IN CHAPTER EXERCISE 1
SOLUTION

1. **0.5 s**

$5 = 5\sin \pi t,\ \sin \pi t = 1 = \sin \frac{\pi}{2}$

or $t = \frac{1}{2} s = 0.5$ s

2. $\mathbf{x = 4\sin 10\pi t}$

amplitude = 4 cm; frequency , v = 5 Hz

angular frequency, $\omega = 2\pi v = 10\pi$ rad s^{-1}

At t = 0, $0 = a\sin\phi$ or $\phi = 0$

Use $x = a\sin(\omega t + \phi)$

3. **(i)** **8 cm s^{-2}** **(ii)** **4 cm s^{-2}**

(i) acceleration $= \omega^2 A = \frac{4\pi^2}{T^2} A$

$= \frac{4\pi^2}{\pi^2} \times 2\,cm\,s^{-2}$

= 8 cm s^{-2}

(ii) acceleration $= \omega^2 x = \frac{4\pi^2}{T^2} x$

$= \frac{4\pi^2}{\pi^2} \times 1\,cm\,s^{-2}$

= 4 cm s^{-2}

4. **(a)** **0.02 m** **(b)** **4s**

(c) **3.142 × 10^{-2} m s^{-1}** **(d)** **4.94 × 10^{-2} ms^{-2}**

Comparing with $x = A\sin(\omega t + \phi_0)$, we get

(a) A= 0.02 m

(b) $\omega = 0.5\pi = \frac{\pi}{2}; \frac{2\pi}{T} = \frac{\pi}{2}$ or T = 4s

(c) $v_{max} = A\omega = 0.02 \times \frac{\pi}{2} ms^{-1}$ = 0.01× 3.142 ms^{-1} = 3.142 × 10^{-2} ms^{-1}

(d) $a_{max} = \omega^2 A = \frac{\pi^2}{4} \times 0.02\,ms^{-2} = \frac{484 \times 0.02}{49 \times 4} ms^{-2} = 4.94 \times 10^{-2}\ ms^{-2}$

IN CHAPTER EXERCISE 2
SOLUTION

1. **(i)** **4.4×10^{-5} J** **(ii)** **3.3×10^{-5} J** **(iii)** **1.1×10^{-5} J**

Total energy $= \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} \times 0.2 \times \frac{4\pi^2}{36} \times (2 \times 10^{-2})^2 J$

$= 4.4 \times 10^{-5}$ J

Kinetic energy $= \frac{1}{2} m\omega^2 (a^2 - x^2) = \frac{1}{2} \times 0.2 \times \frac{4\pi^2}{36} [4 \times 10^{-4} - 1 \times 10^{-4}] J$

$= 3.3 \times 10^{-5}$ J

Potential energy = $(4.4 \times 10^{-5} - 3.3 \times 10^{-5})$ J

$= 1.1 \times 10^{-5}$ J

2. (i) $\mathbf{\frac{A}{\sqrt{2}}}$ (ii) $\mathbf{\pm \frac{\sqrt{3}}{2} A}$

(i) $\frac{1}{2} k(A^2 - x^2) = \frac{1}{2} kx^2$

(ii) when $v = \frac{1}{2} v_{max}$

$K.E. = \frac{1}{4} (K.E.)_{max} = \frac{1}{8} kA^2$

$\therefore$ $\frac{1}{2} k(A^2 - x^2) = \frac{1}{8} kA^2$

3. **(a)** **$0.314\ ms^{-1}$** **(b)** **0.1 J** **(c)** **0.1 J** **(d)** **0.083 J**

(a) $v_{max} = A\omega = 2.5 \times 10^{-2} \times 2 \times \frac{22}{7} \times 2\ ms^{-1} = 0.314 ms^{-1}$

(b) $E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} mv_{max}^2 = \frac{1}{2} \times 2 \times 0.314 J = 0.1$ J

(c) Maximum potential energy = 0.1 J

(d) Kinetic energy $= \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} \times 2 \left(2 \times \frac{22}{7} \times 2\right)^2 [(2.5 \times 10^{-2})^2 - (1 \times 10^{-2})^2]$

$= 0.083$ J

4. (a) $v = \omega\sqrt{A^2 - x^2}$

or $v^2 = \omega^2(A^2 - x^2)$

Now, $0.03^2 = \omega^2(A^2 - 0.04^2)$

and $0.04^2 = \omega^2(A^2 - 0.03^2)$

On simplification, A = 0.05 m and ω = rad s^{-1}

Time period, $T = \frac{2\pi}{\omega} = 2\pi s$ = 2 × 3.142 s = 6.284 s

(b) $\text{Energy} = \frac{1}{2}mA^2\omega^2$

$= \frac{1}{2} \times 50 \times 10^{-3} \times 0.05 \times 0.05 \times 1 \times 1 J$

= 6.25 × 10^{-5} J

IN CHAPTER EXERCISE 3
SOLUTION

1. **8.5 s**

$T = 2\pi\sqrt{\frac{L}{g}}$, $\frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$

Required time period = $\sqrt{\frac{10}{1.6}} \times 3.4$

2. The time period is independent of the mass of bob.

3. Due to electric force of attraction between the bob and the plate, the effective value of g shall increase. Since $T = 2\pi\sqrt{\frac{l}{g}}$ therefore T shall decrease.

4. **0.16 ms^{-1}**

$v = A\omega = 0.05 \times \frac{2\pi}{2}$

5. **$0.02\pi\ ms^{-1}$, $0.02\pi^2 ms^{-2}$**

Time period = time taken in one oscillation = 2 s

$v = A\omega = 2 \times \frac{2\pi}{2}$

$$a = A\omega^2 = \frac{2}{100} \times \left(\frac{2\pi}{2}\right)^2$$

IN CHAPTER EXERCISE 4
SOLUTION

1. **3.33 rad s^{-1}**

$$\omega = \sqrt{\frac{k}{m}}$$

2. $\mathbf{2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$

The equivalent force constant is $\frac{k_1 k_2}{k_1 + k_2}$

3. **0.54 s**

Effective force constant = 40 Nm^{-1}

Time period, $T = 2\pi\sqrt{\frac{0.3}{40}}s = 0.54$ s

4. **(a)** **0.31 s** **(b)** **20 ms^{-2}** **(c)** **1.0 ms^{-1}**

(a) $T = 2\pi\sqrt{\frac{m}{k}}$

(b) max. $acc. = \frac{k}{m}A$

(c) max. velocity = $A\sqrt{\frac{k}{m}}$

5. **1.1 × 10^2 N m^{-1}, 36 kg**

$$T = 2\pi\sqrt{\frac{m}{2k}};\ k = \frac{2\pi^2 m}{T^2}$$

Here m = 12 kg and T = 1.5 s

After the block has been placed on the tray, mass is (M + 12)kg.

Now, $T = 2\pi\sqrt{\frac{M+12}{2k}}$

EXERCISE 1

1. A	2. C	3. D	4. A	5. D	6. D
7. C 8.	D 9.	C 10.	C 11.	D 12.	D 13
C 14. B	15. C	16. B	17. A	18. C	19. B
20. C	21. B	22. D	23. A	24. B	25. A
26. C	27. C	28. B	29. (B)	30. (A)	31. (D)
32. (C)	33. (B)	34. (C)	35. (B)	36. (C)	37. (B)
38. (A)	39. (A)	40. (A)	41. (C)	42. (C)	43. (B)
44. (B)	45. (D)				

SOLUTION

1. **(A)**

$$v = A \times 2\pi \cos\left(2\pi t + \frac{\pi}{3}\right)$$

$\Rightarrow$ $$v = v_{max.} \Rightarrow \cos\left(2\pi t + \frac{\pi}{3}\right) = \pm 1$$

$\Rightarrow$ $$\left(2\pi t + \frac{\pi}{3}\right) = \pi$$

$t = 1/3$

2. **(C)**

$$a = A\omega^2$$

$$v = A\omega \Rightarrow \omega = \frac{v}{A}$$

$$a = A \times \frac{v^2}{A^2}$$

$$a = \frac{v^2}{A} \rightarrow \text{'A' doubled} \rightarrow \text{'a' halved}$$

3. **(D)**

$P.E._{min}$ at mean position = 5 J

T.E. = 9 J

max K.E. = 4 J

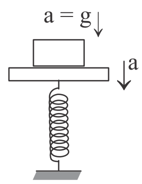
$$\frac{1}{2} mA^2\omega^2 = 4J$$

$\Rightarrow$ $\omega = 200$

$$T = \frac{2\pi}{200} = \frac{\pi}{100} \text{sec.}$$

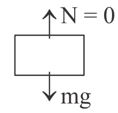
4. **(A)**

max. acceleration of plank should not exceed g



$$A\omega^2 = g$$

$$A = 10/\pi^2$$



a = g (when block leaves contact)

5. **(D)**

Particle starts from mean position.

