IIT – JEE: 2024 **TW TEST (MAIN)** **DATE: 05/10/22**

TOPIC: CALCULUS IN PHYSICS

Solution

1. (B)

2. (B)

3. (B)

4. (B)

$$\frac{de^{(ax+b)}}{dx} = e^{(ax+b)}\frac{d}{dx}e^{(ax+b)} = ae^{(ax+b)}$$

5. (C)

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

$$= 4\pi(10)^2 \times 4$$

$$= 1600\,\pi\ \text{cm}^3/\text{sec}$$

6. (A)

7. (D)

8. (D)

$$\frac{dy}{dx} = \frac{d}{dx}x^{-\frac{2}{3}} = -\frac{3}{2}x^{-\frac{3}{2}-1} = -\frac{3}{2}x^{-\frac{5}{2}}$$

9. (D)

10. (B)

$$\frac{dy}{dx} = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2y}{dx^2} = -A\omega^2\sin(\omega t + \phi)$$

$$= -\omega^2 y$$

11. (C)

12. (D)

$$\frac{dy}{dx}=12x^5+4x^3$$

At $x=1, \frac{dy}{dx}=12+4=16$

13. (C)

$$\frac{df(x)}{dx}=2\lambda x+\mu$$

At $x=2,\ 2\lambda(2)+\mu=11 \Rightarrow 4\lambda+\mu=11$ ...(1)

At $x=4$, $8\lambda+\mu=15$ ...(2)

From (1) & (2),

$\lambda=1, \mu=7$

$\therefore\ \lambda+\mu=8$

14. (C)

$$\frac{dy}{dx}=(6x^2+3)3+(3x+7)(12x)$$

At $x=1, \frac{dy}{dx}=(6+3)+(3+7)(12)=27+120=127$

15. (D)

$$\frac{dx}{dt}=3at^2,\ \frac{dy}{dt}=2bt$$

$$\therefore\ \frac{dy}{dx}=\frac{2bt}{3at^2}=\frac{2b}{3at}$$

At $t=1, \frac{dy}{dx}=\frac{2b}{3a}$

16. (A)

$$\frac{dy}{dx}=x^3\cdot\frac{1}{x}+(\ln x)\cdot 3x^2$$

$$=x^2+3x^2(\ln x)$$

$$=x^2(1+3\ln x)$$

$$=x^2(1+\ln x^3)$$

17. (A)

$$\frac{dy}{dx}=xe^x\cos x+x\sin x\cdot e^x+e^x\sin x$$

$$=e^x(x\cos x+x\sin x+\sin x)$$

$$=xe^x\left(\cos x+\sin x+\frac{\sin x}{x}\right)$$

18. (D)

19. (B)

$$\frac{dy}{dx} = \cos\left(xe^x\right)\cdot\left(xe^x + e^x\right)$$
$$= e^x\left(1+x\right)\cos\left(xe^x\right)$$

20. (B)

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TOPIC: ATOMIC STRUCTURE

SOLUTION

21. (C)

$K.E._{\max} = h\nu - \phi$

$E = h\nu - \phi \Rightarrow \phi = h\nu - E$

$E_2 = 2h\nu - \phi$

$E_2 = 2h\nu - (h\nu - E)$

$E_2 = h\nu + E$

22. (A)

Only one photon is emitted in 1 transition.

23. (D)

$\Delta X \, \Delta\nu \ge \frac{h}{4\pi m}$

$\Delta X \ge \frac{h}{4\pi m \Delta\nu}$

If $\Delta\nu = 0$, then $\Delta X = \infty$

24. (A)

$\lambda = \frac{h}{P}$

$P = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{6626\times10^{-9}} = 10^{-28}$ kg ms^{-1}

25. (A)

No. of orbitals $= 5^2 = 25$

Type of orbitals = *s*, *p*, *d*, *f*, *g*

26. (C)

Nodal plane of P_y orbital is *XZ* plane.

27. (B)

No. of radial nodes $= n - \ell - 1$

No. of radial nodes in $3s = 3 - 0 - 1 = 2$

No. of radial nodes in $3p = 3 - 1 - 1 = 1$

No. of radial nodes in 3*d* = 3 – 2 – 1 = 0

28. (D)
$Sc = [Ar]\ 4s^2\ 3d^1$
$Sc^+ = [Ar]\ 4s^1\ 3d^1$
Higher energy orbital is 3*d*.
$n = 3, \ell = 2$.

29. (B)
K shell = 1st orbit (2 electrons)
L shell = 2nd orbit (8 electrons)
M shell = 3rd orbit (16 electrons)
N shell = 4th orbit (2 electrons)
Atomic no. = 2 + 8 + 16 + 2 = 28

30. (C)
$$4.9 = \sqrt{n(n+2)}$$
$$n(n+2) = 24$$
$$n = 4$$
Cr^+ : [Ar] $4s^0\ 3d^5$ (n = 5)
Ti^{4+} : [Ar] (n = 0)
Fe^{2+} : [Ar] $3s^0\ 3d^6$ (n = 4)

31. (D)
X rays are EM waves.

