IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 15/07/23**

TOPIC: PROPERTIES OF MATTER

SOLUTIONS

1. (B)
Conceptual

2. (A)
$Y = \frac{F\ell}{Ae} \Rightarrow e = \frac{F\ell}{YA}$
$F = T = 6mg$
$e = \frac{6mg\ell}{YA}$

3. (A)
$\frac{-\Delta P}{V} = \left(\frac{\Delta P}{K}\right) = (h\rho g)C \qquad (2700\times10^3\times10)45.4\times10^{-11} = 1.2\times10^{-2}$

4. (C)
$W = \frac{1}{2}Fe$

5. (D)
Conceptual

6. (A)
Conceptual

7. (D)
As $\upsilon \propto r^2$ and the mass of the drop $m \propto r^3$, its momentum $p = m\upsilon \propto r^5$.
Here, $2^5 = 32$.

8. (A)
Breaking stress depends on the material of wire only.

9. (C)
$\eta = \frac{F/A}{x/L} \Rightarrow x = \frac{L}{\eta}\times\frac{F}{A}$

If η and F are constant then $x \propto \frac{L}{A}$

For maximum displacement are at which force applied should be minimum and vertical side should be maximum, this is given in the *R* position of rectangular block.

10. (A)

$$Y = \frac{FL}{\pi r^2 \ell}$$

$$\therefore\ \lambda = \frac{FL}{\pi r^2 Y} \Rightarrow \ell \propto \frac{L}{r^2}$$

11. (A)

Side of the cube

$a = 60$ mm = 60×10^{-3} m.

Change in the pressure

$\Delta P = 2.5 \times 10^7\ \text{N/m}^2$

Bulk modulus $B = 1.25 \times 10^{11}\ \text{N/m}^2$

The volume of cube is

$V = (60 \times 10^{-3})^3$

The change in volume is

$$\Delta V = \frac{\Delta PV}{B} = \frac{\left(2.5\times10^7\right)\times\left(60\times10^{-3}\right)^3}{\left(1.25\times10^{11}\right)}$$

$$= 43.2\times10^{-9}\ \text{m}^3 = 43.2\ \text{mm}^3$$

12. (A)

$$\Delta\lambda = \frac{F\ell}{Ay} \Rightarrow \frac{\Delta\ell}{(F/A)} = \frac{\ell}{y} = \text{slope of curve}$$

$$\frac{\ell}{y} = \frac{(4-2)\times10^{-3}}{4000\times10^3}$$

Given $\ell = 1\,\text{m} \rightarrow y = \frac{4000\times10^3}{2\times10^{-3}} = 2\times10^9\ \text{N/m}^2$

13. (B)

According to Hooke's law; if a body is deformed, the strain produced is directly proportional to the applied stress. When the stress is removed, the material return back to its original dimension.

14. (B)

$$U = \frac{1}{2}\frac{(\text{stress})^2}{Y}$$

$$\text{Stress} = \frac{\text{Tension}}{\text{area}}$$

$$\text{Stress} = \frac{U_A}{U_{\text{middle}}} = \frac{T_A^2}{\left(T_{\text{middle}}\right)^2} = \frac{F^2}{\left(F/2^2\right)} = 4$$

$\therefore$ Ans. (B)

15. (A)

$$\sigma = \frac{\int_0^{\ell} \frac{m}{\ell} \cdot dx\,\omega^2 \cdot x}{A}$$

$$\frac{2\sigma A}{m\ell} = \omega^2$$

$$\omega^2 = \frac{2\sigma}{\ell^2 \rho} \Rightarrow \omega = \frac{1}{\ell}\sqrt{\frac{2\sigma}{\rho}}$$

16. (A)

Viscous force = $mg\sin\theta$

$$\therefore\ \eta A\frac{v}{t} = mg\sin\theta \text{ or } \eta a^2 = \frac{v}{t} = a^3\rho g\sin\theta$$

$$\eta = \frac{r\rho g\sin\theta a}{v}$$

17. (B)

Shear force F = T = mg = 0.020 × 10 = 0.2 N

Shear stress on the fluid = $= \frac{F}{A} = \frac{0.2}{0.1} = 2$

Strain rate $= \frac{\upsilon}{\ell} = \frac{0.090}{0.30 \times 10^{-3}}$

$$\eta = \frac{\text{stress}}{\text{strain rate}} = \frac{2\left(0.30 \times 10^{-3}\right)}{(0.090)} = \frac{20}{3} \times 10^{-3} \text{ Pa s}.$$

