(1) using $\frac{a}{\sin A} = \frac{b}{\sin B}$

$$\sin A = \frac{4}{12}\sin 60^\circ = \frac{1}{2\sqrt{3}}$$

(2) $$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{9+16-4}{24} = \frac{7}{8}$$

(3) $2\left[bc\cos A + ca\cos B + ab\cos C\right]$

$$= 2\left[bc\left(\frac{b^2+c^2-a^2}{2bc}\right) + ca\left(\frac{c^2+a^2-b^2}{2ca}\right) + ab\left(\frac{a^2+b^2-c^2}{2ab}\right)\right]$$

$$= a^2+b^2+c^2$$

(4) $$\frac{c-a\cos B}{b-a\cos C} = \frac{(a\cos B+b\cos A)-a\cos B}{(a\cos C+c\cos A)-a\cos C}$$

$$= \frac{b}{c}$$

$$= \frac{2R\sin B}{2R\sin C} = \frac{\sin B}{\sin C}$$

(5) same as (3)

(6) $$\tan\left(\frac{B-C}{2}\right) = \frac{(\sqrt{3}+1)-(\sqrt{3}-1)}{(\sqrt{3}+1)+(\sqrt{3}-1)}\cot(30^\circ)$$

$$= \frac{2}{2\sqrt{3}}\cdot\sqrt{3}$$

$$= 1$$

(7) $\tan\left(\frac{C-B}{2}\right) = \frac{\sqrt{3}+1-2}{\sqrt{3}+1+2}\cot 30^\circ$

$= \frac{(\sqrt{3}-1)}{\sqrt{3}(\sqrt{3}+1)} \cdot \sqrt{3}$

$= 2-\sqrt{3}$

So, $\frac{C-B}{2} = 15^\circ$

(8) $\left(\frac{b-c}{a}\right)\cos^2\left(\frac{A}{2}\right) = \cancel{\frac{(b-c)}{a}\left(\frac{1+\cos A}{2}\right)}$

$=\left(\frac{b-c}{a}\right)\frac{s(s-a)}{bc}$

$= \frac{s^2(b-c) - s(ab-ac)}{abc} \rightarrow$ (i)

Similarly, $\left(\frac{c-a}{b}\right)\cos^2\left(\frac{B}{2}\right) = \frac{s^2(c-a) - s(bc-ab)}{abc} \rightarrow$ (ii)

& $\left(\frac{a-b}{c}\right)\cos^2\left(\frac{C}{2}\right) = \frac{s^2(a-b) - s(ac-bc)}{abc} \rightarrow$ (iii)

Adding (i),(ii),(iii) we get

$\sum\left(\frac{b-c}{a}\right)\cos^2\left(A/2\right) = \frac{s^2(0) - s(0)}{abc} = 0$

(9) $\frac{b^2+c^2-a^2}{4\cot A} = \frac{2bc\cos A}{4\left(\frac{\cos A}{\sin A}\right)}$

$= \frac{1}{2}bc\sin A$

$= \Delta.$

(10). $(a^2-b^2)\dfrac{\sin A\sin B}{\sin(A-B)} = \dfrac{4R^2(\sin^2 A-\sin^2 B)\sin A\sin B}{\sin(A-B)}$

$= \sin(A+B)\, a\cdot b$

$= 2\left(\frac{1}{2}ab\sin C\right)$

$= 2\Delta$

(11) $\Delta = \sqrt{36\cdot 18\cdot 12\cdot 6}$

$= 36\cdot 3\cdot 2$

$= 216$

$R = \dfrac{18\cdot 24\cdot 30}{4\cdot 216}$

$= 15.$

(12) $a\cos A + b\cos B + c\cos A$

$= 2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C$

$= R(\sin 2A + \sin 2B + \sin 2C)$

$= 4R\sin A\sin B\sin C$

(13) $2R^2\sin A\sin B\sin C$

$= \frac{1}{2}(2R\sin A)(2R\sin B)\sin C$

$= \frac{1}{2}ab\sin C = \Delta$

(14) $\Delta = \sqrt{9\cdot 5\cdot 3\cdot 1} = 3\sqrt{15}$

$r = \dfrac{3\sqrt{15}}{9} = \sqrt{\dfrac{5}{3}}$

(15) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$= 1 + \left(\frac{r}{R}\right)$

(16) $\Delta = \cancel{5}\,216, \quad s = 36$

$$r_1 = \cancel{15}\ \frac{216}{18} = 12 \text{ cm}$$

$$r_2 = \frac{216}{12} = 18 \text{ cm}$$

$$r_3 = \frac{216}{6} = 36 \text{ cm}$$

(17)

$$r = \frac{\left(\frac{\sqrt{3}}{4}a^2\right)}{\left(\frac{3a}{2}\right)} = \frac{a}{2\sqrt{3}}$$

$$R = \frac{a}{2\sin 60^0} = \frac{a}{\sqrt{3}} = \frac{2a}{2\sqrt{3}}$$

$$r_1 = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{a}{2}} = \frac{\sqrt{3}a}{2} = \frac{3a}{2\sqrt{3}}$$

$$r : R : r_1 = 1 : 2 : 3$$

(18)

