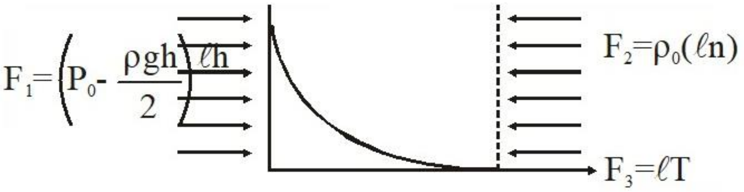
IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 16/07/23**

TOPIC: PROPERTIES OF MATTER

SOLUTIONS

1. (B)

In the FBD only horizontal forces are shown.



$\ell =$ length of side wall of aquarium

In equation

$F_1 - F_2 - F_3 = 0$

$h = \sqrt{\frac{2T}{\rho g}}$.

2. (D)

Strain = $\frac{\Delta L}{L} = \alpha \Delta T$

Elastic potential energy = $E = \frac{1}{2}(stress)(strain)$

Per unit volume

$E = \frac{y}{2}(strain)^2 = \frac{y}{2}\alpha^2 \Delta T^2$

$= \frac{y}{2}\left(\frac{\gamma}{3}\right)T^2$

$$\boxed{E = \frac{y\gamma^2 T^2}{18}}$$

3. (C)

$T = M\omega_0^2 L$

$T' = M\omega_0^2 \frac{L}{2}$

$\sigma = \frac{T}{A} = \frac{T'}{2A}$

$\frac{2\omega_0^2}{\omega^2} = \frac{T}{T'} = \frac{1}{2}$

$\frac{\omega_0}{\omega} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

4. (A)

$$\int_0^{t_1} dt = \frac{-A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2}dy \Rightarrow t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H}-\sqrt{\frac{H}{2}}\right]$$

$$\int_0^{t_2} dt = \frac{-A}{a\sqrt{2g}}\int_{H/2}^{0} y^{-1/2}dy \Rightarrow t_2 = \frac{A}{a}\left[\sqrt{\frac{H}{g}}\right]$$

$$\frac{t_1}{t_2} = \sqrt{2}-1 = 0.414$$

Correct option (A)

5. (A)

$$v = \sqrt{2(g+a)h},\ \ t = \sqrt{\frac{2(H-h)}{(g+a)}}$$

$$x = \sqrt{2(g+a)h}.\sqrt{\frac{2(H-h)}{(g+a)}} = 2\sqrt{h(H-h)} = 2\sqrt{2\times 3}$$

4.9 m

Correct option (A)

6. (ABC)

In the case of dropping from height ‘h’

Speed just before striking = $\sqrt{2gh} = v_0$

Speed just after striking = $\frac{v_0}{\sqrt{2}} = \sqrt{gh} = v_T$

In the case of dropping from height ‘4h’

Just before striking $v_0^1 = \sqrt{8gh}$

Just after striking $v = \frac{v_0^1}{\sqrt{2}} = \frac{2\sqrt{2gh}}{\sqrt{2}} = 2v_T$

Also viscous force belonging to $v_T = mg - \frac{mg}{3} = \frac{2mg}{3}$

$\therefore$ Viscous force belong to $2v_T = \frac{4}{3}mg$

7. (AB)

Let ℓ be the length of the capillary tube. Let x be the length of the capillary tube dipped in the liquid at which the liquid level inside and outside the tube is the same.

$\therefore$ initial pressure $p_1 = p_a$ (atmospheric) final pressure $p_2 = p_a + \frac{2\alpha}{r}$, where σ is surface tension and r is radius of the tube.

Initial volume of air $V_1 = \ell a$, where a is area of cross-section

Final volume of air $V_2 = (\ell - x)a$.

By Boyles law

$$p_1V_1 = p_2V_2$$

$$\therefore \quad p_a \times \ell = p_a \times \ell + p\ell - P_a x - px$$

$$\therefore \quad x = \frac{p\ell}{(p_a + p)} = \frac{\left(\frac{2\sigma}{r}\right)\ell}{pa + \frac{2\sigma}{r}} = \frac{\ell}{1 + \frac{pa}{\left(\frac{2\sigma}{r}\right)}}$$

$\therefore$ excess pressure is 5 kN/m^2 (approximately)

8. (ACD)

$$F = -\eta A \frac{dv}{dx}$$

Viscous force

9. (AB)

$$\Delta l = \frac{Fl}{AY}$$

AY can be increased by making F or l two times.

