

# PACE-IIT & MEDICAL

MUMBAI / AKOLA / DELHI / KOLKATA / GHAZIABAD / NASHIK / GOA / BOKARO / PUNE

IIT – JEE: 2024

TW TEST (ADV)

DATE: 02/10/22

TOPIC: KINEMATICS - II

## Answer Key

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (C)  | 2. (B)  | 3. (D)  | 4. (B)  | 5. (A)  |
| 6. (C)  | 7. (D)  | 8. (B)  | 9. (C)  | 10. (B) |
| 11. (A) | 12. (A) | 13. (A) | 14. (B) | 15. (A) |
| 16. (B) | 17. (C) | 18. (B) | 19. (C) | 20. (B) |
| 21. (A) | 22. (A) | 23. (C) | 24. (A) | 25. (B) |

## SOLUTION

1. (C)

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} \Rightarrow \tan \theta = 0.5 \Rightarrow \cos \theta = \frac{2}{\sqrt{5}}$$

$$\text{Also, } \frac{g}{2u^2 \times \frac{4}{5}} = 0.04 \Rightarrow y = 12.5 \text{ m/s}$$

2. (B)

$$x = 0.3n; y = -0.2n$$

$$\text{Equation of trajectory : } y = -\frac{gx^2}{2u^2} \Rightarrow n = 9$$

3. (D)

$$\text{Time of flight} = 1 + 3 = 4 \text{ sec (using symmetry)}$$

4. (B)

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} \Rightarrow \tan \theta = 1$$

5. (A)

Distance travelled = area under the v-t curve

$$= \frac{20 \times 2}{2} + 20 \times 2 + 20 \times 1 + \frac{20 \times 1}{2} + \frac{20 \times 1}{2} = 100 \text{ m}$$

6. (C)

The displacement of the body during the time  $t$  as it reaches the point of projection

$$\Rightarrow S = 0 \Rightarrow v_0 t - \frac{1}{2} g t^2 = 0 \qquad \Rightarrow t = \frac{2v_0}{g}$$

During the same time  $t$ , the body moves in absence of gravity through a distance  $D' = v_0 t$ , because in absence of gravity  $g = 0$

$$\Rightarrow D' = v_0 \left( \frac{2v_0}{g} \right) = \frac{2v_0^2}{g} \qquad \dots(1)$$

In presence of gravity, the total distance covered is

$$= D = 2H = 2 \frac{v_0^2}{2g} = \frac{v_0^2}{g} \qquad \dots(2)$$

$$(1) \div (2) \Rightarrow D' = 2D.$$

7. (D)

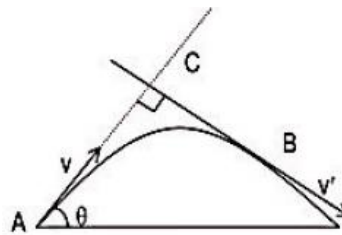
$$\vec{v} = \vec{u} + \vec{a}t$$

Considering along the line AC

$$0 = v - g \sin \theta t \Rightarrow t = \frac{v}{g \sin \theta}$$

Now, consider along the line CB

$$v' = 0 + g \cos \theta \frac{v}{g \sin \theta} = v \cot \theta$$



8. (B)

$$\text{Drift } (\Delta x) = (v_{b,x}) \Delta t = (v_{b,r} \cos \theta + v_r) \Delta t$$

Where  $v_{b,x}$  = velocity of boat w.r.t. ground

$v_{\perp,r}$  = velocity of boat w.r.t. ground

$v_r$  = velocity of river w.r.t. ground

$$\text{For } \Delta x = 0, v_r = -v_{br} \cos \theta$$

$$\Rightarrow (v_r)_{\max} = v_{br}$$

For,  $v_r > v_{br}$  we can not have zero drift.

9. (C)

$$\text{Drift} = \frac{d}{2} = \frac{(V_r - V_s \sin 30)d}{V_s \cos 30}$$

$$\Rightarrow V_s = 4(2 - \sqrt{3})V$$

10. (B)

When the horizontal range is maximum, the maximum height attained is  $\frac{R}{4} = 100\text{m}$ .

The velocity of the projectile is minimum at the highest point.

$\therefore$  Required point is (200, 100).