18. (C)

When two drops of radius r each combine to form a big drop, the radius of big drop will be given by

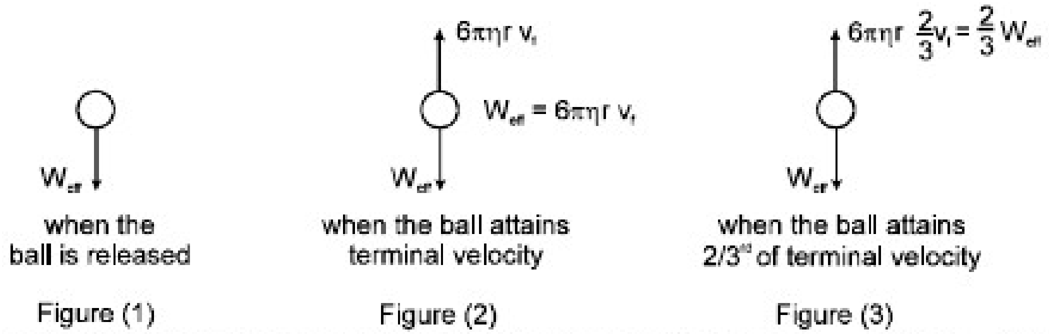
$$\frac{4}{3}\pi R^3 = \frac{4\pi}{3}r^3 + \frac{4\pi}{3}r^3$$

or $R^3 = 2r^3$ or $R = 2^{1/3} r$

Now $\frac{V_R}{V_r} = \left(\frac{R}{r}\right)^2 = 2^{2/3} = 4^{1/3}$

$\therefore$ $V_R = 5 \times 4^{1/3}$ cm/s

19. (A)



When the ball is just released, the net force on ball is

The terminal velocity 'v_f' of the ball is attained when net force on the ball is zerp.

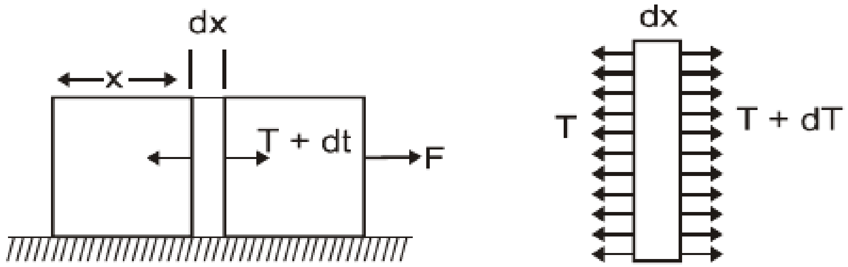
$\therefore$ Viscous force $6\pi\eta r v_f = W_{\text{eff}}$

When the ball acquired $\frac{2}{3}$ rd of its maximum velocity v_f the viscous force is $= \frac{2}{3}$ W_{eff}.

Hence net force is $W_{\text{eff}} - \frac{2}{3}W_{\text{eff}} = \frac{1}{3}W_{\text{eff}}$

$\therefore$ Required acceleration is $= \frac{a}{3}$

20. (B)



Acceleration $a = \frac{F}{m}$

then $T = \frac{mx}{\ell} \times \frac{F}{m} = \frac{Fx}{\ell}$

Extension in 'dx' element – $d\delta = \frac{Tdx}{Ay} = \frac{Fxdx}{\ell Ay}$

total extension $\delta = \int_0^{\ell} \frac{Fxdx}{\ell Ay} = \frac{F\ell}{2Ay}$

21. (13800)

F = shear strength × area on which shear stress acts

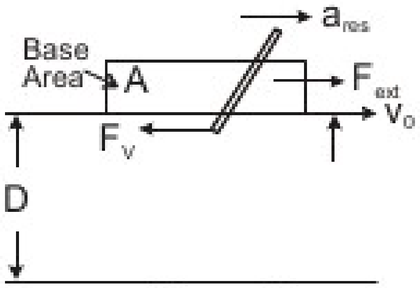
$= 345 \times 10^6 \times 4 \times 10^{-4}$