32. (D)
$$\frac{E_1}{E_2} = \frac{hc/\lambda_1}{hc/\lambda_2} = \frac{1/\lambda_1}{1/\lambda_2} = \frac{\bar{\nu}_1}{\bar{\nu}_2} = \frac{2}{3}$$

33. (B)
$$E_1 = \frac{12400}{4000}\text{eV}$$
$$E_2 = \frac{12400}{6000}\text{eV}$$
$$E = \frac{E_1 + E_2}{2}$$
$$\frac{12400}{\lambda} = \frac{\frac{12400}{4000} + \frac{12400}{6000}}{2}$$
$$\frac{1}{\lambda} = \frac{1}{2}\left(\frac{1}{4000} + \frac{1}{6000}\right)$$
$$\lambda = \frac{2(4000)(6000)}{10000} = 4800\text{Å}$$

34. (A)
Bohr’s model is only applicable to unielectronic species.

35. (B)

$$r = 0.529 \times \frac{n^2}{Z} \text{Å}$$

$$r = (x)\frac{(2)^2}{3} = \frac{4x}{3}$$

36. (C)

Time period $\propto \frac{n^3}{Z^2}$

$$\frac{\text{Time period in 2nd orbit}}{\text{Time period in 3rd orbit}} = \frac{(2)^3}{(3)^3} = \frac{8}{27}$$

37. (C)

P.E. = 2 T.E.

$$= -2 \times 13.6 \times \frac{Z^2}{n^2}$$

$$= -2 \times 13.6 \text{ eV} = -27.2 \text{ eV}$$

38. (B)

Energy difference will be minimum for higher orbits.

39. (B)

Ionisation energy in ground state = 13.6 Z^2

= 13.6 $(2)^2$

= 54.4 eV

40. (B)

$$\frac{1}{\lambda} = R(3)^2\left[\frac{1}{1^2} - \frac{1}{4^2}\right]$$

$$\frac{1}{\lambda} = 9R\left(\frac{15}{16}\right)$$

$$\lambda = \frac{16}{135R}$$

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TOPIC: TRIGONOMETRY EQUATIONS

Solution

41. (D)

$\frac{1-\cos 2\theta}{1+\cos 2\theta}=3 \Rightarrow \frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}=3$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$.

42. (C)

$\sec^2\theta+\tan^2\theta=\frac{5}{3}$, also $\sec^2\theta-\tan^2\theta=1$

$\Rightarrow \tan^2\theta=\frac{1}{3}=\tan^2\left(\frac{\pi}{6}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

43. (C)

$\sin 4\theta = \cos\theta - \cos 7\theta \Rightarrow \sin 4\theta = 2\sin(4\theta)\sin(3\theta)$

$\Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = n\pi$ or $\sin 3\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 3\theta = n\pi + (-1)^n \frac{\pi}{6} \Rightarrow \theta = \frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$.

44. (A)

$\frac{1-\tan^2\theta}{\sec^2\theta}=\frac{1}{2} \Rightarrow \cos^2\theta-\sin^2\theta=\frac{1}{2}$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

45. (D)

$\cos^2\theta - \frac{5}{2}\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = \frac{(5/2)\pm\sqrt{(25/4)-4}}{2} = \frac{5\pm 3}{4}$

Rejecting (+) sign,

$\Rightarrow \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$.

46. (B)

$(2\cos x - 1)(3 + 2\cos x) = 0$

Then $\cos x = \frac{1}{2}$ as $\cos x \neq \frac{-3}{2}$

$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{3}; \begin{Bmatrix} \text{for } n=0,\ x = \frac{\pi}{3}, \frac{5\pi}{3} \\ \text{for } n=1,\ x = \frac{5\pi}{3} \end{Bmatrix}$

47. (A)

We have, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

Now check by options, put $x = \frac{\pi}{6}$

then $(81)^{\sin^2 \pi/6} + (81)^{\cos^2 \pi/6} = 30$

$\Rightarrow (81)^{1/4} + (81)^{3/4} = 30 \Rightarrow 30 = 30$

Hence (a) is the correct answer.

48. (B)

$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = n\pi + \frac{\pi}{3}$

For $-\pi < \theta < 0$

Put $n = -1$, we get $\theta = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$ or $\frac{-4\pi}{6}$.

49. (D)

We have, $\tan\theta + \frac{1}{\sqrt{3}} = 0$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$\because \theta$ lies in between 0° and 360°

$\therefore \theta = 150^\circ$ and 330°.

