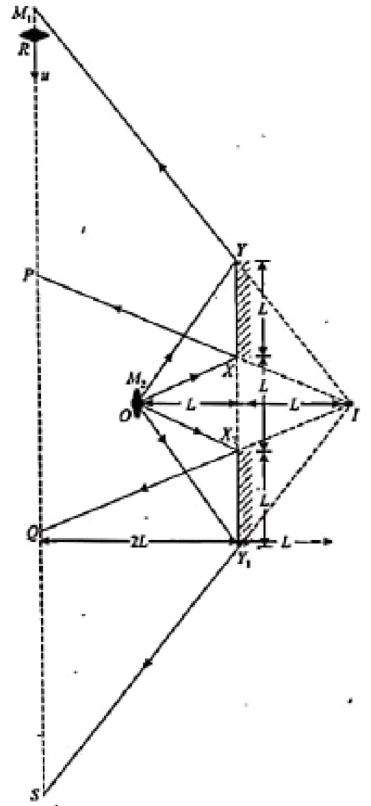
**IIT – JEE: 2024** | **TW TEST (ADV)** | **DATE: 26/02/23**

**TOPIC: RAY OPTICS**

## SOLUTIONS

1. (C)

When the man $M_1$ is in the region RP and SQ, then man $M_1$ sees the image of $M_2$.




From the similar triangle PIQ and $XIX_1$

$\Rightarrow \quad \frac{PQ}{XX_1} = \frac{3L}{L} \Rightarrow PQ = 3L$

From the similar triangle, RIS and PIQ

$\Rightarrow \quad \frac{RS}{YY_1} = \frac{3L}{L} \qquad \Rightarrow RS = 9L$

Then $RP + QS = RS - PQ = 9L - 3L = 6L$

$\Rightarrow \qquad \text{Time} = \frac{6L}{u}$

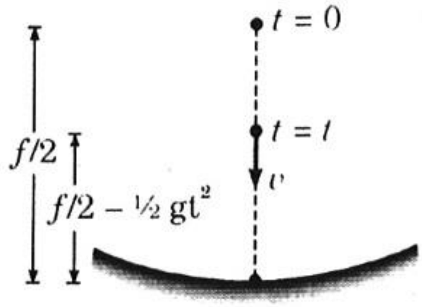
2. (B)

In the situation given in question, we can see that after third reflection the reflected ray becomes parallel to mirror $M_2$ after which no more reflections will take place. Thus light ray can reflect maximum three times in this case.

3. (A)

Velocity of particle after time '$t$' is given by

$$\upsilon = gt$$




Distance travelled in time '$t$' is given by

$$S = \frac{1}{2}gt^2$$

Now, we know that

$$\vec{V}_{I/M} = -m^2\vec{V}_{O/M}$$

Here, $\vec{V}_{O/M} = gt$

Also, $m = \frac{f}{f-u} = \frac{-f}{-f+\left(\frac{f}{2} - \frac{gt^2}{2}\right)} = \frac{2f}{f+gt^2}$

$\therefore$ $V_I$ = Velocity of image

$$= -\left(\frac{2f}{f+gt^2}\right)^2 .gt$$

$$= \frac{4f^2gt}{(f+gt^2)^2}$$ (–ve sign indicates that image will move upward)

For maximum speed of image

$$\frac{d\upsilon_1}{dt} = 0$$

$\Rightarrow$ $t = \sqrt{\frac{f}{3g}}$

$\upsilon_{max} = \frac{3}{4}\sqrt{3fg}$ **Ans.**

4. (C)

Using refraction formula for air–glass interface, we use

$u = -x; R = +10cm; \mu_1 = 1$ and $\mu_2 = 3/2$

By refraction formula, we use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$\Rightarrow$ $\frac{\mu_2}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10}$

$\Rightarrow$ $x = -20cm$

As the second surface is flat, rays must become parallel after first refraction only as from flat surface rays will not suffer any deviation when falls normally.

5. (A)

Ray diagram of the grazing emergence is shown in figure.

