IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 25/02/23**

TOPIC: RAY OPTICS

SOLUTIONS

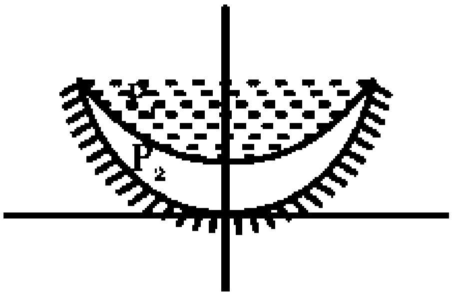
1. (A)

$R = 60\,cm, R_2 = 20\,cm$

Let P_1 be power of lens of water, P_2 be power of glass lens, and P_3 be power of concave mirror.

$P_{eq} = 2P_1 + 2P_2 + P_3$

$\frac{1}{-f_{eq}} = \frac{2}{f_1} + \frac{2}{f_2} - \frac{1}{f_3}$ .... (1)



For the lens of water

$\frac{1}{f_1} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$

$\frac{1}{f_1} = \frac{1}{180}$

For glass lens

$\frac{1}{f_2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$

$\frac{1}{f_2} = \frac{1}{60}$

For concave mirror $f_3 = \frac{R}{2}$

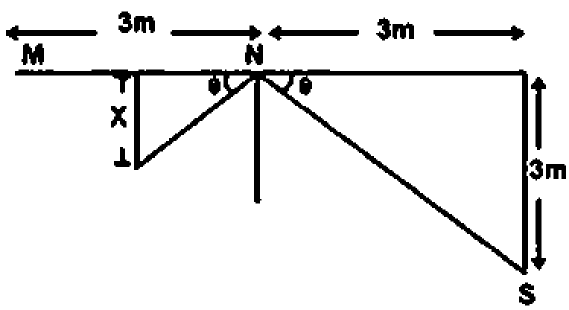
$f_3 = -10\,cm$

$\frac{1}{-f_{eq}} = \frac{2}{180} + \frac{2}{60} - \frac{1}{-10}$

$\frac{1}{-f_{eq}} = \frac{26}{180} \Rightarrow f_{eq} = \frac{-90}{13}\,cm$

2. (D)

$\tan\theta = \frac{x}{(MN/2)}$ (in the smaller triangle)



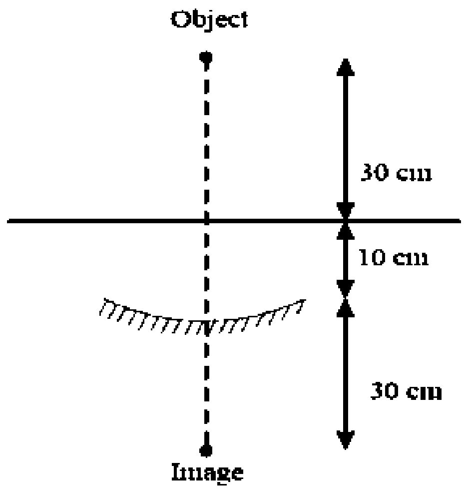
$\tan\theta = \frac{3m}{3m}$ (in the bigger triangle)

$\therefore \quad \frac{x}{1.5} = \frac{3}{3}$

$\therefore \quad x = 1.5\,m$

3. (B)

For partial reflection, the final image should be 30 cm below water surface i.e. 20 cm below the mirror.



For the image 30 cm below surface, its actual depth should be

$h = 30\left(\frac{4}{3}\right) = 40$ cm below surface i.e.

30 cm below the mirror

For mirror, $u = -\left(10 + 30\left(\frac{4}{3}\right)\right)$

$\Rightarrow u = -50\,cm$ and $v = 30\,cm$

$\frac{1}{f} = -\frac{1}{50} + \frac{1}{30} = \frac{-3+5}{150} = \frac{2}{150}$

Or $f = 75\,cm$

Convex mirror of focal length 75 cm.

(Diagram drawn was just for reference actually it should be a convex mirror but it was not known so we assumed concave but at final calculation, we found focal length +ve so it must be a convex mirror)

4. (B)

For Refraction at first surface

$\frac{1.5}{v} - \frac{1}{u} = \frac{1.5-1}{R}$

$u = -2R$

$v = \infty$

Therefore, rays strike the mirror normally. So, by principle of reversibility, O and I coincide.

5. (B)

For the lens L_1, the ray must move parallel to the axis after refraction, for the image to coincide with the object.

$\frac{\mu_1}{\infty} + \frac{\mu_\omega}{x} = \frac{\mu_1 - \mu_\omega}{R_1}$, where x is the distance of the object from the lens L_1

$\frac{4/3}{x} = \frac{2-4/3}{5}$

$\Rightarrow x = 10\,cm$

For the lens L_2, the ray must appear to come from the centre of curvature after refraction, for the image to coincide with the object.

$\frac{\mu_2}{-R_2} + \frac{\mu_\omega}{y} = \frac{\mu_1 - \mu_\omega}{\infty}$, where y is the distance of the object from the lens L_2

$y = R_2\frac{4/3}{3/2} = 8\,cm$

Distance between the two lenses = $10\,cm + 8\,cm = 18\,cm$

6. (A)

For a thin lens,

$|m| = \left|1 - \frac{\upsilon}{f}\right|$

From the graph, slope of the line $m = \frac{1}{f}$

$\frac{c}{b} = \frac{1}{f}$

$f = \frac{b}{c}$

7. (C)

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For no dispersion $d\left(\frac{1}{f}\right) = 0$

$d\left[(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] = 0 \quad \Rightarrow R_1 = R_2$

8. (A)

