

PACE-IIT & MEDICAL

MUMBAI / AKOLA / DELHI / KOLKATA / GHAZIABAD / NASHIK / GOA / BOKARO / PUNE

IIT – JEE: 2024

TW TEST (ADV)

DATE: 28/08/22

TOPIC: CALCULUS IN PHYSICS

Answer Key

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (A) | 2. (B) | 3. (B) | 4. (B) | 5. (C) |
| 6. (D) | 7. (A) | 8. (A) | 9. (D) | 10. (C) |
| 11. (C) | 12. (B) | 13. (A) | 14. (B) | 15. (A) |
| 16. (C) | 17. (D) | 18. (C) | 19. (C) | 20. (D) |
| 21. (D) | 22. (C) | 23. (D) | 24. (A) | 25. (A) |

SOLUTIONS

1. (A)

$$\begin{aligned}\int_0^{\infty} e^{-5x} dx &= \left[\frac{e^{-5x}}{-5} \right]_0^{\infty} \\ &= \frac{0-1}{-5} = \frac{1}{5}\end{aligned}$$

2. (B)

$$\begin{aligned}\int \frac{x^3}{4+x^4} dx \\ &= \frac{\ln(4+x^4)}{4} + C\end{aligned}$$

3. (B)

$$\begin{aligned}\int_0^1 \frac{dx}{(ax+b)} \\ &= \left[\frac{\ln(ax+b)}{a} \right]_0^1 \\ &= \left[\ln(a+b) - \ln b \right] \frac{1}{a} \\ &= \frac{1}{a} \ln \left(\frac{a+b}{b} \right) = \frac{1}{a} \ln \left(1 + \frac{a}{b} \right)\end{aligned}$$

4. (B)

$$3y = 2x^3 + 1$$

$$\frac{3dy}{dt} = 6x^2 \frac{dx}{dt}$$

$$dy = 2x^2 \cdot dx$$

$$8 = 2x^2 \Rightarrow x = \pm 2$$

$$y = \frac{2(\pm 2)^3 + 1}{3} = \pm \frac{16+1}{3}$$

$$= \frac{17}{3}, -\frac{15}{3}$$

5. (C)

$$y = x \cdot \cos x$$

$$\frac{dy}{dx} = x \cdot (-\sin x) + \cos x(1)$$

$$= -x \sin x + \cos x$$

6. (D)

$$\frac{dy}{d\theta} = -2 \sin 2\theta$$

$$\frac{dx}{d\theta} = \frac{1}{2} \cos \frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{-4 \sin 2\theta}{\cos \theta/2} = -16 \sin \frac{\theta}{2} \cdot \cos \theta$$

7. (A)

$$\int_2^5 (3x^2 + 4x + 1) dx$$

$$= [x^3 + 2x^2 + x]_2^5$$

$$= 180 - 18 = 162$$

8. (A)

$$\int_0^{\pi/4} \sec^2 \theta \cdot d\theta = [\tan \theta]_0^{\pi/4} = 1$$

9. (D)

$$\frac{dy_1}{dx} = \cos x + \sin x = 0$$

$$\tan x = -1 \quad \therefore A_1 = \sqrt{2}$$

$$\frac{dy_2}{dx} = 2 \cos x - 4 \sin x = 0$$

$$\tan x = \frac{1}{2}$$

$$B = 2\sqrt{5}$$

10. (C)

$$\frac{dp}{dt} = (10t + 3t^2)$$

$$\Delta p = \int (10t + 3t^2) dt$$

$$= \left(\frac{10t^2}{2} + \frac{3t^3}{3} \right)_0^2$$

$$= 5(4) + 8$$

$$= 28.$$

11. (C)

$$y = e^x - \ln x + \frac{1}{x}$$

$$\frac{dy}{dx} = e^x - \frac{1}{x} - \frac{1}{x^2}$$

12. (B)

$$\frac{dy}{dx} = x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\frac{d^2y}{dx^2} = 2x + 1$$

$$\text{At } x = -4; \frac{d^2y}{dx^2} = -7 < 0 \text{ (maximum)}$$

$$\text{At } x = 3; \frac{d^2y}{dx^2} = 7 > 0 \text{ (minimum)}$$

13. (A)