Here we have $r + \theta_c = A$

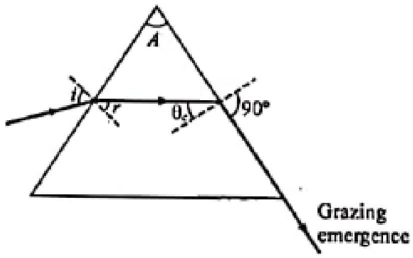
$\Rightarrow \quad r + \theta_c = 60$

Using Snell's law at the second surface, we have

$$\sqrt{2}\sin\theta_c = 1\sin 90$$

$\Rightarrow \quad \sin\theta_c = \frac{1}{\sqrt{2}}; \theta_c = 45^o$

Then, we have $\qquad r = A - \theta_c = 15^o$




Using snell's law at the Ist surface, we have

$$1\sin i = \sqrt{2}\sin r$$

And $\sin i = \sqrt{2}\sin 15$

$\Rightarrow \quad i = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

(As $\sin 15^o = \frac{\sqrt{3}-1}{2\sqrt{2}}$ )

6. (BC)

A plane mirror always produces an image of nature opposite to that of object and the nature of rays after reflection on a plane mirror remain same as that of before incidence hence options (B) and (C) are correct.

7. (BCD)

This is a case of displacement method experiment in which we have studied that the object height is given as

$$S_0 = \sqrt{I_1 I_2} = \sqrt{9 \times 4} = 6\,\text{cm}$$

Magnification in first situation of lens is

$$m = \frac{3}{2}$$

$\Rightarrow \quad \frac{v}{u} = \frac{3}{2} \quad \Rightarrow \quad v = \frac{3}{2}u$

And $v + u = 90$

$\Rightarrow \quad \frac{5}{2}u = 90 \quad \Rightarrow \quad u = 36\,\text{cm}$

By lens formula, we have

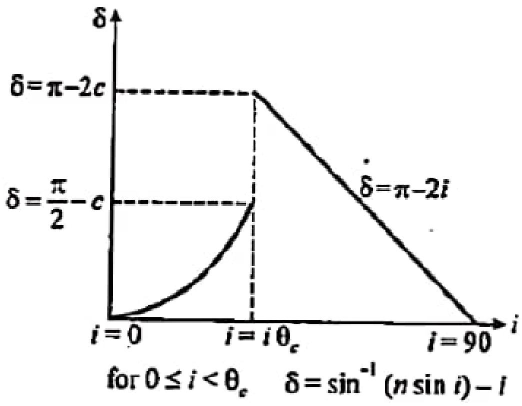
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{54}-\frac{1}{(-36)}=\frac{1}{f} \qquad \Rightarrow \qquad f=21.6\,\text{cm}$$

8. (B)
Cutting a lens in transverses direction doubles their focal length i.e. 2f.
Using the formula of equivalent focal length $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}$
We get equivalent focal length as $\frac{f}{2}$.

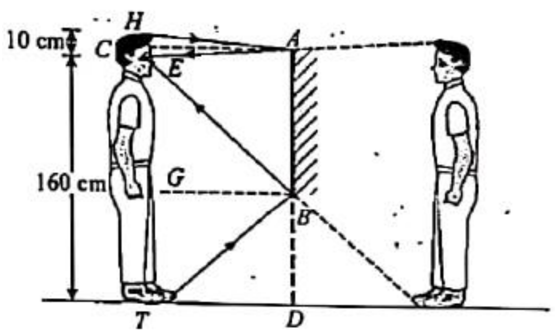
9. (BC)

10. (ABCD)
Shown in figure, explains how deviation angle varies with the incidence angle. With this figure we can analyse that all given options are correct.




11. (BC)
From figure we can see than

$$AB=\frac{HT}{2}=\frac{170}{2}=85\,\text{cm}$$

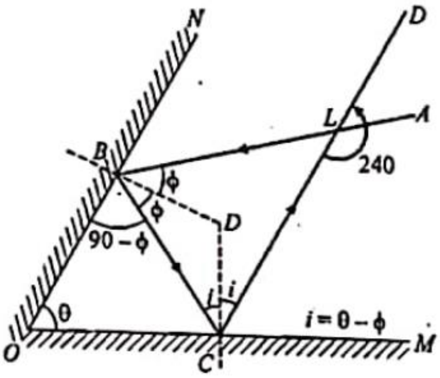



And $\Delta EBT$ we have $EG=GT=BD=\frac{160}{2}=80\,\text{cm}$.