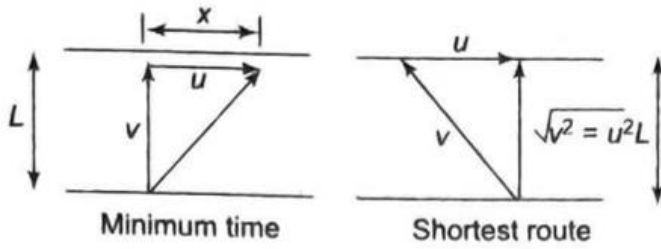
11. (A)

The maximum distance covered by the vehicle before coming to rest =  $\frac{v^2}{2a} = \frac{(15)^2}{2(0.3)} = 375 \text{ m}$

The corresponding time =  $t = \frac{v}{a} = \frac{15}{0.3} = 50 \text{ sec}$

∴ The distance of the vehicle from the traffic signal after one minute =  $400 - 375 = 25 \text{ m}$

12. (A)



$$10 = \frac{L}{v} \quad \dots(i)$$

$$12.5 = \frac{L}{\sqrt{v^2 - u^2}} = \frac{L}{v\sqrt{1 - u^2/v^2}} \quad \dots(ii)$$

From (i) and (ii),

$$\frac{1}{12.5} = \frac{L}{v} \times \frac{v\sqrt{1 - u^2/v^2}}{L}$$

$$\frac{4}{5} = \sqrt{1 - \frac{12^2}{v^2}}$$

$$\frac{16}{25} = 1 - \frac{12^2}{v^2} \Rightarrow \frac{12^2}{v^2} = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\frac{12}{v} = \frac{3}{5} \Rightarrow v = \frac{12 \times 5}{3} = 20 \text{ m/s}$$

13. (A)

Given, acceleration  $a = bt$

$$\Rightarrow \frac{dv}{dt} = bt \Rightarrow v = \frac{bt^2}{2} + c$$

$$\text{At } t = 0, v = v_0 \Rightarrow c = v_0$$

$$\text{So, } v = \frac{bt^2}{2} + v_0$$

$$\Rightarrow \frac{ds}{dt} = \frac{bt^2}{2} + v_0$$

$$\Rightarrow s = \frac{bt^3}{6} + v_0 t$$

14. (B)

**At 4 s**

$$u = at = 8 \text{ m/s}$$

$$s_1 = \frac{1}{2} at^2 = \frac{1}{2} \times 2 \times 4^2 = 16 \text{ m}$$

**From 4 s to 8 s**

$$a = 0, v = \text{constant} = 8 \text{ m/s}$$

$$s_2 = 8 \times 4 = 32 \text{ m}$$

$$u_t = \frac{at^2}{2}$$

**From 8s to 12 s**

$$s_3 = s_1 = 16\text{ m}$$

$$\therefore s_{\text{Total}} = s_1 + s_2 + s_3 = 64\text{ m}$$

15. (A)

At the time of overtaking,

$$s_1 = s_2$$

$$\therefore 2ut + \frac{1}{2}at^2 = ut + \frac{1}{2}(2a)t^2$$

$$\therefore t = \frac{2u}{a}$$

$$\begin{aligned}\therefore s_1 \text{ (or } s_2) &= (2u) \left( \frac{2u}{a} \right) + \frac{1}{2}(a) \left( \frac{2u}{a} \right)^2 \\ &= \frac{6u^2}{a}\end{aligned}$$

16. (B)

$s$  = net area of  $v$ - $t$  graph

At 2s, net area = 0

$$\therefore s = 0$$

and the particle crosses its initial position.

17. (C)

18. (B)

$$\sqrt{x} = t + 3$$

$$\therefore x = (t + 3)^2$$

$$\begin{aligned}\text{or } v &= \frac{dx}{dt} \\ &= 2(t + 3)\end{aligned}$$

$\therefore v$ - $t$  equation is linear.

19. (C)

$\Delta v = v_f - v_i =$  area under  $a$ - $t$  graph

$$v_i = 0$$

$$\begin{aligned}\Rightarrow v_f &= \text{area} \\ &= 40 + 50 \\ &= 90\text{ m/s}\end{aligned}$$

20. (B)

$$v^2 = 25 + 25s$$

$$\text{or } v^2 = (5)^2 + 2(12.5)s$$

Now compare with  $v^2 = u^2 + 2as$

21. (A)

22. (A)

$$a = v \cdot \frac{dv}{ds} = (4)(-\tan 60^\circ)$$

$$= -4\sqrt{3} \text{ m/s}^2$$

23. (C)