$$V = \frac{\pi}{3} r^2 h$$

$$\tan \theta = \frac{r}{h} = \frac{5}{20} = \frac{1}{4}$$

$$\Rightarrow h = \frac{r}{\tan \theta} = 4r$$

$$V = \frac{\pi}{3} r^2 (4r)$$

$$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \cdot \frac{dr}{dt}$$



14. (B)

$$\frac{dy}{dx} = \sec x \cdot \tan x + \sec^2 x$$

$$= 2\sqrt{3} + (2)^2$$

$$= 2(\sqrt{3} + 2)$$

15. (A)

$$\begin{aligned} W &= \int \frac{2}{r^2} dr \\ &= \left[\frac{-2}{r} \right]_{1/2}^{\infty} \\ &= - \left(-\frac{2}{1/2} \right) = 4 \text{ J} \end{aligned}$$

16. (C)

$$\begin{aligned} \int_1^2 y \cdot dx &= \left(\frac{ax^3}{3} + 2x^2 - 3x \right)_1^2 = 10 \\ \Rightarrow a &= 3 \end{aligned}$$

17. (D)

$$\begin{aligned} x &= y^2 + y \\ 1 &= 2y \cdot \frac{dy}{dx} + \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{1}{(2y+1)} \end{aligned}$$

18. (C)

$$\begin{aligned} x &= t^2 - 24t + 6 \\ \frac{dx}{dt} &= 2t - 24 = 0 \\ t &= 12 \\ \frac{d^2x}{dt^2} &= 2 > 0 (\text{minima}) \\ \text{at } t &= 12 \end{aligned}$$

19. (C)

$$\begin{aligned} y &= \ln x^2 \\ \frac{dy}{dx} &= \frac{1}{x^2} \times 2x = \frac{2}{x} = \frac{2}{e} \end{aligned}$$

20. (D)

$$\begin{aligned} v &= bs^2 \\ a &= \frac{v dv}{ds} = bs^2 (2bs) \\ &= 2b^2 s^3 \end{aligned}$$

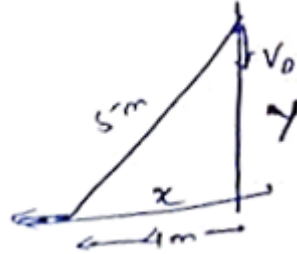
21. (D)

$$A = 4\pi r^2$$

$$\begin{aligned}\frac{dA}{dt} &= 8\pi r \frac{dr}{dt} \\ &= 8\pi(2)(10) \\ &= 160\pi\end{aligned}$$

22. (C)

$$\begin{aligned}x^2 + y^2 &= 5^2 \\ 2x \frac{dy}{dt} + 2y \frac{dy}{dt} &= 0 \\ \frac{dy}{dt} &= \frac{-x}{y} \frac{dx}{dt} \\ &= \frac{4}{3}(3) = 4 \text{ m/s}\end{aligned}$$



23. (D)

$$F = \frac{dP}{dt}$$

24. (A)

$$W = \text{area of } F \cdot x \text{ curve} = \frac{1}{2} \times 20 \times 10 = 100 \text{ J}$$

25. (A)

$$P \propto t$$

$$F = \frac{dP}{dt} = \text{constant}$$

$$\text{i.e. } F \propto t^0$$

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IIT – JEE: 2024

TW TEST (ADV)

DATE: 28/08/22

TOPIC: ATOMIC STRUCTURE

Answer Key

26. (B)	27. (A)	28. (A)	29. (C)	30. (C)
31. (C)	32. (B)	33. (C)	34. (C)	35. (C)
36. (B)	37. (C)	38. (A)	39. (A)	40. (D)
41. (C)	42. (A)	43. (B)	44. (C)	45. (B)
46. (D)	47. (A)	48. (C)	49. (C)	50. (C)

SOLUTIONS

26. (B)

$$E = \frac{1}{2}(E_1 + E_2) \Rightarrow \frac{1}{\lambda} = \frac{1}{2}$$

$$\therefore \lambda = 4800 \text{ \AA}$$

27. (A)

$$E = n \cdot \frac{hc}{\lambda}$$

$$\Rightarrow \frac{E \cdot \lambda}{hc} = \frac{10^{-7} \times 5000 \times 10^{-10}}{6.626 \times 10^{-34} \times 3 \times 10^8} = 2.5 \times 10^{11}$$

28. (A)