12. (ABCD)
If the mirrors OM and ON make an angle $\theta$ with each other as shown in figure, then we have

$$\angle BLC=240-180=60^{o}$$

In $\Delta ABC$, we have




$$2\phi + 2(\theta - \phi) + 60^o = 180^o$$

$\Rightarrow \quad 2\phi + 2\theta - 2\phi + 60^o = 180^o$

$\Rightarrow \quad \theta = 60^o$

Number of images $= \left( \frac{360^o}{60^o} - 1 \right) = 5$

Hence all option are correct.

13. (BC)

We can write,

$$\frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \cot 30^o$$

$$= \sqrt{3} = 1.732.$$

Critical angle, $\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{3}}$

14. (BD)

Ray 1 and Ray 2 may have any angle between them

Similarly ray 5 and ray 6 may have any angle between them.

This depends on angle of incidence on first face.

15. (AC)

A concave or convex mirror is to be placed left of the object. The object and the image both will be real for concave mirror and virtual for convex mirror.

16. (10)

Final image coincides with the object when the image produced by lens is formed at centre of curvature of mirror or itself on the pole of mirror. So there are two possible conditions here.

For the lens, we use $u = -15\text{cm}$ and $f = +10\text{cm}$ so in lens formula, we use

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow \quad \frac{1}{v_1} - \frac{1}{-15} = \frac{1}{10}$

$\Rightarrow \quad v_1 = +30\,\text{cm}.$

Thus image produced must be at the centre of curvature of the mirror as it is not possible at the pole because the distance of pole to lens is only 10cm.

$\Rightarrow \quad R = 20\text{cm}$

$\Rightarrow \quad f = 10\text{cm}$

17. (10)

By lens makers formula, we can find the radius of curvature of the lens surface as

$$\frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{2R} = \frac{1}{10}$$

$$\Rightarrow \quad R = 5\text{cm}$$

For image to be obtained on object, light rays on mirror smust fall normally to retrace the path of incident rays after reflection.

So by refraction formula, we have

$$\Rightarrow \quad \frac{3/2}{\infty} - \frac{I}{-d} = \left(\frac{1}{-d} - \frac{1/2}{5}\right)$$

$$\Rightarrow \quad \frac{1}{d} = \frac{1}{10}$$

$$\Rightarrow \quad d = 10\text{cm}$$

18. (2)

For refraction formula at spherical surface, we use

$$v = 2R; u = \infty; \mu_1 = 1 \text{ and } \mu_2 = \mu$$

Using refraction formula, we have

$$\frac{\mu_2}{v} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{\mu}{2R} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\Rightarrow \quad \mu = 2\mu - 2$$

$$\Rightarrow \quad \mu = 2$$

19. (30)

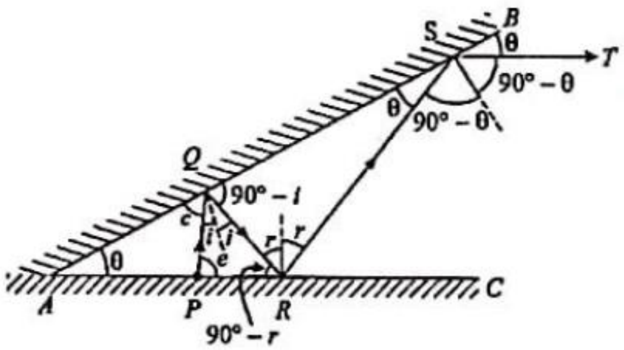
From the figure in triangle QRS, we have