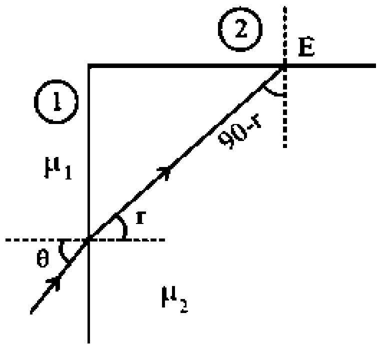
At surface (1)

$\mu_1 \sin\theta = \mu_2 \sin r$ …….. (1)

At surface (2)

$\mu_1 = \mu_2 \sin(90 - r)$

$\mu_1 = \mu_2 \cos r$ ……. (2)



From equation (1) and (2)

$\Rightarrow \sin\theta = \frac{\sin r}{\cos r}$

$\Rightarrow \sin\theta = \frac{\frac{\left(\sqrt{\mu_2^2 - \mu_1^2}\right)}{\mu_2}}{\left(\frac{\mu_1}{\mu_2}\right)}$

$\Rightarrow \sin\theta = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

θ must satisfy,

$\Rightarrow \theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right)$

9. (A)

For critical angle at the glass–liquid interface we have

$\mu_1 \sin C = \mu_2 \sin 90^o$

$\Rightarrow \frac{3}{2}\sin C = \mu$

$\Rightarrow \sin C = \frac{2\mu}{3}$

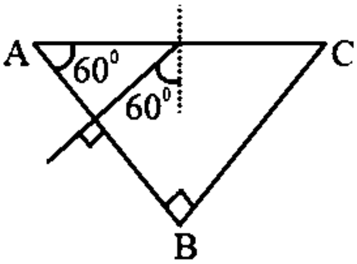
For total internal reflection

$60^o > C$

$\Rightarrow \sin 60^o > \sin C$

$\Rightarrow \frac{\sqrt{3}}{2} > \frac{2\mu}{3}$

$\mu < \frac{3\sqrt{3}}{4}$



10. (C)

From lens equation,

$$\frac{1}{\upsilon} - \frac{1}{(-20)} = \frac{1}{10} \quad \Rightarrow \quad \upsilon = +20\,\text{cm}$$

Magnification,

$$m_1 = \frac{\upsilon}{u} = \left(\frac{+20}{-20}\right) = -1$$

Image is real, inverted, and same size as object.

11. (A)

Consider the figure. The ray will come out from CD if it suffers a total internal reflection at surface AD, i.e., it strikes the surface AD at critical angle C (the limiting case).

Applying Snell's law at P, we get

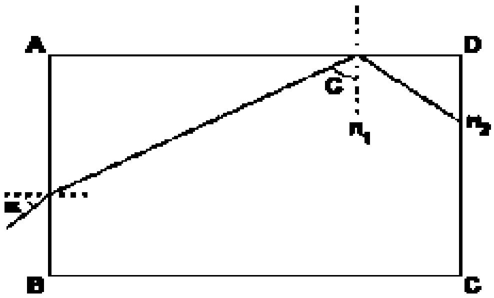
$$n_1 \sin C = n_2 \quad \text{or} \quad \sin C = \frac{n_2}{n_1}$$

Applying snell's law at Q, we get

$n_2 \sin\alpha = n_1 \cos C;$

$$\Rightarrow \quad \sin\alpha = \frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$$

Or $\alpha = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$



12. (C)

$$\frac{\mu_2}{\upsilon} - \frac{\mu_1}{-u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \upsilon = \frac{uR\mu_2}{u(\mu_2 - \mu_1) - R\mu_1}$$

$$\Rightarrow \upsilon = \frac{uR\mu_2}{u\mu_2 - \mu_1(u+R)}$$

We can see that $\mu_1 > \mu_2$ then image distance is negative which means virtual image

13. (A)

$$\frac{1}{f_1} = \frac{1}{2} \times \frac{2}{18}$$

$$\frac{1}{f_2} = \frac{(\mu_1 - 1)}{-18}$$

When μ_1 is filled between lens and mirror,

$$P = \frac{2}{18} - \frac{2}{18}(\mu_1 - 1) = \frac{2 - 2\mu_1 + 2}{18}$$

$$\Rightarrow F_m = -\left(\frac{18}{2 - \mu_1}\right)$$

$$2 = 6 - 3\mu_1$$

$$3\mu_1 = 4$$

$$\mu_1 = \frac{4}{3}$$

14. (A)

Equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{20} + \frac{1}{30}$$

$$\therefore \quad F = \frac{20 \times 30}{20 + 30}$$

$$= \frac{600}{50} = 12\,\text{cm}$$

15. (A)

From lens formula

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{……….. (i)}$$

Also, ${}_\ell\mu_g = \frac{\mu_g}{\mu_\ell} = \frac{1.5}{1.6}$

$$\therefore \frac{1}{f'} = ({}_\ell\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f'}=\left(\frac{15}{16}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{.……… (ii)}$$