$$\cos\theta = \frac{(6+\sqrt{12})^2 + 48 - 24}{2\,(6+\sqrt{12})\sqrt{48}}$$

$$= \frac{48 + 12\sqrt{12} + 24}{2\,(6+\sqrt{12})\,4\sqrt{3}}$$

$$= \frac{12\,(6+\sqrt{12})}{8\sqrt{3}\,(6+\sqrt{12})}$$

$$= \frac{\sqrt{3}}{2}$$

$$\theta = \pi/6.$$

(19) $a^2+b^2+c^2 = ca + ab\sqrt{3}$

$\Rightarrow 4a^2+4b^2+4c^2-4ac-4\sqrt{3}ab=0$

$\Rightarrow (a-2c)^2 + (\sqrt{3}a-2b)^2 = 0$

$a = 2c$ & $\sqrt{3}a = 2b$ only

(20) $$\frac{s}{R} = \frac{a+b+c}{2R}$$

$$= \sin A + \sin B + \sin C$$

(21) $$\Delta = \sqrt{9.4.3.2} = 6\sqrt{6}$$

$$R = \frac{5.6.7}{24\sqrt{6}}$$

$$2R = \frac{35}{2\sqrt{6}}$$

(22) $$\frac{b^2-c^2}{2aR} = \frac{4R^2(\sin^2 B - \sin^2 C)}{2aR}$$

$$= \frac{2R\sin(B-C)\sin(B+C)}{2R\sin A}$$

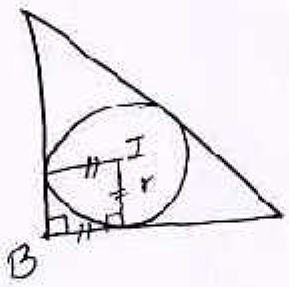
$$= \sin(B-C)$$

(23) $$\frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$
$$= \cos A + \cos B + \cos C - 1$$
$$= \frac{7}{16} \Rightarrow \frac{R}{r} = \frac{16}{7}$$

(24) $\frac{R}{r} = \frac{1}{4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

$= \frac{2}{1}$

(25)



$r = s - b$

$\Rightarrow 2r = a + c - b$

(26) $\cos 60^\circ = \frac{100 + x^2 - 81}{20x}$

$x^2 - 10x + 19 = 0$

$x = 5 \pm \sqrt{6}$

~~$x = 5 - \sqrt{6}$ only ($\because x < 9$)~~

(27) M-I Let $a^2 + b^2 = c^2$

$\Rightarrow 2c^2 = 8R^2$

$\frac{c}{2R} = \sin C = 1 \Rightarrow \angle C = \pi/2$

Hence, verified

M-II $\frac{\cot A + \cot B + \cot C}{R} = \frac{a^2 + b^2 + c^2}{abc}$

$\Rightarrow \frac{a^2 + b^2 + c^2}{8R^2} = \left(\frac{\cot A + \cot B + \cot C}{8R^3}\right) abc$

$= \cos A \cos B \cos C (\tan B \tan C + \tan C \tan A + \tan A \tan B)$

$\Rightarrow 1 = \cos A \cos B \cos C - \cos(A + B + C)$

$A + B + C = \pi \Rightarrow \cos A \cos B \cos C = 0$

(28)

$r_1 r_2 + r_2 r_3 + r_3 r_1$

$= \dfrac{\Delta^2}{(s-a)(s-b)} + \dfrac{\Delta^2}{(s-b)(s-c)} + \dfrac{\Delta^2}{(s-c)(s-a)}$

$= s^2$

(29)

$r_1 + r_2 = \dfrac{\Delta}{s-a} + \dfrac{\Delta}{s-b}$

$= \dfrac{\Delta c}{(s-a)(s-b)}$

$(r_1 + r_2)(r_2 + r_3)(r_3 + r_1) = \dfrac{\Delta^3 abc}{(s-a)^2(s-b)^2(s-c)^2}$

$= \dfrac{\Delta^3 abc\, s^2}{\Delta^4}$

$= 4Rs^2$

(30)

$r_1 - r_2 = r_3 + r$

$\Rightarrow \dfrac{\Delta}{(s-a)} - \dfrac{\Delta}{(s-b)} = \dfrac{\Delta}{s-c} + \dfrac{\Delta}{s}$

$\dfrac{(a-b)}{(s-a)(s-b)} = \dfrac{(a+b)}{s(s-c)}$

$\tan^2\left(\dfrac{B}{2}\right) = \dfrac{a-b}{a+b}$

$\cos B = \dfrac{b}{a} \Rightarrow \angle A = \pi/2$

(31)
$$\frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc}$$
$$= \cos^2 A/2 - \sin^2 A/2$$
$$= \cos A$$

(32)
$$\cos B = \cos 60^\circ = \frac{a^2 + 2k^2 - 3k^2}{2 \cdot a \cdot \sqrt{2}k}$$
$$\Rightarrow a^2 - (\sqrt{2}k)a - k^2 = 0$$
$$a = \left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)k \quad \text{only}$$

using
$$\frac{a}{\sin A} = \frac{b}{\sin 60^\circ}$$
$$\sin A = \frac{a}{b} \sin 60^\circ$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$A = 75^\circ$$

(33) $\cancel{\sin^2 A + (\sin^2 B - \sin^2 C) = 0}$

$\cancel{\sin^2 A + \sin(B+C)\sin(B-C) = 0}$

$\cancel{\sin A\,[\sin(B+C) + \sin(B-C)] = 0}$

$\cancel{2\sin A \sin 2B \cos 2C}$

$\cancel{2\sin A \sin B \cos C = 0}$

(33)
$$\sin^2 A + \sin^2 B = \sin^2 C$$
$$4R^2(a^2 + b^2) = 4R^2 c^2$$
$$a^2 + b^2 = c^2$$
$$\angle C = 90^\circ$$