$$90 - i + \theta + 2r = 180^o$$

$$\Rightarrow \quad r = \left(\frac{90 + i - \theta}{2}\right)$$

$$\Rightarrow \quad p = 180 - (90 - r) - 2i$$

$$\Rightarrow \quad p = 180 - 90 + \left(\frac{90 + i - \theta}{2}\right) - 2i$$




$$\Rightarrow \quad p=\frac{180-4i+90+i-\theta}{2}$$

$$\Rightarrow \quad p=\left(\frac{270-3i-\theta}{2}\right)$$

$$\Rightarrow \quad m=p-\theta=180-(90-i)-2i$$

$$\Rightarrow \quad \left(\frac{270-3i-\theta}{2}\right)-\theta=90-i$$

Substituting $\theta=20^{o}$, we get

$i=30^{o}$

20. (9)

Using formula for equivalent focal length of lens.

**IIT – JEE: 2023** **TW TEST (ADV)** **DATE: 26/02/23**

**TOPIC: CHEMICAL EQUILIBIRUM**

## SOLUTIONS

**21.** **(A)**

$N_2O_4\,(g) \rightleftharpoons 2NO_2\,(g)$

As dissociation increases M.M. decreases

**22.** **(B)**

$$\log_{10} K = -\frac{\Delta H^o}{2.303}\cdot\frac{1}{RT}+\frac{\Delta S^o}{2.303R}$$

For exothermic reaction, the slope should be positive and given $\Delta S^o < 0$ implies the y intercept is negative.

**23.** **(A)**

$K_P = (P_{H2O})^4$

$16 \times 10^{-12} = P_{H2O}$

$2 \times 10^{-3}$ atm $= P_{H2O}$

Saturated V.P = 7.6 torr = 0.01 atm

V = 1 lit

Mole of water absorbed = moles of water in saturated air – mole of water at equilibrium

$$= \frac{0.01\times1}{RT}-\frac{2\times10^{-3}\times1}{RT}=\frac{8\times10^{-3}}{RT}=3.55\times10^{-4}$$

Wt. of water absorbed = $3.55 \times 10^{-4} \times 18 = 6.4 \times 10^{-3}$ g

**24.** **(D)**

$$Ni^{2+} + 6NH_3 \rightleftharpoons Ni(NH_3)_6^{2+} \qquad K_4 = 6\times10^8$$

0.1 1

***L.R.***

***x*** $1 - 6(0.1)$ 0.1

$$6\times10^8 = \frac{(0.1)}{(0.4)^6(x)}$$

On Solving, **x** is roughly equal to $4 \times 10^{-8}$ mole/lit

**25.** **(D)**

$$AB \rightleftharpoons A^+ + B^-$$

$(1-x)-b$ $\quad x \quad$ $x-b$

$$AB \quad + \quad B^- \rightleftharpoons AB_2^-$$

$$(1-x)-b \quad x-b \quad\quad b$$

$$K_1 = \frac{x(x-b)}{(1-x)-b} \qquad K_2 = \frac{b}{[(1-x)-b](x-b)}$$

$$\frac{K_1}{K_2} = \frac{x(x-b)}{[(1-x)-b]} \times \frac{[(1-x)(x-b)]}{b}$$

$$\frac{K_1}{K_2} = \frac{x}{b}(x-b)^2$$

$$\frac{x}{b} = \frac{1}{(x-b)^2}$$

$$\frac{[A^+]}{[AB_2^-]} = \frac{1}{[B^-]^2}$$

**26. (AB)**

The reaction will not shift in any direction because Δn is zero. Moles of all component remain the same but because volume is reduced, their concentrations will increase. Also, because the reaction is not shifting in any direction, overall moles remain constant but increase in pressure means the total pressure at equilibrium will increase.

**27. (ABC)**

$$K_1 = \frac{[NO]^2}{[N_2][O_2]}$$

$$K_2 = \frac{[NO]}{[N_2]^{1/2}[O_2]^{1/2}}$$

$$K_3 = \frac{[N_2][O_2]}{[NO]^2}$$

$$K_4 = \frac{[N_2]^{1/2}[O_2]^{1/2}}{[NO]}$$

**28. (CD)**

Addition of solid species does not shift the reaction equilibrium.