Dividing Equation (i) by Equation (ii), we get

$$\frac{f}{f'}=\frac{\left(\frac{15}{16}-1\right)}{(1.5-1)}=-\frac{1}{8}$$

$$f'=-8f$$

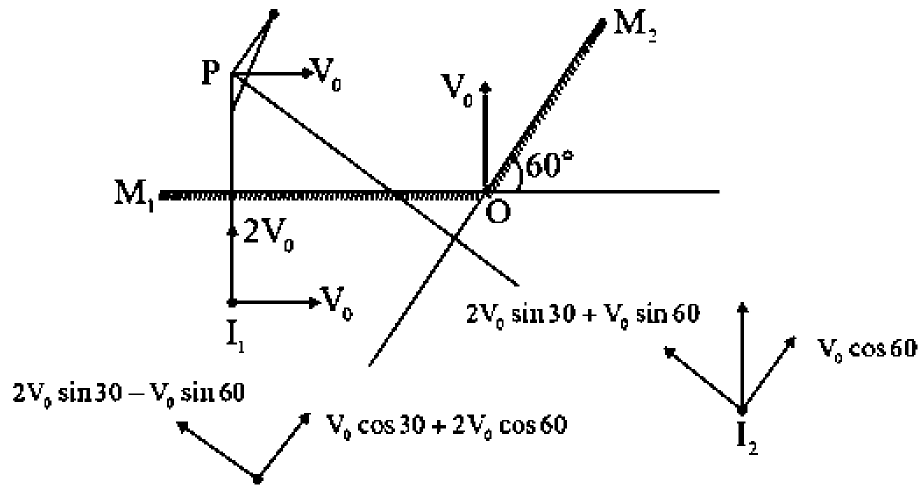
$$=-160\,\text{cm}$$

16. (B)

Velocity of image of the point in the first mirror with respect to the image of the point in the second mirror is

$$\left(\vec{V}_{I_1}\right)_{I_2}=-V_0\sqrt{3}\hat{i}-V_0\sqrt{3}\hat{j}$$

$$\left|\left(\vec{V}_{I_1}\right)_{I_2}\right|=V_0\sqrt{6}$$



17. (A)

Image of the flame formed by the concave mirror on the wall will be real and inverted, hence magnification

$$m=\frac{I}{O}=\frac{-1.5}{0.5}=-3$$

$$m=-\frac{\upsilon}{u}=-3$$

$\therefore\quad \upsilon=3u$

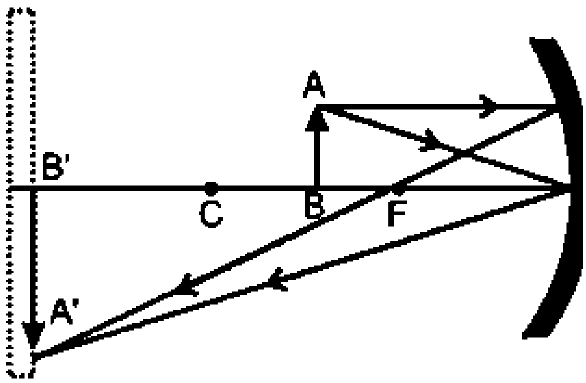
Using $\frac{1}{\upsilon}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{3u}+\frac{1}{u}=-\frac{1}{1.5}$$

$$(f=-1.5\,m)$$

$$\frac{4}{3u}=-\frac{1}{1.5}$$

$\therefore\quad u=\frac{-4\times1.5}{3}=-2m$



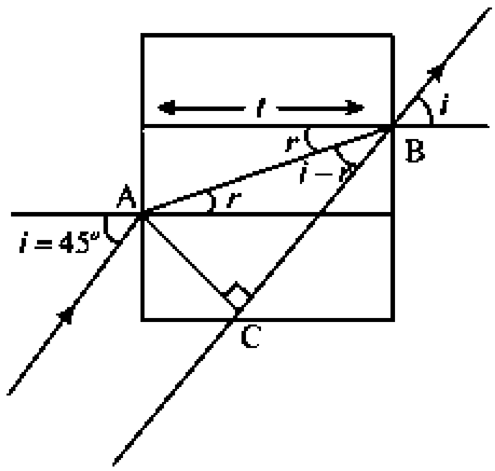
Thus the flame is at distance 2m from the mirror and

$$\upsilon = 3u = -6\,m$$

Hence the distance between wall and mirror is 6 m. It implies that the distance between the flame and the wall will be 4m.

18. (B)

Here, angle of incidence $i = 45^o$



$$\frac{\text{Lateral shift}(d)}{\text{Thickness of glass slab}(t)} = \frac{1}{\sqrt{3}}$$

Lateral shift $d = \frac{t\sin\delta}{\cos r} = \frac{t\sin(i-r)}{\cos r}$

$$\Rightarrow \frac{d}{t} = \frac{\sin(i-r)}{\cos r}$$

or $\frac{d}{t} = \frac{\sin i\cos r - \cos i\sin r}{\cos r}$

or $\frac{d}{t} = \frac{\sin 45^o\cos r - \cos 45^o\sin r}{\cos r} = \frac{\cos r - \sin r}{\sqrt{2}\cos r}$

or $\frac{d}{t} = \frac{1}{\sqrt{2}}(1-\tan r)$

or $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}}(1-\tan r)$

or $\tan r = 1 - \frac{\sqrt{2}}{\sqrt{3}}$

or $r = \tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt{3}}\right)$

19. (B)

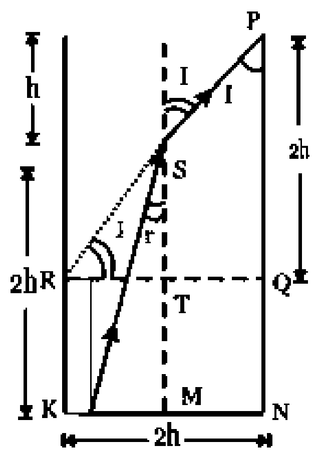
$PQ = QR = 2h$ $\quad\quad \therefore \angle i = 45^o$

$\therefore ST = RT, h = KM = MN$

So, $KS = \sqrt{h^2 + (2h)^2} = h\sqrt{5}$

$\therefore \sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin 45^o}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$



20. (B)

