_{11}$$

$$= 12^3 + 12 + 11^3 - 11$$

$$= 1728 + 12 - 1331 - 11$$

$$= 1740 - 1342$$

$$= 398$$

Q.3

$$C_r = B_{r+1} - B_r$$

$$= A_{r+3} - A_{r+2} - A_{r+2} + A_{r+1}$$

$$= A_{r+3} + A_r - 2A_{r+2}$$

$$= (r+3)^3 + r^3 - 2(r+2)^3 + r + 3 + r - 2r - 4$$

$$= r^3 + 27 + 27r + 9r^2 + r^3 - 2r^3 - 16 - 24r - 12r^2 - 1$$

$$= -3r^2 + 3r + 8$$

$$\frac{d}{r} = -6r + 3.\left|diff\right| = 6$$

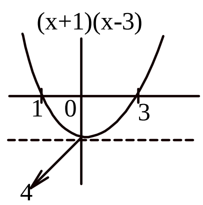
PASSAGE – II

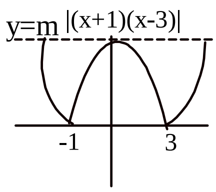
4. $\left|(x+1)(x-3)\right| = m$

clearly, if $\boxed{\begin{array}{l} m < 0 \\ \text{or } m < 4 \end{array}}$

then 4 solutions,
(from graph)

$\text{Ans}:(c)$





5. Clearly again from graph, 3 solutions are when m = 4

$\Rightarrow \left|(x+1)(x-3)\right| = 4$

$$x = \frac{2 \pm \sqrt{32}}{2} = \frac{2 \pm 4\sqrt{2}}{2} = 1 \pm 2\sqrt{2}$$

But anyway,

For 3 solutions, m = 4

12 means inserted = 6 pairs having sum $= \frac{1}{6} + 2$

$$\text{Sum} = 6\left(\frac{1}{6} + 2\right) = 1 + 12 = 13$$

$\boxed{\text{Ans}:(c)}$

6. Two solutions

$\Rightarrow m - 0 \text{ or } m > 4$

$a, ar, ar^2 \equiv GP$

Now, $ar^2 = a^2 \Rightarrow a = r^2$

Also, $a + r = 0$ …………….(Least value of m)

$\Rightarrow r^2 + r = 0b$

$\Rightarrow r = -1$

So $a = 1$

$GP = 1, -1, 1 - 1, 1 - 1$

$\boxed{\text{Ans}:(d)}$

PASSAGE – III

7. $3, B, C, D$

Now, $\frac{2\times 3\times c}{c+3} = b$ or $\frac{c}{c+3} = \frac{b}{6}$

Also, $2c = b + d$

$c = \frac{b+d}{2} = \frac{b+(b+4)}{2} = b + 2$

So $\frac{b+2}{b+5} = \frac{b}{6}$

Or $6b + 12 = b^2 + 5b$

$b^2 - b - 12 = 0$

$c = 3, \quad b = 4,\ c = 6,\ d = 8$

$\boxed{\text{Ans}:(d)}$

8. $4, A_1, A_2, A_3, 6$

$\Rightarrow 4, 4.5, 5, 5.5, 6$

$5.5 = \frac{11}{2} = \frac{11}{4} = \frac{14+8}{4} = \frac{k+8}{4}$

$\boxed{\text{Ans}:(c)}$

9. $k = 14, b = 4, d = 8$

$kb^2 = 14.4^7 . kd^7 = 14.8^7$

$r = \left(\frac{14.8^7}{14.4^7}\right) = 2$

PASSAGE – IV

10. $\frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}+\frac{c}{4}}{4} \geq 10\sqrt{\left(\frac{a}{3}\right)^3\left(\frac{b}{3}\right)^3\left(\frac{c}{4}\right)^4}$

$\boxed{\text{Ans}:(a)}$

11. (D)

$\frac{a+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}+\frac{c}{6}}{10}$

$$\geq \frac{10}{\frac{1}{a}+\frac{3}{b}+\frac{3}{b}+\frac{3}{b}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}+\frac{6}{c}}$$

or $\frac{1}{a}+\frac{9}{b}+\frac{36}{c} \geq \frac{100}{20}$ i.e. 5

12. $$\frac{3(a+x)+4(y+2)+5(z+4)}{12} \geq \sqrt[12]{(1+x)^3 (y+2)^4 (z+4)^5}$$

$$\Rightarrow \frac{5+3+8+20}{12} \geq \sqrt[12]{(1+x)^3 (y+2)^4 (z+4)^5}$$

3^{12} is max value.

$\boxed{\text{Ans}:(b)}$

ASSERTION – RESONING

Q. 32 (C)

$$t_1 = \frac{a_1}{2}, t_2 = \frac{a_2}{4}, t_3 = \frac{a_3}{6}$$

Now, $t_{r+1} - t_{(r)}$

$$= \frac{a_{r+1}}{2^{r+1}} - \frac{a_r}{2^r}$$

$$\frac{= 2^r a_{r+1} - 2^r \times 2a}{2^{2r+1}}$$

$$= \frac{2^r (a_{r+1} - 2a)}{2_{2r+1}} = \left(\frac{1}{2^{r+1}}\right)\begin{bmatrix} \left(r \times 2^{r+3} - (r-1) 2^{r+2}\right) \\ -2(r-1) 2^{r+2} - r(r-2)^{r+1} \end{bmatrix}$$

$$= \left(\frac{1}{2^{r+1}}\right)\left[{}_r 2^{r+3} - 3(r-1) 2^{r+2} + 2(r-2)\right]$$

This is a constant.

Thus A.P.

Q.35 (a) For A.P

$$\frac{t_r - t_p}{t_p = t_p} = \frac{(r-p)}{(q-p)} = \text{rational}$$

Hence it is irrational

Q.36 (d)

$$S = 1 + 3y + 5y^2 + \ldots$$

$Sy = y + 3y^2 + 5y^3 + ...$

$S(1-y) = 1 + 2y + 2y^2 + ...$

$$= 1 + 2\left(\frac{y}{1-y}\right) = \frac{1+y}{1-y}$$

$$\Rightarrow S = \frac{1+y}{(1-y)^2}$$

$$\text{Here, } S = \frac{a+1-\frac{1}{x}}{\left(1-\frac{1}{x}\right)^2} = \frac{2x^2-x}{(x-1)^2} \neq x^2 - x$$

MATRIX MATCH

Q.37 $(A) \rightarrow (P)$

So $\frac{p}{64} = 2^{9-6} = 2^3 = 8$

(B) $\rightarrow (r)$

$$\frac{4^{x+\frac{1}{2}} + 4^{\frac{3}{2}-x}}{4} = \frac{1}{2}\left(\frac{x^{x+\frac{1}{2}} + 4^{\frac{3}{2}-x}}{x+\frac{1}{2}+\frac{3}{2}-x}\right)$$

This is greater than $\sqrt{4^{x+\frac{1}{2}} \times 4^{\frac{3}{2}-x}} = 4$

$\frac{1}{2}(4) = 2$

(C) $\rightarrow (q)$

$(x+2y-2)(2y+z-x)(z+x-y)$

$= (a+2a = 2d - a - 2d)(2a+2d+a+2d-a)(a+2d+a-a-d)$

$= (2a)(2a+4d)(a+d)$

$= 2x \times 2z \times y$

$= 4xyz$

(D) $\rightarrow (p)$ $t_1 = 7(t_2 + ... + ...)$

$$a = 7\left(\frac{a}{1-r} - a\right)$$

$$\Rightarrow 8a = \frac{7a}{1-r}$$

$$1 - r = \frac{7}{8}$$

Q.40 $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ is A.P.

So $\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ is A.P.

So $\frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ is A.P.

$\Rightarrow \frac{b+c-a}{a}, \frac{a+c-b}{b}, \frac{a+b-c}{b}, \frac{a+b+c}{c}$ is A.P.

Thus $\frac{a}{b+c-a}, \frac{b}{a+c-b}, \frac{c}{a+b-c}$ is A.P.