Because the reaction is endothermic, it will move in forward direction upon increase in temperature. Also, increase in pressure shifts the reaction towards lower gaseous moles i.e. towards left.

**29. (ABCD)**

Conceptual

**30. (ABCD)**

Addition of inert gas at constant volume does not shift the equilibrium

**31.** **(CD)**

Upon increasing the volume of container (indirectly done by introducing an inert gas at constant pressure) will move the reaction forward because $\Delta n$ is positive.

**32.** **(C)**

The reaction equilibrium constant is dependent only on the temperature.
In this case, addition of $H_2$ shifts the reaction in backward direction.

**33.** **(AC)**

Adding hot carbon will have no practical effect as it solid in nature.
Also, reducing the volume will shift the reaction to lesser gaseous moles i.e. towards the left.

**34.** **(ABC)**

Adding inert gas at constant volume does not shift the equilibrium

**35.** **(BCD)**

A decrease in the pressure will move the reaction towards generation of more moles (in this case, to the right).
The backward reaction will be endothermic which favore by increase in temperature
Addition of products will also shift the reaction in backward direction

**36.** **(1)**

1 + (2 – 1)x = D/d
x = D/d –1 Thus, intercept = 1 i.e. at point A value of D/d = 1

**37.** **(8)**

K overall $= \frac{k_1}{k_2} \times \frac{k_3}{k_4}$

**38.** **(8)**

| | $A_2 \rightleftharpoons 2A;\ K_1 = x$ atm | |
|---|---|---|
| Initial partial pressure | 1 atm | 0 |
| Equ. Partial pressure | $1-(x+z)$ | $2x$ |

| | $B_2 \rightleftharpoons 2B;\ K_2 = y$ atm | |
|---|---|---|
| Initial partial pressure | 1 atm | 0 |
| Equ. Partial pressure | $1-(y+z)$ | $2y$ |

| | $A_2$ | + $B_2$ | $\rightleftharpoons$ $2AB;\ K_3 = 2$ | |
|---|---|---|---|---|
| Initial partial pressure | 1 | 1 | 0 | |
| Equ. Partial pressure | $1-(x+z)$ | $1-(y+z)$ | $2z = 0.5$ | …… (1) |

From question, $[1-(x+z)]+2x+[1-(y+z)]+2y+2z = 2.75$

$\therefore\ x + y = 0.75 \qquad ...(2)$

Now, $K_3 = \frac{(0.5)^2}{(0.75-x)(0.75-y)} = 2$

$y = 0.50$ or $0.25$

$$\therefore \quad \frac{K_2}{K_1} = \frac{\dfrac{(2y)^2}{1-(y+z)}}{\dfrac{(2x)^2}{1-(x+z)}} = \frac{(2y)^2 \times (0.75-x)}{(2x)^2 \times (0.75-y)} = \frac{1}{8} \text{ or } \frac{8}{1}$$

**39.** **(3)**

$3.6 \times 10^{-3}$ atm

**40.** **(3)**

**IIT – JEE: 2024** **TW TEST (ADV)** **DATE: 26/02/23**

**TOPIC: PROBABILITY**

## SOLUTIONS

41. (D)

Out of (2n + 1) tickets we can select 3 numbers in AP in $n^2$ number of ways

So number of favourable cases = 100

Total number of cases $C_3^{31}$

Required probability = $\frac{10}{133}$

42. (D)

A chess board is a square divided into 64 equal squares.

In 1st diagonal we have only 1 square

In 2nd diagonal we have only 2 squares

In 3rd diagonal we have 3 squares so selection can be done in ${}^3C_3$ ways

In 4 diagonal we have 4 squares so selection can be done in $C_3^4$ ways

And so on

Hence, the total number of ways in which 3 squares can be chosen

$2({}^3C_3 + {}^4C_3 + {}^5C_3 + {}^6C_3 + {}^6C_3 + {}^7C_3) + {}^8C_3$

[Note that we do not have $2.{}^8C_3$ ]

Hence the total number of favourable ways $m = 4({}^3C_3 + {}^4C_3 + {}^5C_3 + {}^6C_3 + {}^7C_3) + 2.{}^8C_3 = 392$.

