IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 28/08/22**

TOPIC: CALCULUS IN PHYSICS

Answer Key

1.	(A)	2.	(B)	3.	(B)	4.	(B)	5.	(C)
6.	(D)	7.	(A)	8.	(A)	9.	(D)	10.	(C)
11.	(C)	12.	(B)	13.	(A)	14.	(B)	15.	(A)
16.	(C)	17.	(D)	18.	(C)	19.	(C)	20.	(D)
21.	(D)	22.	(C)	23.	(D)	24.	(A)	25.	(A)

SOLUTIONS

1. (A)

$$\int_0^\infty e^{-5x}dx = \left[\frac{e^{-5x}}{-5}\right]_0^\infty$$

$$= \frac{0-1}{-5} = \frac{1}{5}$$

2. (B)

$$\int \frac{x^3}{4+x^4}dx$$

$$= \frac{\ln\left(4+x^4\right)}{4} + C$$

3. (B)

$$\int_0^1 \frac{dx}{(ax+b)}$$

$$= \left[\frac{\ln(ax+b)}{a}\right]_0^1$$

$$= \left[\ln(a+b) - \ln b\right]\frac{1}{a}$$

$$= \frac{1}{a}\ln\left(\frac{a+b}{b}\right) = \frac{1}{a}\ln\left(1+\frac{a}{b}\right)$$

4. (B)

$$3y = 2x^3 + 1$$

$$\frac{3dy}{dt} = 6x^2 \frac{dx}{dt}$$

$$dy = 2x^2 \cdot dx$$

$$8 = 2x^2 \Rightarrow x = \pm 2$$

$$y = \frac{2(\pm 2)^3 + 1}{3} = \pm \frac{16+1}{3}$$

$$= \frac{17}{3}, -\frac{15}{3}$$

5. (C)

$$y = x \cdot \cos x$$

$$\frac{dy}{dx} = x \cdot (-\sin x) + \cos x(1)$$

$$= -x \sin x + \cos x$$

6. (D)

$$\frac{dy}{d\theta} = -2 \sin 2\theta$$

$$\frac{dx}{d\theta} = \frac{1}{2} \cos \frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{-4 \sin 2\theta}{\cos \theta/2} = -16 \sin \frac{\theta}{2} \cdot \cos \theta$$

7. (A)

$$\int_2^5 (3x^2 + 4x + 1) dx$$

$$= \left[x^3 + 2x^2 + x \right]_2^5$$

$$= 180 - 18 = 162$$

8. (A)

$$\int_0^{\pi/4} \sec^2 \theta \cdot d\theta = \left[\tan \theta \right]_0^{\pi/4} = 1$$

9. (D)

$$\frac{dy_1}{dx} = \cos x + \sin x = 0$$

$$\tan x = -1 \qquad \therefore A_1 = \sqrt{2}$$

$$\frac{dy_2}{dx} = 2 \cos x - 4 \sin x = 0$$

$$\tan x = \frac{1}{2}$$

$$B = 2\sqrt{5}$$

10. (C)

$$\frac{dp}{dt} = \left(10t + 3t^2\right)$$

$$\Delta p = \int \left(10t + 3t^2\right) dt$$

$$= \left(\frac{10t^2}{2} + \frac{3t^3}{3} \right)_0^2$$

$$= 5(4) + 8$$

$$= 28.$$

11. (C)

$$y = e^x - \ln x + \frac{1}{x}$$

$$\frac{dy}{dx} = e^x - \frac{1}{x} - \frac{1}{x^2}$$

12. (B)

$$\frac{dy}{dx} = x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$\frac{d^2y}{dx^2} = 2x + 1$$

At $x = -4; \frac{d^2y}{dx^2} = -7 < 0$ (maximum)

At $x = 3; \frac{d^2y}{dx^2} = 7 > 0$ (minimum)

13. (A)

$$V = \frac{\pi}{3} r^2 h$$

$$\tan\theta = \frac{r}{h} = \frac{5}{20} - \frac{1}{4}$$

$$\Rightarrow h = \frac{r}{\tan\theta} = 4r$$

$$V = \frac{\pi}{3} r^2 (4r)$$

$$\therefore \frac{dV}{dt} = \frac{4\pi}{3} \times 3r^2 \cdot \frac{dr}{dt}$$

14. (B)

$$\frac{dy}{dx} = \sec x \cdot \tan x + \sec^2 x$$

$$= 2\sqrt{3} + (2)^2$$

$$= 2\left(\sqrt{3} + 2\right)$$

15. (A)

$$W = \int \frac{2}{r^2} dr$$

$$= \left[\frac{-2}{r} \right]_{1/2}^{\infty}$$

$$= -\left(-\frac{2}{1/2} \right) = 4 \text{ J}$$

16. (C)

$$\int_1^2 y \cdot dx = \left(\frac{ax^3}{3} + 2x^2 - 3x \right)_1^2 = 10$$

$$\Rightarrow \ a = 3$$

17. (D)

$$x = y^2 + y$$

$$1 = 2y \cdot \frac{dy}{dx} + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{(2y+1)}$$

