IIT – JEE: 2024 **TW TEST (ADV)** **DATE: 04/09/22**

TOPIC: VECTORS

Solutions

1. (D)

The resultant is

$\overrightarrow{BA}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}$

$=\overrightarrow{BA}+\left(\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DA}\right)$

$=\overrightarrow{BA}+\overrightarrow{BA}$

$=2\overrightarrow{BA}$

2. (C)

$\left|\vec{A}+\vec{B}\right|=\sqrt{\left|\vec{A}\right|^2+\left|B\right|^2+2\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta}$

Since $0\le\theta\le\pi$

$-1\le\cos\theta\le1$

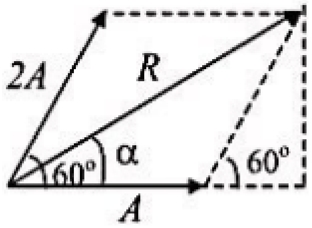
Hence, $\sqrt{\left|\vec{A}\right|^2+\left|B\right|^2-2\left|\vec{A}\right|\left|\vec{B}\right|}\le\left|\vec{A}+\vec{B}\right|\le\sqrt{\left|\vec{A}\right|^2+\left|B\right|^2+2\left|\vec{A}\right|\left|\vec{B}\right|}$

Thus $\left|\vec{A}+\vec{B}\right|\le\left|\vec{A}\right|+\left|\vec{B}\right|$

Alternatively: triangle inequality

Hence (C) is correct.

3. (C)



From the figure,

$$\tan\alpha=\frac{2A\sin60^o}{A+2A\cos60^o}=\frac{\sqrt{3}}{2}$$

4. (C)

P = 2 dyne, Q = 3 dyne, R = 4 dune, $\theta=?$

$R^2=P^2+Q^2+2PQ\cos\theta$

$4^2=2^2+3^2+2\times2\times3\times\cos\theta$

$\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}(0.25)$

5. (C)

$x = 4 + 4\cos 60^o = 6m, \ \ y = 4\sin 60^o = 2\sqrt{3}m$

$r = \sqrt{x^2 + y^2} = \sqrt{36+12} = \sqrt{48}m = 6.9\,m$.

6. (B)

P + Q = 10 N, P – Q = 6 N, 2P = 16 N

Again 8 + Q = 10 or Q = 2 N

$P' = 11\,N, \ \ Q' = 90^o$

$R' = \sqrt{121+25}\ N = 12.1\ N$

Hence (B) is correct.

7. (B)

$\tan\theta = \frac{\sqrt{3}A}{A} = \sqrt{3}$

$\theta = 60^o$

8. (B)

$\vec{A} \perp \vec{B}$

$\vec{A}\cdot\vec{B} = 0 = \left(5\vec{i} + 7\hat{j} + 3\hat{k}\right)\left(2\vec{i} + 2\hat{j} - a\hat{k}\right) = 10 + 14 - 3a$

$\therefore\ 3a = 24 \ \Rightarrow\ a = 8$

Hence (B) is correct.

9. (A)

Calculate $\vec{A}\times\vec{B}$

and Area of a parallelogram $= \left|\vec{A}\times\vec{B}\right|$

Hence (A) is correct.

10. (A)

Calculate torque $(\vec{\tau}) = \vec{r}\times\vec{F}$

Where $\vec{r} = \vec{r}_1 - \vec{r}_2$

Hence (A) is correct.

11. (A)

$\frac{AB\cos\theta}{AB\sin\theta} = \sqrt{3}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

or $\theta = 30^o$

Hence (A) is correct.

12. (B)

$\vec{A}\times\vec{B}$ is a vector $\perp$ to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A}\times\vec{B} = (\hat{i}-2\hat{j}+\hat{k})\times(3\vec{i}+\hat{j}-2\hat{k}) = 3\vec{i}+5\hat{j}+7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$= \dfrac{3\vec{i}+5\hat{j}+7\hat{k}}{\sqrt{3^2+5^2+7^2}} = \dfrac{3\vec{i}+5\hat{j}+7\hat{k}}{\sqrt{83}}$

Hence (B) is correct.

13. (C)

$\vec{A}\perp\vec{B}$. Then,

$\vec{A}\cdot\vec{B} = |A||B|\cos 90^o = 0$

Hence (C) is correct.