SEQUENCE SERIES

EXERCISE – 2 [C]

Q.1 $$\frac{\frac{n}{2}\left[10+(n-1)4\right]}{\frac{n}{2}\left[14+(n-1)2\right]}=\frac{5}{4}$$

$$\frac{4n+6}{2n+12}=\frac{5}{4}$$

$$\frac{2n+3}{n+6}=\frac{5}{4}$$

$$\Rightarrow \boxed{n=6}$$

Q.2 $$\frac{n}{2}\left[2\times 2+(n-1)3\right]=950$$

$$\frac{n}{2}\left[4+3n-3\right]=950$$

$$3n^2+n=1900$$

$$\boxed{n=25}$$

Q.3 Let n be number of terms, then

$$S_{even}+S_{odd}=S_n$$

$$S_{even}=S_{odd}+\left(\frac{n}{2}\right)d$$

$$\Rightarrow 30=24+\frac{nd}{2}$$

$$\Rightarrow nd=12$$

also $a+(n-1)d-q=\frac{21}{2}$

Q.4 $$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a'+(n-1)d'\right]}=\frac{7n+1}{4n+27}$$

For $n=21$

$$\frac{2a+20d}{2a'+20d'}=\frac{147+1}{111}$$

$$\Rightarrow \frac{a+10d}{a'+20d'}=\frac{148}{111}$$

$$\Rightarrow \frac{t_{11}}{t_{11}} = \frac{148}{111} = \frac{4}{3}$$

$$4+3=\boxed{7}$$

Q.5 $LCM = 60$

So (div. by 15) + (div. by 20) – (div. by 60)

$= 20+15-5$

$= \boxed{30}$

Q.6 $\dfrac{\frac{n(n+1)}{2} - r}{n-1} = 5 \Rightarrow n^2 - 9n + 10 = 2r$

Now, $2 \le n^2 - 9n + 10 \le 2n$

(i) $n^2 - 9n + 10 \ge 2$ $\qquad$ $n^2 - 9n + 12 \ge 0$

$\Rightarrow n \le \dfrac{9-\sqrt{51}}{2}$ or $n \le \dfrac{9+\sqrt{51}}{2}$

(ii) $n^2 - 9n + 10 \le 2n$ $\qquad$ $n^2 - 11n + 10 \le 0$

$\Rightarrow 1 \le n \le 5+10$

From (i) & (ii), $\dfrac{9\sqrt{51}}{2} \le n \le 10$

Hence n may be 8, 9 or 10

Hence sum of values of n = 27

Q.7 D = common diff

$-2D + kD^2 + 8D^3 = -6D + 4D^2 - D^3$

$9D^3 + (k-4)D^2 + 4D = 0$

$9D^2 + (k-4)D + 4 = 0$

$(k-4)^2 - 4 \times 4 \times 9 \ge 0$

$(k-4)^2 - (12)^3 \ge 0$

$(k-16)(k+8) \ge 0$

$\boxed{k \ge 16}$

Q.8 Numbers be a – d, a, a + d

$(a^2)^2 = (a-d)^2 (a+d)^2$

$\Rightarrow a^4 = \left(a^2 - d^2\right)^2$

$= a^4 + d^4 + 2a^2d^2$

$\Rightarrow d^4 = 2a^2a^2$

$\Rightarrow d^2 = 2a^2$

$$r = \frac{a^2}{(a-d)^2} = \left(\frac{a}{a-d}\right)^2 = \left(\frac{1}{1-\frac{d}{a}}\right)^2$$

$\frac{d}{a} = \pm\sqrt{2}$

So $r = \left(\frac{1}{a+\sqrt{2}}\right)^2$ or $\left(\frac{1}{1-\sqrt{2}}\right)^2$

$= \left(\sqrt{2}-1\right)^2$ or $\left(\sqrt{2}+1\right)^2$

$r_1 + r_2 = 2(2+1) = 6$

Q.9 Let the number be $100ar^2 + 10at + a$

Now, $100ar^2 + 10at + a - 400 = 100\left(ar^2 - 4\right) + 10ar + a$

Given that $a + ar^2 - 4 = 2ar$

Also as each of a, ar, ar^2 is an integer lying between

1 & 9, hence $r = 2$ or 3

For $r = 2, a + ar^2 - 4 = 2ar \Rightarrow a = 4,$ but $ar^2 > 9$

For $r = 3, a + ar^2 - 4 = 2ar \Rightarrow a = 1$

Hence the number is 931 & sum of digits = 13

Q.10 $\frac{a\left(r^n - 1\right)}{r-1} > 1000$

$\frac{\left(3^n - 1\right)}{2} > 1000$

$3^n > 2001$

$n = 7$

Q.11 n includes all number div by 2, 3 or 5

We have

$$\sum \perp^n = \left(1+\frac{1}{2}+\frac{1}{4}+....\right)+\left(1+\frac{1}{3}+\frac{1}{9}+.....\right)$$

$$+\left(1+\frac{1}{5}+\frac{1}{25}....\right)-\left(1+\frac{1}{6}+\frac{1}{x}....\right)-\left(1+\frac{1}{15}+\frac{1}{225}\right)$$

$$-\left(1=\frac{1}{10}+\frac{1}{100}.....\right)+2\left(1+\frac{1}{30}+\frac{1}{900}+....\right)$$

Q.12 $\frac{a}{r}, a, ar = GP$

$a^3 = 216 \qquad \Rightarrow a = 6$

$\left(\frac{6}{r}\times 6\right)+\left(6\times 6r\right)+\left(6\right)^2 = 156$

$\Rightarrow \frac{36}{r}+36+36r = 156$

$r = 3$

$\text{sum} = 18+6+2$

$= \boxed{26}$

Q.13 $\frac{a}{r}, a, ar, ar^2.....$

$\frac{a\left(\left(r\right)^{24}-1\right)}{\left(r\right)-1} = 1 \qquad \Rightarrow \frac{\left(\frac{1}{b}\right)\left(1-r^{24}\right)}{\left(r^{23}\right)\left(1-r\right)}$

$\Rightarrow ar^{23} = 2$

$t_{24} = 2$

We have $\left(t_1 \times t_{24}\right) = \left(\sqrt{2}\right)^2$

Q.14 We have

$\frac{p}{q} = 1+\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)^2+3\left(\frac{1}{6}\right)^3.....$

Now, $S = 1+x+2x^2+3x^3....$

$Sx = x+x^2+2x^3+3x^4.....$

$S\left(1-x\right) = \left(x^2+x^3+3x^4\right)$

$= \frac{x^2}{1-x}+\frac{\left(1-x\right)^2}{\left(1-x\right)}$

$$S=\frac{x^2}{(1-x)^2}+\frac{1}{1-x}$$

$$\frac{p}{q}=\frac{\frac{1}{36}}{\frac{25}{36}}+\frac{\frac{1}{5}}{\frac{5}{6}}=\frac{1}{25}+\frac{6}{5}=\frac{31}{25}$$

$$\left|p^2-q^2\right|=4$$

Q.15 $=0.9\left(100-1+1000-1+....10^{101}-1\right)$

$$=0.9\left(10^2\ \_....+10^{101}-100\times 1\right)$$

$$=0.9\left(10^3+....10^{101}\right)$$

$$=0.9\frac{\left(10^3\right)\left(10^{99}-1\right)}{10-1}=100\left(10^{99}-1\right)$$

$$=10^{99}-100$$

Q.16 $t_1=b-2$

$$t_3=b+6$$

$$r=\sqrt{\frac{b+6}{b-2}}$$

$$\frac{t_1+t_3}{2}=b+2$$

$$\frac{b+2}{b-2\sqrt{\frac{b+6}{b-2}}}=\frac{5}{3}\Rightarrow\frac{b+2}{\sqrt{(b-2)(b+6)}}=\left(\frac{5}{3}\right)^2$$

$$\Rightarrow\frac{(b+2)^2}{(b-2)(b+6)}=\left(\frac{5}{3}\right)^2\Rightarrow\boxed{b=3}$$

Q.17 After withdrawal of Acid = 729 – 3x

After II withdrawal of Acid $=729-3x-\left(\frac{729-3x}{729}\right)\times 3x$

i.e. $\frac{(729-3x)^2}{729}$

After III withdrawal Acid $=\frac{(729-3x)^2}{729}-\left(\frac{729-3x}{729}\right)^2\times 3x$

i.e. $\dfrac{(729-3x)^3}{729^2}$

After VI withdrawal Acid $=\dfrac{(729-3x)^6}{729^5}=64$

Hence, $(729-3x)^6=2^6 3^{30}$ or $x=81$

Q.18 $y=\dfrac{a+b}{2}$

$x=\dfrac{a+y}{2}=\dfrac{a}{2}+\dfrac{a}{4}+\dfrac{b}{4}=\dfrac{3a}{4}+\dfrac{b}{4}=\dfrac{3a+b}{4}$

$z=\dfrac{a+3b}{4}$

$xyz=55 \Rightarrow 55=\dfrac{(a+b)(3a+b)(a+3b)}{a+b}$

Again, It $1+p, y=\dfrac{2ab}{a+b}$

$x=\dfrac{2ay}{a+y}=\dfrac{2a\times\dfrac{2ab}{a+b}}{a+\dfrac{2ab}{a+b}}=\dfrac{4a^2b}{a^2+3ab}$

$z=\dfrac{4a^2b}{a^2+3ab}$

$xyz=\dfrac{4a^2b\times 4ab^2\times 2ab}{ab(a+b)(b+3a)(a+3b)}$

$\dfrac{343}{55}=\dfrac{32a^3b^3}{ab(a+b)(3a+b)(a+3b)}$

$\therefore 55\times\dfrac{343}{55}=a^3b^3$

$ab=7$

Q.19 $\dfrac{1}{t_1}=2.5=\dfrac{5}{2}$

$\dfrac{1}{t_2}=\dfrac{23}{12}$

Smaller the t r_1, greater the term.