10. (BCD)

$v = v_T\left(1 - e^{-t/\tau}\right)$ Where $\tau = \frac{2\rho r^2}{9\eta}$

11. (ABC)
Conceptual

12. (C)
Conceptual

13. (BD)
Conceptual

14. (ABC)

Tension in the rod at a point $\frac{l}{4}$ from the centre $T = \int_{l/4}^{l/2} \frac{m}{l} x\omega^2 dx = \frac{3}{32} m\omega^2 l$

Elongation in the rod $\Delta l = \frac{m\omega^2}{2AlY} \int_{-l/2}^{l/2} \left(x^2 - \frac{l^2}{4}\right) dx = \frac{m\omega^2 l^2}{12AY}$

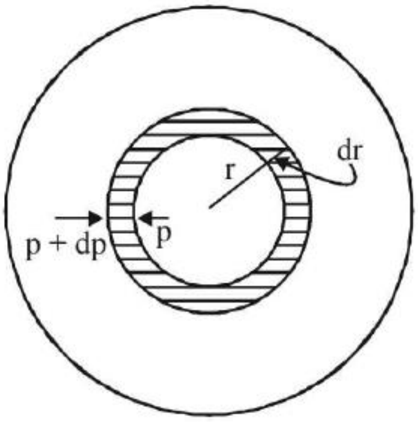
15. (BC)
Let us consider an elemental mass dm shown in the shaded portion.
Here, $P4\pi r^2 - (P + dP)4\pi r^2$

$$= \frac{GMr}{R^3}\rho\left(4\pi r^2\right)dr$$

$$\therefore \quad -\int_0^P dp = \frac{GM\rho}{R^3}\int_R^r r\,dr$$

$$\therefore \quad P = \frac{GM\rho}{2R^3}\left[R^2 - r^2\right]$$

$$\therefore \quad \frac{P\left(r = 3R/4\right)}{P\left(r = 2R/3\right)} = \frac{\left[R^2 - \frac{9R^2}{16}\right]}{\left[R^2 - \frac{4R^2}{9}\right]} = \frac{\frac{7R^2}{26}}{\frac{5R^2}{9}} = \frac{63}{80}$$



$$\text{and} \quad \frac{P\left(r = 3R/5\right)}{P\left(r = 2R/5\right)} = \frac{\left[R^2 - \frac{9R^2}{25}\right]}{\left[R^2 - \frac{4R^2}{25}\right]} = \frac{16}{21}$$

(B) and (C) are correct option(s).

16. (8)

$$\frac{\Delta P_1}{\Delta P_2} = \frac{0.01}{0.02} = \frac{1}{2}$$

$$\text{or} \quad \frac{4T/r_1}{4T/r_2} = \frac{1}{2} \quad or \quad \frac{r_1}{r_2} = 2$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = 8$$

17. (2)

Let v is the volume of balls

$$V\left(\rho_1 + \rho_2\right)g = \left(6\pi\eta rv\right)2 --- (1)$$

$$V\rho_1 g - T = 6\pi\eta rv --- (2)$$

$$\text{Breaking stress} = \frac{T}{A}$$

$$= \frac{\left(\frac{1}{4}\right)10\left(9-5\right)10^3}{0.5\times10^{-4}} = 2\times10^8$$

18. (3)

Here due to force of buoyancy the bubble will move up and so viscous force which opposes the motion will act downward and as weight of bubble is zero, in dynamic equilibrium,

Th = F,

$$\text{i.e.} \ \frac{4}{3}\pi r^3\sigma g = 6\pi\eta r u_T$$

$$\text{or} \ \eta = \frac{2}{9}\frac{\sigma r^2 g}{u_T} = \frac{2}{9}\times\frac{1.47\times\left(0.1\right)^2\times980}{2.1}$$

$$\text{i.e.} \ \eta = 1.524 \text{ poise}$$

19. (2)