14. (D)

Displacement $= 4\hat{k}$

Force $= -\hat{i}+2\hat{j}+3\hat{k}$

Since work W is the scalar product of force and displacement,

$W = (-\hat{i}+2\hat{j}+3\hat{k})\cdot 4\hat{k}$

W = 12 Joule

15. (C)

Equal sides have their opposite angles equal.

$\theta = 45^o = \dfrac{\pi}{4}$

$\alpha = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$ radian

Hence (C) is correct.

16. (D)

$Q^2 = P^2 + Q^2/4$ or $P^2 = \dfrac{3}{4}Q^2$ or $P = \left(\dfrac{\sqrt{3}}{2}\right)Q$

Now $\tan\theta = \dfrac{Q/2}{\left(\dfrac{\sqrt{3}}{2}\right)Q} = \dfrac{1}{\sqrt{3}}$

$\theta = 30^o$.

Required angle $\alpha = 180^o - 30^o = 150^o$.

Hence (D) is correct.

17. (C)

$|\vec{A}| = |\vec{B}|, \vec{A}\neq\vec{B}$

$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B}$ -

$= |\vec{A}|^2 - |\vec{B}|^2 = 0$

Hence (C) is correct.

18. (B)

$|\vec{A}| \neq 0, |\vec{B}| \neq 0, |\vec{C}| \neq 0$

$\vec{A} \times \vec{B} = 0 \Rightarrow \vec{A} \| \vec{B}$

$\vec{B} \times \vec{C} = 0 \Rightarrow \vec{B} \| \vec{C} \Rightarrow \vec{A} \| \vec{B} \| \vec{C}$

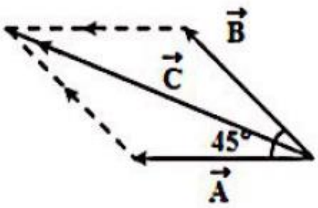
Hence $\vec{A} \times \vec{C} = \vec{0} =$ null vector

Hence (B) is correct.

19. (D)

$\vec{A}.\vec{B} = AB\cos\theta$ .....(1)

$|\vec{A} \times \vec{B}| = AB\sin\theta\hat{n}$ .....(2)



$\therefore \quad AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45^o$

Again given $\vec{C} = \vec{A} + \vec{B}$

$\therefore \quad |\vec{C}| = \left(A^2 + B^2 + 2AB\cos 45^o\right)^{1/2}$

$= \left(A^2 + B^2 + \sqrt{2}AB\right)^{1/2}$

Hence (D) is correct.

20. (C)

Resultant force, $\vec{F} = 15\hat{i} + 0\hat{j} + 25\hat{k}$

Clearly, the particle shall move in x – z plane.

21. (D)

Let $|\vec{A}| = |\vec{B}| = a$, then $|\vec{C}| = \sqrt{2}a$

Given that $\vec{A} + \vec{B} + \vec{C} = 0$ or $\vec{A} + \vec{B} = -\vec{C}$

Taking self product, we have

$(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = (-\vec{C}) \cdot (-\vec{C})$ or $A^2 + B^2 = 2\vec{A}\vec{B} = C^2$

$a^2 + a^2 + 2a^2\cos\theta = 2a^2$

$\therefore \cos\theta = 0$ or $\theta = 90^o$

So, angle between $\vec{A}$ and $\vec{B} = 90^o$

Again $\vec{B} + \vec{C} = -\vec{A}$

or $(\vec{B} + \vec{C}) \cdot (\vec{B} + \vec{C}) = (-\vec{A}) \cdot (-\vec{A})$ or $B^2 + C^2 + 2\vec{B} \cdot \vec{C} = A^2$

or $a^2 + 2a^2 + 2\sqrt{2}a^2\cos\varphi = a^2$

$\cos\varphi = \frac{-1}{\sqrt{2}}$ or $\theta = 135^o$

θ angle between $\vec{B}$ and $\vec{C} = 135^o$

Similarly, angle between $\vec{C}$ and $\vec{A}$ is 135°.

22. (A)

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \left(\sqrt{x^2+y^2}\right)^2 = (x+y)^2 + (x-y)^2 + 2(x+y)(x-y)\cos\theta$

$\Rightarrow \cos\theta = \dfrac{-\left(x^2+y^2\right)}{2\left(x^2-y^2\right)}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{-\left(x^2+y^2\right)}{2\left(x^2-y^2\right)}\right)$

23. (D)

The given vectors are parallel $(\vec{B} = 2\vec{A})$

Hence their cross product is zero. Thus (A) is correct.