And total number of ways $= {}^{64}C_3 = \frac{64.63.62}{1.2.3} = 32.21.62$

Hence the required probability

$= \frac{m}{n} = \frac{392}{32.21.62} = \frac{7}{744}$.

43. (C)

$E_1 =$ denotes selection for 1st bag

$E_2 =$ denotes selection for 2nd bag

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$

$A$ = selected balls are 1 red & 1 black

$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$

$P\left(\frac{A}{E_2}\right) = \frac{{}^3C_1 \times {}^2C_1}{{}^{(n+5)}C_2} = \frac{12}{(n+5)(n+4)}$

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{6}{(n+5)(n+4)}}=\frac{6}{11}$$

$\Rightarrow \quad n=4$

44. (C)

(A) $P(E_1)\cdot P(E_2)=\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}\neq P(E_1\cap E_2)$

(B) $P(E_1'\cap E_2')=1-P(E_1\cup E_2)$

$$=1-\left(P(E_1)+P(E_2)-P(E_1\cap E_2)\right)$$

$$=1-\left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right)=\frac{17}{24}$$

$P(E_1')P(E_2)=\frac{5}{6}\times\frac{1}{4}=\frac{5}{24}$

(C) $P(E_1\cap E_2')=P(E_1)-P(E_1\cap E_2)=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

(D) $P(E_1'\cap E_2)=P(E_2)-P(E_1\cap E_2)=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$

45. (B)

A = event that two defective machines are identified in first two tests out of four machines.

$\therefore \quad P(A)=\frac{{}^2C_2}{{}^4C_2}=\frac{1}{6}$.

46. (ABC)

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.8=0.6+0.4-P(A\cap B)$

$\therefore \quad P(A\cap B)=0.2$

P (A U B U C) $=(0.6+0.4+0.5)-(0.2+P(B(\cap C)+0.3)+0.2$

$=1.5-0.3-P(B\cap C)$

We know, $0.85\leq P(A\cup B\cup C)\leq 1$

or $\quad 0.85\leq 1.2-P(B\cap C)\leq 1$

$\therefore \quad 0.2\leq P(B\cap C)\leq 0.35$.

47. (BC)

At least one $\Rightarrow$ Any one, any two, all the three $=0.75=\frac{3}{4}$.

At least two $\Rightarrow$ Any two, all the three $=50\%=\frac{1}{2}$

Exactly two $\Rightarrow$ Any two $=40\%=\frac{2}{5}$.

$$\Sigma M(1-P)(1-C)+\Sigma MP(1-C)+MPC=\frac{3}{4} \quad \ldots(1)$$

$$\Sigma MP(1-C)+MPC=\frac{1}{2} \quad \ldots(2)$$

$$\Sigma MP(1-C)=\frac{2}{5} \quad \ldots(3)$$

Solving (2) and (3),

$$MPC=\frac{1}{10}\Rightarrow (c) \quad \ldots(4)$$

Solving (2) and (4),

$$M+P+C=\frac{27}{20}$$

48. (ACD)

Let $P(A)\text{and}P(B)$ denote the percentage of city population who read newspapers A and B. Then from given data, we have $P(A)=25\%=\frac{1}{4},\ P(B)=20\%=\frac{1}{5}$

$P(A\cap B)=8\%=\frac{2}{25}$.