18. (C)

$$x = t^2 - 24t + 6$$

$$\frac{dx}{dt} = 2t - 24 = 0$$

$$t = 12$$

$$\frac{d^2x}{dt^2} = 2 > 0 \text{ (minima)}$$

at t = 12

19. (C)

$$y = \ln x^2$$

$$\frac{dy}{dx} = \frac{1}{x^2} \times 2x = \frac{2}{x} = \frac{2}{e}$$

20. (D)

$$v = bs^2$$

$$a = \frac{vdv}{ds} = bs^2 (2bs)$$

$$= 2b^2 s^3$$

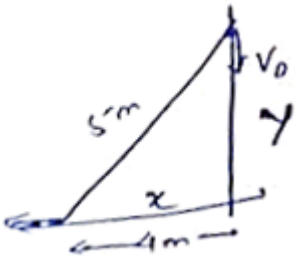
21. (D)

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$
$$= 8\pi(2)(10)$$
$$= 160\pi$$

22. (C)

$$x^2 + + y^2 = 5^2$$
$$2x\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$
$$= \frac{4}{3}(3) = 4\,\text{m/s}$$



23. (D)

$$F = \frac{dP}{dt}$$

24. (A)

$$W = \text{area of } F \cdot x \text{ curve } = \frac{1}{2} \times 20 \times 10 = 100\ \text{J}$$

25. (A)

$$P \propto t$$
$$F = \frac{dP}{dt} = \text{constant}$$

i.e. $F \propto t^o$

IIT – JEE: 2024 | **TW TEST (MAIN)** | **DATE: 28/08/22**

TIME: 1 Hr. | **TOPIC: ATOMIC STRUCTURE** | **MARKS: 100**

Answer Key

26.	(D)	27.	(B)	28.	(C)	29.	(B)	30.	(A)
31.	(C)	32.	(D)	33.	(C)	34.	(D)	35.	(D)
36.	(D)	37.	(B)	38.	(C)	39.	(D)	40.	(D)
41.	(B)	42.	(B)	43.	(B)	44.	(A)	45.	(B)
46.	(D)	47.	(C)	48.	(C)	49.	(A)	50.	(B)

SOLUTIONS

26. (D)
Informative

27. (B)
$\upsilon = \nu \cdot \lambda$
$= 10 \times 2.5 = 25\ \text{m/sec}$
Distance $= 25 \times 40 = 1000\ \text{m}$

28. (C)
$r = 0.53 \frac{n^2}{Z}$ Å
Radius of first Bohr orbital = r
So, radius of first Li^{+2} orbit $= \frac{r}{3}$

29. (B)
Possible radii are 1st and 2nd.
In this energy difference is 10.2 eV.
2nd possible radii are 2nd and 4th.
In this energy difference is 2.55 eV.

30. (A)
$V = 2.185 \times 10^8 \frac{Z}{n}$ cm/sec.

$$= 2.185\times10^{8}\,\frac{(3)}{2}$$

$$= 3.276\times10^{8}\ \text{cm/sec.}$$

31. (C)

$$\frac{nh}{2\pi} = \frac{2h}{\pi} \Rightarrow n = 4$$

$$\text{P.E.} = -27.2\frac{Z^2}{n^2} = -27.2\,\frac{(2)^2}{(4)^2}$$

$$= -6.8\ \text{eV}$$

32. (D)

Bohr model is valid only on unielectron species.

33. (C)

Value of m is $-l$ to $+l$.

34. (D)

$$E = -13.6\frac{Z^2}{n^2}\ \text{eV}$$

$$\frac{E_{2-1}}{E_{3-2}} = \frac{\frac{1}{2^2}-\frac{1}{1^2}}{\frac{1}{3^2}-\frac{1}{2^2}} = \frac{-\frac{3}{4}}{-\frac{5}{36}} = \frac{27}{5}$$

35. (D)

Informative

36. (D)

Number subshell = n

37. (B)

3f does not exist

38. (C)

Informative

39. (D)

An orbital can have maximum 2 electron.

40. (D)

41. (B)

Azimuthal quantum number determine shape of the subshell.