$\Rightarrow \quad x = A\sin(\omega t)$

at t = 1

$$x_1 = A\sin\left(\frac{2\pi}{8}\times 1\right)$$

$$x_1 = \frac{A}{\sqrt{2}}$$

at t = 2

$$x_2 = A$$

distance covered in 1st second $= \frac{A}{\sqrt{2}}$

distance covered in 2nd second $= A - \frac{A}{\sqrt{2}}$

ratio $= \frac{1/\sqrt{2}}{1-\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

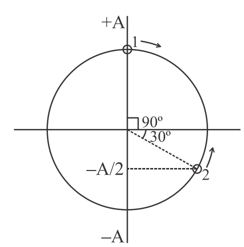
6. **(D)**

Circular representation at t = 0

Phase difference = 2π / 3

Phase covered by each particle = π / 3

Time taken $= \frac{T}{360}\times 60 = \frac{T}{6}$



7. **(C)**

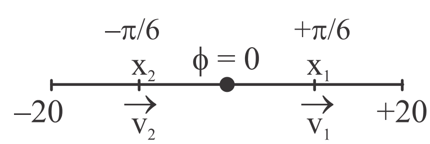
For max distance $V_{res.} = 0$

$v_1 = v_2$

$\Rightarrow \quad x_1 = x_2$

$x_1 + x_2 = 20$ cm

$x_1 = 10$ cm



Phase difference between x = 0 and x = $x_1 \Rightarrow \phi = \sin^{-1}\left(\frac{x_1}{A}\right)$

$$= \frac{\pi}{6}$$

Phase difference between x = 0 and x = $x_2 \Rightarrow \phi = -\frac{\pi}{6}$

Phase difference between x_1 and $x_2 = \pi / 3$

8. **(D)**

Let v_0 is maximum velocity of each particle.

When particles are on opposite sides of x = 0, let their phase by $+\alpha$ & $-\alpha$ ($v_{P_1} = v_{B_1} = v_0 \cos\alpha = 1.2$)

When they cross each other let the phase be β,

$$v_{P_2} = v_{B_2} = v_0 \cos\beta = 1.6$$

Phase travelled by Q is $\alpha + \beta$

Phase travelled by P is $\left(\frac{\pi}{2} - \alpha + \frac{\pi}{2} - \beta\right)$

(Since 'P' goes to one extreme then comes back to cross Q)
since angular frequency is same, phase moved would also be same.

$$\alpha + \beta = \frac{\pi}{2} - \alpha + \frac{\pi}{2} - \beta$$

$$\alpha + \beta = \frac{\pi}{2}$$

$$v_{P_2} = v_0 \cos\left(\frac{\pi}{2} - \alpha\right) = 1.6 = v_0 \sin\alpha$$

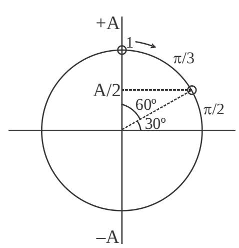
$$v_0 = \sqrt{(1.2)^2 + (1.6)^2}$$

$$v_0 = 2 \text{ m/s}$$

9. **(C)**

From extreme to x = a/2
phase covered = $\pi/3$
time taken = T/6

$$< v > = \frac{a/2}{T/6} = \frac{3a}{T}$$



10. **(C)**

K.E. at D = $\frac{1}{4}$ max K.E.

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{4} \times m\omega^2 A^2$$

$$x_{(CD)} = \frac{\sqrt{3}\,'A'}{2}$$

AE = 2A = 2R

A = 2R

BD = 2CD

$$= 2\times\frac{\sqrt{3}}{2}R = \sqrt{3}R$$

11. **(D)**

Let $v = A\omega\cos(\omega t)$

$a = -A\omega^2\sin(\omega t)$

$$\frac{v^2}{A^2\omega^2}+\frac{a^2}{A^2\omega^4}=1$$

$v^2 = -\frac{1}{\omega^2}a^2 + A^2\omega^2$ Straight line with '–ve' slope

12. **(D)**

Phase moved in T/8 $\phi = \frac{2\pi}{T}\times\frac{T}{8} = \frac{\pi}{4}$

$x = a\sin(\omega t)$

$$x = \frac{a}{\sqrt{2}}$$

13. **(C)**

$\omega^2 x = \omega\sqrt{A^2 - x^2}$
$x = 1$, $A = 2$
$\omega^2\times 1 = \sqrt{4-1}$
$\omega = \sqrt{3}$

frequency $f = \frac{\omega}{2\pi} = \frac{\sqrt{3}}{2\pi}$

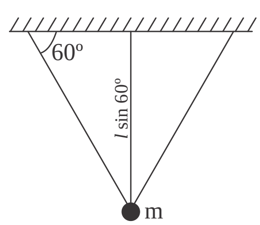
14. **(B)**
K in parallel = 2k
K in series = k/2

15. **(C)**
$g_{el.} = (g + a)$ when the elevator accelerates up.

$T_P = 2\pi\sqrt{\frac{L}{(g+a)}}$ $T_S = 2\pi\sqrt{\frac{M}{k}}$

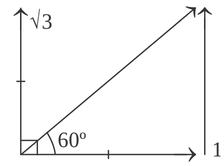
T_P - downwards, T_s - same

16. **(B)**



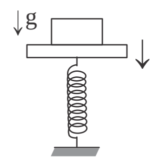
$$T = 2\pi\sqrt{\frac{L\sin 60^\circ}{g}}$$

17. (A)



$$y = \sin(\omega t) + \sqrt{3}\cos(\omega t)$$

$$y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$$



$$\underset{\text{(max. accl.)}}{A\omega^2} = g \qquad \omega = \sqrt{\frac{g}{A}}$$

for maximum acceleration

$$y \to \max \Rightarrow \sin\left(\omega t + \frac{\pi}{3}\right) = 1$$

$$\omega t + \frac{\pi}{3} = \frac{\pi}{2}$$

$$\omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

18. (C)

$$T = 2\pi\sqrt{\frac{I}{mga}}$$

$$= 2\pi\sqrt{\frac{\frac{3}{2}mR^2}{mgR}}$$

$$= 2\pi\sqrt{\frac{3R}{2g}}$$

$$= 2\pi\sqrt{\frac{3}{g}}$$

For equivalent length of simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = 3$$

19. (B)

$$U = -ax^2 + bx^4$$

for equilibrium (mean position)

$$F = -\frac{du}{dx} = 0 = +2ax - 4bx^3 = 0$$

$$x = 0,\ x = \sqrt{\frac{2a}{4b}}$$

let y be the displacement from mean position

$$F = 2ax - 4bx^3$$

putting $x = \left(\sqrt{\frac{a}{2b}} + y\right)$

$$F = 2a\left(\sqrt{\frac{a}{2b}} + y\right) - 4b\left(\sqrt{\frac{a}{2b}} + y\right)^3$$

$$= \left(\sqrt{\frac{a}{2b}} + y\right)\left\{2a - 4b\left(\frac{a}{2b} + y^2 + 2\times\sqrt{\frac{a}{2b}}\times y\right)\right\}$$

$$= \left(\sqrt{\frac{a}{2b}} + y\right)\left(2a - 4b\times\frac{a}{2b} - 4by^2 - 4b.2\sqrt{\frac{a}{2b}}.y\right)$$

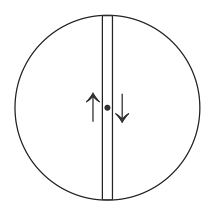
$$= -2\times\frac{a}{2b}.y\times 4b - 4b.2\sqrt{\frac{a}{2b}}y^2 \to 0$$

$$F_m = -4ay$$

$$\omega = \sqrt{\frac{4a}{m}}$$

20. **(C)**

Particle executes SHM of amplitude 'R'. Initially they collide at the centre since their time periods are same $\left(\sqrt{\frac{GM}{R^3}} = \omega\right)$



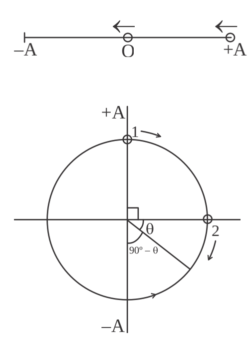
$$2mR\omega - mR\omega = 3mA\omega$$

$$\frac{R}{3} = A \quad (A \to \text{ new amplitude})$$

21. **(B)**

Suppose collision occurs at θ

Phase covered by 1 is $\phi_1 = \frac{\pi}{2} + \theta$



Phase covered by 2 is $\phi_2 = \frac{\pi}{2} + \frac{\pi}{2} - \theta$

$$\phi_1 = \phi_2 \text{ (T-same)}$$

$$\frac{\pi}{2} + \theta = \frac{\pi}{2} + \frac{\pi}{2} - \theta$$

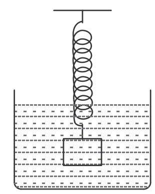
$$\theta = \frac{\pi}{4}$$

phase $\phi_1 = \frac{\pi}{2} + \theta = \frac{3\pi}{4}$

time taken $= \frac{T}{2\pi} \times \frac{3\pi}{4} = \frac{3T}{8}$

22. **(D)**

$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T' = 2\pi\sqrt{\frac{m}{k}}$$

