IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 30/07/23**

TOPIC: CAPACITORS

SOLUTIONS

1. (B)

Since the capacitors are connected in series, the charge on each will be the same = Q (say).
Equivalent capacitance is

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{3\times 9}{3+9} = \frac{9}{4}\,\mu\text{F}$$

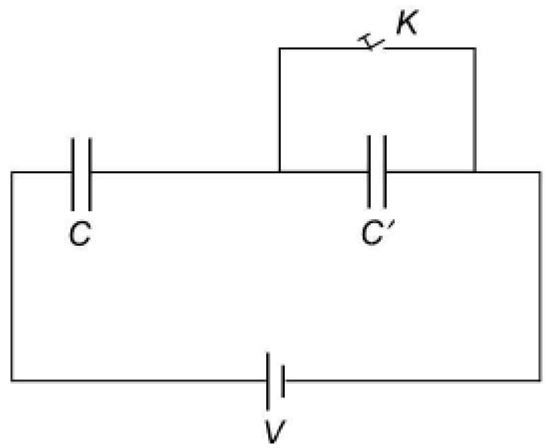
$$\therefore \quad Q = CV = \frac{9}{4}\times 8 = 18\ \mu\text{C}$$

2. (D)

The capacitance of the parallel combination is $C' = 2\ C$. The given circuit can be redrawn as shown in Fig. 21.37. When key K is open, the total capacitance $= \frac{CC'}{C+C'} = \frac{C\times 2C}{C+2C} = \frac{2C}{3}$

$\therefore$ Charge on capacitors is

$$Q_1 = \frac{2CV}{3}$$



When key K is closed, capacitor C' is short-circuited. The capacitance of the circuit now = C. Therefore, the charge on C is

$$Q_2 = CV$$

$\therefore$ Charge flowing through the battery is $(Q_2 - Q_1)$

$= CV - \frac{2CV}{3} = \frac{CV}{3}$, which is choice (d).

3. (C)

Initial charge on the capacitor plates is

$$Q_0 = CV_0 = (5\ \mu\text{F}) \times (120\ \text{V}) = 600\ \mu\text{C}$$

So, the charge of the positive plate is + 600 μC and of the negative plate is – 600 μC. If an additional charge of + 200 μC is given to the positive plate, its charge becomes = 600 + 200 = 800 μC. Let Q be the charge induced on the negative plate. The positive plate loses a charge Q and the negative plate gains a charge Q such that the total positive charge on the positive plate = total negative charge on the negative plate, i.e.

$$800 - Q = -600 + Q$$

$\Rightarrow Q = 700\ \mu\text{C}$. Therefore, the potential difference between the plates now is

$$V = \frac{Q}{C} = \frac{700\mu\text{C}}{5\mu\text{F}} = 140\ \text{V}$$

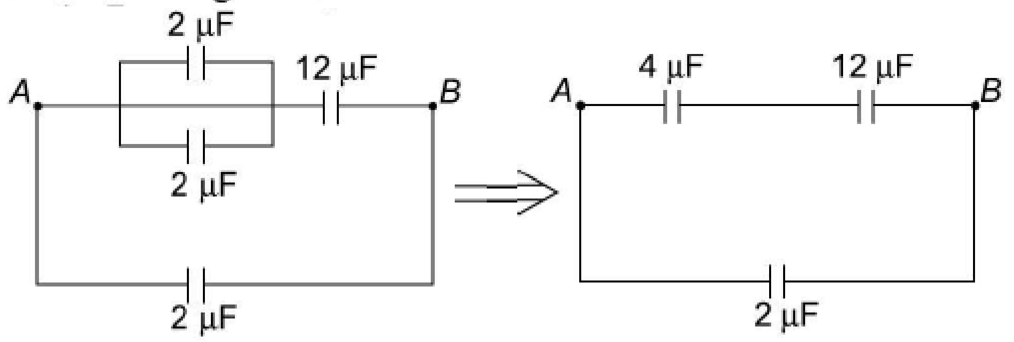
So the correct choice is (c).