$l = 1m,\ A = 2\times10^{-6}m^2,\ F = 20N,\ \Delta l = 0.5\times10^{-4}m$

$$Y = \frac{F/A}{\Delta l/l}$$

$$= \frac{20}{2\times10^{-6}} \times \frac{1}{0.5\times10^{-4}}$$

$$= \frac{20}{1\times10^{-10}}$$

$$= 20\times10^{10}$$

$$= 2\times10^{11}\,N/m^2$$

In $10^{11}\ N/m^2 \quad Y = 2$

20. (6)

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$\therefore hr = \frac{2T\cos\theta}{\rho g} = \text{constant}$$

$$\therefore h_1r_1 = h_2r_2 \text{ or } h_2 = \frac{h_1r_1}{r_2}$$

Substituting the values

$$h_2 = (2.0)\,(3) \quad \left(\frac{r_2}{r_1} = \frac{1}{3}\right)$$

$$= 6.0\text{ cm}$$

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TOPIC: ELECTROCHEMISTRY

SOLUTIONS

21. (B)

$Cl_2(g) + 2Br^-_{(eq)} \longrightarrow 2Cl^-(eq) + Br_{2\,(g)}$

$$Q = \frac{[Cl^-]^2 P_{Br_2}}{P_{Cl_2}.[Br^-]^2} = \frac{(0.01)^2 \times 0.01}{1 \times (0.01)^2} = 0.01$$

$$E = E^\circ - \frac{0.06}{2}\log(0.01)$$

$= 0.29 - 0.03 \log 10^{-2}$

$= 0.35$ V

22. (A)

$2H^+(eq) + 2e^- \longrightarrow H_2(g)$

When $P_{H_2} = 1$ atm

$$E = E^\circ - \frac{0.06}{2}\log\frac{1}{1^2} = E^\circ$$

When $P_{H_2} = 100$ atm

$$E = E^\circ - \frac{0.06}{2}\log\frac{100}{1^2} = E^\circ - 0.06$$

$\therefore$ change in reduction pot. is 0.06 V

23. (B)

$Ag \longrightarrow Ag^+ + e^-$

$$E = E^\circ - \frac{0.06}{1}\log[Ag^+]$$

$$-0.209 = -0.799 - \frac{0.06}{1}\log\frac{Ksp}{[Cl^-]}$$

$$-0.209 = -0.799 - \frac{0.06}{1}\log\frac{Ksp}{0.1}$$

$\therefore Ksp = 10^{-11}$

24. (B)
Eq of A = eq of B = eq of C

$$\frac{2.1}{7/x} = \frac{2.7}{27/y} = \frac{7.2}{48/z}$$

$$x : y : z :: 1 : 3 : 2$$

25. (B)

$$CH_3COOH\ (eq) + NaOH\ (eq) \longrightarrow CH_3COONa\ (eq) + H_2O\ (\ell)$$

$0.015 \times V$	$0.015 \times V$	0	
0	0	$0.015 \times V$	

$$\therefore \quad [CH_3COONa] = \frac{0.015 \times V}{2V} = \frac{0.015M}{2}$$

$$\Lambda_m = \frac{K \times 1000}{M} = \frac{6.3 \times 10^{-4} \times 1000}{0.015/2} = 84\ S\ cm^2\ mole^{-1}$$

26. (AB)
Salt bridge is introduced to keep the solutions of two electrodes separate, such that the ions in electrode do not mix freely with each other. But it cannot stop the process of diffusion. It does not participate in the chemical reaction. However, it is not necessary for occurence of cell reaction, as we know that designs like lead accumulator, there was no salt bridge, but still reactions takes place.

27. (ABD)
The species having less reduction potential with respect to NO_3^- ($E^o = 0.96$ V) will be oxidised by NO_3^-. These species are V, Fe, Hg.

28. (A)
The species having higher reduction potential oxidizes the species having lower reduction potential.

29. (D)

$$2H^+(eq) + 2e^- \longrightarrow H_2(g)$$

Let $P_{H_2} = 1$ bar

$\therefore$ When pH = 0 $\Rightarrow [H^+] = 1$ M

$$E = E^\circ - \frac{0.06}{2} \log 1 = E^\circ$$

& when pH = ? $\Rightarrow [H^+] = 10^{-7}$ M

$$E = E^\circ - \frac{0.06}{2} \log \frac{1}{(10^{-7})^2} = E^\circ - 0.42$$

$\therefore$ Reduction pot, decrease by 0.42 V

30. (AB)
During electro refining of Cu, Cu^{2+} dissolve & deposit at anode & cathode respectively.