$T' = T$

23. **(A)**

$$g_{elevator} = (g + a)$$

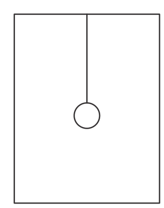
$$T_2 = 2\pi\sqrt{\frac{L}{g+a}}$$

$$T_1 > T_2$$

24. **(B)**

$$x = A\sin(\omega t)$$

$$x = A\sin\left(\frac{2\pi}{T} \times \frac{T}{12}\right)$$



$x = A / 2$

$$\frac{K.E.}{P.E.} = \frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2 x^2} = \frac{3}{1}$$

25. **(A)**

$$v_{max_1} = A\omega \qquad (\omega \to \text{constant})$$

$$v_{max_2} = 2A\omega = 2v$$

26. **(C)**

$$t = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{\mu}{k}} = 1\text{sec}$$

27. **(C)**

$$y_1 = \sin\left(\omega + \frac{\pi}{3}\right)$$

$$y_2 = \sin(\omega t)$$

Phaser $A_{max} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{1+1+2\times\frac{1}{2}}$

$A_{max} = \sqrt{3}$

28. **(B)**

$y_1 = A\sin\omega t$

$y_2 = A\cos\omega t$

$y_1 + y_2 = \sqrt{2}\,A\sin\left(\omega t + \frac{\pi}{4}\right)$

energy $= \frac{1}{2}m\omega^2\left(\sqrt{2}A\right)^2$

$= m\omega^2A^2$

30. **(A)**

Let $x = A\sin(\omega t)$ $a = -A\omega^2$

$\frac{da}{dt} = -A\omega^3\cos(\omega t)$

for max $\frac{da}{dt} \Rightarrow \cos\omega t = \pm 1$

at x $= 0$

for min $\frac{da}{dt} \Rightarrow \cos\omega t = 0$

$x = \pm A$

31. **(D)**

centre of mass falls as water comes out, then suddenly amount regainits original position as total of water goes out.

32. **(C)**

$T = 2\pi\sqrt{\frac{\rho L}{\sigma g}} = 2\pi\sqrt{\frac{L_1}{g}}$

$T = 2\pi\sqrt{\frac{mg}{\sigma Ag}}$

$\sigma \rightarrow$ density of liquid

$\rho \rightarrow$ density of solid

$\rho_x Ag = \sigma L_1 Ag$

$$\frac{\rho L}{\sigma} = L_1$$

33. **(B)**

atteractive force b/w change and metal plate, g_{eff} increases.

34. **(C)**

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2) \qquad \text{T.E.} = \frac{1}{2}m\omega^2A^2 = E$$

at x = A/2

$$\text{K.E.} = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$$

$$= \frac{1}{2}m\omega^2A^2 \times \frac{3}{4} = \frac{3E}{4}$$

35. **(B)**

$$k_1l_1 = k_2l_2 = k\,l_{total}$$

$$k_1 \times \frac{l}{4} = k \times l$$

$$k_1 = 4k$$

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \text{new } T' = 2\pi\sqrt{\frac{m}{4k}} = \frac{T}{2}$$

36. **(C)**

$$\omega^2 = \frac{mgx}{\frac{mL^2}{l^2} + mx^2}$$

for minimum T $\Rightarrow$ ω is maximum

$$\Rightarrow \quad \frac{d\omega^2}{dx} = 0 \Rightarrow mg\frac{\left[\left(\frac{mL^2}{l^2} + mx^2\right) - x.2mx\right]}{\left(\frac{mL^2}{l^2} + mx^2\right)^2} = 0$$

$$x = \frac{L}{\sqrt{12}} = \frac{L}{2\sqrt{3}}$$

37. **(B)**

Super position of two SHM's in the same direction will be another SHM if their frequencies are equal. Resultant equation of option (B) is $y = 5\sin\left(\omega t + \tan^{-1}\frac{3}{4}\right)$

38. (A)

$$y = 10\cos\left(2\pi t + \frac{\pi}{6}\right)$$

$$\frac{dy}{dt} = -20\pi\sin\left(2\pi t + \frac{\pi}{6}\right)$$

at t = 1/6

$$v_p = -20\pi\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$$

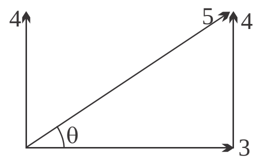
$$= -0.628 \text{ m/s}$$

39. (A)

$$y_1 = 3\sin(\omega t)$$

$$y_2 = 4\sin\left(\omega t + \frac{\pi}{2}\right) + 3\sin(\omega t)$$

using phasor method



$\rightarrow [\sin(\omega t)]$

$$y_2 = 5\sin\left(\omega t + \tan^{-1}\frac{4}{3}\right)$$

phase difference $\phi = \tan^{-1}\left(\frac{4}{3}\right)$

40. (A)

If particle motion starts from extreme

$$x = A\cos(\omega t)$$

at $\omega t = \pi / 6$

$$x = A\cos\left(\frac{\pi}{6}\right)$$

$$V = V_{max}\sin(\omega t)$$

$$\omega t = \pi / 6$$

$$V = V_{max}\sin\left(\frac{\pi}{6}\right)$$

$$V = \frac{V_{max}}{2}$$

$$P = m v$$

$$= \frac{mV_{max}}{2} - 0 = \frac{\sqrt{m2E}}{2} = \sqrt{\frac{mE}{2}}$$

41. (C)

$$\omega_1 = \frac{2\pi}{3} \qquad \omega_2 = \frac{2\pi}{5}$$

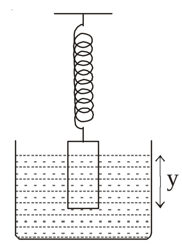
Relative $\omega = \omega_1 - \omega_2$

time taken to come back in same phase $\left(t = \dfrac{2\pi}{\omega_{rel}}\right)$

$$t_1 = \frac{2\pi}{\left(\frac{2\pi}{3} - \frac{2\pi}{5}\right)} = \frac{15}{5-3} = 7.5\text{s}$$

42. **(C)**

Let the spring is further extended by y when the cylinder is given small downward push. Then the restoring forces on the spring are,



(i) Ky due to elastic properties of spring

(ii) upthrust = $yAdg$ = weight of liquid displaced

$\therefore$ Total restoring force $= (K + Adg)\,y$

$\therefore$ $M \times a = -(K + Adg)y$

Comparing with $a = -\omega^2 y$, we get

$$\omega^2 = \left(\frac{K + Adg}{M}\right) \text{ or } \omega = \sqrt{\frac{K + Adg}{M}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K + Adg}{M}}\ .$$

43. **(B)**

Maximum tension in the string is at lowest position.

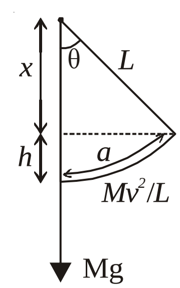
Therefore $T = Mg + \dfrac{Mv^2}{L}$

To find the velocity v at the lowest point of the path, we apply law of conservation of energy i.e.

$$\frac{1}{2}Mv^2 = Mgh = MgL(1-\cos\theta) \qquad [\because \ h = L - x, h = L - L\cos\theta]$$

or $v^2 = 2gL(1-\cos\theta)$

or $v = \sqrt{2gL(1-\cos\theta)}$



$\therefore$ $T = Mg + 2Mg(1-\cos\theta)$

$$T = Mg\left[1 + 2 \times 2\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$T = Mg\left[1 + 4\left(\frac{\theta}{2}\right)^2\right] \qquad [\because \ \sin(\theta/2) = \theta/2 \text{ for small amplitudes}]$$

$$T = Mg[1 + \theta^2]$$

From figure $\theta = \dfrac{a}{L}$ $\therefore$ $T = Mg\left[1 + \left(\dfrac{a}{L}\right)^2\right]$.

44. **(B)**

The small block oscillates along the inclined plane with an amplitude A. As a result the centre of mass of the system undergoes SHM along the horizontal direction:

$$x_{cm} = \frac{mA\sin\omega t}{m+M}\cos 60^o = \frac{1}{2}\frac{m}{m+M}A\sin\omega t$$

The acceleration of the C.M. is $a_{cm} = -\omega^2 x_{cm}$, along the horizontal while the net horizontal force is $=(M+m)a_{cm}$, which is equal to the force of friction acting on it.