$\therefore$ Percentage of those who read A but not B $=P(A\cap B)=P(A)-P(A\cap B)$

$$=\frac{1}{4}-\frac{2}{25}=\frac{17}{100}=17\%.$$

Similarly $P(\overline{A}\cap B)=P(B)-P(A\cap B)$

$$=\frac{1}{5}-\frac{2}{25}=\frac{3}{25}=12\%$$

If $P(C)$ denote the percentage of those who look into advertisement, then from the given data we obtain

$P(C)=30\%\text{ of }P(A\cap B)+40\%\text{ of }P(\overline{A}\cap B)+50\%\text{ of }P(A\cap B)$

$$=\frac{3}{10}\times\frac{17}{100}+\frac{2}{5}\times\frac{3}{25}+\frac{1}{2}\times\frac{2}{25}=\frac{51+48+40}{1000}=\frac{139}{1000}=13.9\%$$

Thus, the percentage of population who read an advertisement is 13.9%

49. (AC)

Let $E_1$ be the event of getting head, $E_2$ be the event of getting tail and let E be the event that noted number is 7 or 8 then

$$P(E_1)=\frac{1}{2};P(E_2)=\frac{1}{2}$$

$P(E/E_1)=P$ (Getting either 7 or 8 when pair of unbaised dice is thrown)

$$=\frac{11}{36}$$

$P(E/E_2)=P$ (Getting either 7 or 8 when a card is picked from the pack of 11 cards)

$$=\frac{2}{11}.$$

$\because$ $E_1$ and $E_2$ are mutually exclusive and exhaustive events

$$P(E)=P(E_1)P(E/E_1)+P(E_2)P(E/E_2)$$

$$=\frac{1}{2}.\frac{11}{36}+\frac{1}{2}.\frac{2}{11}$$

$$=\frac{11}{72}+\frac{1}{11}=\frac{193}{792}$$

50. (ACD)

Probability that defective from A is $\frac{25}{100}\times\frac{5}{100}=\frac{125}{10000}$

Probability that defective from B is $\frac{35}{100}\times\frac{4}{100}=\frac{140}{10000}$

Probability that defective from C is $\frac{40}{100}\times\frac{2}{100}=\frac{80}{10000}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine A is $\frac{125}{125+140+80}=\frac{125}{345}=\frac{25}{69}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine B is $\frac{140}{125+140+80}=\frac{140}{345}=\frac{28}{69}$

A bulb is drawn and is found to be defective then Probability that it is manufactured by machine C is $\frac{80}{125+140+80}=\frac{80}{345}=\frac{16}{69}$

51. (AB)

The probability that both will be alive for 10 years, hence i.e., the prob. That the man and his wife Both be alive 10 years hence $0.83\times 0.37=0.7221$

The prob. that at least one of then will be alive is 1-P(That none of them remains alive 10 years)

$=1-(1-0.83)(1-0.87)$

$=1-0.17\times 0.13$

$=1-0.17\times 0.13$

$=0.9779$

52. (ABCD)

$P(A)=0.7;\ P(B)=00.4$

$P(A-B)=P(A)-P(AB)$

$\Rightarrow\ P(AB)=0.2$

$\Rightarrow\ P(A\cup B)=0.9\ \Rightarrow\ P(B-A)=0.2,\Rightarrow P(\bar{A}\cup\bar{B})=1-P(AB)=0.8$

$\Rightarrow\ P\left(\frac{B}{A\cup\bar{B}}\right)=\frac{P(A\cap B)}{P(A\cup\bar{B})}=\frac{1}{4}$

53. (ABD)

| Urn | Red Marbles | White marbles | Blue marbles |
|---|---|---|---|
| A | 5 | 3 | 8 |
| B | 3 | 5 | 0 |

$P(E_1)=P(R)=\left(\frac{2}{3}\right)\left(\frac{5}{16}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{8}\right)=\frac{10}{48}+\frac{6}{48}=\frac{1}{3}$

$P(E_2) = P(W) = \left(\frac{2}{3}\right)\left(\frac{3}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{5}{8}\right) = \frac{6}{48} + \frac{10}{48} = \frac{1}{3}$

$P(E_3) = P(B) = \left(\frac{2}{3}\right)\left(\frac{8}{16}\right) = \frac{1}{3}$

(C) Let A : event that urn A is chose

$$P\left(\frac{A}{R}\right) = \frac{P(A \cap R)}{P(R)} = \frac{\left(\frac{2}{3}\right)\left(\frac{5}{16}\right)}{\frac{1}{3}} = \left(\frac{10}{48}\right)(3) = \frac{5}{8} \Rightarrow \text{ (C) is incorrect.}$$

(D) $P\left(\frac{A}{W}\right) = \frac{P(A \cap W)}{P(W)} = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{16}\right)}{\frac{1}{3}} = \left(\frac{6}{48}\right)(3) = \frac{3}{8}$

$P\left(\frac{\text{face five}}{W}\right) = \left(\frac{3}{8}\right)\left(\frac{1}{4}\right) = \frac{3}{32} \Rightarrow$ (D) is correct.