Common diff of $AP = \frac{23}{11} - \frac{30}{12} = \frac{-7}{12}$

So we have $\frac{30}{12}, \frac{23}{12}, \frac{16}{12}, \frac{2}{12}, \frac{-5}{12}$

so $\frac{2}{12}$

Reciprocal = $\frac{12}{2} = 6$

Q.20 $\sum_{r=0}^{88} \frac{1}{\cos r^0 \cos(r+1)^0}$

$\frac{1}{\sin 1^0} \sum_{\ell=0}^{88} \frac{\sin 1^0}{\cos r^0 . \cos(r+1)^0}$

$= \frac{1}{\sin 1^0} \sum_{\ell=0}^{88} \frac{\sin[(r+1)-r]}{\cos r^0 . \cos(r+1)^0}$

$= \frac{1}{\sin 1^o} \left(\sum_{r=0}^{88} \frac{\sin(r+1)\cos r^0 - \cos(r+1)\sin r}{\cos r . \cos(r+1)} \right)$

$= \frac{1}{\sin^0} \left(\sum_{\ell=0}^{88} \tan(r+1)^0 - \tan r^0 \right)$

$= \frac{1}{\sin^0} \left(\tan 89^0 - \tan 0^0 \right)$

$= \frac{\cot^0}{\sin 1^0} \qquad \Rightarrow \theta = 1^0$

Q.21 roots are

$-3x, -x, x, 3x$

So, $\quad 10x^2 = 3m + 2$

$9x^4 = m^2$

$\Rightarrow \frac{100}{9} = \frac{9m^2 + 4 + 12m}{m^2}$

$\Rightarrow 100m^2 = 81m^2 + 108m + 36$

$\Rightarrow 19m^2 - 108m - 36 = 0$

$\Rightarrow 19m^2 - 114m + 6m - 36 = 0$

$\Rightarrow (19m + 6)(m - 6) = 0$

$\Rightarrow m = 6$

Q.22 17, 21, 25 ……….. 417

16, 21, 26 ……….. 466

Common terms $\to$

21, 41, 61 ……………….

$T_n = 20n + 1 \le 417$

$\Rightarrow 20n \le 416$

$n \le 20.8$

$\boxed{n = 20} \Rightarrow K = 5$

Q.23 $P2^x + \frac{4}{2^x} = 5$

$\Rightarrow pt^2 - 5t + 4 = 0$

Where $\left(t = 2^x > 0\right)$

So, $D \ge 0$

$\Rightarrow 25 - 16p \ge 0$

$p < \frac{25}{16}$

But, $p > 0$

So, $p = 0, p = 1$

at $p = 1$, 2 solution

at $p = 0$, 1 solution

No. of value of p is $Q = 1$

AP : $1, \frac{1}{x_1}, \frac{1}{x_2} .... \frac{1}{x_{20}}, \frac{1}{6}$

AP : $6, \frac{1}{x_1}, \frac{6}{x_2} ...... \frac{6}{x_{20}}$

AP :

Q.24 $a_1 = \frac{1}{2}$

$(n-1)a_{n-1} = (n+1)a_n$

$\Rightarrow n(n-1)a_{n-1} - n(n+1)a_n = 0$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_{n-1} - n(n+1)a_n = \sum 0$

$\Rightarrow 2.1.a, -n(n+1)a_n = 0$

$\Rightarrow a_n = \frac{1}{n(n+1)}$

$S_n = \sum_{r=1}^{n} \frac{1}{n(n+1)}$

$= \sum_{r=1}^{\infty} \frac{1}{r} - \frac{1}{r+1}$

$= 1 - 0 = 1$

Q.25 (1), (2, 3, 4), (5, 6, 7, 8, 9) ……..

No. of terms in n^{th} jump is N_n

$N_1 = 1, N_2 = 3, N_3 = 5$

$N_n = (2n-1)$

So, total no. of elements till $(n-1)$ groups

is $1+3+5+......(2n-3) = (n-1)^2$

total no. of elements till n groups

is $+3+5......2n-1 = n^2$

sum of elements till $(n-1)$ groups is

$1+2+3+4..........(2-1)^2$

=

$$\frac{(n-1)^2\left((n-1)^2+1\right)}{2}$$

Sum of elements till n groups is

$1+2+3+.........n^2 = \frac{n^2(n^2+1)}{2}$

Sum of elements in 12 groups is

Ans – 4 $\frac{12^2(11^2+1)}{2} - \frac{11^2(11^2+1)}{2} = 6238$

Q.26 t^{th}**M**ean $= (r+1)^{th}$ term

$T_{r+1} = x + r.\frac{(2y-x)}{n+1}$

$= \frac{(n+1-r)x + 2xy}{n+1}$

$$T_{r+1} = 2x + r\frac{(y-23)}{n+1}$$

$$\Rightarrow \frac{(n+1-\ell)2x+2y}{n+1}$$

$$\Rightarrow (n+1-r)x + 2xy = (n+1-r)2x + xy$$

$$\Rightarrow (n+1-\ell)x = xy$$

$$\Rightarrow \frac{y}{x} = \frac{x+1}{r} - 1$$

$$\Rightarrow \frac{n+1}{r} - \frac{y}{z} = 1$$

Ans – 1

Q.27 $\left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+........\infty\right)}$

$$= \left(\sqrt{5}\right)^{-2\log_{\sqrt{5}}\left(\frac{1/4}{1-1/2}\right)}$$

$$= \left(\sqrt{5}\right)^{\log_{\sqrt{5}}(1/2)^{-2}}$$

$$= 4$$

Q.28 $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)........\infty$

$$= \frac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{\left(1-\frac{1}{3}\right)}.........\infty$$

$$= \frac{\left(1-\frac{1}{3^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)}{2/3}..........\infty$$

Q.29 $x = \frac{a+b}{2}$

$$y = \sqrt{ab}$$

$$z = \frac{2ab}{a+b}$$

$$y^2 = xz$$

$$z = \frac{x}{9}(\text{given})$$

$$\Rightarrow y^2 = \frac{x^2}{9}$$

$$\Rightarrow \frac{x^2}{y^2} = 9$$

Q.30 $\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{2ab}{a+b}$

$$\Rightarrow a^{n+1} + b^{n+1} + ab\left(b^{n-1} + a^{n-1}\right)$$

$$= 2ab\left(a^{n-1} + b^{n-1}\right)$$

$$\Rightarrow a^{n+1} + b^{n+1} = ab\left(a^{n-1} + b^{n-1}\right)$$

$$\Rightarrow a^n (a-b) = b^n (a-b)$$

$$\Rightarrow a^n = b^n$$

$$\Rightarrow n = 0$$

Q.31 $S = \frac{1}{a^5} + \frac{1}{a^4} + \frac{3}{a^3} + a^8 + a^{10}$

By $AM \geq GM$

$$\frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3 + 8^{10}}$$

$$\Rightarrow \boxed{S \geq 1}$$

SEQUENCE SERIES

EXERCISE – 3

Q.1

Let the numbers be a – 3d, a – d, a + d, a + 3d.

As given $(a-3d)+(a-d)+(a+d)+(a+3d)=24\,\&\,(a-3d)(a-d)(a+d)(a+3d)=945$

Hence a = 6 & $(a^2-9d^2)(a^2-d^2)=945$.

Substituting the value of a gives $d^4-40d^2+39=0$

Therefore $d=\pm 1$ or $\pm\sqrt{39}$

As numbers are integers hence d must be 1 or – 1.

Hence the number are {3, 5, 7, 9}.

Q.2

Let the numbers be a, ar, ar^2 so that $a+ar+ar^2=70\,\&\,4a, 4ar, 4ar^2$ are in A.P.

Now, $4a(1+r^2)=10ar \Rightarrow 2r^2-5r+2=0 \Rightarrow 2$ or $\frac{1}{2}$

From $a+ar+ar^2=70\,\&$ r = 2 gives a = 10, hence numbers are 10, 20, 40.

Q.3

$$S=\frac{2}{3}+\frac{1}{3}+\frac{2}{3^3}+\frac{1}{3^3}+\frac{2}{3^5}+\frac{1}{3^5}+.....\infty \text{ terms} \Rightarrow S=1+\frac{1}{3^2}+\frac{1}{3^4}+......\infty \text{ terms}$$

$$S=\frac{1}{1-\frac{1}{3^2}}=\frac{9}{8}.$$

Q.4

$$S=\frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+\frac{4}{t^4}-\frac{5}{7^6}.....\infty \text{ terms}$$

$$\Rightarrow S=\left(\frac{4}{7}+\frac{4}{7^3}-\frac{4}{7^5}+....\infty \text{ terms}\right)-\left(-\frac{5}{7^2}+\frac{5}{7^4}+\frac{5}{7^6}+....\infty \text{ terms}\right)$$

$$S=\frac{\frac{4}{7}}{1-\frac{1}{7^2}}-\frac{\frac{5}{7^2}}{1-\frac{1}{7^2}}=\frac{23}{48}.$$

Q.5

$$S=1+(1+b)r+(a+b+b^2)r^2+(1+b+b^3)r^3+....\infty \text{ terms}$$

$$\Rightarrow (a-b)S=(1-b)+(1-b^2)r+(1-b^3)r^2+(1-b^4)r^3+....\infty \text{ terms}$$

$$\Rightarrow (1-b)S=(1+r+r^2+r^3+...\infty \text{ terms})-b(1+br+b^2r^2+b^3r^3+...\infty \text{ terms})$$

$$\Rightarrow S=\frac{1}{(1-r)(1-b)}-\frac{b}{(1-br)(1-b)}s$$

Q.6

Let $b=ar, c=ar^2 \,\&\, d=ar^3$

Now, L.H.S. $=\left(ar-ar^2\right)^2+\left(ar^2-a\right)^2+\left(ar^3-ar\right)^2$

$$=a^2r^2(1-r)^2+a^2(1-r)^2(1+r)^2+a^2r^2(1-r)^2(1+r)^2$$

$$=a^2(1-r)^2\left(r^4+2r^3+3r^2+2r+1\right)$$

$$=a^2(1-r)^2\left(r^2+r+1\right)^2$$

$$=\left(a-ar^3\right)^2=(a-d)^2$$

Q.7

$\mathbf{g}=\sqrt{ab} \,\&\, h=\dfrac{2ab}{a+b}\Rightarrow \dfrac{h}{g}=\dfrac{2\sqrt{ab}}{a+b}$

Hence $\dfrac{a+b}{\sqrt{ab}}=\dfrac{13}{6}$. Let $\sqrt{\dfrac{a}{b}}=x+\dfrac{1}{x}=\dfrac{13}{6}$

Therefore $x=\dfrac{2}{3}\Rightarrow \dfrac{a}{b}=\dfrac{4}{9}$.