Incidence angle on face BC is

$$i = 90 - \theta$$

$i = A = 90 - \theta > \theta_C$ (for light not to cross BC)

$$\cos\theta > \sin\theta_C = \frac{6/5}{3/2} = \frac{4}{5}$$

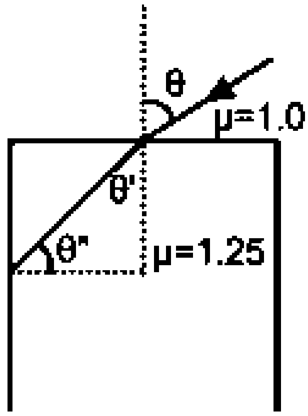
$$\Rightarrow \quad \theta < \cos^{-1}\frac{4}{5} = 37^o$$

21. (3)

The critical angle for this case is

$$\theta'' = \sin^{-1}\frac{1}{1.25} = \sin^{-1}\frac{4}{5}$$

Or, $\sin\theta'' = \frac{4}{5}$



Since $\theta'' = \frac{\pi}{2} - \theta'$, we have

$\sin\theta' = \cos\theta'' = 3/5$. From Snell's law,

$$\frac{\sin\theta}{\sin\theta'} = 1.25$$

Or, $\sin\theta = 1.25 \times \sin\theta'$

$$= 1.25 \times \frac{3}{5} = \frac{3}{4}$$

Or, $\theta = \sin^{-1}\frac{3}{4}$.

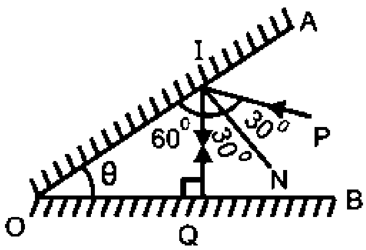
If θ'' is greater than the critical angle, θ will be smaller than this value. Thus, the maximum value of θ, for which total reflection takes place at the vertical surface, is $\sin^{-1}(3/4)$.

22. (30)

$$\angle i = \angle r = 30^o$$

$$\therefore \quad \angle OIQ = 60^o$$

$$\therefore \quad \angle IOQ = 90^o - 60^o = 30^o$$



23. (25)

R.I. of lens,

$$\mu = \frac{c}{\upsilon} = \frac{3\times10^8}{2\times10^8} = \frac{3}{2}$$

From ΔACO, radius of curvature of lens is,

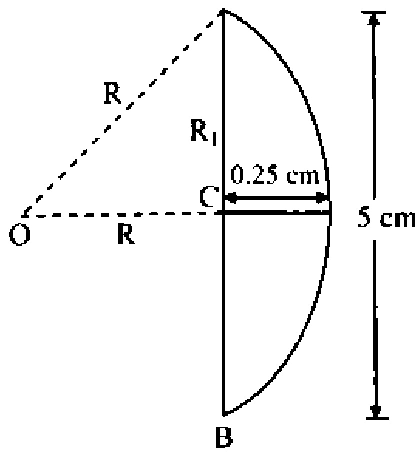
$$R^2 = R_1^2 + (R - 0.25)^2$$

$$\therefore \quad R^2 = (2.5)^2 + (R - 0.25)^2$$

$$\therefore \quad R^2 = 6.25 + R^2 + 0.05 - 0.5R$$

$$\therefore \quad 0.5R = 6.3125$$

$$\therefore \quad R = 12.625\text{cm}$$



$$F=\frac{R}{\mu-1}=\frac{12.625}{\frac{3}{2}-1}=25.25\,\text{cm}$$

24. (60)

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$R_1=\infty$

$R_2=-30\text{cm}$

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{-30}\right)$$

$$\frac{1}{f}=\frac{0.5}{30}$$

$f=60\text{cm}$

25. (4)

$$=-\left(\frac{v^2}{u^2}\right)du=-\frac{(-20)^2}{(-10)^2}\times 0.1=-0.4\,\text{cm}$$

26. (60)

The monochromatic ray of light will retrace its path if it strikes the mirror perpendicularly.

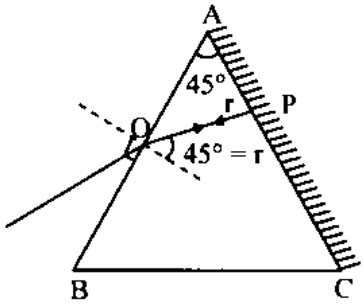
Using Snell's law of refraction,

$1\times\sin\theta=\mu\times\sin 45^{\circ}$

$\Rightarrow \sin\theta=\sqrt{\frac{3}{2}}\times\sin 45^{\circ}$

$\sin\theta=\sqrt{\frac{3}{2}}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{3}}{2}$

$\Rightarrow \theta=60^{\circ}$



27. (25)

We have,

$$m=-\frac{v}{u}=\frac{1}{2}$$

$$v=-\frac{u}{2}$$

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{u}+\frac{1}{-u/2}=\frac{1}{2.5\,m}$$

$$-\frac{1}{u}=\frac{1}{2.5\,m}$$

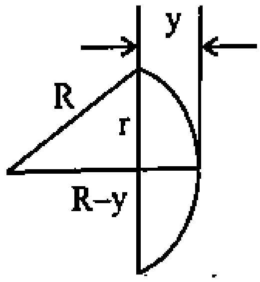
$u=-2.5\,m$

Thus, he should stand at a distance of 2.5 m from the mirror.