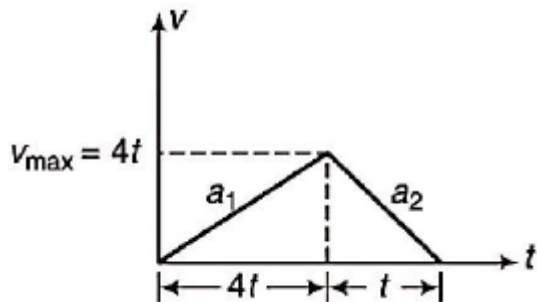
Relative velocity of  $A$  with respect to  $B$  should be along  $AB$  or absolute velocity components perpendicular  $AB$  should be same.

$$\therefore \frac{2u}{3} \sin \theta = u \sin 30^\circ$$

$$\therefore \theta = \sin^{-1}\left(\frac{3}{4}\right)$$

24. (A)

Deceleration is four times. Therefore, deceleration time should be  $\frac{1}{4}$  th.



$$v_{\max} = (a_1)(4t) = (1)(4t) = 4t$$

Area of  $v$ - $t$  graph = displacement

$$\therefore 200 = \frac{1}{2}(5t)(4t)$$

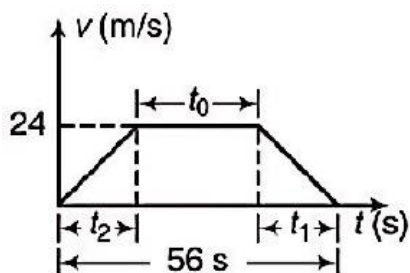
$$\text{or } t = \sqrt{20} \text{ s}$$

$$\text{Total journey time} = 5t = 22.4 \text{ s}$$

25. (B)

Area of  $v$ - $t$  graph = displacement

$$\therefore 1032 = \frac{1}{2}(56 + t_0)(24) \text{ or } t_0 = 30 \text{ s}$$



$$\text{Deceleration time } t_1 = \frac{24}{4} = 6 \text{ s}$$

$$\therefore \text{Acceleration time } t_2 = 56 - t_0 - t_1 = 20 \text{ s}$$

$$\therefore \text{Acceleration} = \frac{24}{20} = 1.2 \text{ m/s}^2$$

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TOPIC: ATOMIC STRUCTURE & MOLE CONCEPT

## Answer Key

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 26. (A) | 27. (A) | 28. (D) | 29. (A) | 30. (D) |
| 31. (A) | 32. (C) | 33. (C) | 34. (D) | 35. (A) |
| 36. (B) | 37. (C) | 38. (B) | 39. (D) | 40. (C) |
| 41. (B) | 42. (A) | 43. (D) | 44. (B) | 45. (C) |
| 46. (A) | 47. (B) | 48. (B) | 49. (A) | 50. (D) |

## SOLUTION

26. (A)

Molar mass of  $C_{12}H_{22}O_{11} = 342$  g

342 g sugar has = 12  $N$  atoms of C

$$\therefore 1.71 \text{ g sugar has} = \frac{12 \times 6.02 \times 10^{23} \times 1.71}{342} \text{ atoms}$$
$$= 3.6 \times 10^{22} \text{ atoms}$$

27. (A)

$$\text{Per cent of oxygen in NaOH} = \frac{16 \times 100}{40} = 40.$$

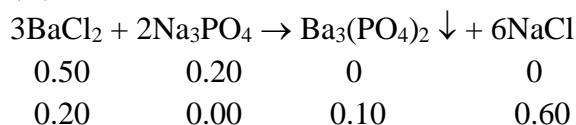
28. (D)

1 mole  $P_4 = N$  molecules of  $P_4 = 4N$  atoms of  $P_4$ .

29. (A)

$$\text{Mole fraction} = \frac{n_1}{n_1 + n_2} = \frac{\text{Molecules of solute}}{\text{Total molecules of solute and solvent}}$$
$$= \frac{10^{24}}{10^{24} + 10^{25}} = \frac{1}{11} = 0.09$$

30. (D)



31. (A)



$$\text{Molality} = \frac{\text{Moles of solute}}{\text{wt. of solvent in kg}}$$

32. (C)

$$\text{milli mole of solution I} = 750 \times 0.5 = 375$$

$$\text{milli mole of solution II} = 250 \times 2 = 500$$

$$\text{total milli mole in mixture} = 375 + 500 = 875$$

$$\therefore \text{Molarity} = \frac{875}{1000} = 0.875 \text{ M}$$

33. (C)

$$\text{Mole of N}_2 \text{ is } = \frac{4}{28} = \frac{1}{7} \text{ (the lowest value)}$$