Q.8

Let $b=ar, c=ar^2 \,\&\, d=ar^3$

Now, $ax^3+bx^2+cx+d=a\left(x^3+rx^2+r^2x^3+r^3\right)=a\left(\dfrac{x^4-r^4}{x-r}\right)$

Also $ax^3+c=a\left(x^2+r^2\right)$

Clearly ax^2+c is a factor of ax^3+bx^2+cx+d

Q.9

Let the numbers be $2, 2+d, 2+2d, \dfrac{(2+2d)^2}{2+d}, 18$

Now, $2+2d, \dfrac{(2+2d)}{2+d}, 18$ are in H.P. hence $\dfrac{(2+2d)^2}{2+d}=\dfrac{2\times(2+2d)\times 18}{(2+2d)+18}$

$\Rightarrow d=-1$ or $\dfrac{1+d}{2+d}=\dfrac{9}{10+d}\Rightarrow d^2+2d-8=0$.

$d=-1\Rightarrow c=0$ but no term of a G>P. may be zero hence $d\neq -1$.

Hence, $d=2$ or -4

$d=2 \Rightarrow a=2, b=4, c=6, d=9, e=18$

$d=-4 \Rightarrow a=2, b=-2, c=-6, d=-18, e=18$.

Q.10

$S_n 1\times 4+2\times 5+3\times 6+4\times 7+....\text{n terms} \Rightarrow t_n = n\times(n+3)$

$$\Rightarrow S_n = \sum_{r=1}^{n} r^2 + 3\times\sum_{r=1}^{n} r$$

$$\Rightarrow S_n = \frac{n(n+1)(2n+1)}{6} + 3\times\frac{n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+5)}{3}$$

Q.11

$S_n = 1\times 2\times 3+2\times 3\times 4+3\times 4\times 5+....\text{n terms} \Rightarrow t_n = n\times(n+1)\times(n+2)$

$$\Rightarrow S_n = \sum_{r=1}^{n} r^3 + 3\times\sum_{r=1}^{n} r^2 + 2\times\sum_{r=1}^{n} r$$

$$\Rightarrow S_n = \frac{n^2(n+1)^2}{4} + 3\times\frac{n(n+1)(2n+1)}{6} + 2\times\frac{n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)(n+2)}{4}$$

Q.12

$$t_n = \frac{1}{(3n-2)(3n+1)} \Rightarrow t_n = \frac{1}{3}\left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)$$

$$\Rightarrow S_n = \frac{1}{3}\sum_{r=1}^{n}\left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$$

$$\Rightarrow S_n = \frac{1}{3}\left(\frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10}\right) + ...+ \frac{1}{3n-2} - \frac{1}{3n+1}$$

$$\Rightarrow S_n = \frac{1}{3}\left(1 - \frac{1}{3n+1}\right) = \frac{n}{3n+1}$$

Q.13 (i)

$t_r = (3r-1)\times(r+1)$

$$\Rightarrow S_n = 3\times\sum_{r=1}^{n} r^2 + 2\times\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1$$

$$\Rightarrow S_n = 3\times\frac{n(n+1)(2n+1)}{6} + 2\times\frac{n(n+1)}{2} - n$$

$$\Rightarrow S_n = \frac{n(2n^2+5n+1)}{2}$$

Q.13 (ii)

$$S_n = \sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$$

$$S_n = \frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{(2r-1)} - \frac{1}{(2r+1)}\right)$$

$$S_n = \frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-....\frac{1}{(2n-1)}-\frac{1}{(2n-1)}\right)$$

$$S_n = \frac{1}{2}\left(1-\frac{1}{(2n+1)}\right) = \frac{n}{2n+1}$$

Q.14

$$S_n = 1.1+2.01+3.001+....\text{n terms}$$

$$S_n = (1+2+3+...\text{n terms})+\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+.....\text{n terms}\right)$$

$$S_n = \frac{n(n+1)}{2}+\frac{(10^n-1)}{9\times 10^n}$$

Q.15

$$\sum_{m=1}^{n}\sum_{r=1}^{m}1 = \sum_{m=1}^{n} m = \frac{n(n+1)}{2}$$

Q.16

$$S_n = 1+|\tan x|+|\tan^2 x|+|\tan^3 x|+...\infty$$

$$S_n = \frac{1}{1-|\tan x|} = \frac{3+\sqrt{3}}{2}$$

$$\Rightarrow |\tan x| = \frac{1+\sqrt{3}}{3+\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan x = K \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \pm\frac{\pi}{6}.$$

Q.17

If side of k^{th} square is a, then side of $(k+1)^{th}$ square will be $\frac{a}{\sqrt{2}}$.

Hence if area of k^{th} square a^2, then area of $(k+1)^{th}$ square will be $\frac{a^2}{2}$.

Now as given, side of 0^{th} square is 1 hence of 1^{st} square will be $\frac{1}{\sqrt{2}}$.

that of second square will be $\frac{1}{2}$, that of third square will be $\frac{1}{2\sqrt{2}}$ ….

so side of k^{th} square will be $\frac{1}{2^{k/2}}$.

Area of k^{th} square will be $\frac{1}{2^k}$.

Sum of areas will be $1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....\infty$ terms

i.e. 2.

Q.18

$S = \sin x + \sin 2x + \sin 3x + ... + \sin nx$

Multiply throughout by $2\sin\frac{x}{2}$ to get

$$\left(2\sin\frac{x}{2}\right)S = 2\sin\frac{x}{2}\sin x + 2\sin\frac{x}{2}\sin 2x + 2\sin\frac{x}{2}\sin 3x + ... + 2\sin\frac{x}{2}\sin nx$$

$$\Rightarrow \left(2\sin\frac{x}{2}\right)S = \cos\frac{x}{2} - \cos\frac{3x}{2} + \cos\frac{3x}{2} - \cos\frac{5x}{2} + \cos\frac{5x}{2} - \cos\frac{7x}{2} + ... + \cos\frac{(2n-1)x}{2}$$

$$\Rightarrow \left(2\sin\frac{x}{2}\right)S = \cos\frac{x}{2} - \cos\frac{(2n+1)x}{2}$$

$$\Rightarrow S = \frac{\cos\frac{x}{2} - \cos\frac{(2n+1)x}{2}}{2\sin\frac{x}{2}} = \frac{\sin\left(\frac{n+1}{2}\right)x\sin\frac{nx}{2}}{\sin\frac{x}{2}}$$

Q.19

$$a + a^2 + a^3 + ...\infty \text{ terms} = \frac{x}{1-x}$$

Differentiate w.r.to x to get

$$1 + 2x + 3x^2 + ...\infty \text{ terms} = \frac{1}{(1-x)^2}$$

Q.20

$$t_n = (2n-1)(2n+1)^2 = 8n^3 + 4n^2 - 2n - 1$$

$$\Rightarrow S_n = 8\times\sum_{r=1}^{n} r^3 + 4\times\sum_{r=1}^{n} r^2 - 2\times\sum_{r=1}^{n} r - n$$

$$\Rightarrow S_n = 8\times\frac{n^2(n+1)^2}{4} + 4\times\frac{n(n+1)(2n+1)}{6} - 2\times\frac{n(n+1)}{2} - n$$

$$\Rightarrow S_n = \frac{1}{3}n\left(6n^3 + 16n^2 + 9n - 4\right)$$

Q.21

$$t_n = \frac{1^3 + 2^3 + 3^3....n \text{ terms}}{1+2+3+....n \text{ terms}} \Rightarrow t_n = \frac{n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{1}{2}\sum_{r=1}^{n} r^2 + \frac{1}{2}\sum_{r=1}^{n} r$$

$$\Rightarrow S_n = \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{6}$$

Q.22

$$t_n = \frac{1}{1+2+3+....n \text{ terms}} \Rightarrow t_n = \frac{2}{n(n+1)}$$

$$\Rightarrow S_n = 2\times\sum_{r=1}^{n}\frac{1}{r(r+1)}$$

$$\Rightarrow S_n = 2\times\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$\Rightarrow S_n = 2\times\left(1-\frac{1}{n+1}\right)=\frac{2n}{n+1}$$

Q.23

$$S_n = 1-\frac{4}{2}+\frac{7}{4}-\frac{10}{8}+....+(-1)^{n-1}\frac{3n-2}{2^{n-1}}$$

$$\frac{1}{2}S_n = \frac{1}{2}-\frac{4}{4}+\frac{7}{8}-....-(-1)^{n-1}\frac{3n-5}{2^{n-1}}+(-1)^{n-1}\frac{3n-2}{2^n}$$

$$\Rightarrow S_n = (-1)^{n-1}\left(\left(\frac{1}{2}\right)^{n-1}+\left(\frac{3n-2}{2^n}\right)\right)$$

Q.24

Total number of terms by the end of n^{th} group $=1+2+3+....+n=\frac{n(n+1)}{2}$