28. (3)

$(R-y)^2 + r^2 = R^2$

$R = \frac{r^2}{2y} = \frac{\left(\frac{6}{2}\right)^2}{2\times 0.3} = 15\,\text{cm}$



$\frac{1}{f} = \frac{\mu-1}{R}$

$\mu = \frac{C}{V} = \frac{3.0\times10^8}{2.0\times10^8} = 1.5$

$\therefore f = 30\,\text{cm}$ converging

Now $f = 30\,\text{cm}$

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{30} = \frac{1}{v} - \frac{1}{u}$

$m = \frac{v}{u} = -2$

$v = -2u$

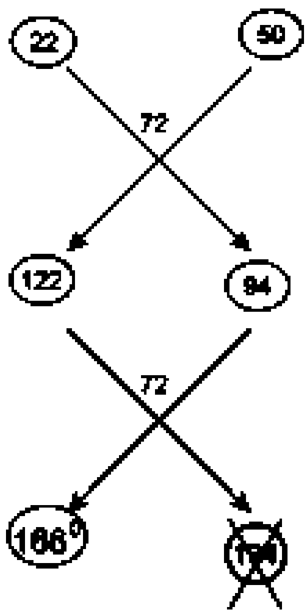
$\frac{1}{30} = \frac{-1}{2u} - \frac{1}{u}$

$\frac{1}{30} = -\frac{3}{2u}$

$u = -45\,\text{cm}$

29. (5)

The above diagram represents the angular positions of possible images that can be formed in the given case. Since, images further than 180° will not be visible, we obtain the maximum number of visible images as 5.



30. (6)

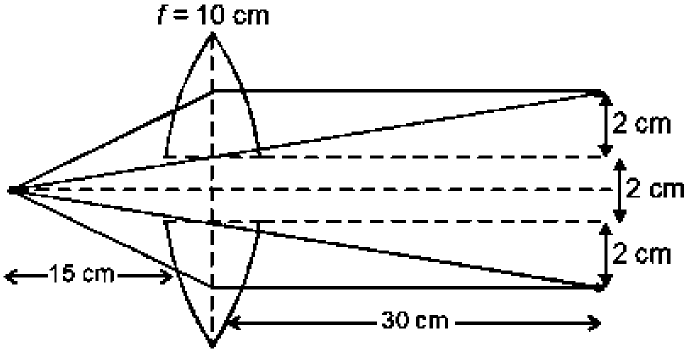
The distance of image formed by each piece can be calculated by

$$\frac{1}{f} = \frac{1}{\upsilon} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{\upsilon} - \frac{1}{-15}$$

$$\Rightarrow \upsilon = 30\,\text{cm}$$

$$|m| = \frac{30}{15} = 2$$



IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 25/02/23**

TOPIC: CHEMICAL EQUILIBIRUM

SOLUTIONS

31. (D)
Equilibrium constant is a function of temperature only.

32. (A)
$$K_P = \frac{1.P_{CO_2}}{1} = P_{CO_2}$$

33. (D)
K_{eq} is independent on concentration.

34. (B)
$$K_P = K_C.(RT)^{\Delta n}{}_g \Rightarrow \frac{K_C}{K_P} = (RT)^{-\Delta n}{}_g = (RT)^{-\left(-\frac{1}{2}\right)}$$
$$= \sqrt{RT}$$

35. (C)
$$K_{eq} = \frac{K_f}{K_b} = \frac{0.16}{4\times10^4} = 4\times10^{-6}$$

36. (C)
For $r_f > r_b$ means the net reaction in forward direction, the reaction quotient Q should be less than K_{eq}.

37. (A)
$$K = \frac{[C_6H_6]}{[C_2H_2]^3} \Rightarrow 4 = \frac{[C_6H_6]}{(0.5)^3} \Rightarrow [C_6H_6] = 0.5M$$

38. (D)
Solubility of gases in liquids : When a gas dissolves in liquid, there is decrease in volume. Thus, increase of pressure will favour the dissolution of gas in liquid.

39. (B)
$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g);\ \ \Delta H = +43.5\,Kcal$$

For endothermic reactions, K_{eq} increases with the increase in temperature.

40. (D)

$$A(s) \rightleftharpoons 2B(g)+3C(g)$$

$\therefore \quad K_C = [B]^2[C]^3 = x^2(y)^3 = x^2y^3$

If concentration of C is two time to that of y at equilibrium *i.e.* $2y$ then if concentration of B is B_2

$$K_C = [B]^2[2y]^3$$

Or $[x]^2[y]^3 = 8[B]^2[y]^3$

$\therefore \quad [B] = \frac{x}{\sqrt{8}} = \frac{x}{2\sqrt{2}}$

41. (A)

From (i) and (ii)

$$2CO(g)+O_2(g) \rightleftharpoons 2CO_2(g)$$

$$K_{2000} = \frac{1}{(4.4)^2 \times 5.31\times10^{-10}} = 9.73\times10^7$$

$$K_{1000} = 2.24\times10^{22}$$

On increasing temperature, K_{eq} is decreased and hence, $\Delta H^\circ = -ve$.

42. (A)

$MCO_3(S) \rightleftharpoons MO(S)+CO_2(g) : K_p = P_{CO_2}$

Now, $\ln\frac{K_2}{K_1} = \frac{\Delta H^\circ}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)$

Or, $2.303\times\log100 = \frac{\Delta H^\circ}{2}\left(\frac{1}{400}-\frac{1}{500}\right)$

$\Rightarrow \Delta H^\circ = 18424\,cal$

43. (D)

$\Delta G^\circ = -2.303\,RT.\log K_{eq}$

Or, $-4.606 = -2.303\times\frac{2}{1000}\times1000\times\log\ K_{eq}$

$\Rightarrow K_{eq} = 10$

For reverse reaction, $K'_{eq} = \frac{1}{10} = 0.1$

44. (C)

	PCl_5	$\rightleftharpoons$ PCl_3	+	Cl_2
	X mole	0		0
Equilibrium	$x(1-\alpha)$	$x.\alpha$		$x.\alpha$

E $P_{PCl_3} = \dfrac{x.\alpha}{x(1+\alpha)} \times P \Rightarrow P_{PCl_3} . P^{-1} = \dfrac{\alpha}{1+\alpha}$