(34) $\cos A = \dfrac{\sin B}{2\sin C}$

$$\frac{b^2+c^2-a^2}{2bc} = \frac{b}{2\cdot 2R\left(\frac{c}{2R}\right)}$$

$$b^2+c^2-a^2 = b^2$$

$$c^2 = a^2 \Rightarrow c=a$$

(35)

$$c^2 = a^2+b^2 \Rightarrow \angle C = 90^\circ$$

$$\Delta = \frac{1}{2}ab = \sqrt{s(s-a)(s-b)(s-c)}$$

$$4s(s-a)(s-b)(s-c) = a^2b^2$$

(36)

$$\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} = \frac{1-\cos(A-B)\cos(A+B)}{1-\cos(A-C)\cos(A+C)}$$

$$= \frac{1-(\cos^2 A-\sin^2 B)}{1-(\cos^2 A-\sin^2 C)}$$

$$= \frac{\sin^2 A+\sin^2 B}{\sin^2 A+\sin^2 C}$$

$$= \frac{a^2+b^2}{a^2+c^2}$$

(37) $rr_1 + r_2 r_3 = \dfrac{\Delta}{s}\cdot\dfrac{\Delta}{(s-a)} + \dfrac{\Delta}{(s-b)}\cdot\dfrac{\Delta}{(s-c)}$

$$= \Delta^2\left[\frac{(s-b)(s-c)+s(s-a)}{s(s-a)(s-b)(s-c)}\right]$$

$$= 2s^2-(a+b+c)s+bc$$

$$= bc$$

(38) $r_1 + r_2 = s\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right)$ (Pg.-10)

$$= \frac{s \sin\left(\frac{A}{2} + \frac{B}{2}\right)}{\cos\frac{A}{2} \cdot \cos\frac{B}{2}}$$

$$= \frac{s \cos\left(\frac{C}{2}\right)}{\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ca}}}$$

$$= c \cot\left(\frac{C}{2}\right)$$

(39.) $16R^2 r r_1 r_2 r_3 = 16\left(\frac{abc}{4\Delta}\right)^2 \cdot \frac{\Delta}{s} \cdot \frac{\Delta}{(s-a)} \cdot \frac{\Delta}{(s-b)} \cdot \frac{\Delta}{(s-c)}$

$$= a^2 b^2 c^2$$

(40) $a \sin(B-C) = 2R \sin A \sin(B-C)$

$$= 2R \sin(B+C) \sin(B-C)$$

$$= 2R(\sin^2 B - \sin^2 C)$$

$\sum a \sin(B-C) = 0$

(41) $\sin A : \sin B : \sin C = a : b : c = 1:2:3$

$b = 4\text{ cm} \Rightarrow a = 2\text{cm},\ c = 6\text{cm}$

$P = a+b+c = 12\text{ cm}$

(42) $r_2 = \frac{2 r_1 r_3}{r_1 + r_3}$

$\Rightarrow b = 8\text{ cm}$

$\Delta = 24\text{ cm}^2 \Rightarrow a = 6\text{ cm},\ c = 10\text{ cm}$

(43)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos 60^\circ = \frac{1}{2} = \frac{c^2 + 9 - 16}{6c}$$

$$c^2 - 3c - 7 = 0$$

(44)

$$\sin A : \sin B : \sin C = \frac{1}{3} : \frac{1}{6} : \frac{1}{2\sqrt{3}}$$

$$\Rightarrow a : b : c = 2 : 1 : \sqrt{3}$$

$$\cos A = \frac{1 + 3 - 4}{2(1)(\sqrt{3})} = 0$$

$$\angle A = 90^\circ$$

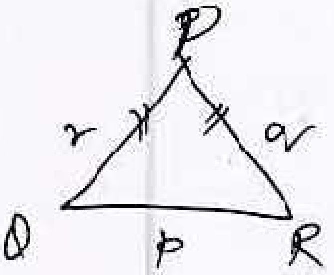
(45)

$$\Delta = \sqrt{8K \cdot 3K \cdot 2K \cdot 3K} = 12K^2$$

$$r = \frac{12K^2}{8K} \Rightarrow 6 = \frac{3K}{2}$$

$$K = 4$$

(46)



$$\frac{r}{\sin R} = 2r$$

$$\sin R = \frac{1}{2}$$

$$\angle R = \frac{\pi}{6} \Rightarrow \angle P = \frac{2\pi}{3}$$

(47)

$$r = \frac{\left(\frac{\sqrt{3}}{4}a^2\right)}{\left(\frac{3a}{2}\right)} = \left(\frac{a}{2\sqrt{3}}\right)$$

Let square has side x then

$$x\sqrt{2} = 2r$$

$$x = r\sqrt{2}$$

$$A_{square} = 2r^2$$

$$\cancel{= 2\left(\frac{a^2}{12}\right)}$$

$$= 2\left(\frac{a^2}{12}\right)$$

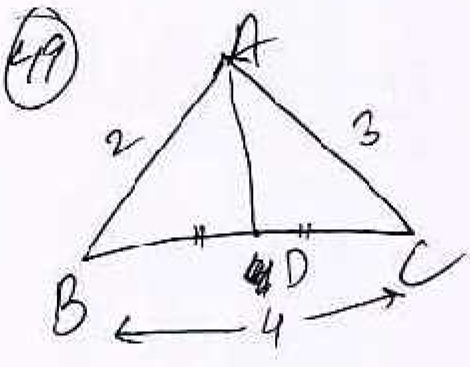
$$= \frac{a^2}{6}$$

(48) ~~Let side be of~~ $2\pi r = na$ (given)

$$\frac{A_1}{A_2} = \frac{\pi r^2}{\frac{na^2}{4}\cot\left(\frac{\pi}{n}\right)}$$

$$= \tan\left(\frac{\pi}{n}\right) : \left(\frac{\pi}{n}\right)$$

(49)



$$\cos B = \frac{4+16-9}{2(2)(4)}$$

$$= \frac{11}{16}$$

(50) $r + r_1 = r_2 + r_3$

$$\frac{\Delta}{s} + \frac{\Delta}{s-a} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$\frac{(s-b)(s-c)}{s(s-a)} = \frac{a}{b+c} \Rightarrow \cot^2\left(\frac{A}{2}\right) = \frac{b+c}{a}$$

$$\frac{\pi}{3} < A < \pi \Rightarrow \frac{\pi}{6} < \frac{A}{2} < \frac{\pi}{2}$$

$$\Rightarrow 0 < \frac{b+c}{a} < 3 \Rightarrow 1 < \frac{a+b+c}{a} < 4$$

$$\Rightarrow \frac{1}{2} < \frac{s}{a} < 2$$

Exercise-1(B)