Sum of all the terms till $(n-1)^{th}$ group $=\dfrac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}$

Sum of n^{th} group $=\dfrac{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2}+1\right)}{2}-\dfrac{\frac{n(n-1)}{2}+1}{2}$

$\Rightarrow$ Sum of n^{th} group $=\frac{n(n^2+1)}{2}$

Q.25

Total number of terms by the end of n^{th} group $=1+2+3+....n=\frac{n(n+1)}{2}$

Sum of all the terms till n^{th} group $=\left(\frac{n(n+1)}{2}\right)^2$

Sum of all the terms till $(n-1)^{th}$ group $=\left(\frac{n(n-1)}{2}\right)^2$

Sum of n^{th} group $=\left(\frac{n(n+1)}{2}\right)^2-\left(\frac{n(n-1)}{2}\right)^2$

$\Rightarrow$ Sum of n^{th} group $=n^3$

Q.26

The given number may be written as

$$N = 9\times 8\times 10^2+8\times 10+8\times 10^3+....+8\times 10^{n-1}+4\times 10^n+4\times 10^{n+1}+....+4\times 10^{2n-1}$$

$$\Rightarrow N = 9+\frac{80(10^{n-1}-1)}{9}+\frac{4\times 10^n(10^n-1)}{9}$$

$$\Rightarrow N = \frac{4\times10^{2n} + 4\times10^{n} + 1}{9}$$

$$\Rightarrow N = \left(\frac{2\times10^{n}+1}{3}\right)^2 = \left(\frac{2\underbrace{000...01}_{n-1 digits}}{3}\right)^3 = \left(\underbrace{666...67}_{n1 digits}\right)^2$$

Q.27

$$t_r = \frac{r^2}{r^4+r^2+1} \Rightarrow t_r = \frac{r^2}{(r^2+r+1)(r^2-1+1)}$$

$$\Rightarrow 2t_r = \frac{(r^2r+1)-(r^2+r+1)}{(r^2+r+1)(r^2-r+1)}$$

$$\Rightarrow 2t_r = \frac{1}{(r^1-r+1)} - \frac{1}{(r^2+r+1)}$$

Hence $2\times\sum_{r=1}^{n}\frac{r^2}{r^4+r^2+1} = 1-\frac{1}{3}+\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{13}+....+\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}$

$$\Rightarrow \sum_{r=1}^{n}\frac{r^2}{r^4+r^2+1} = \frac{1}{2} - \frac{1}{2(n^2+n+1)}$$

Q.28

$S = x + 2x^2 + 3x^3 + ... + nx^n$

$xS = \quad x^2 + 2x^2 + ... + (n-1)x^n + nx^{n+1}$

- -

$(1-x)S = x + x^2 + x^3 + ... + x^n - nx^{n+1}$

$$\Rightarrow S = \frac{x(1-x^n)}{(1-x)^2} - \frac{nx^{n+1}}{1-x}$$

$$\Rightarrow (1+x+2x^2+3x^3+....+nx^n)^2 = \left(\frac{x^2-x+1-(n+1)x^{n+1}-nx^{n+2}}{(1-x)^2}\right)^2$$

Expanding the square and considering only those terms in which index of x

Does't exceed n gives $(x^2-x+1)^2(1-x)^{-4}$

Now coefficient of x^n in $(1-x)^{-4}$ will be ${}^{3+r}C_3$

Hence the coefficient of x^n in $(x^4-2x^3+3x^3-2x+1)(1-x)^{-4}$ will be

${}^{n-1}C_3 - 2\times{}^{n}C_3 + 3\times{}^{n+1}C_3 - 2\times{}^{n+2}C_3 + {}^{n+3}C_3$

i.e. $\frac{n(n^2+11)}{6}$

Q.29

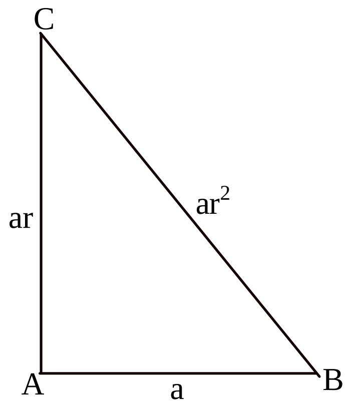
$$S_n = \sum_{r=1}^{n} S_n = \frac{n(n+1)}{2}$$

$$\Rightarrow \sum_{r=1}^{n}\frac{1}{S_r}=\sum_{r=1}^{n}\frac{2}{r(r+1)}$$

$$\Rightarrow \sum_{r=1}^{n}\frac{1}{S_r}=2\times\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$\Rightarrow \sum_{r=1}^{n}\frac{1}{S_r}=2\times\left(1-\frac{1}{n+1}\right)=\frac{2n}{n+1}$$

Q.30



$$\tan B = \frac{ar}{r} = a$$

Also $a^2 + a^2r^2 = a^2r^4$

$\Rightarrow r^4 - r^2 - 1 = 0$

$$\Rightarrow r = \sqrt{\frac{a+\sqrt{5}}{2}}$$

Q.31

$T_1 = \Delta ABC$, where $AB = BC, \angle B = \alpha$, let $\angle BCA = \angle BCA = \alpha_1$, then $\alpha_1 \frac{\pi-\alpha}{2}$

$T_2 \Delta A_1BC$, where $A_1B = A_1C, \angle BA_1C = \alpha_2$, Let $\angle A_1BC = \angle BCA_1 = \alpha_2$, then $\alpha_2 = \frac{\pi-\alpha_1}{2}$.

Hence $\alpha_n = \frac{\pi-\alpha_{n-1}}{2}$. Now Let $\lim\limits_{n\to\infty}\alpha_n = \lim\limits_{n\to\infty}\alpha_{n-1} = \theta$, then $\theta = \frac{\pi-\theta}{2}$ or $\theta = \frac{\pi}{3}$.

As $n\to\infty$, $T_n \to$ an equilateral triangle.

Q.32

$x^4\sin\theta_1 + x^3\sin\theta_2 + ... + \sin\theta_5 = 2 \Rightarrow \left|x^4\sin\theta_1 + x^3\sin\theta_2 + ... + \sin\theta_5\right| = 2$

or $|x|^4|\sin\theta_1| + |x|^3|\sin\theta_2| + ... + |\sin\theta_5| = 2$.

Now for $\theta_i \in \left[-\frac{\pi}{6},\frac{\pi}{6}\right], |\sin\theta_i| \in \left[0,\frac{1}{2}\right]$

$\therefore |x|^4 + |x|^3 + ... + 1 \le 4$ or $\frac{1}{1-|x|} < 4$. Hence $|x| > \frac{3}{4}$.

Q.33

$540 = 3^3 \times 2^2 \times 5$

Sum of all the proper divisors $= \left(1+3+3^2+3^3\right)\left(1+2+2^2\right)(1+5)-1-540$ i.e. 1680.

Q.34

$$\text{Required sum } = \frac{(1+3+5+....+2n-1)^2-\left(1^2+3^2+5^2+....+(2n-1)^2\right)}{2}$$

$$= \frac{n^4-\frac{(4n^2-1)}{3}}{2} \text{ or } \frac{n}{6}\left(3n^3-4n^2+1\right).$$

Q.35

$$\left(\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}\right)+\left(\frac{2}{x^2-1}-\frac{2}{x^2+1}=\frac{4}{x^4-1}\right)+....+\left(\frac{2^n}{x^{2n}-1}-\frac{2^n}{x^{2n}+1}=\frac{2^{n+1}}{x^{2n+1}-1}\right)$$

$$\Rightarrow \frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2+1}-\frac{4}{x^4+1}-....-\frac{2^n}{x^{2^n}+1}=\frac{2^{+ln}}{x^{2^{n+1}}-1}.$$

Hence

$$\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{4}{x^4+1}+....+\frac{2^n}{x^{2n}+1}=\frac{1}{x-1}-\frac{2^{+ln}}{x^{2^{n+1}}-1}$$

Q.36

$$S=\frac{1}{(1-x)(1-x^3)}+\frac{x^2}{(1-x^3)(1-x^5)}+\frac{x^4}{(1-x^5)(1-x^7)}+....+\frac{x^{2n-2}}{(1-x^{2n-1})(1-x^{2n+1})}$$

$$(x^3-x)S=\frac{(1-x)-(1-x^3)}{(1-x)(1-x^3)}+\frac{(1-x^3)-(1-x^5)}{(1-x^3)(1-x^5)}+\frac{(1-x^5)-(1-x^7)}{(1-x^5)(1-x^7)}+....+\frac{(1-x^{2n-1})-(1-x^{2n+1})}{(1-x^{2n-1})(1-x^{2n+1})}$$

Or $(x^3-x)S=\frac{1}{(1-x^3)}-\frac{1}{(1-x)}+\frac{1}{(1-x^5)}-\frac{1}{(1-x^3)}+\frac{1}{(1-x^7)}-\frac{1}{(1-x^5)}+....+\frac{1}{(1-x^{2n+1})}-\frac{1}{(1-x^{2n-1})}$

Or $S=\frac{1}{x^3-x}\left\{\frac{1}{1-x^{2n+1}}-\frac{1}{(1-x)}\right\}.$

Q.37

$A_0A_1=1\Rightarrow AA_1=\cot\theta$

$\therefore A_1A_2=AA_1\sin\theta=\cos\theta,$

$A_2A_3=A_1A_2\cos\theta=\cos^2\theta,$

$A_3A_4=A_2A_3\cos\theta=\cos^3\theta$.....etc.