42. (B)

Third line in Balmer series $5 \rightarrow 2$

43. (B)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{x} = R(2)^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\frac{1}{\lambda} = R(3)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \frac{\lambda}{x} = \frac{1}{9\left(\frac{7}{9\times16}\right)}$$

$$\lambda = \frac{16x}{7}$$

44. (A)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\,\text{K.E.}}}$$

So, order of de-Broglie wavelength is $\lambda_e > \lambda_p = \lambda_\alpha$

45. (B)

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

46. (D)

$$r = 0.53\frac{n^2}{Z}\text{ Å}$$

47. (C)

$\lambda = \frac{h}{mv}$ de-Broglie equation

48. (C)

M^{th} shell $\Rightarrow$ 3^{rd} shell

Maximum electron $= 2n^2$

$= 18$

49. (A)
Last electron is in $5s^1$

50. (B)
Lower energy orbital is filled first violation in second option.

Answer Key

51.	(B)	52.	(B)	53.	(C)	54.	(D)	55.	(D)
56.	(C)	57.	(C)	58.	(D)	59.	(B)	60.	(C)
61.	(A)	62.	(A)	63.	(C)	64.	(C)	65.	(D)
66.	(C)	67.	(B)	68.	(B)	69.	(A)	70.	(B)
71.	(D)	72.	(A)	73.	(D)	74.	(D)	75.	(D)

SOLUTIONS

51. (B)

$\sec\theta+\tan\theta=3$

$\sec\theta-\tan\theta=\frac{1}{3}$

$2\sec\theta=3+\frac{1}{3}=\frac{10}{3}$

$\sec\theta=\frac{5}{3}$

$\therefore \quad \cos\theta=\frac{3}{5}$

52. (B)

$\sin\theta=\frac{4}{5}$ but θ lies in 2^{nd} or 4^{th}

Quadrant Hence, $\sin\theta=\frac{4}{5}$ or $-\frac{4}{5}$

53. (C)

$\Rightarrow \quad \sec^4\theta\left(\cos^2\theta\right)\left(1+\sin^2\theta\right)-2\tan^2\theta$

$\Rightarrow \quad \sec^2\theta\left(1+\sin^2\theta\right)-2\tan^2\theta \Rightarrow \sec^2\theta+\tan^2\theta-2\tan^2\theta$

$\Rightarrow \sec^2\theta-\tan^2\theta=1$

54. (D)

$d=10\theta$ where $\theta=\frac{360^o}{60^o}\times 20=\frac{2\pi}{3}$

$$d = 10 \times \frac{2\pi}{3} \Rightarrow \frac{3d}{10\pi} = 2$$

55. (D)

$\sin 4 < 0$.

56. (C)

$\cos(A+B) = 1 \Rightarrow A = -B$

$\therefore\ 2 + \tan(-B)\cot B = 2 - 1 = 1$

57. (C)

$\sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

$\therefore$ using this $K = \frac{1}{\sqrt{2}}$

$\therefore\ \sqrt{2}K = 1$

58. (D)

If $A + B = 45^\circ$ then $(1+\tan A)(1+\tan B) = 2$

59. (B)

$\operatorname{cosec}^2 x + 25\sec^2 x$

$1 + \cot^2 x + 25(1 + \tan^2 x)$

$1 + \cot^2 x + 25 + 25\tan^2 x$

$26 + \underbrace{25\tan^2 x + \cot^2 x}$

$\geq 2\sqrt{25}$

≥ 10

$\therefore$ Least value of $\operatorname{cosec}^2 x + 25\sec^2 x$ is 36.

60. (C)

$5\cos\theta + 3\left[\cos\theta\cdot\cos\left(\frac{\pi}{3}\right) - \sin\theta\cdot\sin\left(\frac{\pi}{3}\right)\right] + 3$

$5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Now, $\lambda = \sqrt{\frac{169}{4} + \frac{27}{4}} + 3$

$= \sqrt{\frac{196}{4}} + 3$

$= 7 + 3 = 10$

$\mu = -7 + 3 = -4$

$\therefore \quad \lambda - \mu = 14$

61. (A)

$$\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{5\pi}{8}\right)+\cos^4\left(\frac{7\pi}{8}\right)$$

$$\Rightarrow \quad \cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)$$

$$\Rightarrow \quad 2\left[\cos^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[\cos^4\left(\frac{\pi}{8}\right)+\sin^4\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\left(\sin^2\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad 2\left[1-\frac{1}{2}\times\frac{1}{2}\right]=2\left[\frac{3}{4}\right]=\frac{3}{2}$$