34. (D)

$$9.108 \times 10^{-31} \text{ kg} = 1 \text{ electron}$$

$$\therefore 1 \text{ kg} = \frac{1}{9.108 \times 10^{-31}} \text{ electron}$$

$$= \frac{1}{9.108 \times 10^{-31}} \times \frac{1}{6.023 \times 10^{23}} \text{ mole electron}$$

35. (A)

$$558.5 \text{ g Fe } \frac{558.5}{55.85} \text{ mole Fe} = 10 \text{ mole Fe}$$

$$= 2 \times 5 \text{ mole C} = 2 \times 60 \text{ g C}$$

36. (B)

$$M = \frac{\text{moles of urea}}{\text{volume in litre}} = \frac{6.02 \times 10^{20}}{6.02 \times 10^{23} \times \frac{100}{1000}} = 0.01 \text{ M}$$

37. (C)

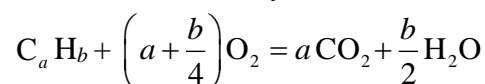
$$m = \frac{\text{moles of CH}_3\text{COOH}}{\text{wt. of solvent in kg}} = \frac{2.05 \times 1000}{897} = 2.285$$

$$\text{Wt. of solvent} = \text{wt. of solution} - \text{wt. of solute}$$

$$= [1000 \times 1.02 - 2.05 \times 60] = 897 \text{ g}$$

38. (B)

Let the formula of hydrocarbon be  $C_aH_b$



$$\text{mole of CO}_2 (a) \text{ formed} = \frac{3.08}{44} = 0.07$$

$$\text{mole of H}_2\text{O formed} \left(\frac{b}{2}\right) = \frac{0.72}{18} = 0.04$$

$$\therefore \frac{a}{b/2} = \frac{0.07}{0.04}$$

$$\text{or } \frac{a}{b} = \frac{0.07}{0.08} = \frac{7}{8}$$

$\therefore$  mole ratio of C and H : : 7 : 8

Thus empirical formula is  $C_7H_8$ .

39. (D)

Average isotopic wt. =  $\Sigma\% \times$  isotopic wt.

$$= \frac{(\text{per cent} \times \text{wt. of isotope}) + (\text{per cent} \times \text{wt. of other isotope})}{100}$$

$$\therefore 20.2 = \frac{a \times 20 + (100 - a) \times 22}{100}$$

$$\therefore a = 90$$

Per cent of heavier isotope =  $100 - 90 = 10$

40. (C)

$$\frac{1}{\lambda_{\text{Lyman}}} = R_H \left[ \frac{1}{1^2} - \frac{1}{\infty^2} \right] = R_H$$

$$\frac{1}{\lambda_{\text{Balmer}}} = R_H \left[ \frac{1}{2^2} - \frac{1}{\infty^2} \right] = \frac{R_H}{4}$$

$$\frac{\lambda_{\text{Balmer}}}{\lambda_{\text{Lyman}}} = 4 \text{ or } \lambda_B = 4 \times 912 \text{ \AA}$$

41. (B)

Angular momentum for  $n$  and  $(n + 1)$  shells are  $\frac{nh}{2\pi}$  and  $(n + 1)\frac{h}{2\pi}$ .

Thus, difference in angular momentum of two successive orbits is  $(n + 1)\frac{h}{2\pi} - \frac{nh}{2\pi} = \frac{h}{2\pi}$

42. (A)

For longest  $\lambda$  of Balmer series  $n_1 = 1$  and  $n_2 = 2$ ,

$$\frac{1}{\lambda} = R_H \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Because  $\Delta E = \frac{hc}{\lambda}$  is minimum when  $\lambda$  is longest.

Thus,  $\Delta E = E_2 - E_1$ .

$$\text{Thus, } \frac{1}{\lambda} = R_H \times \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] = \frac{3}{4} \times 109678$$

$$\therefore \lambda = 1.215 \times 10^{-15} \text{ cm} = 1215 \text{ \AA}$$

43. (D)

$$\Delta u = \frac{0.1}{100} \times 10 = 10^{-2} \text{ m sec}^{-1}; \text{ Now } \Delta u \cdot \Delta X = \frac{h}{4\pi m}$$

$$\therefore \Delta X = \frac{6.625 \times 10^{-34}}{4 \times 10^{-2} \times 3.14 \times 200 \times 10^{-3}} = 2.64 \times 10^{-32} \text{ m}$$

44. (B)

All the three electrons are to be kept in  $1s$ .