$\Rightarrow A_0A_1+A_1A_2+A_2A_3+....\infty$ terms $=\frac{1}{1-\cos\theta}$

Hence $1-\cos\theta=\frac{1}{2\left(2+\sqrt{3}\right)}$ or $\cos\theta=\frac{\sqrt{3}}{2}$ i.e. $\theta=\frac{\pi}{3}$.

Q.38

$a_0=a_1=1\ \&\ n_{n+1}=a_{n-1}a_n+1\Rightarrow a_2=2, a_3=3, a_4=6, a_5=19,...$

Hence a_n is even when n is type 3m – 1 & even otherwise.

As 2007 is not of type 3m – 1 hence a_{2007} is not divisible by 4.

Q.39

A.P. $\rightarrow a_1, a_1+2, a_1+4,..., a_1+4n$

G.P. $\rightarrow (a_1+4n), \frac{a_1+4n}{2}, \frac{a_1+4n}{4},..., \frac{a_1+4n}{2^{2n}}$

$a_{n+1}=a_{3n+1}\Rightarrow a_1+2n=\frac{a_1+4n}{2^n}$ or $a_1=\frac{2n\left(2-2^n\right)}{2^n-1}$

Now $a_{2n+1}=a_1+4n\Rightarrow a_{2n+1}=\frac{2n\left(2-2^n\right)}{2^n-1}+4n$

or $a_{2n+2} = \frac{2^{n+1} n}{2^n - 1}$.

Q.40

Given $a \times b = d \times \ell = 192$ & $\frac{AM}{HM} = \frac{(d+\ell)^2}{2(d \times \ell)} = \frac{25}{48}$.

Hence $(d+\ell)^2 = 2 \times \frac{25}{48} \times 192$ or $d + \ell = 52$

$d + \ell = 52$ & $d \times \ell = 192 \Rightarrow d = a$ & $\ell = 48$ {as d must divide ℓ }

Now gcd {a, b} = 4 & lcm {a, b} = 48, hence

(a, b) can be $(4, 48)$ or $(16,12)$.

Q.41

$$S = \sum_{r=1}^{n} a_r b_r + \sum_{r=1}^{n} (a_r - a)^2$$

$$\Rightarrow S = \sum_{r=1}^{n} a_r (1 - a_r) + na^2 + \sum_{r=1}^{n} a_r^{\,2} - 2a \sum_{r=1}^{n} a_r$$

$$\Rightarrow S = \sum_{r=1}^{n} a_r - \sum_{r=1}^{n} a_r^{\,2} + na^2 + \sum_{r=1}^{n} a_r^{\,2} - 2a \sum_{r=1}^{n} a_r$$

$$\Rightarrow S = na - na^2$$

$$\Rightarrow S = na(1-a)$$

$$\Rightarrow S = nab$$

Q.42

(i) $S = \frac{1}{1.3} + \frac{2}{1.3.5} + \frac{3}{1.3.5.7} + \ldots$ upto n terms

$2S = \frac{1.2}{1.3} + \frac{2.2}{1.3.5} + \frac{3.2}{1.3.5.7} + \ldots$ upto n terms

$2S = \frac{3-1}{1.3} + \frac{5-1}{1.3.5} + \frac{7-1}{1.3.5.7} + \ldots.$ upto n terms

$$2S = 1 - \frac{1}{1.3} + \frac{1}{1.3} - \frac{1}{1.3.5} + \frac{1}{1.3.5} - \frac{1}{1.3.5.7} + \ldots + \frac{1}{1.3.5\ldots(2n-1)} - \frac{1}{1.3.5\ldots(2n+1)}$$

$$S = \frac{1}{2}\left(1 - \frac{1}{1.3.5\ldots(2n+1)}\right).$$

(ii) $S = \sum_{r=1}^{n} \frac{2r+1}{r^2 (r+1)^2} \Rightarrow S = \sum_{r=1}^{n} \frac{(r+1)^2 - r^2}{r^2 (r+1)^2}$

or $S = \sum_{r=1}^{n} \frac{1}{r^2} - \sum_{r=1}^{n} \frac{1}{(r+1)^2}$

$\Rightarrow S = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \frac{1}{3^2} + \ldots + \frac{1}{n^2} - \frac{1}{(n+1)^2}$

or $S = 1 - \frac{1}{(n+1)^2}$

(ii) $\frac{1}{x+1} = 1 - \frac{x}{x+1}$,

$$\frac{2x}{(x+1)(x+2)}=\frac{x}{x+1}-\frac{x^2}{(x+1)(x+2)},$$

$$\frac{3x^2}{(x+1)(x+2)(x+3)}=\frac{x^2}{(x+1)(x+2)}-\frac{x^3}{(x+1)(x+2)(x+3)}$$

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$$\frac{1}{x+1}+\frac{2x}{(x+1)(x+2)}+\frac{3x^2}{(x+1)(x+2)(x+3)}+\ldots\text{upto n terms }=1-\frac{x^n}{(x+1)(x+2)\ldots(x+n)}$$

Q.43

$$\frac{a_1}{a+a_1}=1-\frac{1}{a+a_1}$$

$$\frac{a_2}{(1+a_1)(1+a_2)}=\frac{1}{1+a_1}-\frac{1}{(1+a_1)(1+a_2)}$$

$$\frac{a_3}{(1+a_1)(a+a_2)(1+a_3)}=\frac{1}{(1+a_1)(1+a_2)}-\frac{1}{(1+a_1)(1+a_2)(1+a_3)}$$

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$$\frac{a_1}{1+a_1}+\frac{a_2}{(1+a_1)(1+a_2)}+\frac{a_3}{(1+a_1)(1+a_2)(1+a_3)}+\ldots\text{ upto n terms }=1-\frac{1}{(1+a_1)(1+a_2)\ldots(1+a_n)}$$

Q.44

$$\frac{x}{1-x^2}=\frac{x}{1-x}-\frac{x^2}{1-x^2},$$

$$\frac{x^2}{1-x^4}=\frac{x^2}{1-x^2}-\frac{x^4}{1-x^4},$$

$$\frac{x^4}{1-x^8}=\frac{x^4}{1-x^4}-\frac{x^8}{1-x^8}$$

.

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$$\frac{x}{1-x^2}+\frac{x^2}{1-x^4}+\frac{x^4}{1-x^8}+\ldots\ldots\text{upto n terms }=\frac{x}{1-x}-\frac{x^{2^n}}{1-x^{2^n}}$$

Now $\lim_{n\to\infty}\frac{x^{2^n}}{1-x^{2^n}}=0$ as $|x|<1$.

Hence $S_\infty=\frac{x}{1-x}$.

Q.45

$$S=\sum_{r=1}^{n}(2r+1)f(r)=\sum_{r=1}^{n}\left((r+1)^2-r^2\right)f(r)$$

$$f(r)=1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{r}\Rightarrow (x+1)^2 f(r)=(r+1)^2 f(r+1)-(r+1)$$

$$S=\sum_{r=1}^{n}\left(\left(r+1\right)^2 f\left(r+1\right)-r^2 f\left(r\right)\right)-\sum_{r=1}^{n}\left(r+1\right)$$

$$S=\left(n+1\right)^2 f\left(n+1\right)-\left(n+1\right)-\frac{n\left(n+1\right)}{2}$$

$$S=\left(n+1\right)^2 f\left(n\right)-\frac{n\left(n+1\right)}{2}.$$

Q.46

$$S_n=\sum_{r=1}^{n}\tan\frac{x}{2^r}\sec\frac{x}{2^{r-1}}\Rightarrow S_n=\sum_{r=1}^{n}\frac{\sin\frac{x}{2^r}}{\cos\frac{x}{2^r}\cos\frac{x}{2^{r-1}}}$$

$$\Rightarrow S_n=\sum_{r=1}^{n}\frac{\sin\left(\frac{x}{2^{r-1}}-\frac{x}{2^r}\right)}{\cos\frac{x}{2^r}\cos\frac{x}{2^{r-1}}}$$

$$\Rightarrow S_n=\sum_{r=1}^{n}\frac{\sin\frac{x}{2^{r-1}}\cos\frac{x}{2^r}-\cos\frac{x}{2^{r-1}}\sin\frac{x}{2^r}}{\cos\frac{x}{2^r}\cos\frac{x}{2^{r-1}}}$$

$$S_n=\sum_{r=1}^{n}\left(\tan\frac{x}{r^{r-1}}-\tan\frac{x}{2^r}\right)=\tan x-\tan\frac{x}{2^n}$$

Now $\lim\limits_{n\to\infty}S_n=\tan x$.