4. (B)

Capacitors 1 μF, 2 μF and 3 μF are in parallel, their total capacitance is 6 μF. Thus, we have three capacitors in series each of capacitance 6 μF across the 12 V power supply. So the potential drop across each is 12/3 = 4 V. This is also the potential across 1 μF capacitor and 2 μF capacitor and 3 μF capacitor, because they are in parallel. Therefore, charge on 2 μF capacitor = 2 μF × 4V = 8 μC. Hence the correct choice is (b).

5. (C)

The given network of capacitors can be redrawn as shown in Fig. 21.38.



The series combination of 4 μF and 12 μF gives 3 μF which is in parallel with 2 μF. Hence the effective capacitance between *A* and *B* = 3 + 2 = 5 μF, which is choice (c).

6. (A, C, D)

The charge on the capacitor plates remains unchanged and is given by

$$Q = CV = \frac{\varepsilon_0 AV}{d}$$

Hence choice (a) is correct.

The electric field which was V/d reduces by a factor $1/K$ and becomes

$$E = \frac{V}{Kd}$$

Hence choice (c) is also correct.

Energy stored in the capacitor before the dielectric slab is inserted is given by

$$U_1 = \frac{Q^2}{2C} = \frac{\varepsilon_0^2 A^2 V^2}{2d^2} \cdot \frac{d}{\varepsilon_0 A} = \frac{\varepsilon_0 AV^2}{2d}$$

After the dielectric slab is inserted, energy stored is

$$U_2 = \frac{Q^2}{2C'} \quad \text{where } C' = \frac{\varepsilon_0 KA}{d}$$

$$= \frac{\varepsilon_0 AV^2}{2Kd}$$

$$\therefore \text{Work done } W = U_1 - U_2 = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$$

Hence the correct choices are (a), (c) and (d).

7. (A, D)

Refer to the solution of Q.13 of section I. The force on each plate of the capacitor is

$$F = \frac{Q^2}{2\varepsilon_0 A} \qquad (1)$$

Electric field $E = \frac{V}{d}$

$$\text{Now } V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

$$\therefore \quad E = \frac{Q}{\varepsilon_0 A} \qquad (2)$$

From (1) and (2). we get $F = \frac{1}{2} QE$.

Hence the correct choice are (a) and (d).

8. (A, C)

Original charge on the first capacitor is $Q_0 = C_1 V_0$. The charge is shared by the two capacitors when they are connected. $Q_0 = Q_1 + Q_2$. Using $Q = CV$, we have $C_1 V_0 = C_1 V + C_2 V$, which gives

$$\frac{V}{V_0} = \frac{C_1}{C_1 + C_2} \qquad (1)$$

Since the second capacitor is uncharged, if has no energy. Therefore, total energy before connection is

$$U_0 = \frac{1}{2} C_1 V_0^2 \qquad (2)$$

The total energy after connection is

$$U = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \qquad (3)$$

Using (1) and (2) in (3), we get

$$\frac{U}{U_0} = \frac{C_1}{C_1 + C_2}$$

Hence the correct choices are (a) and (c).

9. (A, B, D)

Due to the introduction of dielectric the capacitance C of the capacitor increases. Since the battery is kept connected, the potential difference between the plates remains unchanged = V, the voltage of the battery. As $Q = CV$, the charge on the plated increases because C increases and V remains unchanged. Electric field = V/d remains unchanged as both V and d remain the same. The energy stored in the capacitor is $U = \frac{1}{2} CV^2$. U increases because C increases and V remains unchanged. Hence the correct choices are (a), (b) and (d).

10. (A, C, D)

Charge Q on the capacitor plates remains unchanged as there is no battery to supply extra change. The capacitance $C = \varepsilon_0 A/d$ becomes half is d is doubled. Therefore, $V = Q/C$ is doubled. The energy stored in the capacitor is $U = Q^2/2C$. U is doubled because C becomes half but Q remains unchanged. The extra energy is supplied by the external agent because work has to be done to pull the plates away from each other. Hence the correct choices are (a), (c) and (d).