45. (C)

$(n + l)$  for rest all is = 7;

For  $3s$  it is  $3 + 0 = 3$ .

46. (A)

$$E_{2(\text{Be}^{3+})} = E_{2(\text{H})} \times Z^2; \quad \text{Also } E_{2(\text{H})} = \frac{E_{1(\text{H})}}{2^2}$$

$$\therefore E_{2(\text{Be}^{3+})} = \frac{E_{1(\text{H})}}{2^2} \times 4^2 = 4 \times E_{1(\text{H})}$$

47. (B)

$$r_n = r_1 \times n^2$$

$$r_3 = 3^2 x = 9x$$

$$\text{Also, } m u r_3 = \frac{3h}{2\pi}$$

$$\text{or } m u = 3 \frac{h}{2\pi \cdot 9x} = \frac{h}{6\pi x}$$

$$\text{or } \lambda = \frac{h}{m u} = \frac{h \cdot 6\pi x}{h} = 6\pi x$$

48. (B)

Electronic configuration of atom with atom no. 105 is :  $1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^2 4p^6 4d^{10}, 5s^2 5p^6 5d^{10} \underline{5f^{14}}, 6s^2 6p^6 \underline{6d^3}, 7s^2$ .

The underlined orbitals have  $(n + l) = 8$ .

49. (A)

Pd :  $[\text{Kr}]4d^{10}$ . An exception.

50. (D)

Ground state of hydrogen atom, *i.e.*,  $1s$

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DATE: 02/10/22

TOPIC: INEQUATIONS & EQUATIONS

## Answer Key

|         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 51. (A) | 52. (C) | 53. (C) | 54. (D) | 55. (A) |
| 56. (D) | 57. (B) | 58. (A) | 59. (B) | 60. (B) |
| 61. (D) | 62. (B) | 63. (C) | 64. (C) | 65. (C) |
| 66. (A) | 67. (D) | 68. (C) | 69. (D) | 70. (A) |
| 71. (D) | 72. (D) | 73. (A) | 74. (A) | 75. (C) |

## SOLUTIONS

51. (A)

$$a = \log_2 3$$

$$b = \log_9 4$$

$$\Rightarrow ab = \log_2 3 \log_3 2 = 1.$$

52. (C)

$$\text{Let } 4 + \log_3(x) = N \Rightarrow \log_2 N = 3 \Rightarrow N = 2^3$$

$$N = 4 + \log_3 x = 8$$

$$\therefore N = 8$$

$$\log_3 x = 4 \Rightarrow x = 81.$$

Sum of digits of  $x = 9$ .

53. (C)

$$\text{Using } \log_{10} p + \log_{10} r - \log_{10} s = \log_{10} \left( \frac{pr}{s} \right)$$

$$\log_{10} \left( \frac{x(x+2)}{5x+4} \right) = 0$$

$$\Rightarrow x^2 + 2x = 5x + 4$$

$$\Rightarrow x = 4, x = -1$$

(reject)

54. (D)

$$x^{(1+\log_{10} x)} = 10^5 \cdot x$$

Taking log on both sides to base 10 :

$$(1 + \log_{10} x)(\log_{10} x) = 5 + \log_{10} x$$

$$\log_{10} x = t \Rightarrow t(1+t) = 5+t$$

$$\Rightarrow t^2 + t = 5+t$$

$$\Rightarrow t = 5^{1/2} \text{ or } t = -5^{1/2}$$

$$\Rightarrow \log_{10} x = 5^{1/2}; \log_{10} x = -5^{1/2}$$

$$\Rightarrow x = 10^{5^{1/2}}; x = 10^{-5^{1/2}}$$

$$\text{Product} = 10^0 = 1$$

55. (A)

Take log on both sides

$$\frac{3}{2} + 2(\ln x)^2 = 4 \ln x$$

$$t = \ln x \quad \dots(1)$$

$$2t^2 - 4t + \frac{3}{2} = 0 \begin{cases} t_1 \rightarrow t_1 \ln x_1 \\ t_2 \rightarrow t_2 \ln x_2 \end{cases}$$

$$t_1 + t_2 = 2$$

$$\ln x_1 + \ln x_2 = 2 \quad (\text{from Eqs. (1)})$$

$$\ln(x_1 x_2) = 2 \Rightarrow x_1 x_2 = e^2$$

Note:  $\log_e x = \ln x$ ; where 'e' is Napier's constant. Its irrational quantity.