Q.47

$$S_n=\sum_{r=1}^{n}\mathrm{cosec}\,2^r\theta\Rightarrow S_n\sum_{r=1}^{n}\frac{\sin\left(2^{r-1}\theta\right)}{\sin\left(2^r\theta\right)\sin\left(2^{r-1}\theta\right)}$$

$$\Rightarrow S_n=\sum_{r=1}^{n}\frac{\sin\left(2^r\theta-2^{r-1}\theta\right)}{\sin\left(2^r\theta\right)\sin\left(2^{r-1}\theta\right)}$$

$$\Rightarrow S_n=\sum_{r=1}^{n}\frac{\sin\left(2^r\theta\right)\cos\left(2^{r-1}\theta\right)-\cos\left(2^r\theta\right)\sin\left(2^{r-1}\theta\right)}{\sin\left(2^r\theta\right)\sin\left(2^{r-1}\theta\right)}$$

$$\Rightarrow S_n\sum_{r=1}^{n}\left(\cot\left(2^{r-1}\theta\right)-\cot\left(2^r\theta\right)\right)=\cot\theta-\mathrm{cor}\left(2^n\theta\right).$$

Q.48

$$S=\sum_{r=1}^{n}\frac{\left(r+2\right)}{r\left(r+2\right)2^r}\Rightarrow S=\sum_{r=1}^{n}\left(\frac{2}{1}-\frac{1}{r+1}\right)\frac{1}{2^r}$$

$$\Rightarrow S=\sum_{r=1}^{n}\left(\frac{1}{2^{r-1}r}-\frac{1}{2^r\left(r+1\right)}\right)$$

$$\Rightarrow S=1-\frac{1}{2^n\left(n+1\right)}$$

Q.49

$$S=\sum_{r=1}^{998}\sqrt{1+\frac{1}{(r+1)^2}+\frac{1}{(r+2)^2}}$$

$$\Rightarrow S=\sum_{r=1}^{998}\sqrt{\frac{(r+1)^2(r+2)^2+(r+1)^2+(r+2)^2}{(r+1)^2(r+2)^2}}$$

$$\Rightarrow S=\sum_{r=1}^{998}\sqrt{\frac{(r^2+4r+3)^2}{(r+1)^2(r+2)^2}}$$

$$\Rightarrow S=\sum_{r=1}^{998}\frac{(r+1)(r+2)+1}{(r+1)(r+2)}$$

$$\Rightarrow S=\sum_{r=1}^{998}\left\{1+\frac{1}{r+1}-\frac{1}{r+2}\right\}=998+\frac{1}{2}-\frac{1}{1000}.$$

Q.50

$$S=\sum_{r=1}^{n}\frac{1}{(1+rx)(1+(r+1)x)}\Rightarrow S=\sum_{r=1}^{n}\frac{(1(r+1)x)-(1+rx)}{(1+rx)(1+(r+1)x)}$$

$$\Rightarrow S=\sum_{r=1}^{n}\left\{\frac{1}{1+rx}-\frac{1}{(r+(r+1)x)}\right\}$$

$$\Rightarrow S=\frac{1}{1+x}-\frac{1}{1+(n+1)x}$$

Q.51

$$\sum_{r=1}^{2n}\frac{1}{r^2}=\frac{1}{4}\sum_{r=1}^{n}\frac{1}{r^2}+\sum_{r=1}^{n}\frac{1}{(2r-1)^2}$$

Now $\lim\limits_{n\to\infty}\sum\limits_{n\to\infty}^{2n}\frac{1}{r^2}=\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{r^2}=\frac{\pi^2}{6}$, hence

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{(2r-1)^2}=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{1}{r^2}-\frac{1}{4}\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r^2}=\frac{3}{4}\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r^2}$$

or $\sum\limits_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$.

Q.52

$$\tan\theta=\cot\theta-2\cot 2\theta\Rightarrow\sum_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\sum_{r=1}^{n}\frac{\cot\frac{x}{r^2}-2\cot\frac{2x}{2^r}}{2^r}$$

or $\sum\limits_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\sum\limits_{r=1}^{n}\left\{\frac{\cot\frac{x}{2^r}}{2^r}-\frac{\cot\frac{x}{2^{r-1}}}{2^{r-1}}\right\}$

$$\Rightarrow\sum_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\frac{\cot\frac{x}{2^n}}{2^n}-\cot x$$

Now $\lim_{n\to\infty}\frac{\cot\frac{x}{2^n}}{2^n}=\frac{1}{x}\lim_{n\to\infty}\frac{\frac{x}{2^n}}{\tan\frac{x}{2^n}}=\frac{1}{x}$, hence $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\frac{1}{x}-\cot x$.

Q.53

Let the common difference be a, b, c, then

$$S_1=\frac{n}{2}\{2+(n-1)a\},S_2=\frac{n}{2}\{2+(n-1)b\},S_3=\frac{n}{2}\{1+(n-1)c\}$$

Or $S_1=n+n(n-1)a, S_2=n+n(n-1)b, S_2=n+n(n-1)c$.

$$\frac{S_1-n}{S_2-n}=\frac{a}{c}\ \&\ \frac{S_1-S_2}{S_2-S_2}=\frac{a-b}{b-c}$$

But $\frac{a-b}{b-c}=\frac{a}{c}$ as a, b, c are in H.P.

Hence $\frac{S_1-n}{S_2-n}=\frac{S_1-S_2}{S_2-S_2}$ or $n=\frac{2S_1S_3-S_1S_2-S_2S_3}{S_1-2S_2+S_3}$

Q.54

$$S=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2k}{k^4+k^2+2}\right)\Rightarrow S=\sum_{r=1}^{n}\left(\frac{(k^2k+1)-(k^2-k+1)}{1+(k^2+k+1)(k^2-k+1)}\right)$$

$$\Rightarrow S=\sum_{k=1}^{n}\left\{\tan^{-1}(k^2+k+1)-\tan^{-1}(k^2-k+1)\right\}$$

$$\Rightarrow S=\tan^{-1}(n^2+n+1)-\tan^{-1}1.$$

Now $\frac{8}{\pi}\sum_{k=1}^{\infty}\tan^{-1}\left(\frac{2k}{k^4+k^2+2}\right)=\frac{8}{\pi}\left(\lim_{n\to\infty}\tan^{-1}(n^2+m+1)-\frac{\pi}{4}\right)$ or 2

Q.55

$$S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2n}{3^m(n.3^m+m.3^n)}\Rightarrow S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\frac{3^m}{m}\left(\frac{3^m}{m}+\frac{3^n n}{}\right)}$$

Also $S=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\frac{3^n}{n}\left(\frac{3^m}{m}+\frac{3^n}{n}\right)}$

$$\Rightarrow 2S=\sum_{m=2}^{\infty}\sum_{n=1}^{\infty}\left(\frac{1}{\frac{3^m}{m}\left(\frac{3^m}{m}+\frac{3^n}{n}\right)}+\frac{1}{\frac{3^n}{n}\left(\frac{3^m}{m}+\frac{3^n}{m}\right)}\right)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m}{3^m}\frac{n}{3^n}$$

$$\Rightarrow 2S = \left(\sum_{m=1}^{\infty}\frac{n}{3^n}\right)\left(\sum_{n=1}^{\infty}\frac{m}{3^m}\right) = \left(\sum_{m=1}^{\infty}\frac{n}{3^n}\right)^2$$

Now Let $s = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + ...\infty$ terms

$$\frac{1}{3}s = \frac{1}{3^2} + \frac{2}{3^3} + \frac{3}{3^4} + \frac{4}{3^5} + ...\infty \text{ terms}$$

Subtracting later from former gives

$$\frac{2}{3}s = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + ...\infty \text{ terms} = \frac{1}{2}$$

Hence $s = \frac{3}{4} \Rightarrow s = \frac{9}{32}$

Q.56

$$a_r = r, d_r = r \Rightarrow S_r = \frac{n}{2}\{2r + (n-1)r\}$$

$$\Rightarrow S_r = \frac{n(n+1)r}{2}$$

Now $\sum_{r=1}^{m} S_r = \frac{n(n+1)}{2}\sum_{r=2}^{m} r$

$$\Rightarrow \sum_{r=2}^{m} S_r \frac{n(n+1)m(m+1)}{4}$$

Q.57

$$22a^2 + 25b^2 + 9c^2 - 75ab - 15bc - 45ca = 0$$

$$\Rightarrow \frac{(15a-5b)^2 + (5b-3c)^2 + (3c-15a)^2}{2} = 0$$

$$\Rightarrow b = 3a \;\&\; c = 5a$$

Hence a, b, c are in A.P.

Q.58

$$\beta = \alpha, \gamma = \alpha r^2, \delta = \alpha r^3$$

$$x^2 - 4x + \lambda = 0 \Rightarrow \alpha + \alpha r = 4, \alpha^2 r = \lambda$$

$$x^2 - 64 + \mu = 0 \Rightarrow \alpha r^2 + \alpha r^3 = 64, \alpha^2 r^5 = \mu$$

$$\frac{\alpha r^2+\alpha r^3}{\alpha+\alpha r}=\frac{64}{4}\Rightarrow r=K\pm 4$$

$$\alpha\pm 4\alpha=4\Rightarrow \alpha=\frac{4}{5}\text{or}\frac{4}{3}$$

Hence $(r,\alpha)=\left(4,\frac{4}{5}\right)\text{or}\left(-4\frac{4}{3}\right)$

Now find $\lambda \& \mu$.