$= 138000 \text{ N} = 138000 \text{ Kg}$

22. (1)

$F_{ext} - F_v = m a_{rms}$

As boat moves with constant velocity $a_{rms} = 0$



$F_{ext} = F_v$

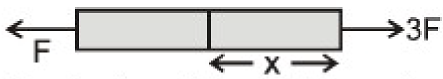
But $F_v = \eta A \frac{dv}{dz}$, but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$

Then $F_{ext} = F_v = \frac{\eta A v_0}{D}$

23. (0)

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta \ell}{\ell} = 0$$

24. (4)



Tension in rod at a distance x from right edge is

$$T = F\left(3 - 2\frac{x}{L}\right)$$

$\therefore$ net extension in rod $= \int_0^L \frac{T}{4A} dx = \frac{2F}{YA} L$

25. (9)

$$R_V = \frac{8\eta L}{\pi R^4} + \frac{8\eta 2L}{\pi (2R)^4} = \frac{8\eta \ell}{\pi R^4}\left[1 + \frac{1}{8}\right]$$

26. (50)

$$B = -V\frac{\Delta P}{\Delta V} = \frac{10^7}{2\times 10^{-4}} = 50\times 10^9 \text{ N/m}^2$$

27. (6)

Magnitude of viscous force, $F = \eta A \frac{dv}{dr}$

$\Rightarrow$ viscous force per unit area $\sigma = \frac{F}{A} = \eta \frac{dv}{dr}$

$$v = v_0\left(1 - \frac{r^2}{R^2}\right) \Rightarrow \frac{dv}{dr} = -\frac{2V_0 r}{R^2} \Rightarrow \sigma = \eta \cdot \frac{2v_0 r}{R^2} \quad \text{...(i)}$$

Volume rate of flow, Q consider an annular element at r from axis, width dr.

$$dA = 2\pi r\, dr \text{ ; } dQ = v.dA = v_0\left(1 - \frac{r^2}{R^2}\right)2\pi r\, dr$$

$$Q=\int dQ=2\pi v_0\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R=\frac{\pi}{2}R^2v_0 \quad\Rightarrow\quad v_0=\frac{2Q}{\pi R^2}$$

$$\therefore \quad \text{(i)} \Rightarrow \ \sigma=\eta\frac{4Q}{\pi R^4}r,\ R=0.1\,\text{m}$$

$$\text{At } r=0.04\text{ m},\ \sigma=(0.75)4\times\frac{\pi}{2}\times10^{-2}\times\frac{0.04}{\pi\times10^{-4}}=6\ \text{Nm}^{-2}$$

28. (8)

$$Mg-f_B=F_v$$

$$\Rightarrow \ \frac{4}{3}\pi r^3(\rho_m-\rho_\ell)g=F_v$$

29. (12)

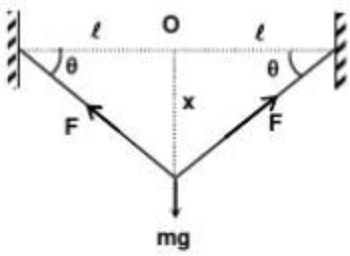
Relative to liquid, the velcoity of spehre is $2v_0$ upwards.

$\therefore$ viscous force on sphere $=6\pi\eta r 2v_0$ downward = $12\pi\eta r v_0$ downward

30. (1)

$$2F\sin\theta=mg$$

$$F=\frac{mg}{2\sin\theta}=\frac{mg\ell}{2x} \qquad [\because \theta \text{ is small}]$$



$$\Delta\ell=\left(\sqrt{\ell^2+x^2}-\ell\right)$$

$$=\ell\left[\left(1+\frac{x^2}{\ell^2}\right)^{1/2}-1\right]$$

$$\Delta\ell=\ell\left[1+\frac{1}{2}\frac{x^2}{\ell^2}-1\right] \qquad [\because\ x<<\ell]$$

$$\therefore \quad \frac{\Delta\ell}{\ell}=\frac{x^2}{2\ell^2}$$

$$\text{Now } \frac{F}{A}=Y\frac{\Delta\ell}{\ell}$$

$$F=AY\frac{\Delta\ell}{\ell}$$

$$\frac{mg\ell}{2x}=AY\left(\frac{x^2}{2\ell^2}\right)$$

$$x^3=\frac{mh\ell^3}{AY}$$

$$\therefore \ x=\ell\left(\frac{mg}{AY}\right)^{1/3}$$

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TOPIC: ELECTROCHEMISTRY

SOLUTIONS

31. (C)
$Pt(s) | H_2(g, 1\,bar) | H^+(aq, 1\,M) || cu^{+2}(1\,M) | Cu$

32. (C)
An intensive property in a bulk property, meaning that its is a physical property which does not depend on the size. An extensive property, on the other hand, depends on the size/amount.