62. (A)

$$(\alpha+\beta)-(\alpha-\beta)=2\beta$$

$$\tan\left[(\alpha+\beta)-(\alpha-\beta)\right]=\tan 2\beta$$

$$\frac{\tan(\alpha+\beta)-\tan(\alpha-\beta)}{1+\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}=\tan 2\beta$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{3}{4} \text{ and } \tan(\alpha-\beta)=\frac{5}{12}$$

$$\Rightarrow \quad \tan 2\beta=\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{4}\times\frac{5}{12}}=\frac{36-20}{48+15}$$

$$\tan 2\beta=\frac{16}{63}$$

63. (C)

$$\alpha=\frac{\pi}{2}-\beta$$

$$\tan\alpha=\cot\beta \Rightarrow \tan\alpha\cdot\tan\beta=1$$

Now, $\alpha-\beta=\gamma$

$$\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}=\tan\gamma$$

$$\tan\alpha-\tan\beta=2\tan\gamma$$

$$\tan\alpha=\tan\beta+2\tan\gamma$$

64. (C)

$$\sin\theta\sin\left(60^\circ-\theta\right)\sin\left(60^\circ+\theta\right)=\frac{\sin 3\theta}{4}$$

$$\therefore\quad \sin 30^\circ\times\frac{\sin 30^\circ}{4}=\frac{1}{2}\times\frac{1}{8}=\frac{1}{16}.$$

65. (D)

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\left(\cos 10^\circ-\cos 50^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-2\cos 50^\circ\sin 30^\circ\sin\left(20^\circ\right)$$

$$\frac{1+\cos 20^\circ}{2}-\cos 50^\circ\sin 20^\circ$$

$$\frac{1+\cos 20^\circ}{2}-\frac{1}{2}\left[\sin\left(70^\circ\right)-\sin\left(30^\circ\right)\right]$$

$$\frac{1+\cos 20^\circ-\sin 70^\circ+\sin 30^\circ}{2}$$

$$\frac{\frac{3}{2}+\cos 20^\circ-\cos 20^\circ}{2}=\frac{3}{4}$$

66. (C)

$$\frac{\sin\left(2^9\times\frac{\pi}{2^{10}}\right)}{2^9\times\sin\left(\frac{\pi}{2^{10}}\right)}\times\sin\left(\frac{\pi}{2^{10}}\right)=\frac{1}{512}$$

67. (B)

$\tan^2 x=\frac{2}{3}$ from the given relation.

Hence, (B).

68. (B)

$$\tan 30^\circ+\tan 15^\circ=-p$$

$$\tan 30^\circ\cdot\tan 15^\circ=q$$

$$\therefore\quad \frac{\tan 30^\circ+\tan 15^\circ}{1-\tan 30^\circ\tan 15^\circ}=1$$

$$\Rightarrow\quad -p=1-q$$

$$\Rightarrow\quad q-p=1$$

$$\therefore\quad 2+q-p=3$$

69. (A)

$$\sin\alpha+\sin\beta=\frac{21}{65}\ \Rightarrow\ 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{21}{65}$$

$$\cos\alpha+\cos\beta=\frac{-27}{65}\ \Rightarrow\ 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-27}{65}$$

$\therefore\quad \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{-21}{27}$

70. (B)

Using componendo and dividendo

$$\frac{1-\cos x}{1+\cos x}=\frac{2-\cos y-2\cos y+1}{2-\cos y+2\cos y-1}$$

$$\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{3(1-\cos y)}{1+\cos y}=\frac{3\times 2\sin^2\frac{y}{2}}{2\cos^2\frac{y}{2}}$$

$\therefore\ \tan^2\frac{x}{2}=3\tan^2\frac{y}{2}$

$$\tan^2\frac{x}{2}\cdot\cot^2\frac{y}{2}=3$$

$$\tan\frac{x}{2}\cdot\cot\frac{y}{2}=\sqrt{3}$$

71. (D)

$$\tan A\cdot\tan B+\tan B\cdot\tan C+\tan C\cdot\tan A=1$$

$$\frac{1}{2}+\frac{\tan C}{2}+\tan C=1$$

$$\frac{3\tan C}{2}=\frac{1}{2}\ \Rightarrow\ \tan C=\frac{1}{3}$$

72. (A)

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{\cos A\cos B+\sin A\sin B}{\cos A\cos B-\sin A\sin B}$$

$$=\frac{1+\tan A\tan B}{1-\tan A\tan B}$$

$$=\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2$$

73. (D)

$$\tan\left[2(A+B)\right]=\frac{2\tan(A+B)}{1-\tan^2(A+B)}\qquad\left[\tan(A+B)=\frac{2+1}{1-2}=-3\right]$$

$\therefore\quad \tan 2(A+B)=\frac{2\times(-3)}{1-(-3)^2}=\frac{-6}{-8}=\frac{3}{4}$

74. (D)

Let $\tan\frac{\theta}{2} = x$ then

$\sin\theta + \cos\theta = -\frac{1}{5}$

$\frac{2x}{1+x^2} + \frac{1-x^2}{1+x^2} = -\frac{1}{5}$

$10x + 5 - 5x^2 = -x^2 - 1$

$4x^2 - 10x - 6 = 0$

$2x^2 - 5x - 3 = 0$

75. (D)

$\frac{(\sin\theta + \cos\theta)^2}{(\cos\theta - \sin\theta)^2} = \frac{1+\sin 2\theta}{1-\sin 2\theta} = \frac{1+P}{1-P}$