Q.59

$$3^x\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^{100}}\right)=5^x\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots+\frac{1}{5^{100}}\right)$$

$$\Rightarrow 3^x\left(\frac{1-\frac{1}{3^{101}}}{1-\frac{1}{3}}\right)=5^x\left(\frac{1-\frac{1}{5^{101}}}{1-\frac{1}{5}}\right)$$

$$\Rightarrow 3^{x-100}\left(\frac{3^{101}-1}{2}\right)=5^{x-100}\left(\frac{5^{101}-1}{4}\right)$$

$$\Rightarrow\left(\frac{5}{3}\right)^{x-100}=\frac{2\left(3^{101}-1\right)}{5^{101}-1}$$

Taking log on both the sides gives

$$\log\left(\frac{5}{3}\right)^{x-100}=\log\frac{2\left(3^{101}-1\right)}{5^{101}-1}$$

$$\Rightarrow(x-100)\log\left(\frac{5}{3}\right)=\log 2\left(3^{101}-1\right)-\log\left(5^{101}-1\right)$$

Q.60

$$a_k=k\,\&\,r_k=\frac{1}{k+1}\Rightarrow S_k=\frac{k}{1-\frac{1}{k+1}}\text{ i.e. }S_k=k+1$$

Now $\sum_{k=1}^{n}S_k=\sum_{k=1}^{n}(k+1)=n+\sum_{k=1}^{n}k$

$$\Rightarrow\sum_{k=1}^{n}S_k=n+\frac{n(n+1)}{2}=\frac{n(n+3)}{2}$$

Q.61

$$t_r = \frac{r \times 2^{r-1}}{(r+1)(r+2)} \Rightarrow t_r \frac{2^r}{r+2} - \frac{2^{r-1}}{r+1}$$

$$\Rightarrow S_n = \frac{3}{2} - \frac{1}{2} + \frac{2^2}{4} - \frac{2}{3} + \frac{2^3}{5} - \frac{2^2}{4} + ... + \frac{2^n}{n+2} - \frac{2^{n-1}}{n+1}$$

$$\Rightarrow S_n = \frac{2^n}{n+2} - \frac{1}{2}$$

Q.62

$$\frac{1}{r(r-1)(r-2)} = \frac{1}{r}\left(\frac{(r-1)-(r-2)}{(r-1)(r-2)}\right)$$

$$= \frac{1}{2}\left(\frac{1}{r} - \frac{2}{r-1} + \frac{1}{r-2}\right)$$

So $\sum_{r=3}^{n} \frac{1}{r^3} < \sum_{r=3}^{n} \frac{1}{r(r-1)(r-2)} = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{n} - \frac{1}{n-1}\right) < \frac{1}{4}$

Now $\sum_{r=1}^{n} \frac{1}{r^3} = 1 + \frac{1}{8} + \sum_{r=3}^{n} \frac{1}{r^3} \Rightarrow \sum_{r=1}^{n} \frac{1}{r^3} < 1 + \frac{1}{8} + \frac{1}{4}$ i.e. $\frac{11}{8}$

Q.63

$$S = \frac{1}{3} + \frac{1.5}{3.7} + \frac{1.5.9.13}{3.7.11.15} + ... + \frac{1.5.9...(4n-3)}{3.7.11...(4n-1)}.$$

$$S = \frac{1}{2}\left[\frac{1.(5-3)}{3} + \frac{1.5(9-7)}{3.7} + \frac{1.5.9.(13-11)}{3.7.11} + \frac{1.5.9.13.(17-15)}{3.7.11.15} + ... + \frac{1.5.9...(4n-3)(4n+1)-(4n-1)}{3.7.11...(4n-1)}\right]$$

$$S = \frac{1}{2}\left[\frac{1.5}{3} - 1 + \frac{1.5.9}{3.7} - \frac{1.5}{3} + \frac{1.5.9.13}{3.7.11} - \frac{1.5.9}{3.7} + \frac{1.5.9.13.17}{3.7.11.15} - \frac{1.5.9.13}{3.7.11} + ... + \frac{1.5.9...(4n+1)}{3.7.11...(4n-1)} \frac{1.5.9...(4n-3)}{3.7.11...(4n-5)}\right]$$

$$S = \frac{1}{2}\left[\frac{1.5.9...(4n+1)}{3.7.11...(4n-1)} - 1\right]$$

Q.64

$$S = 8 + 26 + 54 + 92 + 140 + 198 + ... + t_n$$

$$S = \quad 8 + 26 + 54 + 92 + 410 + ... + t_{n-1} + t_n$$

$$0 = 8 + (18 + 28 + 38 + 48 + 58 + ...(n-1)\text{ terms}) - t_n$$

$$\Rightarrow t_n = 8 + \frac{n-1}{2}\{2 \times 18 + (n-2) \times 10\}$$

$\Rightarrow t_n = 5n^2 + 3n$

$\Rightarrow S = 5\times\sum_{r=1}^{n} r^2 + 3\times\sum_{r=1}^{n} r$

$\Rightarrow S = \frac{5n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}.$

$\Rightarrow S = \frac{n(n+1)(5n+7)}{3}$

Q.66

Let $(x+1)(x+3)(x+5)...(x+99) = a_{50}x^{50} + a_{49}x^{49} + a_{48}x^{48} + ... + a_1x + a_0$

Clearly $a_{50}1$ & sum of the roots $= -\frac{a_{49}}{a_{50}}$

Hence $a_{49} = -(-1-3-5)...-99$

$\Rightarrow a_{49} = 2500$

Q.67

Observe that the series in question can be rewritten as

$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{1319}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{1318}\right)$

or, by factoring out $\frac{1}{2}$ from the later parenthesis

$\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{1319}\right)-\left(-1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{659}\right)$

Which equals $\frac{1}{600}+\frac{1}{661}+\frac{1}{662}+...+\frac{1}{1319}$. Now

$\frac{1}{660}+\frac{1}{1319}=\frac{1979}{660\times1319}, \frac{1}{661}+\frac{1}{1318}=\frac{1979}{661\times1318}, \frac{1}{662}+\frac{1}{1317}=\frac{1979}{662\times1317}$ and so on.

Thus, $\frac{p}{q}$ may be represented as

$\frac{p}{q}=1979\left\{\frac{1}{660\times1319}+\frac{1}{661\times1318}+\frac{1979}{989\times990}\right\}$

That term on the right can in turn be represented as a faction m/n where $n = 660\times661\times662\times...\times1319$. Thus, p must have 1979 as a factor, unless n includes the factors of 1979. But 1979 is a prime number, so n cannot include any factors of 1979, and hence p must be divisible 1979.

By the way, this problem can be generalized primes of the form 3k+2 as if $p/q = 1-1/2+1/3-...-1/(2k)+1(2k+1)$, then 3k+2 divides p.

Q.68

$N = 100010001...1 = 1+10^4+10^8+...n$ terms

$\Rightarrow N = \frac{10^{4n}-1}{9}$. Hence $N = \frac{(10^{2n}-1)(10^{2n}+1)}{9} \Rightarrow N = \frac{\left(\underbrace{99...9}_{2n-1\text{ digits}}\right)\times\left(\underbrace{100...01}_{2n\text{ digits}}\right)}{9}$

$$\Rightarrow N = \left(\underbrace{11...1}_{2n-1 \text{ digits}}\right) \times \left(\underbrace{100...01}_{2n \text{ digits}}\right)$$. Therefore every number of this type is a composite number.

Q.69

Given $x_n = \frac{n-2}{n} x_{n-1} + \frac{1}{n}$. Also $x_0 = 0, x_1 = 1, x_2 = \frac{1}{2}$

Now $x_{n-1} = \frac{n-3}{n-1} x_{n-2} + \frac{1}{n-1} \Rightarrow x_n = \left(\frac{n-2}{n}\right)\left(\frac{n-3}{n-1}\right) x_{n-2} + \frac{1}{n} + \left(\frac{1}{n-1}\right)\left(\frac{n-2}{n}\right)$

$$\Rightarrow x_n = \left(\frac{n-2}{n}\right)\left(\frac{n-3}{n-1}\right)\left(\frac{n-4}{n-2}\right) x_{n-3} + \frac{1}{n} + \left(\frac{1}{n-1}\right)\left(\frac{n-2}{n}\right) + \left(\frac{1}{n-2}\right)\left(\frac{n-2}{n}\right)\left(\frac{n-3}{n-1}\right)$$

Hence $x_n = \frac{2}{n(n-1)} x_2 + \frac{(n-1)+(n-2)+(n-3)...+2}{n(n-1)}$

$\Rightarrow x_n = \frac{1+2+3+...+(n-1)}{n(n-1)} = \frac{1}{2}$. Hence $x_{2009} = \frac{1}{2}$.

Q.70

$f_{n+2} = f_{n+1} + f_n \Rightarrow f_{n+1} - f_{n-1} = f_n$

Now $S = \sum_{n=2}^{\infty} \frac{1}{f_{n-1} \times f_{n+1}} \Rightarrow S = \sum_{n=2}^{\infty} \frac{f_n}{f_{n-1} \times f_n \times f_{n+1}}$

$$\Rightarrow S = \sum_{n=2}^{\infty} \frac{f_{n+1} - f_{n-1}}{f_{n-1} \times f_n \times f_{n+1}} \Rightarrow S = \sum_{n=2}^{\infty}\left(\frac{2}{f_{n-1} \times f_n}\right) - \sum_{n=2}^{\infty}\left(\frac{1}{f_n \times f_{n+1}}\right)$$

Hence $S = \left(\frac{1}{f_1 \times f_2} + \frac{1}{f_2 \times f_3} + \frac{1}{f_3 \times f_4} + ...\infty \text{ terms}\right) - \left(\frac{1}{f_2 \times f_4} + \frac{1}{f_3 \times f_4} + \frac{1}{f_4 \times f_5} + ...\infty \text{ terms}\right)$

$$\Rightarrow S = \frac{1}{f_1 \times f_2} = 1$$