31. (CD)

$$2H_2SO_4 + Pb(s) + PbO_2\ (s) \rightleftharpoons 2PbSO_4(aq) + 2H_2O(\ell)$$

32. (BCD)
(A) : During electrolysis Cu at anode will oxidise as well as Cu^{2+} of solution will reduce at cathode.
(B) : During electrolysis

At cathode : $2H^+(g) + 2e^- \longrightarrow H_2(g)$

At anode : $H_2O(\ell) \longrightarrow \frac{1}{2}O_2(g) + 2H^+(g) + 2e^-$

(C) : During electrolysis

At cathode : $Na^+ + e^- \longrightarrow Na$

At anode : $2OH^- \longrightarrow \frac{1}{2}O_2(g) + H_2O(\ell) + 2e^-$

(D) : During electrolysis

At cathode : $2H^+ (aq) + 2e^- \longrightarrow H_2 (g)$

At anode : $H_2O(\ell) \longrightarrow \frac{1}{2}O_2(g) + 2H^+(g) + 2e^-$

33. (AB)

$H_2O(\ell) \longrightarrow H_2(g) + \frac{1}{2}O_2(g)$

At cathode at anode

Mole of H_2O electrolyzed $= \frac{270}{18} = 15$ mole

Equivalent of H_2O electrolyzed = 15 × 2 = 30

Equivalent of H_2 gas released = Equivalent of O_2 gas released = 30

mole of H_2 gas $= \frac{30}{2} = 15$ mole

mole of O_2 gas $= \frac{30}{4} = 7.5$ mole

Volume of O_2 gas = 7.5 × 22.4 L = 168 L

Total volume of gas = (15 + 7.5) × 22.4 = 504 L

Faraday of electricity consumed $= 30 \times \frac{100}{75} = 40$ F

34. (BC)

Specie giving higher reduction potential can oxidize the specie having lower reduction potential.

35. (AD)

At cathode : $2H^+(aq) + 2e^- \longrightarrow H_2 (g)$

At anode : $Ag(s) + Cl^- (aq) \longrightarrow AgCl (s) + e^-$

Overll cell rxn : $2Ag(s) + 2H^+ (aq) + 2Cl^- (aq) \longrightarrow H_2(g) + 2AgCl (s)$

$$Q = \frac{P_{H_2}}{[H^+][Cl^-]^2}$$

$$E = E^\circ - \frac{0.06}{2}\log Q$$

With increase in value of Q, EMF of cell decrease.

36. (22)

$$\Lambda_m^{\infty} = \frac{K}{1000 \times \text{Solubility}}$$

$$1.5 \times 10^{-4} \times 3 = \frac{9 \times 10^{-6}}{1000 \times S}$$

$S = 2 \times 10^{-5}$

$Ksp = 3^3 \times S^4$

$= 27 \times (2 \times 10^{-5})^4 = 4.32 \times 10^{-18}$

37. (0.2)

Cell reaction :

$$H_{2(g)} + 2AgCl_{(s)} \longrightarrow 2H^+_{(eq)} + 2Ag_{(s)} + 2Cl^-_{(eq)}$$

$$Q_C = \frac{[H^+]^2[Cl^-]^2}{P_{H_2}} = \frac{(10^{-6})^2(10^{-6})^2}{1} = 10^{-24}$$

$$E = E^\circ - \frac{0.06}{n} \log Q$$

$$0.92 = E^\circ - \frac{0.06}{2} \log 10^{-24}$$

$E^\circ = 0.2V$

38. (10)

$Mg\ (s) + Cu^{2+}\ (aq) \longrightarrow Cu\ (s) + Mg^{2+}\ (aq)$

Initially : $E = 2.7 = E^\circ - \frac{RT}{2 \times F} \ell n\left(\frac{1}{1}\right) \Rightarrow E^\circ = 2.7$

When : $[Mg^{2+}] = x$

$$E = 2.67 = 2.7 - \frac{300}{2 \times 11500} \times \ell n\left(\frac{x}{1}\right)$$

$\ell n\ x = 2.3 \Rightarrow x = 10$

39. (2)