45. **(D)**

When the spring undergoes displacement in the downward direction it completes one half oscillation while it completes another half oscillation in the upward direction. The total time period is:

$$T = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$$

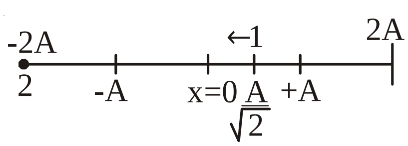
EXERCISE 2

ONE OR MORE THAN ONE OPTION MAY BE CORRECT

1. C	2. C	3. B	4. A, B, C, D
5. B, C, D	6. B, C, D	7. A, B	8. A, B, C
9. B, D	10. D	11. C, D	12. B, C, D
13. A, B, C	14. C	15. B, D	16. A, B, C, D
17. B	18. A, C	19. A, C	20. B, C

SOLUTION

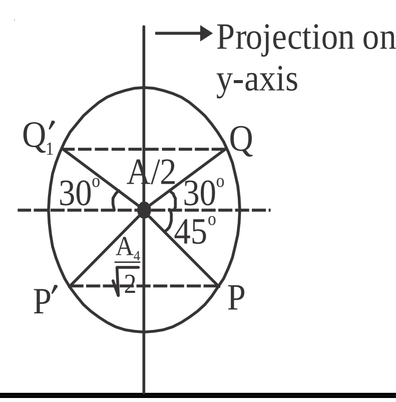
1. **(C)**



Phase dift of first particle from mean position is 45^0.
Phase dift of 2^{nd} particle from mean position is 90^0.
Total phase difference = 90 + 45 = 135

2. **(C)**

Possible angles b/w PQ, P'Q, PQ', P'Q' are 75^0, 165^0, 285^0, 195^0 & 135^0 is not possible.



3. **(B)**

$v^2 = 108 - 9x^2$

$v^2 = 9(12 - x^2)$

$v^2 = \omega^2(A^2 - x^2)$

$\omega = 3$

amplitude $A = \pm\sqrt{12}$

acceleration $a = -\omega^2 x$

at x = 3

$a = -9 \times 0.03$

$= -0.27\ m/s^2$

SHM about $x = 0$

4. **(A, B, C, D)**

The block loses contact with plank when the plank is at its amplitude

acceleration of block $a_b = g$ $(\because\ N = 0)$

acceleration of plank $a_p = a\omega^2$

to just leane $A\omega^2 = g$

the contact $\omega^2 = \dfrac{10}{40 \times 10^{-2}}$

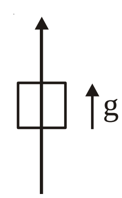
$\omega = 5\, rad / sec$

$T = \dfrac{2\pi}{5}$

at lowest point of SHM.

upward acceleration of block = acceleration of plank

$= A\omega^2 = g$



N = 2mg

at half waydown acceleration of block $= \dfrac{g}{2}\uparrow$

$N = mg + \dfrac{mg}{2} = \dfrac{3}{2}mg$

At mean position, velocity in maximum a = 0

N = mg

5. **(B, C, D)**

$v = \omega\sqrt{A^2 - y^2}$

also $v = \dfrac{dy}{dt}$ $v = 0$ at $t = \dfrac{T}{2}$

$a = -\omega^2 y = max$ at $t = T$

$F = ma = 0$ at $t = \dfrac{3}{4}T$

at $t = \frac{T}{2}$ $v = 0 \Rightarrow$ K.E. =0

$\Rightarrow$ P.E. = T.E.

6. **(B, C, D)**

$U = 5x^2 - 20x$

$F = \frac{-dv}{dx} = -10 + 20 = -10(x-2)$

$k = 10$

F = 0 at x = 2 (mean position)

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{0.1}{10}}$

$T = \frac{\pi}{5}$

7. **(A, B)**

$x = \frac{A}{2} = A\sin(\omega t)$

$\omega t = \frac{\pi}{6} \quad \Rightarrow \quad t = \frac{T}{12}$

$v = v_0\cos(\omega t) = \sqrt{3}\frac{v_0}{2}$

$a = a_0\sin(\omega t) = \frac{-a_0}{2}$

8. **(A, B, C)**

$\frac{1}{2}m\omega^2A^2 = KE_{max}$

$\frac{1}{2}m\omega^2(A^2 - x^2) = 0.64 \times KE_{max}$

$A^2 - x^2 = 0.64A^2$

$x^2 = 0.36A^2$

x = 0.6 A = 6 cm

$K.E. = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

at $x = 5 = \frac{A}{2}$

$$K.E. = \frac{3}{4}\ \frac{1}{2}m\omega^2A^2 = \frac{3}{4}\ \text{max P.E.}$$

9. **(B, D)**

x = 3 sin 100 t + 8 $\cos^2$ 50 t

= 3 sin 100 t + 4 + 4 cos 100 t

$x = 5\sin(100t + \phi) + 4 \rightarrow$ SHM

Amplitude = 5

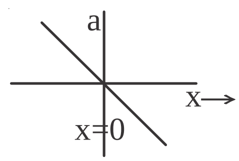
maximum x = 5 + 4 = 9

10. **(D)**

$a = -\omega^2 x$

Slope = $-\omega^2$

straight line



11. **(C, D)**

$$\frac{x}{a} = \sin(\omega t)$$

$$\left(1 - \frac{y}{a}\right) = \cos(\omega t)$$

$$\frac{x^2}{a^2} + \left(1 - \frac{y}{a}\right)^2 = 1 \qquad \Rightarrow \text{uniform circle}$$

$$v_x = \frac{dx}{dt} = a\omega\cos(\omega t)$$

$$v_4 = \frac{dy}{dt} = a\omega\sin(\omega t)$$

$$v = \sqrt{v_x^2 + v_4^2} = \text{constant}$$

distance α time

12. **(B, C, D)**

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1 \qquad \text{ellipse}$$

$$a = -\omega^2 x \qquad \rightarrow \text{straight line}$$

$$\frac{a^2}{A^2\omega^4} + \frac{v^2}{A^2\omega^2} = 1 \qquad \rightarrow \text{ellipse}$$

13. **(A, B, C)**

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

at $x = 0$ $\quad v = A\omega = 1.0$

at v = 0 $\quad x = A = 2.5$

$\omega = 4$

$T = \frac{2\pi}{4} = 1.575$

$a = \omega^2 A$

$= 40\ cm/s^2$

$v = \omega\sqrt{A^2 - x^2}$

$v = 4\sqrt{(2.5)^2 - (1)^2}$

$= 4\sqrt{5.25}$

$= 2\sqrt{21}$

14. **(C)**

$y = A(1 + \cos 2\omega t)$

$y = A(2\sin \omega t + \phi)$

$\frac{A_1}{A_2} = \frac{2}{1}$

$\frac{V_1}{V_2} = \frac{A_1\omega_1}{A_2\omega_2} = \frac{A \times 2\omega}{2A \times \omega}$

$\frac{a_1}{a_2} = \frac{\omega_1^2 A_1}{\omega_2^2 A_2} = \frac{2}{1}$

15. **(B, D)**

Let $x = A\sin(\omega t + \phi)$

at t = 0

$x = +\frac{A}{2}$

$\phi = \frac{\pi}{6}, \frac{5\pi}{6}$

also $v = -v_0 = v_0\cos(\omega t + \phi)$

$\phi = \frac{5\pi}{6}$

$x = A\sin\left(\omega t + \frac{5\pi}{6}\right)$

$x = A\sin\left(\omega t + \frac{\pi}{2} + \frac{\pi}{3}\right)$

$= A\cos\left(\omega t + \frac{\pi}{3}\right)$

16. **(A, B, C, D)**
for equilibrium
kx = mg
x = 1 cm
if released from natural length
A = 2x = 2 cm

$$f = 2\pi\sqrt{\frac{m}{k}} \approx 5$$

frequency doesn't depend on value of g.

17. **(B)**
The block has v_0 at equilibrium

$$A = \frac{v_0}{\omega_0}$$

$$x = \frac{v_0}{\omega_0}\sin(\omega_0 t)$$

initial phase is zero
since the block is moving is +ve direction.