11. (A, B, D)

As the batteries are acting in opposition (becasue their positive terminals are connected together), the effective voltage is

$$V = V_1 - V_2 = 12 - 2 = 10\text{ V}$$

As the capacitors C_1 and C_2 are in series, the effective capacitance of the circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} = \frac{1}{C_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

or $C = \frac{6}{5} = 1.2\ \mu\text{F}$. Therefore, charge on capacitors is

$$Q = CV = 1.2\ \mu\text{F} \times 10\text{ V} = 12\ \mu C$$

$\therefore$ Potential difference across A and B = potential difference across capacitor C_2

$$= \frac{Q}{C_2} = \frac{12\mu\text{C}}{2\mu\text{F}} = 6\text{ V}$$

So the correct choices are (a), (b) and (d).

12. (B, C)

The capacitors are in series. So the combined capacitance is $C' = C/2$. Therefore, energy stored is

$$U = \frac{1}{2}C'V^2 = \frac{1}{4}\ CV^2$$

So the correct choices are (b) and (c).

13. (B, C, D)

Capacitors 1 μF, 2 μF and 3 μF are in parallel, their combined capacitance is 6 μF. Thus, we have three capacitors each of capacitance 6 μF connected in series. The equivalent capacitance of the network is, therefore C = 2 μF. Potential drop across each 6μF capacitor = 12/3 = 4 V. Hence potential drop across 1 μF, 2 μF and 3 μF capacitor is 4V. So the charge on 2 μF capacitor = 2 μF × 4 V = 8 μC.

Energy stored in the circuit = $\frac{1}{2}CV^2$

$= \frac{1}{2} \times (2 \times 10^{-6}) \times (12)^2 = 1.44 \times 10^{-4}$ J

Hence the correct choice are (b), (c) and (d).

14. (B, C, D)

In the steady state, no current flows in branches BC and AD containing the capacitors. So the path of the current is $A \rightarrow B \rightarrow D \rightarrow C \rightarrow A$ through the battery. Hence the three resistors are in series and current in the circuit is

$$I = \frac{16}{2+4+2} = 2\text{ A}$$

Now, p.d across A and C = 16 V and p.d across the 2 Ω resistor in arm AB = 2 Ω × 2 A = 4 V. Hence the p.d across the capacitor in arm BC = 16 – 4 = 12 V which is also the p.d across the capacitor in arm AD. Charge on each capacitor is $Q = CV$ = 2 μF × 12 V = 24 μC.

Energy stored = $\frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 = (2 \times 10^{-6})$ $\times (12)^2 = 2.88 \times 10^{-4}$ J

So the correct choices are (b) ,(c) and (d).

15. (B, D)

At $t = 0$, current in each circuit $= \frac{V}{R}$; V = voltage of the battery. Current decays as $e^{-t/\tau}$ where $\tau = RC$, the time constant. Since the time constant of the second circuit is twice that of the first, it will take longer to lose 50% of the initial charge than the first circuit. Hence the correct choice are (b) and (d).

16. (2)

Capacitors of capacitances 2 μF and 3 μF are in parallel and this combination is in series with 1 μF capacitor. Thus we have 1 μF capacitor in series 5 μF capacitor and the potential difference across this series combination is 6V. Therefore, the potential difference across 5 μF capacitor (which consists of a parallel combination of 2 μF and 3 μF capacitors) is 1 V. Hence the charge on 2 μF capacitor = 2 μF × 1 V = 2 μC

17. (8)

The capacitance of a parallel plate capacitor with air (or vacuum) as dielectric is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

When a dielectric of dielectric constant K is introduced, the capacitance becomes

$$C = \frac{K \varepsilon_0 A}{d}, \quad \therefore \quad \frac{C}{C_0} = K$$

Now $Q = C_0 V_0$ and $Q = CV$. Therefore

$$\frac{V}{V_0} = \frac{C_0}{C} = \frac{1}{K}$$

But $V/V_0 = \frac{1}{8}$. Therefore $K = 8$.