(1) $b \cdot 2\cos^2(\frac{A}{2}) + a \cdot 2\cos^2(\frac{B}{2}) = 3c$

$\Rightarrow \quad b(1+\cos A) + a(1+\cos B) = 3c$

$b + a = 2c$

a, c, b are in AP

(2) $\frac{(a+b-c)+(a-b+c)+(b+c-a)}{3} \geq \sqrt[3]{(a+b-c)(a-b+c)(b+c-a)}$

$\Rightarrow \quad \frac{(a+b+c)^3}{27} \geq (2s-2c)(2s-2b)(2s-2a)$

$\Rightarrow \quad s(s-a)(s-b)(s-c) \leq \frac{s^4}{27}$

$\Rightarrow \quad A \leq \frac{s^2}{3\sqrt{3}}$

(3) $(a+b)^2 \sin^2\frac{C}{2} + (a-b)^2\cos^2\frac{C}{2}$

$= \left[(a+b)^2 - (a-b)^2\right]\sin^2\frac{C}{2} + (a-b)^2$

$= 4ab\left[\frac{(s-a)(s-b)}{ab}\right] + (a-b)^2$

$= (c+a-b)(c-a+b) + (a-b)^2$

$= c^2 - (a-b)^2 + (a-b)^2 = c^2$

(4) $\Delta = \sqrt{34 \cdot 24 \cdot 8 \cdot 2} = 16\sqrt{51}$

$CH = \frac{2\Delta}{AB} = \sqrt{51}$

$CM = \frac{1}{2}\sqrt{2(576+100) - 1024} = \sqrt{132}$

$HM^2 = CM^2 - CH^2$

$HM = 9.$

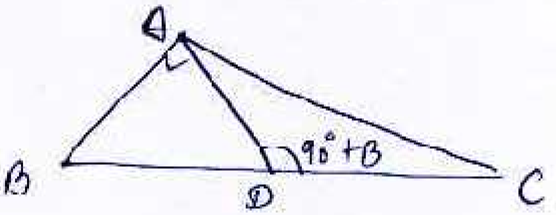
(5)

$$CD = \frac{2ab\cos\left(\frac{C}{2}\right)}{a+b}$$

$$\frac{a+b}{ab} = \frac{2}{3} \times \frac{1}{6}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$$

(6)



$$2\cot(90^\circ + B) = \cot 90^\circ - \cot(A - 90^\circ)$$

$$-2\tan B = \tan A$$

$$\frac{\tan A}{\tan B} = -2$$

(7)

$$a\cos(B-C) = 2R\sin A\cos(B-C)$$

$$= R\ 2\sin(B+C)\cos(B-C)$$

$$= R\ (\sin 2B + \sin 2C)$$

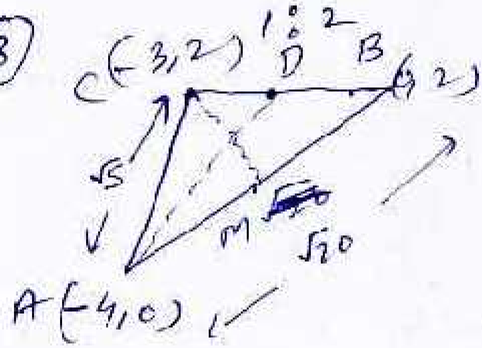
$$\sum a\cos(B-C) = R\ (2\sin 2A + 2\sin 2B + 2\sin 2C)$$

$$= 2R\ (4\sin A\sin B\sin C)$$

$$= 8R \cdot \frac{a}{2R} \cdot \frac{b}{2R} \cdot \frac{c}{2R}$$

$$= \frac{abc}{R^2}$$

(8)



AD is angle bisector then

D $(-2, 2)$

Eqn. of AD is $x - y + 4 = 0$

Mid-point of AB is M $(-2, 1)$

Eqn. of CM is $x + y + 1 = 0$

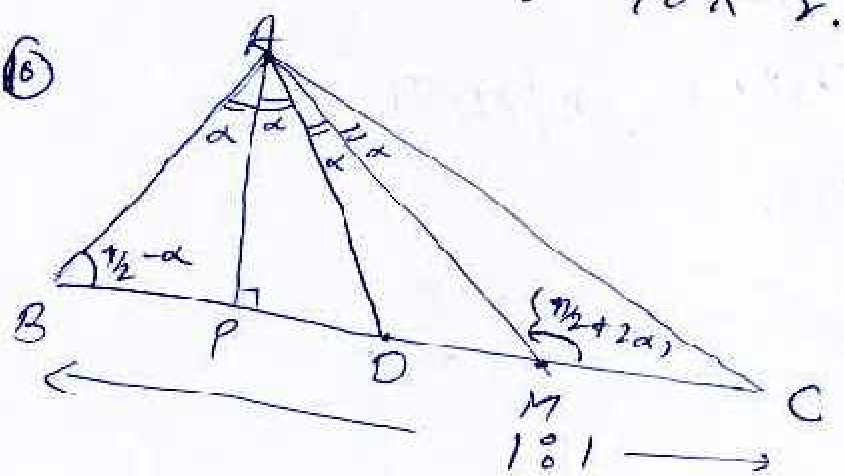
Point of intersection of CM & AD is $\left(-\frac{5}{2}, \frac{3}{2}\right)$

(9)

$$II_1 = I_1A - IA$$
$$= (r_1 - r)\sec(A/2)$$
$$= (s - s + a)\tan(A/2)\sec(A/2)$$
$$= a\sin(A/2)$$
$$= 4R\sin A/2$$

$$II_1 \cdot II_2 \cdot I_3 = (4R\sin A/2)(4R\sin B/2)(4R\sin C/2)$$
$$= 16R^2 r.$$

