STRAIGHT LINES

Exercise – 1(A)

Q.1

Slope of line is $\Rightarrow \dfrac{\sqrt{3}-0}{-2-1} = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{-1}{\sqrt{3}}$, $\theta = 150^{o}$

Q.2

Mid-point of B & C is (4 , 2):

slope is $\dfrac{3-2}{2-4} = -\dfrac{1}{2}$

Q.3

Vertices of integral $\Rightarrow$ Δ never equilateral

Because equate Areas

$$\Rightarrow \frac{\sqrt{3}}{2}\left((x_1-x_2)^2+(y_1-y_2)^2\right)=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

This is irrational & $R\pi S$ is Rational

It is a contradiction $\therefore$ be equilateral

Q.4

Slope is zero : 4^2

$\Rightarrow$ y = 2

Q.5

use slope – intercept form

y = 2x – y

Q.6

$\Rightarrow$ m = $\frac{3}{5}$ & c = – 3

$\therefore$ slope intercept form y = $\frac{3}{5}$x – 3

Q.7

$$m = \frac{4-(-5)}{2-3} = -9$$

line is $y = -9x + c$ put (3 , 4) to get c

Q.8

The information c = – 2 & m = $\sqrt{3}$

Slope intercept $y = \sqrt{3x} - 2$

Q.9

form $\frac{y-0}{x-0} = \frac{a\sin\theta - 0}{a\cos\theta - 0} \Rightarrow \boxed{y = x\tan\theta}$

Q.10

Both a = b

$\therefore$ intercept form $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \boxed{x + y = a}$ ………slope – 1

Q.11

$x\cos\alpha + y\sin\alpha = a$

$\therefore$ for y intercept put x = 0

$\therefore \quad y = a \operatorname{cosec} \alpha$

Q.12

$$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1 \quad \text{or} \quad x + y = a$$

Put (1, – 2) to get ‘a’

$\Rightarrow 1 - 2 = a \Rightarrow a = -1$

Line is x + y + 1 = 0

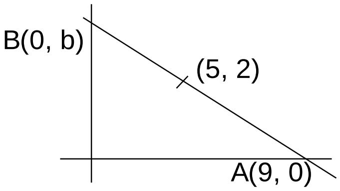
Q.13

Intercept form $\frac{x}{a} + \frac{y}{b} = 1$; here b = 2a

(1, 2) satisfies $\frac{1}{a} + \frac{2}{b} = 1$

$$\Rightarrow \frac{1}{a} + \frac{1}{a} = 1 \quad \Rightarrow \begin{pmatrix} a = 2 \\ b = 4 \end{pmatrix}$$

Q.14

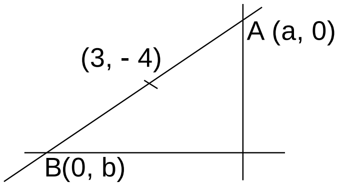


$\Rightarrow \therefore \frac{a}{2} = 5 \;\&\; \frac{b}{2} = 2$

$\Rightarrow a = 10, b = 4$

$\Rightarrow \therefore \text{line } \frac{x}{10} + \frac{y}{4} = 1$

Q.15



By section formula

$$\Rightarrow \frac{2a}{5} = 3 \ \ \& \ \ \frac{3b}{5} = -4$$

$$\Rightarrow a = \frac{15}{2}, b = -\frac{20}{3}$$

Line is $\frac{2x}{15} - \frac{3y}{20} = 1$

Q.16

$$\Rightarrow a + b = -2$$

Let $\frac{x}{a} + \frac{y}{b} = 1$

put (3 – 3)

$$\Rightarrow \frac{2}{a} - \frac{3}{b} = 1$$

$$\Rightarrow 2b - 3a - ab = 0$$

$$\Rightarrow 2b + 3(b+2) + b(b+2) = 0$$

$$\Rightarrow b^2 + 7b + 6 = 6$$

$$\Rightarrow b = -6 \ \ , \ \ b = -1$$

$$\Rightarrow a = 4 \ \ \ , \ \ b = -1$$

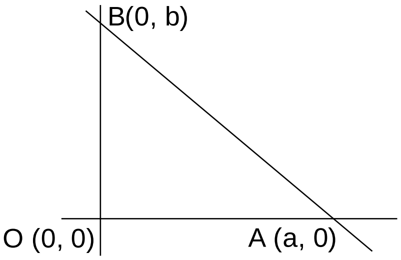
Q.17

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} = 1$$

$A(3a, 0)$; $B(0, 3b)$; $0(0 , 0)$

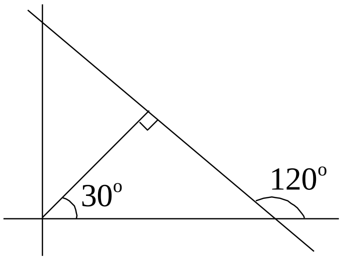
Centroid (a, b)

Q.18



$$\therefore \text{ Area of } \Delta OAB = \frac{1}{2} ab$$

Q.19

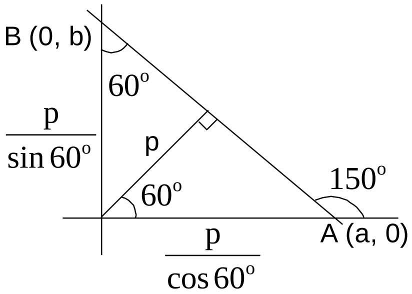


slope angle is 120^o

$\Rightarrow m = \tan 120^o = -\sqrt{3}$

Only option (B)

Q.20



$$\Rightarrow \theta = 150^o \Rightarrow \tan 150^o = \frac{-1}{\sqrt{3}}$$

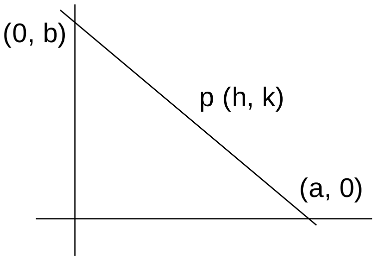
$$\Rightarrow \frac{1}{2} ab = 54\sqrt{3}$$

$$\Rightarrow \frac{p}{\sin 60^o \cos 60^o} = 2 \times 54 \times \sqrt{3}$$

$$\Rightarrow p^2 = 81 \Rightarrow p = 9$$

$$\Rightarrow \text{line } \boxed{x\cos 60^o + y\sin 60^o = P}$$

Q.21.



$$\Rightarrow h = \frac{a}{2}, \quad k = \frac{b}{2}$$

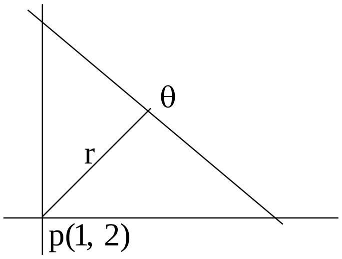
we know a + b = 10

$\Rightarrow$2h + 2k = 10

Replace h by x & k by y

$$\Rightarrow \boxed{x + y = 5}$$

Q.22



Let θ by parametric equation of Line PQ

$\theta(1+r\cos 45^o, 1+r\sin 45^o)$

Now θ satisfies line $x+2y-7=0$

$\therefore$ get r

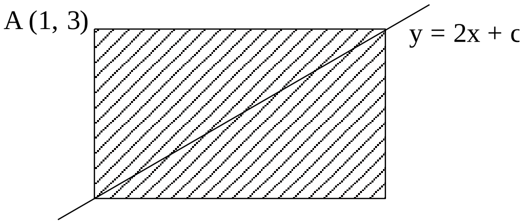
Q.23

$\theta = 60^o$, p at distance 3 units is

By parametric equation $\left(1\pm 3\cos 60^o, 2\pm 3\sin 60^o\right)$

$$\Rightarrow \left(\frac{5}{2}, 2+\frac{3\sqrt{3}}{2}\right) \text{ or } \left(-\frac{1}{2}, 2-\frac{3\sqrt{3}}{2}\right)$$

Q.24.



Slope of AC = $-\frac{1}{2}$

It is parallel to other diagonal

$\therefore$ it is a square

: AC mid point (3 , 2)

Lies on line y = 2x + c

$\Rightarrow 2 = 6 + c \;\Rightarrow\; \boxed{c = -4}$

Q.25

$\Rightarrow m_1 = 1,\ m_2 = \sqrt{3}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right) = \tan^{-1}\left(2-\sqrt{3}\right) = 15^o$$

Q.26

$$\Rightarrow m_1 = -\frac{2}{3}, m_2 = \frac{3}{2}$$

since $m_1 m_2 = -1 \quad \therefore \theta = 90^o$

Q.27

$\Rightarrow m_1 = 2\ ,\ m_2 = -3$

$$\Rightarrow \tan\theta = \left|\frac{2-(-3)}{1-6}\right| = 1 \quad \therefore \theta = \frac{\pi}{4}$$

Q.28

$\Rightarrow m_1 = 0\ ,\ m_2 = 1$

$\Rightarrow \therefore \tan\theta = 1$

$\Rightarrow \therefore$ angle 135^o

Q.29

$\Rightarrow m_1 = 0,\ m_2 = \sqrt{3}$

$\Rightarrow \therefore \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60^o$

Q.30

$\Rightarrow m_1 = \frac{4}{7}$, $m_2 = -1$, $m_3 = -\frac{7}{4}$

Since, $m_1 m_3 = -1$

$\therefore$ lines l_1 & l_3 are perpendicular

$\therefore$ there is the orthocenter which is intersection point of lines l_1 & l_3

$4x - 7y + 10 = 0$

$\Rightarrow \boxed{x = \frac{-10 + 7y}{4}}$

$$7\frac{(7y-10)}{4} + 4y = 15$$

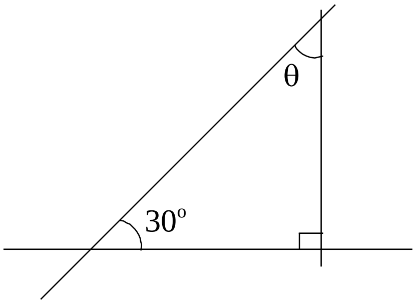
$$49y - 70 + 16y = 60$$

$$65y = 130$$

$$y = 2$$

$$\therefore x = 1$$

Q.31



$\Rightarrow$ slope $m = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 60^{o}$

Q.32

$\Rightarrow m_1 = -\frac{m}{2}$, $m_2 = -\frac{2}{3}$

$\Rightarrow m_1 m_2 = -1$

$\Rightarrow \frac{m}{3} = 1 \Rightarrow m = 3$

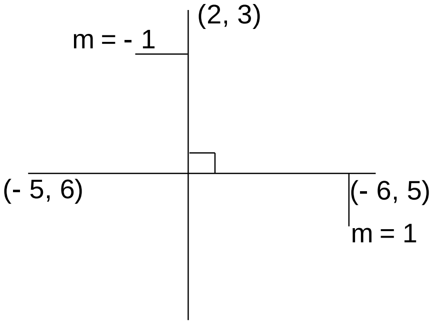
Q.33

$\Rightarrow m_1 = 2 \qquad \therefore m_2 = -\frac{1}{2}$

$\Rightarrow \frac{\lambda - 3}{-2} = -\frac{1}{2}$

$\Rightarrow \boxed{\lambda = 4}$

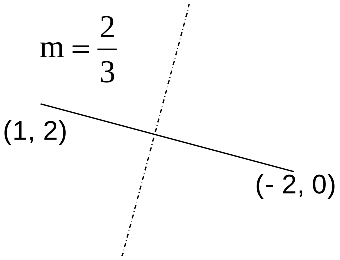
Q.34



Equation y – 3 = – 1 (x – 2)

x + y – 5 = 0

Q.35.

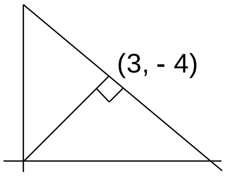


Mid – point $\left(-\frac{1}{2}, 1\right)$

slope $= -\frac{3}{2}$

$$y-1=\frac{-3}{2}\left(x+\frac{1}{2}\right)$$

Q.36.



slope of perpendicular form origin $= -\frac{4}{3}$

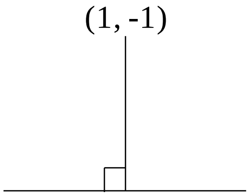
So, slope of the line = $\frac{3}{4}$

Equation : $\frac{y+4}{x-3}=\frac{3}{4}$

$\Rightarrow 4y+16=3x-9$

$\Rightarrow \boxed{3x-4y=25}$

Q.37.



$\Rightarrow 2x-3y-5=0$

$\Rightarrow m=-\frac{3}{2}$

Equation $\frac{y+1}{x-1}=-\frac{3}{2}$

$\Rightarrow 2y+2=-3x+3$

$\Rightarrow \boxed{3x+2y-1=0}$

= 1 or – 1

Q.38

slope $= -\frac{a}{b}$

equation $\frac{y-d}{x-c} = \frac{-a}{b}$

Q.39

slope $= \frac{b}{a}$: $y-b = \frac{b}{a}(x-a)$

Q.40

$\Rightarrow$ m = 0

equation $\boxed{y=-2}$

Q.41

$$\Rightarrow m = \frac{-\frac{1}{a}}{\frac{1}{b}} = -\frac{b}{a}$$

Equation $\boxed{y-b = -\frac{b}{a}(x-a)}$

Q.42

parallel line $x-3y+\lambda=0$

put (2 , 2)

$\Rightarrow \lambda = 4$

$\Rightarrow \therefore x-3y+4=0$

put x = 0,

$\Rightarrow \boxed{y = \frac{4}{3}}$

Q.43

let line be $2x - 3y = \lambda$

$$x_{\text{intercept}} = \frac{\lambda}{2},\ y_{\text{intercept}} = -\frac{\lambda}{3}$$

$$\Rightarrow \therefore A = \frac{1}{2} \times \frac{\lambda^2}{6} = 12$$

$\Rightarrow \boxed{\lambda = \pm 12}$

Q.44

Let line be

$\Rightarrow (x - y + 4) + \lambda(3x + y - 7) = 0$ ....(1)

$$\text{slope} = -\frac{1}{2} = -\frac{(1 + 3\lambda)}{\lambda - 1}$$

$\Rightarrow \lambda - 1 = 2 + 6\lambda$

$\Rightarrow \boxed{\lambda = -\frac{3}{5}}$ put in equation (1)

Q.45

let line be $x + 5y = \lambda$

$$= x_{in} = \lambda_1; y_{in} = \frac{\lambda}{5}$$

Area $= \frac{1}{2} \times \lambda \times \frac{\lambda}{5} = 5 \quad \Rightarrow \boxed{\lambda = \pm 5\sqrt{2}}$

Q.46.

centroid is (1, 1)

Let line be $x - 2y = \lambda$, put (1, 1)

$\Rightarrow \boxed{\lambda = -1}$

line is x – 2y + 1 = 0

Q.47

Let parallel line is $x \cos ec\theta - y \sec\theta = \lambda$

Put $\left(a\cos^3\theta, a\sin^3\theta\right)$ = $\boxed{\lambda = a\dfrac{\cos 2\theta}{\sin\theta\cos\theta}}$

Q.48

Let 'm' be slope

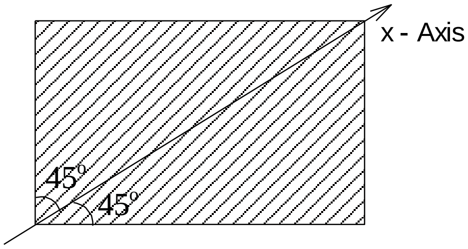
$$\Rightarrow \tan 60^o = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right|$$

$$\Rightarrow \frac{m+\sqrt{3}}{1-\sqrt{3}m} = +\sqrt{3} \quad \text{or} \quad = \sqrt{3}$$

$$\Rightarrow m = 0\ ,\ \ m = \sqrt{3}$$

Lines are $\dfrac{y+2}{x-3} = 0$ or $\sqrt{3}$

Q.49



m = 1 or – 1

$$\frac{y-2}{x-1} = 1 \text{ or } -1$$

Q.50

If Q is angle between lines & slope of lines is M

$$\Rightarrow \therefore\ m = \left|\frac{M-m}{1+Mm}\right|$$

$$\Rightarrow M-m = \pm m(1+Mm)$$

$$\Rightarrow M = 0 \quad \text{or} \quad \frac{2m}{1-m^2}$$

Q.51

$$\Rightarrow p = \left|\frac{2}{\sqrt{3+1}}\right| = 1$$

Q.52

$$\Rightarrow p = \left|\frac{2(2)-4(1)+8}{\sqrt{25}}\right| = 2$$

Q.53

$$\Rightarrow p = \left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right| = \frac{|ab|}{\sqrt{a^2+b^2}}$$

Q.54

$$\Rightarrow p = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{13-(-9)}{\sqrt{5^2+13^2}}\right| = \frac{22}{13}$$

Q.55

$$\Rightarrow p = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{4-\left(-\frac{5}{3}\right)}{\sqrt{5}}\right| = \frac{17}{3\sqrt{5}}$$

Q.56

Formula $\frac{h-7}{2} = \frac{k-8}{3} = -\frac{\left(2(7)+3(8)-4\right)}{13}$

$\Rightarrow h = \frac{23}{13}$, $k = \frac{2}{13}$

Q.57

Image formula

$$\Rightarrow \frac{h+5}{2} = \frac{k-13}{-3} = -2\frac{\left(2(-5)-3(13)-3\right)}{13}$$

$\Rightarrow h = 11 , k = -11$

Q.58

solve $x + y = 2$ ....$x = 1, y = 1$

$\Rightarrow 2x - y = 1$

$\therefore$ line is $\boxed{\frac{y}{x} = 1}$

Q.59

Interface point (– 3, 3)

$\therefore$ line is $\frac{y-3}{x+3} = 2$

Q.60

Let line be $\left(4x-3y+1\right)+\lambda\left(5x-2y-3\right)=0$ ……………………..(1)

Its slope should be $\frac{2}{3}$

$$\Rightarrow \frac{-(4+5\lambda)}{-3-2\lambda} = \frac{2}{3}$$

get λ & put it back in (1)

Q.61

Required line is $\left(x+2y+1\right)+\lambda\left(x-y+7\right)=0$ ……….(1)

line (1) is perpendicular to 5x – 2y + 7 = 0

So, $-\left(\frac{5}{2}\right)\left(\frac{1+\lambda}{2-\lambda}\right)=-1$

Solve for λ & put in equation (1).

Q.62

Let the line be $(3x+y-5)+\lambda(x-y+1)=0$ ………………(1)

It's slope can be 1 or – 3

$\Rightarrow \therefore -\frac{(\lambda+3)}{1-\lambda}=1$ or – 3

Solve for λ & put in equation (1)

Q.63

a + c = 2b or $a-2b+c=0$

We can observe that point (1, – 2) notifies ax + by + c = 0 in respective of any value of

a, b or c

Q.64

$\Rightarrow$Rewrite condition as $\frac{3a}{4}+\frac{b}{2}+c=0$

$\Rightarrow$We can see that lines ax + by + c = 0

$\Rightarrow$Always pan through $\left(\frac{3}{4},\frac{1}{2}\right)$

Q.65

use concurrency condition

$$\Rightarrow \begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix}=0$$

$\Rightarrow$solve determinates to get

$\Rightarrow a+c=2b$

$\Rightarrow \therefore A \cdot P$

Q.66

Rearrange a(x + y – 1) + b (2x + 3y – 1) = 0

$\Rightarrow$It is form $\lambda L_1 + ML_2 = 0$

$\Rightarrow \therefore$ it passe through intersection of $L_1 = 0 \,\&\, L_2 = 0$

Solve x + y = 1

$\Rightarrow$2x + 3y = 1

$\Rightarrow \therefore$(2 , - 1)

Q.67

Bisectors are

$$\Rightarrow \frac{3x-4y+7}{5} = \pm \frac{(12x-5y-8)}{13}$$

Simplify

Q.68

3x – 4y + 7 = 0

$\Rightarrow$– 12x – 5y + 2 = 0

$\Rightarrow a_1a_2 + b_1b_2 = -36 + 20 = -ve$

Equation of acute angle bisector is

$$\Rightarrow \left(\frac{3x-4y+7}{5}\right) = \left(\frac{-12x-5y+2}{13}\right)$$

$\Rightarrow$99x – 27y + 81 = 0

STRAIGHT LINES

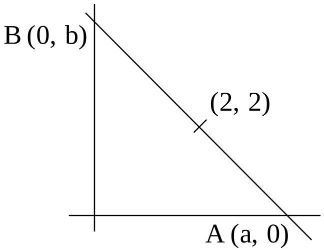
Exercise : 1(B)

Q.1

use formula $\Rightarrow \left|\frac{(c_1 - c_2)(d_1 - d_2)}{a_1b_2 - a_2b_1}\right|$

$$\Rightarrow \left|\frac{((-1)-(-3))(2-1)}{-9-(-16)}\right| = \frac{2}{7}$$

Q.2

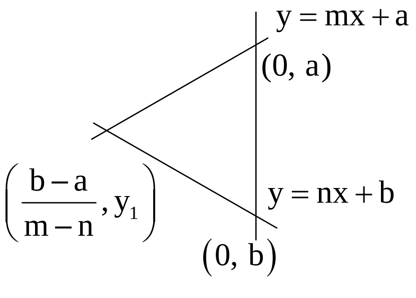


$$\Rightarrow \frac{a}{2} = 2 \ \& \ \frac{b}{2} = 2$$

$$\Rightarrow a = b = 4$$

$$\Rightarrow \text{line } x + y = 4$$

Q.3

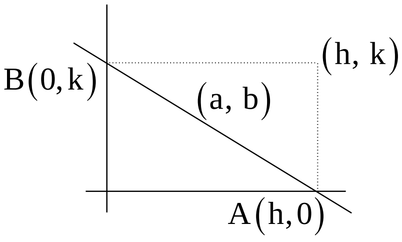


$$\Rightarrow A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow \frac{1}{2} \times |b-a| \times \frac{|b-a|}{|m-n|}$$

$$\Rightarrow \frac{1}{2}\frac{(a-b)^2}{|m-n|}$$

Q.4



variable line is

$$\Rightarrow \frac{x}{h}+\frac{y}{k}=1$$

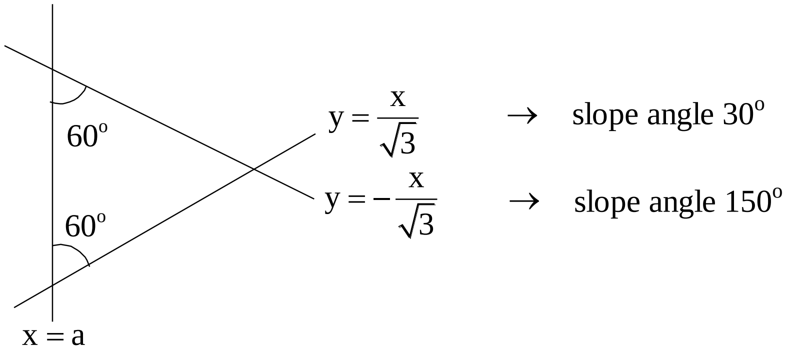
now put (a,b)

$$\Rightarrow \frac{a}{h}+\frac{b}{k}=1$$

$\therefore$ locus is $\boxed{\frac{a}{x}+\frac{b}{y}=1}$

Q.5

$$\Rightarrow y=\pm\frac{1}{\sqrt{3}}x \;\;\&\; x=a$$



Q.6

let line be $\frac{x}{a}+\frac{y}{b}=1$

Now $\frac{1}{a}+\frac{1}{b}=\frac{1}{p} \quad \Rightarrow \quad \frac{p}{a}+\frac{p}{b}=1$

: definitely line passes through (p , p)

Q.7

$$\Rightarrow d_1=\frac{|n-n^1|}{\sqrt{m^2+l^2}} \ , \ d_2=\frac{|n^1-n|}{\sqrt{l^2+m^2}}$$

Distance between both parallel lines are equal

$\therefore$ it must be a Rhombus

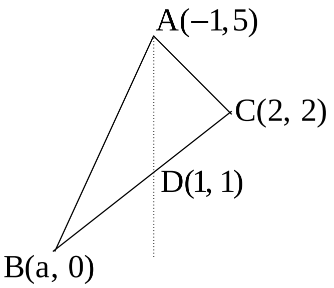
$\therefore \theta=90^\circ$

Q.8

$\frac{y-y_1}{x-x_1}=m$ since y_1 not fixed

$\therefore$ it is a set of parallel lines because slope fixed

Q.9



$\therefore$ slope of AD = – 2

$\therefore$ slope y perpendicular from $B=\frac{1}{2}$

line $\frac{y}{x}=\frac{1}{2}$

Q.10

$$\Rightarrow p=\frac{a}{\sqrt{\sec^2\theta+\cos ec^2\theta}} \ , \ q=\frac{|a\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}$$

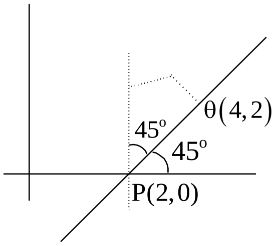
$$\Rightarrow \therefore 4p^2 + q^2 = \frac{4a^2 \sin^2\theta \cos^2\theta}{\sin^2\theta + \cos^2\theta} + \frac{a^2\cos^2 2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= a^2(\cos^2 2\theta + 4\sin^2\theta\cos^2\theta)$$

$$= a^2(\cos^2 2\theta + \sin^2 2\theta)$$

$$= a^2$$

Q.11



slope is 1

Now new slope is tan (90) = ∞

$\therefore$ New line x = 2

Q.12 [C]

use $m_1m_2 = -1$

other diagonal slope – 2

Q.13

concurrency $\Rightarrow$ $\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$

area of (p_1q_1) (p_2q_2) & (p_3q_3) is also zero

$\therefore$ points collinear

Q.14

Let point be (h , k) $\therefore$ h + k = 4

Also $\left|\frac{4h+3k-10}{5}\right|=1$ now 4h + 3k – 10 = 5 or – 5

Solve h = 3 or h = – 7

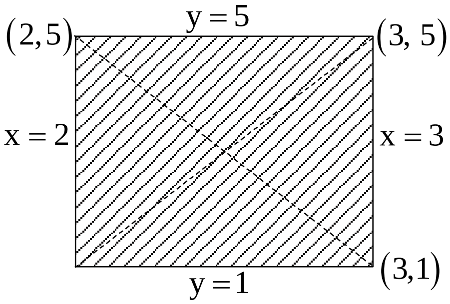
$\Rightarrow$k = 1 or k = 11

Q.15

concurrency = $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

Open the determinant to get $a^3+b^3+c^3-3abc=0$

Q.16



$\Rightarrow D_1 \equiv \frac{y-1}{y-3}=\frac{5-1}{2-3}$

$\Rightarrow D_2 \equiv \frac{y-1}{x-2}=\frac{5-1}{3-2}$

Factorize (x – 2) (x – 3) = 0

$\Rightarrow$x = 2 , x = 3

Similarly,   y = 1 & y = 5

Q.17

Rearrange $\cos\theta(2x+3y-5)+\sin\theta(3x-5y+2)=0$ in the form $\mu L_1+\lambda L_2=0$

$\therefore$ Passes through a fixed point of intersection $L_1=0$ & $L_2=0$

$$\left.\begin{aligned}2x+3y&=5\\3x-5y+2&=0\end{aligned}\right\}\quad \text{solve}\quad \begin{aligned}x&=1\\y&=1\end{aligned}$$

Q.18

$\Rightarrow 2b = a + c$

$\Rightarrow a - 2b + c = 0 \Rightarrow (1\,,\,-2)$

satisfies $\boxed{ax + by + c = 0}$

Q.19

put 3a + 2b = 13 solution for a < b

$\Rightarrow 4b - a = 5$

$\Rightarrow b = 2\,,\ a = 3$

$\therefore$ line is

$$\Rightarrow \frac{y-3}{x-2} = -1$$

$\Rightarrow x + y - 5 = 0$

Q.20

we can write equation as,

$$\Rightarrow \left.\begin{aligned}ax - ay &= p\\bx - by &= p\\cx - cy &= p\end{aligned}\right\}\ \text{they are}$$

$\Rightarrow \therefore$ they don’t intersect

Q.21

solve the lines we get x = 2 , y = – 7

line is $\boxed{\frac{y+7}{x-2} = 12}$ if it satisfies

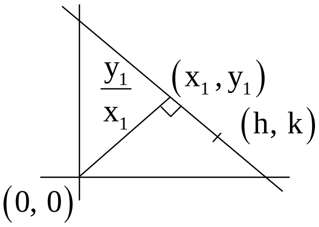
The mid-point of (0, 0) & (3, 4)

$\therefore$ it is equidistant

put (4 , 17)

$\Rightarrow \frac{17+7}{4-2} = 12$. Hence equidistant

Q.22



slope of line is = $-\frac{x_1}{y_1}$

equation of line is = $\frac{y-y_1}{x-x_1} = -\frac{x_1}{y_1}$

Now (h , k) satisfies

$\Rightarrow hx_1 + ky_1 = x_1^2 + y_1^2$

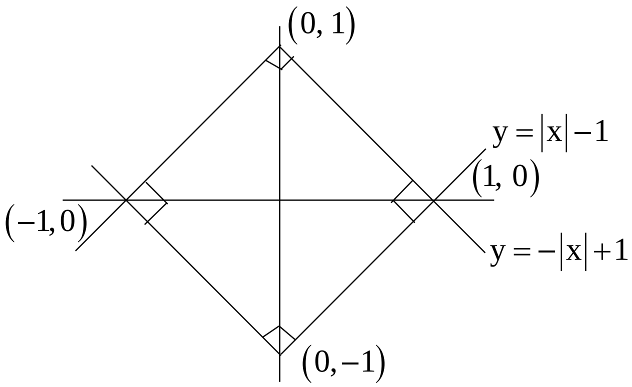
$\Rightarrow yy_1 - y_1^2 = -xx_1 + x_1^2$

$\Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$

Put $x_1 \to x$

$\Rightarrow y_1 \to y$

Q.23



It is a square with side $\sqrt{2}$

$\therefore$ Area $a^2 = 2$

Q.24

put origin in L_1 we get positive sign

put origin in L_2 we get negative sign

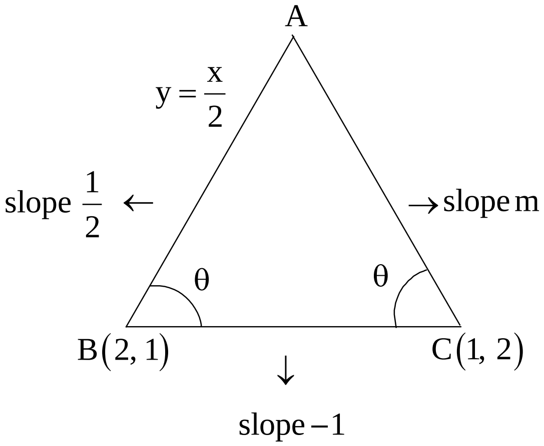
$\therefore$ for $(a^2, a+1)$ to lie in same region nit should follow the same sign

$\Rightarrow 3a^2 - (a+1) + 1 > 0$ & $a^2 + 2(a+1) - 5 < 0$

$\Rightarrow a \in (-\infty\ ,\ 0) \cup (\frac{1}{3}\ ,\infty) \cap (-3,1)$

$\Rightarrow \boxed{(-3,0) \cup (\frac{1}{3}\ ,\ 1)}$

Q.25



Equality angle between lines

$$\Rightarrow \frac{\frac{1}{2}+1}{1+\frac{1}{2}} = \frac{-1-m}{1-m}$$

solve for m we get m = 2

line AC $\frac{y-2}{x-1} = 2$

Q.26

let line be $2x + 6y + \lambda = 0$

Intercept $= \sqrt{x_{int}{}^2 + y_{int}{}^2} = 10$

$$= \frac{\lambda^2}{4} + \frac{\lambda^2}{36} = 100$$

$$\Rightarrow 10\lambda^2 = 36 \times 100$$

$$\Rightarrow \boxed{\lambda = \pm 6\sqrt{10}}$$

2 lines

Q.27

$\Rightarrow \sqrt{3}x - y = 4\sqrt{3}k$ ....(i)

$\Rightarrow \sqrt{3}kx + ky = 4\sqrt{3}$ ....(ii)

eliminate k

$$\Rightarrow (\sqrt{3}x + y)(\frac{\sqrt{3}x - y}{4\sqrt{3}}) = 4\sqrt{3}$$

$\Rightarrow 3x^2 - y^2 = 48$ clearly a hyperbola.

Q.28

put $m_1 m_2 = -1$

$$\Rightarrow \frac{-1}{(a-1)} \times \frac{-2}{a^2} = -1$$

$$\Rightarrow -2 = a^2 - a^2 \text{ or } a^3 - a^2 + 2 = 0$$

Clearly , $a = -1$ is a root $\therefore$ factorize

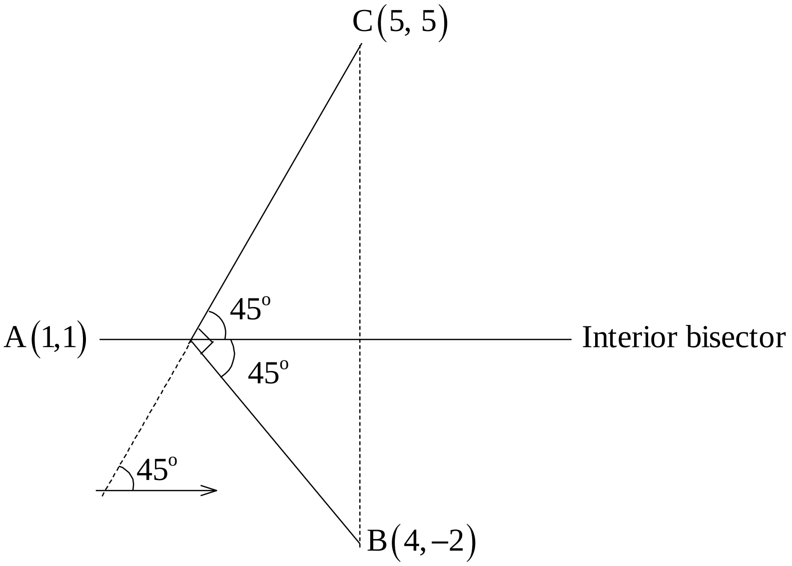
$\Rightarrow (a+1)\ (a^2 - 2a + 2) = 0$ $\therefore$ $a = -1$ is the only answer

Q.29 **[A]**

$$\Rightarrow \alpha = \left|\frac{4}{\sqrt{2}}\right| \ \&\ \beta = \left|\frac{5 + \frac{4}{2}}{5}\right| = \frac{11}{10}$$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{20\sqrt{2}}{11}$$

Q.30



As $m_1 m_2 = -1$

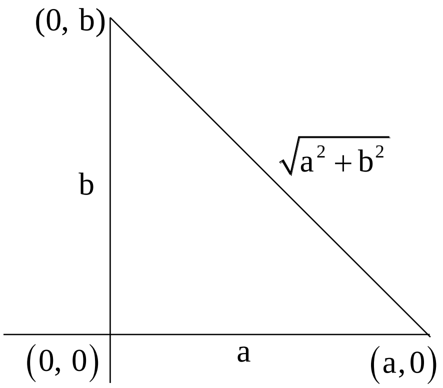
AC perpendicular AB

since slope angle of AC = 45° & slope angle of BA135°

we can see that the interior bisector is parallel to x – axis

$\therefore$ equation x = 5

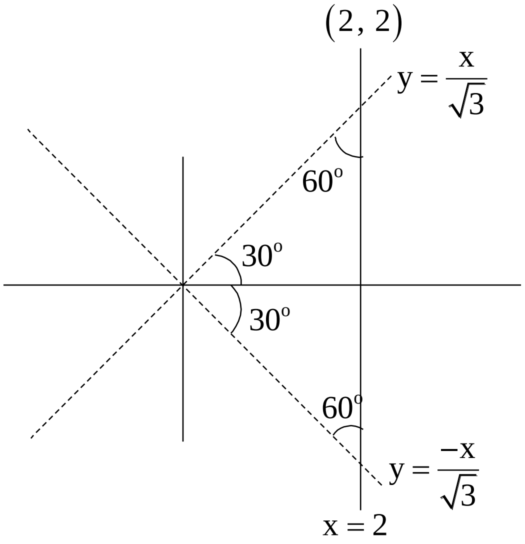
Q.31



are in center formula

$$\Rightarrow\left(\frac{ab}{a+b+\sqrt{a^2+b^2}},\frac{ab}{\sqrt{a^2+b^2}a+b}\right)$$

Q.32



we see diagramatically if we drop a vertical line from (2, 2), it becomes equilateral

$\Rightarrow\therefore$ x = 2

Q.33

$\therefore$ equation of line is

$$\Rightarrow \frac{y-k'}{x-h} = \frac{-h}{k'}$$

or $hx + k'y = h^2 + k'^2$

But line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\therefore \text{ complex } \frac{h}{\frac{1}{a}} = \frac{k'}{\frac{1}{b}} = h^2 + k'^2$$

Put a & b into $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{k^2}$

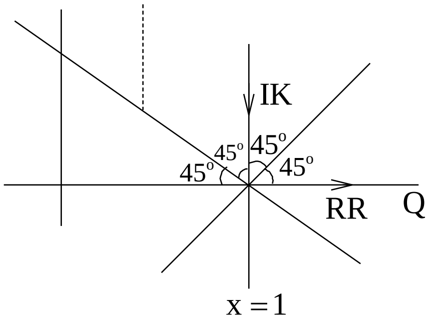
$$\Rightarrow \boxed{\frac{h^2}{(h^2+k'^2)} + \frac{k'^2}{(h^2+k'^2)} = \frac{1}{k^2}}$$

Replace $h \to x$

$k' \to y$

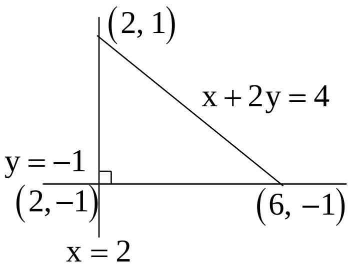
$\Rightarrow \therefore$ equation $x^2 + y^2 = k^2$

Q.34



by law of reflection & diagram we can see it is the x – axis

Q.35

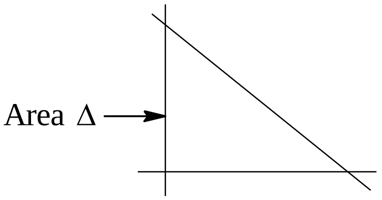


clearly in triangle angled Δ circomcentre is mid-point of hyperbola

$\Rightarrow$ (4, 0)

Q.36

it means $r^2 = pq$



$$\Rightarrow x \text{ is } = \frac{-r}{p}$$

$$\Rightarrow y \text{ is } = \frac{-r}{q}$$

$$\Rightarrow \frac{1}{2} \times \beta \times k$$

$$\Rightarrow \frac{1}{2} \times \left| \frac{r^2}{pq} \right| = \frac{1}{2} = \text{ constant}$$

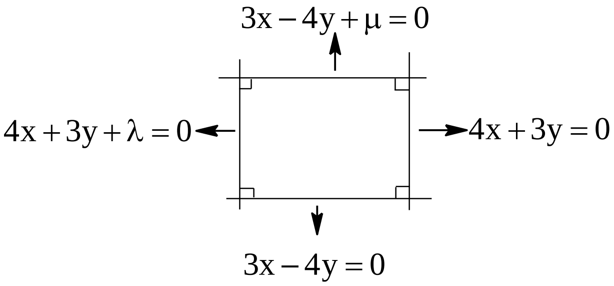
Q.37

rearrange $(4a - 5b)^2 = c^2$

$\Rightarrow 4a - 5b + c = 0$ or $-4a + 5b + c = 0$

$\Rightarrow$ (4, – 5) & (– 4, 5) lie on line

Q.38 **[A]**



Now area = 25

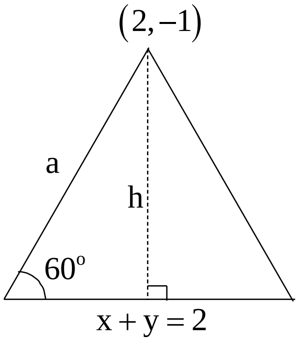
$\therefore$ side = 5

$\therefore$ distance between parallel line 5

$$\Rightarrow \left|\frac{\lambda}{5}\right|=5 \ \& \ \left|\frac{\mu}{5}\right|=5$$

$$\Rightarrow \therefore \lambda = \pm 5 \ \& \ \mu = \pm 5$$

Q.39 **[B]**



$$\Rightarrow A=\frac{\sqrt{3}}{4}\times\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2$$

$$\Rightarrow A=\frac{1}{2\sqrt{3}}$$

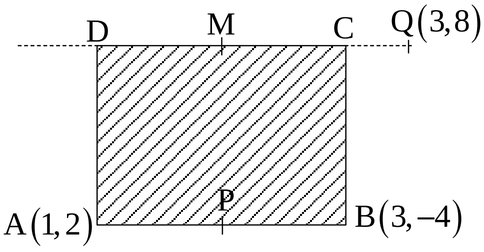
$$\Rightarrow \text{side is } \frac{h}{a}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow a=\frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow h = \left(\frac{2-1-2}{\sqrt{2}} \right)$$

$$\Rightarrow h = \frac{1}{\sqrt{2}}$$

Q.40



MP perpendicular AB

And MQ parallel to AB

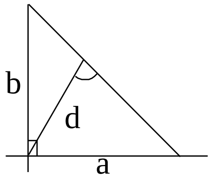
$\therefore$ Only one point satisfies

Q.41

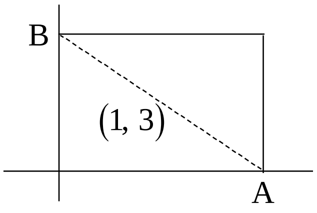
single origin is same

$\therefore$ perpendicular distance from origin should be same after rotation.

we know $\frac{1}{a^2} = \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$



Q.42



Let (h, k) be focus

For rectangular A (h, 0), B (0, k)

and line AB is $\Rightarrow \frac{x}{h}+\frac{y}{k}=1$

Put (1 , 3)

$\Rightarrow \boxed{\frac{1}{h}+\frac{3}{k}=1}$ $\quad h \rightarrow x$

$k \rightarrow y$

Q.43 **[B]**

$3x+3y+7=0$

$$\Rightarrow \therefore \frac{-x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=\frac{7}{3\sqrt{2}}$$

$$\Rightarrow x\cos\left(\frac{5\pi}{4}\right)+4\sin\left(\frac{5\pi}{4}\right)=\frac{7}{3\sqrt{2}}$$

Q.44

Let line be $bx+ay+\lambda=0$

It also passes through where the given line cuts $x-\text{axis}$ at point $(a\,,0)$

$\Rightarrow \therefore \boxed{\lambda=-ab}$

STRAIGHT LINES

Exercise – 1(C)

Q.1

Slope of line is $\Rightarrow \dfrac{\sqrt{3}-0}{-2-1} = \dfrac{-1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{-1}{\sqrt{3}}$, $\theta = 150^o$

Q.2

use slope – intercept form

y = 2x – y

Q.3

$\Rightarrow m = \dfrac{3}{5}$ & c = – 3

$\therefore$ slope intercept form $y = \dfrac{3}{5}x - 3$

Q.4

$\theta = 60^o$, p at distance 3 units is

By parametric equation $\left(1 \pm 3\cos 60^o, 2 \pm 3\sin 60^o\right)$

$\Rightarrow \left(\dfrac{5}{2}, 2+\dfrac{3\sqrt{3}}{2}\right)$ or $\left(-\dfrac{1}{2}, 2-\dfrac{3\sqrt{3}}{2}\right)$

Q.5

slope is zero

so, equation of line is y = 2

Q.6

$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1$ or x + y = a

Put (1, – 2) to get ‘a’

$\Rightarrow 1 - 2 = a$

$\Rightarrow a = -1$

Line is x + y + 1 = 0

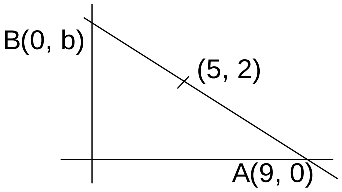
Q.7

Intercept form $\frac{x}{a} + \frac{y}{b} = 1$; here b = 2a

(1, 2) satisfies $\frac{1}{a} + \frac{2}{b} = 1$

$$\Rightarrow \frac{1}{a} + \frac{1}{a} = 1 \Rightarrow \begin{pmatrix} a = 2 \\ b = 4 \end{pmatrix}$$

Q.8



$\Rightarrow \therefore \frac{a}{2} = 5$ & $\frac{b}{2} = 2$

$\Rightarrow$ a = 10, b = 4

$\Rightarrow \therefore$ line $\frac{x}{10} + \frac{y}{4} = 1$

Q.9

Point of intersection of 3x – 2y – 1 = 0 and x – 4y + 3 = 0 is (1 , 1)