Na¹⁰⁺ ion has single electron

29. (C)

$$\frac{(2\pi r)_3}{(2\pi r)_2} = \frac{3^2}{2^2} = \frac{9}{4}$$

30. (C)

$$V_{n,z} = 2.188 \times 10^6 \frac{z}{n} \text{ m/s}$$

$$19.54 \times 11.2 \times 10^3 = 2.188 \times 10^6 \times \frac{1}{n} \Rightarrow n \approx 10$$

31. (C)

$$f_{n,z} = \frac{1}{T_{n,z}} \propto \frac{1}{n^3}$$

32. (B)

33. (C)

$$E_{n,z} = -13.6 \frac{z^2}{n^2} \text{ eV} \Rightarrow -13.6 = -13.6 \times \frac{z^2}{n^2} \Rightarrow z = 3$$

34. (C)

$$\text{P.E.} = -27.2 \times \frac{z^2}{n^2} = -27.2 \text{ eV}$$

35. (C)

$\Delta E_{2 \rightarrow 1}$ is even greater than $\Delta E_{\infty \rightarrow 2}$.

36. (B)

$$\Delta E = 13.6 z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}$$

$$\text{or } 47.2 = 13.6 \times z^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \Rightarrow z \approx 5$$

37. (C)

$$R = \frac{2\pi^2 m e^4}{(4\pi\epsilon_0)^2 h^3 c}$$

$$\Rightarrow R \propto m$$

Here, m becomes $\left(m - \frac{m}{4} = \frac{3m}{4} \right)$, then R becomes $\frac{3}{4} R$.

38. (A)

$$\text{Now, } v = \frac{c}{\lambda} = cRz^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$= cR \times 2^2 \left(\frac{1}{3^2} - \frac{1}{5^2} \right)$$

$$= \frac{64cR}{225}$$

39. (A)

$$\frac{\bar{\nu}_2}{\bar{\nu}_1} = \frac{z_2^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)_1}{z_1^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)_2}$$
$$\Rightarrow \frac{\bar{\nu}_2}{2.5 \times 10^5} = \frac{3^2 \left(\frac{1}{3^2} - \frac{1}{5^2} \right)}{4^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)}$$

$$\therefore \bar{\nu}_2 = 7.2 \times 10^4 \text{ cm}^{-1}$$

40. (D)

$$\Delta x = \frac{h}{4\pi m \cdot \Delta V_{\min}}$$

41. (C)

Number of orbit = Number of waves.

42. (A)

Theoretical

43. (B)

For p -orbital, $l = 1$

$$\therefore \text{Orbital angular momentum} = \sqrt{l(l+1)} \cdot \frac{h}{2\pi}$$
$$= \sqrt{1(1+1)} \cdot \frac{h}{2\pi} = \sqrt{2} \cdot \frac{h}{2\pi}$$

44. (C)

Number of orbitals of a particular type = $2l + 1$ and for g -orbital, $l = 4$

45. (B)

The p -orbital has only one nodal plane.

46. (D)

Informative

47. (A)

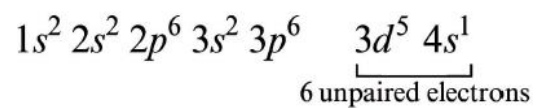
Orbitals are $3p, 4s$.

Number = $3 + 1 = 4$.

48. (C)

Pauli's exclusion principle limits the maximum capacity of electrons in an orbital equal to 2.

49. (C)



50. (C)

Number of unpaired electron in Fe^{2+} is 4.

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DATE: 28/08/22

TOPIC: TRIGONOMETRY - I

Answer Key

51. (B)	52. (B)	53. (C)	54. (D)	55. (D)
56. (C)	57. (C)	58. (D)	59. (B)	60. (C)
61. (A)	62. (A)	63. (C)	64. (C)	65. (D)
66. (C)	67. (B)	68. (B)	69. (A)	70. (B)
71. (D)	72. (A)	73. (D)	74. (D)	75. (D)