18. **(A, C)**
Distance of mean position from water level = immersed length
= maximum amplitude for equilibrium

$$\rho \times 60 \times a \times g = 3\rho L a g$$

maximum amplitude = L = immersed length = 20 cm

$$T = 2\pi\sqrt{\frac{m}{3\rho a g}}$$

19. **(A, C)**

Average total energy = $\frac{1}{2}m\omega^2A^2$

= maximum K.E.

root mean square velocity = $\frac{v_0}{\sqrt{2}}$

mean velocity = 0

20. **(B, C)**

Average KE = $\frac{1}{4}m\omega^2A^2$ = Average P.E.

$$\omega = 2\pi f$$

$$KE = \frac{1}{2}m\omega^2A^2\cos^2(\omega t) = \frac{1}{2}m\omega^2A^2\left(\frac{1+\cos 2\omega t}{2}\right)$$

f_{KE} = 2 f

EXERCISE 3

Comprehension - I

1. **(B)**

in experiment I

frequency = no. of oscillations/sec

$$= \frac{20}{60} / s$$

$$= \frac{1}{3} Hz$$

2. **(C)**

frequency is independent of amplitude

3. **(B)**

frequency is also independent of mass

4. **(D)**

particle stops at extreme so it drops vertically.

Comprehension-II

5. **(B)**

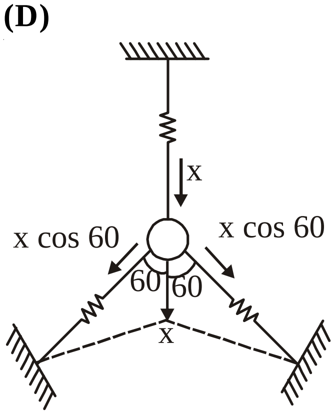
Spring cut into 3 equal parts then spring constant of each part becomes 3k in parallel

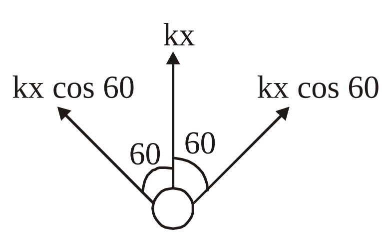
$$k_{eff} = k_1 + k_2 + k_3 = 9k$$

$$T' = 2\pi\sqrt{\frac{m}{9k}}$$

$$T' = \frac{T}{3}$$

6. **(D)**





$$F_{net} = kx + 2kx\cos^2 60$$

$= \frac{3}{2}kx$

$k_{eff} = \frac{3}{2}k$

7. **(C)**

k_{eff} in series = k

k_{eff} in parallel = 9k

<u>Comprehension-III</u>

8. **(D)**

When spring of 2k displaces x, spring of k displaces by 2s (torque balanced about mid point)

mid point displacesby $\frac{3x}{2} = y_0$

$x = \frac{2y_0}{3}$

$F_{net} = 2kx + k2x$

$= 4\ kx$

$F_{net} = \frac{4k2y_0}{3} = \frac{8k}{3}y_0$

energy stored = $\frac{1}{2}\left(\frac{8k}{3}\right)(y_0)^2$

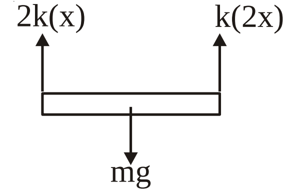
$= \frac{4k}{3}y_0^2$

9. **(A)**

$$T = 2\pi\sqrt{\frac{m}{\left(\frac{8k}{3}\right)}}$$

10. **(B)**

$$\frac{w_{external}}{w_{gravity}} = \frac{w_{gravity} - w_{spring}}{w_{gravity}}$$



$$= \frac{\frac{3m3x}{2} - \frac{1}{2}k(2x)^2 + \frac{1}{2}2kx^2}{\frac{mg3x}{2}}$$

putting mg = 4kx $= \frac{1}{2}$

Comprehension-IV

11. **(A)**
Total energy remains constant

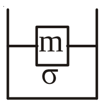
12. **(D)**
$d = A\sin(\omega t)$

$t = \frac{1}{\omega}\sin^{-1}\left(\frac{d}{A}\right)$

13. **(B)**
$v = \omega\sqrt{A^2 - x^2}$
at $x = 0$ $v \rightarrow$ maximum
$x = \pm A$ $v = 0$

Match the Column

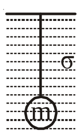
14. **(A) → P, R, (B)→ R, (C) → P, Q, (D) → P, Q**



$F_{res} = \sigma g(Ax)$

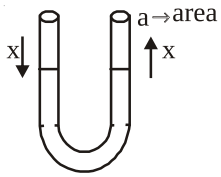
$T = 2\pi\sqrt{\frac{m}{\sigma A g}}$

(B)→ R



$F_{res} = (mg - \sigma v g)\frac{x}{L}$

(C) → P, Q
Liquid will behave as a point mass
(D) → P, Q



$F_{res} = \rho g(2x)a$

$F_{res} = (\rho g 2a)x$

$T = 2\pi\sqrt{\frac{m}{(\rho g 2a)}} = 2\pi\sqrt{\frac{L}{2g}}$

15. **A→ Q, B→R, C→P, D→P**

$T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

$g_{eff} = |\bar{g} - \bar{a}|$

(A) $g_{eff} = \dfrac{3g}{2}$

(B) $g_{eff} = \dfrac{g}{2}$

(C) $g_{eff} = \sqrt{g^2 + \left(\sqrt{3}g\right)^2}$

(d) $g_{eff} = \dfrac{GM}{R^2} = \dfrac{\dfrac{GM}{2}}{\left(\dfrac{R}{2}\right)^2} = 2\left(\dfrac{GM}{R^2}\right) = 2g$

16. **(A) $\rightarrow$ R, (B) $\rightarrow$ S, (C) $\rightarrow$ P, (D) $\rightarrow$ P**

17. **(A) $\rightarrow$ P, Q, (B) $\rightarrow$ P, Q, (C) $\rightarrow$ S, (D) $\rightarrow$ R**

18. **A$\rightarrow$Q, B$\rightarrow$P, C$\rightarrow$R, D$\rightarrow$ S**

$y = A\sin(\omega t)$

$v = A\omega\cos(\omega t)$

(A) $K.E. = \dfrac{1}{2}mv^2 \rightarrow$ max at $t = 0$

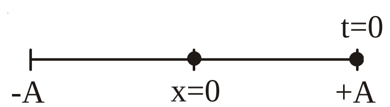
(B) PE = min at t = 0

EXERCISE 4

1. $x = 0.2\cos 5\pi t$

Time period $T = \dfrac{2\pi}{5\pi} = 0.4s$

Particle is at x = 0.2 at t = 0



from x = + A to x = 0

it takes 0.1s

Total distance convered in 0.7 s is

s = 7 × A = 7 × 0.2 = 1.4 m

average speed $< v > = \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{1.4}{0.7} = 2m/s$

2. From the given graph

$x = -\dfrac{\beta}{\alpha}x$

comparing with

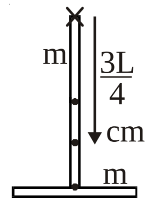
$a = -\omega^2 x$

$\omega = \sqrt{\frac{\beta}{\alpha}}$

frequency $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

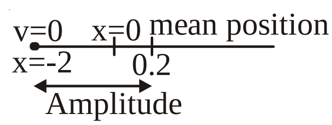
3. $T = 2\pi\sqrt{mgd}$

$= 2\pi\sqrt{\frac{\left(\frac{mL^2}{12} + mL^2 + \frac{mL^2}{3}\right)}{\left(2m \times \frac{3L}{4}\right)}}$



$= 2\pi\sqrt{\frac{17L}{18g}}$

4. F = –10 x + 2
F = – 10 (x – 0.2)
k =10
m = 0.1 kg



$\sqrt{\frac{10}{0.1}}$ = 10 rad/s Time period = $\frac{2\pi}{10}$s

mean position at x = 0.2
Amplitude A = + 2 + 0.2 = 2.2m
equation since particle starts from extreme

$x - 0.2 = -2.2\cos\omega t$

$x = -2.2\cos\omega t + 0.2$

5. $U = x^2 - 4x + 3$

(i) $F = -\frac{dv}{dx} - 2x + 4$

= –2(x –2) (SHM) equilibrium position at x = + 2

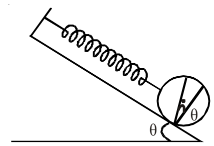
(ii) $T = 2\pi\sqrt{\frac{1}{2}} = 2\pi$

(iii) $V = A\omega$

$2\sqrt{6} = A \times \sqrt{2}$, $2\sqrt{3m} = A$

6. Water doesn't roll as the cylinder so it is treated as point mass a.
about constant poit

$\tau_{restoring} = -k(R\theta)R$



$I\alpha = -kR^2\theta$

$(2MR^2 + \underset{\text{water as point mass}}{\underset{\downarrow}{mR^2}})\alpha = -kR^2\theta$

$\omega^2 = \dfrac{k}{2M+m}$

when water becomes ice (neglecting change in volume)
ice behaves as solid cylinder

$I\alpha = -kR^2\theta$

$\left(2MR^2 + \dfrac{3}{2}mR^2\right)\alpha = -kR^2\theta$

$\omega = \sqrt{\dfrac{k}{2M + \dfrac{3}{2}m}}$

7. **(a)** For small amplitude, the two blocks oscillate together. The angular frequency is

$$\omega = \sqrt{\frac{k}{M+m}}$$

and so the time period $T = 2\pi\sqrt{\dfrac{M+m}{k}}$.