54. (AC)

$P_K = \frac{{}^{15}C_K}{2^{15}}$

It is max where $K = \frac{15-1}{2}$ or $\frac{15+1}{2}$

55. (BC)

Let S be the sample space .Then $|S| = 5.{}^5P_4$. If 0 is present then the number of 5 digit number divisible by 3 is $4\lfloor 4 = 96$. If 0 is absent then the number of 5 digit number divisible by 3 is 120.

Required Probability $= \frac{216}{600}$

56. (4)

The probability that he get marks $= \frac{1}{31}$

The probability that he get marks in second trial is $\frac{30}{31} \times \frac{1}{30} = \frac{1}{31}$

The probability that he get marks in third trial is $\frac{1}{31}$

Continuing this process the probability from $r$ trial is $\frac{r}{31} > \frac{1}{8}$

$\Rightarrow \quad r > \frac{31}{8}$

$r = 4$

57. (2)

$n(X) = k+1$

No. of ways to construct $A = 2^{k+1}$

No. of ways to construct $B = 2^{k+1}$

$\therefore$ Total ways to construct $A$ and $B$ = $2^{k+1} \times 2^{k+1}$

Favourable ways to construct $A = 2^{k+1}$
Favourable ways to construct $B$ such that $B = A^C$ is = 1
$\therefore$ Favourable ways = $2^{k+1} \times 1$

Required Probability = $\dfrac{2^{k+1}}{\left(2^{k+1}\right)^2} = \dfrac{1}{2^{k+1}}$

$\Rightarrow \quad m-1 = k+1$
$\Rightarrow \quad m-k = 2$

58. (479)

$A = \{1, 2, 3, 4\} : P(A) = \dfrac{3}{4} \to$ Correct

$B = \{5, 6, 7, 8, 9, 10\} : P(B) = \dfrac{1}{4}$ Correct

8 Correct Ans:

$(4, 4) : {}^4C_4 \left(\dfrac{3}{4}\right)^4 \cdot {}^6C_4 \left(\dfrac{1}{4}\right)^4 \cdot \left(\dfrac{3}{4}\right)^4$

$(3, 5) : {}^4C_3 \left(\dfrac{3}{4}\right)^3 \cdot \left(\dfrac{1}{4}\right)^1 \cdot {}^6C_5 \left(\dfrac{3}{4}\right)^4 \cdot \left(\dfrac{3}{4}\right)$

$(2, 6) : {}^4C_2 \left(\dfrac{3}{4}\right)^2 \cdot \left(\dfrac{1}{4}\right)^2 \cdot {}^6C_6 \left(\dfrac{1}{4}\right)^6$

Total $= \dfrac{1}{4^{10}} \left[3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2\right]$

$\Rightarrow \quad k = 479$

59. (5)

$p^2 \geq 4q \Rightarrow$

| p | q |
|---|---|
| 2 | 1 |
| 3 | 1, 2 |
| 4 | 1 to 4 |
| 5 | 1 to 6 |
| 6 | 1 to 9 |
| 7, 8, 9, 10 | 1 to 10 |

The total number of pairs ($p$, $q$) is 1 + 2 + 4 + 6 + 9 + 40 = 62

Probability $= \dfrac{62}{10.10} = \dfrac{31}{50}$

60. (33)

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
Divisible by 3.
Case – I : All 1 $\to$ (1)
Case – II : All 8 $\to$ (1)
Case – III : 3 ones & 3 eights

$\dfrac{6!}{3! \times 3!} = 20$

Required probability $\therefore\ p = \frac{22}{64}$

$$96p = 96 \times \frac{22}{64} = 33$$