So, line passing through (1 , 1) and $(\pi, 0)$ is

$\Rightarrow (y-1) = \frac{1-0}{1-\pi}(x-1) \;\; \Rightarrow \;\; x - y = \pi(1-y)$

Q.10

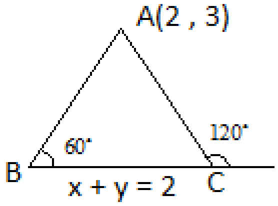
x + y = 2

$\Rightarrow$slope = – 1

i.e. angle w.r.t. x – axis is 135^o

Therefore AG will make angle = 135 + 120 = 255

$\Rightarrow$slope $= \tan^{-1}(255) = 2+\sqrt{3}$



$\Rightarrow\; y-3=(2+\sqrt{3})(x-2)$ is equation of AC

And AB will make angle $=135+60=195^\circ$

$\Rightarrow$slope $= \tan^{-1}(195) = 2-\sqrt{3}$

$\therefore$ equation of AB is $\;y-3=2-\sqrt{3}(x-2)$

$\Rightarrow\; y-3=2\pm\sqrt{3}(x-2)$ is the equation of sides

Q.11

$\Rightarrow p = \left|\frac{2}{\sqrt{3+1}}\right| = 1$

Q.12

$\Rightarrow \frac{x}{3a} + \frac{y}{3b} = 1$

A(3a, 0) ; B(0, 3b) ; 0(0, 0)

Centroid (a , b)

Q.13

solve $x + y = 2$ ....$x = 1, y = 1$

$\Rightarrow 2x - y = 1$

$\therefore$ line is $\boxed{\frac{y}{x} = 1}$

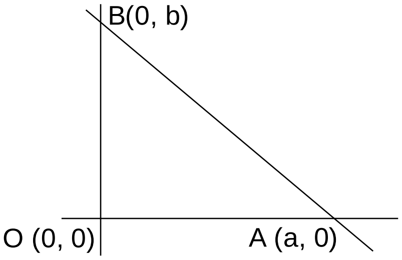
Q.14

$x\cos\alpha + y\sin\alpha = a$

$\therefore$ for y intercept put x = 0

$\therefore \quad y = a\,\text{cosec}\,\alpha$

Q.15



$\therefore$ Area of $\Delta OAB = \frac{1}{2}ab$

Q.16

form $\frac{y-0}{x-0} = \frac{a\sin\theta - 0}{a\cos\theta - 0} \Rightarrow y = x\tan\theta$

Q.17

$\Rightarrow m_1 = 0 \text{ , } m_2 = 1$

$\Rightarrow \therefore \tan\theta = 1$

$\Rightarrow \theta = 45^o$

So obtuse angle between the lines = $180^o - 45^o = 135^o$

Q.18

$\Rightarrow m_1 = 2 \text{ , } m_2 = -3$

$\Rightarrow \tan\theta = \left|\frac{2-(-3)}{1-6}\right| = 1 \quad \therefore \theta = \frac{\pi}{4}$

Q.19

$\Rightarrow m_1 = 2 \qquad \therefore m_2 = -\frac{1}{2}$

$\Rightarrow \frac{\lambda-3}{-2} = -\frac{1}{2}$

$\Rightarrow \lambda = 4$

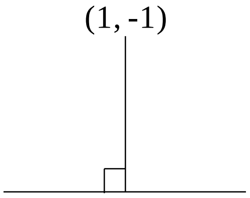
Q.20

$\Rightarrow m_1 = 0, \ m_2 = \sqrt{3}$

$\Rightarrow \therefore \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60^{o}$

Q.21



$\Rightarrow 2x - 3y - 5 = 0$

$\Rightarrow m = -\frac{3}{2}$

Equation $\frac{y+1}{x-1} = -\frac{3}{2}$

$\Rightarrow 2y + 2 = -3x + 3$

$\Rightarrow 3x + 2y - 1 = 0$

Q.22

Interface point (– 3, 3)

$\therefore$ line is $\frac{y-3}{x+3}=2$

Q.23

Let line be $\left(4x-3y+1\right)+\lambda\left(5x-2y-3\right)=0$ ……….(1)

Its slope should be $\frac{2}{3}$

$\Rightarrow\frac{-(4+5\lambda)}{-3-2\lambda}=\frac{2}{3}$

get λ & put it back in (1)

Q.24

slope $=-\frac{a}{b}$

$\Rightarrow$equation $\frac{y-d}{x-c}=\frac{-a}{b}$

Q.25

Let the line be $\left(3x+y-5\right)+\lambda\left(x-y+1\right)=0$ …………………………(1)

It’s slope can be 1 or – 3

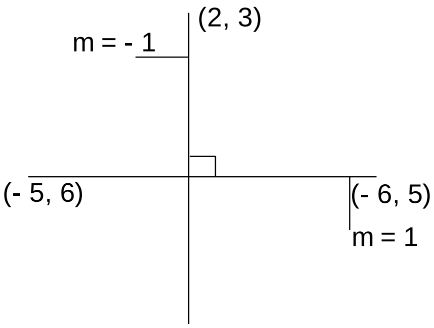
$\Rightarrow\therefore-\frac{\left(\lambda+3\right)}{1-\lambda}=1$ or – 3

Solve for λ & put in equation (1)

Q.26

slope $=\frac{b}{a}$: $y-b=\frac{b}{a}(x-a)$

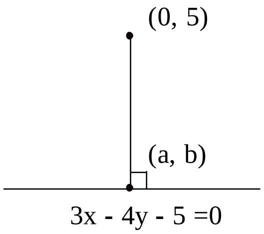
Q.27



Equation y – 3 = – 1 (x – 2)

$\Rightarrow$x + y – 5 = 0

Q.28 [D]



$\Rightarrow$3a – 4b – 5 = 0 ……(1)

$$\Rightarrow \left(\frac{5-b}{0-a}\right)\left(\frac{3}{4}\right) = -1$$

$\Rightarrow$3b – 15 = – 4a

$\Rightarrow$4a + 3b = 15 ……(2)

by (1) & (2)

$\Rightarrow$a = 3, b = 1

Q.29

let line be $x + 5y = \lambda$

$$\Rightarrow x_{in} = \lambda_1; y_{in} = \frac{\lambda}{5}$$

$$\Rightarrow \text{Area } = \frac{1}{2} \times \lambda \times \frac{\lambda}{5} = 5$$

$\Rightarrow \lambda \pm 5\sqrt{2}$

Q.30

$$\Rightarrow p = \left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right| = \left|\frac{4 - \left(-\frac{5}{3}\right)}{\sqrt{5}}\right| = \frac{17}{3\sqrt{5}}$$

Q.31

$12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$

For pair of straight lines, $12x^2 + 7xy - py^2 = 0$ should have two factors.

$$\Rightarrow -p\left(\frac{y}{x}\right)^2 + 7\left(\frac{y}{x}\right) + 12 = 0$$

$\Rightarrow$ D = 49 – 4 (12) (– P) = perfect square.

$\Rightarrow$ 49 + 48p = perfect square

$\Rightarrow$ p = – 1

So, $12x^2 + 7xy + y^2 - 18x + qy + 6 = 0$

Now, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow (12)(1)(6) + 2\left(\frac{q}{2}\right)(-9)\left(\frac{7}{2}\right) - (12)\left(\frac{q}{2}\right)^2 - (-9)^2 - 6\left(\frac{7}{2}\right)^2 = 0$$

$$\Rightarrow q = -5, -\frac{11}{2}$$

Q.32

$$\Rightarrow m_1 = -\frac{m}{2}\ ,\ m_2 = -\frac{2}{3}$$

$\Rightarrow m_1 m_2 = -1$

$$\Rightarrow \frac{m}{3} = 1 \Rightarrow m = 3$$

Q.33

Bisectors are

$$\Rightarrow \frac{3x-4y+7}{5}=\pm\frac{(12x-5y-8)}{13}$$

Simplify

Q.34

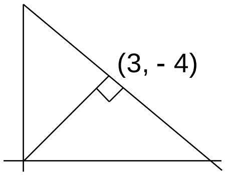
$$\Rightarrow x\cos\alpha_1+y\sin\alpha_1=p_1 \quad ; \Rightarrow x\cos\alpha_2+y\sin\alpha_2=p_2$$

$$\Rightarrow \text{slope } m_1=\tan\alpha_1 \qquad ; \Rightarrow m_2=\tan\alpha_2$$

$$\Rightarrow \tan\theta=\frac{m_1-m_2}{1+m_1m_2}=\frac{\tan\alpha_1-\tan\alpha_2}{1+\tan\alpha_1\tan\alpha_2}=\tan(\alpha_1-\alpha_2)$$

$$\Rightarrow \text{angle } \theta=\alpha_1-\alpha_2$$

Q.35



$\Rightarrow$slope of the perpendicular line from origin = $-\frac{4}{3}$

$\Rightarrow$Now, slope of the line = $\frac{3}{4}$

Equation : $\frac{y+4}{x-3}=\frac{3}{4}$

$$\Rightarrow 4y+16=3x-9$$

$$\Rightarrow 3x-4y=25$$

Q.36

$$x^2+2xy-y^2=0$$

$\Rightarrow$ h = 1, a = 1, b = – 1

$$\Rightarrow \theta = \tan^{-1}\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = 90^o$$

Q.37 [A]

$\Rightarrow$ x – y + 1 = 0 ……(1)

$\Rightarrow$ 3x + y – 5 = 0 ……(2)

Equation of line passing through (1) & (2) is

$$\Rightarrow (x - y + 1) + \lambda(3x + y - 5) = 0$$

$$\Rightarrow x(1+3\lambda) + y(\lambda - 1) + 1 - 5\lambda = 0$$

if (1) & (3) are perpendicular

$$\Rightarrow -\left(\frac{1+3\lambda}{\lambda - 1}\right)(1) = 1$$

$$\Rightarrow \lambda = -1$$

If (2) & (3) are parallel

$$\Rightarrow -\left(\frac{1+3\lambda}{\lambda - 1}\right)(-3) = -1$$

$$\Rightarrow \lambda = -\frac{1}{5}$$

So equation of lines are

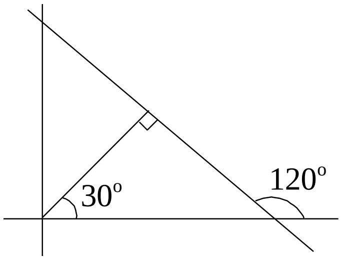
$\Rightarrow$ (x – y + 1) – 1 (3x + y – 5) = 0

$\Rightarrow$ 2x + 2y – 6 = 0

$\Rightarrow$ x + y – 3 = 0

$$\Rightarrow (x - y + 1) - \frac{1}{5}(3x + y - 5) = 0$$

Q.38

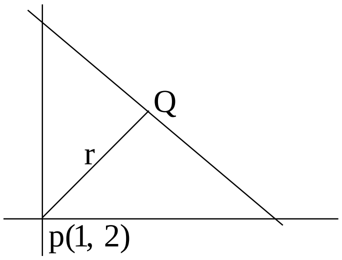


slope angle is 120°

$\Rightarrow m = \tan 120^{o} = -\sqrt{3}$

Only option (B)

Q.39



Let Q by parametric equation of Line PQ

$\Rightarrow Q(1 + r\cos 45^{o}, 1 + r\sin 45^{o})$

Now Q satisfies line $x + 2y - 7 = 0$

$\therefore$ get r

Q.40 [D]

$3x - 4y + 7 = 0$

$\Rightarrow - 12x - 5y + 2 = 0$

$\Rightarrow a_1a_2 + b_1b_2 = -36 + 20 = -ve$

Equation of acute angle bisector is

$$\Rightarrow \left(\frac{3x - 4y + 7}{5}\right) = \left(\frac{-12x - 5y + 2}{13}\right)$$

$\Rightarrow 99x - 27y + 81 = 0$

Q.41

$\Rightarrow 2x + 3y - 4 = 0\ ; 6x + 9y + 8 = 0$

$\Rightarrow (2\times 8 + 3(-9) - 4)\ (6(8) + 9(-9) + 8) = (-15)(-25) > 0$

$\Rightarrow \therefore\ (8, -9)$ will lies on the same side of lines.

Q.42

Rearrange $\cos\theta(2x + 3y - 5) + \sin\theta(3x - 5y + 2) = 0$ in the form $\mu L_1 + \lambda L_2 = 0$

$\therefore$ Passes through a fixed point of intersection $L_1 = 0$ & $L_2 = 0$

$$\left.\begin{array}{l} \Rightarrow 2x + 3y = 5 \\ \Rightarrow 3x - 5y + 2 = 0 \end{array}\right\} \quad \text{solve} \quad \begin{array}{l} x = 1 \\ y = 1 \end{array}$$

Q.43

$\Rightarrow$ a + c = 2b or $a - 2b + c = 0$

We can observe that point (1, – 2) notifies ax + by + c = 0 in respective of any value of a, b or c

Q.44

$\Rightarrow x^2 - y^2 - 2y - 1 = 0$

Angle between pairs of lines $\Rightarrow \tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a + b}$

$$\Rightarrow \tan\theta = \frac{2\sqrt{0 - 1}}{1 - 1} = \infty \Rightarrow \theta = 90^\circ$$

Q.45

$\Rightarrow x^2 + y^2 + 2gx + 2fy + 1 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0 \qquad \begin{array}{l} \Rightarrow 1(1 - f^2) - 0(0 - gf) + g(0 - g) = 0 \\ \Rightarrow 1 - f^2 - g^2 = 0 \ \Rightarrow\ f^2 + g^2 = 1 \end{array}$$

Q.46

$$\Rightarrow x^2 + xy = 0$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{\frac{1}{4} - 0}}{1+0} = \frac{2\times\frac{1}{2}}{1} = 1$$

$$\Rightarrow \theta = 45^\circ$$

Q.47

$$\Rightarrow ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right)$$

Q.48

$$\Rightarrow \lambda x^2 + 2y^2 - 5xy + 5x - 7y + 3 = 0$$

Represent pair of lines if $\beta = 0$

$$\Rightarrow \begin{vmatrix} \lambda & -\frac{5}{2} & \frac{5}{2} \\ -\frac{5}{2} & 2 & -\frac{7}{2} \\ \frac{5}{2} & -\frac{7}{2} & 3 \end{vmatrix} = 0$$

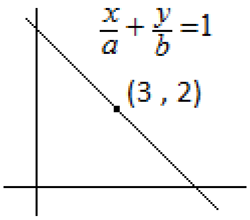
$$\Rightarrow \lambda(6 - \frac{49}{4}) + \frac{5}{2}\left(\frac{-13}{2} + \frac{35}{4}\right) + \frac{5}{2}\left(\frac{35}{4} - 5\right) = 0$$

$$\Rightarrow \lambda\left(\frac{24-49}{4}\right) + \frac{5}{2}\left(\frac{5}{2}\right) + \frac{5}{2}\left(\frac{13}{4}\right) = 0$$

$$\Rightarrow -50\lambda + 100 = 0$$

$\Rightarrow \lambda = +2$

Q.49



Line passing through (3, 2)

such that a + b = 10

let line be $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow$a + b = 10, b = 10 – a

$\Rightarrow \frac{x}{a} + \frac{y}{10 - a} = 1$

Point (3 , 2) satisfies this

$\Rightarrow \frac{3}{a} + \frac{2}{10 - a} = 1$

$\Rightarrow$3(10 – a) + 2a = a (10 – a)

$\Rightarrow 30 - 3a + 2a = 10a - a^2$

$\Rightarrow a^2 - 11a + 30 = 0$

$\Rightarrow$a = 5 , 6

$\therefore$ equation of the line is

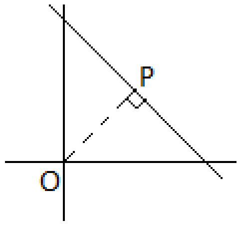
$\Rightarrow \frac{x}{5} + \frac{y}{10-5} = 1$ & $\frac{x}{6} + \frac{y}{10-6} = 1$

$\Rightarrow$x + y = 5 & $\frac{x}{6} + \frac{y}{4} = 1$

Q.50

$\Rightarrow$slope = – 2

slope of normal from origin is $\frac{1}{2}$



slope of OP = $\frac{1}{2}$

$\Rightarrow \frac{1}{2} = \tan\alpha$ α ranges from $[0\ ,\ 2\pi]$

So $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\sin\alpha = \frac{-1}{\sqrt{5}}$

and $\cos\alpha = \frac{2}{\sqrt{5}}$ or $\cos\alpha = \frac{-2}{\sqrt{5}}$

$\Rightarrow \therefore\ \ x\cos\alpha + y\sin\alpha = p$

$\Rightarrow \frac{2x}{\sqrt{5}} + \frac{y}{\sqrt{5}} = 4\sqrt{5}$ or $\frac{-2x}{\sqrt{5}} - \frac{y}{\sqrt{5}} = 4\sqrt{5}$

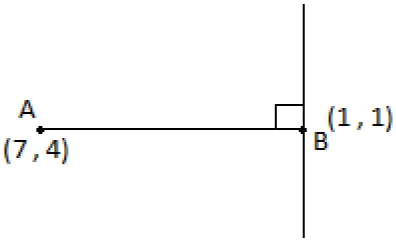
$\Rightarrow 2x + y = 20$ or $-2x - y = 20$

$2x + y = -20$

STRAIGHT LINES

Exercise – 2(A)

Q.1.



$(2x+3y-5)+\lambda(x+y-2)=0$ represents family of straight lines passing through

intersection of

2x + 3y – 5 = 0 i.e. (1 , 1)

x + y – 2 = 0

the straight line furthest away from (7 , 4) with be perpendicular to the line

joining A (7 , 4) and B (1 , 1)

slope of line : $(2x+3y-5)+\lambda(x+y-2)=0$

i.e. $(2+\lambda)x+(3+\lambda)y-(5+2\lambda)=0$

is $\dfrac{-(2+\lambda)}{(3+\lambda)}$

slope of line is $\dfrac{-(2+\lambda)}{(3+\lambda)}$

slope of line joining (7 , 4) and (1 , 1) is $\dfrac{4-1}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$

$\therefore$ slope of line perpendicular to AB = – 2

$= \dfrac{-(2+\lambda)}{(3+\lambda)}$

$\therefore \dfrac{2+\lambda}{3+\lambda} = 2$

$$2+\lambda=6+2\lambda$$

$\lambda=-4$ (C)

Q.2. Line equidistant from 2x + y = 5 and x + 2y = 4 is the angle bisector of the 2 lines.

i.e.

$$\left(\frac{2x+y-5}{\sqrt{5}}\right)=\pm\left(\frac{x+2y-4}{\sqrt{5}}\right)$$

$$(2x+y-5)\pm(x+2y-4)=0$$

i.e. 3x + 3y – 9 = 0 or x – y – 1 = 0

i.e. x + y – 3 = 0 or x – y – 1 = 0 (A)

Q.3. 3x + y + 2 = 0

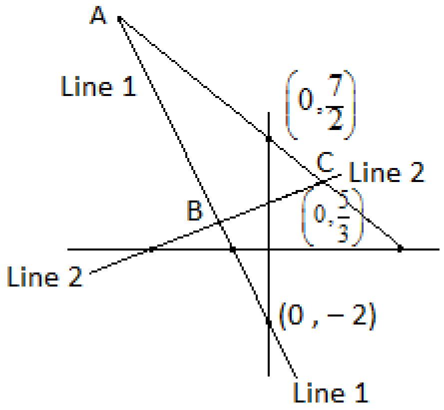
$$\frac{x}{-\frac{2}{3}}+\frac{y}{-2}=1$$

2x – 3y + 5 = 0

$$\frac{x}{-\frac{5}{2}}+\frac{y}{\frac{5}{3}}=1$$

x + 4y = 14

$$\frac{x}{14}+\frac{y}{\frac{7}{2}}=1$$

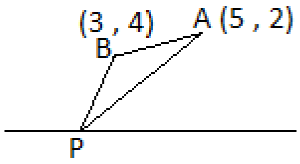


2y – intercepts $\left(0\ ,\ \frac{5}{3}\right)$ and $\left(0\ ,\ \frac{7}{2}\right)$

So point $(0\ ,\ \lambda)$

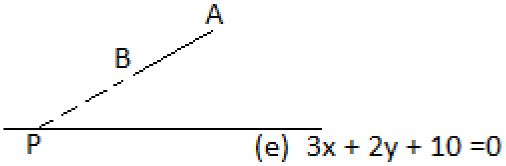
$$\frac{5}{3}<\lambda<\frac{7}{2}$$ (B)

Q.4. $|PA-PB|$



$AB > |PA - PB|$ (triangle law)

So PA – PB will be max if PBA are collinear, then PA = PB = AB



Slope of AB $= \frac{2-4}{5-3} = -1$

Equation of AB ≡ (y – 2) = (– 1) (x – 5)

y – 2 = 5 – x

y = 7 – x ………………….(1)

Intersection of 3x + 2y + 10 = 0 and (AB) x + y – 7 = 0

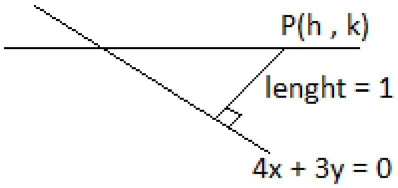
$$\frac{x}{\begin{vmatrix} 2 & 10 \\ 1 & -7 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 10 \\ 1 & -7 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{\lambda}{-24} = \frac{-y}{-31} = \frac{1}{1}$$

x = – 24 , y = 31

(x , y) ≡ (– 24 , 31) (A)

Q.5. Point P lies on x + y = 4



$$\left| \frac{4h+3k}{\sqrt{4^2+3^2}} \right| = 1 \qquad \begin{bmatrix} h+k=4 \because \text{point lies} \\ \text{on x + 4 = 4} \\ \therefore\ \text{k = 4 – h} \end{bmatrix}$$

$$\left|\frac{4h+3(4-h)}{5}\right|=1$$

$$|4h+12-3h|=5$$

$$|12+h|=5$$

$$12+h=\pm 5$$

$$h=-12 \pm 5$$

h = − 7 , − 17

If h = −7 , h = −17

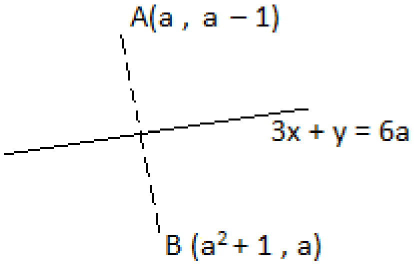
k = 4 −h , k = 21

k = 11

(−7 , 11) (−17,21)

Ans. : (D)

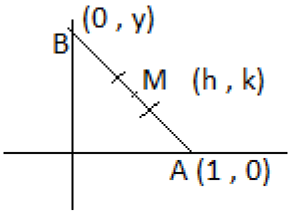
Q.6.



Using formula for reflection about line

Q.7. A (1 , 0)

B (0 , α)

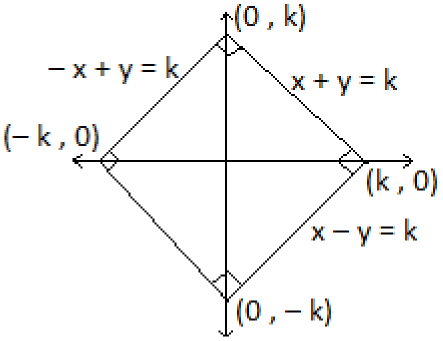


$M \equiv \left(\frac{1+0}{2}, \frac{\alpha+0}{2}\right)$ $\therefore h = \frac{1}{2}$ and $k = \frac{\alpha}{2}$

$M \equiv \left(\frac{1}{2}, \frac{\alpha}{2}\right)$ $\therefore x = \frac{1}{2}$ is the locus of M

Ans. : (A)

Q.8. $|x| + |y| = k$



Length of the side $= \sqrt{(k-0)^2 + (0-k)^2}$

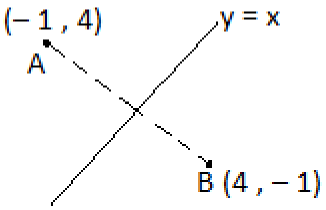
$= \sqrt{2k}$

So Area of square $= 2k^2$ = 8 sq. unit.

$\therefore k^2 = 4$

k = 2 (C)

Q.9.



A is (− 1 , 4)

Distance AB $= \sqrt{(-1-4)^2 + (4-(-1))^2}$

$$=\sqrt{25+25}$$

$$=5\sqrt{2}$$ sq. unit. Ans. : (A)

Q.10. a + c = 2b c = 2b – a

ax + by + c = 0

ax + by + 2b – a = 0

a (x – 1) + b (y + 2) = 0

$$(x-1)+\frac{b}{a}(y+2)=0$$

Represents family of straight lines passing through the intersection of the 2 lines

x – 1 and y = – 2

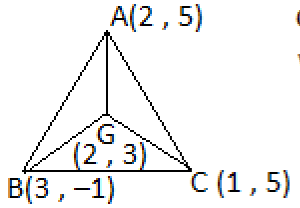
i.e. x = 1 and y = – 2

it of intersection is (1 , – 2)

So, ax + by + c = 0 always passes through (1 , – 2)

Ans. : (C)

Q.11. G is centroid of
Δ ABC



Geometrically the areas will be equal but
we will find the areas none the loss

$$S_1=\Delta GBC=\frac{1}{2}\begin{vmatrix}2 & 3 & 1 & 2\\ 3 & -1 & 5 & 3\end{vmatrix}$$

$$=\frac{1}{2}\left|(-2+15+3)-(9-1+10)\right|$$

$$=\frac{1}{2}\left|16-(18)\right|=1$$ sq. units.

$$S_3=\Delta GBA=\frac{1}{2}\begin{vmatrix}2 & 2 & 3 & 2\\ 5 & 3 & -1 & 5\end{vmatrix}$$

$$=\frac{1}{2}\left|(6-2+15)-(10+9-2)\right|$$

$$=\frac{1}{2}\left|19-17\right|=1 \text{ sq. units.}$$

$$S_2=\Delta GAC=\frac{1}{2}\begin{vmatrix}2 & 1 & 2 & 2\\ 5 & 5 & 3 & 5\end{vmatrix}$$

$$=\frac{1}{2}\left|(10+3+10)-(5+10+6)\right|$$

$$=\frac{1}{2}\left|23-21\right|=1 \text{ sq. units.}$$

$S_1=S_2=S_3$ Ans. : (D)

Q.12. (p + q) x + (2p + q) y = p + 2q

p (x + 2y – 1) + q (x + y – 2) = 0

This represents a family of straight lines passing through the intersection of

x + 2y – 1 = 0 &

x + y – 2 = 0

i.e.

$$\frac{x}{\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}}=\frac{-y}{\begin{vmatrix}1 & -1\\ 1 & -2\end{vmatrix}}=\frac{1}{\begin{vmatrix}1 & 2\\ 1 & 1\end{vmatrix}}$$

$$\frac{x}{-3}=\frac{-y}{-1}=\frac{1}{-1}$$

x = 3 , y = – 1

(3 , – 1)

x_1 = 3 , y_1 = – 1

x_3 + y_1 = 2 Ans. : (A)

Q.13. $2x^2+5x-4=0:\alpha,\ \beta$

Centroid of $\left(\alpha, \frac{1}{\alpha}\right), \left(\beta + \frac{1}{\beta}\right), (0,0)$

$$= \left(\frac{\alpha + \beta + 0}{3}, \frac{\frac{1}{\alpha} + \frac{1}{\beta} + 0}{3} \right)$$

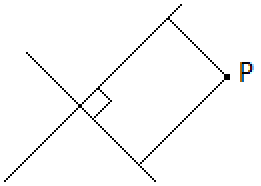
$$\alpha + \beta = \frac{-5}{2}$$

$$\alpha\beta = \frac{-4}{2}$$

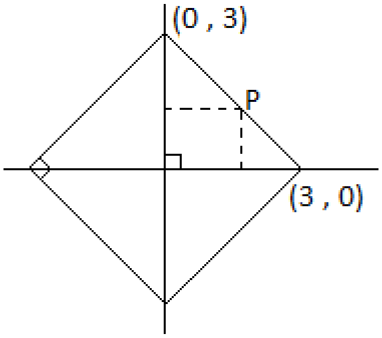
So, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{-5}{2}}{\frac{-4}{2}} = \frac{5}{4}$

Centroid $\equiv \left(\frac{-5}{6}, \frac{5}{12} \right)$ Ans. : (B)

Q.14.



Without loss of generality we can assume the perpendicular lines as the x and y



So the condition becomes

$|x| + |y| = 3$

Area enclosed = Area of square

= $\left(3\sqrt{2}\right)^2$ = 18 sq. units. Ans. : (D)

Q.15. $x=-2+\frac{r}{\sqrt{10}}$, $y=1+\frac{3r}{\sqrt{10}}$

$x=x_1+r\cos\theta$, $y=y_1+r\sin\theta$

Slope = $\tan\theta=\frac{3}{1}$

Point on the line is (– 2 , 1)

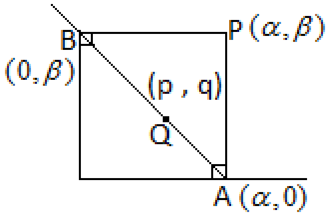
So, equation is

(y – 1) = 3 (x + 2)

y = 3x + 7

y intercept is 7 Ans. : (B)

Q.16.



Hence x – co – ordinate of P is α

y – co – ordinate of P is β

equation of the line (double intercept is)

$\frac{x}{\alpha}+\frac{y}{\beta}=1$

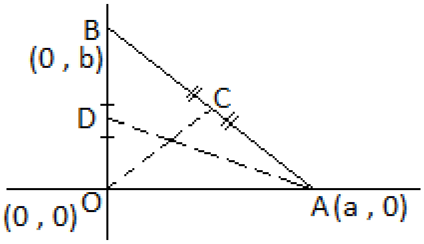
Substituting co – ordinates of Q we get

$\frac{p}{\alpha}+\frac{q}{\beta}=1$

Replacing α,β by x and y to get the equation of P.

$\frac{p}{x}+\frac{q}{y}=1$ Ans. : (C)

Q.17.



Co – ordinates of D $\equiv\left(\frac{0+0}{2}\,,\,\frac{0+b}{2}\right)$

$$\equiv\left(0,\frac{b}{2}\right)$$

Co – ordinates of C $\equiv\left(\frac{0+a}{2},\frac{b+0}{2}\right)$

$$\equiv\left(\frac{a}{2},\frac{b}{2}\right)$$

AD perpendicular OC

Slope of AD $=\frac{0-\frac{b}{2}}{a-0}=\frac{-b}{2a}$

Slope of OC $=\frac{\frac{b}{2}-0}{\frac{a}{2}-0}=\frac{b}{a}$

slope AD × slope BC = − 1

$\frac{-b^2}{2a^2}=-1$ $\therefore\ b^2=2a^2$ Ans. : (B)

Q.18. $x\sec\theta+y\cos ec\theta=a\quad(L_1)$

Perpendicular distance from origin

$$P_1 = \left| \frac{-a}{\sqrt{\sec^2\theta + \cos ec^2\theta}} \right|$$

$$P_2 = |a\cos\theta\sin\theta| = \left| \frac{a\sin^2\theta}{2} \right|$$

$$(L_2) \quad x\cos\theta - y\sin\theta = a\cos 2\theta$$

Perpendicular distance of origin from line

$$P_2 = \left| \frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} \right|$$

$$P_2 = |a\cos 2\theta|$$

$$4P_1^2 + P_2^2 = a^2\cos^2 2\theta + 4\left(\frac{a^2\sin^2 2\theta}{4} \right)$$

$$= a^2$$

Independent of θ Ans. : (A)

Q.19. y = mx (L$_1$) y – mx + 0 = 0

Equation of L$_2$ is (y – 3) = 2m (x – 2)

Y = 2mx – 4m + 3 , y – 2mx + 4m – 3 = 0 (L$_2$)

Intersection is $x = \frac{1}{m}$ and y = 1

$$\frac{x}{\begin{vmatrix} 1 & 0 \\ 1 & 4m-3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -m & 0 \\ -2m & 4m-3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} -m & 1 \\ -2m & 1 \end{vmatrix}}$$

$$\frac{x}{4m-3} = \frac{-y}{-m(4m-3)} = \frac{1}{m}$$

$$x = \frac{4m-3}{m} \quad , \quad y = 4m-3$$

$$\therefore x = 4 - \frac{3}{m} \qquad \frac{y+3}{4} = m \qquad \text{................(ii)}$$

$$\frac{3}{m} = 4 - x$$

$$\frac{3}{4-x} = m \qquad \text{................(i)}$$

Equating (i) and (ii) we get

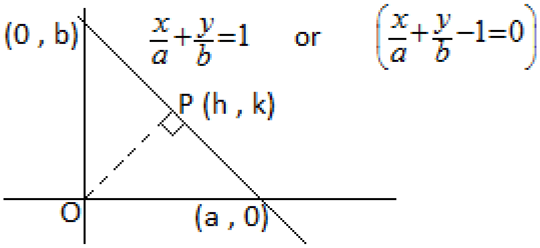
$$\frac{3}{4-x} = \frac{y+3}{4}$$

12= (y + 3) (4 – x)

$$\cancel{12} = 4y - xy + \cancel{12} - 3x$$

$$\boxed{3x - 4y + xy = 0}$$ Ans. : (C)

Q.20.



$$\frac{h-0}{\frac{1}{a}} = \frac{k-0}{\frac{1}{b}} = -\frac{\left(\frac{0}{a}+\frac{0}{b}-1\right)}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$$h = \frac{\frac{1}{a}}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)} = \frac{c^2}{a}$$

$$k = \frac{\frac{1}{b}}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)} = \frac{c^2}{b}$$

$$\therefore\ h^2+k^2 = c^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

$$= c^4\left(\frac{1}{c^2}\right)$$

$$h^2+k^2 = c^2$$

Locus is $\boxed{x^2+y^2=c^2}$ Ans. : (A)

Q.21. $x^2+y^2\le 36$ and 3x – 4y = 25

Let $x = r\cos\theta$

$y = r\sin\theta$

So we get

$$r^2\cos^2\theta + r^2\sin^2\theta \le 36$$

$$r^2 \le 36$$

$$r \le 6$$

and

$$3(r\cos\theta)-4(r\sin\theta)=25$$

$$r(3\cos\theta - 4\sin\theta) = 25$$

$$r = \frac{25}{3\cos\theta-4\sin\theta}$$

$$r \le 6$$

$$\therefore\ \frac{25}{3\cos\theta-4\sin\theta} \le 6$$

$$3\cos\theta - 4\sin\theta \ge \frac{25}{6}$$

$$\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta \ge \frac{5}{6}$$

$$\sin 37^{\circ} \underline{n} \ \frac{3}{5}$$

$\sin(37^{\circ}-\theta) \geq \frac{5}{6}$ no. of angles. Between θ and 360° are infinite.

So infinitely many solutions. Ans. : (D)

Q.22.Let A (x_1 , y_1) , B(x_2 , y_2) , C (x_3 , y_3) be the points.

ax + by + c = 0 be the line

Given

$$\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} + \frac{ax_2+by_2+c}{\sqrt{a^2+b^2}} + \frac{ax_3+by_3+c}{\sqrt{a^2+b^2}}$$

$$a(x_1+x_2+x_3) + b(y_1+y_2+y_3)+3c=0$$

Dividing by 3 we get

$$a\left(\frac{x_1+x_2+x_3}{3}\right) + b\left(\frac{y_1+y_2+y_3}{3}\right)+c=0$$

Co – ordinates of the centroid satisfy

ax + by + c = 0

passes through the centroid Ans. : (B)

Q.23. $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{x}{b}+\frac{y}{a}=1$$

Point of intersection is

$$\frac{x}{\begin{vmatrix} \frac{1}{a} & -1 \\ \frac{1}{b} & -1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} \frac{1}{b} & -1 \\ \frac{1}{a} & -1 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} \frac{1}{a} & \frac{1}{b} \\ \frac{1}{b} & \frac{1}{a} \end{vmatrix}}$$

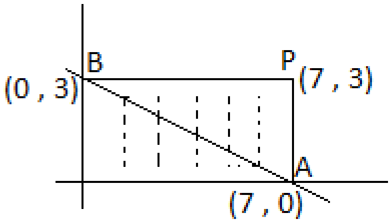
$$\frac{x}{\left[\frac{1}{b}-\frac{1}{a}\right]}=\frac{-y}{\left[\frac{1}{a}-\frac{1}{b}\right]}=\frac{-1}{\left[\frac{1}{a^2}-\frac{1}{b^2}\right]}$$

$$x = +\left(\frac{1}{a}+\frac{1}{b}\right)=y$$

So point $\left(\frac{1}{a}+\frac{1}{b}\ ,\ \frac{1}{a}+\frac{1}{b}\right)$ lies on the line y = x Ans. : (A)

Q.24. 3x + 7y = 21

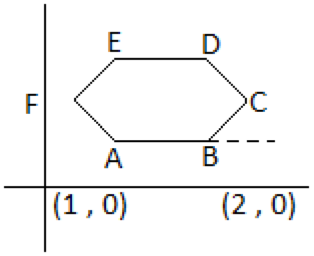
$\frac{x}{7}+\frac{y}{3}=1$

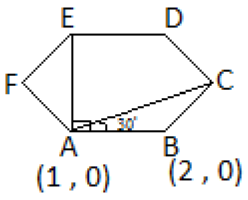


Number of lattize points is $=2\times 6$

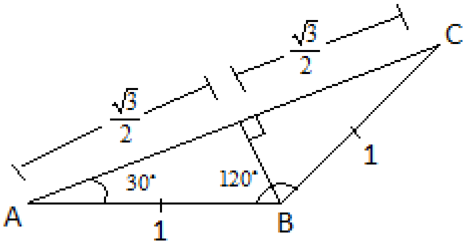
= 12 Ans. : (A)

Q.25.





AC = AE = $\sqrt{3}$



AC = $\sqrt{3}$

Rotate B about A by 30° and scale by $\sqrt{3}$ to get C i] $\left(2 + 0\cdot\text{i}\right) - \left(1 + 0\cdot\text{i}\right)$ ……………(subtract)

= $1 + 0\cdot\text{i}$

ii] rotate/ scale

$$\left(1+0\cdot i\right)\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\sqrt{3}$$

$$\equiv \frac{3}{2}+\frac{\sqrt{3}i}{2}$$

iii] Add co – ordinates of A

$$\frac{3}{2}+\frac{\sqrt{3}i}{2}+1+0\cdot i=\frac{5}{2}+\frac{\sqrt{3}i}{2}$$

$$C\equiv\left(\frac{5}{2},\frac{\sqrt{3}}{2}\right)$$

Get co – ordinates of E by rotating B 90° anticlockwise about A and scale by $\sqrt{3}$

1) Subtract
   $(2+0\cdot i)-(1+0\cdot i)$
   $=(1+0\cdot i)$
2) Scale/rotate
   $(1+0\cdot i)(0+i)(\sqrt{3})$
   $=\sqrt{3}i$
3) Add
   $(0+\sqrt{3}i)+(1+0\cdot i)$
   $\equiv 1+\sqrt{3}i$
   $E\equiv(1,\sqrt{3})$

Slope of CE is $\dfrac{\sqrt{3}-\dfrac{\sqrt{3}}{2}}{1-\dfrac{5}{2}} = \dfrac{\dfrac{\sqrt{3}}{2}}{\dfrac{-3}{2}} = \dfrac{-1}{\sqrt{3}}$

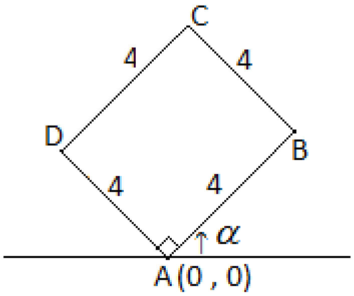
Equation of CE

$$(y-\sqrt{3})=\frac{-1}{\sqrt{3}}(x-1)$$

$$\sqrt{3}y-3+x-1=0$$

$$\sqrt{3}+x=4$$ Ans. : (A)

Q.26.



Co – ordinates of B $\equiv\ (4\cos\alpha\,,4\sin\alpha)$

Co – ordinates of D $\equiv\ (4\cos(90+\alpha),4\sin(90+\alpha))$

$$\equiv(-4\sin\alpha\,,4\cos\alpha)$$

Equation of BD = ?

Equation of BD $=\dfrac{4\cos\alpha-4\sin\alpha}{-4\sin\alpha-4\cos\alpha}$

$$=\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)$$

Equation is

$$(y-4\sin\alpha)=\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)(x-4\cos\alpha)$$

$$(y-4\sin\alpha)(\sin\alpha+\cos\alpha)=(\sin\alpha-\cos\alpha)(x-4\cos\alpha)$$

$$4\cos\alpha(\sin\alpha-\cos\alpha)-4\sin\alpha(\sin\alpha+\cos\alpha)=x(\sin\alpha-\cos\alpha)-y(\sin\alpha+\cos\alpha)$$

$$4\sin\alpha\cos\alpha-4\cos^2\alpha-4\sin^2\alpha-4\sin\alpha\cos\alpha=x(\sin\alpha-\cos\alpha)\ -y(\sin\alpha+\cos\alpha)$$

$$-4=x(\sin\alpha-\cos\alpha)-y(\sin\alpha+\cos\alpha)$$

$4 = x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha)$ Ans. : (C)

Q.27. $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \quad \Rightarrow \quad \frac{1}{c} = \frac{2}{b} - \frac{1}{a}$

bcx + acy + ab = 0

$$\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$$

$$\frac{x}{a} + \frac{y}{b} + \frac{2}{b} - \frac{1}{a} = 0$$

$$(x-1)\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right)(y+2) = 0$$

Family of straight line passing through (1 , − 2) Ans. : (D)

Q.28. Lines : 4x – 7y + 10 = 0 and 7x + 4y = 15 are perpendicular. So, it is a right angled triangle whose orthocenter is at the right angle. i.e. intersection of

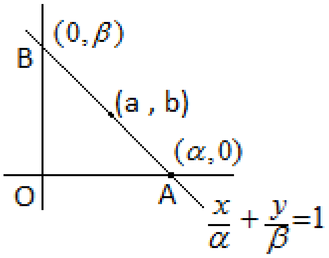
4x – 7y + 10 = 0 and 7x + 4y – 15 = 0

$$\frac{x}{\begin{vmatrix} 10 & -7 \\ -15 & 4 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 4 & 10 \\ 7 & -15 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 4 & -7 \\ 7 & 4 \end{vmatrix}}$$

$$\frac{x}{(-65)} = \frac{y}{(-130)} = \frac{1}{65}$$

$x = -1 \quad , \quad y = -2$

Q.29.



Equation of line is passing through (a , b). So,

$$\frac{a}{\alpha} + \frac{b}{\beta} = 1 \qquad \text{..................(i)}$$

Now centroid of $\Delta OAB = (h , k)$

$$(h , k) \equiv \left(\frac{0 + 0 + \alpha}{3} , \frac{0 + 0 + \beta}{3} \right)$$

$$= \left(\frac{\alpha}{3} , \frac{\beta}{3} \right)$$

$\alpha = 3h$, $\beta = 3k$

So putting in equation (i) we get

$$\frac{a}{3h} + \frac{b}{3k} = 1$$

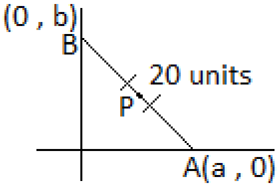
Replacing h by x and k by y we get

$$\frac{a}{3x} + \frac{b}{3y} = 1$$

ay + bx = 3xy

bx + ay – 3xy = 0 Ans. : (A)

Q.30.



Equation of AB is $\frac{x}{a} + \frac{y}{b} = 1$

Let co – ordinates of P be (h , k) mid – point of AB

$$(h , k) \equiv \left(\frac{0 + a}{2} , \frac{b + 0}{2} \right)$$

$$= \left(\frac{a}{2} \ , \ \frac{b}{2} \right)$$

$h = \frac{a}{2}$, $k = \frac{b}{2}$, a = 2h , b = 2k

$|AB| = 20$ units.

$$|AB| = 20 = \sqrt{(a-0)^2 + (0-b)^2}$$

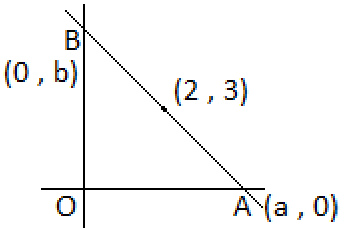
$$400 = a^2 + b^2$$

$$400 = 4h^2 + 4k^2$$

$\Rightarrow h^2 + k^2 = 100$

$\Rightarrow x^2 + y^2 = 100$ is the locus of P Ans. : (D)

Q.31.