(b) The acceleration of the blocks at displacement x from the mean position is

$$a = -\omega^2 x = \left(\frac{-kx}{M+m}\right)$$

The resultant force on the upper block is, therefore,

$$ma = \left(\frac{-mkx}{M+m}\right)$$

This force is provided by the friction of the lower block.

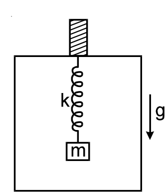
Hence, the magnitude of the frictional force is $\left(\dfrac{mk|x|}{M+m}\right)$

(c) Maximum force of friction required for simple harmonic motion of the upper block is $\dfrac{mkA}{M+m}$ at the extreme positions. But the maximum frictional force can only be μ mg. Hence

$$\frac{mkA}{M+m} = \mu\, mg$$

or, $\quad A = \dfrac{\mu(M+m)g}{k}$

8. When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx which should balance the weight of the block.

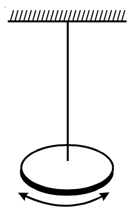


Thus, x = mg/k. As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a psuedo force mg upward on the block.

This force will 'balance' the weight. Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by x = mg/k, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is mg/k.

9. The situation is shown in figure. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\,kg)(5.0\times10^{-2}m)^2}{2} = 2.5\times10^{-4}\ kg\text{-}m^2.$$



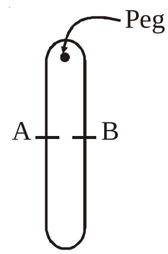
The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

or, $$C = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(2.5\times10^{-4}kg-m^2)}{(0.20s)^2} = 0.25\ \frac{kg-m^2}{s^2}.$$

10. If the string is displaced slightly downward by x, we can write,the net (restoring)force

$$= (\mu x - 2\mu x)2g$$
$$= -2\mu xg$$



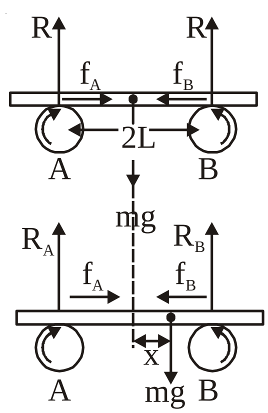
$$\therefore\ (5\mu\ell)\cdot\ddot{x} = -2\mu xg$$

or $$\ddot{x} = -\frac{2g}{5\ell}\cdot x$$

$$\therefore\ \omega = \sqrt{\frac{2g}{5\ell}}$$

or $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{5\ell}{g}}$$

11. When the plank is situated symmetrically on the drums, the reactions on the plank from the drums will be equal and so the force of friction will be equal in magnitude but opposite in direction and hence, the plank will be in equilibrium along vertical as well as in horizontal direction.



Now if the plank is displaced by x to the right, the reaction will not be equal. For vertical equilibrium of the plank

$R_A + R_B = mg$ ...(i)

And for rotational of plank, taking moment about center of mass we have

$R_A(L+x) = R_B(L-x)$ ...(ii)

Solving Eqns. (i) and (ii), we get

$$R_A = mg\left(\frac{L-x}{2L}\right)$$

and $$R_B = mg\left(\frac{L+x}{2L}\right)$$

Now as $f = \mu R$, so friction at B will be more than at A and will bring the plank back, i.e., restoring force here

$$F = -(f_B - f_A) = -\mu(R_B - R_A) = -\mu\frac{mg}{L}x$$

As the restoring force is linear, the motion will be simple harmonic motion with force constant

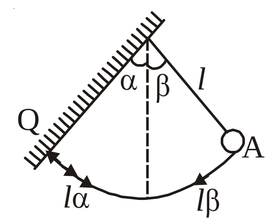
$$k = \frac{\mu mg}{L}$$

So that $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{\mu g}}$.

12. **(a)** If $\alpha > \beta$, the ball does not collide with the wall and it performs full oscillations like a simple pendulum.

$\Rightarrow$ period $= 2\pi\sqrt{\frac{\ell}{g}}$

(b) If $\alpha < \beta$, the ball collides with the wall and rebounds with same speed. The motion of ball from A to Q is one part of a simple pendulum.



time period of ball $= 2(t_{AQ})$.
Consider A as the starting point (t = 0)
Equation of motion is $x(t) = A\cos\omega t$

$x(t) = \ell\beta\cos\omega t,$

$x(t) = \ell\beta\cos\omega t,$ because amplitude $= A = \ell\beta$

time from A to Q is the time t when x becomes $-\ell\alpha$

$\Rightarrow \quad -\ell\alpha = \ell\beta\cos\omega t$

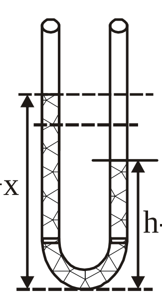
$\Rightarrow \quad t = t_{AQ} = 1/\omega\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$

The return path from Q to A will involve the same time interval.
Hence time period of ball $= 2t_{AQ}$

$$= \frac{2}{\omega}\cos^{-1}\left(-\frac{\alpha}{\beta}\right) \Rightarrow 2\sqrt{\frac{\ell}{g}}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

$$= 2\pi\sqrt{\frac{\ell}{g}} - 2\sqrt{\frac{\ell}{g}}\cos^{-1}\left(\frac{\alpha}{\beta}\right)$$

13. Suppose that the liquid is displaced slightly from equilibrium so that its level rises in one arm of the tube, while it is depressed in the second arm by the same amount, x.
If the density of the liquid is ρ, then, the total mechanical energy of the liquid column is :



$$E = \frac{1}{2}\left\{A(h+x)\rho + A(h-x)\rho\right\}.\left(\frac{dx}{dt}\right)^2$$

$$+\left[A(h+x)\rho\cdot g\cdot\frac{h+x}{2} + A(h-x)\cdot\rho\cdot g\cdot\frac{h-x}{2}\right]$$

$$= \frac{1}{2}(2Ah\rho)\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}(2A\rho g)(h^2 + x^2) \qquad \text{(i)}$$

After differentiating the total energy and equating it to zero, one finds acceleration

$= -\omega^2 x$

The angular frequency of small oscillations, ω, is:

$$\omega = \sqrt{\frac{2A\rho g}{2Ah\rho}} = \sqrt{\frac{g}{h}} \qquad \text{(ii)}$$

14. Suppose that the plank is displaced from its equilibrium position by x at time t, the centre of the cylinder is, therefore, displaced by $\frac{x}{2}$

$\therefore$ the mechanical energy of the system is given by,

$E = K.E.\ E = K.E.$ (Plank) + P.E.(spring) + K.E. (cylinder)

$$E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 + \frac{1}{2}2m\left\{\frac{d}{dt}\left(\frac{x}{2}\right)\right\}^2 + \frac{1}{2}\left(\frac{1}{2}2m.R^2\right)\left\{\frac{1}{R}\frac{d}{dt}\left(\frac{x}{2}\right)\right\}^2$$

$$= \frac{1}{2}\left(\frac{7}{4}m\right)\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2$$

After differentiating the total energy and equating it to zero, one finds acceleration $-\omega^2 x$

The angular frequency, $\omega = \sqrt{\frac{4k}{7m}}$

15. Suppose that the particle is displaced from its equilibrium position at O, and that its x-coordinate at time t is given by x.

The total energy of the particle at time t is given by,

$$E = \frac{1}{2}m\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} + mgy \qquad \text{(i)}$$

Differentiating the equation of the curve, we get,

$$2x\frac{dx}{dt} = 4a\frac{dy}{dt}$$

$$\therefore \quad E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2\left[1 + \frac{x^2}{4a^2}\right] + \frac{mg}{4a}x^2$$

$\because$ $\because$ The oscillations are very small, both x and $\frac{dx}{dt}$ are small. We ignore terms which are of higher order than quadratic terms in x or, $\frac{dx}{dt}$ or, mixed terms.

$$\therefore \quad E \simeq \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\left(\frac{mg}{2a}\right)x^2 \qquad \text{(ii)}$$

After differentiating the total energy and equating it to zero, one finds acceleration = $-\omega^2 x$

The angular frequency of small oscillations is, consequently,

$$\omega = \sqrt{\frac{mg}{2a.m}} = \sqrt{\frac{g}{2a}} \qquad \text{(iii)}$$

16. At equilibrium the net force on the cylinder is zero in the vertical direction:

$F_{net} = B - W = 0$, $B \equiv$ the buoyancy and $W \equiv$ the weight of the cylinder.