56. (D)

57. (B)

Wavy curve method

58. (A)

$$\text{Domain: } 1 - \sqrt{1+x} > 0 \text{ \& } 3 - \sqrt{1+x} > 0$$

$$\Rightarrow \sqrt{1+x} < 1$$

$$\text{Put } \sqrt{1+x} = t \Rightarrow t \geq 0$$

$$(1-t)^2 = 3-t$$

$$t^2 - t - 2 = 0$$

$$t = 2, -1 \text{ (both rejected)}$$

$$\Rightarrow \text{No real solution}$$

59. (B)

$$t = \log_3 x$$

$$\sqrt{(1+2t)\left(2+\frac{t}{2}\right)} = \frac{3}{2}t$$

$$t > 0$$

Squaring

$$2 + \frac{t}{2} + 4t + t^2 = \frac{9}{4}t^2$$

$$5t^2 - 18t - 8 = 0$$

$$5t^2 - 20t + 2t - 8 = 0$$

$$(5t + 2)(t - 4) = 0$$

$$\Rightarrow t = 4$$

$$\Rightarrow \log_3 x = 4$$

$$x = 81$$

60. (B)

$$7^x = a, 7^{x+12} = b$$

$$a^2 - 2ab + b^2 = 0$$

$$(a - b)^2 = 0$$

$$\Rightarrow a = b$$

$$\Rightarrow x^2 = x + 12$$

$$x^2 - x - 12 = 0$$

$$x^2 - 4x + 3x - 12 = 0$$

$$(x + 3)(x - 4) = 0$$

$$x = 4, -3.$$

61. (D)

$$\frac{x-3}{11-x} \geq 0$$

$$\frac{x-3}{x-11} \leq 0$$

$$x \in [3, 11)$$

62. (B)

$$0 < x^2 - 5x + 6 \leq 2$$

63. (C)

$$x = \log_2 \sqrt{\sqrt{\sqrt{\sqrt{56 + \sqrt{56 + \sqrt{56 + \sqrt{56 + \dots \infty}}}}}}}$$

$$2^x = \sqrt{\sqrt{\sqrt{\sqrt{56 + \sqrt{56 + \sqrt{56 + \dots}}}}}}$$

$$2^x = \sqrt{56 + 2^x}$$

$$2^{2x} - 2^x = 56 \Rightarrow 2^{2x} - 2^x - 56 = 0$$

Let  $2^x = t \quad \therefore t^2 - t - 56 = 0$

$$t = 8 \quad \& \quad t = -7x$$

$$2^x = 8 \quad \therefore 2^x = 2^3 \Rightarrow x = 3$$

$$\therefore 2 < x < 4$$

64. (C)

$$-2 < x - 5 < 2$$

$$3 < x < 7$$

65. (C)

Let  $\log_2 7$  be rational then  $\log_2 7 = \frac{p}{q}$  ( $q \neq 0$ )

$$7 = 2^{p/q} \Rightarrow 2^p = 7^q$$

Here all power of 2 is even but 7 is odd.

$\therefore$  our assumption is wrong

$\therefore \log_2 7$  is irrational.

66. (A)

**Case (1) :**  $t < 2$

$$-2t + 4 = 2 - t + 3$$

$$t = -1$$

$$\log_5 x = -1$$

$$\Rightarrow x = \frac{1}{5}$$

**Case (2) :**  $2 \leq t < 3$

$$2t - 4 = 2 - t + 3$$

$$3t = 9$$

$$t = 3$$

$$\log_5 x = 3$$

**Case (3) :**  $t \geq 3$

$$2t - 4 = 2 + t - 3$$

$$t = 3$$

$$\log_5 x = 3$$

$$x = 125$$

67. (D)

68. (C)