(10)



$$2\cot(\pi/2 + 2\alpha) = \cot 3\alpha - \cot\alpha$$
$$\Rightarrow -2\tan 2\alpha = \frac{\sin(\alpha - 3\alpha)}{\sin\alpha\sin 3\alpha}$$
$$\Rightarrow 2\sin\alpha\sin 3\alpha = \cos 2\alpha$$
$$\Rightarrow \cos 4\alpha = 0$$
$$\alpha = \frac{\pi}{8}.$$

(11)

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\left(\frac{1}{r_2} + \frac{1}{r_3}\right)\left(\frac{1}{r_3} + \frac{1}{r_1}\right) = \frac{KR^3}{a^2b^2c^2}$$

$$\left(\frac{c}{\Delta}\right)\left(\frac{a}{\Delta}\right)\left(\frac{b}{\Delta}\right) = K\frac{a^3b^3c^3}{64\Delta^3 (a^2b^2c^2)}$$

$$K = 64$$

(12)

$$\frac{a\cos A + b\cos B + c\cos C}{a+b+c} = \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2s}$$
$$= \frac{4R^2 \sin A\sin B\sin C}{R(2s)} = \frac{\Delta}{sR} = \frac{r}{R}$$

(13) $\cancel{r_1 + r_2 = \Delta}$

$$(r_1+r_2)(r_2+r_3)(r_3+r_1) = 4Rs^2$$

(from Q.29 / Ex-1A)

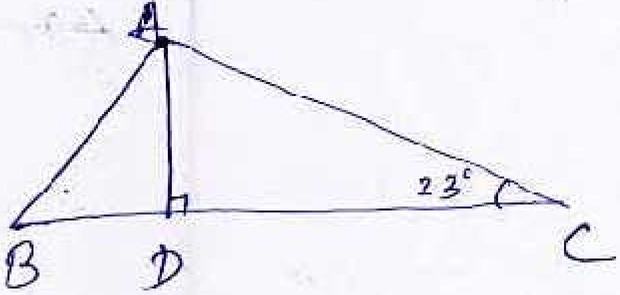
$$r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$$

(from Q.28 / Ex-1A)

$$\Rightarrow K \frac{(r_1+r_2)(r_2+r_3)(r_3+r_1)}{r_1 r_2 + r_2 r_3 + r_3 r_1} = K[4R]$$

$$\Rightarrow K = \frac{1}{4}$$

(14)



$$AD = \frac{abc}{b^2 - c^2}$$

$$\cancel{c \sin B = \frac{a\,(2R\sin B)(2R\sin C)}{4R^2(\sin^2 B - \sin^2 C)}}$$

$$\cancel{c \sin B = a \sin C}$$

$$\Rightarrow c\sin B = \frac{(2R\sin A)(2R\sin B)\,c}{4R^2(\sin^2 B - \sin^2 C)}$$

$$\Rightarrow \sin(B+C)\sin(B-C) = \sin A$$

$$\sin(B - 23^\circ) = 1 \Rightarrow B = 113^\circ$$

(15)

$$\cos 3A + \cos 3B + \cos 3C = 1$$

$$\Rightarrow 2\cos\left(\frac{3A+3B}{2}\right)\cos\left(\frac{3A-3B}{2}\right) + 1 - 2\sin^2\left(\frac{3C}{2}\right) = 1$$

$$\Rightarrow -2\sin\left(\frac{3C}{2}\right)\left[\cos\left(\frac{3A-3B}{2}\right) + \sin\left(\frac{3C}{2}\right)\right] = 0$$

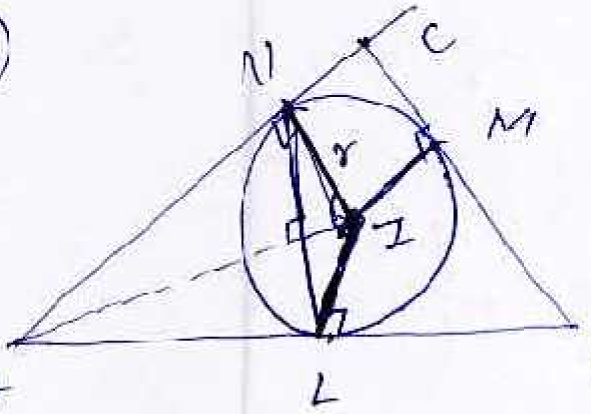
$$\Rightarrow \sin\left(\frac{3C}{2}\right)\left[\cos\left(\frac{3A}{2} - \frac{3B}{2}\right) + \cos\left(\frac{3A}{2} + \frac{3B}{2}\right)\right] = 0$$

$$\Rightarrow \cancel{\sin\left(\frac{3A}{2}\right)} \quad \sin(3A/2)\sin(3B/2)\sin(3C/2) = 0$$

$\Rightarrow \frac{3A}{2}$ or $\frac{3B}{2}$ or $\frac{3C}{2} = \pi$

~~atleast~~ one of A or B or $C = \frac{2\pi}{3}$

(16)



$LN = 2r\sin\left(\frac{\pi - A}{2}\right)$

($LINA$ is cyclic quadrilateral)

$$2y = \frac{LN}{\sin(\pi - A)}$$

$$y = \frac{2r\cos A/2}{2\sin A} = \frac{r}{2\sin A/2}$$

$$xyz = \frac{r^3}{8\sin A/2 \sin B/2 \sin C/2} = \frac{r^3 R}{8 \cdot r}$$

$$= \frac{1}{2} r^2 R$$

(17) ΔDEF is pedal triangle

$$\frac{\text{Perimeter }(\Delta DEF)}{\text{Perimeter }(\Delta ABC)} = \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2s}$$

$$= \frac{R(4\sin A\sin B\sin C)}{2s}$$

$$= \frac{\left(\frac{1}{2}ab\sin C\right)}{Rs}$$

$$= \frac{r}{R}$$

(18.) $b+c = 3a$

$2R(\sin B + \sin C) = 3(2R\sin A)$

$2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 3\left(2\sin\frac{A}{2}\cos\frac{A}{2}\right)$

$\cos\left(\frac{B-C}{2}\right) = 3\cos\left(\frac{B+C}{2}\right)$

$4\sin\frac{B}{2}\sin\frac{C}{2} = 2\cos\frac{B}{2}\cos\frac{C}{2}$

$\cot\frac{B}{2}\cot\frac{C}{2} = 2$

(19.)