When the cylinder is depressed slightly by x, the buoyancy increases from B to $B + \delta B$ where:

$\delta B = |x|\rho_\ell Ag$

while the weight W remains the same.

$\therefore$ the net force, $F'_{net} = B + \delta B - W$

$= \delta B = |x|\rho_\ell Ag$

The equation of motion is, therefore, $\rho_s Ah\dfrac{d^2x}{dt^2} = -x\rho_\ell Ag$

the minus sign takes into account the fact that x and restoring force are in opposite directions.

$\therefore \quad \dfrac{d^2x}{dt^2} = -x\dfrac{\rho_\ell g}{\rho_s h}$

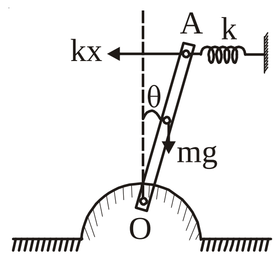
and the angular frequency, ω, is

$$\omega = \sqrt{\frac{g\rho_\ell}{h\rho_s}}$$

17. Suppose that the rod is displaced by a small angle θ as shown in the figure. The total mechanical energy of the system is given by,

$$E = \frac{1}{3}m\ell^2\dot{\theta}^2 - mg\frac{\ell}{2}(1-\cos\theta) + \frac{1}{2}k(\ell\theta)^2$$

$$= \frac{1}{3}m\ell^2\dot{\theta}^2 + \frac{1}{2}\left(k\ell^2 - \frac{mg\ell}{2}\right)\theta^2 \qquad \text{(i)}$$



$\therefore$ the angular frequency of small oscillations is,

$$\omega = \sqrt{\frac{k\ell^2 - \frac{mg\ell}{2}}{\frac{1}{3}m\ell^2}} = \sqrt{\frac{3k}{m} - \frac{3g}{2\ell}} \qquad \text{(ii)}$$

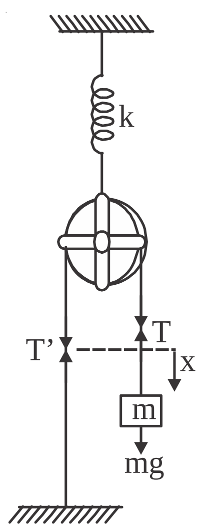
The condition for the system to be oscillation is,

$$\frac{3k}{m} > \frac{3g}{2\ell} \qquad \text{or,} \qquad k > \frac{mg}{2\ell} \qquad \text{(iii)}$$

18. Suppose that the block is depressed by x. The pulley (owing to the constraint) is depressed by $\dfrac{x}{2}$. Suppose that the tension in the string are $T \,\&\, T'$ on both sides. We can write:

For block: $mg - T = m\ddot{x}$ ...(i)

For pulley: $T + T' + mg - k(x + x_0) = m\dfrac{\ddot{x}}{2}$ … (ii)



The angular acceleration of the pulley, $\alpha = \dfrac{\ddot{x}/2}{R}$ … (iii)

$(T - T')\cdot R = I\cdot\dfrac{\ddot{x}}{2R}$ … (iv)

From (i), (ii), (iii) and (iv) we get,

$$3mg - k(x + x_0) = \left(\frac{5m}{2} + \frac{I}{2R^2}\right)\ddot{x} \qquad \text{… (v)}$$

The frequency of small oscillation,

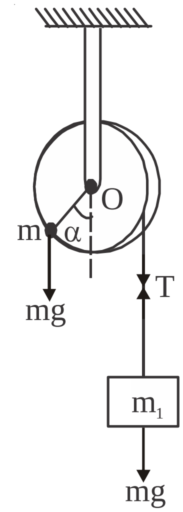
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{5m}{2} + \frac{I}{2R^2}}}$$

19. **(a)** At equilibrium, the net torque on the pulley is zero.

$m_1 g \cdot R = mg \cdot R\sin\alpha$ … (i)

or, $\sin\alpha = \frac{m_1}{m}$

or, $\alpha = \sin^{-1}\frac{m_1}{m}$ … (ii)



(b) If the system is displaced slightly from the equilibrium position, it oscillates. Suppose that the position of the particle is given by the angular variable θ, at some instant. The total mechanical energy is given by:

$E = K.E. + P.E.$

where, $K.E. = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 + \frac{1}{2}(mR^2)\dot{\theta}^2 + \frac{1}{2}m_1 R^2\dot{\theta}^2$

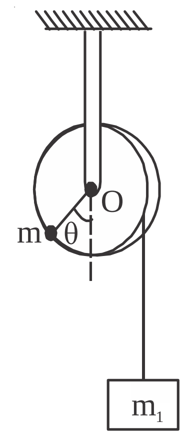
and, P.E.= loss in P.E. of m_1 + gain in P.E. of m

$= -m_1 gR(\theta - \alpha) + mgR(\cos\alpha - \cos\theta)$

$= -m_1 gR(\delta\theta) + 2mgR\sin\frac{\alpha+\theta}{2}\sin\frac{\theta-\alpha}{2}$

$= -m_1 gR\delta\theta + 2mgR\sin\left(\alpha + \frac{\delta\theta}{2}\right)\sin\frac{\delta\theta}{2}$

$\simeq -m_1 gR\cdot\delta\theta + 2mgR\cdot\frac{\delta\theta}{2}\cdot\sin\alpha + 2mgR\cos\alpha\left(\frac{\delta\theta}{2}\right)^2$



where $\delta\theta$ is defined by the expression : $\theta = \alpha + \delta\theta$, $\delta\theta$ being a small quantity. Since the frequency depends only on terms which are quadratic in $\delta\theta$, we can write,

$E = \frac{1}{2}\left(\frac{1}{2}M + m + m_1\right)R^2\dot{\theta}^2 + \frac{1}{2}mgR\cos\alpha(\delta\theta)^2$ + terms linear in $\delta\theta$ or, constants.

After differentiating the total energy and equating it to zero, one finds acceleration = $-\omega^2 x$

$\therefore$ the angular frequency, $\omega = \sqrt{\frac{mgR\cos\alpha}{\left(\frac{1}{2}M + m + m_1\right)R^2}}$

and the frequency, $f = \frac{1}{2\pi}\sqrt{\frac{mg\cos\alpha}{\left(\frac{1}{2}M + m + m_1\right)R}}$.

20. **(a)** Since the system is in equilibrium, we can write the tension in the string, T as:

$$T = m_1\omega_0^2 r$$

and, $T = m_2 g$

$\therefore$ $m_1\omega_0^2 r = m_2 g$ … (i)

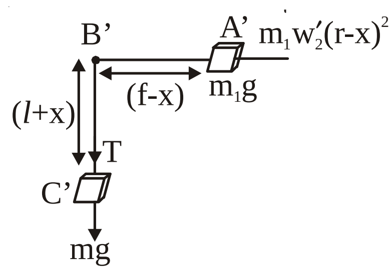
(b) Suppose that the block m_2 is depressed by x. The radius of the circle of rotation is now given by,

$$r' = r - x.$$

and the angular speed ω' is given by,

$$m_1 r^2\omega_0 = m_1(r-x)^2\omega'$$

or, $\omega' = \frac{\omega_0 r^2}{(r-x)^2}$ (ii)



The free body diagram as well as the geometry of the problem are as shown in the adjacent figure.

$$m_1\frac{d^2}{dt^2}(r-x) = m_1\omega'^2(r-x) - T \quad \text{(iii)}$$

$$m_2\frac{d^2}{dt^2}(\ell + x) = m_2 g - T \quad \text{(iv)}$$

The first term on the RHS of the equation (iii) can be rewritten as,

$$m_1\omega'^2(r-x) = \frac{m_1\omega_0^2 r^4}{(r-x)^3} = m_1\omega_0^2 r\left(1 - \frac{x}{r}\right)^{-3}$$

$\simeq m_1\omega_0^2 r\left(1 + \frac{3x}{r}\right)$ (after binomial expansion and assuming $x << r$)

Equation (iii) and (iv) become

$$-m_1\ddot{x} = m_1\omega_0^2 r\left(1 + \frac{3x}{r}\right) - T$$

$$-m_2\ddot{x} = T - m_2 g.$$

Adding, $-(m_1 + m_2)\ddot{x} = m_1\omega_0^2 r\left(1 + \frac{3x}{r}\right) - m_2 g$

$\therefore \qquad \ddot{x} = -\dfrac{3m_1\omega_0^2}{m_1 + m_2}x \qquad$ (v)