50. (D)

We have, $\cos^2\theta + \sin\theta + 1 = 0$

$\Rightarrow 1 - \sin^2\theta + \sin\theta + 1 = 0$

$\Rightarrow \sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = 2$, which is not possible and $\sin\theta = -1$.

Therefore, solution of given equation lies in the interval $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$.

51. (A)

$\sin\left(\theta + \frac{\pi}{6}\right) = 1 = \sin\left(\frac{\pi}{2}\right) \Rightarrow \theta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.

52. (A)

$\cos A \sin\left(A - \frac{\pi}{6}\right) = \frac{1}{2}\left[\sin\left(2A - \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\right]$

But $\sin\left(2A - \frac{\pi}{6}\right) - \frac{1}{2}$ attain maximum value at $2A - \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow A = \frac{\pi}{3}$.

53. (A)

Here $\cos\theta = 1 - 2\cos^2 40^o = -(2\cos^2 40^o - 1)$

$= -\cos(2 \times 40^o) = -\cos 80^o$

$= \cos(180^o + 80^o) = \cos(180^o - 80^o)$

Hence, $\cos 260^\circ$ and $\cos 100^\circ$ *i.e.*, $\theta = 100^\circ$ and $260°$.

54. (A)

Since A.M. $\geq$ G.M. $\frac{1}{2}(2^{\sin x} + 2^{\cos x}) \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$\Rightarrow \; 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1+\frac{\sin x + \cos x}{2}}$

and we know that $\sin x + \cos x \geq -\sqrt{2}$

$\therefore \; 2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$, for $x = \frac{5\pi}{4}$.

55. (B)

$(1+\tan\theta)\,(1+\tan\phi) = 2 \Rightarrow \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi} = 1$

$\Rightarrow \; \tan(\theta+\phi) = 1 \; \Rightarrow \; \theta + \phi = \frac{\pi}{4} = 45^{o}$.

56. (C)

$\sec x \cos 5x = -1 \Rightarrow \cos 5x = -\cos x$

$\Rightarrow \; 5x = 2n\pi \pm (\pi - x) \; \Rightarrow \; x = \frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{4}$

Hence $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{6}, \frac{7\pi}{4}, \frac{9\pi}{6}, \frac{11\pi}{6}$.

57. (C)

$\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x + \sin 2x + \alpha = 0$

$\Rightarrow \sin^2 2x - 2\sin 2x - 2 - 2\alpha = 0$

Let $\sin 2x = y$. Then the given equation becomes $y^2 - 2y - 2(1+\alpha) = 0$,

where $-1 \leq y \leq 1$, $\quad (\because \; -1 \leq \sin 2x \leq 1)$

For real, discriminant $\geq 0 \Rightarrow 3 + 2\alpha \geq 0 \; \Rightarrow \; \alpha \geq -\frac{3}{2}$

Also $-1 \leq y \leq 1 \Rightarrow -1 \leq 1 - \sqrt{3+2\alpha} \leq 1$

$\Rightarrow \; 3 + 2\alpha \leq 4 \Rightarrow \alpha \leq \frac{1}{2}$. Thus $-\frac{3}{2} \leq \alpha \leq \frac{1}{2}$.

58. (B)

$3\cos\theta + 4\sin\theta = 5\left[\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right] = 5\cos(\theta - \alpha)$

where $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$

Now $3\cos\theta + 4\sin\theta = k$

$\therefore \; 5\cos(\theta-\alpha) = k \Rightarrow \cos(\theta - \alpha) = \pm 1$

$\Rightarrow \; \theta - \alpha = 0^o, 180^o \Rightarrow \theta = \alpha,\; 180^o + \alpha$.

59. (C)

$3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow \; 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow \; 3\sin(\sin x - 2) - (\sin x - 2) = 0$

$\Rightarrow \; (3\sin x - 1)(\sin x - 2) = 0 \Rightarrow \; \sin x = \frac{1}{3}$ or 2

$\Rightarrow \; \sin x = \frac{1}{3}$, $(\because \sin x \neq 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha$, $0 < \alpha < \frac{\pi}{2}$ are the solutions in $[0, 5\pi]$. Then $\alpha,\; \pi - \alpha,\; 2\pi + \alpha,\; 3\pi - \alpha,\; 4\pi + \alpha$, $5\pi - \alpha$ are the solutions in $[0, 5\pi]$.

$\therefore$ Required number of solutions = 6.

60. (D)

Given equation is $\sqrt{3}\sin x + \cos x = 4$

which is of the form $a\sin x + b\cos x = c$ with $a = \sqrt{3}, b = 1, c = 4$.

Here $a^2 + b^2 = 3 + 1 = 4 < c^2$, therefore the given equation has no solution.