18. (5)

When a dielectric slab of thickness t and dielectric constant K is introduced between the plates of a parallel plate capacitor, the potential difference between the plates is given by

$$V = E_0\left[d - t\left(1 - \frac{1}{K}\right)\right]$$

In order to maintain the same value of V, the separation between the plates should be increased by d' given by

$$d' = t\left(1 - \frac{1}{K}\right) \text{ or } K = \frac{t}{t - d'}$$

$$= \frac{2\,\text{mm}}{2\,\text{mm} - 1.6\,\text{mm}} = 5$$

19. (3)

The capacitance before the introduction of the slab is $C = \dfrac{\varepsilon_0 A}{d}$

If Q is the charge on the plates, the potential difference is

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \qquad (1)$$

Let d' be the new separation between the plates. When a slab of thickness t and dielectric constant K is introduced, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

Since charge Q remains the same, the new potential difference is

$$V' = \frac{Q}{C'} = \frac{Q\left[d' - t\left(1 - \frac{1}{K}\right)\right]}{\varepsilon_0 A} \qquad (2)$$

Given $V' = V$. Equating Eqs. (1) and (2), we get

$$d = d' - t\left(1 - \frac{1}{K}\right) \text{ or } d' - d = t\left(1 - \frac{1}{K}\right)$$

Given $d' = d = 2$ mm and $t = 3$ mm. Thus

$$2 = 3\left(1 - \frac{1}{K}\right)$$

which gives $K = 3$.

20. (9)

The maximum charge the first capacitor can hold is

$$Q_1 = C_1 V_1 = 1 \times 10^{-6} \times 6000 = 6 \times 10^{-3}\ \text{C}$$

The maximum charge the second capacitor can hold is

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4000 = 8 \times 10^{-3}\ \text{C}$$

We know that in a series combination, the charge on each capacitor is the same. Now the first capacitor cannot hold a charge of 8×10^{-3} C; it can hold a maximum charge of 6×10^{-3} C. Therefore, the charge on the second capacitor must also be 6×10^{-3} C. Hence, the voltage across the second capacitor is

$$V_2 = \frac{6 \times 10^{-3}\ \text{C}}{2 \times 10^{-6}\ \text{F}} = 3000 \text{ volts} = 3 \text{ kilovolts}$$

Thus, the maximum voltage the system can withstand $= V_1 + V_2 = 6$ kilovolts $+ 3$ kilovolts $=$ 9 kilovolts.