Line AB is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\Delta\, AOB = 12$ sq. units = $\frac{1}{2}\,|\,ab\,|$

$$\therefore |\,ab\,| = 24$$

Line AB passes through (2 , 3). So,

$$\frac{2}{a} + \frac{3}{b} = 1$$

2b + 3a = ab …………………..(i)

Case 1 : ab = + 24

then $b = \frac{24}{a}$

$\therefore$ putting in (i) we get,

$$\frac{48}{a} + 3a = 24$$

$$\frac{16}{a} + a = 8$$

$$a^2 + 16 - 8a = 0$$

(a – 4) (a – 4) = 0

a = 4 $\therefore$ b = 6

$$\frac{x}{4} + \frac{y}{6} = 1$$ line is one

Case 2 : ab = – 24

$$b = \frac{-24}{a} \qquad \therefore \frac{2(-24)}{a} + 3a = -24$$

$$\frac{-48}{a} + 3a = -24$$

$$\frac{-16}{a} + a = -8$$

$$a^2 - 16 = -89$$

$$a^2 + 8a - 16 = 0$$

$$a = \frac{-8 \pm \sqrt{64+64}}{2}$$

$$a = \frac{-8 \pm 8\sqrt{2}}{2}$$

$$a = -4 \pm 4\sqrt{2}$$

If $a = -4 + 4\sqrt{2} = 4 - 4\sqrt{2}$

$$b = \frac{-24}{-4 + 4\sqrt{2}} = \frac{-24(4 + 4\sqrt{2})}{16}$$

$b = -6(\sqrt{2}+1)$

$a = 4(\sqrt{2}-1)$

$b = -6(\sqrt{2}+1)$

If $a = -4 - 4\sqrt{2}$

i.e. $a = -4(1+\sqrt{2})$

then $b = \frac{-24}{a}$

$b = 6(\sqrt{2}-1)$

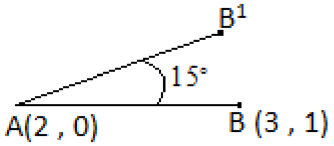
Lines can be

$$\frac{x}{4(\sqrt{2}-1)} + \frac{y}{-6(\sqrt{2}-1)} = 1$$

$$\frac{x}{-4(\sqrt{2}+1)} + \frac{y}{6(\sqrt{2}-1)} = 1$$

Number of possibilities = 3 Ans. : (C)

Q.32.



Slope of AB $= \frac{1-0}{3-2} = 1$

Slope of AB^1 $= \frac{m + \tan\alpha}{1 - m\tan\alpha}$ $(\alpha = 15^\circ)$

$$= \frac{1 + 2 - \sqrt{3}}{1 - (1)(2-\sqrt{3})}$$

$$= \frac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$$

$\therefore$ Equation of AB^1 is

$$y - 0 = \sqrt{3}\ (x-2)$$

$$y = \sqrt{3}x - 2\sqrt{3}$$

$$0 = \sqrt{3}x - y - 2\sqrt{3}$$ Ans. : (C)

Q.33.

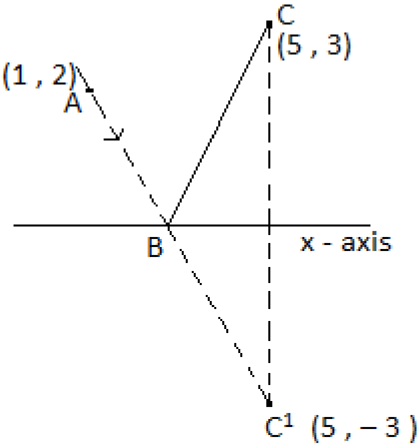


Image of C in x – axis is C^1

ABC^1 are collinear. So, equation of AB and AC^1 is the same

Slope of AC^1 is $\frac{-3-2}{5-1} = \frac{-5}{4}$

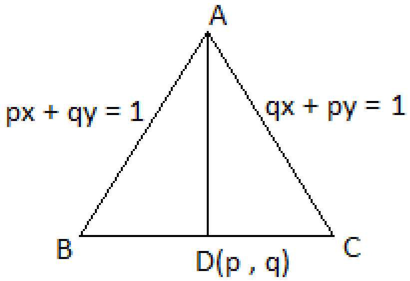
Equation is $y - 2 = \frac{-5}{4}(x-1)$

4 (y – 2) + 5 (x – 1) = 0

4y – 8 + 5x – 5 = 0

4y + 5x = 13 Ans. : (A)

Q.34. Family of straight lines passing through A is



$(px + qy - 1) + \lambda(qx + py - 1) = 0$

But this passing through (p , q)

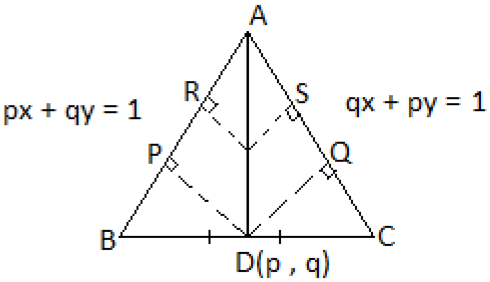
So $(p^2 + q - 1) + \lambda(pq + qp - 1) = 0$

$\therefore \lambda = \frac{-(p^2 + q^2 - 1)}{(2pq - 1)}$

$\therefore$ equation of AD is

$$(px + qy - 1) - \frac{(p^2 + q^2 - 1)}{(2pq)}(qx + py - 1) = 0$$

Alternate way



Let x be any point on AD

Using similarity we get

$$\frac{RX}{PD} = \frac{XS}{DQ} \qquad \therefore \qquad \frac{RX}{XS} = \frac{PD}{DQ}$$

$$PD = \left|\frac{p^2 + q^2 - 1}{\sqrt{p^2 + q^2}}\right| \qquad RX = \left|\frac{px + qy - 1}{\sqrt{p^2 + q^2}}\right|$$

$$DQ = \left|\frac{2pq - 1}{\sqrt{p^2 + q^2}}\right| \qquad XS = \left|\frac{qx + py - 1}{\sqrt{p^2 + q^2}}\right|$$

Substituting we get

$$\left|\left(\frac{px + qy - 1}{qx + py - 1}\right)\right| = \left|\frac{p^2 + q^2 - 1}{2pq - 1}\right|$$

$$\frac{px+qy-1}{qx+py-1} = +\left(\frac{p^2+q^2-1}{2pq-1}\right)$$

∵ x and (p , q) lie on the same side w.r.t. to the 2 lines.

$$(2pq-1)(px+pq-1)=(p^2+q^2-1)(qx+py-1)$$ Ans. : (C)

Q.35. $x^2 - 8x + 12 = 0$

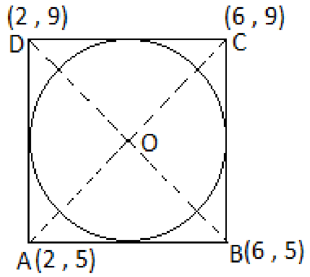
$\Rightarrow (x - 6)(x - 2) = 0$

$\Rightarrow x = 6 , x = 2$

$y^2 - 14y + 45 = 0$

$\Rightarrow (y - 9)(y - 5) = 0$

$\Rightarrow y = 9 , y = 5$

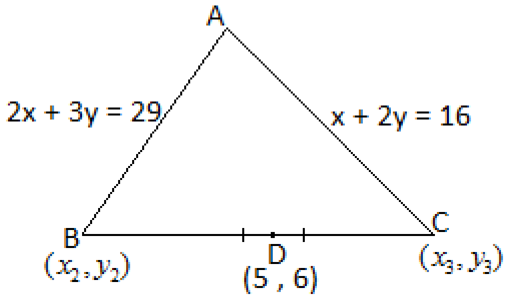


Center = mid – point of AC

$$= \left(\frac{2+6}{2}, \frac{5+9}{2}\right)$$

$= (4 , 7)$ Ans. : (B)

Q.36.



$x_3 + 2y_3 = 16$ ....(i)

$2x_2 + 3y_2 = 29$ ....(ii)

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) \equiv (5,6)$$

$x_2 + x_3 = 10 , \quad y_2 + y_3 = 12$

$x_3 = 10 - x_2 , \quad y_3 = 12 - y_2$

Putting in (i) we get

$(10-x_2)+2(12-y_2)=16$

$10-x_2+24-2y_2=16$

$34-x_2-2y_2=16$

$18=x_2+2y_2$

$36=2x_2+4y_2$ ....(iii)

(iii) – (ii)

$y_2=7$

And $\therefore x_2=18-2y_2$

$x_2=18-14$

$x_2=4$

B (4 , 7)

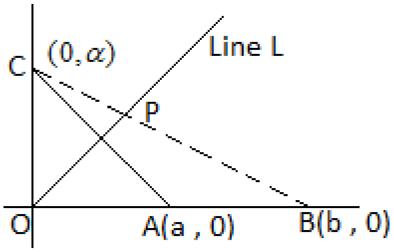
Equation of BC = Equation of BD

Slope BD $=\dfrac{7-6}{4-5}=-1$

(y – 6) = (– 1) (x – 5)

Y = 5 – x + 6 ⇒ y = 11 – x Ans. : (C)

Q.37.



Equation of AC is

$$\frac{x}{a} + \frac{y}{\alpha} = 1$$

So equation of OP is

$$\frac{x}{\alpha} - \frac{y}{a} = 0$$ (perpendicular to AC passing through 0)

Or $ax = \alpha y$ ……….(i)

Equation of BC is

$$\frac{x}{a} + \frac{y}{\alpha} = 1$$ ………..(ii)

Let $P \equiv (h\ ,\ k)$ is the intersection of (i) and (ii)

$$\frac{h}{\begin{vmatrix} 0 & \frac{-1}{a} \\ +1 & \frac{1}{\alpha} \end{vmatrix}} = \frac{k}{\begin{vmatrix} \frac{1}{\alpha} & 0 \\ \frac{1}{b} & +1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} \frac{1}{\alpha} & \frac{-1}{a} \\ \frac{1}{b} & \frac{1}{\alpha} \end{vmatrix}}$$

$$\frac{h}{\frac{+1}{a}} = \frac{k}{\frac{+1}{\alpha}} = \frac{1}{\frac{1}{\alpha^2} + \frac{1}{ab}} = \frac{ab\alpha^2}{ab + \alpha^2}$$

$$\therefore\ h = \frac{+b\alpha^2}{ab + \alpha^2}$$

$$k = \frac{+ab\alpha}{ab + \alpha^2}$$

$$h - (ab + \alpha^2) = b\alpha^2$$

$$hab = \alpha^2\ (b - h)$$

$$\alpha^2 = \frac{hab}{b - h}$$ …………….(A)

$$ab\alpha = (ab + \alpha^2)\,k$$

$$= \left(ab + \frac{hab}{b-h}\right)k$$

$$ab\alpha = ab\left(\frac{b}{b-h}\right)k$$

$$\alpha = \frac{bk}{b-h}$$ ....(B)

From (A) & (B) we get

$$\alpha^2 = \frac{hab}{b-h} = \frac{b^2k^2}{(b-h)^2}$$

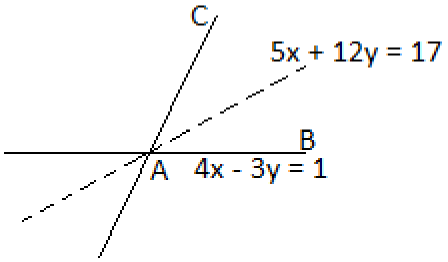
$$ha\,(b-h) = bk^2$$

$$hab - h^2a = bk^2$$

$$hab = bk^2 + h^2a$$

$$\therefore \boxed{xab = ax^2 + by^2}$$ Ans. : (C)

Q.38.



AP is the angle bisector of AB and AC

$$\frac{(5x+12y-17)}{(\sqrt{5^2+12^2})} = \pm\left(\frac{4x-3y-1}{\sqrt{4^2+3^2}}\right)$$

$$\frac{5x+12-17}{13} = \pm\left(\frac{4x-3y-1}{5}\right)$$

$$5(5x+12y-17) \pm 13(4x-3y-1) = 0$$

i] $(25x+60y-85) \pm (52x-39y-13) = 0$

1) 77x + 21y – 98 = 0 i.e.

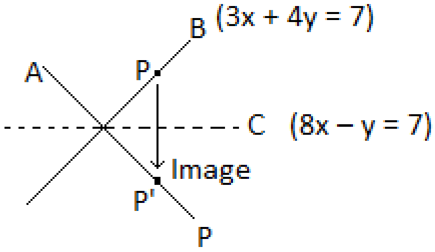
OR 11x + 3y – 14 = 0

2) –27x + 99y – 72 = 0

– 3x + 11y – 8 = 0 Ans. : (A) (B)

Q.39. 3x + 4y = 7 (AB)

8x – y = 7 (AC)



Any point online AB P can be written as

(1 – 4t , 1 + 3t) [parameter is t and it satisfies the equation]

Take the image of it write AC to get P' (h , k)

$$\frac{h - (1 - 4t)}{8} = \frac{k - (1+3t)}{-1} = \frac{-2(8(1-4t)-(1+3t)-7)}{(8^2+1^2)}$$

$$\frac{h+4t-1}{8} = \frac{k-3t-1}{-1} = \frac{-2\,(8-1-7-32t-3t)}{65}$$

$$= \frac{-2(-35t)}{65} = \frac{14t}{13}$$

$$\frac{h+4t-1}{8} = \frac{14t}{13}$$

$$h + 4t - 1 = 8\left(\frac{14}{13}\right)t$$

$$h = 1 + \left(\frac{112}{13} - 4\right)t$$

$$h = 1 + \left(\frac{60}{13}\right)t \qquad \Rightarrow \quad \frac{t}{13} = \frac{h-1}{60}$$

$$k = 1 + 3t - \frac{14}{13}t$$

$$k = 1 + \left(\frac{25}{13}\right)t \qquad \Rightarrow \quad \frac{t}{13} = \frac{k-1}{25}$$

$$\therefore \frac{h-1}{\cancel{60}} = \frac{k-1}{\cancel{25}}$$

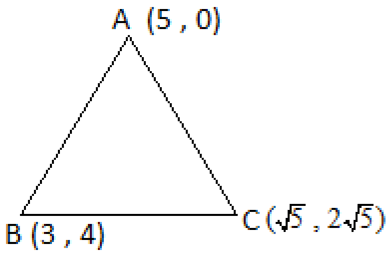
$$12 \qquad\quad 5$$

$$5(h-1) = 12(k-1)$$

$$5h - 5 = 12k - 12$$

$$\therefore \boxed{5x+7=129}$$ Ans. : (A)

Q.40.

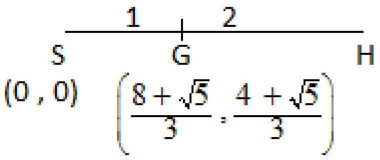


Circumcenter of ΔABC is (0 , 0)

OA = 5 units $\qquad OC^2 \equiv \left((\sqrt{5})^2 + (2\sqrt{5})^2\right)$

OB = 5 units $\qquad OC^2 = 25$ units.

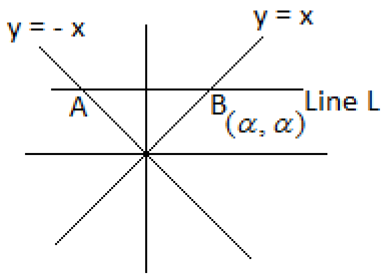
$$\text{Centroid} \equiv \left(\frac{8+\sqrt{5}}{3}, \frac{4+\sqrt{5}}{3}\right)$$



Co – ordinates of H

$\equiv (8 + \sqrt{5}\ , 4 + 2\sqrt{5})$ Ans. : (A)

Q.41.



$$\begin{pmatrix} x = \alpha + r\cos\theta \\ y = \alpha + r\sin\theta \end{pmatrix}$$

For A y = – x

$$\therefore\ \alpha + r\cos\theta = -\,(\alpha + r\sin\theta)$$

$$r(\cos\theta + \sin\theta) = -\,2\alpha$$

$$r = \left(\frac{-\,2\alpha}{\cos\theta + \sin\theta}\right)$$

$$|r| = \left|\frac{2\alpha}{\cos\theta + \sin\theta}\right| = \text{constant k}$$

Mid – point of AB is by taking

$$r = \frac{-\,\alpha}{\cos\theta + \sin\theta}$$

$$x = \alpha - \frac{\alpha}{\cos\theta + \sin\theta}$$

$$x = \frac{\alpha\sin\theta}{\cos\theta + \sin\theta}$$

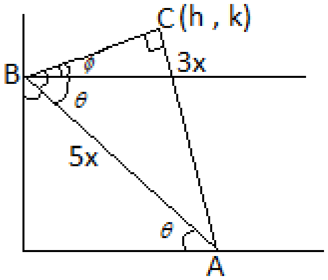
$$x = \alpha - \frac{\alpha \sin\theta}{\cos\theta + \sin\theta}$$

$$y = \frac{\alpha\cos\theta}{\cos\theta + \sin\theta}$$

$$x^2 + y^2 = \frac{\alpha^2}{(\sin\theta + \cos\theta)^2} = \frac{k^2}{4}$$

$4x^2 + 4y^2 = k^2$ is the locus of mid – point of AB

Q.42.



$\angle CBA\ \underline{n}\ 37^{\circ}$

$\phi = 37^{\circ} - \theta$

$B \equiv (0\ ,\ 5x\sin\theta)$

Inclination of BC is $37^\circ - \theta$

$C\ \equiv\ (h\ ,\ k)$ using parameter locus we get

$$\frac{h - 0}{\cos\,(37^\circ - \theta)} = \frac{k - 5x\sin\theta}{\sin\,(37^\circ - \theta)} = 4x$$

$$h = 4x\cos(37^\circ - \theta)$$

$$k = 4x\sin(37^\circ - \theta) + 5x\sin\theta$$

$$h = 4x\left(\frac{4}{5}\cos\theta + \frac{3}{5}\sin\theta\right) \qquad \text{.................(i)}$$

$$k = 4x\left(\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta\right) + 5x\sin\theta$$

$$k = \left(\frac{12}{5}\cos\theta + \frac{9}{5}\sin\theta\right)x$$

$k = 3x\left(\frac{4\cos\theta}{5} + \frac{3\sin\theta}{5}\right)$ ………………..(ii)

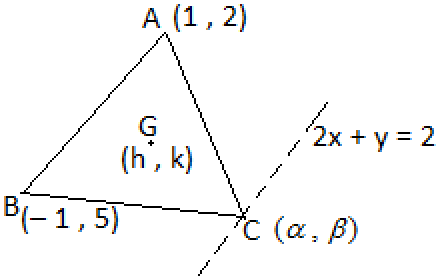
$\therefore \frac{h}{k} = \frac{4}{3}$

3h = 4k

$\therefore$ Locus of P is

$\boxed{3x = 4y}$ Ans. : (A)

Q.43.



Co – ordinates of Centroid

$$(h\ ,\ k) \equiv \left(\frac{1-1+\alpha}{3}\ ,\ \frac{2+5+\beta}{3}\right)$$

$$(h\ ,\ k) \equiv \left(\frac{\alpha}{3}\ ,\ \frac{7+\beta}{3}\right)$$

C lines on 2x + y = 2

$\therefore 2\alpha + \beta = 2$

$\therefore \beta = 2 - 2\alpha$

$$(h,\ k) \equiv \left(\frac{\alpha}{3}\ ,\ \frac{7+(2-2\alpha)}{3}\right)$$

$$\equiv \left(\frac{\alpha}{3}\ ,\ \frac{9-2\alpha}{3}\right)$$

$h = \frac{\alpha}{3}$ | $k = \frac{9-2\alpha}{3}$

$\alpha = 3h$ $3x = 9 - 2\alpha$

Now $\alpha = 3h$

$\therefore 3k = 9 - 6h$

$\therefore k = 3 - 2h$

$k + 2h = 3$

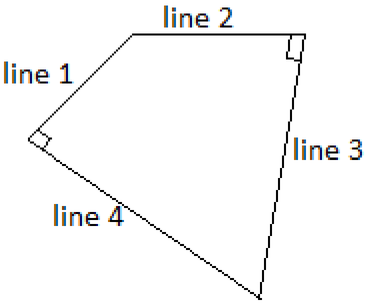
$\therefore$ locus of centroid is $2x + y = 3$ Ans. : (B)

Q.44. $(a + b)x + (a - b)y = 2a$ ....line 1

$(a - b)x - (a + b)y = 2b$ ....line 4 } are perpendicular

$(a + b)x - (a - b)y = 2b$ ....line 2

$(a - b)x + (a + b)y = 2a$ ....line 3 } lines are perpendicular



Whether line 1 & line 2 are perpendicular depends on the value of a & b Ans. : (D)

Q.45. $2x - y = 0$ L_1

$2x - y - 8 = 0$ L_2

$2x + y - 4 = 0$ L_3

$2x + y + 4 = 0$ L_4

L_1 & L_2 are parallel lines

L_3 & L_4 are parallel lines

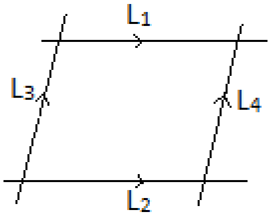
Angle between L_1 & L_3

$m_1 = 2$

$m_2 = -2$ $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{2 - (-2)}{1 + 2(-2)}\right| = \frac{4}{3}$$

So it's not perpendicular



parallel distance between L_1 & L_2

$$\left|\frac{C - C^1}{\sqrt{a^2 + b^2}}\right| = \left|\frac{0 - (-8)}{\sqrt{2^2 + 1^2}}\right|$$

$$= \frac{8}{\sqrt{5}} \text{ units}$$

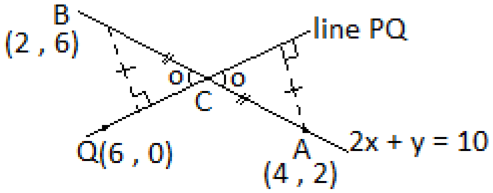
Perpendicular distance between L_3 & L_4 is

$$\left|\frac{4 - (-4)}{\sqrt{2^2 + 1^2}}\right| = \frac{8}{\sqrt{5}}$$

So, lines L_1 & L_2 and L_3 & L_4 are separated equally

∴ is is rhombus. Ans. : (B)

Q.46.



Let equation of the line be ax + by + c = 0

By similarity mid – point of AB lies on PQ

So $C \equiv \left(\frac{2 + 4}{2}, \frac{6 + 2}{2}\right) \equiv (3, 4)$ lies on PQ

Equation of PQ = Equation of CQ

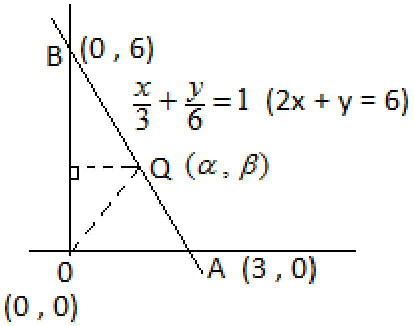
Slope CQ $= \dfrac{4-0}{3-6} = \dfrac{4}{-3}$

$\therefore$ Equation is $y - 0 = \dfrac{4}{-3}(x-6)$

$-3y = 4x - 2y$

$\boxed{24 = 4x + 2y}$ Ans. : (C)

Q.47.



Area of $\Delta BOQ = \dfrac{1}{2} \times$ base $\times$ height

base = OB = 6 units.

height $= |\alpha|$

$\therefore \dfrac{1}{2} \times 6 \times |\alpha| = 12$ sq. units

$|\alpha| = 4$

$\alpha = \pm 4$

$\therefore$

If $\alpha = 4$	If $\alpha = -4$
$\beta = 6 - 2\alpha$	$\beta = 6 - 2\alpha$
$\beta = -2$	$\beta = 6 - 2(-4)$
Q (4 , – 2)	$\beta = 14$

Q (– 4 , 14)

Equation of OQ	Equation of OQ
$y = \frac{-2}{4} x$	$y = \frac{14}{-4} x$
$y = -\frac{x}{2}$	$y = \frac{7x}{-2}$
2y + x = 0 or	2y + 7x = 0

Ans. : (A)

Q.48. Equation of the line passing through the intersection of

2x – y – 4 = 0 and 3x + 2y – 13 = 0

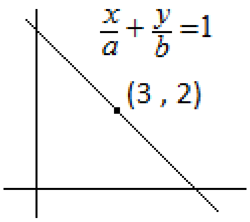
2x – y = 4

3x + 2y = 13

$$\frac{x}{\begin{vmatrix} 4 & -1 \\ 13 & 2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 4 \\ 3 & 13 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}}$$

$\frac{x}{21} = \frac{y}{14} = \frac{1}{7}$

x = 3 , y = 2



line passing through (3 , 2) such that a + b = 10

Let line be $\frac{x}{a} + \frac{y}{b} = 1$

A + b = 10, b = 10 – a

$$\frac{x}{a} + \frac{y}{10 - a} = 1$$

Point (3 , 2) satisfies this

$$\frac{3}{a} + \frac{2}{10 - a} = 1$$

3(10 – a) + 2a = a (10 – a)

$30-3a+2a=10a-a^2$

$a^2 - 11a + 30 = 0$

a = 5 , 6

∴ equation of the line is

$\frac{x}{5}+\frac{y}{10-5}=1$ & $\frac{x}{6}+\frac{y}{10-6}=1$

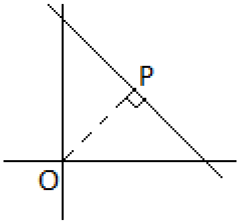
x + y = 5 & $\frac{x}{6}+\frac{y}{4}=1$ Ans. : (A)

Q.49. sum number v49 is the same as sum number 48.

Q.50.

slope = – 2

slope of normal from origin is $\frac{1}{2}$



slope of OP = $\frac{1}{2}$

$\frac{1}{2} = \tan\alpha$ α ranges from $[0\ ,\ 2\pi]$

So $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\sin\alpha = \frac{-1}{\sqrt{5}}$

And $\cos\alpha = \frac{2}{\sqrt{5}}$ or $\cos\alpha = \frac{-2}{\sqrt{5}}$

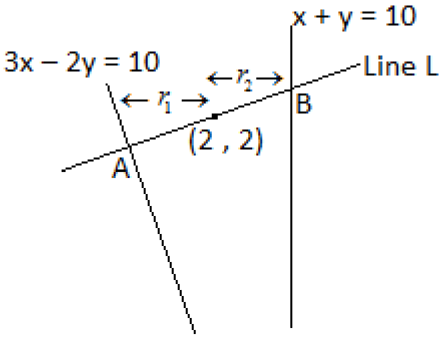
$\therefore\ x\cos\alpha + y\sin\alpha = p$

$\frac{2x}{\sqrt{5}}+\frac{y}{\sqrt{5}} = 4\sqrt{5}$ $\frac{-2x}{\sqrt{5}}-\frac{y}{\sqrt{5}} = 4\sqrt{5}$

2x + y = 20 – 2x – y = 20

2x + y = – 20 Ans. : (A)

Q.51.



Parametric equation of line 1

$$\frac{x-2}{\cos\theta} = \frac{y-2}{\sin\theta} = 0$$

$B \equiv (2 + r_1\cos\theta\ ,\ 2\ + r_2\sin\theta)$ r₁ and r₂ are equal in magniside &

$A \equiv (2 + r_2\cos\theta\ ,\ 2 + r_2\sin\theta)$ opposite indirection $r_1 = -r_2$

A satisfies 3x – 2y = 0

$3(2\ +\ r_2\cos\theta) - 2(2 + r_2\sin\theta)$

$6 - 4 + 3r_2\cos\theta - 2r_2\sin\theta\ =\ 0$

$2\ =\ 2r_2\sin\theta - 3r_2\cos\theta$

$2\ =\ r_2(2\sin\theta - 3\cos\theta)$ ....(A)

B satisfies x + y = 10

$(2 + r_1\cos\theta) + (2\ +\ r_1\sin\theta) = 0$

$r_1\ (\cos\theta + \sin\theta) = -\ 4$ ....(B)

Dividing (A) by (B)

$$\frac{2}{-4} = \frac{r_2\ (2\sin\theta - 3\cos\theta)}{r_1(\cos\theta + \sin\theta)}$$

$$\frac{-1}{2} = (-1)\left(\frac{2\sin\theta - 3\cos\theta}{\cos\theta + \sin\theta}\right)$$

$$= -\left(\frac{2\tan\theta - 3}{1 + \tan\theta}\right)$$

$$(1 + \tan\theta) = 2(2\tan\theta - 3)$$

$$(1 + \tan\theta) = 4\ \tan\theta - 6$$

$$\tan\theta = \frac{7}{3}$$

$$y - 2 = \frac{7}{3}(x - 2)$$

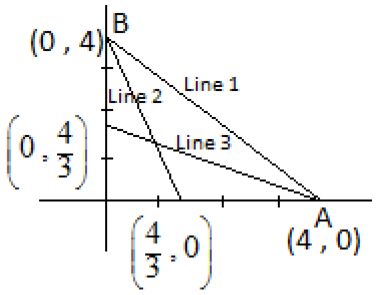
3y – 14 = 7x – 14

3y = 7x Ans. : (D)

$$\text{Q.52. } \left.\begin{array}{l} x + y = 4 \\ 3x + y = 4 \\ x + 3y = 4 \end{array}\right\}$$

None of the lines are perpendicular

So, it is not right angled



Line 1 : $\frac{x}{4} + \frac{y}{4} = 1$

Line 2 : $\frac{x}{\frac{4}{3}} + \frac{y}{4} = 1$

Line 3 : $\frac{x}{4} + \frac{y}{\frac{4}{3}} = 1$

Meeting of line 2 and 3

i.e. 3x + y = 4

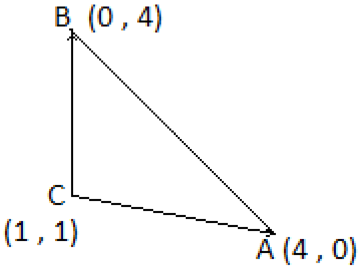
x + 3y = 4

$$\frac{x}{\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 4 \\ 1 & 4 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}}$$

$\frac{x}{8} = \frac{y}{8} = \frac{1}{8}$ x = y = 1

$C \equiv (1,1)$

C is equidistant from A (4 , 0) and B (0 , 4). So triangle is isosceles.



rector $\overline{CA} = (0-1)\hat{i} + (4-1)\,j$

$= -\hat{i} + 3\,j$

rector $\overline{CB} = (4-1)\hat{i} + (0-1)\,j$

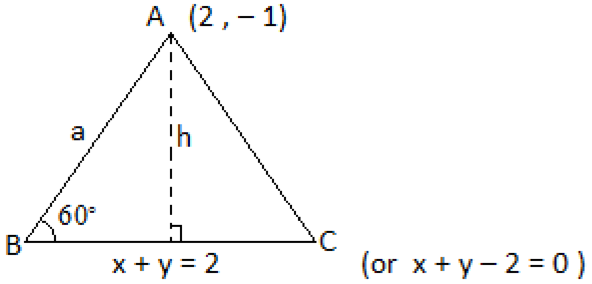
$= 3\hat{i} - j$

Dot product $\overline{CA} \cdot \overline{CB} = 3\,(-1) + 3\,(-1)$

= – 6 (negative)

So $\angle\, BCA$ is obtuse Ans. : (B)

Q.53.



$$\sin 60^{\circ} = \frac{h}{a}$$

$$a = \frac{h}{\frac{\sqrt{3}}{2}} = \frac{2h}{\sqrt{3}}$$

$$h = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{2 + (-1) - 2}{\sqrt{1^2 + 1^2}} \right|$$

$$= \frac{1}{\sqrt{2}}$$

$$a = \frac{2h}{\sqrt{3}}$$

$$a = \frac{2 \times \frac{1}{\sqrt{2}}}{\sqrt{3}}$$

$$a = \sqrt{\frac{2}{3}}$$

Ans. : (A)

Q.54. $(-a, -b)$, $(0, 0)$, (a, b) , (a^2, ab) , (ab, b^2)

All these points lie on the line

$\frac{y}{x} = \frac{a}{b}$ i.e. $\boxed{by = ax}$

Collinear. Ans. : (C)

Q.55. $ax + by + c = 0$ L_1

$bx + cy + a = 0$ L_2

$cx + ay + b = 0$ L_3

If a , b , c not all are same the L_1 , L_2 , L_3 cannot be co – incident lines.

If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is zero the lines are concurrent

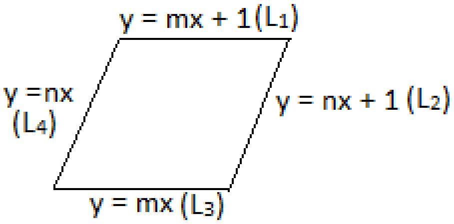
i.e. if $a^3 + b^3 + c^3 - 3abc = 0$ (value of the determinant) then lines are concurrent.

But it is given that,

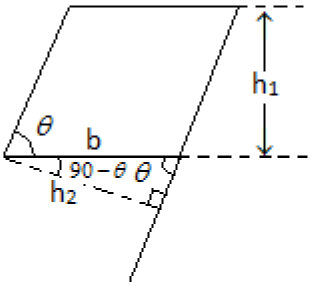
$$a^3 + b^3 + c^3 - 3abc \neq 0$$

Lines from a triangle. Ans. : (A)

Q.56.



Area = b× h1



h1 and h2
are perpendicular disatnce between
the parallel lines

$$\cos(90-\theta) = \frac{h_2}{b}$$

$$b = \frac{h_2}{\sin\theta} \qquad \therefore \text{ Area = } = \frac{h_1 \times h_2}{\sin\theta}$$

θ is the angle between the lines

h_1 = distance between L_1 and L_3

$$= \left|\frac{1}{\sqrt{1+m^2}}\right|$$

h_2 = distance between L_2 and L_4

$$= \left|\frac{1}{\sqrt{1+n^2}}\right|$$

$$\tan\theta = \left|\frac{m-n}{1+mn}\right|$$ ………………….(acute angle formula)

$$\Rightarrow \sin\theta = \frac{|m-n|}{\sqrt{(m-n)^2 + (1+mn)^2}}$$

$$= \frac{|m-n|}{\sqrt{m^2 + n^2 + 1 + m^2n^2}}$$

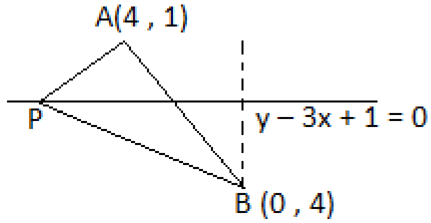
$$= \frac{|m-n|}{\sqrt{(1+m^2)(1+n^2)}}$$

$$\text{Area} = \frac{h_1h_2}{\sin\theta}$$

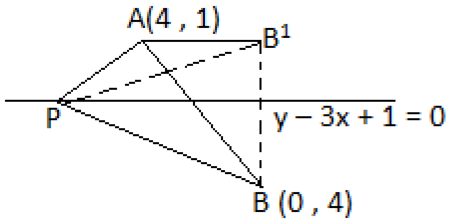
$$= \frac{\frac{1}{\sqrt{1+m^2}} \times \frac{1}{\sqrt{1+n^2}}}{\frac{|m-n|}{\sqrt{1+m^2}\sqrt{1+n^2}}}$$

$$A = \frac{1}{|m-n|}$$ Ans. : (A)

Q57.

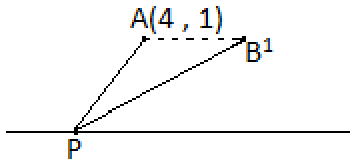


<u>Construction</u> : Reflect B about the line y – 3x + 1 = 0 to get B^1

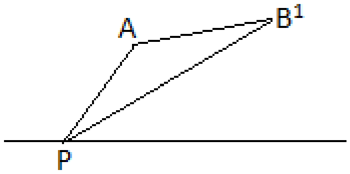


$PB = PB^1$

Symmetry so

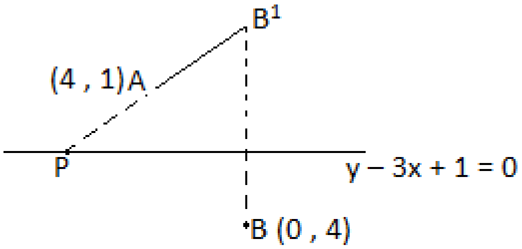


$$|PA - PB| = |PA - PB^1|$$



$AB^1 > |PA - PB^1|$ triangle inequality

So $|PA - PB^1| = AB^1$ if PAB^1 are collinear



$$\frac{x-0}{-3} = \frac{y-4}{1} = \frac{-2(4-3(0)+1)}{1^2+3^2}$$

$$= \frac{-10}{10} = -1$$

$\therefore x = 3$, $y = 4 - 1$,

y = 3

Equation of $AB^1 = (y - 3) = \left(\frac{3-1}{3-4}\right)(x - 3)$

$$y - 3 = \frac{2}{-1}(x-3)$$

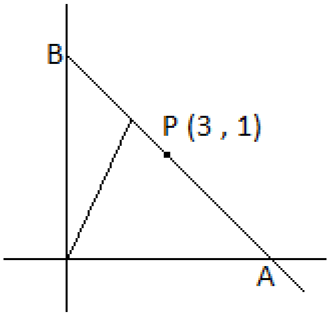
y – 2x + 3 = 0

Intersection of AB^1 y – 2x + 3 = 0 and y – 3x + 1 = 0 is

x = – 2 , y = – 7

P (– 2 , – 7) Ans. : (D)

Q.58.



Equation of the line is

$x \cos \alpha + y \sin \alpha = P$ (Normal from of line)

$P = x \cos \alpha + y \sin \alpha$

Point P (3 , 1) satisfies this so,

$P = 3\cos \alpha + \sin \alpha$. We have to choose α so, that P is maximum.

$-\sqrt{10} \le 3\cos \alpha + \sin \alpha \le \sqrt{10}$

So if $3\cos \alpha + \sin \alpha = \sqrt{10}$ then P is max.

$(P = \sqrt{10})$

i.e. if $\cos \alpha = \frac{3}{\sqrt{10}}$ and $\sin \alpha = \frac{1}{\sqrt{10}}$

i.e. $\tan \alpha = \frac{1}{3}$ and $P = \sqrt{10}$

So slope of line is – 3.

$\therefore$ Equation of line is

$$\frac{3x}{\sqrt{10}} + \frac{1}{\sqrt{10}} y = \sqrt{10}$$

$\therefore \boxed{3x + y = 10}$

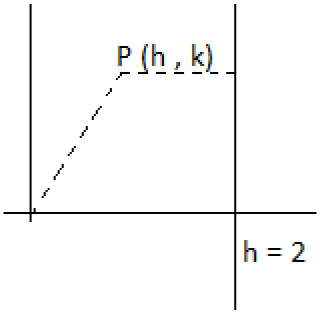
$$\left[\begin{array}{l} a\cos\theta + b\sin\theta \text{ is max} \\ \text{when } \cos\theta = \dfrac{a}{\sqrt{a^2+b^2}} \text{ and} \\ \sin\theta = \dfrac{b}{\sqrt{a^2+b^2}} \end{array}\right]$$

$$\frac{x}{\frac{10}{3}} + \frac{y}{10} = 1$$

$$A \equiv \left(\frac{10}{3}, 0\right) \; B \equiv (10, 0)$$

$\Delta\ ADB$ area $\equiv \frac{1}{2} \times 10 \times \frac{10}{3} = \frac{50}{3}$ sq. units Ans. : (A)

Q.59.



Distance from origin = $\sqrt{h^2+k^2}$

Distance from line x – 2 = 0 is

$$\left|\frac{h-2}{\sqrt{1}}\right| = |h-2|$$

$$\therefore \sqrt{h^2+k^2} + |h-2| = 4$$

<u>Case I</u> : $h \geq 2$ then

$$\sqrt{h^2+k^2} + h - 2 = 4$$

$\sqrt{h^2+k^2} = 6 - h$

$h \leq 6$

Squaring we get

$h^2 + k^2 = h^2 + 36 - 12h$

$k^2 = 36 - 12h$

$k^2 = 12(3 - h) \geq 0$

$\therefore 3 - h \geq 0$

$h \leq 3$ i.e. for $2 \leq x \leq 3$

Locus is $y^2 = 12(3 - x)$

<u>Case II</u> : $h \leq 2$

Then,

$\sqrt{h^2+k^2} + 2 - h = 4$

$\sqrt{h^2+k^2} = h + 2$

Squaring we get

$h^2 + k^2 = h^2 + 4h + 4$

$k^2 = 4(h + 1)$

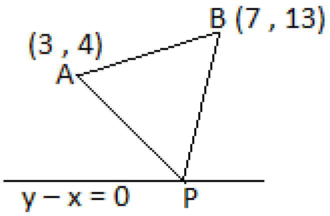
$k^2 \geq 0$

$\therefore h + 1 \geq 0$

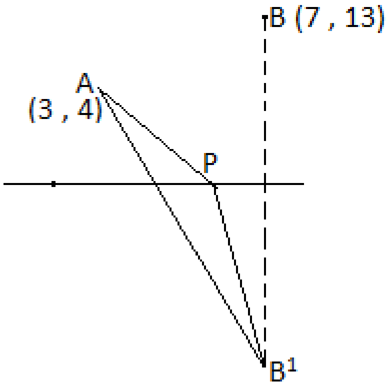
$h \geq -1$

So for $-1 \leq x \leq$ locus is $y^2 = 4(x+1)$ Ans. : (C)

Q.60.



PA = PB > AB

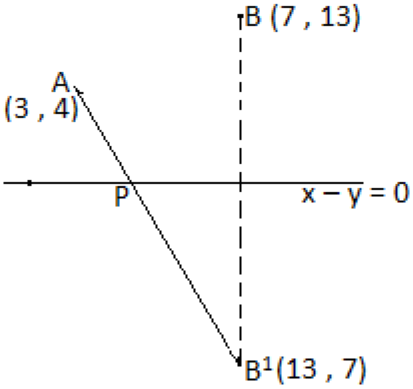


B^1 is the image of B in the line

PB = PB^1 (symmetry)
PA + PB^1 > AB^1
So
PA + PB^1 will be least

If APB^1 are collinear.

Least find image of B in line y = x



Equation of AB^1 is

$$y - 4 = \left(\frac{7-4}{13-3}\right)(x-3)$$

$$y - 4 = \frac{3}{10}\,(x-3)$$

$10y - 40 = 3x - 9$

$10y = 3x + 31$ ……………Equation of AB^1

Finding the intersection of AB^1 and $x - y = 0$

We get,

$P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)$ Ans. : (A)

EXERCISE 2 (A)

61. Compare with $Ax^2+2Hxy+By^2+2ax+2fy+C=0$
A = 1, B = 1, H = 0, G = g, F = f, C = 1
$\therefore \Delta = 0$

$$\Rightarrow \quad \begin{vmatrix} 1 & 0 & g \\ 0 & 1 & f \\ g & f & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \left(1-f^2\right)+g\left(-g\right)=0$$

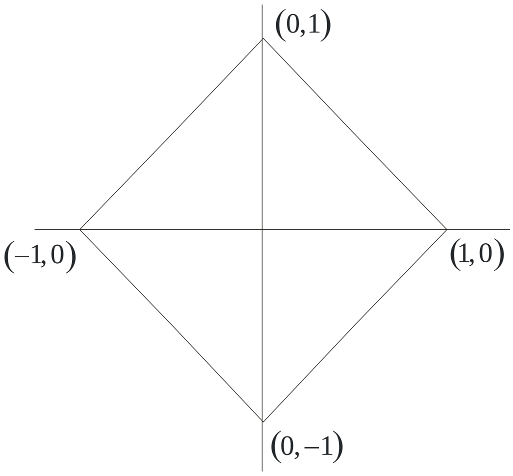
$$f^2+g^2=1$$

62. $A=\lambda,\ B=2,\ H=-5/2,\ G=5/2,\ f=-7/2,\ C=3$
Put $\Delta=0$

$$\Rightarrow \quad \begin{vmatrix} \lambda & -5/2 & 5/2 \\ -5/2 & 2 & -7/2 \\ 5/2 & -7/2 & 3 \end{vmatrix} = 0$$

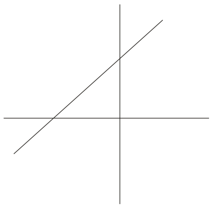
Solve to get $\lambda=2$

63. Area enclosed by $|x-1|+|y-3|=1$ is same as enclosed by $|x|+|y|=1$ (shift of origin)

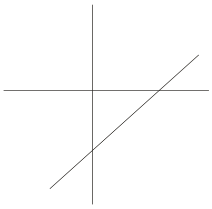


$\therefore \quad A=2$

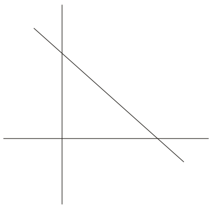
64. (D)



OR



OR

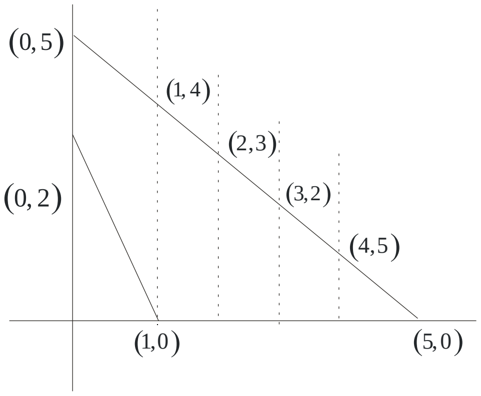


Here

$\left(\frac{-c}{a}\right)<0$	$\frac{-c}{a}>0$	$\frac{-c}{a}>0$
$\&\frac{-c}{b}>0$	$\frac{-c}{b}<0$	$\&\frac{-c}{b}>0$

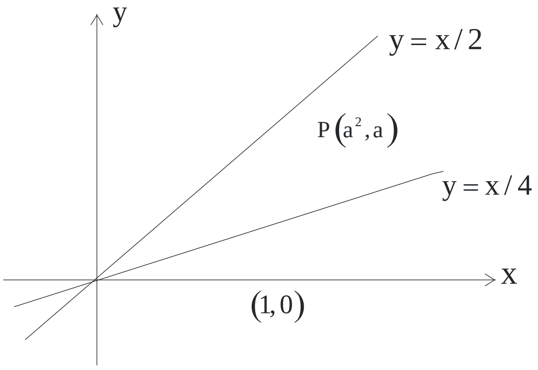
65. Equation of pair of bisection is $h\left(x^2-y^2\right)=xy/\left(a-b\right)$
$\therefore$ of live pair is coordinate Axes $\Rightarrow h=0$
So that equation is $xq=0$ as $x=0, y=0$

66.