45. (A)

	N_2	+	$3H_2$	$\rightleftharpoons$	$2NH_3$
	0.2 mole		0.6 mole		0
Equilibrium	$0.2 - 0.2 \times \dfrac{40}{100}$		$0.6 - 0.08 \times 3$		2×0.08
	$= 0.12$		$= 0.36$		$= 0.16$

$$\therefore \frac{V_{final}}{V_{initial}} = \frac{n_{final}}{n_{initial}} = \frac{0.64}{0.80} = \frac{4}{5}$$

46. (A)

	CH_3COOH	+	C_2H_5OH	$\rightleftharpoons$	$CH_3COOC_2H_5$	+	H_2O
	1		1		0		0
Equilibrium	$1 - \dfrac{1}{4}$		$1 - \dfrac{1}{4}$		$\dfrac{1}{4}$		$\dfrac{1}{4}$
	$= \dfrac{3}{4}$		$\dfrac{3}{4}$				

$$K = \frac{\frac{1}{4} \times \frac{1}{4}}{\frac{3}{4} \times \frac{3}{4}} = \frac{1}{9}$$

47. (A)

At equilibrium , $[A] = [B]$ and $[C] = [D]$ and hence

$$K = \frac{[C][D]}{[A][B]} = \frac{[C]^2}{[A]^2} \Rightarrow \frac{[C]}{[A]} = \sqrt{2.25}$$

$$= 1.5 \Rightarrow \frac{[A]}{[C]} = \frac{2}{3}$$

48. (A)

	$H_2(g)$	+	$S(s)$	$\rightleftharpoons$	$H_2S(g)$
	0.3 mole		2 mole		0
Equilibrium	$(0.3 - x)$ mole		$(2 - x)$ mole		x mole

$$K_C = \frac{\frac{x}{2}}{\frac{(0.3 - x)}{2}} = 0.08 \Rightarrow 0.022$$

Now, $P_{H_2S} = \dfrac{x \times 0.08 \times 360}{2} = 0.32\,\text{atm}$

49. (C)

K_{eq} is a function of temperature only.

50. (D)

For forward reaction (exothermic), temperature should be low. As mole of gas is decreasing, the pressure should be high.

51. (1)

$\Delta G^o = \Delta H^o - T\Delta S^o$

$= -30 - 300 \times (-0.1)$

$= -30 + 30 = 0$

$\therefore \quad \Delta G^o = 2.303RT \log K_p$

$\therefore \quad K_p = 1$

52. (4)

$2A(s) + nB(g) \rightleftharpoons 3C(g)$

$\frac{K_p}{K_c} = (RT)^{\Delta x}$

$\left(\frac{0.0105}{0.45}\right) = (0.0821 \times 523)^{\Delta x}$

$\therefore \quad \Delta x = -1$

$\therefore \quad \Delta x = 3 - (n) = -1$

$\therefore \quad n = 4$

53. (4)

$\frac{K_f}{K_b} = 4$

$\therefore \quad K_C = \frac{K_f}{K_b} = 4$

54. (1)

$AB_2(g)$	$\rightleftharpoons$	$AB(g)$	+	$B(g)$
P		0		0
$P - P'$		P'		P'

Given $\quad P = 6\,atm$

$P + P' = 8\,atm$

$\therefore \ P' = 2\,atm$

$K_P = \frac{2 \times 2}{4} = 1$

55. (27)

$$2AB_3(g) \rightleftharpoons A_2(g) + 3B_2(g)$$

$t = 0$	8	0	0
At equilibrium	$(8 - a)$	$\frac{a}{2}$	$\frac{3a}{2}$

Thus, $K_c = \frac{[A_2][B_2]^3}{[AB_3]^2}$; Also $\frac{a}{2} = 2 \quad \therefore\ a = 4$

$\therefore\ [AB_3] = \frac{4}{1};\ [A_2] = \frac{2}{1};\ [B_2] = \frac{6}{1}$

Thus, $K_c = \frac{2 \times 6^3}{4^2} = 27\ \text{mol}^2\text{L}^{-2}$

56. (3)

57. (8)

$K_p = P'_{NH_3} \times P'_{HCl} = 16$

$K_p = P \times P = 16$

$\therefore\ P = 4$

$\therefore$ Total pressure at eq. $= P + P = 4 + 4 = 8$

58. (4)

K_{eq} is independent on catalyst.

59. (1)

K_C does not depend on concentration of reactants or products.

$K_C = \frac{[D][C]}{[A][B]} = \frac{1.5 \times 1.5}{1.5 \times 1.5} = 1$

60. (23)

$K_P = \left(P_{H_2O}\right) = 23\ \text{mm}$

IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 25/02/23**

TOPIC: PROBABILITY

SOLUTIONS

61. (D)

Let C, S, B, T be the events of the person going by car, scooter, bus or train respectively.

Then as per the given information P(C) = $\frac{1}{7}$, P(S) = $\frac{3}{7}$, P(B) = $\frac{2}{7}$, and P(T) = $\frac{1}{7}$.

Let O be the event of the person reaching the office in time.