$H_2(g) + M^{4+}(aq) \longrightarrow 2H^+(aq) + M^{2+}(aq)$

$$Q = \frac{[H^+]^2[M^{2+}]}{P_{H_2}[M^{4+}]} = \frac{1 \times [M^{2+}]}{1 \times [M^{4+}]} = 10^x$$

$$E = 0.092 = 0.151 - \frac{0.059}{2} \log 10^x \Rightarrow x = 2$$

40. (–11.62)

$$2A\,(s) + B^{2n+}\,(aq) \longrightarrow 2A^{n+}\,(aq) + B(s)$$

Given $\Delta H^\circ = 2\Delta G^\circ$

$\because \quad \Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\therefore \quad \Delta G^\circ = 2\,\Delta G^\circ - T\Delta S^\circ$

$$\Delta G^\circ = T\Delta S^\circ$$

$\because \quad \Delta G = \Delta G^\circ + RT\,\ell n\,Q = 0$

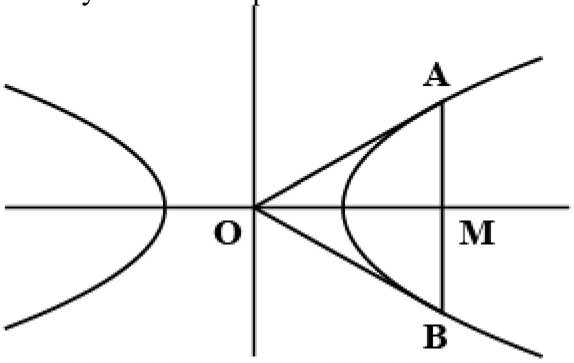
$$\Delta G^\circ = -8.3 \times 300 \times \ell n\left(\frac{2^2}{1}\right) = 300 \times \Delta S^\circ$$

$$\Delta S^\circ = -\,8.3 \times \ell n\,4 = -8.3 \times 2 \times 0.7 = -11.62 \text{ J/K}$$

TOPIC: HYPERBOLA

SOLUTIONS

41. (D)
Let the length of the double ordinate be 2ℓ
$\therefore\ AB = 2\ell$ and AM = BM = ℓ
Clearly ordinate of point A is ℓ.



The abscissa of the point A is given by
$\frac{x^2}{a^2} - \frac{l^2}{b^2} = 1 \Rightarrow x = \frac{a\sqrt{b^2+l^2}}{b}$
$\therefore$ A is $\left(\frac{a\sqrt{b^2+l^2}}{b}, l\right)$
Since ΔOAB is equilateral triangle, therefore
OA = AB = OB = $2l$
Also, $OM^2 + AM^2 = OA^2$
$\therefore\ \frac{a\left(b^2+l^2\right)}{b} + l^2 = 4l^2$
We get $l^2 = \frac{a^2b^2}{3b^2-a^2}$
Since $l^2 > 0\ \ \therefore\ \frac{a^2b^2}{3b^2-a^2} > 0 \Rightarrow 3b^2 - a^2 > 0$
$\Rightarrow 3a^2\left(e^2-1\right) - a^2 > 0 \Rightarrow e > \frac{2}{\sqrt{3}}$

42. (A)
The equation of the hyperbola is $x^2 - 2y^2 - 2x + 8y - 1 = 0$
Or $(x-1)^2 - 2(y-2)^2 + 6 = 0$
Or $\frac{(x-1)^2}{-6} + \frac{(y-2)^2}{3} = 1$ or $\frac{(y-2)^2}{3} - \frac{(x-1)^2}{6} = 1 \to 1$

Or $\frac{Y^2}{3}-\frac{X^2}{6}=1$, where $X = x - 1$ and $Y = y - 2 \to 2$

$\therefore$ the centre = (0, 0) in the X-Y coordinates.