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TOPIC: HYDROCARBON

SOLUTIONS

21. **(B)**

22. **(B)**

23. **(B)**

24. **(B)**

25. **(A)**

26. **(A, B)**

27. **(A, B, C)**

28. **(A, D)**

29. **(B, C)**

30. **(C)**

31. **(D)**

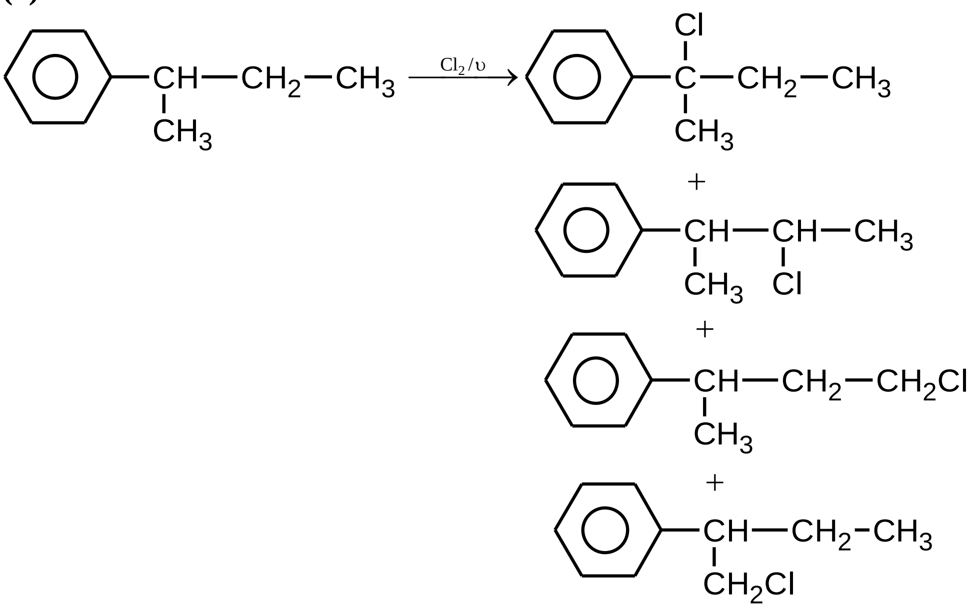
32. **(ABCD)**

33. **(ABCD)**

34. **(AB)**

35. **(ABC)**

36. **(4)**



37. **(3)**

38. **(3)**

H_2 / Ni_2B, $H_2 / Pd - BaSO_4$, $Na / \ell iq.\, NH_3$

39. **(7)**

40. **(3)**

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TOPIC: APPLICATION OF DERIVATIVES

SOLUTIONS

41. **(B)**

$e^{2y} = 1 + 4x^2$

$2y = \log_e (1 + 4x^2)$

$y = \frac{1}{2}\log_e (1 + 4x^2)$

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{1+4x^2} \times 4 \times 2x = \frac{4x}{1+4x^2}$

$\frac{dy}{dx} = \frac{4x}{1+4x^2} = m$

$1 + 4x^2$ will be positive for $x \in R$

Hence, $\frac{dy}{dx} = m = \frac{4x}{1+4x^2} < 1$, for all $x \in R$

$\Rightarrow |m| \le 1$

$\therefore$ Option [B] is correct answer.

42. **(C)**

$y' = +\frac{ye^{xy} - 1}{1 - xe^{xy}}$

$1 = xe^{xy}$

$0 = \ell n\, x + xy$

$y' \to \infty$ at (1, 0)

43. **(C)**

$y = \frac{ax+b}{x^2 - 5x + 4}$

$y' = \frac{(ax^2 - 5ax + 4a) - (ax+b)(2x-5)}{(x-1)^2\ (x-4)^2}$

$= \frac{-ax^2 + 4a - 2bx + 5b}{(x-1)^2\ (x-4)^2}$

$= y'_{(2,-1)} = 0$

$\Rightarrow -4a + 4a - b = 0 \Rightarrow b = 0$

& $2a + b = 2$

Put $b = 0$ in above equations

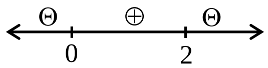
$\Rightarrow$ $a = 1, b = 0$

44. **(A)**

$$f'(x)=\frac{x^2-3x+3-(x-1)(2x-3)}{(x^2-3x+3)^2}$$

$$=\frac{-x^2+2x}{(x^2-3x+3)^2}$$

$$=\frac{-x(x-2)}{(x^2-3x+3)^2}$$



f(x) increase in [0, 2]

f(x) decrease $x \in [-\infty, 0] \cup [2, \infty)$

45. **(A)**

$$f(x)=x^3+4x^2+\lambda x+1$$

Differentiating w.r.t. *x*, we get

$f'(x) = 3x^2 + 8x + \lambda + 0 < 0$ … (1)

Also, $x \in (-2, -2/3)$

$\Rightarrow (x + 2)\left(x+\frac{2}{3}\right) < 0 \Rightarrow x^2 + 2x + \frac{2}{3}x + \frac{4}{3} < 0$

$\Rightarrow x^2 + \frac{8}{3}x + \frac{4}{3} < 0$

$\Rightarrow 3x^2 + 8x + 4 < 0$ … (2)

Comparing equation (1) and (2),

we get $\lambda = 4$

$\therefore$ Option (A) is correct answer.