33. (C)
Conductivity depends upon solvation of ion present in solution.

34. (B)
Cathode $\rightarrow Cl_2 + 2e^- \rightarrow 2Cl^-$ $1P_2$
Anode $\rightarrow 2Cl^- \rightarrow 2e^- + Cl_2$ $1P_1$
$$E_{cell} = E^o_{cell} - \frac{0.0591}{2}\log\frac{P_1}{P_2}$$
As per concentration cell E^o_{cell} in O.
$$E_{cell} = \frac{-0.0591}{2}\log\frac{P_1}{P_2}$$
$\Delta G = -nF\,E_{cell}$
for a reaction to be spontaneous, ΔG is to be negative.
Thus E_{cell} must be positive.
$$\Rightarrow E_{cell} = -\ln\frac{P_1}{P_2}$$
$$\Rightarrow E_{cell} = \frac{0.0591}{2}\log\frac{P_2}{P_1}$$
For E_{cell} to be positive.
$P_2 > P_1$

35. (C)
Cathode $\rightarrow 4H_2O + 4e^- \rightarrow 2H_2 + 4OH^-$
Anode $\rightarrow 2H_2O \rightarrow O_2 + 4H^+ + 4e^-$
$2H_2 : O_2 \Rightarrow 2:1$

36. (B)

Putting the electrolysis of dilute sulphuric acid. Using platinum electrode, oxygen gas is liberated at anode.

37. (D)

At STP, 1 mole oxygen $\rightarrow$ 22400 mL

No. of moles of oxygen corresponding to 5600 ml

$$\frac{5600}{22400} = \frac{1}{4}$$

1 mol of oxygen $\xrightarrow{\text{gives}}$ 4 moles of electron

1 mol of silver $\xrightarrow{\text{reacts}}$ 1 mole of electron.

$$\frac{W_{Ag}}{M_{Ag}} \times 1 = \frac{W_{O_2}}{M_{O_2}} \times 4$$

$$\frac{W_{Ag}}{M_{Ag}} \times 1 = \frac{1}{4} \times 4$$

$$\frac{W_{Ag}}{108} = \frac{1}{4} \times 4$$

$$W_{Ag} - = 108\text{ g}$$

38. (A)

$$E^{o}_{(Cu^{2+}/Cu)} > E^{o}_{(Sn^{2+}/Sn)}$$

$\therefore$ Cu electrode act an cathode.

Sn electrode act an anode.

$\therefore$ Cu^{+2} ion can be reduced by H_2.

39. (C)

$M > B > A$

40. (B)

$$\Lambda^{m}_{BaSO_4} = \Lambda^{m}_{BaCl_2} + \Lambda^{m}_{H_2SO_4} - 2\Lambda^{m}_{HCl}$$

$$= x_1 + x_2 - 2x_3$$

41. (C)

$K = 1.382 \times 10^{-6}\ S\,cm^{-1}$ (given)

$\Lambda_{AgCl} = 61.9 + 76.3 = 138.2$

Let the solubility be S.

$$138.2 = \frac{1000 \times 1.382 \times 10^{-6}}{S}$$

$S = 1 \times 10^{-5}$ mol litre^{-1}

42. (A)

The correct option is **A**

$[Cr(NH_3)_6]Cl_3$

Explanation for the correct option

(A) $[Cr(NH_3)_6]Cl_3$

Conductivity: -

- Here Conductivity is the ability of an aqueous solution to carry electric current.
- Conductivity can be determined by concerning the number of ions present in the molecule i.e., the higher the number of ions, the higher the conductivity.