Put x = 1,   3 pH here
Put n = 2,   2 pH
Put n = 3,   1 pt here
∴ Total 6 gourd point

67. Plot lines



Let $L_1 = 2y - x$, $L_2 = 4y - x$

∴ pt (1, 0) gives -lve sign to both lines

∴ P must give + lve to L_1 4∈ lve to L_2

∴ $2a - a^2 < 0$ & at$(0, 4)$

$(-\infty, 0) \cup (3, \infty)$

A (2,4)

68. Let the live by y – mx – c = 0

∴ adol is algebraic distance

$$\frac{1-2m-c}{\sqrt{Hn2}} + \frac{2-3m-c}{\sqrt{Hn2}} + \frac{7+4m-c}{\sqrt{Hn2}}$$

10 – m – 3c = 0

$$\frac{10}{3} = m + c$$

∴ pares through (1, 10/3)

69. $\frac{ds}{dx} : ax + hg + g = 0$ if n eqn. put y = 0

$\frac{dg}{dy} ; hx + by + f = 0$ $\frac{-g}{a} = \frac{-f}{h}$

Hg = gf

70. Let P^2 (x, y)

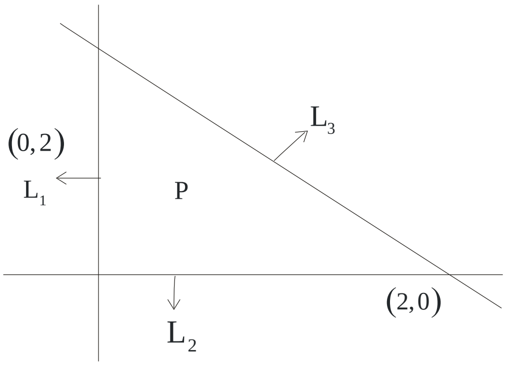
$\therefore$ Alfa f_1 P^2 is (y, x)

Alfa f_2 is becomes $\left(y+3x, x_1\right)$

Alfa f_3 is becomes $\left(\frac{y_1+2x_1}{2}, \frac{y_1+4x_1}{2}\right)$

P^2 (A) becomes (0,0)
P^2 (B) (4,0) becomes (4, 8)
P^2 (C)(4,2) becomes (5, 9)
P^2 (D) (0,2) becomes (1, 1)
Now pb form 1Lgm

71.



For P to be insides it must give signs (+) $\in$lve & $\in$ lve w.r.t lives L_1, L_2, L_3 respectively

$\therefore$ $a > 0,\ \ a^2 > 0\ \ \&\ \ \text{at}\ a^2 - 2 < 0$

$a > 0\ \ \&\ \ \text{at}\ (-2,1)$

$\Rightarrow$ $\text{at}\left(0,1\right)$

72. (B)

here $L_1 : x\cos\alpha + y\sin\theta = p$

$\Delta\ L_2 = x\sin\alpha - y\cos\alpha = 0$

$ax \perp r\ \ \&\ \ ax + by \perp p\ \ \text{is @}\ \frac{\pi}{4}\ \text{with}\ L_1 \Rightarrow ax + by \qquad cp = 0\ \text{is angle}$

_______ of $L_1 \& L_2$

$= \left(x\cos\alpha + y\sin\alpha = p\right) = 1\left(x\sin\alpha - y\cos\alpha\right)$

Take (+lve & y_n)

$x\left(\cos\alpha - \sin\alpha\right) + y\left(\sin\alpha + \cos\alpha\right) - p = 0$

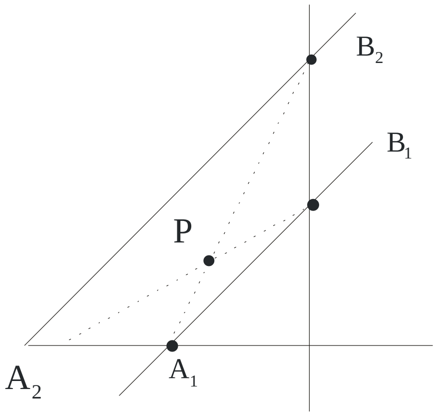
Compare with

$ax + by + p = 0$

$$\frac{a}{\cos\alpha - \sin\alpha} = \frac{b}{\sin\alpha + \cos\alpha} = \frac{-p}{p}$$

$a^2 + b^2 = 2$

73.



$A_1 A_2 B_1 B_2$

Co cyclic

$= m_1 m_2 = 1$ or $m^2 = 1$

$\Rightarrow m = 1 \quad m \leftarrow R^{-1}$

$A_1\left(\frac{-C_1}{m}, 0\right), A_2\left(\frac{-C_2}{m}, 0\right)$

$B_1(0, C_1) \quad B_2(0, C_2)$ let p be (h, k)

$\therefore$ A, P & B_2 collinear A_2 P & B_1 collinear

$\frac{k}{h+C_1} = \frac{C_2}{C_1}$ $\frac{k}{h+C_2} = \frac{C_1}{C_2}$

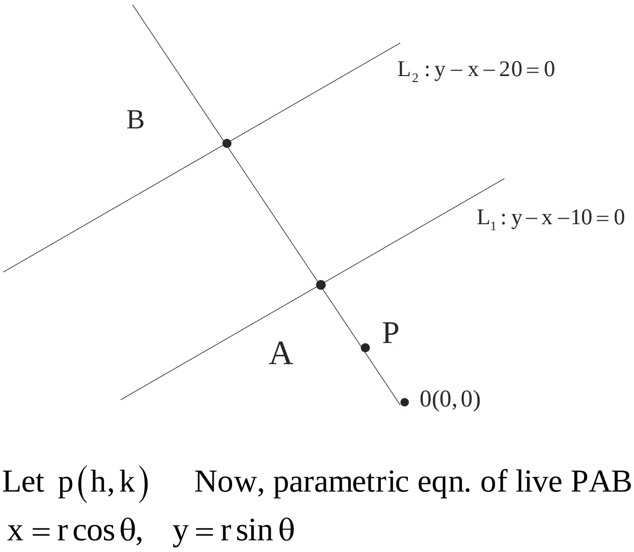
$\rightarrow \frac{k}{C_2} = \frac{H}{C_1} + 1$ & $\frac{k}{C_1} = \frac{H}{C_2} + 1$

Subtract to get

$k\left(\frac{1}{C_2} = \frac{1}{C_1}\right) = h\left(\frac{1}{C_1} = \frac{1}{C_2}\right)$

$k = -h$

74.



Let $p(h, k)$ Now, parametric eqn. of live PAB can be taken as

$x = r\cos\theta, \quad y = r\sin\theta$

Let OP = r_1 $\therefore$ $h = r_1\cos\theta, \quad k = r\sin\theta$

OA = r_2 $A(r_2\cos\theta, r_2\sin\theta)$

OB = r_3 $B(r_3\cos\theta, r_3\sin\theta)$

Now put pts on L_1 & L_2

$r_2(\sin\theta - \cos\theta) = 10$ $r_3(\sin\theta - \cos\theta) = 20$

$$r_2 \frac{10}{\sin\theta - \cos\theta}$$

π is given the

$$\frac{2}{r_1} = \frac{1}{r_2} + \frac{1}{r_3}$$

$$\frac{2}{r_1} = \frac{3}{20}(\sin\theta - \cos\theta)$$

$$r_3 = \frac{20}{\sin\theta - \cos\theta}$$

$$\frac{40}{3} = r_1 \sin\theta - r_2 \cos\theta$$

$$\frac{40}{1} = y - x$$

75. From 74 put

$$r_1^2 = r_2 r_3$$

$$r_1^2 = \frac{200}{(\sin\theta - \cos\theta)^2}$$

$$x - (r_1 \sin\theta - r_1 \cos\theta)^2 = 200$$

$$(x - y)^2 = 200$$

76. From 74

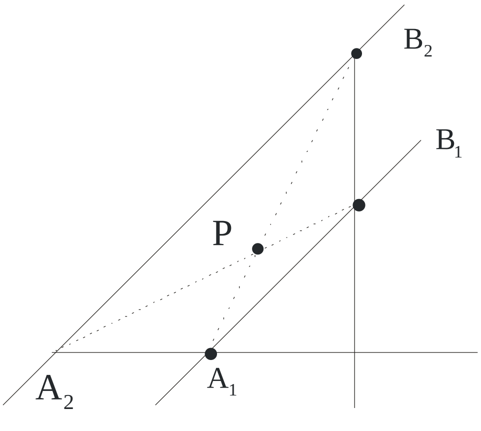
$$\frac{1}{r_1^2} = \frac{1}{r_2^2} + \frac{1}{r_3^2}$$

$$\frac{1}{r_1^2} = \frac{(\sin\theta - \cos\theta)^2}{400}$$

$$80 = (r_1 \sin\theta = r_1 \cos\theta)^2$$

$$(x - y)^2 = 80$$

77.



$A_1\left(\frac{-C_1}{2}, 0\right)$ $B_1(0, C_1)$

$A_2\left(\frac{-C_2}{2}, 0\right)$ $B_2(0, C_2)$

P(h, k) is collinear with A_1 B_2

$$\frac{k - C_2}{h} = \frac{2C_2}{C_1} \quad \text{or} \quad \frac{k}{C_2} - 1 = \frac{2h}{C_1}$$

Also p(h, k) collinear with A_2 & B_2

$$\Rightarrow \quad \frac{k}{C_1} - 1 = \frac{2h}{C_2}$$

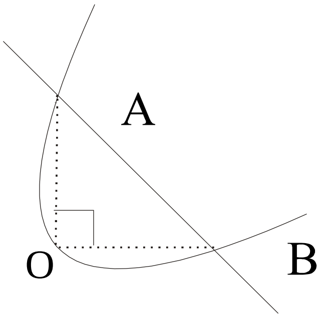
$\therefore$ Subtract

$$k\cancel{\left(\frac{1}{C_2}-\frac{1}{C_1}\right)}=2h\cancel{\left(\frac{1}{C_1}-\frac{1}{C_2}\right)}$$

$k+2h=0$

78. Let live be y = mx – c = 0
Homogenize live with curve

$$3x^2-y^2+\left(4g-2n\right)\left(\frac{y-mx}{c}\right)=0$$



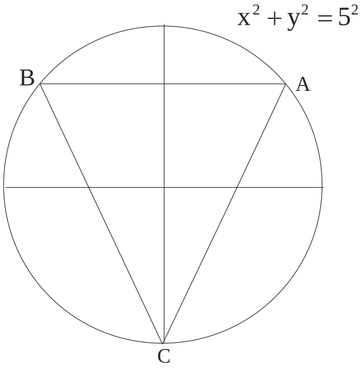
Sin OA & OB

$\therefore \cos ec\ \ x^2+\cos ec\ \ y^2=0$

$\left(3c+2m\right)+\left(-c+4\right)=0$

$m+c=2$ $\quad$ p^2 is (1, -2)

79. (D)
A(3, 4) $\quad B\left(\cos\theta,\ 5\sin\theta\right)$ $\quad C\left(5\sin\theta,\ -5\cos\theta\right)$



Now, 5(0,0) & centride is $C_1\left(\frac{3+5\cos\theta+5\sin\theta}{3},\frac{4+5\sin\theta-5\cos\theta}{3}\right)$

$\therefore\ H=\left(3+5\cos\theta+5\sin\theta\right),\left(4+5\sin\theta-5\cos\theta\right)$

$h-3=5\cos\theta+5\sin\theta$

$k-4=5\sin\theta=5\cos\theta$

$x+y-7=10\sin\theta$ also $x-y+1=10\cos\theta$

$\left(x+y-7\right)^2+\left(x-y+1\right)^2=100$

80.

By parametric live	Also
$k=0+4\sin\theta$	$h=0+3\cos\left(\theta-90\right)$
$k=4\sin\theta$	$h=3\sin\theta$

$\therefore \frac{K}{4}=\frac{H}{3}$

Paper – 2(A)

18. (A, C)

Let P be (a_1, -39)

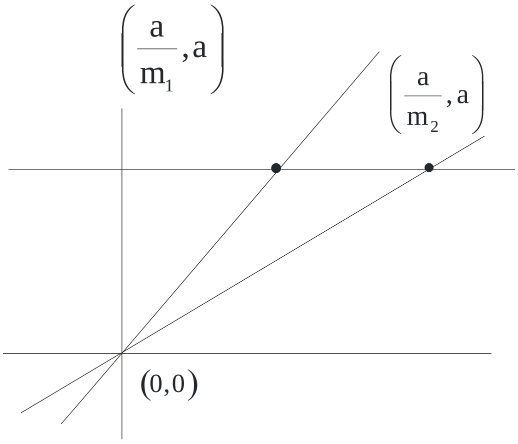
(3, 4) given (- lve sign with 3x – 4y – 8 = 0

$\therefore \left(a_1 - 3a\right)$ must give (+lve sign with line

$3a + 12a - 8 > 0$

$a > \frac{8}{15}$

19. (A, C, D)



$m_1 + m_2 = a$

$m_1 m_2 = -(a+1)$

$\therefore A = \frac{1}{2}\left|\frac{a^2}{m_1} - \frac{a^2}{m_2}\right| = \frac{a^2}{2}\frac{|m_1 - m_2|}{m_1 m_2}$

$A = \frac{a^2}{2}\frac{\sqrt{(m_1+m_2)^2 - 4m_1 m_2}}{|m_1 m_2|} = \frac{a^2}{2}\frac{\left(\sqrt{a^2 + 4(a+1)}\right)}{|a+1|}$

$A = \frac{a^2}{2}\left|\frac{a+2}{a+1}\right|$

20. (A,B, C, D)

If they intersect @ 4 concylic points

$\therefore m_1 m_2 = 1$

$\left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = 1$

$ac = bd$

Now, $(a-1)^2 = (b-d)^2 \Rightarrow (a-c) = 7(b-d)$

Now subtract lives

$x\left(\frac{1}{a} - \frac{1}{c}\right) + y\left(\frac{1}{b} - \frac{1}{d}\right) = 0$

$x\left(\frac{c-a}{ac}\right) + y\left(\frac{b-d}{bd}\right) = 0$

$\Rightarrow \qquad x \pm y = 0$

21. (A, C)

Let line be y = mx + c

Put (1,0) $\Rightarrow$ c = - m

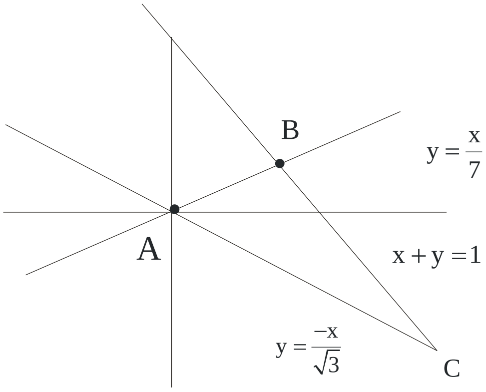
$\therefore \quad y = m(x-1)$ or $mx - y - 1 = 0$

$\therefore$ distance = $\frac{\sqrt{3}}{2}$

$$\frac{1}{\sqrt{Hm_2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{4}{3} = m^2 + 1$$

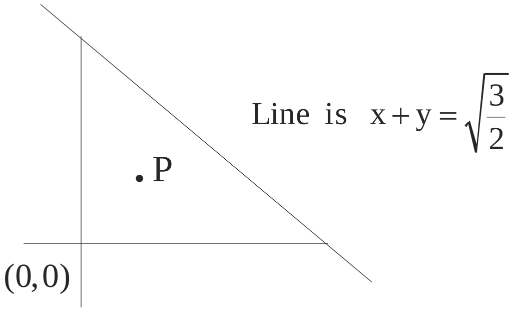
$$m = \pm\frac{1}{\sqrt{3}}$$

22. (B, C)



The Δ is obtuse $\Rightarrow$ interior are In centre & centroid

23. (A, B)



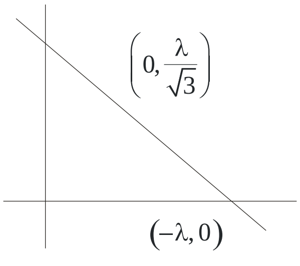
If $P(\sin\theta, \cos\theta)$ inside the Δ

$$\sin\theta > 0 \ \& \cos\theta > 0 \quad \& \quad \sin\theta + \cos\theta < \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \quad q \in 1^{st} \text{ quadrant } \& \sin\left(\theta + \frac{\pi}{4}\right) < \sin\frac{\pi}{3}$$

24. (B,D)

Let the line be $x - \sqrt{3y} + \lambda = 0$



$\therefore$ length of intercept

$$\Rightarrow \quad \lambda^2 + \frac{\lambda^2}{3} = 100$$

$$\lambda = y5\sqrt{3}$$

25. (C, D)

$$\begin{vmatrix} 1 & 1 & -1 \\ m-1 & m^2-7 & -5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = 0$$

By property

0

$$\begin{vmatrix} 0 & 0 & -1 \\ -m^2+m+6 & m^2-12 & -5 \\ -m+3 & 2m-5 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad (2m-5)(-m^2+m+6)+(m-3)(m^2-12)=0$

$\Rightarrow \quad (m-3)\left[-(2m-5)(m12)+(m^2-12)\right]=0$

m = 3 or $m^2-m+2=0$ discard

fro m = 3 lives are parallel

26. (B, C)

Let live have slope ‘m’

$= \frac{1}{2} = \left|\frac{m+2}{1-2m}\right|$

$\frac{m+2}{1-2m} = \frac{1}{2}$ or $\frac{-1}{2}$

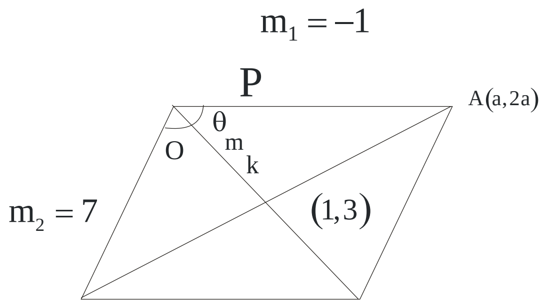
$2m+4=1-2m$ or $2m+2=-1+2m$

4m = - 3 $\Rightarrow m=\infty$

$m=\frac{-3}{4}$

$\therefore$ lives G.M. $\frac{y-3}{x-2}=\frac{-3}{4}$ or ∞

27.



Now diagonal bisects the angle

$\therefore$ equality $\tan\theta$

$\frac{m+1}{1-m}=\frac{7-m}{1+7m}$

$\Rightarrow \quad m=\frac{1}{3}$ or 3

Diagonal are

$\frac{y-3}{x-1}=\frac{1}{3}$ or -3

Put (a, 2a) on there live

$A=\frac{8}{5}$ or $\frac{6}{5}$

28. (A,B,C,D)

Given $\frac{m_1}{m_2}=\frac{9}{2}$

& $\frac{m_1-m_2}{1+m_1m_2}=\pm\frac{7}{9}$

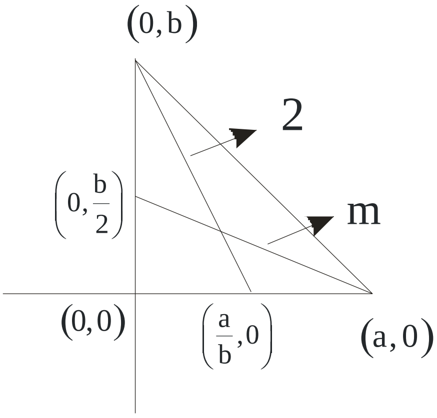
$\frac{7m_2}{2+gm_2}=\frac{7}{9}$ or $\frac{-7}{9}$

$9m_2=2+9m_2{}^2$ or $-9m_2=2+9m_2^2$

$9m_2^2-9m_2^2+2=0$ $\quad$ $9m_2^2+9m_2+2=0$

$m_2=\frac{2}{3}\text{or}\frac{1}{3}$ or $m_2=\frac{-2}{3}\text{or}\frac{-1}{3}$

29. Consider (0,b)



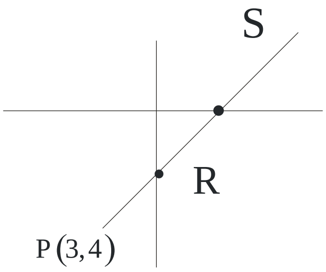
Here $\frac{-2b}{a}=2$ $\quad$ Or $\frac{-b}{2a}=2$

$\Rightarrow\frac{b}{a}=-1$ $\quad$ $\Rightarrow\frac{-b}{a}=4$

$m=\frac{-b}{2a}=\frac{1}{2}$ $\quad$ & $m=\frac{-2b}{a}$

= + 8

30. (A, B, C, D)



Let PR = r_1

PS = r_2

$\therefore$ $R\left(3+r,\cos\theta,\ 4+r,\sin\theta\right)$

$\Rightarrow \quad 3 + r, \cos\theta = 6$

$r_1 = 3\sec\theta$

The $S\left(3 + r_2, \cos\theta,\ 4 + r_2, \sin\theta\right)$

$\Rightarrow \quad 4 + r_2 \sin\theta = 8$

R = $4\operatorname{cosec}\theta$

31. (A, B, & C)

Put $\frac{y}{x} = m$

$1 + m - m^2 = m^3$

Or $m^3 + m^2 = m + 1$

$\cancel{(m+1)}m^2 = \cancel{(m+1)}$

$m = -1$ or $m^2 = 1$

m = -1 $m = \pm 1$

32. $x^2 + mxy - 2y^2 + 3y - 1 = 0$

A = 1, B = - 2, G = 0, $f = \frac{3}{2}$ C = -1 $H = \frac{M}{2}$

$$\Delta = 0 \quad \begin{vmatrix} 1 & \frac{m}{2} & 0 \\ \frac{m}{2} & -2 & \frac{3}{2} \\ 0 & \frac{3}{2} & -1 \end{vmatrix} = 0$$

$$1\left(2 - \frac{9}{4}\right)\frac{-m}{2}\left[\frac{-m}{2}\right] = 0$$

$$\frac{m^2}{4} - \frac{1}{4} = 0 \Rightarrow m = \pm 1$$

Find intersection Ph

33. (A, B, D)

$\Rightarrow$ Angle between lives must be $180^o - 2\alpha$

$$\Rightarrow \left|\tan 2\alpha\right| = \left|\frac{2\sqrt{h^2 - 1}}{2}\right|$$

$\tan^2 2\alpha = h^2 - 1$

$h = \left|8a^2\alpha\right|$

34. (A, B, C, D)

Use condition of both roots common

$3\left(\frac{y}{x}\right)^2 + p\left(\frac{y}{x}\right) + 2 = 0$ has roots m & m_1

$-3\left(\frac{y}{x}\right)^2 + 9\left(\frac{y}{n}\right) + 2 = 0$ has roots m & m_2

$m + m_2 = \frac{-p}{3}, mm_1 = \frac{2}{3}$ & $m + m_2 = \frac{9}{3}$

Also $m_1m_2 = 1$ $mm_2 = \frac{-2}{3}$

From here $m^2 = \frac{4}{9}$

$7m = \frac{2}{3}$ OR $7m = \frac{-2}{3}$

$7m = \frac{2}{3}$

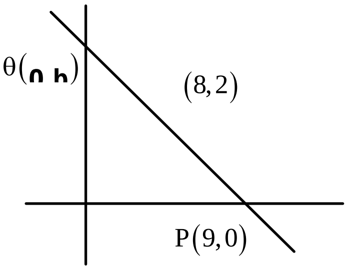
$m_1 = 1$ & $m_2 = -1$

$P = -5$ $q = -1$

$m_1 = -1$ & $m_2 = 1$

$p = 5, \quad q = +1$

23.



a, b (+) ve

also $\frac{9}{a}+\frac{2}{b}=1$

Weighted A.M $\geq$ Weighted H.M

$$\frac{\left(2\sqrt{2}\right)\left(\frac{2\sqrt{2}}{a}\right)+\left(\sqrt{2}\right)\left(\frac{\sqrt{2}}{b}\right)}{2\sqrt{2}+\sqrt{2}}$$

$$\frac{1}{3\sqrt{2}}\geq\frac{3\sqrt{2}}{a+b}$$

$a+b\geq 18$

M, n 18

24. From Q. 23

Use A.M $\geq$ G.M

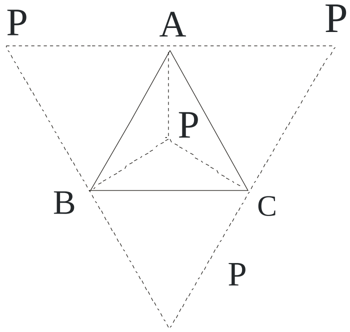
$$\frac{\frac{8}{a}+\frac{1}{b}}{2}\geq\sqrt{\frac{16}{ab}}$$

$$\frac{1}{4}\geq\frac{16}{4b}$$

$ab\geq 64$

$A\geq 32$

25. 4 position possible



Here P such that P,A, B, & C form is any order vertices taken & 4H position is centriod

26. given ax $x^2-(y-2)^2=0$

$X+y-2=0$ or $x-y+2=0$

Bisection Are

$$\frac{x+y-2}{\cancel{\sqrt{2}}}=1\frac{(x-y+2)}{\cancel{\sqrt{2}}}$$

i.e. $\cancel{x}+y-2=\cancel{x}-y+2$ $\quad$ $x+\cancel{y}-2=-x+\cancel{y}-2$

$y=2$ $\quad$ $x=0$

3 lives $x+y=3$, $y=2$ $x=0$

3 vertices (0,2) (0,3)(1,2) $A = \frac{1}{2}$

27. Given $2x + 3y = 6$

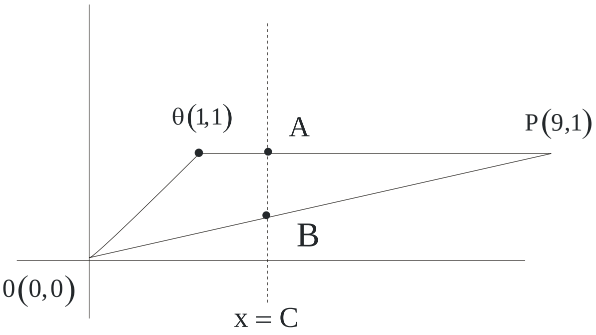
Use equality – Inequality

$\left(2x + 3y\right) \le \sqrt{2^2 + 3^2}\sqrt{x^2 + y^2}$

$\Rightarrow \quad \frac{6}{\sqrt{13}} \le \sqrt{x^2 + y^2}$

$m = \frac{6}{\sqrt{13}}$

28.



A is (C , 1)

B is $\left(C_1, \frac{C}{3}\right)$

$\therefore$ Are of Δ ABP should be 2

Area of ΔOPQ

$\therefore \frac{1}{2}\left|\left(\frac{C-9}{3}\right)^2\right| = 2$

Discard C = 15 or C = 3

Take C = 3

29. $a\left(2x + y - 3\right) + b\left(x + 3y + 1\right) = 0$

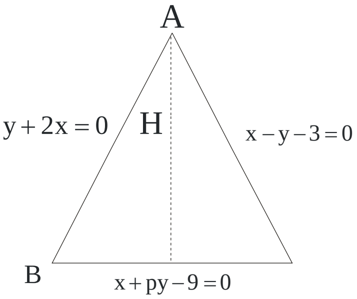
This live passes through intersection of $\begin{matrix} 2x + y - 3 = 0 \\ x + 3y + 1 = 0 \end{matrix}$ i.e. $\left(2, -1\right)$

It also satisfies live mx + 2y + 6 = 0

2m – 2 + 6 = 0

m = - 2

30.



Altitudes from A is $\left(y + 2n\right) + \lambda\left(x - y - 3\right) = 0$

It slope should be P Also H satisfies it

$-\frac{(\lambda+2)}{1-\lambda}=p$

$7+\lambda(-7)$

$\lambda=\frac{7}{4}$

$\Rightarrow$ P is 5

Similarly get q = 45

$\frac{p+q}{10}=5$

STRAIGHT LINES

Exercise – 2(B)

Q.1 (C)(D)

$y+1=\lambda^2 x \;,\;\; x+1=\lambda^2 y \;,\;\; x+y=\lambda^2$

are concurrent line. Let they intersect at $(x_\circ \;,\; y_\circ)$

Therefore,

$y_\circ+1=\lambda^2 x_\circ$ ....(1)

$x_\circ+1=\lambda^2 y_\circ$ ....(2)

$x_\circ+y_\circ=\lambda^2$ ....(3)

$\Rightarrow \lambda^2=-1 \quad or \quad x_\circ-y_\circ=0$

not possible $x_\circ=y_\circ$

from (1) & (2)

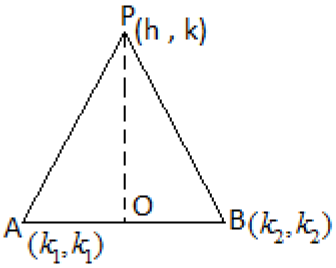
$\Rightarrow 2x_\circ=2y_\circ=\lambda^2$

$$\Rightarrow \frac{\lambda^2}{2}+1=\lambda^2\cdot\frac{\lambda^2}{2} \Rightarrow \lambda^4-\lambda^2-2=0$$

$$\Rightarrow (\lambda^2-2)(\lambda^2+1)=0 \Rightarrow \lambda^2=2 \;\Rightarrow \lambda=\pm\sqrt{2}$$

Q.2 (A)(C)

$$\text{AB}=\sqrt{(\text{k}_1-\text{k}_2)^2+(\text{k}_1-\text{k}_2)^2}=\sqrt{2(\text{k}_1-\text{k}_2)^2}$$
$$=2\sqrt{2}$$



$$\text{Now Area}=\frac{1}{2}(\text{AB})(\text{PQ})$$

$$\Rightarrow 2=\frac{1}{2}\times 2\sqrt{2}\times PQ$$

$$\Rightarrow PQ=\sqrt{2}$$

Equation of line AB is $y-k_1=\frac{k_2-k_1}{k_2k_1}(x-k_1)$

$y-x=0$.

Perpendicular distance from P to AB is $\sqrt{2}$.

$\Rightarrow \sqrt{2}=\left|\frac{h-k}{\sqrt{2}}\right| \Rightarrow h-k=\pm 2$

$\Rightarrow x-y=\pm 2$

and $(x-y-2)(x-y+2)=0$

Q.3 (B)(C)

$(b + c)(b - c) = 4a(a + c)$

$4a^2 + 4ac + (c^2 - b^2) = 0$

$\Rightarrow a=\frac{-c \pm b}{2}$

Substituting the value of a in ax + by + c = 0.

$(b - c)x + 2by + 2c = 0$	$(b + c)x - 2by - 2ac = 0$
$b(x + 2y) + c(2 - x) = 0$	$b(x - 2y) + c(x - 2) = 0$
$b+c \neq 0$	
$\therefore$ x = 2 y = – 1	x = 2 y = 1
(x , y) = (2 , – 1)	(x , y) = (2 , 1)

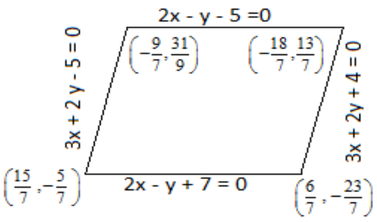
Q.4 (A)(B)

Let center of circle be M (a , b). Then the distance between M and x – axis, distance between M and y – axis and the distance between M and line 3x + 4y = 120 will be same.

$\Rightarrow k=h=\frac{|3h+4k-120|}{5}$

$$\begin{array}{l|l} 5h = 3h + 4k - 120 & -5h \neq 3h + 4k - 120 \\ 5h = 3h + 4k - 120 & -5h = 3h + 4k - 120 \\ h = 60\ ,\ k\ =\ 60 & h = 10\ \ ,\ \ k = 10 \end{array}$$

Q.5 (A)(B)(C)



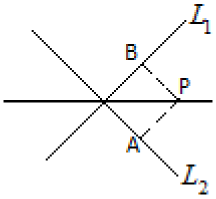
Solving the equations get the vertices and then the equations of diagonals as 6x + 11y = 5 & 18x + 5y + 1 = 0.

Area of parallelogram $= \left|\dfrac{(d_1 - d_2)\ (c_1 - c_2)}{a_1 b_2 - a_2 b_1}\right|$

$$= \left|\frac{(7+5)(4+5)}{4+3}\right| \quad = \frac{180}{7}$$

Q.6 (A)(B)(C)(D)

$L_1 : x \cos\alpha + y \sin\alpha - c$
$L_2 : x \sin\alpha - y \cos\alpha$
$L : ax + by + c = 0$ is the angle bisector of L_1 and L_2



Then, $P(x_\circ\ ,\ y_\circ)$ any point on L will be equidistance from L₁ and L₂

i.e. PA = PB { and $P(x_\circ\ ,\ y_\circ)$ lie on ax + by + c = 0 $\Rightarrow y_\circ = -\dfrac{ax_\circ - c}{b}$ }

$$PA = PB \Rightarrow |x_\circ \cos\alpha + y_\circ \sin\alpha - 0| = |x_\circ \sin\alpha - y_\circ \cos\alpha|$$

$$\Rightarrow \left|x_\circ \cos\alpha - \frac{(ax_\circ + c)}{b}\sin\alpha - c\right| = \left|x_\circ \sin\alpha - \frac{(ax_\circ + c)}{b}\cos\alpha - c\right|$$

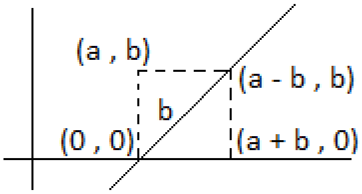
Solving we get $a^2 + b^2 = 2$

$$\Rightarrow -\sqrt{2} \le a \le \sqrt{2} \quad , \quad -\sqrt{2} \le b \le \sqrt{2}$$

Q.7 (A)(B)(C)

A is (a, b)

A cannot lie on y = x – a.



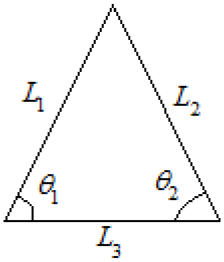
Q.8 (A)(B)

L_1 : (a + b)x + (a – b)y = 2ab

L_2 : (a - b)x + (a + b)y = 2ab

By solving them, we get x = y = b

Let the slope of third line be M



Then L_3 : y – a + b = m (x – b + a)

Isosceles triangle i.e. $\tan\theta_1 = \tan\theta_2$

$$\Rightarrow \left|\frac{m_1-m}{1+m_1m}\right| = \left|\frac{m_2-m}{1+m_2m}\right|$$

$$\Rightarrow \left|\frac{\frac{a+b}{b-c}-m}{1+m\left(\frac{a+b}{b-a}\right)}\right| = \left|\frac{\left(\frac{b-c}{a+b}\right)-1}{1+m\left(\frac{b-a}{a+b}\right)}\right|$$

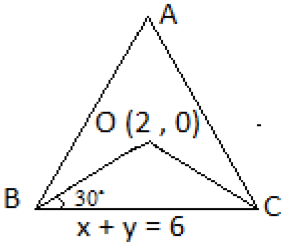
$$\Rightarrow \left|\frac{(a+b)-mb+ma}{b-a+ma+ma}\right| = \left|\frac{b-a-ma-mb}{a+b+mb-ma}\right|$$

$$(a+b)^2 - m^2(b-a)^2 = (b-a)^2 - m^2(a+b)^2$$

$$\Rightarrow 4ab = m^2 4ab \quad \Rightarrow \quad m = \pm 1$$

$$\therefore L_3 : x+y=0 \qquad as \quad L_3 : x-y+2(a-b)=0$$

Q.9 (A)(B)(C)



Let M₁ = slope (OB)

M₂ = slope (OC)

M = slope (BC) = - 1

Therefore, $\left|\frac{M_1-m}{1+M_1m}\right| = \left|\frac{M_2-m}{1+M_2m}\right| = \tan 30^\circ$

$$\left|\frac{M_1+1}{1-m}\right| = \frac{1}{\sqrt{3}} \quad \& \quad \left|\frac{M_2+1}{M_2-1}\right| = \frac{1}{\sqrt{3}}$$

$$M_1 = \frac{-(1+\sqrt{3})}{\sqrt{3}-1} \ \& \ M_2 = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}$$

OB : (y – 2) $= \frac{-(1+\sqrt{3})}{\sqrt{3}-1}(x-2)$

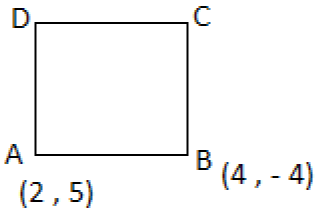
OC : (y – 2) $= \frac{-(\sqrt{3}-1)}{\sqrt{3}+1}(x-2)$

$\therefore B \equiv ((3-\sqrt{3}),\ (3+\sqrt{3})),\ C \equiv (3+\sqrt{3}),\ (3-\sqrt{3})$ and $A \equiv (0\ ,\ 0)$

$$a = \sqrt{(0-(3-\sqrt{3})^2)+(0-(3+\sqrt{3})^2)} \ = \ 2\sqrt{6}$$

Area $= \frac{3\sqrt{3}}{4}a^2 = 6\sqrt{3}$

Q.10 (A)(B)



$A \equiv (2,3) \qquad B \equiv (4,-4)$

$\overrightarrow{AB} = (2,-9)$

By 90° Rotation

$\overrightarrow{AC} \equiv (9,2)$

$$\therefore (C_x - 2\ ,\ C_y - 5) = (9,2)$$

$$C_x = 11 \ \ ,\ \ C_y = 7$$

$$C \equiv (11,7)$$

$(D_x - 4\ ,\ D_y + 4)\ =\ (4,2)\ \ D \equiv (13\ ,\ -2)$

$\overrightarrow{BA} = (-2,9)$

$\overrightarrow{BC} = (-9,-2)$

$(C_x - 4, C_y + 4) = (-9,-2)$ $\qquad C \equiv (-5,0)$

$(D_x - 2, D_y - 5) = (-9,-2)$ $\qquad D \equiv (-7,3)$

Q.11 (A)(B)

Altitudes are $y = m_1x$, $y = m_2x$, $y = m_3x$.

$\Rightarrow$ orthocenter is origin.

And we know that orthocenter, centroid and circumcenter lie on the same line.

Y = hx is the equation of line passing through orthocenter and circumter and centroid will lie on this line. But orthocenter will be fixed.

Q.12 (A)(B)

$A \equiv (a\cos\theta\ ,\ b\sin\theta)$ $\qquad B \equiv (-a\sin\theta\ ,\ b\cos\theta)\backslash$

$C \equiv (-a\cos\theta\ ,\ -b\sin\theta)$ $\qquad D \equiv (a\sin\theta\ ,\ -b\cos\theta)$

$$AB = \sqrt{a^2(\cos\theta + \sin\theta)^2 + b^2(\sin\theta - \cos\theta)^2}$$

$$= \sqrt{a^2\sin 2\theta - b^2\sin 2\theta}\ = CD$$

$$AD = \sqrt{a^2(\cos\theta - \sin\theta)^2 + b^2(\cos\theta + \sin\theta)^2}$$

$$= \sqrt{a^2 - a^2\sin 2\theta + b^2\sin 2\theta}\ = BC$$

$\therefore$ ABCD is parallelogram

And Area is function of θ

Q.13 (B)

$h_1 = |a\cos\theta - p|$ & $h_2 = |a\cos\theta - p|$

$$h_1h_2 = k^2 \Rightarrow (a\cos\theta + p)(a\cos\theta - p) = k^2$$

$$a^2\cos^2\theta - p^2 = k^2$$

Foot of perpendicular from A.

$$P_1 = \left(-a - \frac{\cos\theta(-a\cos\theta - p)}{1}, 0 - \frac{\sin\theta(-a\cos\theta - p)}{1}\right)$$

$$P_1 = (-a + a\cos^2\theta + p\cos\theta \ , \ a\sin\theta\cos\theta + p\sin\theta)$$

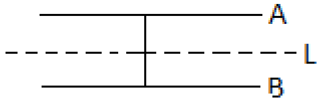
For locus of P_1

$$x^2 - y^2 = a^2 + a^2\cos^2\theta + p^2 - 2a^2\cos^2\theta - 2ap\cos\theta + 2ap\cos^3\theta + 2ap\sin^2\theta\cos\theta$$

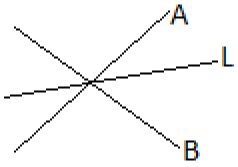
Similarly, locus of B : $x^2 + y^2 = a^2 + k^2$.

Q.14 (A)(B)(C)

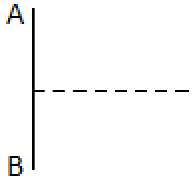
(a)



(b)



(c)

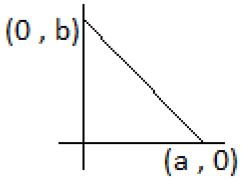


Q.15 (A)(B)(C)

$$L : \frac{x}{a} + \frac{y}{b} = 1$$

$$s = \left| \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| \qquad \Rightarrow ab = 5\sqrt{a^2 + b^2} \ \ or \ \ a^2b^2 = 25(a^2 + b^2)$$

mid – point $p \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$



$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{a^2}+\frac{4}{b^2}=4\left(\frac{1}{25}\right)=\frac{4}{25}$$

$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{25}$$

Foot of $\perp$ is at distance = 5 = constant

Locus is area of radius s and centre (0 , 0)

$\therefore x^2+y^2=25$

Maximum area obtained when a = b

$\Rightarrow a^2 = 25(2a^2) \;\Rightarrow a = 5\sqrt{2}$

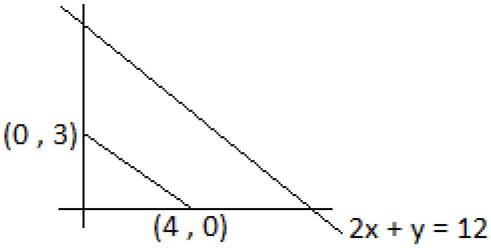
Area $=\frac{1}{2}a^2 = 25$

Q.16 (A)(B)

A lie on 2x + y = 12

A $\equiv (x, 12-2x)$

Centroid $\equiv \left(\frac{4+x}{3}, \frac{15-2x}{3}\right)$



$\frac{4+x}{3}=h$, $\frac{15-2x}{3}=k$

$\Rightarrow x = 3h-4$

Substituting this in $\frac{15-2x}{3}=k$

$\Rightarrow \frac{15-2(3h-4)}{3}=k \;\Rightarrow 3k = 15-64+8$

$\Rightarrow 6h+3h=23$

$\Rightarrow 6x+3y-23=0$

$$\text{Area }(\Delta ABC) = \left|\frac{1}{2}\begin{vmatrix} 4 & 0 & 1 \\ 0 & 3 & 1 \\ x & 12-3x & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|4(3-(2+2x)+1(-3x))\right| = \frac{1}{2}\left|5x-36\right|$$ not constant.