46. **(A, B, C)**

$f(x) = \tan^{-1}(\sin x + \cos x)$

Differentiating w.r.t. x, we get

$f'(x) = \frac{1}{1+(\sin x+\cos x)^2}(\cos x - \sin x)$

$f'(x) = \frac{\cos x-\sin x}{1+(\sin x+\cos x)^2} > 0$

$= \frac{(\cos x-\sin x)}{2+\sin 2x} > 0$

$-1 \le \sin 2x \le 1 \Rightarrow -1 + 2 \le 2 + \sin 2x \le 1 + 2$

$\Rightarrow 1 \le 2 + \sin 2x \le 3$

Hence, $(\cos x - \sin x) > 0$

$\cos x > \sin x$

$\Rightarrow 1 > \tan x$

$\Rightarrow \tan x < 1$

$\Rightarrow x < \pi/4$

$\Rightarrow x \in \left(\frac{-\pi}{2}, \frac{\pi}{4}\right)$

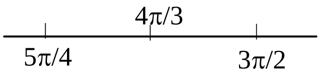
Since, $-\infty < \tan x < \infty \Rightarrow x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Options A and B are correct answers.

We have to check options (C) and (D) as well.

For C :

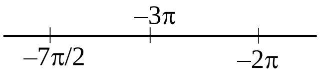
$\frac{5\pi}{4}, \frac{3\pi}{2}$



$$\frac{\cos x - \sin x}{2 + \sin 2x} = \frac{\cos(\pi + \pi/3) - \sin(\pi + \pi/3)}{2 + \sin(2\pi + 2\pi/3)}$$

$$= \frac{-\frac{1}{2} + \sqrt{3}/2}{2 + \sqrt{3}/2} > 0$$

Option (C) is also correct answer.

For D : 

$$\frac{\cos x - \sin x}{2 + \sin 2x} = \frac{\cos(-3\pi) - \sin(-3\pi)}{2 + \sin 2(-3\pi)}$$

$$= \frac{\cos(2\pi + \pi) + \sin(2\pi + \pi)}{2 - \sin 6\pi}$$

$$= \frac{-1 + 0}{2 - 0} = -\frac{1}{2}$$

Option (D) is not correct answer.

$\therefore$ Options A, B, C only correct answer.

47. **(A, B)**

$y' = \sec^2 t - \text{cosec}^2 t$

$$\frac{dx}{dt} = \frac{-2\,\text{cosec}^2 t}{\cot\ t}$$

$(y')_{(\pi/4)} = 0$

48. **(A, B, C, D)**

$y = ax,\ x^2 + y^2 = b^2$

$y = ax \Rightarrow (dy/dx)_1 = a$

$x^2 + y^2 = b^2 \Rightarrow 2x \times 1 + 2y \times \frac{dy}{dx} = 0$

$$\Rightarrow \left(\frac{dy}{dx}\right)_2 = -\frac{x}{y}$$

$$\left(\frac{dy}{dx}\right)_1 \times \left(\frac{dy}{dx}\right)_2 = \frac{-ax}{y} = \frac{-ax}{ax} = -1$$

For all $a, b \in R$

Hence, all options are correct answers.

49. **(A, B, C, D)**

$h(x) = 3f\left(\frac{x^2}{3}\right) + f(3 - x^2)\ \forall\ x \in (-3, 4),\ f''(x) > 0\ \forall\ x \in (-3, 4)$

Differentiating above function w.r.t. x, we get

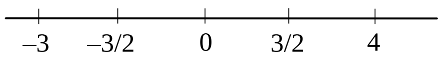
$h'(x) = 3f'\left(\frac{x^2}{3}\right) \times \frac{2x}{3} + f'(3 - x^2) \times (-2x)$

$= 2x\left[f'\left(\frac{x^2}{3}\right) - f'(3 - x^2)\right]$

Since, $f'(x)$ is increasing in $(-3, 4)$

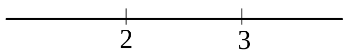
For $x > 0$, $h'(x)$ would be increasing.