$\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \Sigma\left(\frac{2R\sin A}{R\cos A}\right)$

$= 2\,\Sigma(\tan A)$

$= 2\tan A\tan B\tan C$

$= 2\left(\frac{abc}{fgh}\right) \Rightarrow \lambda = 2$

(20)

$2R\cos B\cos C = R\cos A$

~~$2\cos(B\cos(C)$~~

$2\cos B.\cos C = -\cos(B+C)$

$\tan B\tan C = 3$

(21)

$IA\cdot IB\cdot IC = r\,\mathrm{cosec}\,\frac{A}{2}\cdot r\,\mathrm{cosec}\,\frac{B}{2}\cdot r\,\mathrm{cosec}\,\frac{C}{2}$

$= r^3\,\frac{abc}{(s-a)(s-b)(s-c)}$

$= r^2\,\frac{\Delta\ abc}{s(s-a)(s-b)(s-c)} = r^2\left(\frac{abc}{\Delta}\right) = 4Rr^2$

(22) $\tan\left(\frac{A-B}{2}\right) = \left[\frac{a - a(\sqrt{3}-1)}{a + a(\sqrt{3}+1)}\right] \cot(15^\circ)$

$$= \frac{(2-\sqrt{3})}{\sqrt{3}}(2+\sqrt{3})$$

$A - B = 60^\circ$

where $A + B = 150^\circ$

$\Rightarrow A = 105^\circ$

(23) $b = a_2 = \frac{5}{3} a_1 \left(2 - \frac{2}{5} a_1\right)$

~~$= \frac{10}{3}\left(\frac{13}{5}\right) = 2$~~

~~Similarly, $c = a_3 = 25\, a_2 \left(1 - \frac{a_2}{5}\right)$~~

~~$= 30$~~

~~$\Delta = \sqrt{17.15.15}$~~

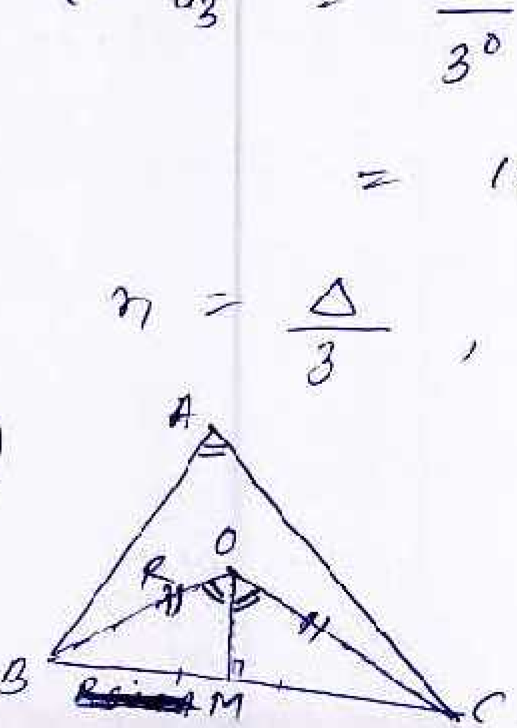
$$= \frac{10}{3}\left(2 - \frac{4}{5}\right) = 4$$

$c = a_3 = \frac{25}{30} a_2 \left(20 - \frac{6a_2}{25}\right)$

$$= 100\left(1 - \frac{24}{25}\right) = 4$$

$r_1 = \frac{\Delta}{3}, \quad r_3 = \frac{\Delta}{1} \Rightarrow r_3 = 3r_1$

(24)



$2R_1 = \frac{a}{\sin 2A}$

$\Rightarrow \frac{a}{R_1} = 2\sin 2A$

$$\sum \frac{a}{R_1} = 2(\sin 2A + \sin 2B + \sin 2C)$$

$$= 2(4\sin A \sin B \sin C)$$

$$= 2\left(\frac{a}{R}\cdot\frac{b}{R}\sin C\right)$$

$$= \frac{4\Delta}{R^2}$$

(25) We can see here

$$m_A^2 + m_B^2 = m_C^2$$

$$\Rightarrow \frac{1}{4}(2b^2+2c^2-a^2) + \frac{1}{4}(2c^2+2a^2-b^2) = \frac{1}{4}(2a^2+2b^2-c^2)$$

$$\Rightarrow \quad a^2+b^2 = 5c^2$$

using $\quad m_A^2 + m_B^2 + m_C^2 = \frac{3}{4}(a^2+b^2+c^2)$

$$c^2 = 100 \Rightarrow c = 10 \Rightarrow AB = 10\text{cm}$$

$$GA = \frac{2}{3}m_A = 6\text{cm}, \quad GB = \frac{2}{3}m_B = 8\text{cm}$$

So, ΔGAB is right angled Δ with

$$Ar(\Delta GAB) = \frac{1}{2}\times 6\times 8 = 24\ \text{cm}^2$$

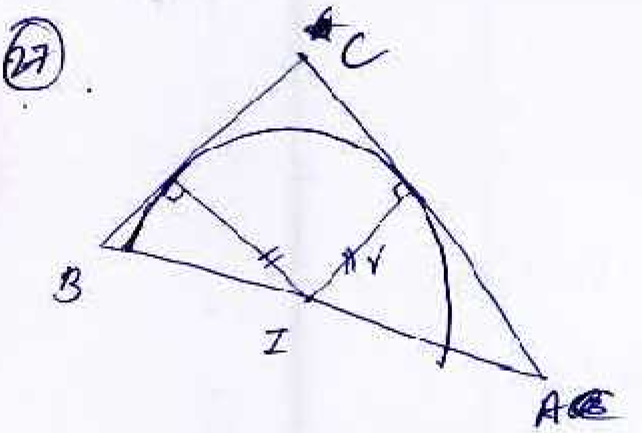
$$\therefore Ar(\Delta ABC) = 3\,Ar(\Delta GAB) = 72\ \text{cm}^2$$