$\therefore$ the centre = (1, 2) in the x-y coordinates. using $\to 2$

If the transverse axis be of length $2a$, then $a=\sqrt{3}$, since in the equation (1) the transverse axis is parallel to the y-axis. If the conjugate axis is of length $2b$, then $b=\sqrt{6}$

But $b^2=a^2\left(e^2-1\right)$

$\therefore \quad 6=3\left(e^2-1\right)$

$\therefore \quad e^2=3$ or $e=\sqrt{3}$

The length of the transverse axis $=2\sqrt{3}$

The length of the conjugate axis $=2\sqrt{6}$

Latus rectum $4\sqrt{3}$

43. (D)

$y^2=8k, xy=-1$

Let $P\left(t,\frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy=-1$ at $P\left(t,\frac{-1}{t}\right)$ is

$\frac{xy_1+yx_1}{2}=-1$

$\frac{-x}{t}+yt=-2$

$y=\frac{x}{t^2}+\left(\frac{-2}{t}\right)$.................(1)

If (1) is tangent to the parabola $y^2=8x$ then

$\frac{-2}{t}=\frac{2}{1/t^2}\Rightarrow t^3=-1$

$t=-1$

$\therefore$ Common tangent is y = x+2

44. (A)

$y=4x^2$ and $\frac{1}{4}y=x^2$

Using $\frac{1}{4a^2}y-\frac{y^2}{16}=1$

$\Rightarrow 4y-a^2y^2=16a^2$

$\Rightarrow a^2y^2-4y+16a^2=0$

$\Rightarrow D\geq 0$ for intersection of two curves

$\Rightarrow 16-4a^2\left(16a^2\right)\geq 0$

$\Rightarrow 1-4a^4\geq 0$

$\Rightarrow \left(2a^2\right)\leq 1$

$\Rightarrow \left|\sqrt{2}a\right| \le 1 \Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

45. (B)

Normal at (6, 3) is

$\frac{a^2 x}{6} + \frac{b^2 y}{3} = a^2 + b^2$

$\Rightarrow \frac{9a^2}{6} = a^2 + b^2 \Rightarrow \frac{3}{2} = 1 + \frac{b^2}{a^2}$

$\therefore \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow e^2 - 1 = \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

46. (A)

The difference between the focal distances is a constant for a hyperbola.

For a rectangular hyperbola latusrectum = transverse axis.

$S(2, 0)$ $S^{|}(h, k)$ $P(0, 0)$ $\left|S^{|}p - Sp\right| = 4 \Rightarrow \left|\sqrt{h^2 + k^2} - 2\right| = 4 \Rightarrow \sqrt{h^2 + k^2} = 6 \Rightarrow h^2 + k^2 = 36$

Locus of (h, k) is $x^2 + y^2 = 36$

47. (BC)

$y^2 = 32x$

Let equation of tangent $y = mx + \frac{8}{m}$

$\frac{64}{m^2} = \frac{8}{9}m^2 - \frac{8}{9}$

$m = \pm 3$, $y = \pm 3x \pm 8/3$.

48. (ABC)

Any point on $xy = c^2$ is $\left(ct, \frac{c}{t}\right)$. As it lies on the given circle, we get

$c^2t^2 + \frac{c^2}{t^2} = a^2 \Rightarrow c^2t^4 - a^2t^2 + c^2 = 0$

Thus $t_1t_2t_3t_4 = 1$, $t_1 + t_2 + t_3 + t_4 = 0$, $\Sigma t_1t_2 = - -\frac{a^2}{c^2}$, $\Sigma t_1t_2t_3 = 0$

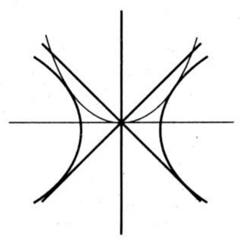
Thus, (a), (b), (c) are true.

49. (CD)

(α, α^2) lie on the parabola $y = x^2$

(α, α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ in 1st and 2nd quadrant

$\therefore$ Asymptotes are $y = \pm 2x$



$\therefore \quad 2\alpha < \alpha^2$

$\Rightarrow \quad \alpha < 0$ or $\alpha > 2$

and $-2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore\ \alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

50. (BC)

$S_1P = S_2P \Rightarrow a - e\alpha = E\alpha - \left(\frac{a}{2}\right)$. Also, $\alpha = \frac{ae + \frac{a}{2}E}{2}$

Eliminating α we get $E^2 + 3eE + (2e^2 - 6) = 0 \ \Rightarrow\ E = \frac{\sqrt{e^2+24} - 3e}{2}$.