If Area = 7

$$\Rightarrow \left|5x-36\right| = 14$$

$$\Rightarrow 5x-36 = 14\, or \qquad 5x-36 = -14$$

$$\Rightarrow x = 10 \quad y = -8 \qquad or \qquad x = \frac{22}{5} \quad y = \frac{10}{5}$$

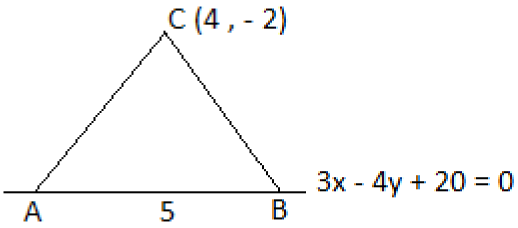
Q.17 (A)(B)(C)

Let A $\equiv (x, y)$

B $\equiv \left(x+5\times\frac{4}{5}\,,\, y+5\times\frac{3}{5}\right)$

$\equiv (x+4\,,\, y+3)$

Centroid $\equiv \left(\frac{x+x+4+4}{3}\,,\,\frac{y+y+3-2}{3}\right)$



$$\equiv \left(\frac{2x+8}{3}\,,\,\frac{2y+1}{3}\right)$$

$$h = \frac{2x+8}{3},\ k = \frac{2y+1}{3}$$

$$3x-4y = 20$$

$$\Rightarrow k = \frac{\frac{3x+20}{2}+1}{3} = \frac{9h-24+44}{12}$$

$$\Rightarrow 12k = 9h+20$$

$$\Rightarrow 9x-12y+20 = 0$$

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} 4 & -2 & 1 \\ x & y & 1 \\ x+4 & y+3 & 1 \end{vmatrix} \right|$$

$$= \left| \frac{1}{2}(4(y-y-3)+2(x-x-4)+(xy+3x-xy-4y)) \right|$$

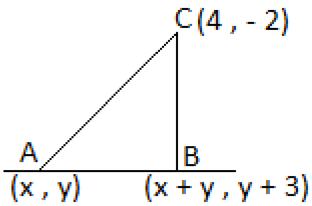
$$= \left| \frac{3x-4y-20}{2} \right| = \left| \frac{-20-20}{2} \right| = 20$$

$$\text{Equation of CB : } y+2 = \frac{-4}{3}(x-4)$$

$$3y+6 = -4x+16$$

$$4x+3y = 10$$

$$3x-4y = -20$$



We get,

$$x+4 = \frac{-4}{5} \ \& \ y+3 = \frac{22}{5}$$

$$A \in \left(\frac{-24}{5}, \frac{-4}{5} \right) \cup \left(\frac{7}{5}, \frac{22}{5} \right)$$

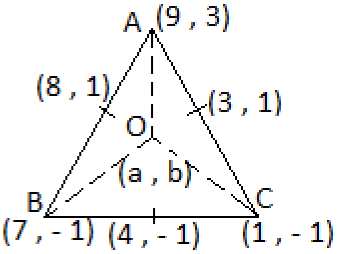
COMPREHENSION TYPE

Passage – 1

Q.1 (D)

$$\textbf{Area of } \Delta\textbf{DEF} = \frac{1}{2} \begin{vmatrix} 8 & 1 & 1 \\ 4 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} |8(-2) - 1(-1) + 1(9)| = \frac{1}{2} |-16+10| = \frac{1}{2} |-6| = 3$$



Q.2 (B)

$$OA = OB \quad \Rightarrow \quad (a-4)^2 + (b-3)^2 = (a-7)^2 + (b+1)^2$$

$$4a + 8b = 40 \quad \Rightarrow \quad 2a + 4b = 20 \quad \text{.............(1)}$$

$OA = OC \quad \Rightarrow \quad (a-a)^2+(b-3)^2 \;=(a-1)^2+(b+1)^2$

2a + b = 11 ....(2)

From (1) and (2) we get,

a = 4 b = 3

$\therefore R = \sqrt{(4-a)+(3-3)} \;=\; \sqrt{(5)^2} \;=5$

a + b + R = 3 + 4 + 5 = 12

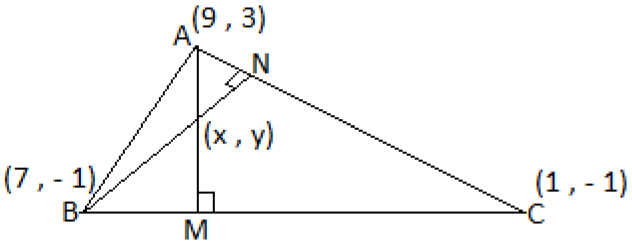
Q.3 (D)

slope of BC $=\dfrac{-1+1}{1-7}=0$

slope of AM $=\dfrac{1}{0}$

equation of AM $\Rightarrow x=9$ ....(1)

slope of AC $=\dfrac{4}{8}=\dfrac{1}{2}$



Slope of BN = – 2.

Equation of BN $\Rightarrow y+1=-2(x-7)$ ....(2)

Solving (1) and (2)

x = 9 y = – 5

Q.4 (A)(B)(C)(D)

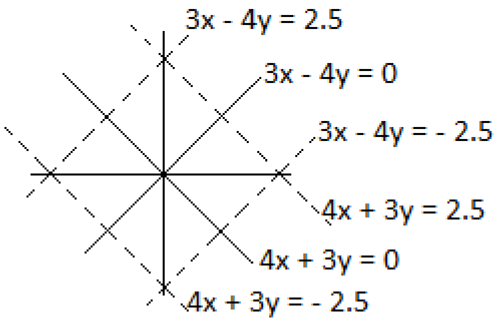
Area = 25

side = 5

O (0 , 0) is mid - point

So distance 0 to vertices will be 2.5

From inspection



Passage – 2

Q.5 (A)

$$4(a^2+b^2)=(c_1-c_2) \quad \Rightarrow \quad \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}=2$$

L_1 : ax + by +c_1 = 0

L_2 : ax + by +c_2 = 0 distance between them = 2.

$\therefore t \in \{0 , 2\}$

Q.6 (A)(C)

Equation of line passing through P and parallel L_1

Let L_p : ax + by $+\lambda$

L_1 : ax + by + c_1

$$\text{diastance} = \frac{c_1-\lambda}{\sqrt{a^2+b^2}} = t$$

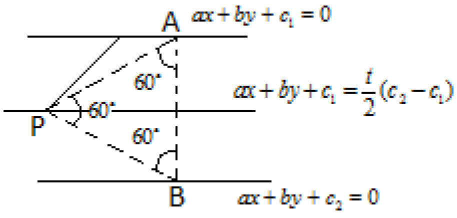
$$\Rightarrow \lambda = c_1 \pm \frac{t}{2}(c_1-c_2)$$

$$\therefore LP : ax+by+c_1 = \frac{t}{2}(c_1-c_2)$$ ………………..(A)

$$ax+by+c_1 = \frac{t}{2}(c_2-c_1)$$ ………………..(C)

Q.7 (B)

$$\text{Area of } \Delta ABP = \frac{1}{\sqrt{3}}\left(t^2-2t+4\right)$$



Q.8 (A)

Area of equilateral triangle $\frac{\sqrt{3}}{4}a^2 = \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$

$$a^2 = \frac{4}{\sqrt{3\times\sqrt{3}}}(t^2 - 2t + 4) \quad \Rightarrow \quad a = \frac{2}{\sqrt{3}}\sqrt{t^2 - 2t + 4}$$

Q.9 (A)(B)(C)(D)

$t \in \{0, 2\}$

$\Rightarrow \ AB = \frac{2}{\sqrt{3}}\sqrt{t^2 - 2t + 4}$

$\Rightarrow \ AB \le \frac{4}{\sqrt{3}}$

$\therefore 2 \le AB \le \frac{4}{\sqrt{3}}$

Area $= \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$ $\qquad \sqrt{3} \le$ Area $\le \frac{4\sqrt{3}}{3}$

When AB = 0, P can obtain four position.

When AB = 2, P can obtain 2 position.

Q.10 (A)(B)(C)

Area $= \frac{1}{\sqrt{3}}(t^2 - 2t + 4)$

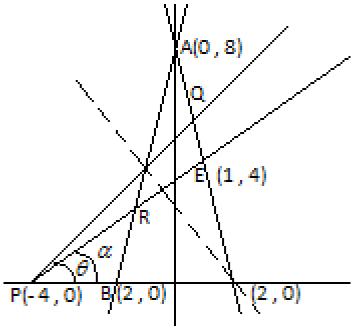
Max when t = 0 (A)

Min when t = 2 (B)

$\sqrt{3} <$ Area $< \frac{4\sqrt{3}}{3}$ (C)

Passage – 3

Q.11 (A)



$\cdot\ A_1 = \text{AreaBCQD}$

$= \text{Area}CQD + \text{Area}BCD$

$AC: y = -4x + 8$

$PQ: y = \tan\theta x + \tan\theta + 4$

solvong them we get,

$\theta \equiv \left(\frac{4 - \tan\theta}{4 + \tan\theta}, \frac{16 + 12\tan\theta}{4 + \tan\theta} \right)$

$\perp$ distance from θt BC is

BC : 4x + 3y – 8 = 0

$$\perp \text{distance} = \frac{32 + 24\tan\theta}{5(4 + \tan\theta)}$$

$$\text{Area CQD} = \frac{1}{2} \times 5 \times \frac{(32 + 24\tan\theta)}{5(4 + \tan\theta)}$$

$$= \frac{32 + 24\tan\theta}{2(4 + \tan\theta)}$$

Area of BCD = 8

$$\therefore A_1 = \frac{48 + 20\tan}{4 + \tan\theta}$$

Q.12 (A)

AB : y = 4x – 8

$$\text{PE} \ : y = \tan\alpha\, x + 4 - \tan\alpha$$

$$R \equiv \left(\frac{\tan\alpha - 12}{\tan\alpha - 4}, \frac{4\tan\alpha + 16}{\tan\alpha - 4} \right)$$

BE : 4x – 3y + 8 = 0

$$\perp \text{Distance from R to BE} \equiv \frac{24\tan\alpha - 32}{\tan\alpha - 4}$$

$$\therefore \text{ Area BER} = \frac{12\tan\alpha - 16}{\tan\alpha - 4}$$

Area BCE = 8

$$\therefore A_2 = \frac{48 - 20\tan\alpha}{4 - \tan\alpha}$$

Q.13 (A)

$\text{when} \quad \tan\theta = 0 \qquad\qquad \min \quad A_1 = \frac{48}{4} = 12$

$\text{when} \tan\theta = 4 \qquad \max \quad A_1 = \frac{128}{8} = 16$

(12 , 16)

Q.14 (A)

$\tan\alpha = \frac{4}{3} \qquad \min \quad A_2 = 8$

$\tan\alpha = 0 \qquad \max \quad A_2 = 12$

A_2 : (8 , 12)

Q.15 (A)

$$\frac{48 + 20\tan\theta}{4 + \tan\theta} \times \frac{4 - \tan\alpha}{48 - 20\tan\alpha}$$

$$\frac{A_1}{A_2} \equiv (1, 2)$$

Passage – 4

Q.17 (A)

$\text{M}(\beta, \beta + 1)$ satisfies y = x + 1.

Q.18 (C)

BC : 8y = x + 2

(24 – 2)(8(β – 1) – β – 2) > 0

$\Rightarrow \beta > -2/3.$

Q.19 (C)

AC : 3y + x = 9

$$(3(\beta+1)+\beta-9)\ (-2-9)>0$$

$$4\beta-6<0 \qquad \beta<\frac{3}{2}$$

AB = 2y = 3x + 6 = 0

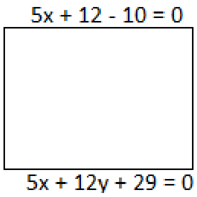
$$(3\beta-2\beta-2+6)\ (18-2+6)>0$$

$$-\frac{6}{7}<\beta<\frac{3}{2}$$

Passage – 5

Q.20 (B)

M = (- 3, 5)
So, one line will pass through (- 3 , 5) and other have two options.
So, Two square are possible



Q.21 (A)

$$\text{distance between lines } =\frac{-10-29}{\sqrt{5^2+12^2}} = \frac{39}{13} = 3$$

Area = 9

Q.22 (B)

$$12x-5y+\lambda=0$$

Will pass through (- 3 , 5)

$$\Rightarrow (-36)-25+\lambda=0$$

$$\lambda=61$$

$12x - 5y + \lambda = 0$

Other line parallel to $12x - 5y + \lambda = 0$ will be $12x - 5y + c = 0$

distance $3 = \frac{|\lambda - c|}{13} \Rightarrow c = 22$

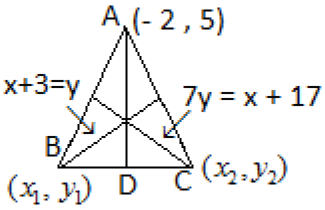
Passage – 6

Q.23 (A)

$x+3=y$

$x+17=y$

$x=-\frac{2}{3} \quad y=\frac{7}{3}$



Q.24 (A)

Centroid divide median in 2 : 1 ratio.

D (0 , 1)

Q.25 (A)

(x_1 , y_1) will pass through x + 3 = y

i.e. $x_1 + 3 = y_1$ and $7y_2 = x_2 + 17$

(0 , 1) is the mid – point of (x_1 , y_1) and (x_2 , y_2)

$\frac{x_1 + x_2}{2} = 0 \qquad \Rightarrow \; x_1 + x_2 = 0$

$\frac{y_1 + y_2}{2} = 2 \qquad \Rightarrow \; y_1 + y_2 = 0$

Solving all four equations we get

$(x_1 , y_1) \equiv (-4, -1)$

$(x_2 , y_2) \equiv (4, 3)$

Q.26 (B)(C)

slope of AB $= \frac{6}{2} = 3$

slope of BC = $\frac{4}{8} = \frac{1}{2}$

$$\tan\theta = \frac{\left|3 - \frac{1}{2}\right|}{\left|1 + \frac{1}{2} \times 3\right|} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$\theta = 45^\circ$

slope of AC $= -\frac{1}{3}$

$\therefore AB \perp AC$

$\therefore$ ABC is right angle and isosceles

Q.27. Area ABC $= \frac{1}{2}[-2(-1,-3)+(-4)(3-5)+4(5+1)]$

$= 20$

Assertion and Reason Type

Q.1 (A)

s – 2 is correct and explain s – 1

Q.2 (A)

s – 2 is correct and explain s – 1

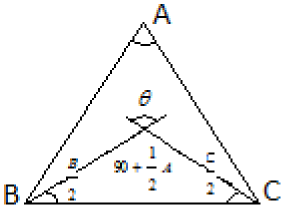
As we know

$$90 + \frac{1}{2}A = \theta$$

and $\frac{A}{2} + \frac{C}{2} + \theta = 180$

$$\frac{B}{2} + \frac{C}{2} = 180 - \theta$$

and $B + C = 180 - A$



Q.3 (A)

Formula to find in-center is $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+ay_2+cy_3}{a+b+c}\right)$

S – 2 is correct explanation of s – 1.

Q.4 (A)

$$S_2 \Rightarrow \quad y=m_1x+\frac{a}{m_1}$$

$$y=m_2x+\frac{a}{m_2}$$

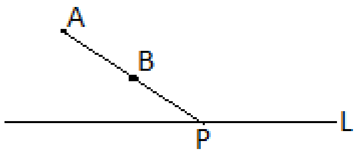
$$\Rightarrow x=\frac{a}{m_1m_2} \quad y=\frac{a}{m_1}+\frac{a}{m_2}$$

So vertices are $\left(\frac{a}{m_1m_2}, \frac{a}{m_1}+\frac{a}{m_2}\right), \left(\frac{a}{m_2m_3}, \frac{a}{m_2}+\frac{a}{m_3}\right)$ and $\left(\frac{a}{m_3m_1}, \frac{a}{m_3}+\frac{a}{m_1}\right)$

And Area $=a^2d^3$

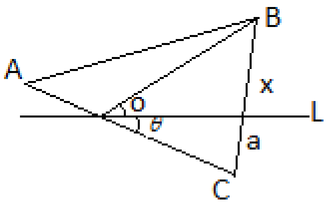
Q.5 (A)

max value of
It will occur when P , A , B
are collinear



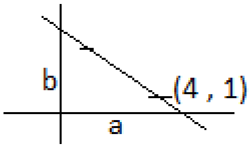
Q,6 (A)

PB = PC
$|PA+PB| \geq AB$



Q.7 (A)

$\frac{x}{a}+\frac{y}{b}=1$ passe through (4 , 1)
$\Rightarrow 4b+a=ab$
$a=\frac{-4b}{1-b}$



$a+b \;=\; b-\frac{4b}{1-b} \;=\; b^2-3b$ is minimum when b = 3 $\Rightarrow a=6$

$\therefore$ a + b = 9

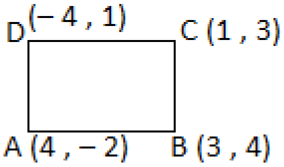
$-\frac{4b^2}{1-b}$ is minimum when b = 2

a = 8.

ab = 16

Q.8 (A)

equation AC : 5x + 3y – 14 = 0
(– 20 + 3 + 4) (15 + 12 – 14) < 0
So, B and D lie on opposite side.
equation BD : – -y + 3x + 19 = 0

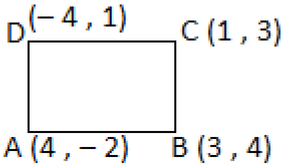


(+ 3 – 21 + 19) (12 + 14 + 19) > 0

So, A and C lie on same side.

$\therefore$ ABCD is concurrent quadrilateral I – true.

AC : 5x – 4y + 12 =0
(15 – 16 + 12) (– 20 – 4 + 12) < 0
So, B and D lie on opposite side.
BD : – 7y + 3x + 19 = 0



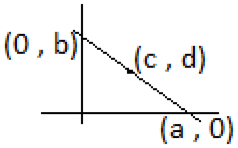
(- 21 + 19) (12 + 14 + 19) < 0

Opposite side.

$\therefore$ ABCD is concurrent quadrilateral.

Q.9 (A)

a > c and b > b
Minimum are of triangle will from
when $\frac{a}{2}=c \;\; \frac{b}{2}=d$



$\therefore \frac{1}{2}ab = 2cd$ Area.

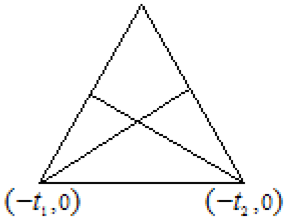
Q.10 (A)

Intersection point of Altitude is orthocenter

$t_2 x-(1+t_2)y+t_1 t_2=0$

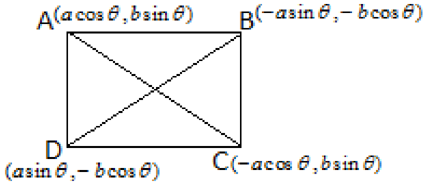
$t_1 x-(1+t_1)y+t_1 t_2=0$

solving then, x = + y



Q.11 (A)

(0 , 0) satisfy AC and BC.



Area = $\begin{vmatrix} 0 & 0 & 1 \\ x_1 & y_2 & 1 \\ x_1 & y_2 & 1 \end{vmatrix}=\frac{1}{2}(x_1 y_2-x_2 y_1)$

Area of OAB $=\frac{1}{2}(-ab\cos^2\theta-ab\sin^2\theta)=+\frac{ab}{2}$

Area of OBC $=\frac{1}{2}(-ab\cos^2\theta+ab\sin^2\theta)=\frac{ab}{2}$

Area of OAB $=\frac{1}{2}(-ab\cos^2\theta-ab\sin^2\theta)=+\frac{ab}{2}$

Area of ODC $=\frac{1}{2}(ab\sin^2\theta+ab\cos^2\theta)=\frac{ab}{2}$

Area ABCD = 2ab.

Q.12 (A)

$\begin{vmatrix} 1 & 1 & 1 \\ k & 1 & 3 \\ 1 & k & 5 \end{vmatrix}=0$ mean lines are concurrent as parallel.

From (J) we know $k \neq 1$ $\therefore$ lines are not parallel and k = 7.

Q.13 (A)

AB slope $=\frac{1-5}{1+1}=\frac{-4}{2}=-2$

(2x + y – 10) is parallel to AB (J)

And if not parallel then passé through mid – point of AB (0 , 3) (3x – 2y + 6) = 0 is equation.

$\therefore$ (2x + y – 10) (3x – 2y + 6) = 0 is the equation

Q.14 (A)

(1 , 2) is the concurrent point.

(2x – 3y + 4) – k (x – 2y + 3) = 5x – 3y + 1

$$\frac{5}{3} = \frac{2+k}{3+2h} \quad \Rightarrow \quad k = \frac{-9}{7}$$

Match the Column

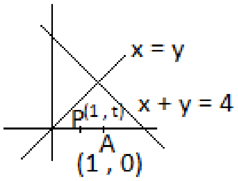
Q.1 $\text{A} \to \text{r, B} \to \text{p, C} \to \text{u, D} \to \text{u}$

(A) P divides A and B in t : 1 – t ratio hence 0 < t < 1

(B) x + 2y = 1 & $x + 2y = \frac{1}{2}$

$$1 \le 1 + \frac{t}{\sqrt{2}} + 2\left(2 + \frac{t}{\sqrt{2}}\right) \le \frac{15}{2}$$
$$\Rightarrow -\frac{4\sqrt{2}}{3} \le t \le \frac{5\sqrt{2}}{6}$$

(C) Point P and A are same on side of x + y = 4
$\Rightarrow$ (1 + t - 4) (1 + 0 - 4) > 0
t < 3
$\Rightarrow$ And P and A are also same side of x = y



$$\Rightarrow (1-t)(1-0) > 0 \Rightarrow t < 1$$

(D) $\begin{vmatrix} 1 & -t & -m \\ 0 & 1 & 2 \\ m & -1 & 0 \end{vmatrix} = 0 \quad \Rightarrow \quad t = \frac{m^2+2}{2}$

Q.2 $\text{A} \to \text{r, B} \to \text{q, C} \to \text{p, q, s D} \to \text{p, q, s}$

(A) $k = 0 \quad \Rightarrow \quad mx - y = 0 \qquad mx - y = 0$

$x - my = 0 \quad x - my = 0$

(B) mx – y = 0

mx – y = 2k slope = + m pependiculer line.

x – my = k

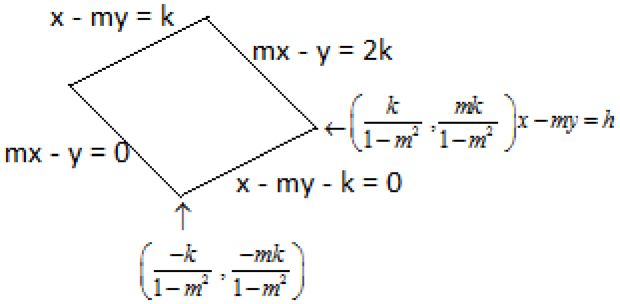
x – my = – k slope $= \frac{1}{m}$ pependiculer line.

$\therefore$ are sides of rhombus

side $= \frac{2k\sqrt{m^2+1}}{m^2-1}$

and diagonal are parallel with

+ 1 , - 1



(C) PQ , S same.

(D) PQ , S same.

Q.3 $A \rightarrow s$, $B \rightarrow r$, $C \rightarrow q$, $D \rightarrow p$

(A) 2x + y = 0

2x + y = 8

2x – y = 4 Rhombus

2x – y = 4

(B)

$$\left.\begin{array}{l} 3x-4y+11=0 \\ 3x-4y-9=0 \end{array}\right\} \quad \text{disatnce} = \frac{20}{5}$$

$$\left.\begin{array}{l} 4x+3y+17=0 \\ 4x+3y-3=0 \end{array}\right\} \quad \text{distance} = \frac{20}{5}$$

(C) 3x + 2y = 0

3x + 2y = 10

2x – 3y = 0

2x – 3y = 35

(D) 2x + y = 0

$2x + y = 0$

$x - y = 8$

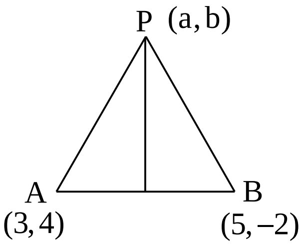
$x - y = 14$

STRAIGHT LINES

EXERCISE - 2(C)

Q.1

A (3, 4) B (5, – 2)



$\Rightarrow PA = PB = \sqrt{(a-3)^2 + (b-4)^2} = \sqrt{(a-5)^2 + (b+2)^2}$

$\Rightarrow 1 - 6a + 9 - 8b + 16 = - 10a + 25 + 4b + 4$

$\Rightarrow 4a - 12b = 4$

$\Rightarrow a - 3b = 1$ …………(1)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a & b & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 10$$

$\Rightarrow a\ (6) - b\ (-2) + 1\ (-26) = 10$

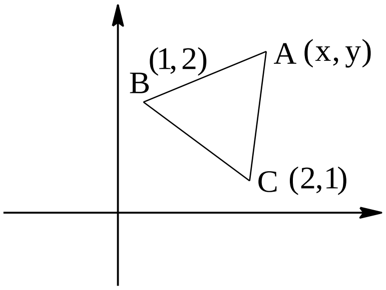
$\Rightarrow 6a + 2b = 46$ ………...(2)

solve (1) & (2)

$\Rightarrow a = 7, b = 2$

So $a + b = 9$

Q.2



$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 6$$

$$\Rightarrow \frac{1}{2}\left(x(2-1)-y(-1)+1(-3)\right)=6$$

$\Rightarrow$x + y = 15

Q.3

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ x & 2 & 1 \\ 1 & y & 1 \end{vmatrix} = \pm 6$$

$$\Rightarrow (2-y)-2(x-1)+(xy-2)=\pm 12$$

$\Rightarrow$– 2x – y + xy = 10 or – 14

Now $-2x-y+xy=10 \Rightarrow (x-1)(y-2)=12$

We can write 12 as a product of two integers in 12 ways.

Further $-2x-y+xy=-14 \Rightarrow (x-1)(y-2)=-12$

We can write -12 as a product of two integers in 12 ways.

Hence total 24 pairs of (x, y) are possible.

Q.4

By the property of parabola.

Ratio of area of triangle by three points of parabola to the area of triangle by intersection points of tangent are those point $=\frac{1}{2}$

So required answer = $4\times 2=8$

Q.5

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2a & a^3 & 1 \\ 2b & b^3 & 1 \\ 2c & c^3 & 1 \end{vmatrix}=A_1$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2(a-c) & a^3-c^3 & 0 \\ 2(b-c) & b^3-c^3 & 0 \\ 2c & c^3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\times 2\times(a-c)(b-c)\begin{vmatrix} 1 & a^2+c^2+ac & 0 \\ 1 & b^2+c^2+bc & 0 \\ c & c^3 & 1 \end{vmatrix}=A_1$$

$$\Rightarrow A_1=(a-c)(b-c)\left(b^2+c^2+bc-a^2-c^2-ac\right)$$

$$\Rightarrow A_1=(a-c)(b-c)(b-a)(b+a+c)$$

$$\Rightarrow A_1=(a-b)(b-c)(c-a)(a+b+c)$$

$$\Rightarrow A_2=\frac{1}{2}\begin{vmatrix} a & bc & 1 \\ b & ca & 1 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a-c & b(c-a) & 0 \\ b-c & a(c-b) & 0 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}(a-c)(b-c)\begin{vmatrix} 1 & -b & 0 \\ 1 & -a & 0 \\ c & ab & 1 \end{vmatrix}$$

$\Rightarrow \frac{1}{2}(a-c)(b-c)(b-a)$

$\Rightarrow A_2 = \frac{1}{2}(a-b)(b-c)(c-a)$

$\Rightarrow \therefore A_1 : A_2 = 2(a+b+c) = 8$

Q.6

$\Rightarrow 5y = x \Rightarrow \tan\alpha = \frac{1}{5}$

$\Rightarrow 5y = 5x \Rightarrow \tan\beta = \frac{5}{k}$

Given that $\beta = 2\alpha$

$\Rightarrow \tan\beta = \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$$\Rightarrow \frac{5}{k} = \frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}} = \frac{10}{24}$$

$\Rightarrow k = 12$

Q.7

$\Rightarrow 2ax - 3y - a = 0 \Rightarrow$ slope $m_1 = \frac{2a}{3}$

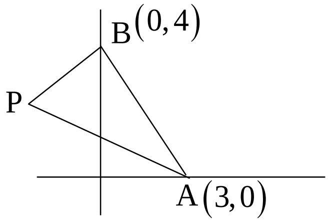
$\Rightarrow 3x + 4y + 1 = 0 \Rightarrow$ slope $m_2 = \frac{-3}{4}$

$\Rightarrow \therefore m_1 m_2 = -1$

$\Rightarrow a = 2$

Q.8

$$\Rightarrow \left.\begin{array}{ll} 3x+4y=9 & \text{........(1)} \\ 4x-3y=-12 & \text{........(2)} \end{array}\right\} \text{ Intersection point } P\left(\frac{-21}{25}, \frac{72}{25}\right)$$



$\because$ Both the lines are perpendicular to each other so they intersect at point P at 90^{o}.

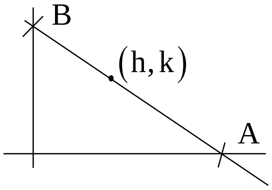
$\Rightarrow \therefore \left| AB \right| = \text{diameter circumcircle of } \Delta PAB$

$\Rightarrow \qquad = 5$

Q.9

Equation of variable straight lines is $\left(x+2y-1\right)+\lambda\left(2x-y-1\right)=0$

$\Rightarrow A\left(\frac{\lambda+1}{1+2\lambda},0\right),\ B\left(,\frac{\lambda+1}{2-\lambda}\right)$



$\Rightarrow \therefore\ 2h=\frac{\lambda+1}{1+2\lambda},\ 2k=\frac{\lambda+1}{2-\lambda}$

By elimination of λ we get

$\Rightarrow x + 3y = 10\ xy$

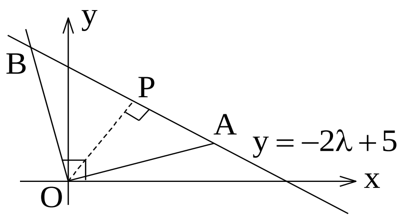
So k = 10

Q.10

$\Rightarrow \because\ \angle AOB = 90^{o}$

So OP is angle bisector of $\angle AOB$ because Δ AOB is iscoceles triangle.

$\Rightarrow \therefore\ OP = \left|\frac{5}{\sqrt{1+4}}\right| = \sqrt{5}$



$\Rightarrow \therefore$ Area of triangle $\Delta AOB = 2(\text{Area of } AOP)$

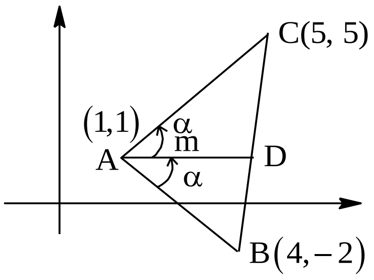
$= 2\left(\frac{1}{2} \times \sqrt{5} \times \sqrt{5}\right) = 5$

Q.11

AD is angle bisector

$\Rightarrow m_{AB} = \frac{-3}{3} = -1$

$\Rightarrow m_{AC} = 1$



So clearly m = 0

So equation of perpendicular AD form C is

$\Rightarrow y - 5 = \infty(x - 5)$

$\Rightarrow$x = 5; so k = 5

Q.12

$\Rightarrow y^2 = 4x, y = 2x + 3$

Let suppose a point P (t2, 2t) on parabola. & Image of point P about given line is Point Q.

So locus of point Q is given by

$$\left(4x+3y+6\right)^2+\ K\left(3x-4y+12\right)\ =0$$

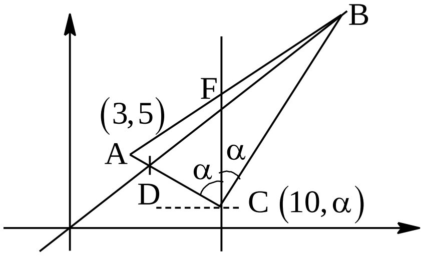
Coordinate of Point Q are given by $\frac{x-t^2}{2}=\frac{y-2t}{-1}=\frac{-(2t^2-2t+3)}{5}$

Eliminate “t” to find the locus, which is given by

$$\left(4x+3y+6\right)^2+\ 20\left(3x-4y+12\right)\ =0$$

So K=20.

Q.13



Equation of BC is 6y – x + k = 0

By figure its clear that

$\Rightarrow m_{BC}=-m_{CA}$

$\Rightarrow\left(\frac{1}{6}\right)=-\left(\frac{a-5}{10-3}\right)$

$\Rightarrow$7 = – 6a + 30

$\Rightarrow$6a = 23

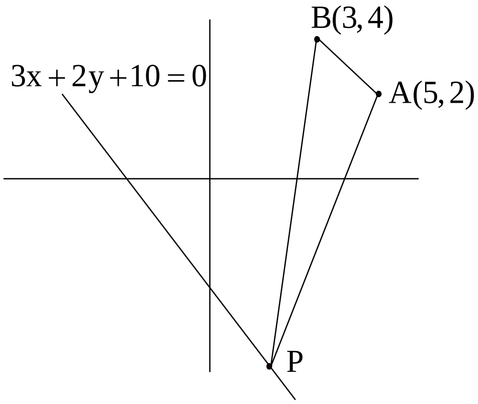
So, $C\left(10,\frac{23}{6}\right)$will satisfy equation of BC.

$\Rightarrow$6(a) – (10) + k = 0

$\Rightarrow$23 – 10 + k = 0

$\Rightarrow$k = – 13

Q.14



$\Rightarrow \left|PA - PB\right| \le AB$

$\Rightarrow \left|PA - PB\right|_{max} = AB$ and that will occur only if points P,A & B are collinear.

$\Rightarrow 3h + 2k + 10 = 0$ …….(1)

$\Rightarrow \left(\frac{k-2}{h-5}\right) = \frac{4-2}{3-5} = -1$

$\Rightarrow k - 2 = - h + 5$

$\Rightarrow (h + k) = 7$

Q.15

Line AB is $y - 1 = m(x - 1)$

$\Rightarrow A\left(1 - \frac{1}{m}, 0\right), B\left(0, 1 - m\right)$

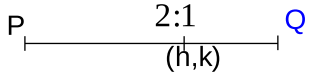
$\Rightarrow C\left(1 - \frac{1}{m}, 1 - m\right)$

Line PQ is $y - \left(1 - m\right) = m\left(x - \left(1 - \frac{1}{m}\right)\right)$

$\Rightarrow y = (1 - m) + mx - m + 1$

$\Rightarrow y = mx + 2(1 - m)$

$$\Rightarrow P\left(-2\frac{(1-m)}{m},0\right),\ Q(0,2(1-m))$$



So $\dfrac{\frac{-2(1-m)}{m}}{3}=h$ ……..(2), $\dfrac{4(1-m)}{3}=k$ …….(1)

$\Rightarrow \dfrac{-1}{2(m)}=\dfrac{h}{k}\Rightarrow m=-\dfrac{k}{2h}$ put it in (1)

$$\Rightarrow\left(1+\frac{k}{2h}\right)=\frac{3k}{4}$$

$\Rightarrow 4h + k = 3kh$

$\Rightarrow 3xy = 2\ (2x + y)$

So, $k = 2$

Q.16

$\Rightarrow -3x + 2y + 1 = 0$

$\Rightarrow 2x - 3y + 1 = 0$

$\Rightarrow a_1a_2+b_1b_2=-6-6<0$

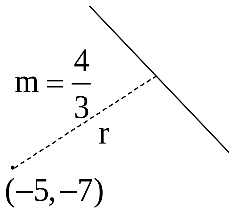
Equation of angle bisectors are

$$\Rightarrow\frac{-3x+2y+1}{\sqrt{13}}=\pm\left(\frac{2x-3y+1}{\sqrt{13}}\right)$$

Negative sign will produce obtuse angle bisector

$\Rightarrow(-3x + 2y + 1) = -(2x - 3y + 1)$

$\Rightarrow x + y - 2= 0$ …………(1)



Let's take point is at distance r

$$\Rightarrow \tan\alpha = \frac{4}{3}$$

So $\left[\left(-5 + r\cos\alpha\right), \left(-7 + r\sin\alpha\right)\right]$is on (1)

$$\Rightarrow -5 + r\left(\frac{3}{5}\right) + \left(-7 + r \times \frac{4}{5}\right) - 2 = 0$$

$$\Rightarrow r\left(\frac{7}{5}\right) = 14$$

$\Rightarrow$r = 10

Q.17

For concurrent lines

$$\Rightarrow \begin{vmatrix} 4 & -1 & 6 \\ 3 & -4 & -6 \\ 1 & 6 & k \end{vmatrix} = 0$$

$\Rightarrow$k = 20

Q.18

Point A $\equiv$ (intersection of AB & AD)

$$\equiv \left(y = 2x + 1 \ \ \& \ \ y = \frac{2x + 7}{4} \right)$$

$$\equiv \left(\frac{1}{2}, 2\right)$$

Point C $\equiv$(intersection of 3y = x + 1 & y = 4x – 7)

So equation of AC $\equiv y-1=\frac{2-1}{\frac{1}{2}-2}(x-2)$

$\Rightarrow y-1=\frac{-2}{3}(x-2)$

$\Rightarrow 2x + 3y - 5 = 0$

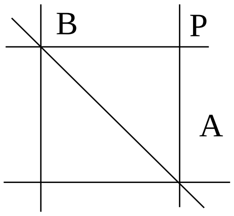
$\Rightarrow a = 2, b = 3, c = 5$

$\Rightarrow |a+b+c|=10$

Q.19

Variable line through intersection point is

$\Rightarrow (2x+3y-1)+\lambda(3x+2y-1)=0$



$\Rightarrow A\left(\frac{\lambda+1}{2+3\lambda},0\right), B\left(0,\frac{\lambda+1}{3+2\lambda}\right), P\left(\frac{\lambda+1}{2+3\lambda},\frac{\lambda+1}{3+2\lambda}\right)$

$\Rightarrow h=\frac{\lambda+1}{2+3\lambda}$ …….(1)

$\Rightarrow k=\frac{\lambda+1}{3+2\lambda}$ …….(2)

By elimination of λ by two equation we get

$\Rightarrow \frac{1}{h}+\frac{1}{k}=5$ or $\frac{1}{x}+\frac{1}{y}=5$

So k = 5

Q.20

Variable point on the Line QPR is

$\Rightarrow \left\{\left(\sqrt{2}\sec\theta+r\cos\alpha\right),\ \left(\sqrt{3}\tan\theta+r\sin\alpha\right)\right\}$ …..Slope of line QPR= $\tan\alpha=2$

$\Rightarrow \therefore$ equation of pair of straight lines is

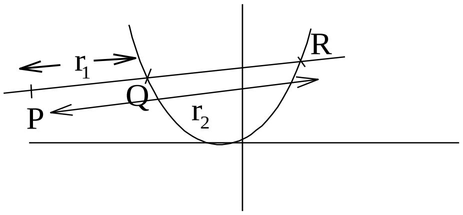
$$\Rightarrow 3x^2-2y^2=0$$

$$\Rightarrow \frac{x^2}{2}-\frac{y^2}{3}=0$$

$$\Rightarrow 3\left(2\sec^2\theta+r^2\cos^2\alpha+2\sqrt{2}\,r\sec\theta\cot\alpha\right)-2\left(3\tan^2\theta+r^2\sin^2\alpha+2\sqrt{3}r\sin\alpha\tan\theta\right)=0$$

$$\Rightarrow r^2\left(3\cos^2\alpha-2\sin^2\alpha\right)+2\sqrt{2}\left(3\sec\theta\cos\alpha-\sqrt{6}\sin\alpha\tan\theta\right)r+6\sec^2\theta-6\tan^2\theta=0\Rightarrow$$

$$\therefore r_1r_2=\frac{6\sec^2\theta-6\tan^2\theta}{3\cos^2\alpha-2\sin^2\alpha}=\frac{6\left(\sec^2\alpha\right)}{3-2\tan^2\alpha}$$



$$\Rightarrow r_1r_2=\frac{6\left(1+4\right)}{3-2\left(4\right)}=-6$$

$$\Rightarrow \left|r_1r_2\right|=6$$

Q.21

Pair of lines passing through the points in which the lines $6x^2+xy-y^2-6x+7y-12=0$ meet the coordinate axes will be given by

$6x^2+xy-y^2-6x+7y-12+\lambda xy=0$ or $6x^2+\left(1+\lambda\right)xy-y^2-6x+7y-12=0$

As its equation of a pair of lines hence

$$6\left(-1\right)\left(-12\right)+2\left(\frac{7}{2}\right)\left(-3\right)\left(\frac{1+\lambda}{2}\right)-6\left(\frac{7}{2}\right)^2-\left(-1\right)\left(-3\right)^2-\left(-12\right)\left(\frac{1+\lambda}{2}\right)^2=0$$

$$\Rightarrow 2\left(1+\lambda\right)^2-7\left(1+\lambda\right)+5=0\Rightarrow\lambda=\frac{3}{2}$$

Hence the required pair of lines is $12x^2+5xy-2y^2-12x+14y-24=0$.

Value of 'a' is 12.

Q.22

Equation of any line through (4, 2) will be y – 2 = m (x – 4) or $\frac{mx-y}{4m-2}=1$.

Homogenize $2x^2+5xy-3y^2+4x+19y-6=0$ using this with equation of above line to get the pair of lines joining P & Q to the origin as

$$2x^2+5xy-3y^2+4x\left(\frac{mx-y}{4m-2}\right)+19y\left(\frac{mx-y}{4m-2}\right)-6\left(\frac{mx-y}{4m-2}\right)^2=0$$

Now as angle POQ is right angle hence in above equation coefficient of x^2 + coefficient of y^2 = 0.

Q.23

Pair of angle bisectors of the lines $x^2-2pxy-y^2=0$ will be

$$\frac{x^2-y^2}{2}=\frac{xy}{-p} \text{ or } px^2+2xy-py^2=0$$

Which must be same as $x^2+2qxy-y^2=0$.

Comparing the two equations gives pq = 1.

Q.24

$\Rightarrow 9x^2+24xy+16y^2+45x+60y-100=0$ … (1)

Pair of straight lines (parallel) then $9x^2+24xy+16y^2=0$ has to have two factor, which are equal.

$\Rightarrow (3x+4y)^2=0$

So 2h = 24

$\Rightarrow$h = 12

So, $(3x+4y+c_1)(3x+4y+c_2)=0$ ... (2)

Equation (1) & (2) are same so by comparing

$\Rightarrow 3c_1 + 3c_2 = 45 \Rightarrow c_1 + c_2 = 15$

$\Rightarrow 4c_1 + 4c_2 = 60 \Rightarrow c_1 + c_2 = 15$

$\Rightarrow c_1 c_2 = -100$

$\Rightarrow c_1 = 20,\ c_2 = -5$

$\therefore$ two lines are 3x + 4y + 20 = 0 & 3x + 4y – 5 = 0

So distance $= \frac{|20-(-5)|}{\sqrt{3^2+4^2}} = 5$

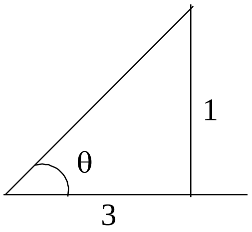
Q.25

$\Rightarrow x^2 - 3xy + ay^2 + 3x - 5y + 2 = 0$

Pair of straight line if $x^2 - 3xy + ay^2$ have two factors

So a = 2

$$\Rightarrow \tan\theta = \frac{2\sqrt{\frac{9}{4}-2}}{1+2} = \frac{1}{3}$$



So, $\sin\theta = \frac{1}{\sqrt{10}}$

$\Rightarrow \operatorname{cosec}^2\theta = 10$

Q.26

$\Rightarrow ax^3 + bx^2y + cxy^2 + dy^3 = 0$ ………..(1)

Clearly above equation represents 3 straight lines, which passes through (0, 0).