Conductivity of $[Cr(NH_3)_6]Cl_3$

- $[Cr(NH_3)_6]Cl_3$ dissociates as $[Cr(NH_3)_6]^{3+} + 3Cl^-$
- Here the number of ions present in the complex is $1 + 3 = 4$
- Conductivity of $[Cr(NH_3)_6]Cl_3$is **high.**

Explanation for incorrect options

B) $[Cr(NH_3)_3Cl_3]$

- The number of ions present in the complex is 1
- As the number of ions is low the conductivity is low.

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B) $[Cr(NH_3)_3Cl_3]$

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- As the number of ions is low the conductivity is low.

(C) $[Cr(NH_3)_4Cl_2]Cl$

- $[Cr(NH_3)_4Cl_2]Cl$dissociates as $[Cr(NH_3)_4Cl_2]^+ + Cl^-$
- The number of ions present in the complex is $1 + 1 = 2$
- Here the number of ions produced is 2 so comparatively the conductivity is low

(D) $[Cr(NH_3)_5Cl]Cl_2$

- $[Cr(NH_3)_5Cl]Cl_2$dissociates as $[Cr(NH_3)_5Cl]^{2+} + 2Cl^-$
- The number of ions present in the complex is $1 + 2 = 3$
- Here the number of ions produced is 3 so comparatively the conductivity is low

Order of conductivity is

$[Cr(NH_3)_6]Cl_3 > [Cr(NH_3)_5Cl]Cl_2 > [Cr(NH_3)_4Cl_2]Cl > [Cr(NH_3)_3Cl_3]$

Therefore, the correct option is **(A)** $[Cr(NH_3)_6]Cl_3$

43. (B)

$$Pb + SO_4^{2-} \rightarrow PbSO_4 + 2e^-$$

$$PbO_2 + 4H^+ + SO_4^{2-} + 2e^- \rightarrow PbSO_4 + 2H_2O$$

The acid reacts with the positive electrods to relear electron.

In this process the H_2SO_4 in consumed and residual water is left behing (or evaporated).

So, the density of sulphuric acid decreases.

44. (D)

$$\frac{\text{Mass of A}}{\text{Eq. mass of A}} = \frac{\text{Mass of B}}{\text{Eq. mass of B}} = \frac{\text{Mass of C}}{\text{Eq. mass of C}}$$

$$\frac{45}{15} = \frac{2.7}{27} = \frac{9.6}{48}$$

$n_1 \quad n_2 \quad n_3$

$n_1 : n_2 : n_3 \Rightarrow 0.3 : 0.1 : 0.2$

$\Rightarrow 3 : 1 : 2$

45. (B)

$i = 9.65 \text{ A}$

$t = 1000 \text{ sec}$

$\text{M.W.} = 40 \text{ g/mol}.$

$$W_{\text{depaited}} = Z \times i \times t = \frac{E}{96500} \times 9.65 \times 1000$$

$$= \frac{(40/1)}{96500} \times 9.65 \times 1000 = 4 \text{ g}$$

46. (D)

$$AgNO_3 + H_2O \rightarrow H^+ + Ag + + NO_3^- + \frac{1}{2}O_2$$

As $[H^+]\uparrow$, $pH\downarrow$

$$LiCl + H_2O \rightarrow Li^+ + OH^- + \frac{1}{2}Cl_2 + \frac{1}{2}H_2$$

As $[OH^-]\uparrow$, $pH\uparrow$

47. (A)

If salt bridge of KCl will not be there then $ZnCl_2$ and $AgNO_3$ will real together to give a ppt. of AgCl.

48. (B)

$$Mn^{+2} + 2H_2O \rightarrow MnO_2 + 2H^+ + H_2$$

Amount of current is given by

$$I = m\frac{F \times Z}{t \times m}$$

$$= \frac{1000 \text{ g} \times 9650 \times 2}{24 \times 60 \times 60 \times 86.9}$$

$I = 25.70 \text{ A}$

Now, current efficiency $= \frac{25.7}{27} \times 100 = 95.185\%$

49. (C)

From NCERT

50. (D)

As the circuit will be incompletes, the ion will have no direction and then they will more randomly.