51. (BCD)

Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Here $\frac{3}{4} = 1 - e^2 \Rightarrow e = \frac{1}{2}$

Foci are $(\pm 1, 0)$

Now the hyperbola is having same focus i.e. $(\pm 1, 0)$. Let e_1 be the eccentricity of hyperbola

$2ae_1 = 2$

But $2a = \frac{1}{2}$ So, $e_1 = 4$

$b^2 = a^2\left(e_1^2 - 1\right) = \frac{1}{16}(16-1) = \frac{15}{16}$

So, the equation of the hyperbola is

$\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{15}{16}} = 1 \Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = \frac{1}{16}$

Its distance between the directrices $= \frac{2a}{e_1} = \frac{1}{2\times 4} = \frac{1}{8}$ units

Length of latus-rectum $= \frac{2b^2}{a}$

$= \frac{2\times 15\times 4}{16\times 1} = \frac{15}{2}$ units

52. (C)

Equation of chord joining θ and ϕ

$\frac{x}{a}\cos\frac{\theta-\phi}{2} - \frac{y}{b}\sin\frac{\theta+\phi}{2} = \cos\frac{\theta+\phi}{2}$

It passes through (ae, 0)

$\therefore\ e\cos\frac{\theta-\phi}{2} = \cos\frac{\theta+\phi}{2}$

$$\therefore \quad \frac{\cos\frac{\theta-\phi}{2}}{\cos\frac{\theta+\phi}{2}} = \frac{1}{e}$$

$$\frac{\cos\frac{\theta-\phi}{2} - \cos\frac{\theta+\phi}{2}}{\cos\frac{\theta-\phi}{2} + \cos\frac{\theta+\phi}{2}} = \frac{1-e}{1+e}$$

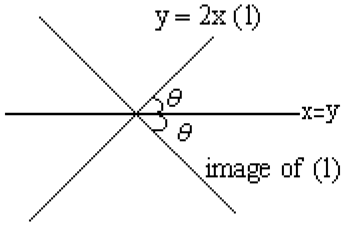
$$\frac{2\sin\frac{\theta}{2}\sin\frac{\phi}{2}}{2\cos\frac{\theta}{2}\cos\frac{\phi}{2}} = \frac{1-e}{1+e} \quad \Rightarrow \quad \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{1-e}{1+e}$$

Since the chord also passes thru (–ae, 0)

Similarly as above, we get $\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{1+e}{1-e}$

53. (BD)

Other asymptote is the image of $y = 2x$ in the line $x = y$ .i.e, $x = 2y$



$\Rightarrow$ Hyperbola is $(x-2y)(2x-y) = K$

$\because$ It passes through (3, 4) $\Rightarrow K = -10$

$\because$ angle between asymptotes $= 2\sec^{-1} e$

$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right) = 2\sec^{-1} e \Rightarrow e = \frac{\sqrt{10}}{3}$

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54. (ABD)

For the ellipse: a = 5 & $e = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$\therefore$ ae = 4

$\therefore$ the focii are (- 4, 0) and (4, 0)

For the hyperbola

ae = 4, e = 2

$\therefore$ e = 2

$b^2 = 4(4-1) = 12$

$b = \sqrt{12}$

55. (ABCD)

Given hyperbola can be written as $\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

$\Rightarrow \quad \frac{X^2}{16} - \frac{Y^2}{9} = 1 \quad$ {where X = x – 1, Y = y = 1}

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$$

Directrices are $X = \pm\frac{a}{e}$

$\Rightarrow \quad x-1=\pm\frac{16}{5} \quad \Rightarrow \quad x=\frac{21}{5}$ and $x=-\frac{11}{5}$

Length of Latus rectum $=\frac{2b^2}{a}=\frac{9}{2}$

and focii are

$\Rightarrow \quad X=\pm\,ae, Y=0 \;\Rightarrow \quad$ (6, 1) and (–4, 1)

56. (8)

Given that $H:\frac{y^2}{25}-\frac{x^2}{16}=1$

Equation of tangent to the hyperbola is : $y=mx+\sqrt{25-16m^2}$

Passes through (4, 1)