For $x < 0$, $h'(x)$ would be decreasing.
For increasing function
$x > 0 \Rightarrow h'(x) > h'(0)$
$\Rightarrow f'\left(\frac{x^2}{3}\right) - f'(3 - x^2) > 0$
$\Rightarrow f'\left(\frac{x^2}{3}\right) > f'(3 - x^2) \Rightarrow \frac{x^2}{3} > 3 - x^2$
$\Rightarrow \frac{4}{3}x^2 > 3 \Rightarrow x^2 > 9/4 \Rightarrow x < -3/2$ or $x > 3/2$



Hence, h(x) would be increasing in (3/2, 4)
h(x) would be decreasing in (0, 3/2)
h(x) would be increasing in (–3/2, 0)
h(x) would be decreasing in (–3, –3/2)
Hence, options (A), (B), (C) and (D) are correct answers.

50. **(A, C)**
When $x < 2$
$f(x) = (x - 1)(2 - x)(3 - x)$



$= (x - 1)(x - 2)(x - 3)$
$= (x^2 - x - 2x + 2)(x - 3)$
$= (x^2 - 3x + 2)(x - 3)$
$= (x^3 - 3x^2 + 2x - 3x^2 + 9x - 6)$
$= (x^3 - 6x^2 + 11x - 6)$
$f(x) = (x^3 - 6x^2 + 11x - 6)$
Differentiating w.r.t. x, we get
$f'(x) = 3x^2 - 12x + 11 - 0$
$f'(x) = (3x^2 - 12x + 11)$
$f'(x) < 0$
$\Rightarrow (3x^2 - 12x + 11) < 0$
We can find roots of above quadratic equation as follows
$x = \frac{12 \pm \sqrt{144 - 132}}{2 \times 3}$

51. **(A, B)**
$y = e^{-x}$
$y' = e^{-x_1}$
tangent $y - e^{-x_1} = - e^{-x_1}(x - x_1)$
Area $A = \frac{1}{2}e^{-x_1}(x_1 + 1)^2 \Rightarrow A_{max} \Rightarrow x_1 = 1$
$P \equiv \left(1, \frac{1}{e}\right)$ curve is symmetric so other point $\left(-1, \frac{1}{e}\right)$ will also

52. **(D)**
$2x - x^2$ in (0, 2) is symmetric about $x = 1$ and also $\sin\frac{\pi x}{2}$
so (A) is true also for both at $x = 1$ point of maxima so C is true.

53. **(A, B)**

$$f'(x) = \frac{1}{1+x^2} \quad ; \quad -1 < x < 1$$
$$= -1 \quad ; \quad x > 1$$
$$= 1 \quad ; \quad x < -1$$

54. **(B, C)**

We have, $f(x) = 8x^3 - 6x^2 - 2x + 1$

$$f(0) = 1$$

$$f(1) = 8 - 6 - 2 + 1 = 1$$

$$f(0) = f(1)$$

By Rolle's theorem, we have

$$f'(c) = 0, \forall\, c \in (0,1)$$

$$24c^2 - 12c - 2 = 0$$

$$\Rightarrow \quad 12c^2 - 6c - 1 = 0$$

$$\Rightarrow \quad c = \frac{6 \pm \sqrt{36+48}}{24} = \frac{6 \pm \sqrt{84}}{24}$$

$$= \frac{2\left(3 \pm \sqrt{21}\right)}{24}$$

$$= \frac{3 \pm \sqrt{21}}{12}$$

$$c = \frac{3+\sqrt{21}}{12} \in (0,1)$$

and $c = \frac{3-\sqrt{21}}{12} \notin (0,1)$

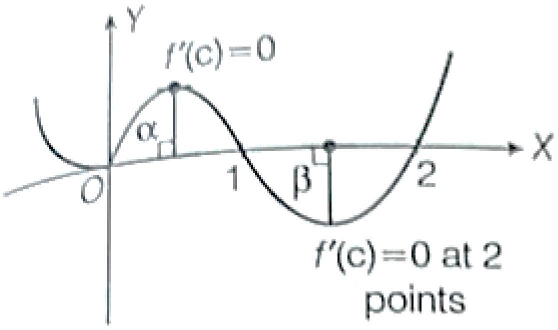
Hence, $f'(c)$ vanishes for some $c \in (0,1)$.

$$f\left(\frac{3+\sqrt{21}}{12}\right) < 0$$

$\therefore$ $f(x) = 0$ has atleast one root in (0, 1).