$\left|\vec{A}\right| = \sqrt{(3)^2 + (4)^2} = 5$

$\left|\vec{B}\right| = \sqrt{(6)^2 + (8)^2} = 10$

(B) is correct.

(C) is also correct.

$\vec{A}\cdot\vec{B} = 50$. Thus option (D) is incorrect.

24. (A)

$$v = \vec{\omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 4\hat{k}$$

Now, $\vec{v}.\vec{v} = (-2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(-2\hat{i} + 3\hat{j} + 4\hat{k})$

or $v^2 = 4 + 9 + 16 = 29$

$\therefore$ $v = \sqrt{29}$ units.

25. (B)

Here, $x = 3m, y = 4m$ and z = 5 m

$\therefore |s| = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2}$

$= \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$ m

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TOPIC: MOLE CONCEPT

Solutions

1. (D)
As the mole of atoms is same, the number of atoms is also same.

2. (C)
Average atomic mass of $C = \frac{12\times98+14\times2}{100} = 12.04$
Number of C^{12} atoms $= \frac{98}{100}\times\frac{12}{12.04}\times6.022\times10^{23} = 5.88\times10^{23}$

3. (D)

4. (C)
New molecular mass of H_2O = 2 × 0.5 + 1 × 20 = 21
Percentage increase in molecular mass of H_2O $= \frac{21-18}{18}\times10 = 16.67\%$

5. (D)
Number of electrons lost $= \frac{13.5}{27}\times6.02\times10^{23}\times3$
$= 9.03\times10^{23}$

6. (C)
$3.72 = 0.02\times M_X + 0.04\times24 \Rightarrow M_X = 138$

7. (B)
$\frac{100}{M} = \frac{90}{2}+\frac{10}{4} \Rightarrow M = 2.105$

8. (A)
$\frac{(40\times32)+(60+48)}{100}$

9. (A)

$$\frac{1.8}{18n} = \frac{5-1.8}{96} \Rightarrow n = 3$$

10. (B)

Mass of nitrogen in the compound

$$= \frac{14}{22400} \times 28 = \frac{0.035\ \text{g}}{2}$$

Percentage of nitrogen in the compound

$$= \frac{0.035}{0.4} \times 100 = 8.75\ \%$$

11. (D)

$$2.5 \times 22.4 = (1 \times 12 + 2 \times 1) \times n \Rightarrow n = 4$$

$\Rightarrow$ Molecular formula $= (CH_2)_4 = C_4H_8$

12. (D)

Molecular mass of oxide $= \frac{100}{30.4} \times 28 = 92.1$

Hence, density of oxide relative to oxygen,

$$\frac{d_{\text{oxide}}}{d_{\text{oxygen}}} = \frac{M_{\text{oxide}}}{M_{\text{oxygen}}} = 2.88$$

13. (C)

$$C_3H_6(g) + \frac{3}{2}NO(g) \rightarrow C_3H_3N(g) + \frac{3}{2}H_2O(g) + \frac{1}{4}N_2(g)$$

42 g   53 g

$\therefore$ 420 kg   530 kg

14. (A)

$2Al + 3H_2SO_4 \rightarrow Al_2(SO_4)_3 + 3H_2$

$2Al + 2NaOH + 2H_2O \rightarrow 2NaAlO_2 + 3H_2$

15. (D)

Molecular mass of $Ag_2S = 2 \times 108 + 32 = 248$

Now, $w \times \frac{2.296}{100} \times \frac{216}{248} = 2 \Rightarrow w = 100\ \text{g}$

16. (D)

The excess reactant is Sulphur and the mass fraction of Sulphur remained $= \frac{56-32}{56} = 0.4285$.