$\Rightarrow\; y=mx+\sqrt{25-16m^2}$

$\Rightarrow\; 1=4m+\sqrt{25-16m^2}$

$\Rightarrow\; 4m^2-m-3$

$\Rightarrow\; m_1=1,\; m_2=\frac{-3}{4}$

$\Rightarrow\; |m_1|=1,\; |m_2|=\frac{3}{4}$

Equation of tangents with slopes 1 & $\frac{3}{4}$

$\Rightarrow\; y=x-3 \qquad ...(\text{i})$

$\Rightarrow\; y=\frac{3}{4}x-4 \qquad ...(\text{ii})$

(i) & (ii), $\Rightarrow\; Q\equiv(-4,-7)$

$P\equiv(4,1)$

$\Rightarrow\; (PQ)^2=8^2+8^2=128$

On converting (i) and (ii) in intercept form we get,

(i) $\Rightarrow \alpha=3$, (ii) $\Rightarrow \beta=\frac{16}{3}$

$\Rightarrow\; \frac{(PQ)^2}{\alpha\beta}=\frac{128}{3\times\frac{16}{3}}=8$

57. (306)

Given, equation of hyperbola,

$H_n:\frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n\in\mathbb{N}$

Now using the eccentricity formula we get,

$e^2=1+\frac{3+n}{1+n}$

$\Rightarrow\; e^2=\frac{2n+4}{n+1}=\frac{2(n+2)}{n+1}$

Now check when

$(n+1) = 9, 25, 49, ...$ is perfect square and then at the same time $2(n+2)$ should also be perfect square

So, checking for $n = 8 \rightarrow e^2 = \frac{20}{9}$

$n = 24 \rightarrow e^2 = \frac{52}{25}$

$n = 48 \rightarrow e = \frac{100}{49} \Rightarrow e = \frac{10}{7}$

Hence, for $n = 48$ both $n + 1$ and $2(n + 2)$ are perfect square.

Now we know that, Length of latusrectum is given by

$l = \frac{2b^2}{a} = \frac{2(n+3)}{\sqrt{n+1}} = \frac{2 \times 51}{7}$

Hence, $21l = \frac{42 \times 51}{7} = 306$

58. (3)

$H: x^2 - y^2 = 1,\ E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$e_H = \sqrt{2}$ & $e_E = \frac{1}{e_H} = \frac{1}{\sqrt{2}}$

For hyperbola

$e^2 = 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

Also given that the common tangent of *H* & *E* is $y = \sqrt{\frac{5}{2}}x + k$ (i.e. $m = \sqrt{\frac{5}{2}}$)

We know that the condition for common tangent of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ is

$a^2m^2 + b^2 = A^2m^2 - B^2$

Now, for common tangency :

$\frac{5}{2}a^2 + b^2 = \frac{5}{2} - 1$

$\frac{5}{2} + \frac{b^2}{a^2} = \frac{3}{2a^2} \Rightarrow a^2 = \frac{1}{2}$

$\therefore\ a^2 + b^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\Rightarrow 4(a^2 + b^2) = 3$

59. (20)

Given equation of the hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If *e* is eccentricity of the hyperbola, then

$e^2 = 1 + \frac{b^2}{a^2}$

$1 + \frac{b^2}{a^2} = \frac{5}{2} \Rightarrow b^2 = \frac{3a^2}{2}$

Length of the latus-rectum is given by $\frac{2b^2}{a}$

i.e., $\frac{2b^2}{a}=6\sqrt{2}$

$\frac{2}{a}\times\frac{3a^2}{2}=6\sqrt{2}$

$\Rightarrow\ a=2\sqrt{2}$ and $b^2=\frac{3}{2}\times 8=12$

$\Rightarrow\ b=2\sqrt{3}$

If $y=mx+c$ is a tangent to the hyperbola, then $c^2=a^2m^2-b^2$

$\therefore\quad c^2=4\times 8-12$

Hence $c^2=20$

60. (4)

Given that $\lambda x-2y=\mu$ is a tangent to the curve $a^2x^2-y^2=b^2$

So, $a^2x^2-\left(\frac{\lambda x-\mu}{2}\right)^2=b^2$

$\Rightarrow\ \left(4a^2-\lambda^2\right)x^2+2\lambda\mu x-\mu^2-4b^2=0$

So for the line to be tangent, the condition is discriminant = 0

i.e. $4\lambda^2\mu^2+4\left(4a^2-\lambda^2\right)\left(\mu^2+4b^2\right)=0$

$4\lambda^2b^2-4a^2\mu^2=16a^2b^2$

Hence, $\frac{\lambda^2}{a^2}-\frac{\mu^2}{b^2}=4$