(26) $$\frac{a}{x} = \frac{2R\sin A}{r\,\text{cosec}\,A/2} = \frac{\sin A}{2\sin B/2 \sin C/2}$$

$$\frac{a}{x}\cdot\frac{b}{y}\cdot\frac{c}{z} = \prod\left(\frac{\sin A}{2\sin^2 A/2}\right) = \prod\cot(A/2)$$

$$= \sum \cot(A/2)$$

(27)



$$\frac{r}{IA} = \sin A$$

$$IA = r\,\mathrm{cosec}(A)$$

Similarly, $IB = r\,\mathrm{cosec}(B)$

$$IA + IB = c$$

$$r = \frac{c\sin A\sin B}{\sin A + \sin B}$$

$$= \frac{ac\sin B}{a+b}$$

$$= \frac{2\Delta}{a+b}$$

(28)

$$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$$

$$\Rightarrow \tan A = \tan B = \tan C$$

$$\Rightarrow A = B = C$$

(29)

$$\cos A + \cos B = 2(1 - \cos C)$$

$$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2\left(2\sin^2\frac{C}{2}\right)$$

$$\cos\left(\frac{A-A}{2}\right) = 2\sin\left(\frac{C}{2}\right)$$

$$\frac{\cos\left(\frac{A}{2} - \frac{B}{2}\right)}{\cos\left(\frac{A}{2} + \frac{B}{2}\right)} = \frac{2}{1}$$

$$\frac{2\cos\frac{A}{2}\cos\frac{B}{2}}{2\sin\frac{A}{2}\sin\frac{B}{2}} = \frac{3}{1}$$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{1}{3}$$

$\Rightarrow \quad 3(s-c) = s$

$\Rightarrow \quad a+b = 2c$

(30)

$A = B$ & $C = \frac{\pi}{4}$ satisfies given relation

(31) We have to go optionwise

(A) $(\Sigma p)(\Sigma \frac{1}{p}) = \left(\frac{2\Delta}{a} + \frac{2\Delta}{b} + \frac{2\Delta}{c}\right)\left(\frac{a+b+c}{2\Delta}\right)$

$= (\Sigma \frac{1}{a})(\Sigma a)$

(B) $(\Sigma p)\Sigma(a) = 2\Delta\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(a+b+c)$

$= 4\Delta^2 (\Sigma \frac{1}{a})(\Sigma \frac{1}{p})$

$\neq \Sigma(\frac{1}{p})\Sigma(\frac{1}{a})$

(32) $\left(\frac{a+b+c}{3}\right)\frac{3}{\left(\frac{a}{2\Delta}+\frac{b}{2\Delta}+\frac{c}{2\Delta}\right)} = 2\Delta$

(33) $\sin A : \sin C = \sin(A-B) : \sin(B-C)$

$\Rightarrow \sin(B+C)\sin(B-C) = \sin(A-B)\sin(A+B)$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$b^2 - c^2 = a^2 - b^2$

(34) . $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$

$$\frac{b^2+c^2-a^2}{abc} + \frac{c^2+a^2-b^2}{2abc} + \frac{a^2+b^2-c^2}{abc} = \frac{a^2+b^2}{abc}$$

$$3b^2 + c^2 + a^2 = 2a^2 + 2b^2$$

$$b^2+c^2 = a^2$$

$$\angle A = \pi/2$$

(35) $r_1 = 2r_2 = 2r_3$

$$\left(\frac{\Delta}{s-a}\right) = 2\left(\frac{\Delta}{s-b}\right) = 2\left(\frac{\Delta}{s-c}\right)$$

$3a = 3b + c$ & $b = c$

$$\Rightarrow 3a = 4b$$

(36) $\left(\frac{a^2}{\sin A} + \frac{b^2}{\sin B} + \frac{c^2}{\sin C}\right) \sin A/_2 \sin B/_2 \sin C/_2$

$$= 2R\,(a+b+c)\,\frac{r}{4R}$$

$$= 2\,(2s)\,\frac{\Delta}{4s} = \Delta$$

(37) $5\theta + 4\theta + 3\theta = \pi \Rightarrow \theta = \frac{\pi}{12}$

$$\frac{a}{\sin\frac{5\pi}{12}} = \frac{b}{\sin\frac{\pi}{3}} = \frac{c}{\sin\frac{\pi}{4}} = 8$$

$$\Rightarrow a = (\sqrt{3}+1)\,2\sqrt{2}, \quad b = 4\sqrt{3}, \quad c = 4\sqrt{2}$$

$$\therefore \Delta = 4(3+\sqrt{3})$$

(38)

$$\cot\frac{B}{2} + \cot\frac{C}{2} = 2\cot\frac{A}{2}$$

$$\frac{\sin\left(\frac{B+C}{2}\right)}{\sin\frac{B}{2}\cdot\sin\frac{C}{2}} = 2\cot\frac{A}{2}$$

$$\sin\frac{A}{2} = 2\sin\frac{B}{2}\cdot\sin\frac{C}{2}$$

$$\cancel{\sin A = \cos\left(\frac{B-C}{2}\right) - \cos\frac{B+C}{2}}$$

$$\sin A = \left[\cos\left(\frac{B-C}{2}\right) - \cos\left(\frac{B+C}{2}\right)\right] 2\cos\left(\frac{A}{2}\right)$$

$$\sin A = \sin B + \sin C - \sin A$$

$$b + c = 2a$$

$\Rightarrow b + c > a$. A moves on ellipse (focal distance property)