55. **(B, D)**

$$f(0) = f(1) = f(2) = 0$$



As, $f(x)$ is continuous and differentiable between $x = 0$ and $x = 2$.

$\therefore$ $f'(c) = 0$ for two points $x = \alpha, \beta$.

$\Rightarrow\ f'(c)=0$ for atleast two $c\in(0,2)$.

56. (27)

Let $g(x) = 1 + 12x - 3x^2$

So, $g'(x)=12-6x=0 \Rightarrow x=2$

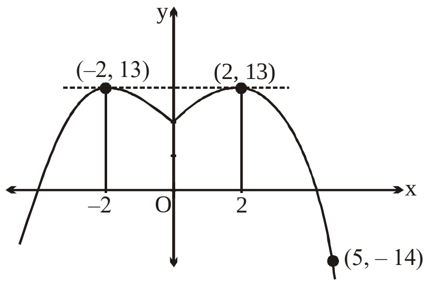
Also, $f(x)=g(|x|)=1+12|x|-3x^2$

$\therefore$ global maximum of $f(x)$ at (x = 2 or – 2) = 13

and global minimum of $f(x)$ occurs at x = 5.

Note that $f(5) = 61 - 75 = -14$.

Hence difference = 13 – (– 14) = 27.



Graph of $f(x) = 1 + 12|x| - 3x^2$

57. (4)

$y=\dfrac{(x-a)^2}{4}$ and $y=e^x$

$\dfrac{dy}{dx}=\dfrac{x-a}{2}$ and $\dfrac{dy}{dx}=e^x$

$\therefore\ \dfrac{x-a}{2}=e^x \Rightarrow (x-a)=2e^x \qquad ...(1)$

Also $\dfrac{(x-a)^2}{4}=e^x \qquad ...(2)$

$\therefore\ \dfrac{4e^{2x}}{4}=e^x$ [Using (1)]

$\Rightarrow\ e^x(e^x-1)=0$. Hence $e^x=1 \Rightarrow x=0 \Rightarrow a=-2$

Hence, sum of the squares of all possible values of a is 4.

58. (1)

$f(x)=3\tan x+x^3$ then $f'(x)=3\sec^2 x+3x^2>0$ hence $f(x)$.

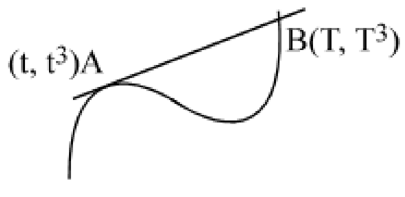
Thus $f(x)$ assumes the value 2 exactly once. Also $f(0)=0$ and $f\left(\dfrac{\pi}{4}\right)>2$ so by intermediate value theorem $f(c)=2$ for some c in $\left(0,\dfrac{\pi}{4}\right)\Rightarrow$

59. (4)

$\dfrac{dy}{dx}=3x^2=3t^2$ at ‘A’

$\therefore\ 3t^2=\dfrac{T^3-t^3}{T-t}=T^2+Tt+t^2$

$=T^2+Tt+t^2$



$T^2+Tt-2t^2=0$

$(T-t)(T+2t)=0 \Rightarrow T=t$ or $T=-2t$ ($T = t$ is not possible)

Now, $m_A=\dfrac{t^3}{t}=t^2;\ m_B=T^2$

$\dfrac{m_B}{m_A}=\dfrac{T^2}{t^2}=\dfrac{4t^2}{t^2}$ (using $T=-2t$)

$K = 4$

60. (1)