Let y = mx is straight line

$\Rightarrow \frac{y}{x} = m$ ……….(2)

by (1) & (2)

$\Rightarrow m^3d + cm^2 + bm + a = 0$ ……….(3) $\rightarrow m_1m_2m_3$

$\Rightarrow m_1m_2m_3 = \frac{-a}{d}$

But $m_1m_2 = -1$ (given)

So, $m_3 = \frac{a}{d}$ put in equation (3)

$\Rightarrow \left(\frac{a^3}{d^3}\right)d + c\left(\frac{a^2}{d^2}\right) + b\left(\frac{a}{d}\right) + a = 0$

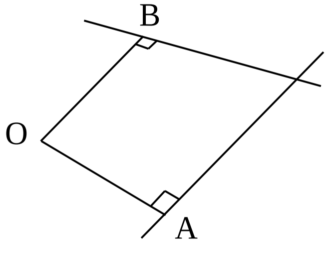
$\Rightarrow a^3 + cd^2 + bad + ad^2 = 0$

$\Rightarrow a^2 + ac + bd + d^2 = 0$ ……… $(\because a \neq 0)$

Q.27 Lets take two lines are

$\Rightarrow y = m_1x + c_1$ ……..(1)

$\Rightarrow y = m_2x + c_2$ ……..(2)



Given OA = OB

$\Rightarrow \frac{c_1}{\sqrt{1+m_1^2}} = \frac{c_2}{\sqrt{1+m_2^2}}$

$\Rightarrow c_1^2\left(1+m_2^2\right) = c_2^2\left(1+m_1^2\right)$

$$\Rightarrow (c_1 - c_2)(c_1 + c_2) = (c_2m_1 + c_1m_2)(c_2m_1 - c_1m_2) \qquad \text{……….(1)}$$

$$\Rightarrow (m_1x - y + c_1)(m_2x - y + c_2) = 0$$

$$\Rightarrow m_1m_2x^2 + y^2 - xy(m_1 + m_2) + (c_1m_2 + c_2m_1)x - (c_1 + c_2)y + c_1c_2 = 0 \qquad \text{……….(2)}$$

$$\Rightarrow ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0 \qquad \text{……….(3)}$$

equation (3) & (2) are same so by comparing ……..

$$\Rightarrow \frac{m_1m_2}{a} = \frac{1}{b} = \frac{-(m_1 + m_2)}{2h} = \frac{c_1m_2 + c_2m_1}{2g} = \frac{-(c_1 + c_2)}{2f} = \frac{c_1c_2}{c}$$

$$\Rightarrow (c_1 + c_2) = \frac{-2f}{b};\ m_1m_2 = \frac{a}{b}$$

$$\Rightarrow (c_1c_2) = \frac{c}{b} \qquad \text{………(4)}$$

$$\Rightarrow (c_1 - c_2) = \sqrt{\frac{4f^2}{b^2} - \frac{4c}{b}} = \frac{2}{b}\sqrt{f^2 - cb} \qquad \text{……….(5)}$$

$$\Rightarrow c_1m_2 + c_2m_1 = \frac{2g}{b} \qquad \text{……….(6)}$$

$$\Rightarrow (c_2m_1 - c_1m_2) = \sqrt{\frac{4g^2}{b^2} - 4(c_1m_2)(c_2m_1)} = \sqrt{\frac{4g^2}{b^2} - 4\left(\frac{c}{b}\right)\left(\frac{a}{b}\right)}$$

$$= \frac{2}{b}\sqrt{g^2 - ac} \qquad \text{………(7)}$$

put values form (4), (5), (6), (7) into (1)

$$\Rightarrow \left(\frac{-2f}{b}\right)\left(\frac{2}{b}\right)\sqrt{f^2 - cb} = \left(\frac{2g}{b}\right)\left(\frac{2}{b}\sqrt{g^2 - ac}\right)$$

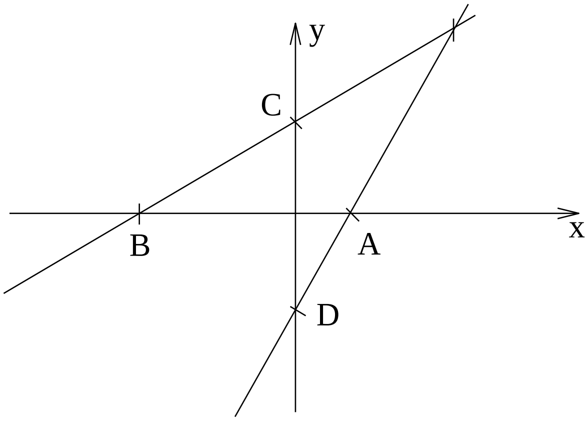
$$\Rightarrow f^2(f^2 - cb) = g^2(g^2 - ac)$$

$$\Rightarrow f^4 - g^4 = c(bf^2 - ag^2)$$

$$\Rightarrow f^{2k} - g^{2k} = c(bf^k - ag^k)$$

By comparing k = 2

Q.28



Required equation of pair of straight line in General form is

$\Rightarrow U+\lambda\left(xy\right)=0$ ………..(1)

$\Rightarrow U\equiv\left(ax^2+by^2+2hxy+2gx+2fy+c\right)=0$ ………..(3)

$\Rightarrow \because U$ is pair of straight line so

$\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$ ………..(2)

By equation (1)

$\Rightarrow \left(ax^2+by^2+\left(2h+\lambda\right)xy+2gx+2fy+c\right)=0$ will be pair of straight line if $\Delta=0$

$$\Rightarrow \Delta=abc+2fg\left(\frac{2h+\lambda}{2}\right)-af^2-bg^2-c\left(\frac{2h+\lambda}{2}\right)^2=0$$

$$\Rightarrow \Delta=\left(abc+2fgh-af^2-bg^2-ch^2\right)+fg\lambda-h\lambda c-\frac{c\lambda^2}{4}=0$$

$$\Rightarrow \Delta=0+\left(fg-hc-\frac{c\lambda}{4}\right)\lambda=0$$

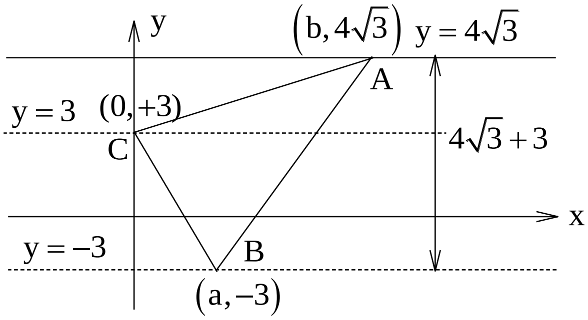
$$\Rightarrow \lambda=\frac{4\left(fg-ch\right)}{c}$$

So equation (1) will become

$\Rightarrow$cU + 4 (fg – ch) xy = 0

$\Rightarrow \therefore k = 4$

Q.29



In triangle ABC

$$\Rightarrow \frac{(b-0)+\left[4\sqrt{3}-3\right]i}{(a-0)+\left[-3-(3)\right]i} = e^{i\frac{\pi}{3}}$$

$$\Rightarrow b+\left(4\sqrt{3}-3\right)i = (a-6i)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$$

$$\Rightarrow b+\left(4\sqrt{3}-3\right)i = \left(\frac{a}{2}+3\sqrt{3}\right)+i\left(-3+\frac{a\sqrt{3}}{2}\right)$$

$$\Rightarrow \therefore 4\sqrt{3}-3 = -3+\frac{a\sqrt{3}}{2}$$

$\Rightarrow$a = 8

So, $CB = \sqrt{a^2+6^2} = \sqrt{8^2+6^2} = 10$

Q.30

$$\Rightarrow \frac{x}{a}+\frac{y}{b} = 1$$

$\Rightarrow$bx + ay = ab ……….(1)

$\Rightarrow$ax + by = 1 ……….(2)

Intersection point of (1) & (2) is (h, k)

So $h=\frac{ab^2-a}{b^2-a^2}$, $k=\frac{a^2b-b}{a^2-b^2}$

$$\Rightarrow k=\frac{b-a^2b}{b^2-a^2}$$

$$\Rightarrow \therefore h^2+k^2+hk=\frac{\left(ab^2-a\right)^2+\left(b-a^2b\right)^2+\left(ab^2-a\right)\left(b-a^2b\right)}{\left(b^2-a^2\right)^2}$$

$$=\frac{a^2\left(b^4+1-2b^2\right)+b^2\left(1+a^4-2a^2\right)+ab\left(1^2-6^2a^2-1+a^2\right)}{\left(b^2-a^2\right)^2}$$

$$=\frac{a^2b^2\left(a^2+b^2\right)+\left(a^2+b^2\right)-4a^2b^2+ab\left(ab-b^2a^2-1\right)}{\left(b^2-a^2\right)^2}$$

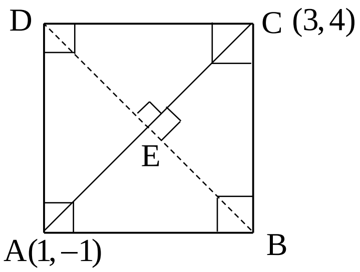
$$=\frac{-3a^2b^2}{\left(b^2-a^2\right)^2}=\frac{-3a^2b^2}{\left(b^2+a^2\right)^2-4a^2b^2}=\frac{-3a^2b^2}{a^2b^2-4a^2b^2}$$

= 1

STRAIGHT LINES

EXERCISE - 3

Q.1



Mid point of A, C is $E\left(2,\frac{3}{2}\right)$

Point $D \equiv$ rotation of point C(3, 4) by angle 90° about E.

$$\equiv\left\{(3-2)+i\left(4-\frac{3}{2}\right)\right\}\operatorname{cis}\left(90^{o}\right)+\left(2+\frac{3}{2}i\right)$$

$$\equiv\left(1+\frac{5i}{2}\right)(i)+\left(2+\frac{3}{2}i\right)$$

$$\equiv\frac{5}{2}i-\frac{1}{2}$$

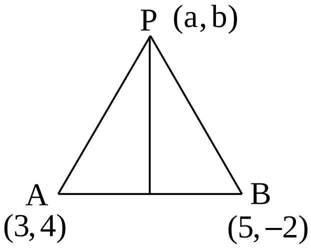
$$\Rightarrow D\left(-\frac{1}{2},\frac{5}{2}\right)$$

point $E \equiv$ mid point of B & D.

So calculate B

Q.2

A (3, 4) B (5, – 2)



$\Rightarrow PA = PB = \sqrt{(a-3)^2+(b-4)^2} = \sqrt{(a-5)^2+(b+2)^2}$

$\Rightarrow$1 – 6a + 9 – 8b + 16 = – 10a + 25 + 4b + 4

$\Rightarrow$4a – 12b = 4

$\Rightarrow$a – 3b = 1 …………(1)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} a & b & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = 10$$

$\Rightarrow$a (6) – b (– 2) + 1 (– 26) = 10

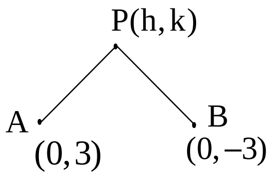
$\Rightarrow$6a + 2b = 46 ………...(2)

Solve (1) & (2)

$\Rightarrow$a = 7, b = 2

So a + b = 9

Q.3



$\Rightarrow$PA + PB = 8

$\Rightarrow$PA = 8 – PB

$\Rightarrow PA^2 = 64 + PB^2 - 16PB$

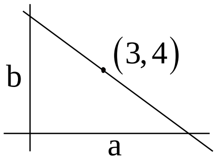
$$\Rightarrow \left(h^2+(k-3)^2\right) = 64 + \left(h^2+(k+3)^2\right) - 16\sqrt{h^2+(k+3)^2}$$

$$\Rightarrow 6k + 6k + 64 = 16\sqrt{h^2+(k+3)^2}$$

Again take square & simplicity for (h, k)

So locus of (h, k) is $\frac{x^2}{16} + \frac{y^2}{7} = 1$

Q.4



$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$ given a + b = 14 ……..(1)

$\Rightarrow \frac{3}{a}+\frac{4}{b}=1$ …….(2)

After solving (1) & (2) we get a = b = 7 or a = 6, b = 8

So equation of line is $\frac{x}{7}+\frac{y}{7}=1$ or $\frac{x}{6}+\frac{y}{8}=1$

Q.5

$\Rightarrow$Q point $\equiv\left(3+r\cos 45^{o}-5+r\sin 45^{o}\right)$

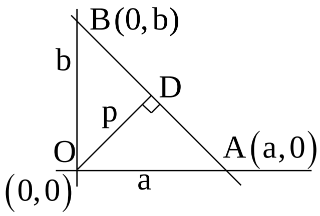
Q point lies on x + y = 6

So, $\left(3+r\cos 45^{o}\right)+\left(-5+r\sin 45^{o}\right)=6$

$\Rightarrow r=4\sqrt{2}$

So length PQ = r = $4\sqrt{2}$

Q.6

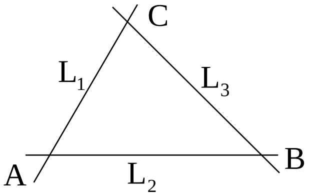


Equation of line AB is $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow OD = p = \frac{|0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \text{ Proved.}$$

Q.7



$$\Rightarrow L_1 \equiv x + y - 6 = 0$$

$$\Rightarrow L_2 \equiv 2x + y - 4 = 0$$

$$\Rightarrow L_3 \equiv x + 2y - 5 = 0$$

$$\Rightarrow A(-2,8),\ B(1,2),\ C(7,-1)$$

So area of triangle $= \frac{1}{2}\begin{vmatrix} -2 & 8 & 1 \\ 1 & 2 & 1 \\ 7 & -1 & 1 \end{vmatrix} = \frac{27}{2}$

Q.8

$$\Rightarrow 3x - 4y + 2 = 0 \Rightarrow m_1 = \frac{3}{4}$$

Let slope of line is m, which is making $\alpha = 45^o$ angle with given line.

So, $m = \frac{m_1 + \tan\alpha}{1 - m_1\tan\alpha}$ or $m = \frac{m_1 - \tan\alpha}{1 + m_1\tan\alpha}$

$$\Rightarrow m = \frac{\frac{3}{4}+1}{1-\frac{3}{4}} = 7 \text{ or } m = \frac{\frac{3}{4}-1}{1+\frac{3}{4}} = \frac{-1}{7}$$

So equation of line is (y – 2) = 7 (x + 3) or y – 2 = $\frac{-1}{7}\left(x+3\right)$

$\Rightarrow$ 7x – y + 23 = 0 ……(1) or 7y + x – 11 = 0 ………(2)

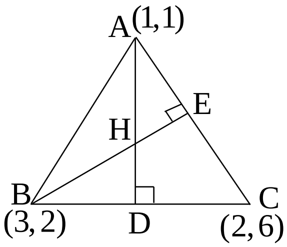
Q.9

Equation of line perpendicular to $\frac{x}{a}+\frac{y}{b}=1$ is $\frac{x}{a}+\frac{y}{b}=k$ ………(1)

$\Rightarrow$ k = 2

So line is $\frac{x}{a}+\frac{y}{b}=2$

Q.10



$\Rightarrow m_{AC}=\frac{6-1}{2-1}=5$

$\Rightarrow m_{BC}=-4$

$\Rightarrow \therefore$ Equation of BE is

$\Rightarrow \left(y-2\right)=\frac{-1}{5}\left(x-3\right)$

$\Rightarrow$ 5y + x = 13 ………..(1)

Equation of AD is

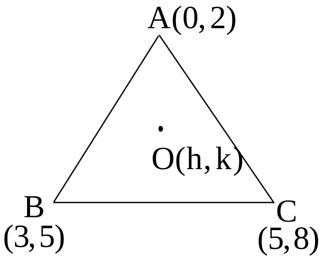
$\Rightarrow \left(y-1\right)=\frac{1}{4}\left(x-1\right) \Rightarrow$ x – 4y = – 3 ………(2)

On solving (1) & (2)

$\Rightarrow x=\frac{37}{9},\ y=\frac{16}{9}$

So orthocenter $H\left(\frac{37}{9},\frac{16}{9}\right)$

Q.11



For Circumcentre.

$\Rightarrow$OA = OB

$$\Rightarrow \sqrt{h^2+(k-2)^2}=\sqrt{(h-3)^2+(k-5)^2}$$

$\Rightarrow$– 4 k + 4 = – 6h + 9 + 25 – 10k

$\Rightarrow$6h + 6k = 30

$\Rightarrow$h + k = 5 …………..(1)

$\Rightarrow$OA = OC

$$\Rightarrow \sqrt{h^2+(k-2)^2}=\sqrt{(h-5)^2+(k-8)^2}$$

$\Rightarrow$– 4k + 4 = 1 – 10h + 25 + 64 – 16k

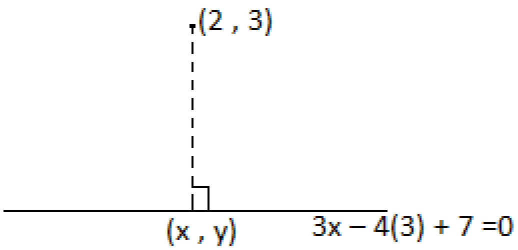
$\Rightarrow$10 h + 12 k = 85

By solving (1) & (2)

$$\Rightarrow h=-\frac{25}{2},k=\frac{35}{2}$$

So, $O\left(\frac{-25}{2},\frac{35}{2}\right)$

Q.12



$$\frac{x-2}{3} = \frac{y-3}{-4} = \frac{-(3(2)-4(3)+7)}{3^2+4^2}$$

$$= -\left(\frac{1}{25}\right)$$

$$x = 2 - \frac{3}{25} \qquad y = 3 + \frac{4}{25}$$

$$x = \frac{47}{25} \qquad y = \frac{79}{25}$$

$$(x\ ,\ y) \equiv \left(\frac{47}{25}, \frac{79}{25}\right)$$

Q.13

$$2x^2 - xy - 3y^2 = 0 \qquad a = 2\ ,\ b = -3,\ h = \frac{-1}{2}$$

$$\Rightarrow 3y^2 + xy - 2x^2 = 0$$

$$\Rightarrow 3\left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) - 2 = 0$$

$$\Rightarrow \frac{y}{x} = \frac{-1 \pm \sqrt{1+24}}{6} = \frac{-1 \pm 5}{6}$$

$$= -1, \frac{2}{3}$$

$\boxed{y = -x \text{ or } y = \frac{2}{3}x}$ are the lines

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \left|\frac{2\sqrt{\frac{1}{4}+6}}{2-3}\right|$$

$$=\left|\frac{5}{-1}\right|$$

$$\boxed{\tan\theta=5 \text{ acute angle between lines}}$$

Q.14

Equation of bisectors of

3x + 4y – 1 = 0 and 5x – 12y + 6 = 0

$$\frac{3x + 4y - 1}{\sqrt{3^2 + 4^2}} = \pm\left(\frac{5x - 12y + 6}{\sqrt{5^2 + 12^2}}\right)$$

$$\therefore (3x + 4y - 1)(13) \pm 5\ (5x - 12y + 6) = 0$$

$$(39x + 52y - 13) \pm (25x - 60y + 30) = 0$$

So <u>case I</u> :

$$(39x + 52y - 13) \pm (25x - 60y + 30) = 0$$

<u>Positive sign</u>

64x – 8y + 17 = 0 ………….(A)

<u>Case II</u> :

<u>Negative sign</u>

14x + 122y – 43 = 0 …………..(B)

Lines are

$$\boxed{64x - 8y + 17 = 0\ \ and\ \ 14x + 112y - 43 = 0}$$

Q.15

$$6x^2 + 11xy - 10y^2 + x + 31y - 15 = 0$$

$a = 6$, $h = \frac{11}{2}$, $b = -10$, $g = \frac{1}{2}$, $f = \frac{31}{2}$, $c = -15$

$$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$$

$$= \left| \frac{2\sqrt{\frac{121}{4} + 60}}{6 - 10} \right|$$

$= \left| \frac{19}{-4} \right|$ Acute angle between lines is $\tan^{-1}\left(\frac{19}{4}\right)$

Point of intersection is given by the formula.

$$\left(\frac{bg - fh}{h^2 - ab} , \frac{af - gh}{h^2 - ab} \right)$$

$$= \left(\frac{(-10)\left(\frac{1}{2}\right) - \left(\frac{31}{2}\right)\left(\frac{11}{2}\right)}{\frac{121}{4} + 60} , \frac{6\left(\frac{31}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{11}{2}\right)}{\frac{121}{4} + 60} \right)$$

$$= \left(\frac{-20 - 341}{361} , \frac{12 \times 31 - 11}{361} \right)$$

$$= (-1 , 1)$$

To find the lines

$$6x^2 + 11xy - 10y^2 + x + 31y - 15 = 0$$

$$6x^2 + x(11y + 1) - 10y^2 + 31y - 15 = 0$$

$$x = \frac{-(11y + 1) \pm \sqrt{(11y + 1)^2 - 4(6)(-10y^2 + 31y - 15)}}{12}$$

$$x = \frac{-(11y + 1) \pm \sqrt{(121y^2 + 1 + 22y + 240y^2 - 24 \times 31y + 360)}}{12}$$

$$x = \frac{-(11y+1) \pm \sqrt{361y^2 - 722y + 361}}{12}$$

$$x = \frac{-(11y+1) \pm 19(y-1)}{12}$$

$$x = \frac{-(11y+1) - 19(y-1)}{12} \quad or \quad \frac{-(11y+1) + 19(y-1)}{12}$$

$$x = \frac{-30y+18}{12} \quad or \quad \frac{8y-20}{12}$$

$$x = \frac{-5y+3}{2} \quad or \quad x = \frac{2y-5}{3}$$

$$\boxed{2x+5y=3 \quad or \quad 3x=2y-5}$$

Q.16

Meeting of

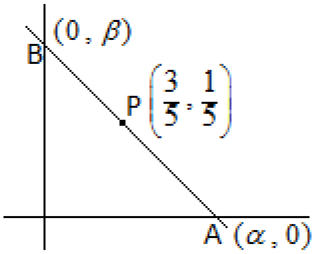
x + 2y – 1 = 0 and

2x – y – 1 = 0 is

$$\frac{x}{\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}}$$

$$\frac{x}{-3} = \frac{y}{-1} = \frac{1}{-5}$$

$$x = \frac{3}{5}, y = \frac{1}{5}$$



Equation of line with intercepts A & B is

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

Mid – point of AB is (h , k)

$$(h , k) \equiv \left(\frac{0+\alpha}{2}, \frac{0+\beta}{2}\right) = \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

So $\alpha = 2h$ and $\beta = 2k$

$\left(\frac{3}{5}, \frac{1}{5}\right)$ lies on $\frac{x}{\alpha} + \frac{y}{\beta} = 1$

So $\frac{x}{5\alpha} + \frac{y}{5\beta} = 1$

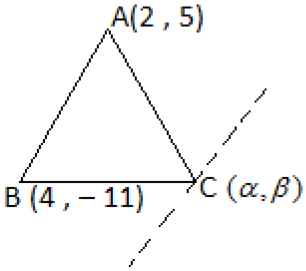
$\alpha = 2h$ And $\beta = 2k$ $\therefore$

$$\frac{3}{10h} + \frac{1}{10k} = 1$$

$\therefore$ Locus is

$$\frac{3}{10x} + \frac{1}{10y} = 1 \qquad \text{i.e.} \boxed{\frac{3}{x} + \frac{1}{y} = 10}$$

Q.17



C lies on

9x + 7y + 4 = 0

$\therefore 9\alpha + 7\beta + 4 = 0$

Centroid is (h , k)

$$(h , k) \equiv \left(\frac{2+4+\alpha}{3}, \frac{5-11+\beta}{3}\right)$$

$$\equiv \left(\frac{6+\alpha}{3} , \frac{\beta - 6}{3} \right)$$

$6 + \alpha = 3h \qquad \frac{\beta - 6}{3} = k$

$\alpha = 3h - 6 \qquad \beta = 3k + 6$

$\therefore 9\alpha + 7\beta + 4 = 0$

$9(3h - 6) + 7(3k + 6) + 4 = 0$

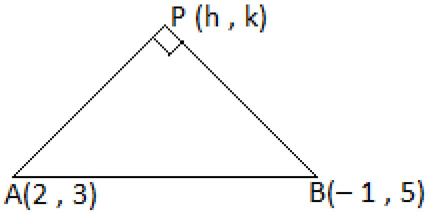
$27h - 54 + 21k + 42 + 4 = 0$

$27h + 21k - 8 = 0$

So locus of centroid is

$\boxed{27x + 21y - 8 = 0}$

Q.18



Slope PA × Slope PB = – 1

$\left(\frac{k-3}{h-2} \right) \left(\frac{k-5}{h+1} \right) = -1 \qquad h \neq 2, -1$

(k – 3) (k – 5) + (h – 2) (h + 1) = 0

$\therefore$ locus of P is

(x – 2) (x + 1) + (y – 3) (y – 5) = 0

$x^2 - x - 2 + y^2 - 8y + 15 = 0$

$\boxed{x^2 + y^2 - x - 8y + 13 = 0} \qquad x \neq 2 , -1$

Q.19

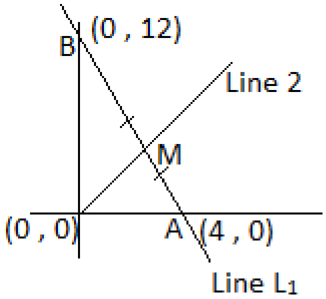
3x + y = 12

$$\frac{x}{4}+\frac{y}{12}=1$$

M is the mid – point of AB so

$$M \equiv \left(\frac{4+0}{2},\frac{12+0}{2}\right)$$

M = (2, 6)



So Equation of line 2 is

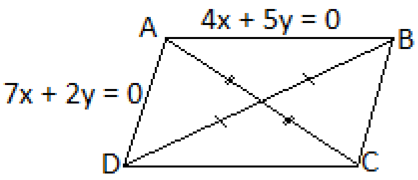
y = 3x slope $= 3\ (m_2)$

slope of $L_1 = -3$

$$\text{So } \tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{3-(-3)}{1+3(-3)}\right| = \left|\frac{6}{-8}\right|$$

$$\tan\theta = \frac{3}{4} \quad \boxed{\theta = \tan^{-1}\frac{3}{4}}$$

Q.20



$A \equiv (0 , 0)$

Equation of BD is 11x + 7y = 9

D is intersection of	B is intersection of
7x + 2y = 0 and	4x + 5y = 10 and

11x + 7y = 9

$$\frac{x}{\begin{vmatrix} 0 & 2 \\ 9 & 7 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 7 & 0 \\ 11 & 9 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 7 & 2 \\ 11 & 7 \end{vmatrix}}$$

$$\frac{x}{-18} = \frac{y}{63} = \frac{1}{27}$$

$$x = \frac{-2}{3} \ , \ y = \frac{7}{3}$$

$$D \equiv \left(\frac{-2}{3} \ , \ \frac{7}{3} \right)$$

11x + 7y = 9

$$\frac{x}{\begin{vmatrix} 0 & 5 \\ 9 & 7 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 4 & 0 \\ 11 & 9 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 4 & 5 \\ 11 & 7 \end{vmatrix}}$$

$$\frac{x}{(-\ 45)} = \frac{y}{36} = \frac{1}{-27}$$

$$x = \frac{5}{3} \ , \ y = \frac{-4}{3}$$

$$B \equiv \left(\frac{5}{3} \ , \frac{-4}{3} \right)$$

Mid – point of BD is $\left(\dfrac{\frac{5}{3} - \frac{2}{3}}{2} \ , \dfrac{\frac{7}{3} - \frac{4}{3}}{2} \right)$

$$P \equiv \left(\frac{1}{2} \ , \frac{1}{2} \right)$$

Equation of OP and OC are the same

Equation of OP is y = x

Q.21

L$_1$: 2x + 3y = 6

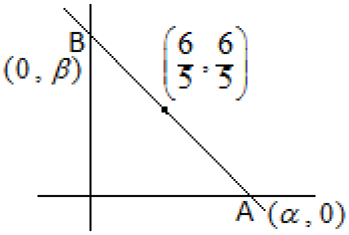
L$_2$: 3x + 2y = 6

Intersection is

$$\frac{x}{\begin{vmatrix} 6 & 3 \\ 6 & 2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 6 \\ 3 & 6 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}}$$

$$\frac{x}{-6} = \frac{y}{-6} = \frac{1}{-5}$$

$$\therefore x = y = \frac{6}{5}$$



Equation of AB is

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

AB passes through $\left(\frac{6}{5}, \frac{6}{5}\right)$. So it satisfies

$$\frac{6}{5\alpha} + \frac{6}{5\beta} = 1 \qquad \ldots\ldots\ldots\ldots(A)$$

Now mid – point of AB is (h , k)

So

$$(h, k) \equiv \left(\frac{\alpha + 0}{2}, \frac{0 + \beta}{2}\right) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$$

$$\therefore h = \frac{\alpha}{2} \qquad k = \frac{\beta}{2}$$

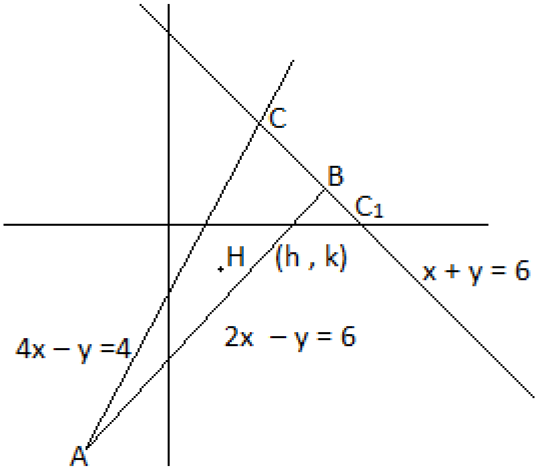
$\alpha = 2h$ $\qquad$ $\beta = 2k$ Putting in (A) we get

$$\frac{6}{10h} + \frac{6}{10k} = 1$$

$\therefore$ Locus is

$$\frac{3}{5x} + \frac{3}{5y} = 1 \qquad \therefore \boxed{3(x + y) = 5xy}$$

Q.22



A $\equiv$ intersection of 4x – y = 4 and 2x – y = 6

$$\frac{x}{\begin{vmatrix} 4 & -1 \\ 6 & -1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 4 & 4 \\ 2 & 6 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 4 & -1 \\ 2 & -1 \end{vmatrix}}$$

So $\frac{x}{2} = \frac{y}{16} = \frac{1}{-2}$ $\quad \therefore x = -1 , y = -8$

A $\equiv$ (– 1 , –8)

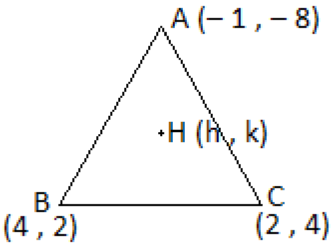
B $\equiv$ intersection of 2x – y = 6 & x +y = 6

is x = 4 & y = 2

B $\equiv$ (4, 2)

C $\equiv$ intersection of 4x – y = 4 & x + y = 6

is x = 2 and y = 4



AH perpendicular to BC

So

$$\left(\frac{k+8}{h+1}\right)\left(\frac{4-2}{2-4}\right)=-1$$

$\left(\frac{k+8}{h+1}\right)=1$ k + 8 = h + 1 ……………(i)

BH perpendicular to AC

So

$$\left(\frac{k-2}{h-4}\right)\left(\frac{4+8}{2+1}\right)=-1$$

$$\left(\frac{k-2}{h-4}\right)\times 4=-1$$

4k – 8 = 4 – h ……………(ii)

k + 8 = h + 1

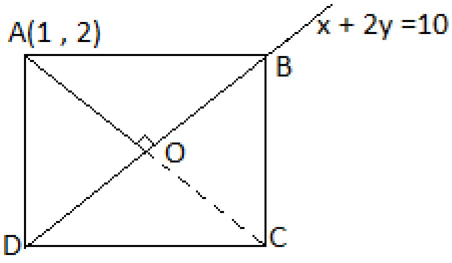
$\therefore$ 5k = 5 and k = 1

h = k + 7

h = 8

Orthocenter is $\boxed{H\,(8\,,1)}$

Q.23



AC perpendicular to BD

Slope of BD = $=-\frac{1}{2}$

So slope of AC = 2

i)Equation of AC is

(y – 2) = 2 (x – 1)

y – 2 = 2x – 2

y = 2x

ii) Perpendicular distance of A to BD is

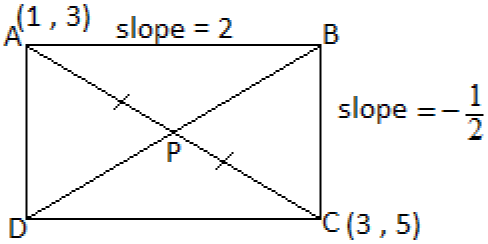
$$\left|\frac{1+2(2)-10}{\sqrt{1^2+2^2}}\right| = \left|\frac{-5}{\sqrt{5}}\right| = \sqrt{5}\text{ Units.}$$

In the figure AO' $=\frac{1}{\sqrt{2}}$(side) ($\frac{1}{2}$ the diagonal)

So (side) $=\sqrt{10}$ units.

$\therefore$ $\boxed{\text{Area } = 10 \text{ sq.units}}$

Q.24



Equation of AB is : y – 3 = 2 (x – 1)

y – 2x = – 1 …………..(i)

Equation of BC is : y – 5 $= -\frac{1}{2}$ (x – 3)

2y + x = 13 …………….(ii)

$$\text{Intersection B} \equiv \frac{y}{\begin{vmatrix} 1 & -2 \\ 13 & 1 \end{vmatrix}} = \frac{x}{\begin{vmatrix} 1 & 1 \\ 2 & 13 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}}$$

$$= \frac{y}{27} = \frac{x}{11} = \frac{1}{5}$$

$x = \frac{11}{5}$, $y = \frac{27}{5}$

$$\text{P is mid – point of AC} \equiv \left(\frac{1+3}{2}, \frac{3+5}{2}\right)$$

$$\equiv (2, 4)$$

Equation of BD = Equation of BP

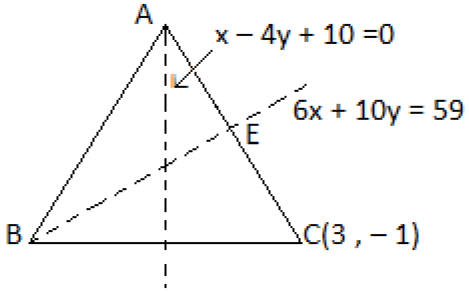
$$(y-4) = \left(\frac{4-\frac{27}{5}}{2-\frac{11}{5}}\right)(x-2)$$

$$(y-4) = \left(\frac{-7}{-1}\right)(x-2)$$

$$(y-4) = 7x-14$$

$$\boxed{7x-y-10=0}$$

Q.25



If A (x_1, y_1)

Then $x_1 - 4y_1 + 10 = 0$ ………..(i) E is the mid – point of AC

$$E \equiv \left(\frac{3+x_1}{2}, \frac{-1+y_1}{2}\right)$$

E satisfies 6x + 10y = 59

$$\therefore 6\left(\frac{3+x_1}{2}\right) + 10\left(\frac{-1+y_1}{2}\right) = 59$$

$3(3 + x_1) + 5(-1 + y_1) = 59$

$3x_1 + 5y_1 = 55$ …………..(ii)

$x_1 - 4y_1 = -10$ ……………(i)

Solving we get

$$\frac{x_1}{\begin{vmatrix} 55 & 5 \\ -10 & -4 \end{vmatrix}} = \frac{y_1}{\begin{vmatrix} 3 & 55 \\ 1 & -10 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & 5 \\ 1 & -4 \end{vmatrix}}$$

$$\frac{x}{-170} = \frac{y}{-85} = \frac{1}{-17}$$

x = 10 , y = 5

A (10, 5)

Image of (13 , – 1) in angle bisector x – 4y + 10 = 0 lies on AB

So

$C^1(\alpha , \beta)$

$$\frac{\alpha - 3}{1} = \frac{\beta + 1}{-4} = \frac{-2(3+4+10)}{1^2 + 4^2} = -2$$

$\alpha = 3 - 2$ $\beta = 8 - 1$

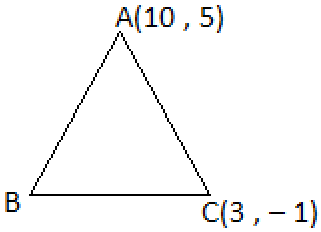
$\alpha = 1,$ $\beta = 7$

$C^1(1 , 7)$ $\therefore$ Equation of AB = Equation AC’

$$(y-7) = \left(\frac{7-5}{1-10}\right)(x-1)$$

$$y - 7 = \frac{2}{-9}(x-1)$$

$$2x - 2 + 9y - 63 = 0$$



$$\boxed{AB: 2x + 9y = 65}$$

Equation of AC :

$$y+1=\left(\frac{5+1}{10-3}\right)(x-3)$$

$$7y+7=6x-18$$

$$\boxed{AC: 6x-7y=25}$$

Intersection of 2x + 9y = 65 and 6x + 10y = 59 is B

$$B \equiv \frac{x}{\begin{vmatrix} 65 & 9 \\ 59 & 10 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 65 \\ 6 & 59 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 9 \\ 6 & 10 \end{vmatrix}}$$

$$\equiv \frac{x}{(650-531)} = \frac{y}{(118-390)} = \frac{1}{-34}$$

$$\equiv \frac{x}{119} = \frac{y}{-272} = \frac{1}{-34}$$

$$x=\frac{-7}{2}\ ,\ y=8$$

$$B \equiv \left(\frac{-7}{2}\ , 8\right) \qquad C \equiv (3, -1)$$

Equation of BC is

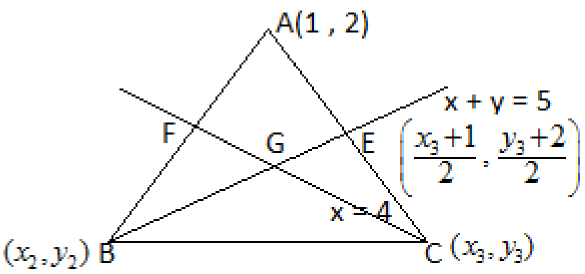
$$(y+1) = \frac{-1-8}{\left(3+\frac{7}{2}\right)}(x-3)$$

$$(y+1) = \frac{-9}{\frac{13}{2}}(x-3)$$

$$(y+1) = \frac{-18}{13}(x-3)$$

13y + 18x = 41

Q.26



Centroid is G is given by intersection of x + y = 5 & x = 4

G ≡ x = 4 and y = 5 – x

y = 1

G ≡ (4 , 1)

$$E \equiv \left(\frac{x_3 + 1}{2} , \frac{y_3 + 2}{2} \right)$$

E lies on x + y = 5. So,

$$\frac{x_3 + 1}{2} + \frac{y_3 + 2}{2} = 5$$

$\therefore x_3 + y_3 = 7$ ………(i)

B lies on x + y = 5. So,

$x_2 + y_2 = 5$ ……….(ii)

$$G \equiv \left(\frac{1 + x_2 + \ x_3 +}{3} , \frac{2 + y_2 \ + y_3}{3} \right) \equiv (4 , 1)$$ ………..(iii)

C lies on x = 4 $\therefore x_3 = 4$

$\therefore x_3 = 4$ Then $y_3 = 7 - x_3$ from (ii)

$y_3 = 3$

C : (4 , 4) ………….(iv)

From (iii)

$x_2 + x_3 = 12 - 1$, $y_2 + y_3 = 3 - 2$

$x_2 = 7$ and $y_2 = -2$

B (7 , – 2).Equation of BC is

B (7 , – 2)   C(4 , 3)

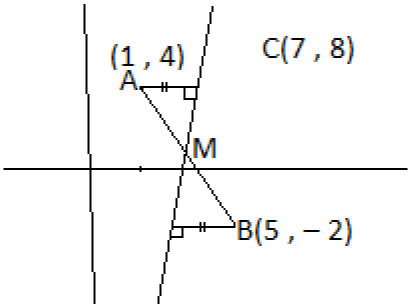
Equation of BC is

$$(y-3)=\left(\frac{3+2}{4-7}\right)(x-4)$$

$$(y-3)(-3) = 5(x-4)$$

$$\boxed{5x+3y=29}$$

Q.27



Perpendicular of AB to line = perpendicular distance of B to line

Hence by symmetry line must pass through mid – point of AB

i.e. $\left(\frac{1+5}{2}, \frac{4-2}{2}\right)$

M $\equiv$ (3 , 1)

Line must be perpendicular to CM so that it is furthest.

Slope of CM is

$$\frac{8-1}{7-3} = \frac{7}{4}$$

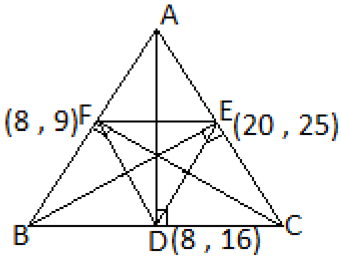
$\therefore$ Equation of line is $(y-1)-(x-3)\times\frac{-4}{7}$

$(7y - 7) = - 4\ (x - 3)$

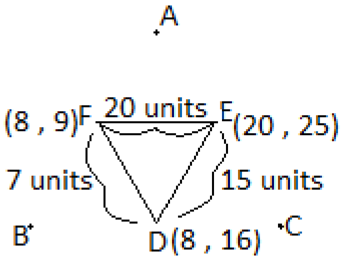
$7y - 7 = - 4x + 12$

$\boxed{7y+4x=19}$

Q.28



A is the ex-centre of Δ DFE



$$A \equiv \left(\frac{-20(8)\ +\ 20(7)+8(15)}{-20+15+7}\ ,\ \frac{-20(16)\ +\ 25(7)+9(15)}{-20+15+7} \right)$$

$$A \equiv \left(\frac{100}{2}\ ,\ \frac{-10}{2} \right) \equiv (50\ ,\ -5)$$

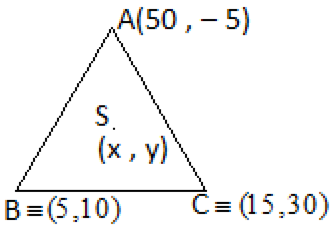
B is also ex-center of $\Delta\,DEF$

$$B \equiv \left(\frac{-\ 140+160+120}{35-7} , \frac{-175+320+135}{35-7} \right)$$

$B \equiv (5,10)$

$$C \equiv \left(\frac{-120+140\ +\ 160}{12}\ ,\ \frac{-135+175+\ 320}{12} \right)$$

$C \equiv \left(\frac{150}{12}, \frac{360}{12}\right) \equiv (15,30)$



Circumcenter be S (x , y)

$(x-5)^2+(4-10)^2 = (x-15)^2+(y-30)^2 = (x-50)^2(y+5)^2$

$\therefore (2x-20)(10)+(2y-40)(20)=0$

x – 10 + 2y – 40 = 0

x + 2y = 50 ……….(i)

and (2x – 65)(35) + (2y – 25) (– 35) = 0

2x – 65 – 2y + 25 = 0

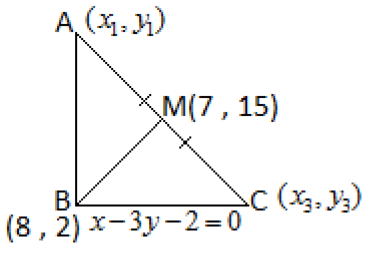
2x – 2y = 40

x – y = 20 …………(ii)

∴ y = 10 & x = 30

S ≡(30 , 10) co – ordinates of circumcenter.