51. (10)

$$E_{cell} = E^o_{cell} - \frac{RT}{2F}\ln\frac{1}{1}$$

$$= 2.7 - 0 = 2.7 \text{ V} \qquad \text{... Case 1}$$

$$E_{cell} = E^o_{cell} - \frac{RT}{2F}\ln\frac{x}{1}$$

$$2.67 = 2.7 - \frac{R \times 300}{2F} \ln x$$

$$-0.03 = \frac{-R \times 300}{2F} \ln x$$

$$\ln x = \frac{0.03 \times 2F}{300 \times R} = 2.3 - \ln 10$$

$$x = 10$$

52. (3)

Given $\frac{\Lambda_m(HX)}{\Lambda_m(HY)} = \frac{1}{10}$

$$\frac{\alpha_1}{\alpha_{10}} = \frac{\Lambda_m(HX)}{\Lambda_m^o(HX)} \Big/ \frac{\Lambda_m(HY)}{\Lambda_m^o(HY)} = \frac{1}{10}$$

$$\frac{K_a(HX)}{K_a(HY)} = \frac{C_1\alpha_1^2}{C_2\alpha_2^2}$$

$$\Rightarrow \frac{0.01}{0.1} \times \left(\frac{1}{10}\right)^2 = \frac{1}{1000}$$

$$\log K_a(HX) - \log k_a(HY) = -3$$

$$\therefore \; pK_a(HX) - pK_a(HY) = 3$$

53. (4)

$$M^+ \rightarrow M^{3+} + +2e^-$$

$\Delta G^o = -nFE^o$ for 1 mol of M^+

$$\Delta G^o = -2 \times 96500 \times (-0.25) J$$

$$= +48.25 \text{ kJ/mol.}$$

Energy released by conversion 1 mole of $X \rightarrow Y \Rightarrow \Delta G = -193$ kJ

Hence mole of M^+ converted $= \frac{193}{48.25} = 4$

54. (6)

Conductivity $K = \frac{GI}{a}$

$$K = \frac{5 \times 10^{-7} S \times 120 \text{ cm}}{1 \text{ cmh}2} = 0.00006 \text{ S/cm}$$

Molar conductivity $\Lambda_m = \frac{1000 \text{ cm}^3/L \times 0.00006 \text{ S/cm}}{0.0015 \text{ mol/L}}$

$$\Lambda_m = 40 \text{ S cm}^2 / \text{mol}$$

$$pH = 4$$

$$[H^+] = 10^{-4} M$$

The degree of dissociation $\alpha = \frac{\Lambda_m}{\Lambda_m^o}$

$$\Lambda_m^o = \frac{\Lambda_m}{\alpha} = \frac{405 \text{ cm}^2 / \text{mol}}{0.06667}$$

$$\Lambda_m^o = 6 \times 10^2 \text{ S cm}^2 / \text{mol}.$$

$\Rightarrow\ Z=6$

55. (2)

$Mg+2A\eta+\rightarrow Mg^{+2}+2Ag$

$$E_{cell}=E^{o}_{cell}-\frac{0.0591}{n}\log\frac{\left[Mg^{+2}\right]}{\left[Ag^{+}\right]^{2}}$$

$$\Rightarrow\ E^{o}_{cell}-E_{cell}=\frac{0.0591}{n}\log\frac{\left[Mg^{2+}\right]}{\left[Ag^{+}\right]^{2}}$$

In the first case $E^{o}_{cell}-E_{cell}=\frac{0.0591}{2}\log\frac{0.1}{(0.5)^{2}}$

In the first case $E^{o}_{cell}-E_{cell}=\frac{0.0591}{2}\log\frac{0.01}{(0.25)^{2}}$

$$=\frac{0.0591}{2}\log\frac{\left((0.1)^{2}\right)}{\left((0.5)^{2}\right)}$$

$$=\frac{0.0591}{2}\times2\times\log\frac{(0.1)}{(0.05)^{2}}$$

$=2$ times

56. (7)

$$E^{o}_{cell}=\frac{0.0591}{2}\log K$$

$$\Rightarrow\ 0.235=\frac{0.0591}{2}\log K$$

$\log K=7.9526$

$K=8.966\times10^{7}$

y = 7

57. (6)

The cell reaction in

$3Mg+2Al^{3+}\rightarrow3Mg^{2+}+2Al$

n = 6

58. (2)

$pH=10$

$\Rightarrow\ \left[H^{++}\right]=10^{-pH}=10^{-10}\ M$

The presence of hydrogen gas in one atm.
The oxidation potential of electrodes would be

$$E_{cell}=\frac{0.059}{n}\log\frac{\left(p_{H_2}\right)^{0.5}}{[H^{+}]}$$

$$=\frac{0.059}{1}\log\frac{1}{10^{-10}}$$

$=+0.59\ V$

$= 590$ mili volts

$= 5.9 \times 10^2$ milivolt.

x = 2

59. (9)

3 L of 0.5 m $K_2Cr_2O_7$ will contain

$0.5 \times 3 = 1.5$ moles

Now, the reduction reaction.