17. (C)

18. (D)

19. (B)

$$2C_{57}H_{110}O_6(s)+163O_2(g)\rightarrow 110H_2O(\ell)+114CO_2(g)$$

Molecular mass of $C_{57}H_{110}O_6$

$=2\times(12\times57+1\times110+16\times6)g=1780\,g$

Molecular mass of $110\,H_2O=110(2+16)=1980\ g$

$1780\,g\ of\ C_{57}H_{110}O_6$ Produced = $1980\,g\ of\ H_2O$

$445\,g\ of\ C_{57}H_{110}O_6$ Produced = $\frac{1980}{1780}\times445\,g\ of\ H_2O$

$=495\ of\ H_2O$

20. (B)

$$(60\times d)\times\frac{60}{100}=50\Rightarrow d=1.38\ \text{g/mL}$$

21. (C)

$$\frac{w\times1000}{208\times125}\times2=\frac{3.78\times1000}{58.5\times100}\times1\ \Rightarrow\ w=8.4\ \text{g}$$

22. (C)

Mass of solvent (in g) = Volume of solution (in ml)

$$(1000\times d-1\times60)=1000\Rightarrow\ \text{d}=1.06\ \text{g/mL}$$

23. (D)

Mass of solvent is same in all.

24. (C)

Mole of solvent is minimum for benzene.

25. (C)

Mass of 1 atom $=1.8\times10^{-22}\,g$

Mass of 6.02×10^{23} atoms

$= 6.02 \times 10^{23} \times 1.8 \times 10^{-22} g$

$= 6.02 \times 1.8 \times 10 g$

$= 108.36 g$

$\therefore$ Atomic mass of element = 108.36

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TOPIC: QUADRATIC EQUATIONS

Solutions

1. (A)

Given equation $4x^2+3x+7=0$, therefore $\alpha+\beta=-\frac{3}{4}$ and $\alpha\beta=\frac{7}{4}$

Now $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-3/4}{7/4}=\frac{-3}{4}\times\frac{4}{7}=-\frac{3}{7}$

2. (C)

α,β are roots of $ax^2+bx+c=0 \Rightarrow \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$.

Let the roots of $cx^2+bx+a=0$ be α',β' then $\alpha'+\beta=-\frac{b}{c}$ and $\alpha'\beta'=\frac{a}{c}$ but $\frac{\alpha+\beta}{\alpha\beta}=\frac{-b/a}{c/a}=\frac{-b}{c} \Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}=\alpha'=\frac{1}{\alpha}$ and $\beta'=\frac{1}{\beta}$

3. (A)

Here, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$.

It roots are $\alpha+\frac{1}{\beta}$, $\beta+\frac{1}{\alpha}$, then sum of roots are

$=\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=(\alpha+\beta)+\frac{\alpha+\beta}{\alpha\beta}=-\frac{b}{ac}(a+c)$ and product $=\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

$=\alpha\beta+1+1+\frac{1}{a\beta}=2+\frac{c}{a}+\frac{a}{c}=\frac{2ac+c^2+a^2}{ac}=\frac{(a+c)^2}{ac}$

Hence required equation is given by

$x^2+\frac{b}{ac}(a+c)x+\frac{(a+c)^2}{ac}=0 \Rightarrow acx^2+(a+c)bx+(a+c)^2=0$

Trick : Let $a=1, b=-3, c=2$, then $\alpha=1, \beta=2$

$\therefore \quad \alpha+\frac{1}{\beta}=\frac{3}{2}$ and $\beta+\frac{1}{\alpha}=3$

Therefore, required equation must be $(x-3)(2x-3)=0$

i.e. $2x^2-9x+9=0$

Here (a) gives this equation on putting $a=1, b=-3, c=2$.

4. (B)

$\alpha+\beta=-6$ ...(i)

$\alpha\beta=\lambda$ ...(ii)

and given $3\alpha+2\beta=-20$ ...(iii)

Solving (i) and (iii), we get

$\beta=2, \alpha=-8$

Substituting these values in (ii), we get

$\lambda=-16$

5. (C)

It is an identity in x, if

$a^2-3a+2=0, a^2-5a+6=0, a^2-4=0$

$\Rightarrow$ $a=1, 2$ and $a=2, 3$ and $a=2, -2$

$\therefore$ Equation is identity, if $a=2 \Rightarrow$ only one solution.

Hence, (C) is the correct answer.