SOLUTIONS

51. (B)

$$\sec \theta + \tan \theta = 3$$

$$\sec \theta - \tan \theta = \frac{1}{3}$$

$$2\sec \theta = 3 + \frac{1}{3} = \frac{10}{3}$$

$$\sec \theta = \frac{5}{3}$$

$$\therefore \cos \theta = \frac{3}{5}$$

52. (B)

$$\sin \theta = \frac{4}{5} \text{ but } \theta \text{ lies in } 2^{\text{nd}} \text{ or } 4^{\text{th}}$$

$$\text{Quadrant Hence, } \sin \theta = \frac{4}{5} \text{ or } -\frac{4}{5}$$

53. (C)

$$\Rightarrow \sec^4 \theta (\cos^2 \theta) (1 + \sin^2 \theta) - 2 \tan^2 \theta$$

$$\Rightarrow \sec^2 \theta (1 + \sin^2 \theta) - 2 \tan^2 \theta \Rightarrow \sec^2 \theta + \tan^2 \theta - 2 \tan^2 \theta$$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = 1$$

54. (D)

$$d = 10\theta \text{ where } \theta = \frac{360^\circ}{60^\circ} \times 20 = \frac{2\pi}{3}$$

$$d = 10 \times \frac{2\pi}{3} \Rightarrow \frac{3d}{10\pi} = 2$$

55. (D)
 $\sin 4 < 0$.

56. (C)
 $\cos(A+B) = 1 \Rightarrow A = -B$
 $\therefore 2 + \tan(-B)\cot B = 2 - 1 = 1$

57. (C)
 $\sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$
 \therefore using this $K = \frac{1}{\sqrt{2}}$
 $\therefore \sqrt{2}K = 1$

58. (D)
 If $A+B = 45^\circ$ then $(1 + \tan A)(1 + \tan B) = 2$

59. (B)
 $\operatorname{cosec}^2 x + 25 \sec^2 x$
 $1 + \cot^2 x + 25(1 + \tan^2 x)$
 $1 + \cot^2 x + 25 + 25 \tan^2 x$
 $26 + 25 \tan^2 x + \cot^2 x$
 $\geq 2\sqrt{25}$
 ≥ 10
 \therefore Least value of $\operatorname{cosec}^2 x + 25 \sec^2 x$ is 36.

60. (C)
 $5 \cos \theta + 3 \left[\cos \theta \cdot \cos \left(\frac{\pi}{3} \right) - \sin \theta \cdot \sin \left(\frac{\pi}{3} \right) \right] + 3$
 $5 \cos \theta + \frac{3}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 3$
 $\frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 3$
 Now, $\lambda = \sqrt{\frac{169}{4} + \frac{27}{4}} + 3$
 $= \sqrt{\frac{196}{4}} + 3$
 $= 7 + 3 = 10$
 $\mu = -7 + 3 = -4$

$$\therefore \lambda - \mu = 14$$

61. (A)

$$\begin{aligned} & \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) \\ \Rightarrow & \cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right) \\ \Rightarrow & 2\left[\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right)\right] \\ \Rightarrow & 2\left[\cos^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{\pi}{8}\right)\right] \\ \Rightarrow & 2\left[1 - 2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right] \\ \Rightarrow & 2\left[1 - \frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)\right] \\ \Rightarrow & 2\left[1 - \frac{1}{2} \times \frac{1}{2}\right] = 2\left[\frac{3}{4}\right] = \frac{3}{2} \end{aligned}$$

62. (A)

$$\begin{aligned} & (\alpha + \beta) - (\alpha - \beta) = 2\beta \\ & \tan[(\alpha + \beta) - (\alpha - \beta)] = \tan 2\beta \\ & \frac{\tan(\alpha + \beta) - \tan(\alpha - \beta)}{1 + \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} = \tan 2\beta \\ \therefore & \tan(\alpha + \beta) = \frac{3}{4} \text{ and } \tan(\alpha - \beta) = \frac{5}{12} \\ \Rightarrow & \tan 2\beta = \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \times \frac{5}{12}} = \frac{36 - 20}{48 + 15} \\ & \tan 2\beta = \frac{16}{63} \end{aligned}$$

63. (C)

$$\begin{aligned} & \alpha = \frac{\pi}{2} - \beta \\ & \tan \alpha = \cot \beta \Rightarrow \tan \alpha \cdot \tan \beta = 1 \\ & \text{Now, } \alpha - \beta = \gamma \\ & \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta} = \tan \gamma \\ & \tan \alpha - \tan \beta = 2 \tan \gamma \\ & \tan \alpha = \tan \beta + 2 \tan \gamma \end{aligned}$$