Then from the given information $P\left(\frac{O}{C}\right)=\frac{7}{9}, P\left(\frac{O}{S}\right)=\frac{8}{9}, P\left(\frac{O}{B}\right)=\frac{5}{9}$ and $P\left(\frac{O}{T}\right)=\frac{8}{9}$

Probability that he will reach office on time $P(O)=\left(\frac{1}{7}\times\frac{7}{9}\right)+\left(\frac{3}{7}\times\frac{8}{9}\right)+\left(\frac{2}{7}\times\frac{5}{9}\right)+\left(\frac{8}{9}\times\frac{1}{7}\right)$

Probability that he will reach office on time if he travel by car $\left(\frac{1}{7}\times\frac{7}{9}\right)$

Here we need to find probability of

$$P\left(\frac{C}{O}\right)=\frac{\left\{P\left(\frac{O}{C}\right)\times P(C)\right\}}{P(O)}=\frac{\left\{\left(\frac{1}{7}\times\frac{7}{9}\right)\right\}}{\left\{\left(\frac{1}{7}\times\frac{7}{9}\right)+\left(\frac{3}{7}\times\frac{8}{9}\right)+\left(\frac{2}{7}\times\frac{5}{9}\right)+\left(\frac{8}{9}\times\frac{1}{7}\right)\right\}}=\frac{1}{7}$$

62. (B)

Out of every 16 events 3 are favorable and events not favorable is 13 hence odd against the event is $\frac{13}{3}$.

In other words here sample space n(S) = 16, favorable event n(E) = 3 and n(E’) = 13 hence odd against that event is n(E’)/n(E) = 13/3

63. (A)

Sample space in this case is $6 \times 6 = 36$

Sum of two result is a prime number we have following cases-

Case (i) if sum is 2 then we have only one option (1, 1)

Case (ii) If sum is 3 then we have following 2 cases (1, 2) and (2, 1)

Case (iii) If sum is 5 then we have following 4 cases (1, 4), (2, 3), (3, 2) and (4, 1)

Case (iv) If sum is 7 then we have following 6 cases (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1)

Case (v) If sum is 11 then we have following 2 cases (5, 6), and (6, 5)

So total number of ways is 1 + 2 + 4 + 6 + 2 = 15

So required probability is $\frac{15}{36}=\frac{5}{12}$

64. (B)

Probability of getting a head in a single throw is $\frac{1}{2}$.

Hence required probability is $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

65. (A)

Probability that none of them will appear tail is $\left(\frac{1}{2}\right)^5=\frac{1}{32}$

So, required probability that at least one tail will appear is $1-\frac{1}{32}=\frac{31}{32}$

66. (A)

From the definition odds in favor of an event is $\frac{n(E)}{n(E')}$ and odds against an event is $\frac{n(E')}{n(E)}$ and from the given information is $\frac{\left\{\frac{n(E)}{n(E')}\right\}}{\left\{\frac{n(E')}{n(E)}\right\}}=\frac{9}{25}$ or $\frac{n(E)}{n(E')}=\frac{3}{5}$

Hence, required probability is $\frac{3}{8}$

67. (D)

Sum is multiple of 3 so we have following cases-

Case (i) sum is 3 then cases are (1, 2), (2, 1)

Case (ii) sum is 6 then cases are (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1)

Case (iii) sum is 9 then cases are (3, 6), (4, 5), (5, 4), and (6, 3)

Case (iv) sum is 12 then only case is (6, 6)

So total number of elements in sample space is n(S) = 12

Number of favorable cases are 4

So required probability $=\frac{4}{12}=\frac{1}{3}$

68. (C)

Since A & B are mutually exclusive so $P(A \cap B) = 0$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{8}+\frac{2}{5}=\frac{5+16}{40}=\frac{21}{40}$

69. (C)

Here P(A) = 1 – 0.6 = 0.4 and P(B) = 1 – 0.4 = 0.6

Since $P(A\cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.4 - (0.6) \times (0.4) = 1 - 0.24 = 0.76$

70. (C)

Since $P(A')=\frac{9}{11}$ then P(A) = $\frac{2}{11}$ similarly P(B) = $\frac{2}{5}$

From the formula P(A ∪ B) = P(A) + P(B) – P(A ∩ B) $=\frac{2}{11}+\frac{2}{5}-\frac{1}{5}=\frac{10+22-11}{55}=\frac{21}{55}$

71. (C)

There are two cases when they will contradict-

Case (i) A speak truth and B speak lie then probability is $\frac{4}{5}\times\frac{1}{4}=\frac{1}{5}$

Case (ii) A speak lie and B speak truth then probability is $\frac{1}{5}\times\frac{3}{4}=\frac{3}{20}$

Hence, required probability is $\frac{1}{5}+\frac{3}{20}=\frac{7}{20}$

72. (C)

Let probability that A, B and C will pass the exam is P(A), P(B) and P(C) respectively then from the given information P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{3}$ and P(C) = $\frac{1}{4}$ and also $P(A')=1-\frac{1}{2}=\frac{1}{2}$, $P(B')=1-\frac{1}{3}=\frac{2}{3}$ and $P(C')=1-\frac{1}{4}=\frac{3}{4}$

Now we have three cases-

Case (i) A and B will pass the exam and C fails that exam then probability is

$P(A)\times P(B)\times P(C')=\frac{1}{2}\times\frac{1}{3}\times\frac{3}{4}=\frac{1}{8}$

Case (ii) A and C will pass the exam and B fails that exam then probability is

$P(A)\times P(B')\times P(C)=\frac{1}{2}\times\frac{2}{3}\times\frac{1}{4}=\frac{1}{12}$

Case (iii) B and C will pass the exam and A fails that exam then probability is

$P(A')\times P(B)\times P(C)=\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}=\frac{1}{24}$

Total probability is $\frac{1}{8}+\frac{1}{12}+\frac{1}{24}=\frac{1}{4}$

73. (B)

Probability of getting a six is $\frac{1}{6}$

If Sanchita starts the game then the probability that she wins is

$$\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+.......\infty=\frac{\frac{1}{6}}{1-\frac{25}{56}}=\frac{6}{11}$$

And if Raj starts the game then probability that Sanchita wins the game is $1-\frac{6}{11}=\frac{5}{11}$

74. (C)