Thus the angular frequency of small oscillations, ω, is given by,

$$\omega = \omega_0\sqrt{\frac{3m_1}{m_1 + m_2}} \qquad \text{(vi)}$$

EXERCISE 5

1. **(A)**

Potential energy $\qquad V(x) = k\,|\,x\,|^3$

By the conservation of energy, $\frac{1}{2}mv^2 + kx^3 = ka^3$ (in the region $x > 0$)

$$v = \sqrt{\frac{2k}{m}(a^3 - x^3)}$$

$$\frac{dx}{dt} - \sqrt{\frac{2k}{m}(a^3 - x^3)}$$

$$\sqrt{\frac{m}{2k}}\int_a^0 \frac{dx}{\sqrt{a^3 - x^3}} = \int_0^t dt$$

Substituting $x = a\sin^2\theta$

$$\sqrt{\frac{m}{2k}} \times \int_{\frac{\pi}{2}}^{0} \frac{2a\sin\theta\cos\theta d\theta}{a^{3/2}\sqrt{1 - \sin^6\theta}} = -t$$

$$t = -\frac{1}{\sqrt{a}}\sqrt{\frac{m}{k}}\int_{\frac{\pi}{2}}^{0} f(\theta)\,d\theta$$

$$T \propto \frac{1}{\sqrt{a}}$$

2. **(D)**

$U_x = k(1 - e^{-x^2})$

$$F = -\frac{dU}{dx} = ke^{-x^2}.(-2x)$$

$$= -k2xe^{-x^2}$$

$F \propto (-x)$ for $x \rightarrow$ small

3. **(A), (C)**

$$y_1 = a\sin\theta t;\; y_2 = a\sin\left(\omega t + \frac{\pi}{4}\right);\; y_3 = a\sin\left(\omega t - \frac{\pi}{4}\right)$$

$y_1 + y_2 + y_3 = a\left(1+\sqrt{2}\right)\sin\omega t$

Energy $\propto$ (amplitude)^2

Energy $\propto a^2(1+\sqrt{2})^2 = a^2(3+2\sqrt{2})$

4. **(A)**

Effective value of $g : g' = g\cos\alpha$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$

5. $\dfrac{F}{A} = \dfrac{Y\Delta L}{L}$

$F = \dfrac{YA\,\Delta L}{L}$

Frequency $f = \dfrac{1}{2\pi}\sqrt{\dfrac{YA}{LM}}$

6. **(A)**

From $0 \to A/2$

Phase covered as $\dfrac{\pi}{6}$

From $0 \to A$

Phase covered as $\dfrac{\pi}{2}$

$T_1 = \dfrac{\pi}{6\omega}$

$T_2 = \dfrac{\frac{\pi}{2} - \frac{\pi}{6}}{\omega}$

$\dfrac{T_2}{T_1} = 2$

$T_2 > T_1$

7. **(A)**

$x = A\cos\omega t$. At t = 0, the particle is at the extreme position.

At the extreme position, the potential energy is maximum and displacement is maximum.

8. $m_1x_1 = m_2x_2 = (m_1 + m_2)v_0t$

$x_2 = v_0t + \dfrac{Am_1}{m_2}(1-\cos\omega t)$

$v_1 = v_0 - A\omega \sin(\omega t)$

When $v_1 = v_0$

$\sin \omega t = 0$

$\cos \omega t = \pm 1$

$$x_2 = v_0 t + \frac{Am_1}{m_2} \times 2$$

$x_1 = v_0 t - 2A$

$$x_2 - x_1 = 2l_0 = 2A\left(1 + \frac{m_1}{m_2}\right)$$

$$l_0 = A\left(\frac{m_1 + m_2}{m_2}\right)$$

9. **(B)**

$$\omega = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of P

for P friction provides $F_{resortoring}$

$F_F = m\omega^2 A$

$$= m\frac{k}{2m}A = \frac{kA}{2}$$

10. **(B)**

We have, $y = kt^2, \dfrac{d^2y}{dt^2} = 2k = 2m/s^2$

$$\therefore \qquad \frac{T_1^2}{T_2^2} = \frac{g+2}{g} = \frac{6}{5}$$

11. Speed of block at $y^* = \omega\sqrt{A^2 - y^2}$

Height h attaind by block after detachment = $\dfrac{\omega^2(A^2 - y^{*2})}{2g}$

Total height attached by the block $H = \dfrac{\omega^2(A^2 - y^{*2})}{2g} + y^*$

For H to be maximum, $\dfrac{dH}{dy^*} = 0$

$\Rightarrow \qquad y^* = g/\omega^2$

12. **(B), (C)**

If $A \neq B$ and $C \neq 0$ then $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ (Not SHM)

If $A = B$ and $C = 2B$ then $x = B + 2B\sin\omega t\cos\omega t = B + B\sin 2\omega t$ (SHM)

If $A = -B$ and $C = 2B$ then $x = B\cos 2\omega t + B\sin 2\omega t$ (SHM)

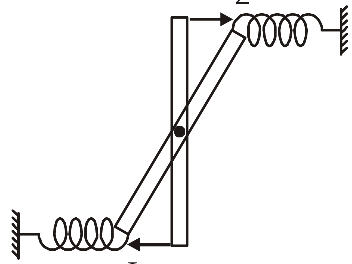
If $A = B$ and $C = 0$ then $x = A$ (Not SHM)

13. **(C)**

$$\tau_{res} = 2 \times k\frac{L}{2}(\theta) \times \left(\frac{L}{2}\right)$$

$$= \frac{kL^2}{2}\theta$$

$$\omega = \sqrt{\frac{\frac{KL^2}{2}}{\frac{ML^2}{12}}} = \sqrt{\frac{6k}{M}}$$



14. **(D)**

$$k_{eff} = \frac{k_1k_2}{k_1 + k_2}$$

$$k_{eff} \times A = k_1 \times A_1$$

$$A_1 = \frac{k_2A}{k_1 + k_2}$$

15. **(D)**

$x = A\sin(\omega t)$ $\quad T = 8s$

$t = \frac{4}{3}$, $\quad x = 1\sin\left(\frac{\pi}{4}t\right)$, $\quad \omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$$x = 1\sin\left(\frac{\pi}{4} \times \frac{4}{3}\right)$$

$$x = \frac{\sqrt{3}}{2}$$

Acceleration $a = -\omega^2 x$

$$= -\left(\frac{\pi}{4}\right)^2 \frac{\sqrt{3}}{2}$$

$$= -\frac{\pi^2\sqrt{3}}{32}\text{cm}/\text{s}^2$$

16. **(C)**

At first only left spring (k) is compressed by x

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 = \frac{1}{2}4ky^2$$

$$\frac{x}{2} = y$$

$$\frac{y}{x} = \frac{1}{2}$$

COMPREHENSION TYPE QUESTIONS

PASSAGE-1 (QNO 17 TO 19)

17. **(C)**

Energy should not exceed V_0

$$V_0 > E > 0$$

18. **(B)**

$$\frac{1}{2}mv^2 = \alpha(A^4 - x^4)$$

$$v = \sqrt{\frac{2\alpha}{m}}\sqrt{A^4 - x^4}$$

$$\int_0^x \frac{dx}{\sqrt{A^4 - x^4}} = \sqrt{\frac{2\alpha}{m}}\int_0^t dt$$

$$t \propto \frac{1}{A}\sqrt{\frac{m}{\alpha}}$$

19. **(D)**

for $|x| > x_0$

potential energy is constant

$F = 0$

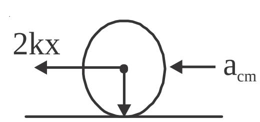
$\Rightarrow \quad a = 0$

PASSAGE 2(Q NO 20 TO 23)

20. **(C)**

$v_0 = A\omega$

At a displacement (x)



$$2kxR = \frac{3}{2}mR^2\alpha$$

$$\left(\frac{4k}{3M}\right)x = R\alpha = a_{cm}$$

$$\omega = \sqrt{\frac{4k}{3m}}$$

at $x = A$

$$2kA - \mu mg = m\omega^2 A$$

$$2kA - \mu mg = \frac{m4k}{3m}A$$

$$A = \frac{3\mu mg}{2k}$$

$$v_0 = A\omega = \mu g\sqrt{\frac{3m}{k}}$$

21. **(D)**

$$\omega = \sqrt{\frac{4k}{3m}}$$

22. **(D)**

$$F_{net} = -m\omega^2 x = -\frac{m4k}{3m}x$$

$$= -\frac{4k}{3}x$$

23. **(A)**

F = QE is a constant force

Hence no change in time period

24. **(A, D)**

In case A $mg(l/2)\sin\theta + Mgl\sin\theta = \left(\frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2\right)\alpha$

In case B $mg(l/2)\sin\theta + Mgl\sin\theta = \left(\frac{ml^2}{3} + Ml^2\right)\alpha$

torque A = torque B

frequency A < frequency B