Q.29



slope of BC $=\frac{1}{3}$ so slope of AB is – 3

Equation of AB is y – 2 = – 3 (x – 8)

y + 3x = 26 : AC

$y_1 + 3x_1 = 26$ (equation of AC) …..(i)

$x_3 - 3y_3 - 2 = 0$ (equation of BC) …..(ii)

$\frac{x_1 + x_3}{2} = 7$, $\frac{y_1 + y_3}{2} = 15$ …………(iii) (mid – point)

$\therefore x_1 + x_3 = 14$ & $y_3 + y_3 = 30$

$x_3 = 14 - x_1$ & $y_3 = 30 - y_1$ putting in (ii) we get

$(14 - x_1) - 3(30 - y_1) - 2 = 0$

$14 - x_1 - 90 + 3y_1 - 2 = 0$

$14 - x_1 + 3y_1 - 42 = 0$

$3y_1 - x_1 = 78$ ………..(A)

$y_1 + 3x_1 = 26$ from(i)

$$\therefore \frac{y_1}{\begin{vmatrix} 78 & -1 \\ 26 & 3 \end{vmatrix}} = \frac{x_1}{\begin{vmatrix} 3 & 78 \\ 1 & 26 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix}}$$

$y_1 = 26$, $x_1 = 0$

A $\equiv$ (0 , 26)

C $\equiv$ (14 , 4)

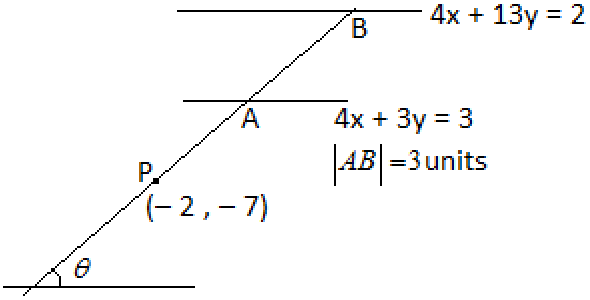
Equation of AC is $y - 26 = \left(\frac{26-4}{0-14}\right)(x-0)$

$$= \frac{-11}{7}(x)$$

$$7y + 11x = 26 \times 7$$

$\boxed{7y + 11x = 182 : \text{equation of AC}}$

Q.30



Using parametric locus equation of line is

$x = -2 + r\cos\theta$

$y = -7 + r\sin\theta$

For point A : Let $r = r_1$ and

$\therefore$ A $(-2 + r_1\cos\theta \ , \ -7 + r_1\sin\theta)$

For point B : Let $r = r_2$

$\therefore$ B $(-2 + r_2\cos\theta \ , \ -7 + r_2\sin\theta)$

$\therefore \ 4(-2 + r_1\cos\theta) + 3(-7 + r_1\sin\theta)$

$r_1(4\cos\theta + 3\sin\theta) = 3 + 8 + 21 = 32$

$4(-2 + r_2\cos\theta) + (-7 + r_2\sin\theta) = 32$

$r_2(4\cos\theta + 3\sin\theta) = 41$

$\therefore (r_2 - r_1)(4\cos\theta + 3\sin\theta) = 9$

$| \ r_2 - r_1 \ | = 9$

$\therefore$ squaring we get

$9(16\cos^2\theta + 9\sin^2\theta + 24\sin\theta\cos\theta) = 81$

$16\cos^2\theta + 9\sin^2\theta + 24\sin\theta\cos\theta = 9$

$$7\cos^2\theta + 24\sin\theta\cos\theta = 0$$

$$\cos\theta = 0 \quad or \quad \tan\theta = \frac{-7}{24}$$

<u>Case I</u> : $\cos\theta = 0$

$\Rightarrow \theta = 90$

Equation is $\boxed{x = -2}$

or

<u>Case II</u> :

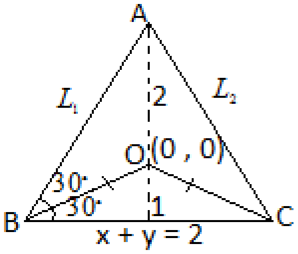
$\tan\theta = \frac{-7}{24}$

$$(y+7) = \frac{-7}{24}(x+2)$$

i.e. $24y + 7x = 182$

$$\boxed{24y + 7x + 182 = 0}$$

Q.31



L_1 & L_2 are inclined at 60° w.r.t. x + y = 2

$$\therefore \tan\theta = \left|\frac{m-(-1)}{1+m(-1)}\right| \quad \theta = 60^\circ$$

$$\sqrt{3} = \left|\frac{m+1}{1-m}\right| \quad \Rightarrow \frac{m+1}{1-m} = \pm\sqrt{3}$$

$$m + 1 = \pm\sqrt{3}(m-1)$$

$$m(1\pm\sqrt{3}) =- (1\pm\sqrt{3})$$

$$m = \frac{-(1\pm\sqrt{3})}{(1\pm\sqrt{3})}$$

$$m = \frac{-(1 \pm \sqrt{3})^2}{(1 - \sqrt{3})}$$

$$m = 2 \pm \sqrt{3}$$

Line trough the centroid

A $\equiv (\alpha\ ,\ \beta)$ then

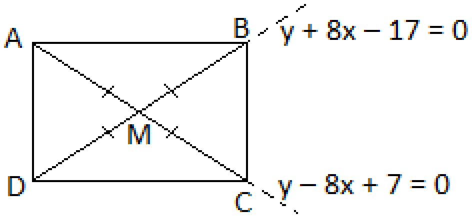
$$\frac{\alpha - 0}{1} = \frac{\beta - 0}{1} = +2\frac{(0+0-2)}{(1^2+1^2)}$$

$$\alpha = \beta = -2$$

A$\equiv(-2,-2)$

$$\boxed{y+2=(2\pm\sqrt{3})(x-2)}$$

Q.32



M is intersection of y + 8x – 17 = 0 & y – 8x + 7 = 0

i.e. y = 5 and x $= \frac{3}{2}$

M $\equiv \left(\frac{3}{2}, 5\right)$

Slope of BM = – 8 = $\tan\theta$

$$\therefore \sin\theta = \frac{8}{\sqrt{65}}\ ,\ \cos\theta = \frac{-1}{\sqrt{65}}$$

$$\therefore B \equiv \left(\frac{3}{2} - \frac{r}{\sqrt{65}}, 5 + \frac{8r}{\sqrt{65}}\right)$$

$$D \equiv \left(\frac{3}{2} + \frac{r}{\sqrt{65}} \, , 5 - \frac{8r}{\sqrt{65}} \right)$$

slope of y -8 x + 7 = 0 is 8

$\therefore \tan\theta = 8$

$$\sin\theta = \frac{8}{\sqrt{65}} \;\; , \cos\theta = \frac{1}{\sqrt{65}}$$

$$\text{A} \equiv \left(\frac{3}{2} + \frac{r}{\sqrt{65}} \, , 5 + \frac{8r}{\sqrt{65}} \right)$$

$$\text{C} \equiv \left(\frac{3}{2} - \frac{r}{\sqrt{65}} \, , 5 - \frac{8r}{\sqrt{65}} \right)$$

2r = length of diagonal

$\therefore$ Area $= 8 = |AB| \times |BC|$

$$|AB| \equiv \left| \frac{2r}{\sqrt{65}} \right|$$

$$|BC| \equiv \left| \frac{16r}{\sqrt{65}} \right|$$

$$\therefore \frac{32r^2}{65} = 8$$

$r^2 = \dfrac{65}{8} \qquad r = \dfrac{\sqrt{65}}{2}$ (taking positive sign)

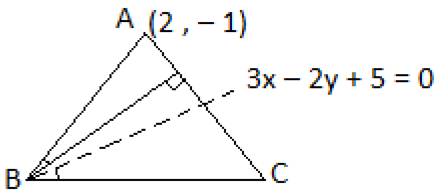
$\therefore$ A $\equiv$ (2 , 9)

B $\equiv$ (1 , 9)

C $\equiv$ (1 , 1)

D $\equiv$ (2 , 1)

Q.22.



Equation of Altitude : 7x – 10y + 1 = 0

Intersection of 3x – 2y + 5 = 0 &

7x – 10y + 1 = 0 gives us B

$$B \equiv \frac{x}{\begin{vmatrix} -5 & -2 \\ -1 & -10 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & -5 \\ 7 & -1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 3 & -2 \\ 7 & -10 \end{vmatrix}}$$

$$\frac{x}{(50-2)} = \frac{y}{(32)} = \frac{1}{-16}$$

$$x = -3 \ , \quad y = -2$$

$$B \equiv (-3, -2)$$

Reflection of A in 3x – 2y + 5 = 0 lies of BC

$\therefore$ Reflection A’ is given by

$$\frac{x-2}{3} = \frac{y+1}{-2} = -\ 2\frac{(6+2+5)}{9+4}$$

$$\frac{x-2}{3} = \frac{y+1}{-2} = -2$$

$\therefore$ A’ $(-4, 3)$ $\therefore$ A’ lies on BC

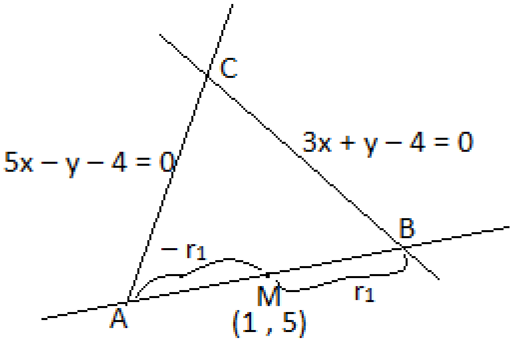
Equation of BC is

$$y+2 = \left(\frac{3+2}{-4+3}\right)(x+3)$$

(y + 2)+ 5 (x + 3) = 0

$$\boxed{BC : y + 5x + 17 = 0}$$

Q.34



Let inclination of line be θ

$\therefore$ parametric locus of line is

$x = 1 + r\cos\theta$

$y = 5 + r\sin\theta$

B $\equiv (1 + r_1\cos\theta, 5 + r_1\sin\theta)$

$\therefore 3(1 + r_1\cos\theta) + (5 + r_1\sin\theta) = 4$

$r_1(3\cos\theta + \sin\theta) = -4$ …………..(A)

$\text{A} \equiv (1 - r_1\cos\theta, 5 - r_1\sin\theta)$

$5(1 - r_1\cos\theta) - (5 - r_1\sin\theta) = 4$

$r_1\sin\theta - 5r_1\cos\theta = 4$

$r_1(\sin\theta - 5\cos\theta) = 4$ …………..(B)

$\frac{(A)}{(B)}$ we get

$$\frac{3\cos\theta + \sin\theta}{\sin\theta - 5\cos\theta} = -1$$

$$\frac{3 + \tan\theta}{\tan\theta - 5} = -1$$

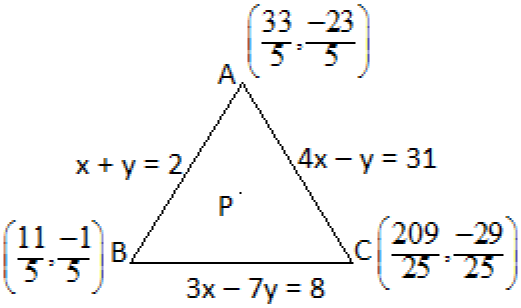
$2\tan\theta - 2 = 0$

$\tan\theta = 1$

$\therefore$ equation of line is y – 5 = x – 1

i.e. $\boxed{y = x + 4}$

Q.35



Point $(\alpha, 2)$

C ≡ 4x – y = 31 &

3x – 7y = 8

$$\frac{x}{\begin{vmatrix}31 & -1\\ 8 & -7\end{vmatrix}} = \frac{y}{\begin{vmatrix}4 & 1\\ 3 & 8\end{vmatrix}} = \frac{1}{\begin{vmatrix}4 & -1\\ 3 & -7\end{vmatrix}}$$

$$\frac{x}{-209} = \frac{y}{29} = \frac{1}{-25}$$

$$x = \frac{209}{25}, \; y = \frac{-29}{25}$$

$$B \equiv x = \frac{11}{5}, \; y = \frac{-1}{5}$$

$$B \equiv \left(\frac{11}{5}, \frac{-1}{5}\right)$$

For point A sign of 3x – 7y – 8 = 0

is $\left(\frac{99+161}{5} - 8\right)$ = 42 – 8

= 34 (positive)

$\therefore 3\alpha - 14 - 8 > 0$

$\alpha > \frac{22}{3}$ ……..(A)

For point B and line 4x – y – 31 = 0

Sign of 4x – y – 31 is

$\frac{44}{5} + \frac{1}{5} - 31 = 9 \ \ 31 = -22$ (negative)

$\therefore 4\alpha - 2 - 31 < 0$

$4\alpha < 33$

$\alpha < \frac{33}{4}$ …………(B)

For point C and line x + y – 2 = 0

Sign of x + y – 2

is $\frac{209}{25} - \frac{29}{25} - 2$

= 7.22 = 5.2 (positive)

$\therefore \alpha + 2 - 2 > 0$

$\alpha > 0$ …………..(C)

Intersection of (A) , (B) , (C) gives

$$\boxed{\frac{22}{3} < \alpha < \frac{33}{4}}$$

Q.36

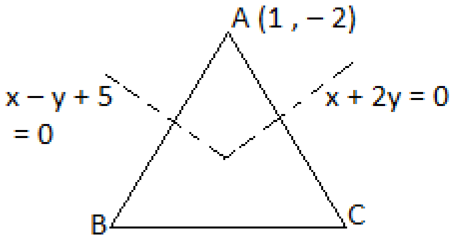


Image of A(1 , – 2) in x – y + 5 = 0 is B

$\therefore$ point B

$$\frac{x-1}{1}=\frac{y+2}{-1}=\frac{-2(1+2+5)}{1^2+1^2}=-8$$

$\therefore x=-7$

y = 6

$B \equiv (-7,6)$

Image of A (1 , – 2) in x + 2y = 0 is C

$\therefore$ point C

$$\frac{x-1}{1}=\frac{y+2}{-1}=-2\left(\frac{1-4}{1^2+2^2}\right)=\frac{6}{5}$$

$$x=\frac{11}{5}\ ,\ y=\frac{2}{5}$$

$$C \equiv \left(\frac{11}{5},\frac{2}{5}\right)$$

Equation of BC is

$$y-6=\left(\frac{6-\frac{2}{5}}{-7-\frac{11}{5}}\right)(x+7)$$

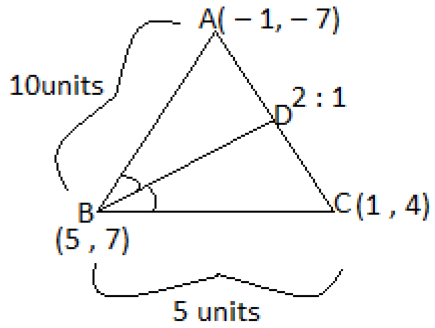
$$y-6=\left(\frac{28}{-46}\right)(x+7)=\frac{14}{-23}(x+7)$$

$$\therefore 23(y-6)+14(x+7)=0$$

$$23y+14x=138-98$$

$$\boxed{23y+14x=40}$$

Q.37



From angle bisector theorem

AD : DC = 2 : 1

$$D \equiv \left(\frac{2(1)+1(-1)}{3}, \frac{8-7}{3} \right)$$

$$D = \left(\frac{1}{3}, \frac{1}{3} \right)$$

Equation of AD $\equiv$

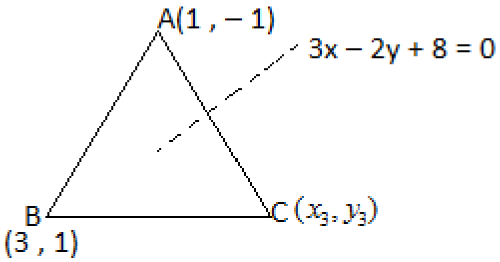
$$y-1=\left(\frac{1-\frac{1}{3}}{5-\frac{1}{3}} \right)(x-5)$$

$$=\frac{2}{14}(x-5)=\frac{(x-5)}{7}$$

7y – 7 = x – 5

$$\boxed{7y=x+2}$$

Q.38



Reflection of a in 3x -2y + 8 = 0 gives C

$$\therefore \frac{x_3-1}{3} = \frac{y_3+1}{-2} = -2\frac{(3+2+8)}{9+4}$$

$$\frac{x_3-1}{3} = \frac{y_3+1}{-2} = -2\frac{(13)}{(13)} = -2$$

$x_3 - 1 = -6$ $\qquad$ $y_3 + 1 = 4$

$x_3 = -5$ $\qquad$ $y_3 = 3$

C $\equiv (-5, 3)$

Equation of BC

$$(y-1) = \left(\frac{3-1}{-5-3}\right)(x-3)$$

$$(y-1) = \left(\frac{2}{-8}\right)(x-3)$$

4(y – 1) + (x – 3) = 0

$$\boxed{4y + x = 7}$$

Q.39

A line through the intersection of $\frac{x}{a} + \frac{y}{b} = 1$ & $\frac{x}{b} + \frac{y}{a} = 1$

is of the form

$$\left(\frac{x}{a} + \frac{y}{b} - 1\right) + \lambda\left(\frac{x}{b} + \frac{y}{a} - 1\right) = 0$$

$$x\,\frac{(b + a\lambda)}{ab} + \left(\frac{a + b\lambda}{ab}\right)y = \lambda + 1$$

$$\frac{x}{\frac{ab(\lambda+1)}{a\lambda+b}} + \frac{y}{\frac{ab(\lambda+1)}{a+b\lambda}} = 1$$

A is the x – intercept

A $\equiv \left(\dfrac{ab(\lambda+1)}{a\lambda+b} , 0 \right)$

B is the y – intercept

B $\equiv \left(0 , \dfrac{ab(\lambda+1)}{a+b\lambda} \right)$

Mid – point of AB

$$\equiv \left(\frac{ab(\lambda+1)}{2(a\lambda+b)} , \frac{ab(\lambda+1)}{2(a+b\lambda)} \right)$$

$$\equiv (h, k)$$

$$\frac{1}{h} + \frac{1}{k}$$

$$= \frac{2(a\lambda+b)}{ab(\lambda+1)} + \frac{2(a+b\lambda)}{ab(\lambda+1)}$$

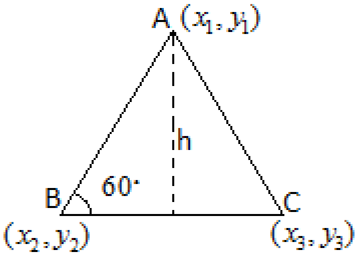
$$= \frac{(a+b)(\lambda+1)2}{ab(\lambda+1)}$$

$$\therefore \frac{1}{h} + \frac{1}{k} = \frac{2(a+b)}{ab}$$

Locus is $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2(a+b)}{ab}$

i.e. $\boxed{(x+y)ab = 2xy(a+b)}$

Q.40



Assume angle = 60º

Proof using vectors

$$\overline{BA} = (x_1 - x_2)\hat{i} + (y_1 - y_2)\, j$$

$$\overline{BC} = (x_3 - x_2)\hat{i} + (y_3 - y_2)\, j$$

$\frac{1}{2}\left|\overline{BA} \times \overline{BC}\right| =$ Area of triangle $= \frac{1}{2}\left|\overline{BA}\right|\left|\overline{BC}\right| \sin 60^\circ$

Area of the triangle

$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ which is rational.

If $x_1 x_2 x_3 ....y_1 y_2 y_3$ are rational

$$\therefore \left|\overline{BA} \times \overline{BC}\right| = 2\Delta \quad \in \theta = \left|\overline{BA}\right|\left|\overline{BC}\right| \sin 60^\circ \in \theta$$

$$\overline{BA} \cdot \overline{BC} = (x_1 - x_2)(x_3 - x_2) + (y_1 - y_2)(y_3 - y_2)$$

$\overline{BA} \cdot \overline{BC}$ is also rational.

$$\therefore \overline{BA} \cdot \overline{BC} = \left|\overline{BA}\right|\left|\overline{BC}\right| \cos 60^\circ \in \theta$$

$\therefore \frac{\left|\overline{BA} \times \overline{BC}\right|}{\left|\overline{BA} \cdot \overline{BC}\right|}$ must be rational.

But ratio of

$\frac{\left|\overline{BA} \times \overline{BC}\right|}{\overline{BA} \cdot \overline{BC}}$ is $\tan 60^\circ$ which is irrational.

$\therefore$ not possible that angle is 60°

Q.41

A (ae , 0) , P (h , k)

B (– ae , 0)

$$\sqrt{(h - ae)^2 + k^2} + \sqrt{(h + ae)^2 + k^2} = 2a$$

$$\sqrt{(h-ae)^2+k^2} = 2a - \sqrt{(h+ae)^2+k^2}$$

Squaring we get

$$(h-ae)^2+k^2 = 4a^2+(h+ae^2)+k^2-4a\sqrt{(h+ae)^2+k^2}$$

$$\therefore 4a\sqrt{(h+ae)^2+k^2} = 4a^2+4aeh$$

$$\sqrt{(h+ae)^2+k^2} = a+eh$$

$$h^2+a^2e^2+2aeh+k^2 = a^2+2aeh+e^2h^2$$

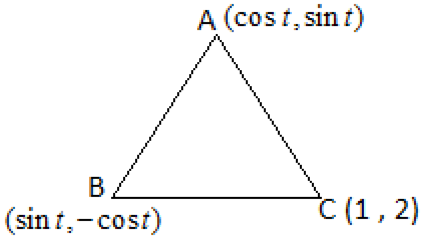
$$h^2(1-e^2)+k^2 = a^2(1-e^2)$$

$$\frac{h^2}{a^2}+\frac{k^2}{a^2(1-e^2)}=1$$

Locus is

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad b^2=a^2(1-e^2)}$$

Q.42



$$\text{Centroid C} \equiv \left(\frac{\cos t+\sin t+1}{3}, \frac{\sin t-\cos t+2}{3}\right)$$

$$\equiv (h , k)$$

$$\therefore 3h-1=\cos t+\sin t \qquad \ldots\ldots\ldots\ldots\ldots(A)$$

$$3k-2=\sin t-\cos t \qquad \ldots\ldots\ldots\ldots\ldots(B)$$

Squaring and adding we get

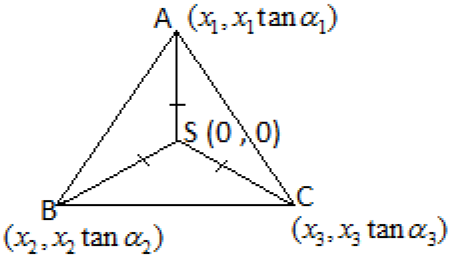
$$(3h-1)^2+(3k-2)^2=1+1+2\cos t\sin t-2\cos t\sin t$$

$$= 2$$

$\therefore$ Locus is

$$\boxed{(3x-1)^2+(3y-2)^2=2}$$

Q.43



SA = SB = SC (circumcentre)

$$\therefore \sqrt{x_1^2 + x_1^2\tan^2\alpha_1} = \sqrt{x_2^2 + x_2^2\tan^2\alpha_2} = \sqrt{x_3^2 + x_3^2\tan^2\alpha_3}$$

$$\therefore x_1^2\sec^2\alpha_1 = x_2^2\sec^2\alpha_1 = x_3^2\sec^2\alpha_3$$

$$x_1\sec\alpha_1 = x_2\sec\alpha_2 = x_3\sec\alpha_3 = k$$

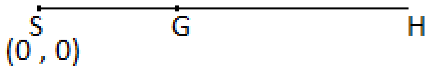
$$\text{Centroid G} \equiv \left(\frac{\sum x_1}{3}, \frac{\sum x_1\tan\alpha}{3}\right)$$

$$\therefore \frac{x_1}{\cos\alpha_1} = \frac{x_2}{\cos\alpha_2} = \frac{x_3}{\cos\alpha_3} = \frac{\sum x_1}{\sum\cos\alpha_1} = k$$

$$G \equiv \left(\frac{k\sum\cos\alpha_1}{3}, \frac{\sum\frac{x}{\cos\alpha_1}\sin\alpha_1}{3}\right)$$

$$G \equiv \left(\frac{k\sum\cos\alpha_1}{3}, \frac{\sum k\sin\alpha_1}{3}\right)$$

$$G \equiv \left(\frac{k\sum\cos\alpha_1}{3}, k\frac{\sum\sin\alpha_1}{3}\right)$$



S , G , H are collinear

Equation of SG is $y-0=\dfrac{\dfrac{k\sum\sin\alpha_1}{3}}{\dfrac{k\sum\cos\alpha_1}{3}}(x-0)$

$$y=\frac{\sum\sin\alpha_1}{\sum\cos\alpha_1}x$$

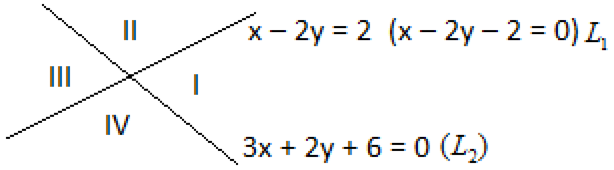
H lies on SG.

$\therefore\ H(\overline{x}\ ,\ \overline{y})$ satisfies equation of SG

$\therefore \overline{y}\ =\ \overline{x}\dfrac{\sum\sin\alpha_1}{\sum\cos\alpha_1}$

$\therefore \overline{y}(\sum\cos\alpha_1)\ =\ \overline{x}\ (\sum\sin\alpha_1)$

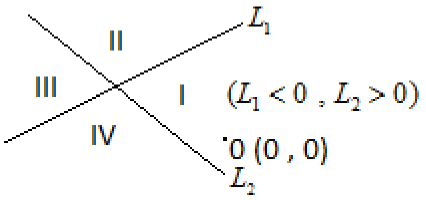
Q.44



$L_1: x-2y-2=0$

$L_2: 3x+2y+6=0$ Assume origin to lie in Area 1.

For (0 , 0) $L_1 < 0$ $L_2 > 0$



Region I : $L_1 < 0$ $L_2 > 0$

Region II : $L_1 > 0 \quad L_2 > 0$

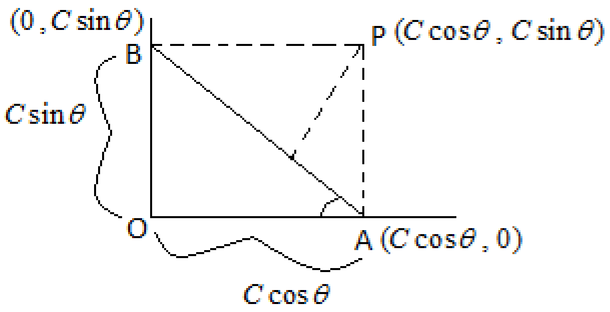
Region III : $L_1 > 0 \quad L_2 < 0$

Region IV : $L_1 < 0 \quad L_2 < 0$

Point	L_1	L_2	Region
(1 , 3)	1 – 6 – 2	3 + 6 + 6	I
(3 , – 2)	3 + 4 – 2	9 – 4 + 6	II
(– 1 , – 4)	– 1 + 8 – 2	– 3 – 8 + 6	III
(– 4 , 1)	– 4 – 2 – 2	– 12 + 2 + 6	IV

So they all lie in different regions.

Q.45



$$|AB| = C$$

Equation of AB is

$$\frac{x}{C\cos\theta} + \frac{y}{C\sin\theta} = 1$$

$$x\sin\theta + y\cos\theta = C\sin\theta\cos\theta$$

$$x\sin\theta + y\cos\theta - C\sin\theta\cos\theta = 0$$

Foot of perpendicular is Q(h , k) given by

$$\frac{h - C\cos\theta}{\sin\theta} = \frac{k - C\sin\theta}{\cos\theta} = -\left(\frac{C\cos\theta\sin\theta + C\sin\theta\cos\theta - c\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}\right)$$

$$\therefore \frac{h - C\cos\theta}{\sin\theta} = \frac{k - C\sin\theta}{\cos\theta} = -C\sin\theta\cos\theta$$

$$h = C\cos\theta - C\cos\theta\sin^2\theta$$

$$h = C\cos^3\theta$$

$$k = C\sin\theta - C\sin\theta\cos^2\theta$$

$$k = C\sin^3\theta$$

$$\therefore \left(\frac{h}{C}\right)^{\frac{1}{3}} = \cos\theta$$

$$\left(\frac{k}{C}\right)^{\frac{1}{3}} = \sin\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\therefore \frac{h^{\frac{2}{3}}}{C^{\frac{2}{3}}} + \frac{k^{\frac{2}{3}}}{C^{\frac{2}{3}}} = 1$$

$$\therefore \boxed{x^{\frac{2}{3}} + y^{\frac{2}{3}} = C^{\frac{2}{3}}}$$

Q.46

$$h = (u\cos\alpha)t$$

$$k = (u\sin\alpha)t - py^2$$

$$t = \frac{h}{u\cos\alpha}$$

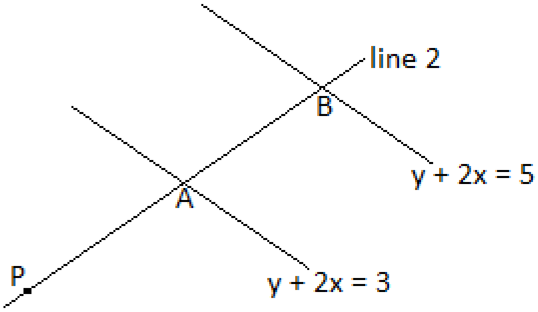
$$\therefore k = (u\cos\alpha)\left(\frac{h}{u\cos\alpha}\right) - p\left(\frac{h}{u\cos\alpha}\right)^2$$

$$k = h\tan\alpha - \frac{ph^2}{u^2}\sec^2\alpha$$

$\therefore$ locus is

$$\boxed{y = x\tan\alpha - \frac{px^2\sec^2\alpha}{u^2}}$$

Q.47



AB = 2 units.

Parameter Equation of line L : $x=2+r\cos\theta$

$y=3+r\sin\theta$

$PA=r_1$

$PB=r_1+2 \qquad \because\ AB=2$

$A\equiv(2+r_1\cos\theta\ ,\ 3+r_1\sin\theta)$

A satisfies t + 2x = 3

$3+r_1\sin\theta+2(2+r_1\cos\theta)=3$

$r_1(\sin\theta+2\cos\theta)=-4 \quad ....(A)$

$B\equiv(2+(r_1+2)cos\theta\ ,\ 3+(n+2)\sin\theta)$

B satisfies y + 2x = 5

$3+(r_1+2\sin\theta+2(2+(n+2)\cos\theta))=5$

$\Rightarrow r_1(\sin\theta+2\cos\theta)+3+2\sin\theta+4+4\cos\theta=5$

$\Rightarrow r_1(\sin\theta+2\cos\theta)=-2-2\sin\theta-4\cos\theta \quad .....(B)$

$\therefore$ form (A) , (B)

we get

$-4=-2-2\sin\theta-4\cos\theta$

$2=1+\sin\theta+2\cos\theta$

$1=\sin\theta+2\cos\theta$

$$1=\frac{2t}{1+t^2}+\frac{2-2t^2}{1+t^2}$$

$1+t^2=2t+2-2t^2$

$3t^2-2t-1=0$

$t = 1\ ,\ \dfrac{-1}{3}$

Case I : If $\tan\dfrac{\theta}{2} = 1$

$\theta = 90^\circ$

So $\boxed{\text{Equation of line } x = 2 \quad \text{..................Ans. : } (\text{i})}$

Case II : $\tan\dfrac{\theta}{2} = \dfrac{-1}{3}$

$$\tan\theta = \frac{2\left(\frac{-1}{3}\right)}{1-\frac{1}{9}} = \frac{\frac{-2}{3}}{\frac{8}{9}}$$

$$\tan\theta = \frac{-6}{8}$$

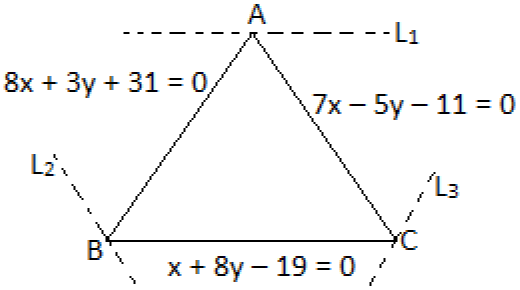
$$\tan\theta = \frac{-3}{4}$$

So equation of L is

$$y - 3 = \frac{-3}{4}(x-2)$$

$\boxed{4y + 3x = 18\text{..............Ans. : } (\text{ii})}$

Q.48



Let equation of line parallel to BC passing through A be called L_1.

$L_1 \equiv 8x+3y+31+\lambda(7x-5y-11)=0$

$\equiv x(8+7\lambda)+(3-5\lambda)y+(31-\lambda(11))=0$

slope condition we get

$\frac{3-5\lambda}{8+7\lambda}=8 \quad \therefore \ 3-5\lambda=64+56\lambda$

$\lambda=-1$

$\therefore L_1 \equiv (8x+3y+31)-(7x-5y-11)$

Let equation of line parallel to AC passing through B be called L$_2$.

$L_2 \equiv 8x+3y+31+\lambda(x+8y-19)=0$

$\equiv (8+\lambda)x+(3+\lambda 8)y+(31-\lambda 19)=0$

L_2 parallel to AC

$\therefore \ \frac{8+\lambda}{3+8\lambda}=\frac{7}{-5} \quad \therefore \ -40-5\lambda=21+56\lambda$

$\lambda=-1$

$L_2: 8x+3y+31-(x+8y-19)=0$

Let equation of L$_3$ be line parallel to AB passing through C.

$L_3 \equiv x+8y-19+\lambda(7x-5y-11)=0$

$\equiv (1+7\lambda)x+(8-5\lambda)y=(19+11\lambda)$

L$_3$ parallel to 8x + 3y + 31 = 0. So,

$\frac{(1+7\lambda)}{(8-5\lambda)}=\frac{8}{3} \quad \therefore \lambda=1$

	Substituting co – ordinates of origin (0 , 0)	

∴

3x

L_1 ,	L_1 : 31 – (– 11) = 46 positive	0 lies between L_1 & BC
	BC : – 19 negative	
L_2 ,	L_2 : 31 – (– 19) = 50 positive	0 lies between L_2 & AC
	AC : – 11 negative	
L_3 ,	L_3 : – 19 – 11) = 0 negative	0 lies between L_3 & AB
	AB : 31 positive	

$L_3: x+8y-19+7x-5y-11=0$

All 3 conditions are satisfied $\Rightarrow$

$\boxed{0 \text{ lies in the triangle}}$

Q.49

5x + 3y = 4

3x + 8y = – 13

H intersection orthocenter

$$\frac{x}{\begin{vmatrix} 4 & 3 \\ -13 & 8 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 5 & 4 \\ 3 & -13 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 5 & 3 \\ 3 & 8 \end{vmatrix}}$$

$$\frac{x}{71} = \frac{y}{-77} = \frac{1}{31}$$

$$x = \frac{71}{31}, y = \frac{-77}{31}$$

slope of BC $= \frac{3}{5}$

Equation of BC $= y+5 = \frac{3}{5}(x+4)$

$$= 5y + 25 = 3x + 12$$

$$\boxed{BC: 5y - 3x + 13 = 0}$$

Equation of AB = ? slope of AB is $\frac{8}{3}$

Equation of AB $= y + 5 = \frac{8}{3}(x + 4)$

$$\boxed{AB: 3y - 8x = 17}$$

$$\text{H} \equiv \left(\frac{71}{31}, \frac{-77}{31}\right)$$

Equation of BH is

$$y + 5 = \left(\frac{-5 + \frac{77}{31}}{-4 - \frac{71}{31}}\right)(x + 4)$$

$$y + 5 = \frac{-78}{-195}(x + 4)$$

$$= \frac{2}{5}(x + 4)$$

5y + 25 = 2x + 8

So slope of AC $= \frac{-5}{2}$ (perpendicular to BH) ………(A)

Let C (h , k)

slope CH $= \frac{-3}{8} = \left(\frac{k + \frac{77}{31}}{h - \frac{71}{31}}\right)$

$$31 \times 8k + 3h \times 31 + 77 \times 8 - 71(3) = 0$$

$$31(8k + 3h) = -403$$

$8k+3h=-13$ …………..(B)

Also BC equation is satisfied by C

$5k - 3h + 13 = 0$ …………..(C)

$\therefore$ h = 1 , k = – 2

C $\equiv$ (1 , – 2)

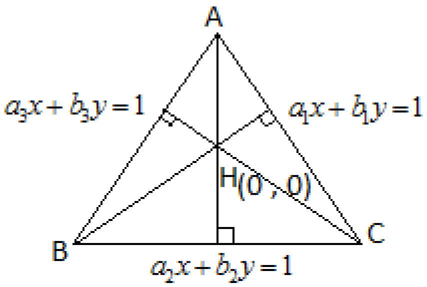
Equation of AC is from (A)

$$y+2=\frac{-5}{2}(x-1)$$

2y + 4 + 5x – 5 = 0

$\boxed{AC:2y+5x=1}$

Q.50



Slope of AH $=\frac{b_2}{a_2}$ perpendicular to BC

Equation of AH or HD is y $=\frac{b_2}{a_2x}$ (passing through origin)

Now $a_3x+b_3y-1=0$

$a_1x+b_1y-1=0$

$b_2x-a_2y=0$ are concurrent

So

$$\begin{vmatrix} a_3 & b_3 & -1 \\ a_1 & b_1 & -1 \\ b_2 & -a_2 & 0 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} a_3 & b_3 & -1 \\ a_1 - a_3 & b_1 - b_3 & 0 \\ b_2 & -a_2 & 0 \end{vmatrix} = 0$$

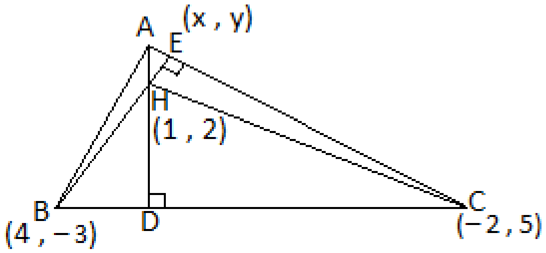
$$\therefore (a_1 - a_3)(-a_2) - (b_2)(b_1 - b_3) = 0$$

$$a_1a_2 - a_2a_3 = -(b_1b_2 - b_2b_3)$$

$$\therefore \boxed{a_1a_2 + b_2b_1 = a_2a_3 + b_2b_3}$$

Using the equation of BH we can prove the other conditions.

Q.51



AH perpendicular to BC

$$\therefore \left(\frac{y-2}{x-1}\right)\left(\frac{5+3}{-2-4}\right) = -1$$

$$\left(\frac{y-2}{x-1}\right) = \frac{3}{4} \qquad 4(y-2) = 3(x-1) \qquad \text{..............(i)}$$

$$4y - 8 = 3x - 3$$

$$3x - 4y = -5 \qquad \text{..............(i)}$$

BH perpendicular to AC

$$\therefore \left(\frac{y-5}{x+2}\right)\left(\frac{2+3}{1-4}\right) = -1$$

$\frac{y-5}{x+2}=\frac{3}{5}$ $\quad$ $5(y-5)-3(x+2)$ ………………(ii)

$5y-25=3x+6$

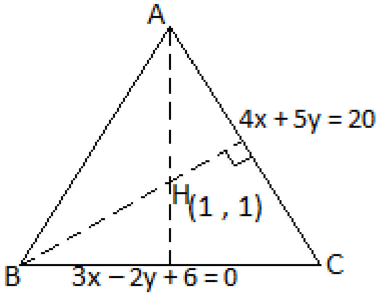
$-31=3x-5y$ ………………(ii)

$\therefore$ solving (i) and (ii) we get

y = 26 , x = 23

A (33 , 26)

Q.52



slope of BC $=\frac{3}{2}$	slope of AC $=\frac{-4}{5}$
$\therefore$ slope of AH $=\frac{-2}{3}$	$\therefore$ slope of BH $=\frac{5}{4}$
Equation of AH = y – 1 = $=\frac{-2}{3}$ (x – 1)	Equation of BH = (y – 1) $=\frac{5}{4}$ (x – 1)
3(y – 1)+2(x – 1) = 0	4(y – 1) = 5(x – 1)
2x + 3y = 5 ……….(i)	1 = 5x – 4y ………(ii)
A is intersection of (AH) 2x + 3y = 5 &	B is intersection of (BH) 1 = 5x – 4y &
(AC) 4x + 5y = 20	(BC) – 6 = 3x – 2y
$\frac{x}{\begin{vmatrix} 6 & 3 \\ 20 & 5 \end{vmatrix}}=\frac{y}{\begin{vmatrix} 2 & 5 \\ 4 & 20 \end{vmatrix}}=\frac{1}{\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}}$	$\frac{x}{\begin{vmatrix} 1 & -4 \\ -6 & -2 \end{vmatrix}}=\frac{y}{\begin{vmatrix} 5 & 1 \\ 3 & -6 \end{vmatrix}}=\frac{1}{\begin{vmatrix} 5 & -4 \\ 3 & -2 \end{vmatrix}}$

$$\frac{x}{-35}=\frac{y}{20}=\frac{1}{-2}$$

$$x=\frac{35}{2}\ ,\ \mathrm{y}=-10$$

$$\mathrm{A}\ \equiv\left(\frac{35}{2},-10\right)$$

$$\frac{x}{-26}=\frac{y}{-33}=\frac{1}{2}$$

$$\mathrm{x}\ =-13\ ,\ y=\frac{-33}{2}$$

$$\mathrm{B}\equiv\left(-13\ ,\ \frac{-33}{2}\right)$$

Equation of slope AB $= \dfrac{\frac{-33}{2}+10}{-13-\frac{35}{2}} = \dfrac{-13}{-61} = \dfrac{13}{61}$

$$y+10=\frac{13}{61}\left(x-\frac{35}{2}\right)$$

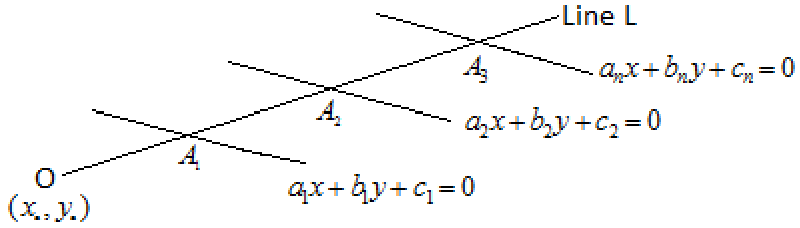
$$\therefore 122(y+10)=26x-35\times 13$$

$$26x-122y=1220+35\times 13$$

Equation of third side is

$$\boxed{26x-122y=1675}$$

Q.53



Let inclination of line L be θ

A_1 , A_2 ...., A_n be points of intersection.

Using parametric form

$$x=x_\circ+r\cos\theta$$

For $A_1(x_1\ ,\ y_1)$. Take length as r_1

$$\therefore x_1 = x_\circ + r_1 \cos\theta$$

$$y_1 = y_\circ + r_1 \sin\theta$$

Now $A_1(x_1\ ,\ y_1)$ satisfies $a_1 x + b_1 y + c_1 = 0$

$$\therefore a_1(x_\circ + r_1 \cos\theta) + b_1(y_\circ + r_1 \sin\theta) + c_1 = 0$$

$$\therefore r_1\,(a_1 \cos\theta + b_1 \sin\theta) = -\,(a_1 x_\circ + b_1 y_\circ + c_1)$$

$$r_1 = \frac{-\,(a_1 x_\circ + b_1 y_\circ + c_1)}{(a_1 \cos\theta + b_1 \sin\theta)} = OA_1$$

$\therefore$ similarly $OA_2 = -\left(\dfrac{a_2 x_\circ + b_2 y_\circ + c_2}{a_2 \cos\theta + b_2 \sin\theta}\right)$

$$OA_n = \frac{-\,(a_n x_\circ + b_n y_\circ + c_n)}{(a_n \cos\theta + b_n \sin\theta)}$$

A be any point of the line L

$\therefore$ A(h , k)

$$h = x_\circ + r\cos\theta$$

$$k = y_\circ + r\sin\theta$$

$$\frac{n}{OA} = \sum \frac{1}{OR_r}$$

$$\frac{n}{r} = \sum - \left(\frac{(a_n \cos\theta + b_n \sin\theta)}{(a_n x_\circ + b_n y_\circ + c_n)}\right)$$

$$-n = \sum - \left(\frac{(a_n r \cos\theta + b_n r \sin\theta)}{(a_n x_\circ + b_n y_\circ + c_n)}\right)$$

$$-n = \sum \left(\frac{a_n (h - x_\circ) + (k - y_\circ) b_n}{a_n x_\circ + b_n y_\circ + c_n}\right)$$

Locus of A is

$$-n = \sum\left(\frac{a_n(x-x_\circ)+(y-y_\circ)b_n}{a_n x_\circ + b_n y_\circ + c_n}\right)$$

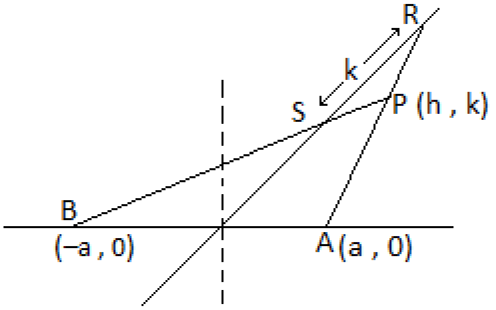
a_1...........a_n 's constant

b_1............b_n 's constant

c_1............c_n 's constant

$x_\circ$, $y_\circ$ is given

Q.54



Equation of PB : $y-0=\left(\frac{k-0}{h+a}\right)(x+a)$

$yh+ay=kx+ak$

$yh-kx+ak-ay$

$yh-kx=ak-ay$

$y(h+a)-kx=ak$ …………..(i)

Equation of PA will be

$y(h-a)-kx=-ak$ …………(ii)

S is intersection of PA & y = x

Co – ordinates of S :

$y(h-a)-kx=-ak$

$y-x=0$

$$\frac{y}{\begin{vmatrix} -ak & -k \\ 0 & -1 \end{vmatrix}} = \frac{x}{\begin{vmatrix} h-a & -ak \\ 1 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} h-a & -k \\ 1 & -1 \end{vmatrix}}$$

$$\frac{y}{(ak)} = \frac{x}{(ak)} = \frac{1}{k-(h-a)}$$

$$x = y = \frac{ak}{k+a-h}$$

$$\text{S} \equiv \left(\frac{ak}{k+a-h} , \frac{ak}{k+a-h} \right)$$

Co – ordinates of R will be

$$\text{R} \equiv \left(\frac{ak}{h+a-k} , \frac{ak}{h+a-k} \right)$$ (replace a by – a)

Length SR

$$= \sqrt{2} \left| \left(\frac{ak}{h+a-k} \right) - \left(\frac{ak}{k+a-h} \right) \right|$$

$$= \sqrt{2} \mid ak \mid \left| \frac{(k+a-h)-(h+a-k)}{a^2-(h-k)^2} \right|$$

Length SR = K

$$\therefore \quad K^2 = 2ak^2 \left(\frac{\left(2k-2h\right)^2}{a^2-(k-h)^2} \right)$$

Replacing k by y , h by x

$$K^2 = \frac{8ay^2(x-y)^2}{\left(a^2-(y-x)^2\right)^2}$$ is the locus of point P.