Probability that none of them hit the plane is

$(1-0.4)(1-0.3)(1-0.2)(1-0.1)=0.6\times0.7\times0.8\times0.9=0.3024$

So required probability is 1 – 0.3024 = 0.6976

75. (C)

Let P(A) = probability that a randomly selected student likes tea = 0.3

Let P(B) = probability that a randomly selected student likes coffee = 0.2

Then P(A ∩ B) = 0.1

Now we have to find $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{0.1}{0.2}=\frac{1}{2}$

76. (C)

There are two cases when Abhay will say 'Yes'-

Case (i) The number that came out is a prime and Abhay is speaking truth, probability for this case is

$$P(P)\times P(T)$$

Here, P(P) = probability of getting a prime = $\frac{3}{6}=\frac{1}{2}=0.5$

P(T) is probability that Abhay is speaking truth and P(T) = 0.6

So, probability for this case is 0.5 × 0.6 = 0.3

Case (ii) The number that came out is not a prime and Abhay is not speaking truth, probability for this case is

$$P(P')\times P(T')=0.5\times 0.4=0.2$$

So total probability for the given case is 0.3 + 0.2 = 0.5

New sample space is 0.5 and we have to find the probability of case (i) which is $\frac{0.3}{0.5}=0.6$

77. (A)

Total number of experiments = n = 5

Number of favorable cases = r = 3

Probability of getting favorable case = P(F) = (1/2)

Probability of not getting favorable case = $P(F')=\frac{1}{2}$

Required probability is $\left({}^5C_3\right)(F)^3(F')^2=\left({}^5C_3\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{10}{32}=\frac{5}{16}$

78. (B)

Total number of experiments = n = 5

Number of favorable cases = r = 0 (means he will not hit the target)

Probability of getting favorable case = P(F) = 0.8

Probability of not getting favorable case = $P(F')$ = 0.2

Required probability is $({}^5C_0)(0.8)^0(0.2)^5=\frac{1}{3125}$

So, probability that the target is hit at least once is $1-\frac{1}{3125}=\frac{3124}{3125}$

79. (A)

Total number of experiments = n = 5

Number of favorable cases = r = 4

Probability of getting favorable case = P(F) = $\frac{2}{6} = \frac{1}{3}$

Probability of not getting favorable case = $P(F') = \frac{2}{3}$

Required probability is $\left({}^{5}C_4\right)\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 = \frac{5\times 2}{3^5} = \frac{10}{243}$

80. (C)

Total number of experiments = n=12

Number of favorable cases = r = 10

Total sample space in one such trial is n(S) = 6 × 6 = 36

Favorable cases are (4, 6), (5, 5) and (6, 4)

Probability of getting favorable case = P(F) = $\frac{3}{36} = \frac{1}{12}$

Probability of not getting favorable case = $P(F') = \frac{11}{12}$

Required probability is $\left({}^{12}C_{10}\right)\left(\frac{1}{12}\right)^{10}\left(\frac{11}{12}\right)^2$

81. (20)

Event G = original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

Now total probability such that signal received by B is green is

$= P(GE_1E_2) + P[G(E_1' E_2')] + P[E_1(G'E_2')] + P[E_2(G'E_1')]$

$P(E) = \frac{46}{80}$

$$P\left(\frac{G}{E}\right) = \frac{\left(\frac{40}{5}\times 16\right)}{\left(\frac{46}{5}\times 16\right)} = \frac{20}{23}$$

82. (38)

Required probability $= \frac{475}{90} = \frac{19}{36}$

83. (7)

$$P(A\cup B\cup C) = \sum P(A) - \sum P(A\cap B) + \sum P(A\cap B\cap C)$$

$$= \frac{7}{16}$$

84. (39)

As per the given condition P(A) = 0.25, P(B) = 0.50 and P(A∩B) = 0.14

And P(AUB) = P(A) + P(B) – P(A∩B) = 0.25 + 0.50 – 0.14 = 0.61

Hence required probability is P(AUB)′ = 1 – P(A ∪ B) = 1 – 0.61 = 0.39

85. (4)

$P(A' \cap B') = 1 - P(A \cup B)$

$$= 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{18}$$

$72P = 72\left(\frac{1}{18}\right) = 4$

86. (2)

Here sample space is selecting 2 out of 10 i.e $^{10}C_2$ and 2 red balls can be selected in $^{4}C_2$ ways hence required probability is $\frac{^{4}C_2}{^{10}C_2} = \frac{4\times3}{10\times9} = \frac{2}{15}$

87. (31)

Probability that none of them will appear tail is $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

So required probability that at least one tail will appear is $1 - \frac{1}{32} = \frac{31}{32}$

88. (75)

Out of 12 students 5 can be selected in $^{12}C_5 = \frac{12\times11\times10\times9\times8}{120} = 792$ ways and the number of ways of selecting 3 girls and 2 boys is $^{6}C_3 \times {}^{6}C_2 = 20 \times 15 = 300$.

So required probability is $\frac{300}{792} = \frac{75}{198}$

89. (18)

We have the ratio of the ships A, B and C for arriving safely to be 2:5, 3:7 and 6:11, respectively.

The probability of ship A for arriving safely = $\frac{2}{2+5} = \frac{2}{7}$

Similarly, for B = $\frac{3}{3+7} = \frac{3}{10}$ and for C = $\frac{6}{6+11} = \frac{6}{17}$

Probability of all the ships for arriving safely = $\frac{2}{7}\times\frac{3}{10}\times\frac{6}{17} = \frac{18}{595}$.

90. (11)

Here, P(X) = 0.3; P (Y) = 0.2

Now $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

Since these are independent events, so P (X ∩ Y) = P (X) * P (Y)

Thus required probability = 0.3 + 0.2 − 0.06 = 0.44