6. (B)

Given equation can be written as $x^2+x(p+q-2r)+pq-pr-qr=0$ ...(i)

whose roots are α and $-\alpha$, then the product of roots

$-\alpha^2=pq-pr-qr=pq-r(p+q)$ ...(ii)

and sum $0=p+q-2r \Rightarrow r=\frac{p+q}{2}$ ...(iii)

From (ii) and (iii), we get

$-\alpha^2=pq-\frac{p+q}{2}(p+q)=-\frac{1}{2}\left\{(p+q)^2-2pq\right\}$

$=-\frac{(p^2+q^2)}{2}$.

7. (A)

$\alpha=7+5\ell$ then $\beta=7-5\ell$, $\alpha+\beta=14$, $\alpha.\beta=74$

$x^2-(\alpha+\beta)x+\alpha\beta=0$ *or* $x^2-14x+74=0$

8. (C)

Roots are equal, then $B^2-4AC=0 \Rightarrow 4a^2m^2=4(m^2+1)(a^2-b^2) \Rightarrow a^2-b^2(m^2+1)=0$

9. (B)

Since, the roots of the given equation are of opposite sign, product of the roots <0

$\Rightarrow$ $\frac{p(p-1)}{3}<0$

$\Rightarrow \quad p(p-1)<0$

$\Rightarrow \quad p\in(0,1)$

Hence, (B) in the correct answer.

10. (C)

Given equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ can be re-written as

$3x^2-2(a+b+c)x+(ab+bc+ca)=0$

$\Delta=4\{(a+b+c)^2-3(ab+bc+ca)\}$ $(\because\ b^2-4ac=\Delta)$

$=4(a^2+b^2+c^2-ab-bc-ac)=2\{(a-b)^2+(b-c)^2+(c-a)^2\}\geq 0$

Hence both roots are always real.

11. (D)

$ax^2+x+b=0$ has real roots

$\Rightarrow\ (1)^2-4ab\geq 0\ \Rightarrow\ -4ab\geq -1$ or $4ab\leq 1$ ….. (i)

Now second equation is $x^2-4\sqrt{ab}x+1=0$.

Therefore $D=16ab-4$, from (i) $D\leq 0$

Hence roots are Complex.

12. (C)

Roots of $x^2-8x+(a^2-6a)=0$ are real.

So $D\geq 0\ \Rightarrow 64-4(a^2-6a)\geq 0\ \Rightarrow\ 16-a^2+6a\geq 0\ \Rightarrow\ a^2-6a-16\leq 0\ \Rightarrow\ (a-8)(a+2)\leq 0$

Now we have two cases: Case I : $(a-8)\leq 0$ and $(a+2)\geq 0\ \Rightarrow\ a\leq 8$ and $a\geq -2$

Case II : $(a-8)\geq 0$ and $(a+2)\leq 0\ \Rightarrow\ a\geq 8$ and $a\leq -2$ but it is impossible

Therefore, we get $-2\leq a\leq 8$

Aliter : Students should note that the expression $(x-a)(x-b)\{a<b\}$ will be less than or equal to zero if $x\in[a,b]$ or otherwise $x\notin[a,b]$.

Therefore $(a-8)(a+2)\leq 0$

i.e. $\{a-(-2)\}(a-8)\leq 0\ \Rightarrow\ a\in[-2,8]$.

13. (A)

Here, $(b+c-2a)+(c+a-2b)+(a+b-2c)=0$

Therefore the roots are rational.

14. (B)

Given equation can be written as

$(6k+2)x^2+rx+3k-1=0$ ...(i) and

$2(6k+2)x^2+px+2(3k-1)=0 \qquad ...(ii)$

Condition for common roots is

$\frac{12k+4}{6k+2}=\frac{p}{r}=\frac{6k-2}{3k-1}=2$ or $2r-p=0$

15. (D)

Let a be a common root, then

$\alpha^2+a\alpha+10=0 \qquad ...(i)$ and

$\alpha^2+b\alpha-10=0 \qquad ...(ii)$

form (i) - (ii),

$(a-b)\alpha+20=0 \Rightarrow \alpha=-\frac{20}{a-b}$

Substituting the value of a in (i), we get

$\left(-\frac{20}{a-b}\right)^2+a\left(-\frac{20}{a-b}\right)+10=0$

$\Rightarrow 400-20a(a-b)+10(a-b)^2=0$

$\Rightarrow 40-2a^2+2ab+a^2+b^2-2ab=0$

$\Rightarrow a^2-b^2=40$

16. (C)

Let $y=\frac{x^2-3x+4}{x^2+3x+4} \Rightarrow (y-1)x^2+3(y+1)x+4(y-1)=0$

For x is real $D\ge 0 \Rightarrow 9(y+1)^2-16(y-1)^2\ge 0$

$\Rightarrow -7y^2+50y-7\ge 0 \Rightarrow 7y^2-50y+7\le 0 \Rightarrow (y-7)(7y-1)\le 0$

Now, the product of two factors is negative if one in –ve and one in +ve.