69. (D)

$$(\log_2 x)^2 + 4(\log_2 x) - 1 = 0$$

Let  $\log_2 x = t$

$$t^2 + 4t - 1 = 0$$

$$t = \frac{-4 \pm \sqrt{16 + 4}}{2}$$

$$t = -2 \pm \sqrt{5}$$

$$\log_2 x = -2 \pm \sqrt{5}$$

$$x = 2^{-2 \pm \sqrt{5}}$$

$$\therefore \beta = 2^{-2 - \sqrt{5}}$$

$$\alpha = 2^{-2 + \sqrt{5}}$$

$$\alpha\beta = 2^{-4} = \frac{1}{16}$$

70. (A)

$$\log_{0.3}(x-1) < \frac{1}{2} \log_{0.3}(x-1)$$

$$x-1 > (x-1)^{1/2}$$

$$x^2 + 1 - 2x > x - 1$$

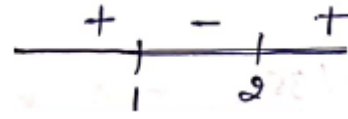
$$x^2 - 3x + 2 > 0 \Rightarrow (x-2)(x-1) > 0$$

$$x \in (-\infty, 1) \cup (2, \infty) \quad \dots(1)$$

$$\& x-1 > 0 \Rightarrow x > 1 \Rightarrow x \in (1, \infty) \quad \dots(2)$$

$$\therefore (1) \& (2)$$

$$\Rightarrow (2, \infty)$$



71. (D)

$$-5x + 6 \leq x + 1 \quad x \geq \frac{5}{6}$$

$$x^2 - 5x + 6 \geq 0 \quad x \in (-\infty, 2] \cup [3, \infty)$$

Intersection

$$\left[ \frac{5}{6}, 2 \right] \cup [3, \infty) \text{ Ans.}$$

72. (D)

$$\text{Domain: } x-3 > 0, x-3 \neq 1 \& x^3 - 3x^2 - 4x + 8 > 0$$

$$x^3 - 3x^2 - 4x + 8 = (x-3)^3$$

$$6x^2 - 31x + 35 = 0$$

$$x = \frac{31 \pm \sqrt{(31)^2 - 4(6)(35)}}{12}$$

$$\Rightarrow x = \frac{31 \pm 11}{12} \Rightarrow x = \begin{cases} \frac{42}{12} = \frac{7}{2} \\ \frac{20}{12} = \frac{5}{3} \end{cases}$$

$\therefore$  No. of solution = 1 as  $x = \frac{5}{3}$  is rejected

73. (A)

$$\text{Domain: } \log_2 x > 0 \text{ i.e. } x > 1$$

$$9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$$

$$3^{2 \log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$$

$$(\log_2 x)^2 + (\log_2 x)^2 - (\log_2 x) - 1 = 0$$



$\therefore$  By property  $a^{\log_a} = 1$

Let  $\log_2 x = t$

$$\therefore 2t^2 - t - 1 = 0$$

$$2t(t-1) + 1(t-1) = 0$$

$$t = 1, -\frac{1}{2} \Rightarrow \text{only } t = 1 \text{ acceptable}$$

$$\Rightarrow \log_2 x = 1 \Rightarrow x = 2 \Rightarrow \text{Sum of roots} = 2$$

74. (A)

$$\frac{1}{2} \log_5 x^2 + (\log_5 x)^2 - 2 < 0$$

Let  $\log_5 x = t$

$$\log_5 x + (\log_5 x)^2 - 2 < 0$$

$$t^2 + t - 2 < 0$$

$$(t+2)(t-1) < 0$$

$$t \in (-2, 1)$$

$$\therefore \log_5 x \in (-2, 1)$$

$$\therefore x \in \left( \frac{1}{25}, 5 \right)$$



75. (C)

$$2 \cdot \log_9 (3^{1-x} + 2) = 1 + \log_3 (4 \cdot 3^x - 1)$$

$$\frac{2}{2} \cdot \log_3 \left[ \frac{3}{3^x} + 2 \right] = \log_3 3 + \log_3 (4 \cdot 3^x - 1)$$

Let  $3^x = t \Rightarrow t > 0$

$$\log_3 \left[ \frac{3}{t} + 2 \right] = \log_3 [3 \cdot (4 \cdot 3^x - 1)]$$

$$3 + 2t = 12t^2 - 3t$$

$$12t^2 - 5t - 3 = 0$$

$$(4t-3)(3t+1) = 0$$

$$t = \frac{3}{4}, -\frac{1}{3} \quad \therefore 3^x = \frac{3}{4}$$

$$x \log 3 = \log 3 - \log 4$$

$$\Rightarrow x = 1 - \log_3 4$$