64. (C)

$$\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{\sin 3\theta}{4}$$

$$\therefore \sin 30^\circ \times \frac{\sin 30^\circ}{4} = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$$

65. (D)

$$\frac{1 + \cos 20^\circ}{2} - \cos 50^\circ (\cos 10^\circ - \cos 50^\circ)$$

$$\frac{1 + \cos 20^\circ}{2} - 2 \cos 50^\circ \sin 30^\circ \sin(20^\circ)$$

$$\frac{1 + \cos 20^\circ}{2} - \cos 50^\circ \sin 20^\circ$$

$$\frac{1 + \cos 20^\circ}{2} - \frac{1}{2} [\sin(70^\circ) - \sin(30^\circ)]$$

$$\frac{1 + \cos 20^\circ - \sin 70^\circ + \sin 30^\circ}{2}$$

$$\frac{\frac{3}{2} + \cos 20^\circ - \cos 20^\circ}{2} = \frac{3}{4}$$

66. (C)

$$\frac{\sin\left(2^9 \times \frac{\pi}{2^{10}}\right)}{2^9 \times \sin\left(\frac{\pi}{2^{10}}\right)} \times \sin\left(\frac{\pi}{2^{10}}\right) = \frac{1}{512}$$

67. (B)

$$\tan^2 x = \frac{2}{3} \text{ from the given relation.}$$

Hence, (B).

68. (B)

$$\tan 30^\circ + \tan 15^\circ = -p$$

$$\tan 30^\circ \cdot \tan 15^\circ = q$$

$$\therefore \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} = 1$$

$$\Rightarrow -p = 1 - q$$

$$\Rightarrow q - p = 1$$

$$\therefore 2 + q - p = 3$$

69. (A)

$$\sin \alpha + \sin \beta = \frac{21}{65} \Rightarrow 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{21}{65}$$

$$\cos \alpha + \cos \beta = \frac{-27}{65} \Rightarrow 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) = \frac{-27}{65}$$

$$\therefore \tan \left(\frac{\alpha + \beta}{2} \right) = \frac{-21}{27}$$

70. (B)

Using componendo and dividendo

$$\frac{1 - \cos x}{1 + \cos x} = \frac{2 - \cos y - 2 \cos y + 1}{2 - \cos y + 2 \cos y - 1}$$

$$\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \frac{3(1 - \cos y)}{1 + \cos y} = \frac{3 \times 2 \sin^2 \frac{y}{2}}{2 \cos^2 \frac{y}{2}}$$

$$\therefore \tan^2 \frac{x}{2} = 3 \tan^2 \frac{y}{2}$$

$$\tan^2 \frac{x}{2} \cdot \cot^2 \frac{y}{2} = 3$$

$$\tan \frac{x}{2} \cdot \cot \frac{y}{2} = \sqrt{3}$$

71. (D)

$$\tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1$$

$$\frac{1}{2} + \frac{\tan C}{2} + \tan C = 1$$

$$\frac{3 \tan C}{2} = \frac{1}{2} \Rightarrow \tan C = \frac{1}{3}$$

72. (A)

$$\frac{\cos(A - B)}{\cos(A + B)} = \frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{1 + \tan A \tan B}{1 - \tan A \tan B}$$

$$= \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$$

73. (D)

$$\tan[2(A + B)] = \frac{2 \tan(A + B)}{1 - \tan^2(A + B)} \quad \left[\tan(A + B) = \frac{2+1}{1-2} = -3 \right]$$

$$\therefore \tan 2(A + B) = \frac{2 \times (-3)}{1 - (-3)^2} = \frac{-6}{-8} = \frac{3}{4}$$

74. (D)

Let $\tan \frac{\theta}{2} = x$ then

$$\sin \theta + \cos \theta = -\frac{1}{5}$$

$$\frac{2x}{1+x^2} + \frac{1-x^2}{1+x^2} = -\frac{1}{5}$$

$$10x + 5 - 5x^2 = -x^2 - 1$$

$$4x^2 - 10x - 6 = 0$$

$$2x^2 - 5x - 3 = 0$$

75. (D)

$$\frac{(\sin \theta + \cos \theta)^2}{(\cos \theta - \sin \theta)^2} = \frac{1 + \sin 2\theta}{1 - \sin 2\theta} = \frac{1 + P}{1 - P}$$