(39)

(40)

$$\cos A + \cos B = 4\sin^2\left(\frac{C}{2}\right)$$

$$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 4\cos^2\left(\frac{A+B}{2}\right)$$

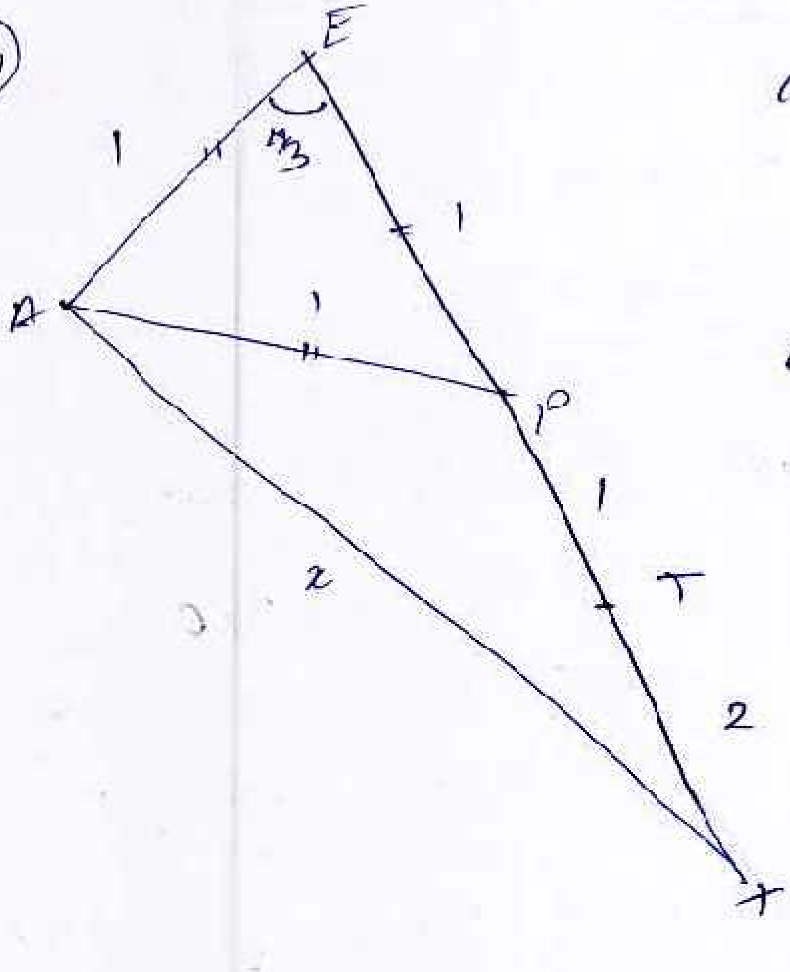
$$\cos\left(\frac{A}{2} - \frac{B}{2}\right) = 2\cos\left(\frac{A}{2} + \frac{B}{2}\right)$$

$$\boxed{\cot\left(\frac{A}{2}\right)\cdot\cot\left(\frac{B}{2}\right) = 3}$$

$$\Rightarrow \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}\sqrt{\frac{s(s-b)}{(s-c)(s-a)}} = 3$$

$$\boxed{a + b = 2c}$$

(41)



$$\cos\frac{\pi}{3} = \frac{1+16-x^2}{2(4)(1)}$$

$$\boxed{x = \sqrt{13}}$$

$\cancel{\cos(\angle XAE)}$

$$\cos(\angle XAE) = \frac{1+13-16}{2\sqrt{13}\cdot 1}$$

$$= -\frac{1}{\sqrt{13}}$$

$$AT = \frac{1}{2}\sqrt{2(13+1)-16}$$

$$= \sqrt{3}$$

$$AT^2 + AE^2 = 4 = ET^2$$

$$\angle EAT = 90^\circ$$

~~(42) Conceptual. (use formulae.)~~

(12) (A) $rs + r_1(s-a) + r_2(s-b) + r_3(s-c)$

$= \Delta + \Delta + \Delta + \Delta = 4\Delta$

(B) $$\frac{(a+b+c)^2}{\cot A/2 + \cot B/2 + \cot C/2}$$

$$= \frac{4R^2(\sin A + \sin B + \sin C)^2}{\cot A/2 \cdot \cot B/2 \cdot \cot C/2}$$

$$= \frac{4R^2 (4\cos A/2 \cos B/2 \cos C/2)^2}{\cot A/2 \cot B/2 \cdot \cot C/2}$$

$= 8R^2 \sin A \sin B \sin C$

$= 4\left(\frac{1}{2}ab\sin C\right) = 4\Delta$

~~(C) $(a^2+b^2-c^2)\tan B = (a^2+b^2-c^2)\frac{\sin B}{\cos B}$~~

~~$= \frac{(a^2+b^2-c^2)\,2ac}{(a^2+b^2-c^2)}\,\frac{b}{2R}$~~

(C) $(a^2+b^2-c^2)\tan B = \frac{(a^2+b^2-c^2)\sin B}{\cos B}$

$= \frac{(a^2+b^2-c^2)\,2ac \cdot b}{(a^2+c^2-b^2)\,2R} = 4\Delta\left(\frac{a^2+b^2-c^2}{a^2+c^2-b^2}\right)$

(D) $b^2\sin 2C + c^2\sin 2B = b^2(2\sin C\cos C) + c^2(2\sin B\cos B)$

$= \frac{bc}{R}(b\cos C + c\cos B) = 4\Delta$