Case I : $(y-7)\ge 0$ and $(7y-1)\le 0 \Rightarrow y\ge 7$ and $y\le\frac{1}{7}$.

But it is impossible Case II : $(y-7)\le 0$ and $(7y-1)\ge 0$

$\Rightarrow y\le 7$ and $y\ge\frac{1}{7} \Rightarrow \frac{1}{7}\le y\le 7$

Hence maximum value is 7 and minimum value is $\frac{1}{7}$

17. (A)

$x^2-3x+3=\left(x-\frac{3}{2}\right)^2+\frac{3}{4}$

Therefore, smallest value is $\frac{3}{4}$, which lie in $\left(-3,\frac{3}{2}\right)$

18. (A)

Given equation is $x^2-2ax+a^2+a-3=0$

If roots are real, then $D\ge 0 \Rightarrow 4a^2-4(a^2+a-3)\ge 0$

$\Rightarrow\ -a+3\geq 0\ \Rightarrow\ a-3\leq 0\ \Rightarrow\ a\leq 3$

As roots are less than 3, hence $f(3)>0$ $\quad 9-6a+a^2+a-3>0\ \Rightarrow\ a^2-5a+6>0$

$\Rightarrow\ (a-2)(a-3)>0\ \Rightarrow$ either $a<2$ or $a>3$

Hence, $a<2$ satisfy all.

19. (A)

Let $f(x)=ax^2-bx+c$

$f(0)=c>0$ and $f(2)=4a-2b+c<0$

So that $f(x)=0$ has a root in the interval $(0,2)$.

Hence, (A) is the correct answer.

20. (C)

If a, b, γ are the roots of the equation.

$$x^3-px^2+qx-r=0$$

$$\therefore\quad (\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1}=\frac{p^2+q}{pq-r}$$

Given, $p=0, q=4,\ r=-1\ \Rightarrow\ \dfrac{p^2+q}{pq-r}=\dfrac{0+4}{0+1}=4$

21. (D)

We know that the expression $ax^2+bx+c>0$ for all x, if $a>0$ and $b^2<4ac$

$\therefore\quad (a^2-1)x^2+2(a-1)x+2$ is positive for all x if

$a^2-1>0$ and $4(a-1)^2-8(a^2-1)<0\ \Rightarrow\ a^2-1>0$ and $-4(a-1)(a+3)<0$

$\Rightarrow\ a^2-1>0$ and $(a-1)(a+3)>0$

$\Rightarrow\ a^2>1$ and $a<-3$ or $a>1$

$\Rightarrow\ a<-3$ or $a>1$

22. (C)

Changing x to $\dfrac{1}{x}$ in given equation,

$\Rightarrow\qquad x^2+6x+9=0$

23. (C)

$mx^2+3x+4<5(x^2+2x+2)$

$\Rightarrow(m-5)x^2-7x-6<0\ \forall\ x\in\mathbf{R}$

$\Rightarrow m-5<0$ and $(-7)^2+24(m-5)<0$

$\Rightarrow m<5$ and $24m-71<0\Rightarrow m<\dfrac{71}{24}$

24. (C)

$$\frac{2x}{2x^2+5x+2} > \frac{1}{x+1} \Rightarrow \frac{2x}{(2x+1)(x+2)} > \frac{1}{(x+1)} \Rightarrow \frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$$

$$\Rightarrow \frac{2x(x+1)-(2x+1)(x+2)}{(x+1)(2x+1)(x+2)} > 0 \Rightarrow \frac{2x^2+2x-2x^2-4x-x-2}{(x+1)(x+2)(2x+1)} > 0 \Rightarrow \frac{-3x-2}{(x+1)(x+2)(2x+1)} > 0$$

Equating each factor equal to 0,

We have $x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$.

It is clear that $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$.

25. (B)

Subtract one equation from the other to obtain $x = 1$.