Q.55

$$y + 2at = t(x - at^2)$$

Let t_1 & t_2 be 2 lines in the family such that l_1 & l_2 are perpendicular.

$$L_1 \equiv y + 2at_1 = t_1x - at_1^3$$

$$L_2 \equiv y + 2at_2 = t_2x - at_2^3$$

$\because$ L_1 perpendicular L_2 slope $L_1 \times L_2 = -1$

$$\therefore \; t_1 \times t_2 = -1$$

$$t_1t_2 = -1$$

$$L_1: \qquad y - t_1x = -2at_1 - at_1^3$$

$$L_2: \qquad y - t_2x = -2at_2 - at_2^3$$

Intersection is

$$\frac{y}{\begin{vmatrix} -2at_1 - at_1^3 & -t_1 \\ -2at_2 - at_2^3 & -t_2 \end{vmatrix}} = \frac{x}{\begin{vmatrix} 1 & -2at_1 - at_1^3 \\ 1 & -2at_2 - at_2^3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & -t_1 \\ 1 & -t_2 \end{vmatrix}}$$

$$y = \frac{(t_2(2at_1 + at_1^3)) - t_1(2at_2 + at_2^3)}{(t_1 - t_2)}$$

$$x = \frac{(2at_1 + at_1^3) - (2at_2 + at_2^3)}{(t_1 - t_2)}$$

$$y = \frac{2at_1t_2 - 2at_1t_2 + at_1t_2(t_1^2 - t_2^2)}{(t_1 - t_2)}$$

$$y = at_1t_2(t_1 + t_2) \qquad [t_1t_2 = -1]$$

$$y = -a(t_1 + t_2)$$

$$x = 2a + a(t_1^2 + t_2^2 + t_1t_2)$$

$$\frac{y^2}{a^2} = (t_1 + t_2)^2 = t_1^2 + t_2^2 + 2t_1t_2$$

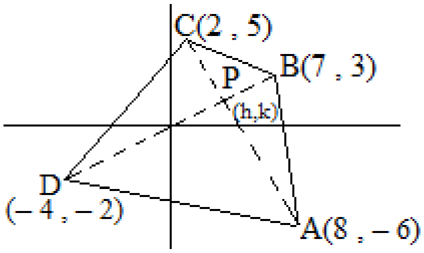
$$\frac{y^2}{a^2} + 1 = t_1^2 + t_2^2 + t_1t_2 = \frac{x - 2a}{a}$$

$$y^2 + a^2 = ax - 2a^2$$

$$y^2 = ax - 3a^2$$

$$\boxed{y^2 = a(x - 3a)}$$

Q.56



Δ ABCD

$$\equiv \frac{1}{2}\begin{vmatrix} 8 & 7 & 2 & -4 & 8 \\ -6 & 3 & 5 & -2 & -6 \end{vmatrix}$$

$$= \frac{1}{2}[\,24 + 35 - 4 + 24 - (-42 + 6 - 20 - 16)]$$

$$= \frac{1}{2}[79 - (-72)\,]$$

$$= \frac{1}{2}(151)$$

$$\Delta PDA = \frac{1}{2}\begin{vmatrix} -4 & 8 & h & -4 \\ -2 & -6 & k & -2 \end{vmatrix}$$

$$= \frac{1}{2}\,[(24 + 8k - 2h) - (-16 - 6h - 4k)]$$

$$= \frac{1}{2}[40 + 4h + 12k]$$

$$= 2[10 + h + 3k]$$

Now the line $x + 3y + 10 = 0$ passes through D does not pass through $\square ABCD$.

So for any point inside the Quadrilateral $10 + h + 3k$ will be positive.

$\therefore\ \Delta PDA = 2[10 + h + 3k]$

ΔPBC

$$= \frac{1}{2}\begin{vmatrix} 2 & 7 & h & 2 \\ 5 & 3 & k & 5 \end{vmatrix}$$

$$= \frac{1}{2}[6 + 7k + 5h - (35 + 3h + 2k)]$$

$$= \frac{1}{2}[-29 + 2h + 5k]$$

$2x + 5y - 29 = 0$ is the equation of BC.

$\therefore$ for all points inside the Quadrilateral $-29 + 2h + 5k$ will be negative.

$$\therefore \Delta PBC = \frac{1}{2}[29 - 2h - 5k]$$

$\therefore \Delta PBC + \Delta PDA$

$$= (20 + 2h + 6k) + \frac{29}{2} - h - \frac{5}{2}k$$

$$= \left(\frac{69}{2} + h + \frac{7k}{2}\right)$$

$$= \frac{151}{4} \quad \left(\frac{1}{2}\square ABCD\right)$$

$138 + 4h + 14k = 151$

$\therefore 4h + 14k = 13$

$\boxed{4x + 14y = 13}$

Q.57

3x + 4y = 9

mx – y = – 1

$$\frac{x}{\begin{vmatrix} 9 & 4 \\ -1 & -1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 3 & 9 \\ m & -1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 9 & 4 \\ m & -1 \end{vmatrix}}$$

$$\frac{x}{-5} = \frac{y}{-3-9m} = \frac{1}{-3-4m}$$

$$x = \frac{5}{3+4m} \quad , \qquad y = \frac{3+9m}{3+4m}$$

x and y must be integers.

$x = \frac{5}{3+4m}$ 3 + 4m must be a factor of 5 or – 5

3 + 4m can be : 1, – 1, 5, – 5

For $3+4m = 1 \quad \Rightarrow \quad m = \frac{-1}{2} \notin \text{I}$

For $3+4m = -1 \quad \Rightarrow \quad m = -1 \in \text{I}$

For $3+4m = 5 \quad \Rightarrow \quad m = \frac{1}{2} \notin \text{I}$

For $3+4m = -5 \quad \Rightarrow \quad m = -2 \in \text{I}$

Put $m = -1 \& -2$ in y and see if y is also integer.

<u>m = – 1</u> then $y = \frac{3-9}{3-4} = \frac{-6}{-1} = 6 \in \text{I}$

<u>m = – 2</u> then $y = \frac{3-18}{3-8} = \frac{-15}{-5} = 3 \in \text{I}$

so m is $-1, -2$

Q.58

$dx + cy = cd$ passes through (a , b) parametric from of the line.

$$\left.\begin{aligned} x &= a + r\cos\theta \\ y &= b + r\sin\theta \end{aligned}\right\} \quad \tan\theta = \frac{-d}{c}$$

When $x=0$, $a+r\cos\theta=0$

$$\therefore r=\frac{-a}{\cos\theta}$$

$\therefore y=b-a\tan\theta=d$ (y – intercept)

When y = 0

$$r=\frac{-b}{\sin\theta}$$

$$\therefore x=a-\frac{b}{\sin\theta}\cos\theta$$

$x=a-b\cot\theta=c$ (x – intercept)

$c=(a-b\cot\theta)$

$d=(b-a\tan\theta)$

$c+d=a+b-b\cot\theta-a\tan\theta$

$c>0$ so $a>b\cot\theta \Rightarrow \dfrac{a\sin\theta-b\cos\theta}{\sin\theta}>0$ …………..(1)

$d>0$ so $b>a\tan\theta \Rightarrow \dfrac{b\cos\theta-a\sin\theta}{\cos\theta}>0$ …………..(2)

Multiplying (1) & (2)

$\Rightarrow \sin\theta\cos\theta<0$

$\tan\theta<0$

Now

$c+d=a+b+b(-\cot\theta)+a(-\tan\theta)$

AB makes 45° with $3x-4y-49=0$

AP also makes 45° with $3x-4y-49=0$

$$\left(\text{slope}=\frac{3}{4}\right)$$

$$\tan 45^\circ = \left| \frac{m - \frac{3}{4}}{1 + \frac{3m}{4}} \right|$$

$$\frac{4m-3}{4+3m} = \pm 1$$

$$(4m-3) \pm (4+3m) = 0$$

$$7m + 1 = 0 \qquad or \qquad m = 7$$

$$m = \frac{-1}{7}$$

$$m = \frac{-1}{7} \qquad or \qquad 7$$

So equation of AP or AB is

$$(y-1) = \frac{-1}{7}(x-1) \qquad and \qquad (y-1) = 7(x-1)$$

$$7y - 7 + x - 1 = 0 \qquad\qquad y = 7x - 6 \qquad \text{…………..(ii)}$$

$$7y + x = 8 \qquad \text{……..(i)} \qquad 7x - y = 6 \qquad \text{…………..(iii)}$$

Intersection with $3x - 4y = 49$ will be give P & B

$$x + 7y = 8$$

$$3x - 4y = 49$$

$$\frac{x}{\begin{vmatrix} 8 & 7 \\ 49 & -4 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1 & 8 \\ 3 & 49 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 7 \\ 3 & -4 \end{vmatrix}}$$

$$= \frac{x}{-375} = \frac{y}{25} = \frac{1}{-25}$$

$$x = 15, \; y = -1$$

(15 , −1) …………(A)

A(1 , 1)

Intersection of $7x-y=6$ & $3x-4y=49$

$$\frac{x}{\begin{vmatrix} 6 & -1 \\ 49 & -4 \end{vmatrix}}=\frac{y}{\begin{vmatrix} 7 & 6 \\ 3 & 49 \end{vmatrix}}=\frac{1}{\begin{vmatrix} 7 & -1 \\ 3 & -4 \end{vmatrix}}$$

$$=\frac{x}{25}=\frac{y}{325}=\frac{-1}{25}$$

$=x=1,\ y=-13$ ……………… $(1,-13)$ (B)

Case : I

A(1 , 1)

$\text{P}\equiv(15,-1)$, B $(1,\ -13)$

Then Q $\equiv(x,\ y)$

Mid – point PQ = mid- point of AB

$\therefore 15+x=1+1$

$x=-13$

and

$-1+y=1-13$

$\therefore y=-11$

So $\text{Q}(-13,-11)$

One possibility A (1 , 1) , B$(1,-13)$, P$(15,-1)$, Q$(-13,-11)$

Case : II

A$(1,1)$, P $(1,-13)$, B $(15,-1)$

Let Q $=(x,y)$

Mid – point PQ = Mid – point AB

$\therefore 1+x=15+1$

$x=15$

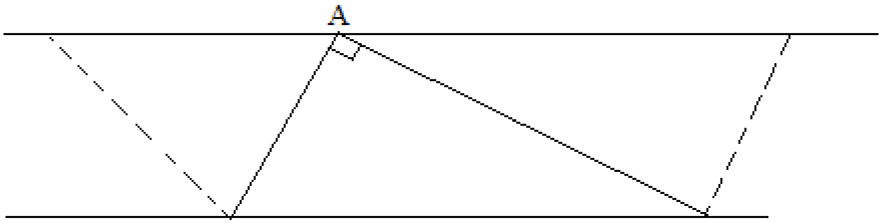
and

$y-13=1-1$

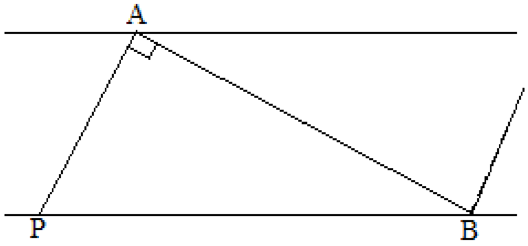
$y=13$

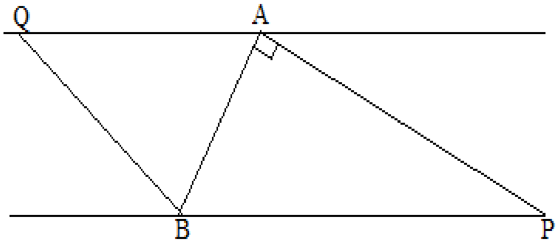
Q $(15,13)$

A $(1,1)$, P $(1,-13)$, Q $(15,13)$, B $(15,-1)$. There are 2 parallelograms possible.



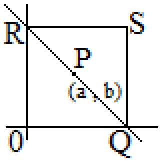
Represented by the dotted lines





Q.59

Q.60



Let line be

$$\frac{x}{\alpha}+\frac{y}{\beta}=1$$

$Q(\alpha\ ,\ 0)\quad and\quad S\equiv(\alpha\ ,\ \beta)$

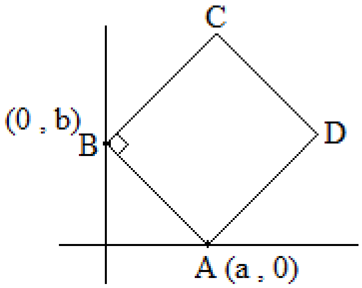
$R(0,\beta)$

Now $P(a,b)$ satisfies the line so

$$\frac{a}{\alpha}+\frac{b}{\beta}=1$$

$$\boxed{\frac{a}{x}+\frac{b}{y}=1}$$

Q.61



Rotate A anticlockwise about B to get

co – ordinates of C

$$C\equiv(a-ib)\,(0+i)+\,0+ib$$

$$C\equiv(ai+b)+ib$$

$$C\equiv b+i(a+b)$$

$$C\equiv(b,a+b)$$

$\therefore$ Mid – point of $AC=\left(\frac{a+b}{2}\,,\frac{a+b}{2}\right)$

$\therefore$ Mid – point of AC lies on the line $\boxed{y=x}$

Q.62

Condition for co – cyclic points $m_1m_2=1$

$x-2y+3=0 \qquad m_1=\frac{1}{2}$

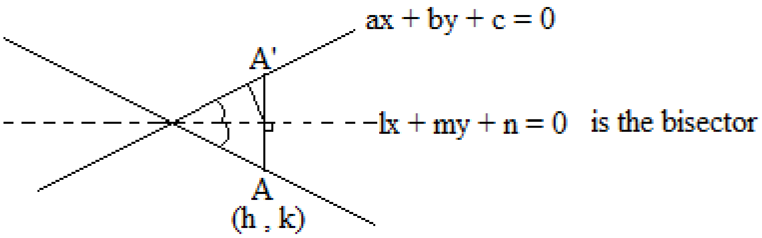
$y=kx+1 \qquad m_2=k$

$\therefore m_1m_2=1$

$$\frac{k}{2}=1$$

$\boxed{k = 2}$

Q.63



Reflection of A (h , k) in lx + my + n = 0 gives A'

$$\frac{x-h}{l}=\frac{y-k}{m}=\frac{-2(lh+my+n)}{l^2+m^2}$$

Let $\frac{lh+mk+n}{l^2+m^2} = p$

$x = h - 2pl$

$y = k - 2pm$

$\because$ A' lies on $ax+by+c=0$

$\therefore a(h-2pl)+b(k-2pm)+c=0$

$ah+bk+c = p(2al+2bm)$

$$\therefore ah+bk+c=\left(\frac{lh+mk+n}{l^2+m^2}\right)^2(al+bm)$$

$\therefore$ locus is

$$\boxed{(l^2+m^2)(ax+by+c)=2(al+bm)(lx+my+n)}$$

Q.64

$L_1 : 2x - y = 5$

$L_2 : 3x - y = 8$

Point P intersection given by

$$\frac{x}{\begin{vmatrix} 5 & -1 \\ 8 & -1 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix}}$$

$$\frac{x}{3} = \frac{y}{1} = \frac{1}{1}$$

$x = 3 \quad y = 1 \qquad P \equiv (3\ ,\ 1)$

L_2 rotated $\frac{\pi}{4}$ clockwise so $m = 3$

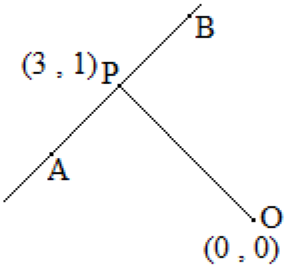
New slope $= \dfrac{3 + \tan\alpha}{1 - m\tan\alpha}$ ....$(\alpha = 45^\circ)$

$$= \frac{3+1}{1-3(1)} = \frac{4}{-2} = -2$$

$\therefore$ Equation of $L_2 : y - 1 = -2(x-3)$

$: y - 1 + 2x - c = 0$

$: y + 2x = 7$



$\mathrm{OP} = \sqrt{10}$ units

slope of L_2 is -2

$$\sin\theta = \frac{2}{\sqrt{5}}$$

$$\cos\theta = \frac{-1}{\sqrt{5}}$$

$$r = \pm\sqrt{10}$$

$$\therefore \quad x = 3 \pm \sqrt{10}\left(\frac{-1}{\sqrt{5}}\right)$$

$$y = 1 \pm \sqrt{10}\left(\frac{2}{\sqrt{5}}\right)$$

$$x = 3 \mp \sqrt{2}$$

$$y = 1 \pm 2\sqrt{2}$$

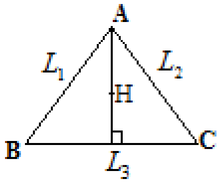
A $(3-\sqrt{2}, 1+2\sqrt{2})$ or B $(3+\sqrt{2}, 1-2\sqrt{2})$

Q.65

$$L_k = x\cos\theta_k + y\sin\theta_k - p_k = 0$$

$$k = 0, 1, 2, 3$$

$$L_1\cos(\theta_2 - \theta_3) = L_2\cos(\theta_3 - \theta_1) = L_3\cos(\theta_1 - \theta_2)$$



slope of BC $= -\cot\theta_3$

Family of lines through A is $L_1 + \lambda L_2 = 0$

slope of AH : $x(\cos\theta_1 + \lambda\cos\theta_2) + (\sin\theta_1 + \lambda\sin\theta_2) = p_1 + \lambda p_2$

$$\therefore \text{ slope of AH } \equiv \frac{-(\cos\theta_1 + \lambda\cos\theta_2)}{\sin\theta_1 + \lambda\sin\theta_2}$$

$\therefore$ slope AH × slope BC $= -1$

$$\therefore \frac{(\cos\theta_1 + \lambda\cos\theta_2)}{(\sin\theta_1 + \lambda\sin\theta_3)}\cot\theta_3 = -1$$

$$\lambda(\cos\theta_2\cot\theta_3 + \sin\theta_3) = -(\sin\theta_1 + \cos\theta_1\cot\theta_3)$$

$$\lambda = \left(\frac{\sin\theta_1\sin\theta_3 + \cos\theta_1\cos\theta_3}{\cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3}\right)(-1)$$

$$\lambda = \frac{-\cos(\theta_3 - \theta_1)}{\cos(\theta_2 - \theta_3)}$$

$\therefore$ Equation of AH is

$$L_1 + \lambda L_2 = 0$$

i.e.

$$L_1 - \frac{\cos(\theta_3 - \theta_1)}{\cos(\theta_2 - \theta_3)} L_2 = 0$$

$L_1 \cos(\theta_2 - \theta_3) = L_2 \cos(\theta_3 - \theta_1)$ and H satisfies this

Similarly

$$L_2 \cos(\theta_3 - \theta_1) = L_3 \cos(\theta_1 - \theta_2)$$

Can be proves using slope BH × slope AC $= -1$

Q.66

$y = x \tan\theta_1$ meets the circle $x^2 + y^2 = a^2$ at $(a\cos\theta_1, a\sin\theta_1)$ at A (solving simultaneously)

A $\equiv (a\cos\theta_1, a\sin\theta_1)$ similarly we get

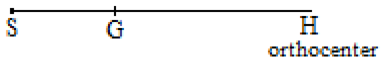
B $\equiv (a\cos\theta_2, a\sin\theta_2)$

C $\equiv (a\cos\theta_3, a\sin\theta_3)$

Circumcentre is the origin S(0 , 0)

Centroid co – ordinates

$$G \equiv \left(\frac{\sum a\cos\theta i}{3}, \frac{\sum a\sin\theta i}{3} \right)$$

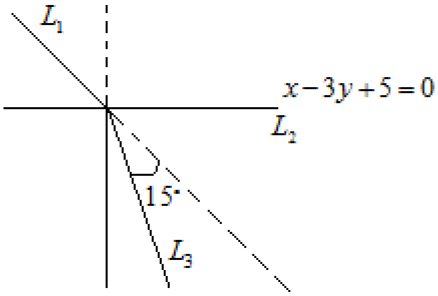


S , G and H are collinear

Equation of SG is $$y - 0 = \left(\frac{\frac{\sum a\sin\theta}{3} i - 0}{\frac{\sum a\cos\theta}{3} i - 0} \right)(x - 0)$$

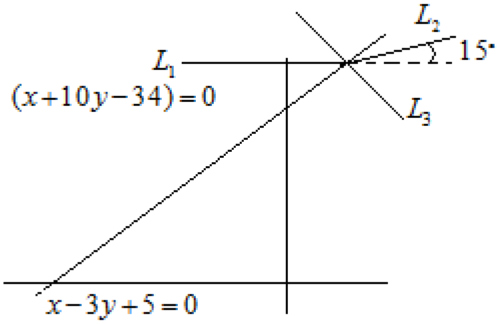
Equation of SG is $y = \left(\dfrac{\sum \sin\theta i}{\sum \cos\theta i} \right) x$ and H lies on this line

Q.67



Acute angle between L_1 & L_2 will be less than the acute angle between L_3 & L_2.

To get the slope of L_3 rotate L_1 clockwise by 15°



rotation is 15° clockwise

$$m_3 \text{ (slope of } L_3\text{)} = \frac{m - \tan\alpha}{1 + m\tan\alpha} = \left[\frac{\frac{-1}{10} - (2\sqrt{3})}{1 + \left(\frac{-1}{10}\right)(2-\sqrt{3})} \right]$$

$$= \frac{-1 - 20 + 10\sqrt{3}}{10 - 2 + \sqrt{3}}$$

$$= \frac{10\sqrt{3} - 21}{8 + \sqrt{3}}$$

$$m_3 = \frac{(10\sqrt{3} - 21)\ (8 - \sqrt{3})}{61}$$

$$m_3 = \frac{-30-168+21\sqrt{3}+80\sqrt{3}}{61}$$

$$m_3 = \frac{101\sqrt{3}-198}{61}$$

Point N is given by intersection of

$x+10y=34$ and $x-3y=-5$

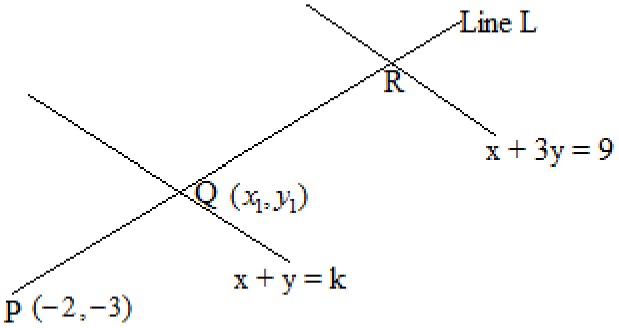
$$\frac{x}{\begin{vmatrix} 34 & 10 \\ -5 & -3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1 & 34 \\ 1 & -5 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 10 \\ 1 & -3 \end{vmatrix}}$$

$$\frac{x}{-52} = \frac{y}{-39} = \frac{1}{-13}$$

x = 4 , y = 3   N(4 , 3)

Equation of L_3 is $(y-3) = \left(\frac{101\sqrt{3}-198}{61}\right)(x-4)$

Q.68



Parametric form of Line

$x = -2 + r\cos\theta$

$y = -3 + r\sin\theta$

For point Q Let PQ $= r_1$

$\therefore x_1 = -2 + r_1\cos\theta$

$y_1 = -3 + r_1 \sin\theta$

Also $x_1 + y_1 = k$

$\therefore -2 + r_1 \cos\theta - 3 + r_1 \sin\theta = k$

$r_1 (\sin\theta + \cos\theta) = k + 5$

$$r_1 = \frac{k+5}{(\sin\theta + \cos\theta)}$$

For point R let PR $= r_2$

$x_2 = -2 + r_2 \cos\theta$

$y_2 = -3 + r_2 \sin\theta$

$x_2 + 3y_2 = 9$

$-2 + r_2 \cos\theta + 3(-3 + r_2 \sin\theta) = 9$

$r_2 (\cos\theta + 3\sin\theta) = 20$

$$r_2 = \frac{20}{(\cos\theta + 3\sin\theta)}$$

$r_1 r_2 = 20$

$$\therefore \frac{k+5}{(\sin\theta + \cos\theta)(\cos\theta + 3\sin\theta)} = 1$$

$k + 5 = (\sin\theta + \cos\theta)(\cos\theta + 3\sin\theta)$

If $\mathrm{k} = -\ 2$

We get

$3 = 3\sin^2\theta + \cos^2\theta + 4\sin\theta\cos\theta$

$\quad = 1 + 2\sin^2\theta + 4\sin\theta\cos\theta$

$2 = 2\sin^2\theta + 4\sin\theta\cos\theta$

$\therefore\ 2\cos^2\theta = 4\sin\theta\cos\theta$

$\cos\theta = 0$ i.e. $\theta = 90^\circ$

OR

$2\cos\theta = 4\sin\theta$

$\tan\theta = \frac{1}{2}$

Equation is $x = -2$ OR $(y+3) = \frac{1}{2}(x+2)$

$2y + 6 = x + 2$

$\boxed{x = -2\ OR\ 2y - x + 4 = 0}$

Now,

$k + 5 = (\sin\theta + \cos\theta)(\cos\theta + 3\sin\theta)$

If $\cos\theta \neq 0$ then divide by $\cos^2\theta$

$(k+5)(\sec^2\theta) = (1 + \tan\theta)(1 + 3\tan\theta)$

$(k+5)(1 + \tan^2\theta) = 1 + 4\tan\theta + 3\tan^2\theta$

For unique line $\tan\theta$ must be unique so roots must be equal

$\therefore (k + 5 - 3)\tan^2\theta - 4\tan\theta + (k+4) = 0$

$(k+2)\tan^2\theta - 4\tan\theta + (k+4) = 0$

$$\tan\theta = \frac{4 \pm \sqrt{16 - 4(k+2)(k+4)}}{2(k+2)}$$

If D = 0 value is unique

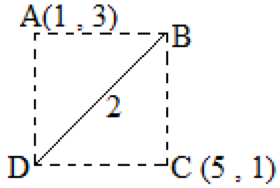
$4(k^2 + 6k + 8) = 16$

$k^2 + 6k + 4 = 0$

$$k = \frac{-6 \pm \sqrt{36-16}}{2}$$

$$\boxed{k = -3 \pm \sqrt{5}}$$

Q.69



Slope of AC is

$$\frac{3-1}{1-5} = \frac{-1}{2}$$

$\therefore$ AC and BD are perpendicular

ABCD is a square.

Rotate AC clockwise 45° scale $\frac{1}{\sqrt{2}}$ to get D (say)

$$C \equiv (4-2i)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1-i}{\sqrt{2}}\right) + \left(1+3i\right)$$

$$C \equiv (2-i)(1-i) + (1+3i)$$

$$C \equiv 2 - i - 2i - 1 + 1 + 3i$$

$$C \equiv 1 - 3i + 1 + 3i$$

C is (2 , 0)

Rotate AC anticlockwise 45° scale $\frac{1}{\sqrt{2}}$ to get B (say)

$$B \equiv (4-2i)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1+i}{\sqrt{2}}\right) + \left(1+3i\right)$$

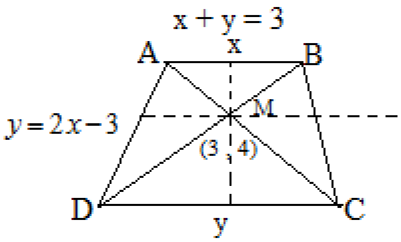
$$B \equiv (2-i)(1+i) + (1+3i)$$

$B \equiv 2 - i + 2i + 1 + 1 + 3i$

$B \equiv 4 + 4i$

$B \equiv (4, 4)$

Q.70



$\frac{XM}{MY} = \frac{1}{2}$ line parallel to x + y = 3 passing through (3 , 4) is x + y = 7

Let equation of CD : x + y = C

length $MY = \left|\frac{C-7}{\sqrt{2}}\right| = \frac{C-7}{\sqrt{2}} \;\; (C > 7)$

$$MX = \left|\frac{4}{\sqrt{2}}\right| = \frac{4}{\sqrt{2}}$$

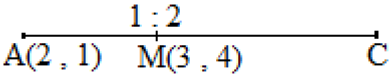
$$\frac{MX}{MY} = \frac{4}{C-7} = \frac{1}{2}$$

∴ Equation of x + y = 15

Co – ordinates of A

A : intersection of

$$\left.\begin{array}{l} x + y = 3 \\ 2x - y = 3 \end{array}\right\} \qquad \begin{array}{l} x = 2 \\ y = 1 \end{array}$$



$$\frac{AC}{CM} = \frac{-3}{2}$$

$$C \equiv \left(\frac{-9+4}{-3+2} , \frac{-12+2}{-3+2} \right)$$

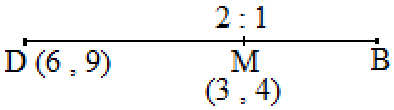
$$C \equiv (5,10)$$

D is intersection of

2x – y = 3 and x + y = 15

$\therefore$ x = 6 , y = 9

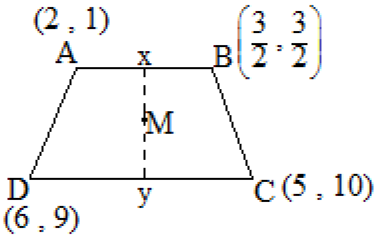
$$D \equiv (6,9)$$



DB : BM = 3 : (– 1)

$$B \equiv \left(\frac{9-6}{3-1} , \frac{12-9}{3-1} \right)$$

$$B \equiv \left(\frac{3}{2} , \frac{3}{2} \right)$$



Equation of BC is

$$y-10 = \left(\frac{10-\frac{3}{2}}{5-\frac{3}{2}} \right)(x-10)$$

$$y-10 = \frac{17}{7}(x-10)$$

BC : $\boxed{7y+10=17x}$

$|AB| = \frac{1}{\sqrt{2}}$ units

$|DC| = \sqrt{2}$ units

$|MX| = 6\sqrt{2}$ units

$$\text{Area } = \frac{1}{2}|MX| \times (|AB| + |DC|)$$

$$= \frac{1}{2} 6\sqrt{2} \times \left(\frac{3}{\sqrt{2}}\right) = 9$$

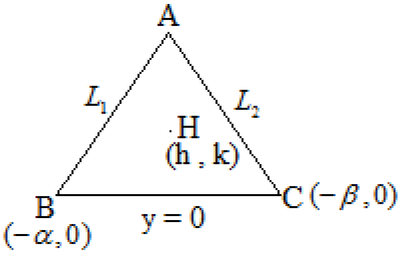
$\boxed{\text{A } = \text{ 9 sq units}}$

Q.71

Let $t = \alpha$; β represent the 2 sides.

$(1+\alpha)x + \alpha y \ = \ -\alpha(1+\alpha) \ \ (L_1)$

$(1+\beta)x + \beta y \ = \ -\beta(1+\beta) \ \ (L_2)$



B and C are the x - intercepts of L_1 & L_2

slope of L_1 is $\frac{-(1+\alpha)}{\alpha}$

slope of L_2 is $\frac{-(1+\beta)}{\beta}$

$\therefore$ CH perpendicular L_1

$$\therefore \frac{k-0}{h+\beta} = \frac{\alpha}{1+\alpha}$$

$k(1+\alpha)-h\alpha=\alpha\beta$ ………..(i)

Similarly using CH perpendicular to L_2 we get

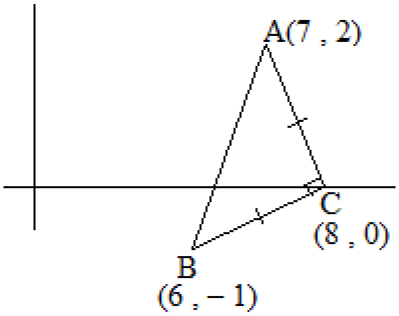
$k(1+\beta)-h\beta=\alpha\beta$

Solving we get $h=k=\alpha\beta$

$\therefore$ the orthocenter lies on the line

$\boxed{y=x}$

Q.72



AC = BC $=\sqrt{5}$ units. And AB = $=\sqrt{10}$ units.

Δ ABC is a right angled isosceles triangle

Equation of AC is $y-0=\left(\frac{0-2}{8-7}\right)(x-8)$

AC : y + 2x = 16

Equation of BC is $y-0=\frac{(0+1)}{8-6}(x-8)$

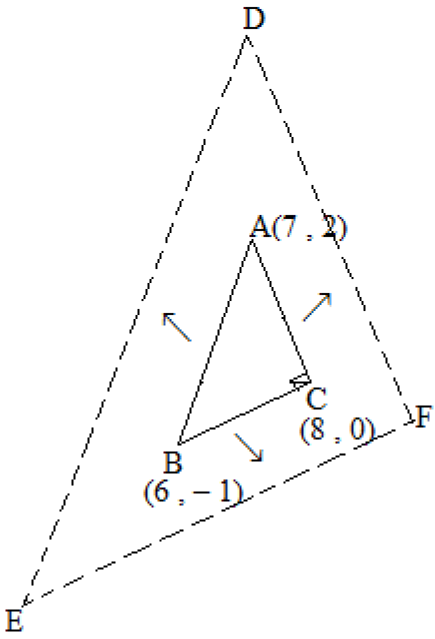
BC : x – 2y = 8

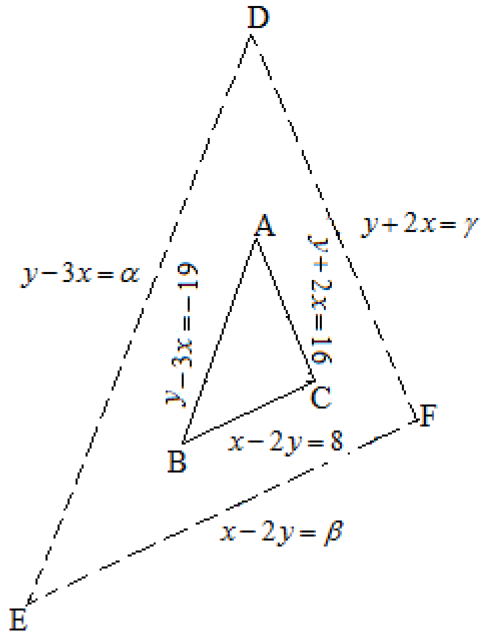
Equation of AB is $y+1=\left(\frac{2+1}{7-6}\right)(x-6)$

$$y+1=3(x-6)$$

AB : $y-3x=-19$

$\Delta ABC \;\square\; \Delta DEF$





distance BC and EF is $\left|\frac{\beta-8}{\sqrt{5}}\right| = \sqrt{5}$

$\therefore \beta = 8 \pm 5$

$\beta = 3 \; or \; \beta$

β we take as $-\beta$ (decreasing y – intercept of EF) $\beta = \beta$

distance AB & DE is $\left|\frac{\alpha+19}{\sqrt{10}}\right| = \sqrt{10}$

$\therefore \alpha = -19 \pm 10$

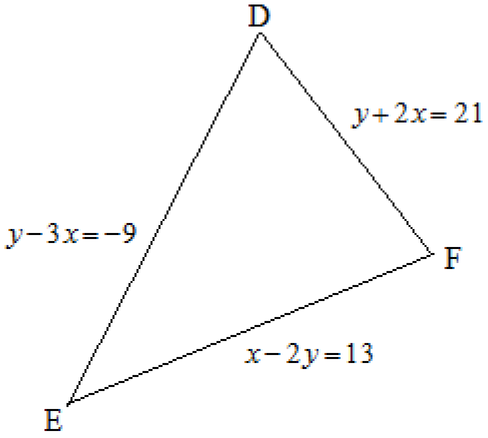
$\alpha = -9 \ or \ -29$

We take $\alpha = -9$ $\therefore$ DE is shifted upward so y – intercept is decreasing $\alpha = -9$

distance of DF to AC is $\left|\frac{\gamma-16}{\sqrt{5}}\right| = \sqrt{5}$

$\therefore \gamma = 11 \ or \ 21$

$\gamma = 21 \because$ DF y – intercept in increasing $\gamma = 21$

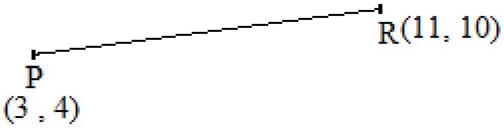


D ≡ (6, 9)

F ≡ (11, – 1)

E ≡ (1, 6)

Q.73



slope of PR = $= \frac{10-4}{11-3} = \frac{6}{8} = \frac{3}{4}$

$$\tan\theta = \frac{3}{4}$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

$$\therefore\ x = 3 + r\cos\theta$$

$$\left.\begin{aligned} x &= 3 + \frac{4r}{5} \\ y &= 4 + \frac{3r}{5} \end{aligned}\right\} \qquad \text{parametric Equation of PR}$$

$$|PQ| = 2.5 \qquad \therefore r = \pm 25$$

$$x = 3 \pm 4\frac{(2.5)}{5}$$

$$\Rightarrow x = 3 \pm 2$$

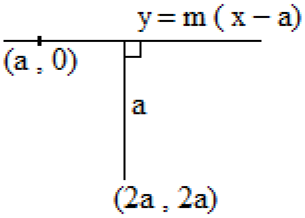
$$\Rightarrow x = 5 \ or\ 1$$

$$y = 4 \pm 3\frac{(2.5)}{5}$$

$$\Rightarrow y = 5.5 \quad or\ 2.5$$

$$\therefore Q \equiv \left(5, \frac{11}{2}\right)\ or\ \left(1, \frac{5}{2}\right)$$

Q.74



Let this be equation of the line y – mx + ma = 0

$$\left|\frac{2a-2ma+ma}{\sqrt{1+m^2}}\right| = a$$

$$\left|\frac{2a-ma}{\sqrt{1+m^2}}\right| = a$$

Squaring we get

$$(2a-ma)^2 = a^2(1+m^2)$$

$$4a^2+m^2a^2-4ama = a^2+a^2m^2$$

$$4a^2 = 4a^2m$$

$$\therefore m = 1$$

$\therefore$ equation of line is $y = x-a$

Q.75

$$9a^2+b^2+6ab-c^2 = 0$$

$$(3a+b)^2-c^2 = 0$$

$$\therefore (3a+b-c)(3a+b+c) = 0$$

$\therefore$ Either 3a + b – c = 0 OR

3a + b + c = 0

Case I : 3a + b – c = 0 $\Rightarrow c = 3a+b$

The line ax + by + c = 0

$$\Rightarrow ax+by+3a+b = 0$$

$$a(x+3)-b(y+1) = 0$$

Always passes through $(-3,-1)$

Case II : $3a+b+c = 0$ $\Rightarrow c = -3a-b$

The line $ax+by+c = 0$

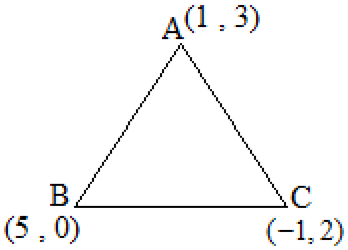
$$\Rightarrow ax+by-3a-b = 0$$

$\Rightarrow a(x-3)+b(y-1)=0$

Always passes through (3 , 1)

$\therefore$ Either line passes through $(-3,-1)$ or (3 , 1) or both.

Q.76



Equation of AB : $y-0=\left(\dfrac{3-0}{1-5}\right)(x-5)$

– 4y = 3 (x – 5)

– 4y = 3x – 15

15= 3x + 4y

c) y – 2x for a is 3 – 2 = 1

y – 2x for B is 0 – 5 = – 5

y – 2x for C is 2 + 2 = 4

A & C positive B negative so this line passes through the interior of the triangle (y – 2x = 0).

So not always true.

c) is false.

For A : 2x + 3y = 2 + 9 = 11

For B : 2x + 3y = 10 + 0 = 10

For C : 2x + 3y = – 2 + c = 4

So positive for all points A , B , C. $\therefore$ it will be positive for all points inside

a) is true

b) For A : 2x – 3y is 2 – 9 = – 7

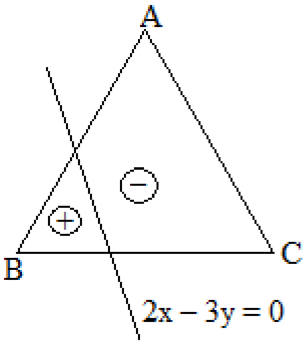
for B : 2x – 3y is 10

C : 2x – 3y is – 2 – 6 = – 8

So for A & C it is negative for B it is positive.

So A & C lie on one side and B lies on the other side.

So the line passé as shown.

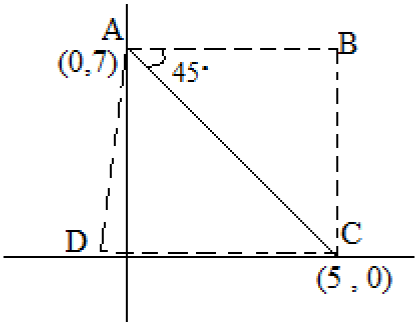


$\therefore$ not always true

b) is false.

Q.77

$\frac{x}{5}+\frac{y}{7}=1$



Rotate AC by 45° anticlockwise scale by $\frac{1}{\sqrt{2}}$ to get B.

Co – ordinates of B

$$(5-0+0-7i)\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}+(0+7i)$$

$$= (5-7i)\frac{(1+i)}{2}+7i$$

$$= \left(\frac{5-7i+5i+7}{2}\right)+7i$$

$$= 6-i+7i$$

$= 6+6i$ B (6 , 6)

Co – ordinates of D : rotate by 45° clockwise scale by $\frac{1}{\sqrt{2}}$ to get D

$$(5-0+0-7i)\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}+(0+7i)$$

$$= (5-7i)\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}+7i$$

$$= \frac{(5-7i)}{2}(1-i)+7i$$

$$= \frac{5-7i-5i-7}{2} + 7i$$

$$= -6i-1+7i$$

$$= -1+i$$

$$D \equiv (-1,1)$$

$$\boxed{\begin{array}{l} B(6\ ,\ 6) \\ D(-1,1) \end{array}}$$

Q.78

$$(2\cos\theta+3\sin\theta)x + (3\cos\theta-\sin\theta)y = 5\cos\theta-2\sin\theta$$

$\therefore$ divide by $\cos\theta$ we get $\cos\theta \neq 0$

$$(2+3\tan\theta)x+(3-5\tan\theta)y = 5-2\tan\theta$$

$$\therefore (2x+3y-5)+ (3x-5y+2)\tan\theta = 0$$

Represents family of straights passing through the intersection of

$(2x+3y-5)0 \qquad \& \qquad (3x-5y+2)=0$

$2x+3y-5=0$

$3x-5y+2=0$

It of intersection.

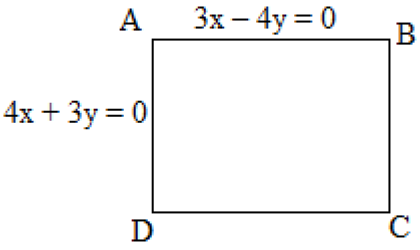
$$\frac{x}{\begin{vmatrix} 5 & 3 \\ -2 & -5 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 5 \\ 3 & -2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 3 \\ 3 & -5 \end{vmatrix}}$$

$$\frac{x}{-19}=\frac{y}{-19}=\frac{1}{-19}$$

$\therefore \; x=1, y=1$

$\boxed{\text{point of intersection is } (1,1)}$

Q.79



Area = 25 units sq.

Side = 5 unit

Distance between AB & CD is 5 unit

Equation of CD is of the form

3x – 4y + c = 0 (∵ CD ++ AB)

Distance AB and CD is

$$=\left|\frac{C-0}{\sqrt{3^2+4^2}}\right|=5$$

$\therefore |C| = 25$

$\therefore C = \pm 25$

$\therefore$ Equation of CD is $3x - 49 \pm 25 = 0$

similarly Equation of BC is of the form 4x + 3y + C’ = 0

distance between AD & BC is

$$\left| \frac{C' - 0}{\sqrt{4^2 + 3^2}} \right| = 5$$

$|C'| = 25$

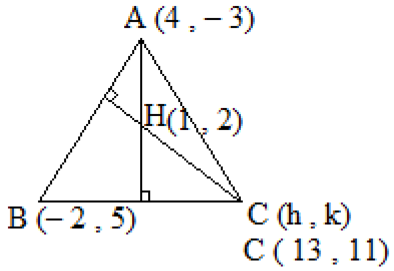
$\therefore C' = \pm 25$

$\therefore$ Equation of BC is $4x + 3y \pm 25 = 0$

Q.80

A (4, – 3)

B(– 2, 5)



AH perpendicular BC

So $\left(\frac{-3-2}{4-1}\right)\left(\frac{k-5}{h+2}\right) = -1$

$$\left(\frac{-5}{3}\right)\left(\frac{k-5}{h+2}\right) = -1$$

$5(k-5) = (h+2)3$

$5k - 3h = 31$ …………(i)

CH perpendicular to AB

$$\therefore\left(\frac{-3-5}{4+2}\right)\left(\frac{k-2}{h-1}\right)=-1$$

$$\frac{-4}{3}\left(\frac{k-2}{h-1}\right)=-1$$

$$4(k-2)=3(h-1)$$

$4k-3h=5$ …………..(ii)

$$\frac{k}{\begin{vmatrix}31 & -3\\ 5 & -3\end{vmatrix}}=\frac{h}{\begin{vmatrix}5 & 31\\ 4 & 5\end{vmatrix}}=\frac{1}{\begin{vmatrix}5 & -3\\ 4 & -3\end{vmatrix}}$$

$$\therefore\ h=33$$

$$\therefore\ \frac{k}{-78}=\frac{h}{-99}=\frac{1}{-3}$$

$$k=26$$

$$h=33$$

$$C\,(33,26)$$

$$\therefore\ \text{centroid}\quad G\equiv\left(\frac{4-2+33}{3}\ ,\ \frac{5-3+26}{3}\right)$$

$$\boxed{G\equiv\left(\frac{35}{3},\frac{28}{3}\right)}$$