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow Cr^{+3} + 7H_2O$

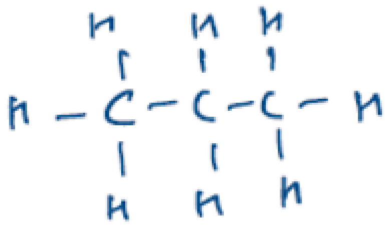
$\because$ 1 mole of $Cr_2O_7^{2-}$ requires 6F of electricity.

$\therefore$ 1.5 mole will require $\Rightarrow 6 \times 1.5$

= 9F

60. (5)

No. of mole of electron involved for 1 mole of $C_3H_8 = 20$



$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

$\therefore$ No. of moles of electron involved for 0.25 mole of C_3H_8.

$\Rightarrow 0.25 \times 50$

$\Rightarrow 5$

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TOPIC: HYPERBOLA

SOLUTIONS

61. (A)
e is greater than 1 for hyperbola

62. (B)
Being a four degree equation

63. (C)
One eccentricity must be greater than 1.

64. (A)
Any tangent to $y^2 = 4x$ is
$y = mx + \frac{1}{m}$
$\Rightarrow x\left(mx + \frac{1}{m}\right) = -2$ put $D = 0$, we get $m = \frac{1}{2}$

65. (C)
$PS_1 - PS_2 = S_1S_2$

66. (D)
Convert the equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ the line as $y = mx + c$ it will be a tangent if $c^2 = a^2m^2 - b^2$

67. (A)
Use condition for tangency. Take $m = 1$

68. (B)
Use $T = S_1$

69. (B)
Use condition for normality

70. (B)

Since foci hyperbola =foci of given ellipse

$\therefore\ 2a = 4$

$\Rightarrow a = 2 \because e = 2$

$\therefore b^2 = a^2\left(e^2 - 1\right) = 12$

Hence equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

71. (B)

If $P(\alpha, \beta)$ lies on the line $\frac{x}{a} + \frac{y}{b} = 1$, then $\frac{\alpha}{a} + \frac{\beta}{b} = 1$ … (i)

The mid-point of AB is $h = \frac{a}{2}$, $k = \frac{b}{2}$. Thus from (i), $\frac{\alpha}{h} + \frac{\beta}{k} = 2$.

So the locus of (h, k) is $\frac{\alpha}{x} + \frac{\beta}{y} = 2$.

$\Rightarrow \left(x - \frac{\alpha}{2}\right)\left(y - \frac{\beta}{2}\right) = \frac{\alpha\beta}{4}$

Which is a rectangular hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Hence (B) is the correct answer.

72. (C)

$\angle LOS = 45° \Rightarrow OS = LS$

$\therefore\ ae = \frac{b^2}{a} \Rightarrow e = e^2 - 1$

$\therefore\ e = \frac{\sqrt{5} + 1}{2}$

Hence (C) is the correct answer.

73. (B)

For $y = \frac{1}{x}, \frac{dy}{dx} = -\frac{1}{x^2} \Rightarrow$ the slope of the normal $= x^2 = -\frac{a}{b}$

$\Rightarrow -\frac{a}{b}$ is positive $\Rightarrow$ a and b are of opposite signs.

Hence (B) is the correct answer.

74. (A)

Let $M\left(x_1, y_1\right)$ be the midpoint of the chord PQ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Equation of PQ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} \Rightarrow y = -\frac{b^2 x_1 x}{a^2 y_1} + \frac{b^2}{y_1}\left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}\right)$

This is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If $\frac{b^4}{y_1^2}\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}\right)^2 = a^2\frac{b^4x_1^2}{a^4y_1^2} - b^2 \Rightarrow \left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}\right)^2 = \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}$

Hence locus of (x_1, y_1) is $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2}-\frac{y^2}{b^2}$

75. (C)

Let the equation of the hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Then, the conjugate hyperbola is $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

Their eccentricities e and e' are given by

$b^2 = a^2(e^2-1)$ and $a^2 = b^2(e'^2-1)$

Multiplying the corresponding sides, we have

$a^2b^2 = a^2b^2(e^2-1)(e'^2-1) \Rightarrow 1 = (e^2-1)(e'^2-1)$

$\Rightarrow 1 = e^2e'^2 - e'^2 - e^2 + 1 \Rightarrow e^2 + e'^2 = e^2e'^2 \Rightarrow \frac{1}{e^2}+\frac{1}{e'^2}=1$

76. (B)

We know that the equation

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

represent a rectangular hyperbola if D ¹ 0, $h^2 > ab$ and $a + b = 0$.

$\therefore$ The given equation represents a rectangular hyperbola if $4 + k = 0$ i.e. $k = -4$

77. (B)

Equation of hyperbola + Equation of conjugate hyperbola = 2 Equations of asymptotes

$\therefore$ Equation of conjugate hyperbola = 2

Equation of Asymp. – Equation of hyperbola

$C(x, y) = 2A(x, y) - H(x, y)$ ....(1)

$\because$ $H(x, y) \equiv x^2 + 3xy + 2y^2 + 2x + 3y = 0$

$\therefore$ Equation of asymptotes is

$x^2 + 3xy + 2y^2 + 2x + 3y + \lambda = 0$

$\because$ $\Delta = 0, abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, then $\lambda = 1$

$\therefore$ $A(x, y) \equiv x^2 + 3xy + 2y^2 + 2x + 3y + 1 = 0$

$\therefore$ $C(x, y) \equiv x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$

78. (B)

Let $(a\sec\theta, b\tan\theta)$ be the any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

The equations of the asymptotes of the given hyperbola are $\frac{x}{a}-\frac{y}{b}=0$ and $\frac{x}{a}+\frac{y}{b}=0$

Now, $p_1 =$ length of the perpendicular from

$\left(a\sec\theta, b\tan\theta\right)$ on $\frac{x}{a}-\frac{y}{b}=0$

$$=\frac{\sec\theta-\tan\theta}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

and, $p_2 =$ length of the perpendicular from $\left(a\sec\theta, b\tan\theta\right)$ on $\frac{x}{a}+\frac{y}{b}=0$

$$=\frac{\sec\theta+\tan\theta}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\therefore\ p_1p_2=\frac{\sec^2\theta-\tan^2\theta}{\frac{1}{a^2}+\frac{1}{b^2}}=\frac{a^2b^2}{a^2+b^2}$$

79. (B)
Use the formula $T=S_1$.

80. (A)
Convert the equation of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

81. (2)
We get a second degree equation in slope of the tangent

82. (6)
Number of point of intersection is 4.

83. (4)
Asymptotes pass through the center of the hyperbola

84. (2)
Perpendicular lines

85. (9)
Write the given equation in standard form i.e. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ by making perfect squares then latus rectum $=\frac{2b^2}{a}$

86. (7)
Let the line $x-y-1=0$ touches the hyperbola $3x^2-4y^2-12=0$ at $\left(x_1, y_1\right)$

Eqn. of tangent at $\left(x_1, y_1\right)$ is $\frac{xx_1}{4}-\frac{yy_1}{3}-1=0$

By comparing we get $x_1 = 4, y_1 = 3$

$\therefore$ Point of contact is (4, 3).

87. (12)

Eqn. of director circle is $x^2 + y^2 = a^2 - b^2 \Rightarrow x^2 + y^2 = 12$

88. (8)

Product of perpendicular $= \dfrac{a^2b^2}{a^2 + b^2}$

89. (9)

The hyperbola is $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

Let P be $(4\sec\theta, 3\tan\theta)$

Now the line $x = 4 \sec\theta$ intersects the asymptote $y = \dfrac{3}{4}x$ at $Q(4\sec\theta, 3\sec\theta)$ and the asymptote $y = -\dfrac{3}{4}x$ at $R(4\sec\theta, -3\sec\theta)$

So, $PQ = 3|\sec\theta - \tan\theta|$ and $PR = 3|\sec\theta + \tan\theta|$

$\therefore PQ.PR = 9$

90. (0)

A tangent can touch a second degree curve at a single point.